

# Step scaling function of the 2-flavor SU(3) sextet model – Wilson fermion approach

(arXiv: 1507.08260)

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KITP

August, 2015

# Basic formulae

$$\beta_s(\tilde{g}_c^2; L) = \frac{\tilde{g}_c^2(sL; a) - \tilde{g}_c^2(L; a)}{\log(s^2)}$$

$$g_{\text{GF}}^2(\mu) = \frac{1}{\mathcal{N}} \langle t^2 E(t) \rangle$$

$$E(t) = -\frac{1}{2} \text{ReTr} [G_{\mu\nu}(t) G^{\mu\nu}(t)]$$

$$\mu = 1/\sqrt{8t}$$

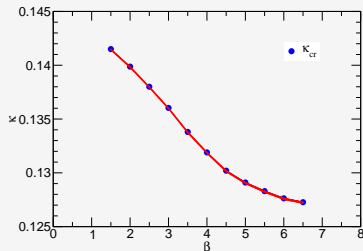
$$c = \sqrt{8t}/L$$

$$\tilde{g}_{\text{GF}}^2(\mu; a) = g_{\text{GF}}^2(\mu; a) \frac{\langle E(t + \tau_0 a^2) \rangle}{\langle E(t) \rangle}$$

# Numerical details

- nHYP smeared clover-Wilson fermion action with smearing coefficients ( $\alpha_1 = 0.75, \alpha_2 = 0.6, \alpha_3 = 0.3$ ), plaquette gauge action and an NDS term with coefficient  $\gamma = (\gamma_1 = \gamma_2 = \gamma_3 =) 0.075$ .
- $L^4$  volumes with  $L/a = 8 - 28$ .
- Standard boundary conditions (periodic for the gauge fields and periodic in space, antiperiodic in time for the fermions).
- 11  $\beta$  values between 1.5 and 6.5 with 0.5 increment.

# Tuning to the critical surface



$$\partial_t \sum_{\mathbf{x}} \langle A_0^a(\mathbf{x}, t) \mathcal{O}^a \rangle = 2m_q \sum_{\mathbf{x}} \langle P^a(\mathbf{x}, t) \mathcal{O}^a \rangle$$

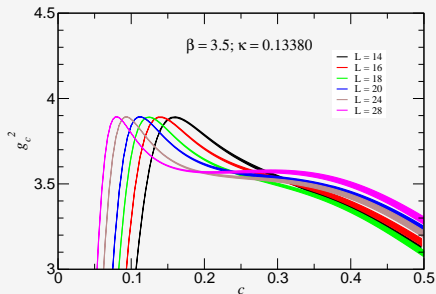
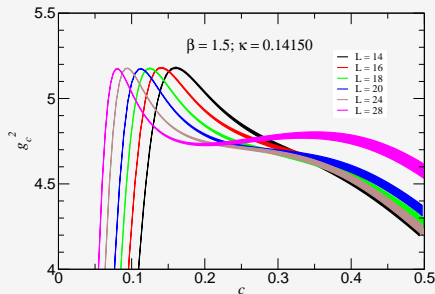
$$\sum_{\mathbf{x}} \langle P^a(\mathbf{x}, t) \mathcal{O}^a \rangle = \sum_{n=0}^{N-1} C_{n,PP}^2 \left[ e^{-En t} + e^{-En(T-t)} \right]$$

$$\sum_{\mathbf{x}} \langle A_0^a(\mathbf{x}, t) \mathcal{O}^a \rangle = \sum_{n=0}^{N-1} C_{n,PP} C_{n,AP} \left[ e^{-En t} - e^{-En(T-t)} \right]$$

$$m_q = -\frac{E_0}{2} \frac{C_{0,AP}}{C_{0,PP}}$$

$\beta$	$\kappa$	$m_q$	$\beta$	$\kappa$	$m_q$
1.5	0.14150	0.0012(3)	4.5	0.13020	0.0020(2)
2.0	0.13987	0.0007(3)	5.0	0.12910	-0.0007(1)
2.5	0.13800	0.0018(3)	5.5	0.12830	-0.0008(1)
3.0	0.13603	-0.0005(3)	6.0	0.12763	0.0028(1)
3.5	0.13380	0.0040(3)	6.5	0.12727	0.0016(1)
4.0	0.13190	0.0004(2)			

# Volume squeezed versus mass deformed

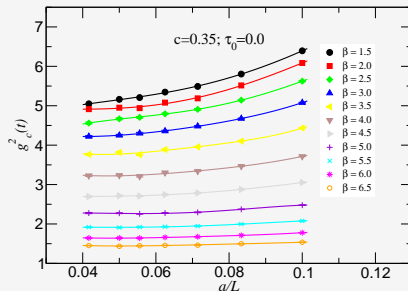


Left plot shows that  $L/a = 28$  at  $\beta = 1.5$  and  $\kappa = 0.14150$  moved into mass deformed regime.  $L/a = 28$  data were not used in the final analysis.

# Analysis of the step scaling function

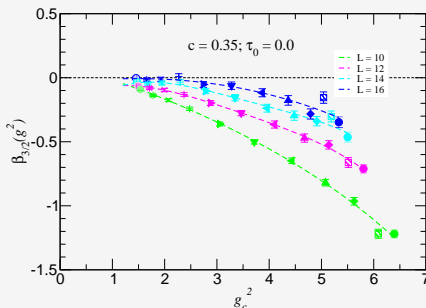
- Volume interpolation of  $g_c^2(t)$  at fixed  $\beta$
- Step scaling function  $\beta_s(g^2; L)$  interpolation at fixed  $L/a$
- $(a/L) \rightarrow 0$  extrapolation

# Volume interpolation



- Interpolating functions with polynomials of degree 3 or 4 in  $\log(L/a)$  or  $a/L$ .
- Reduce fluctuations among independent volumes and provide  $g_c^2$  on  $L/a = 15$  and  $21$ .

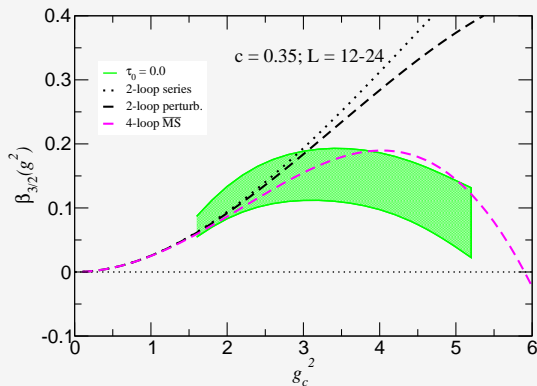
# $g_c^2$ interpolation



- Polynomials of degree 3 or 4.
- The hopping parameter  $\kappa$  is mistuned on  $\beta = 2.0$ .
- Mistuning increases the step scaling function in the continuum limit.



# Result after $(a/L) \rightarrow 0$ extrapolation

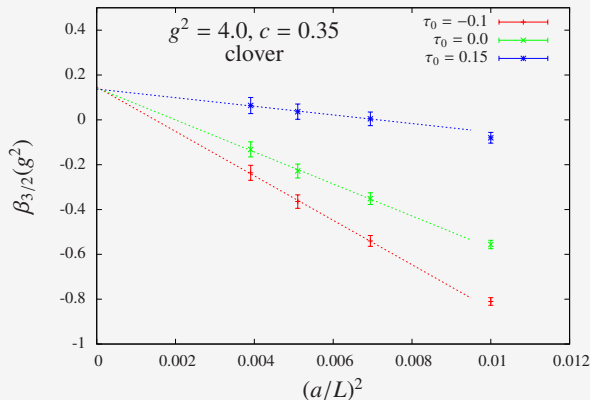


- The continuum extrapolated step scaling function follows 4-loop  $\overline{\text{MS}}$  perturbative predictions.

# Cut-off effects and consistency check

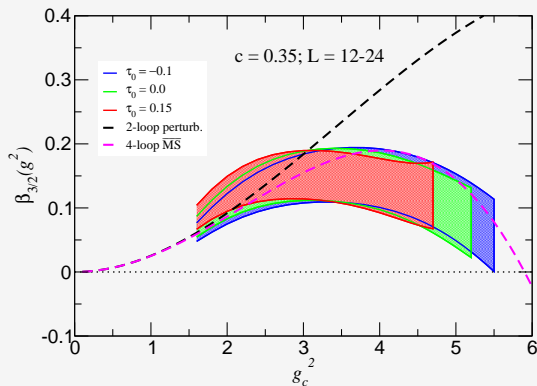
- For fixed  $c$  ( $c = 0.35$ ):
  - Dependence on the t-shift ( $\tau_0 = -0.1, 0.0$ , and  $0.15$ )
  - Dependence on the energy density operator: clover and plaquette.
  - Dependence on the lattice volume:  $L/a \geq 12$  and  $L/a \geq 10$ .
- Different  $c$  values ( $c = 0.25, 0.3, 0.4$ ).

# t-shift dependence



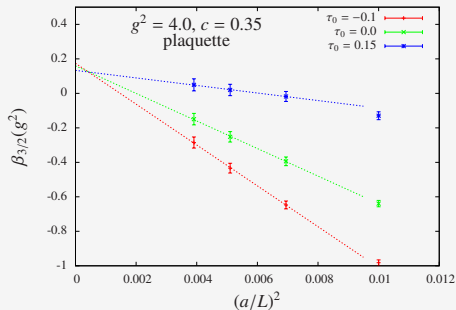
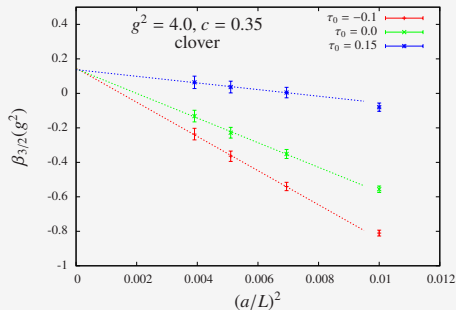
- Different  $\tau_0$  values extrapolate to the same point with  $c = 0.35$ .

# t-shift dependence



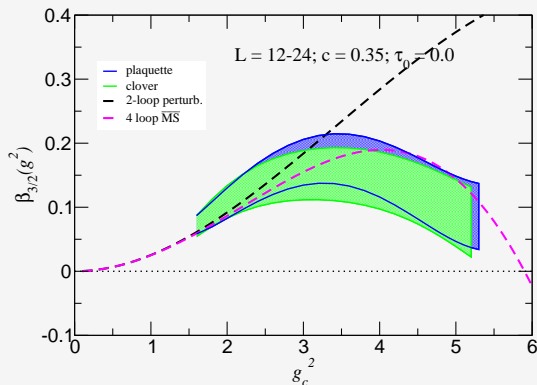
- Different  $\tau_0$  values give the same step scaling function with  $c = 0.35$ .

# Operator dependence



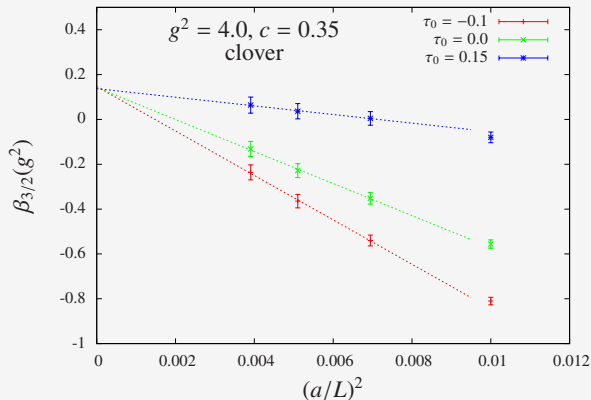
- The clover and plaquette operators give consistent result with  $c = 0.35$ , indicating that  $(a/L)^2 \rightarrow 0$  extrapolation removes most cut-off effects.

# Operator dependence



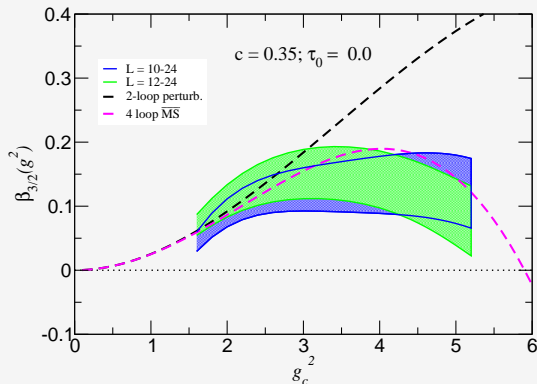
- The clover and plaquette operators give consistent result with  $c = 0.35$  and  $\tau_0 = 0.0$ , indicating that  $(a/L)^2 \rightarrow 0$  extrapolation removes most cut-off effects.

# Volume dependence



- $L/a = 10 \rightarrow 15$  fits well with the other volumes.

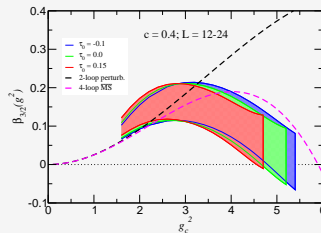
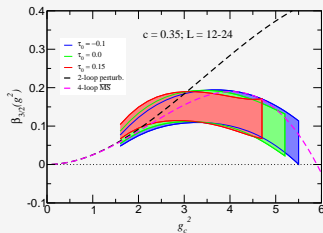
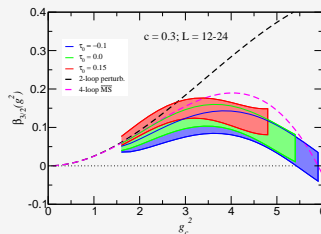
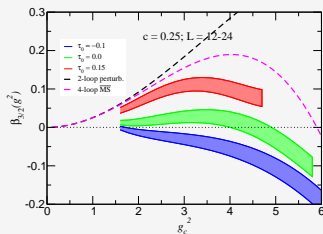
# Volume dependence



- Including  $L/a = 10 \rightarrow 15$  in the extrapolation has negligible effect on the step scaling function.



# c parameter dependence



- Cut-off effects remain for  $c = 0.25$  but disappear within statistical errors for  $c \geq 0.35$  with the clover (and plaquette) operator.

# Conclusion and discussion

- The step scaling function from Wilson fermions follows closely the 4-loop(non-universal)  $\overline{\text{MS}}$  prediction that predicts an IRFP at  $g^2 \approx 5.90$ .
- There is an over  $3\sigma$  tension of the step scaling function at strong couplings between Staggered and Wilson fermions.
- For 4 dimensional conformal and near conformal systems, could different lattice actions approach different universality properties of the system?

# References

Thank you for your attention!



Anna Hasenfratz, Yuzhi Liu, Cynthia Yu-Han Huang

*The renormalization group step scaling function of the 2-flavor  $SU(3)$  sextet model.*

*arXiv: 1507.08260.*



Zoltan Fodor, Kieran Holland, Julius Kuti, Santanu Mondal, Daniel Nogradi, Chik Him Wong

*The running coupling of the minimal sextet composite Higgs model.*

*arXiv:1506.06599.*

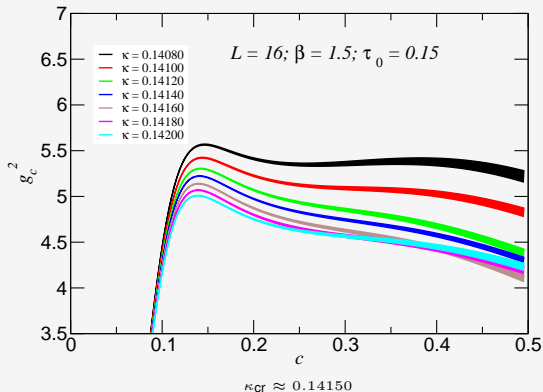


Thomas DeGrand, Yigal Shamir, Benjamin Svetitsky

*Suppressing dislocations in normalized hypercubic smearing.*

*Phys. Rev. D 90, 054501 (2014).*

# Volume squeezed versus mass deformed



The changes between  $\kappa = 0.14120$  and  $0.14100$ , and between  $\kappa = 0.14160$  and  $0.14180$  signal that the system moved from the volume squeezed to the mass deformed regime.