The running coupling of the minimal sextet composite Higgs model

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in collaboration with

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The model

Composite Higgs

$SU(3) \qquad N_f = 2 \qquad R = 2S$

massless, Dirac fermions

almost QCD but fundamental $\longrightarrow$ sextet

Same as what Julius talked about this morning

Motivations

See Ayana's talk yesterday

On the lattice very important:

control all systematic errors before conclusions!

Plan

- Define gradient flow based running coupling scheme
- Lattice discretizations
- Continuum extrapolation
- Assess systematic effects
- Final result

By the way: first fully controlled non-perturbative continuum result on the model :)

Previous results

DeGrand-Shamir-Svetitsky, 0803.1707, 1006.0707 (no continuum)

Hasenfratz-Svetitsky: Nagoya 2015, LLNL 2015, USQCD 2015

Continuum running coupling scheme

Work in finite box, T^4, use gradient flow, $\sqrt{8t}/L = c$ fix

$$g^2(L) \sim \langle t^2 E(t) \rangle$$

Gauge fields periodic, massless fermions anti-periodic in all 4 directions

Step scaling $L \to sL$, discrete β-function $\frac{g^2(sL)-g^2(L)}{\log(s^2)}$, $s = 3/2$

Lattice discretization

Fermions: $m = 0$, rooted staggered with stout

Gauge links: 3 ingredients

- Flow (Wilson and tree level Symanzik)
- Dynamical gauge action (tree level Symanzik)
- Observable E (clover)

Terminology: flow-action-observable: WSC and SSC.

Continuum should agree for both!

Wait, what???

Rooted staggered fermions with $m = 0$???

Rooting and $m = 0$

Golterman, Shamir, Sharpe, . . .:

Rooting is okay for $m > m_*$, where m_* depends on the lattice spacing, a decreases m_* decreases

But remember: above is for infinite volume!

In infinite volume, m is the only IR regulator

We have finite volume L which is itself an IR regulator

Rooting and $m = 0$

Modified Golterman, Shamir, Sharpe, ...:

As long as we have a large enough IR regulator rooting is okay!

Key insight: rooting fails due to small Dirac eigenvalues

m fixes this, finite volume and anti-periodic fermions ditto
$\sim 1/L^{\alpha}$

Lower bound on m (HMC fails for too small m anyway)

Upper bound on L (HMC fails for too large L anyway)

Step scaling

$\frac{g^2(sL)-g^2(L)}{\log s^2}$ discrete β-function

$8 \to 12$, $12 \to 18$, $16 \to 24$, $20 \to 30$, $24 \to 36$

for many fixed β bare couplings

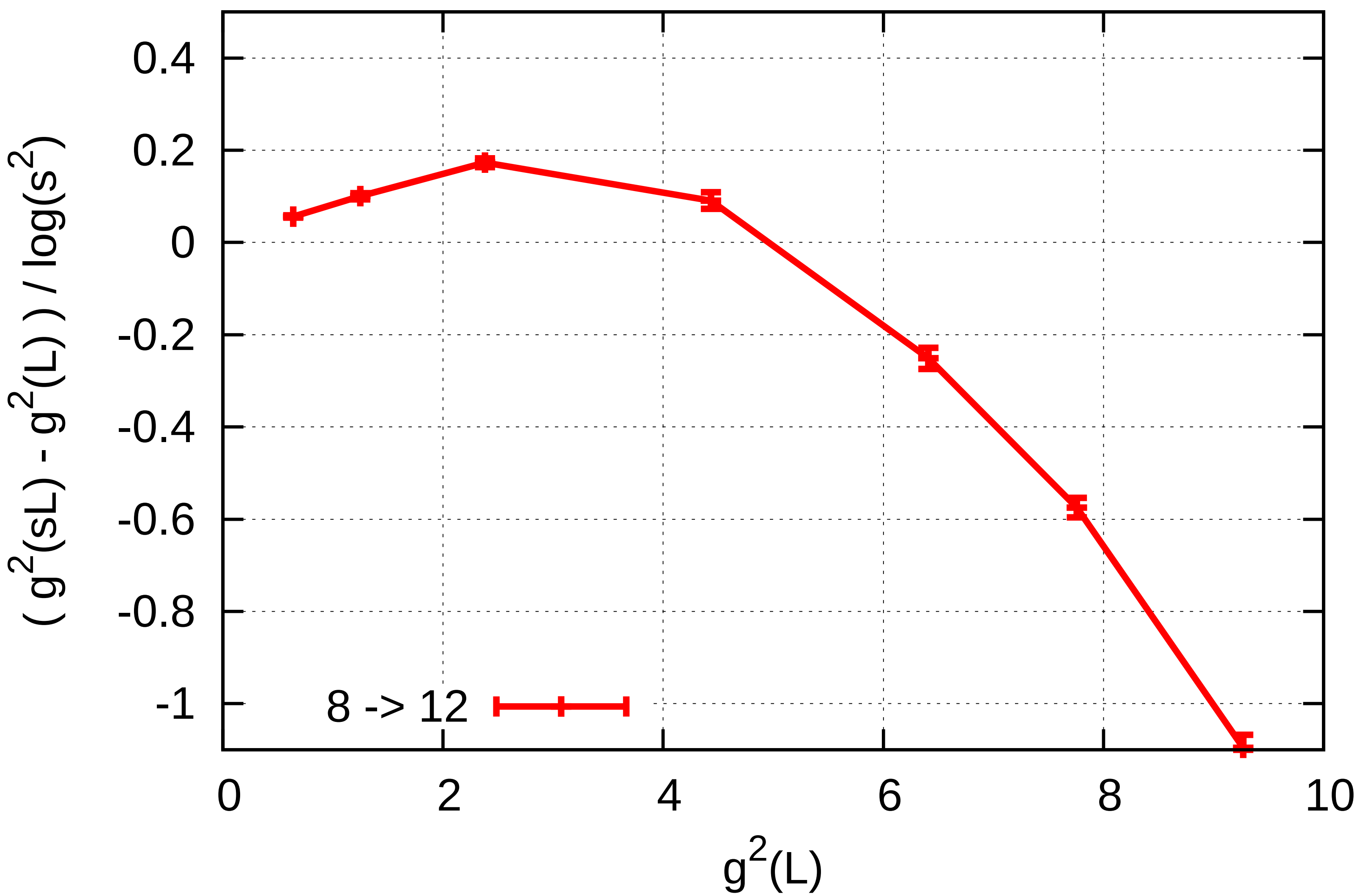
Plot discrete β-function as a function of $g^2(L)$

5 steps: 5 lattice spacings $\to$ can quantify systematic error from continuum extrapolation

Results - raw data

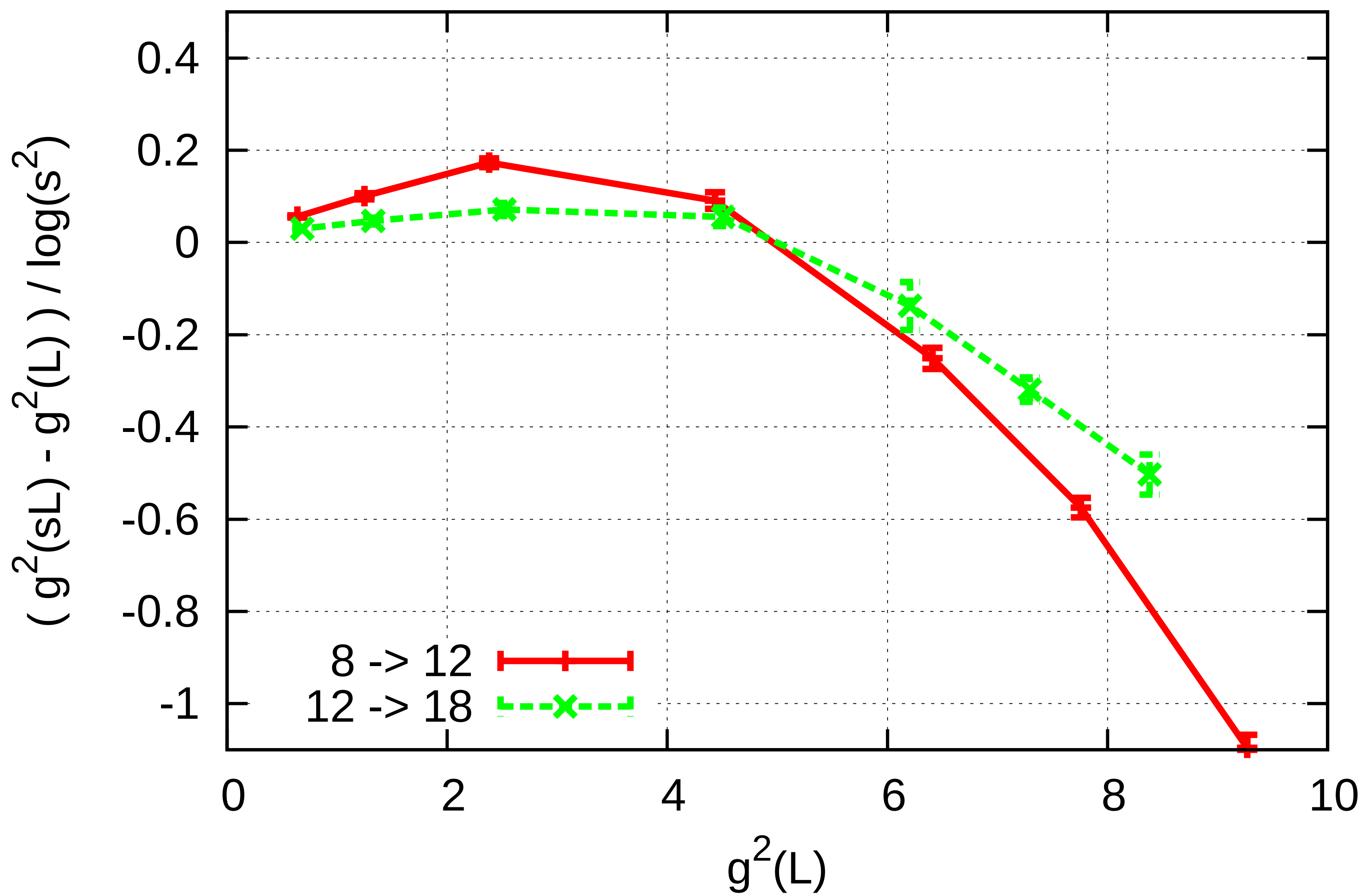
Flow-Action-Observable = WSC

Results, WSC
$8 \rightarrow 12$,



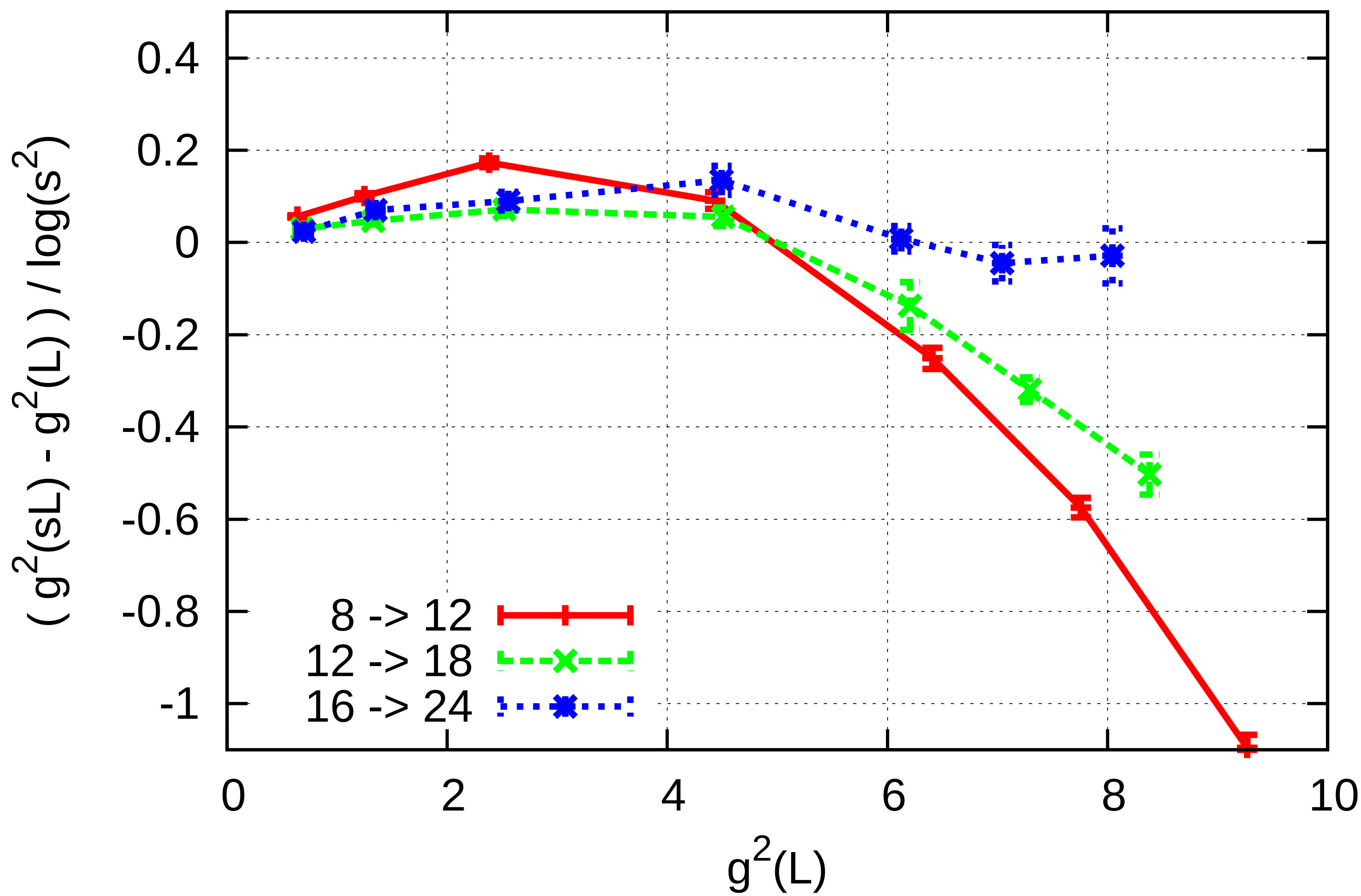
Results, WSC

$8 \to 12,\ 12 \to 18$



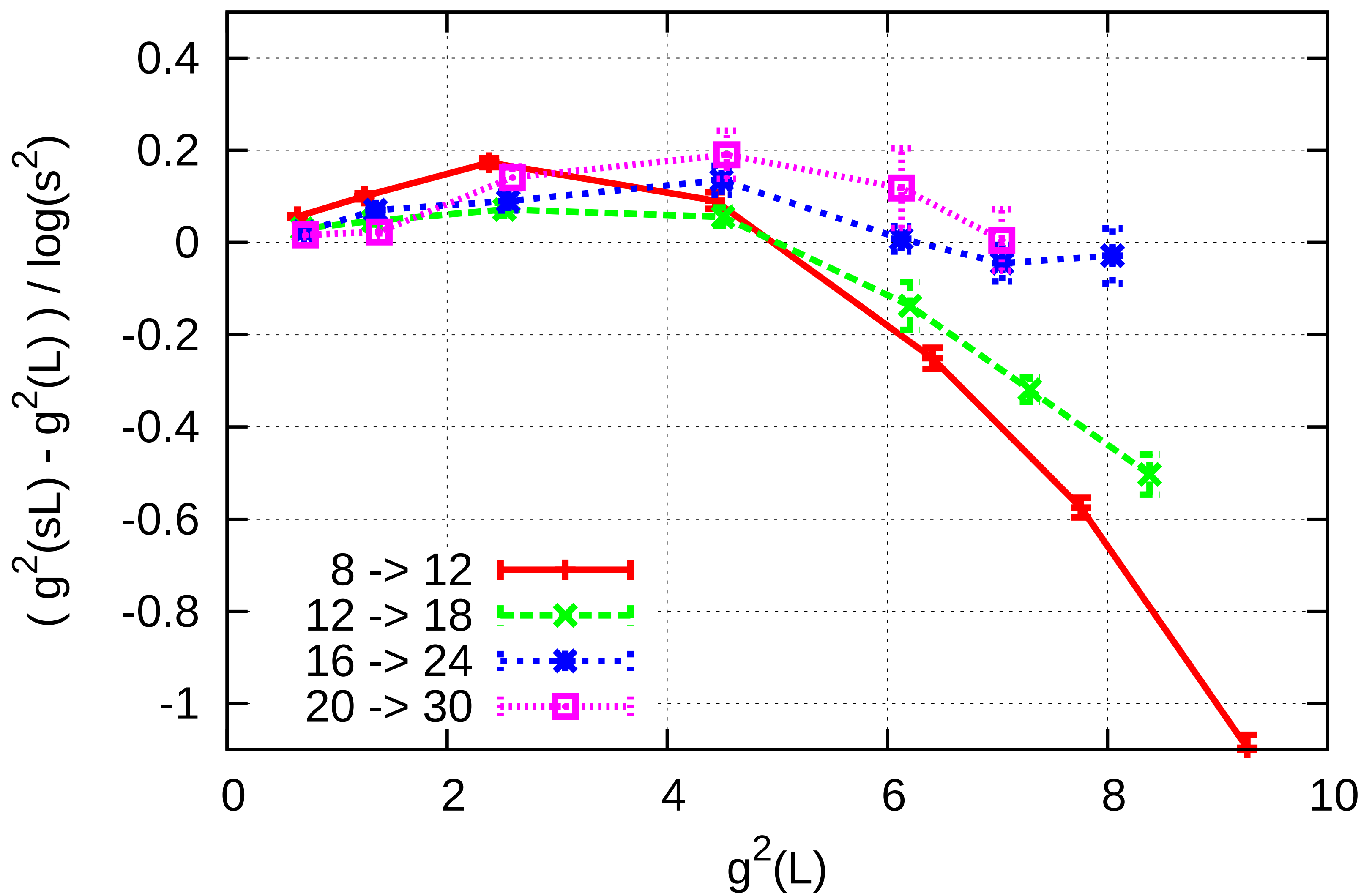
Results, WSC

$8 \to 12,\ 12 \to 18,\ 16 \to 24$



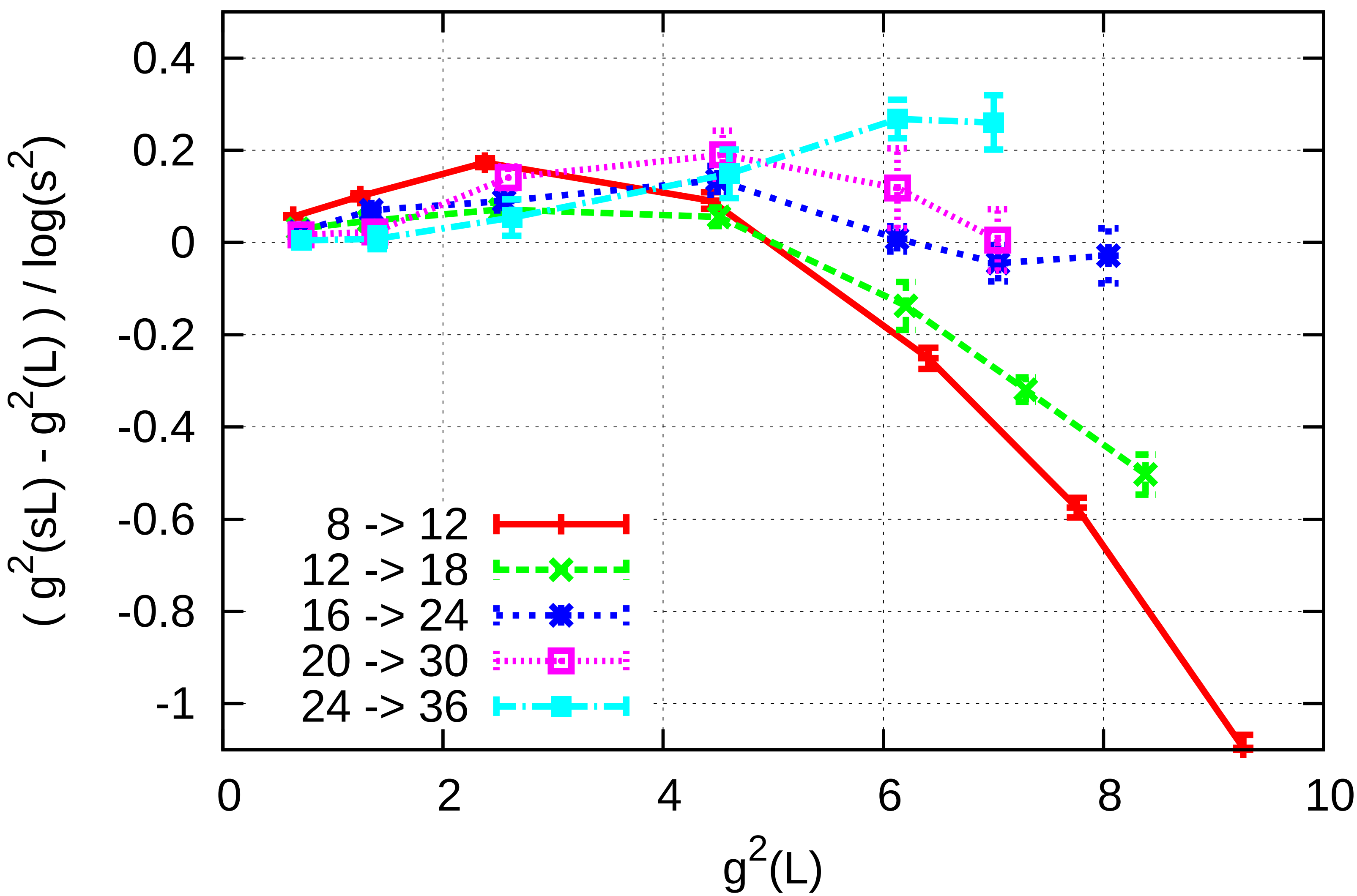
Results, WSC

$8 \rightarrow 12,\ 12 \rightarrow 18,\ 16 \rightarrow 24,\ 20 \rightarrow 30$



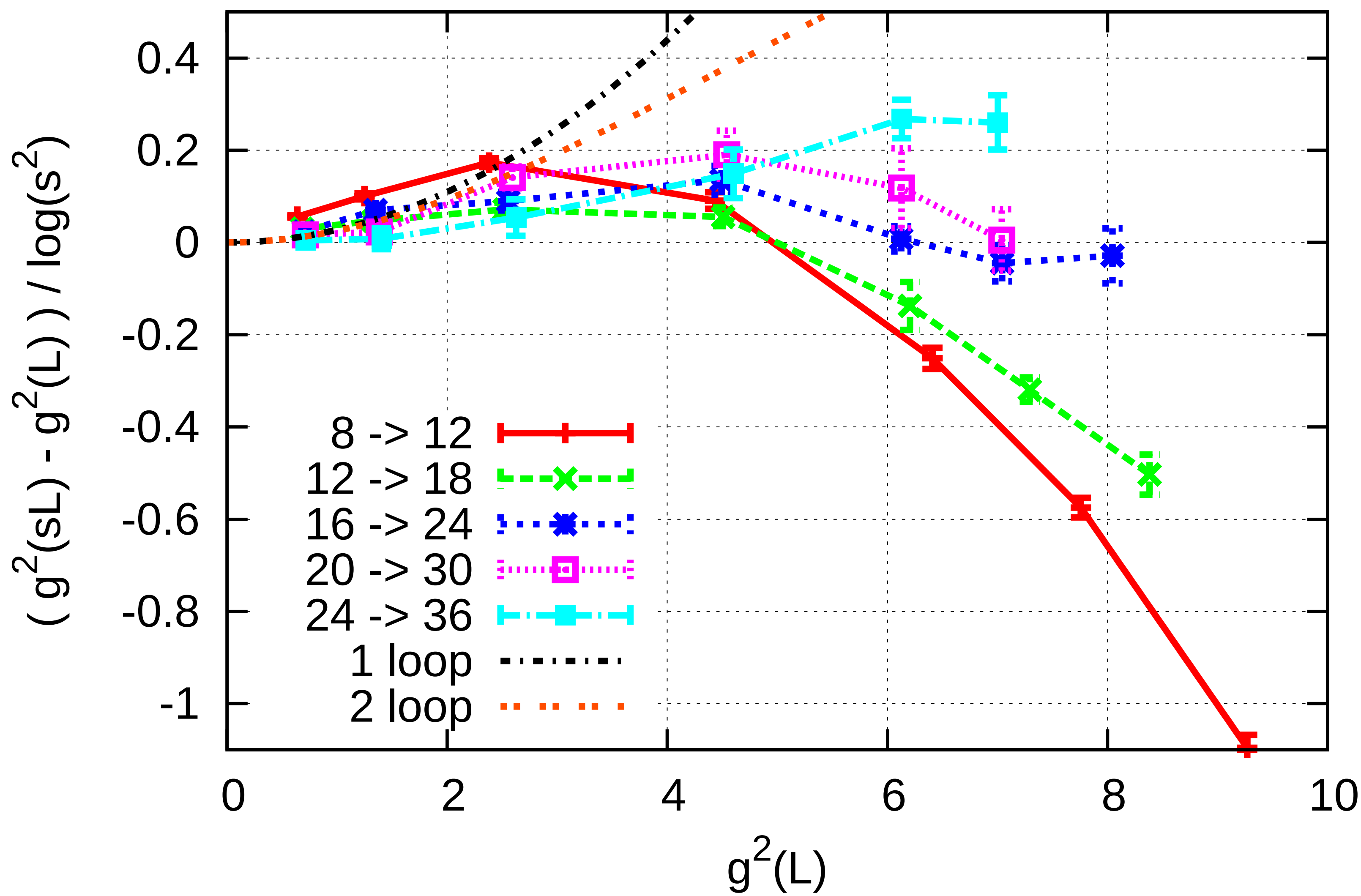
Results, WSC

$8 \to 12,\ 12 \to 18,\ 16 \to 24,\ 20 \to 30,\ 24 \to 36$



Results, WSC

$8 \to 12,\ 12 \to 18,\ 16 \to 24,\ 20 \to 30,\ 24 \to 36$

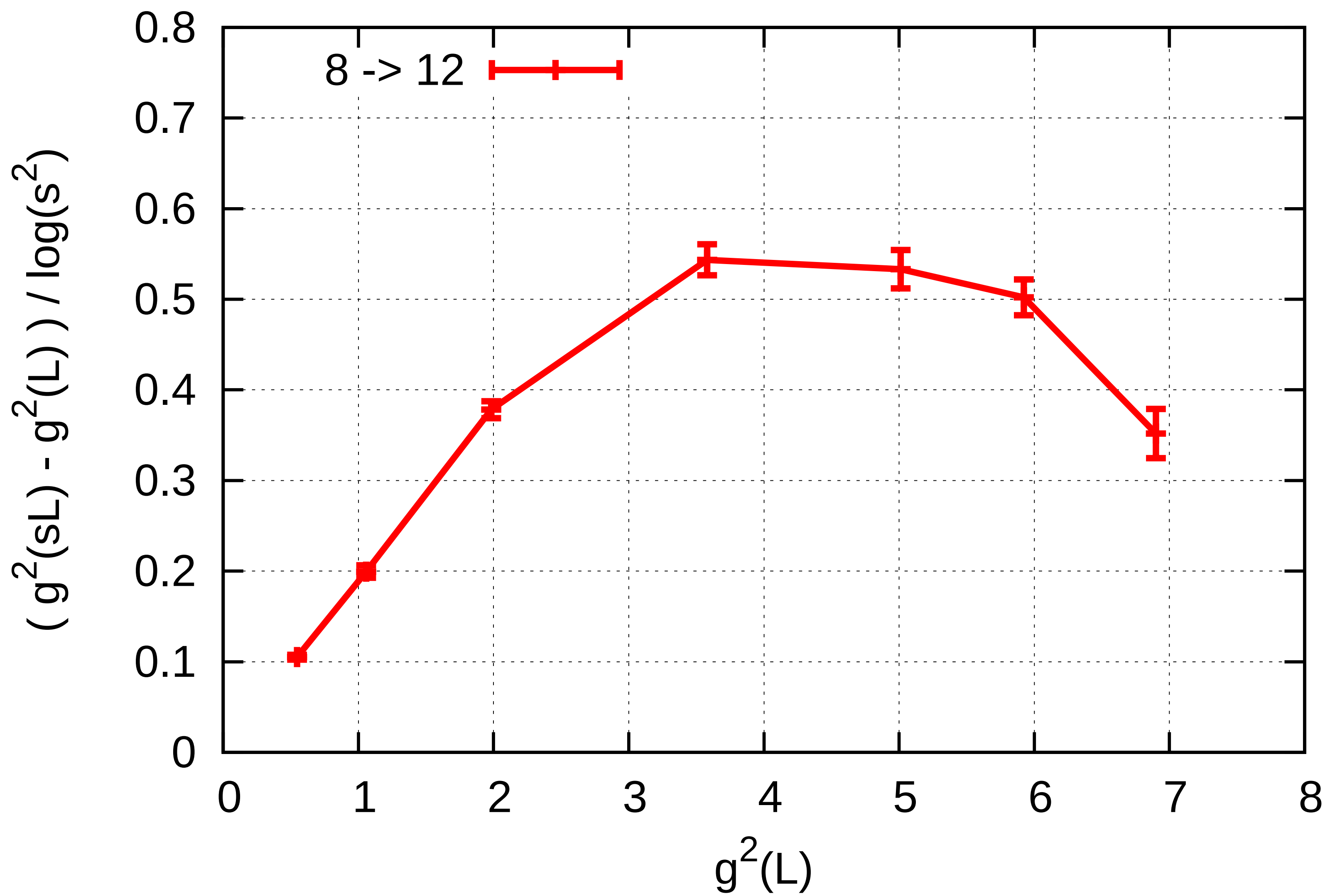


Results - raw data

Flow-Action-Observable = SSC

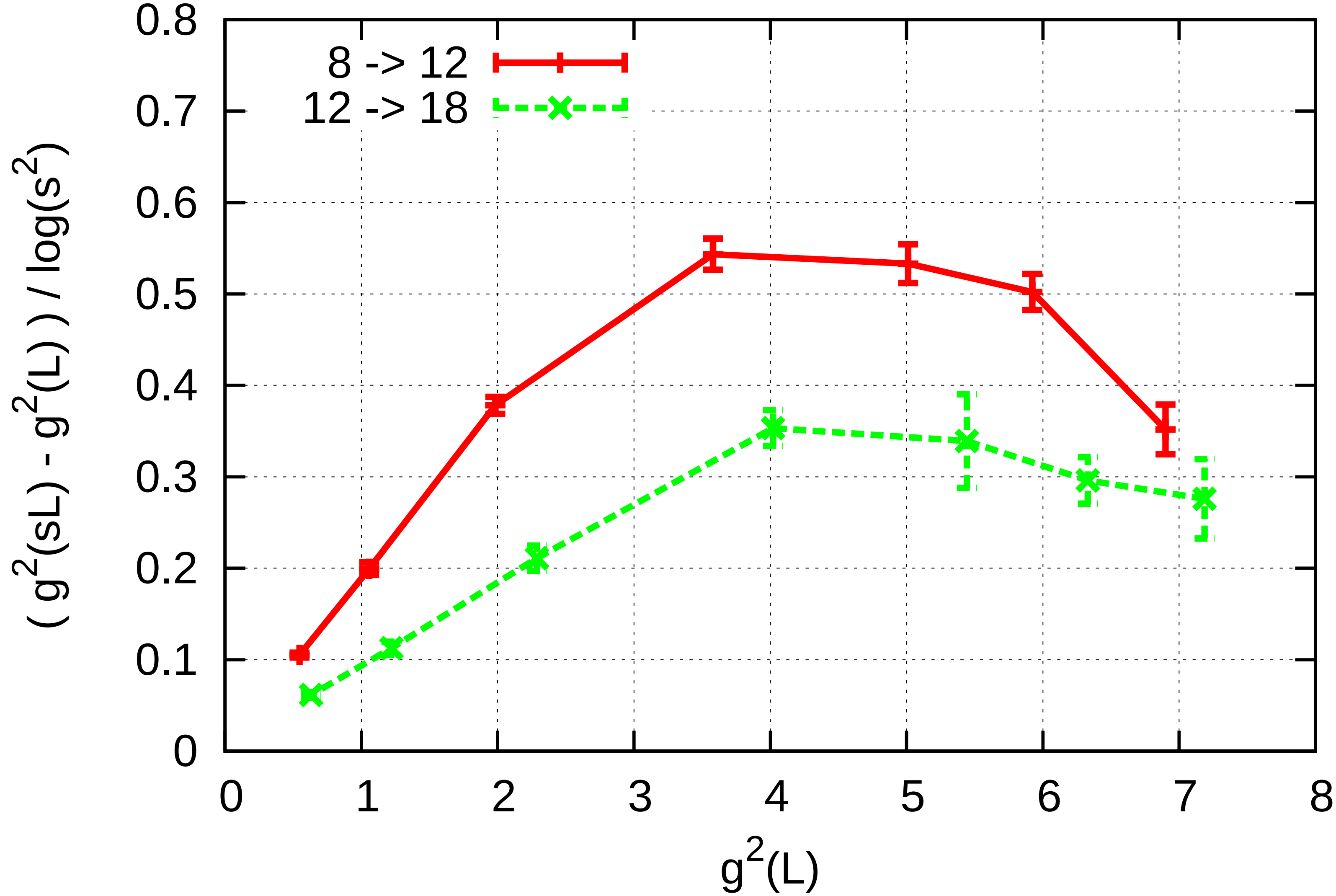
Results, SSC

$8 \to 12$,



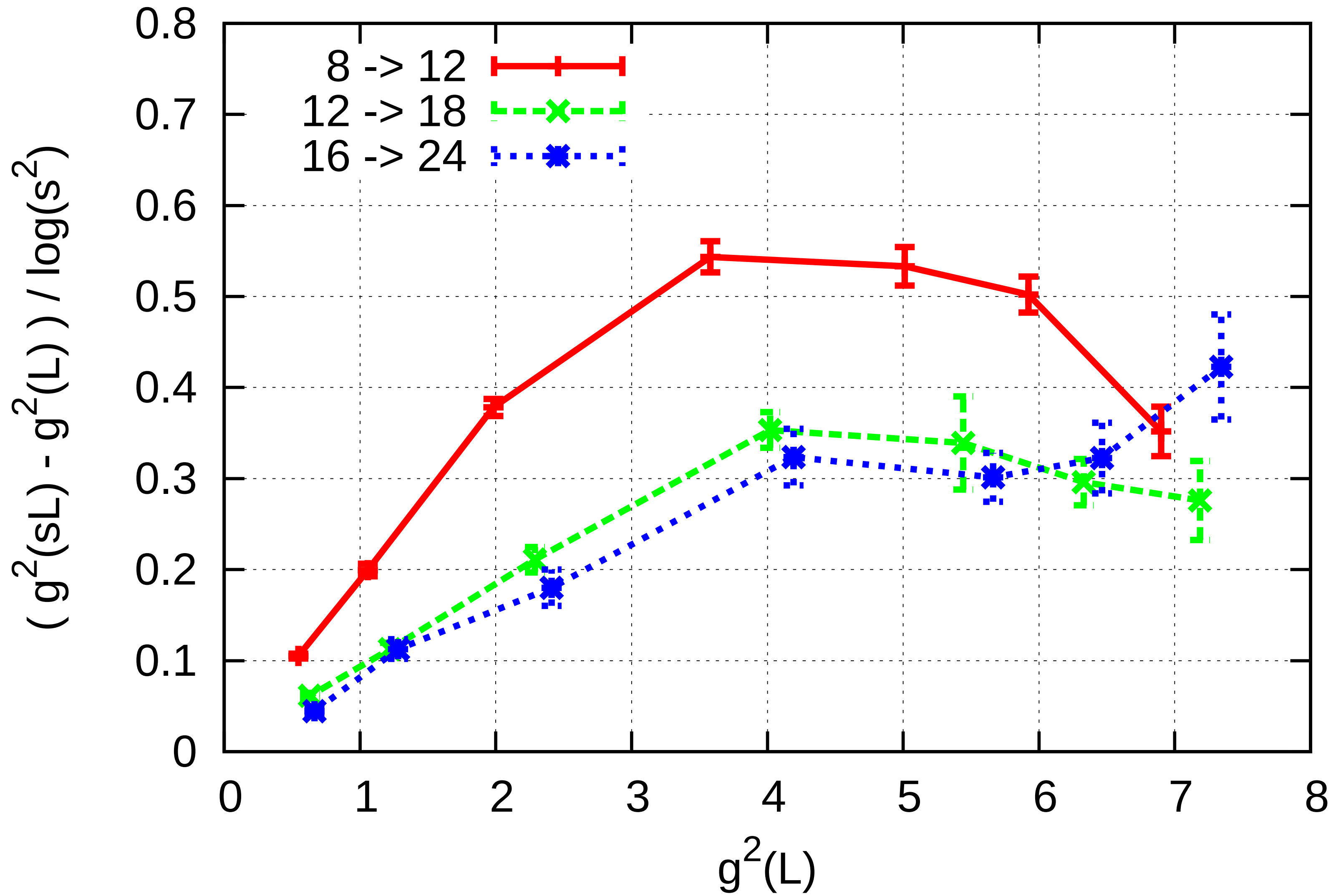
Results, SSC

$8 \to 12,\ 12 \to 18$



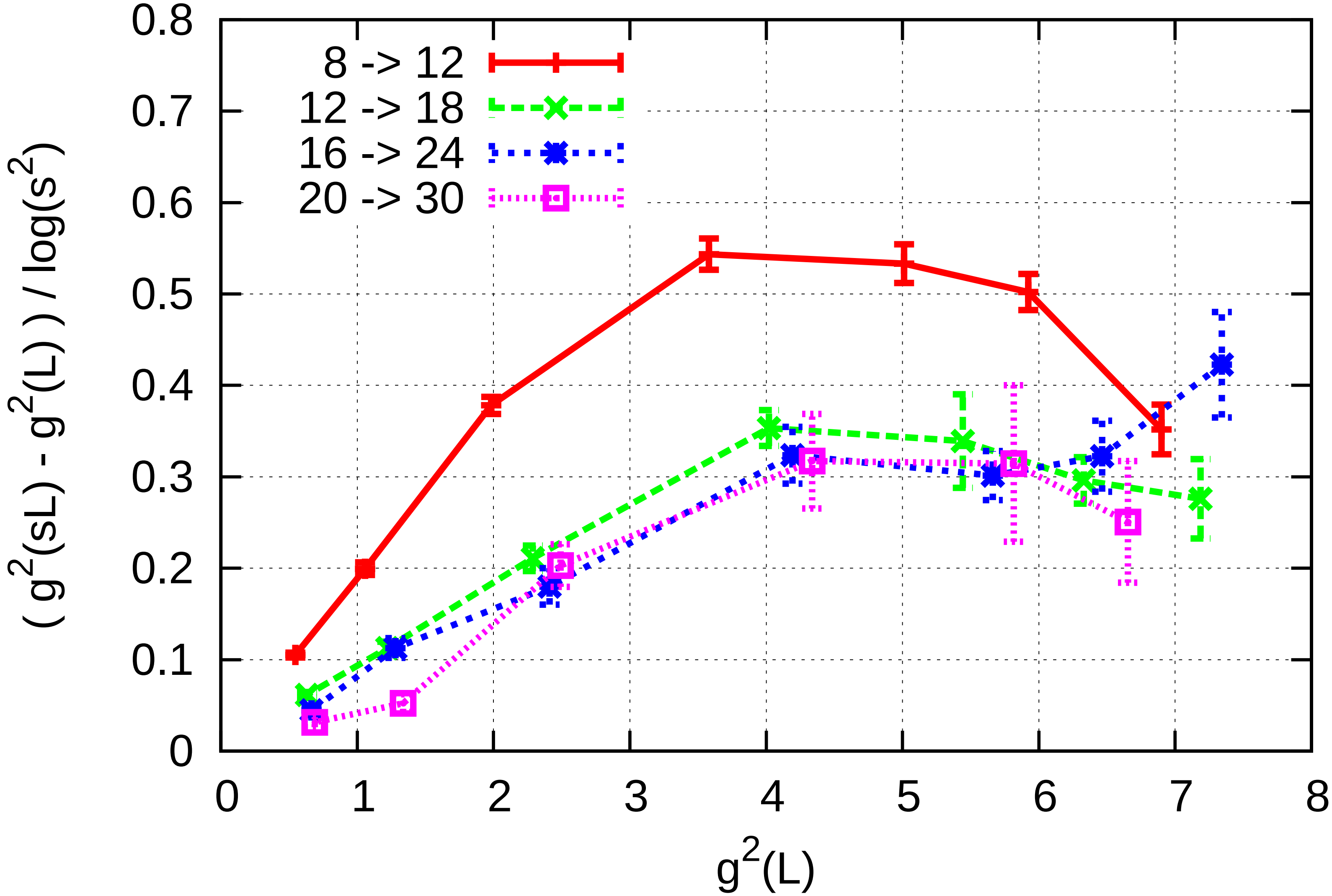
Results, SSC

$8 \rightarrow 12,\ 12 \rightarrow 18,\ 16 \rightarrow 24$



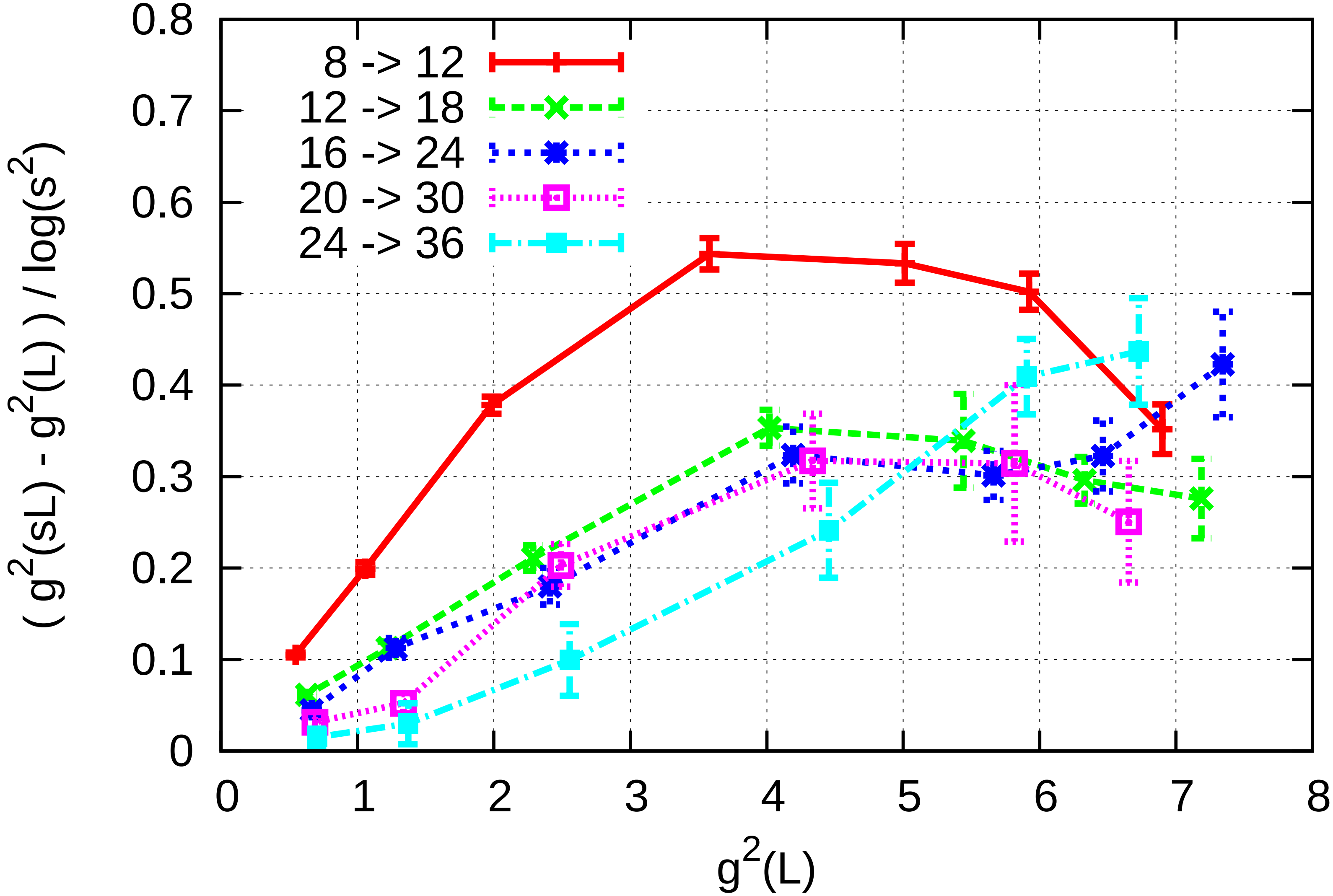
Results, SSC

$8 \to 12,\ 12 \to 18,\ 16 \to 24,\ 20 \to 30$



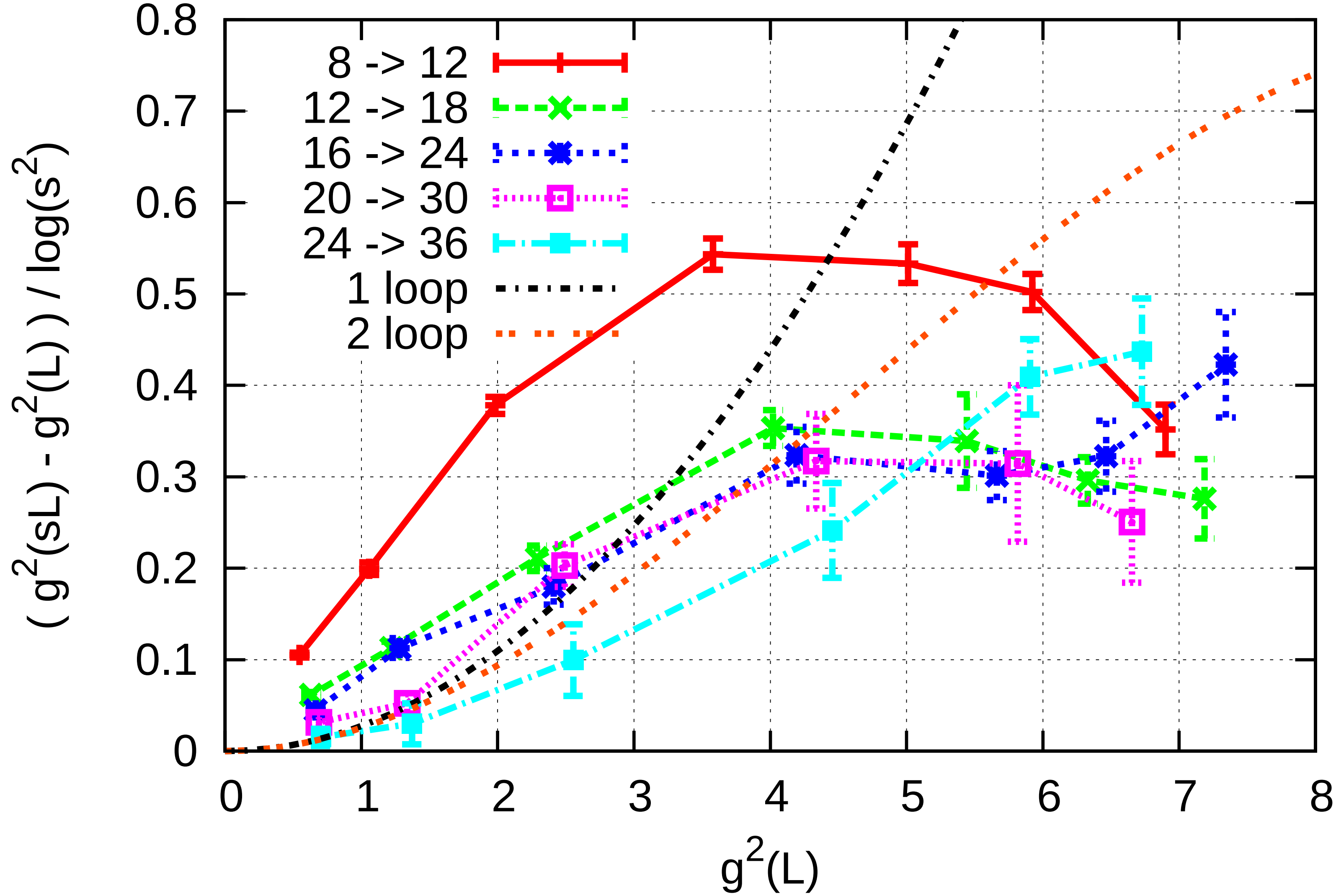
Results, SSC

$8 \rightarrow 12,\ 12 \rightarrow 18,\ 16 \rightarrow 24,\ 20 \rightarrow 30,\ 24 \rightarrow 36$



Results, SSC

$8 \to 12,\ 12 \to 18,\ 16 \to 24,\ 20 \to 30,\ 24 \to 36$



Continuum extrapolation

Interpolation $g^2(\beta)$ at fixed L/a

$$\frac{\beta}{6} - \frac{1}{g^2(\beta)} = \sum_{m=0}^{n} c_m \left(\frac{6}{\beta}\right)^m$$

Can now interpolate all data

Can pick fixed $g^2(L)$ and read off 5 discrete β-function values corresponding to 5 steps, i.e. 5 lattice spacings and continuum extrapolate linearly in a^2/L^2

8 lattice volumes $\rightarrow$ 8 independent interpolations

Systematic uncertainty from continuum extrapolation

Twofold:

- Interpolation orders $n = 3, 4, 5$
- Number of points in continuum extrapolation: $4, 5$

Systematic uncertainty 1: interpolation orders

There is no theory for choosing interpolation order n

These are purely empirical fits

What should n be?

Systematic uncertainty 1: interpolation orders

Sometimes what people do:

Manually select interpolation orders to get good χ^2/dof values

Problem 1: χ^2 follows χ^2-distribution, peak for χ^2/dof around 1, but it's perfectly okay to have χ^2/dof away from 1 sometimes, if we have many independent fits

I.e. it's very unlikely that many independent χ^2 fits will all give values close to $\chi^2/dof = 1$

Problem 2: manually choosing orders introduces bias

Systematic uncertainty 1: interpolation orders

One solution: consider all interpolation orders and impose a statistical test and demand at least a certain prescribed probability (30%) for a particular assignments of orders

Kolmogorov-Smirnoff test is simple and works very well

Originally from Budapest-Wuppertal collaboration, finite temperature QCD (interpolations in T, etc.)

Systematic uncertainty 1: interpolation orders

χ^2 are χ^2-distributed, goodness of fit q-values are uniformly distributed on $[0, 1]$.

Do a Kolmogorov-Smirnoff test on the q-values

How likely or unlikely it is to have the 8 q-values drawn from a uniform distribution?

Kolmogorov-Smirnoff test

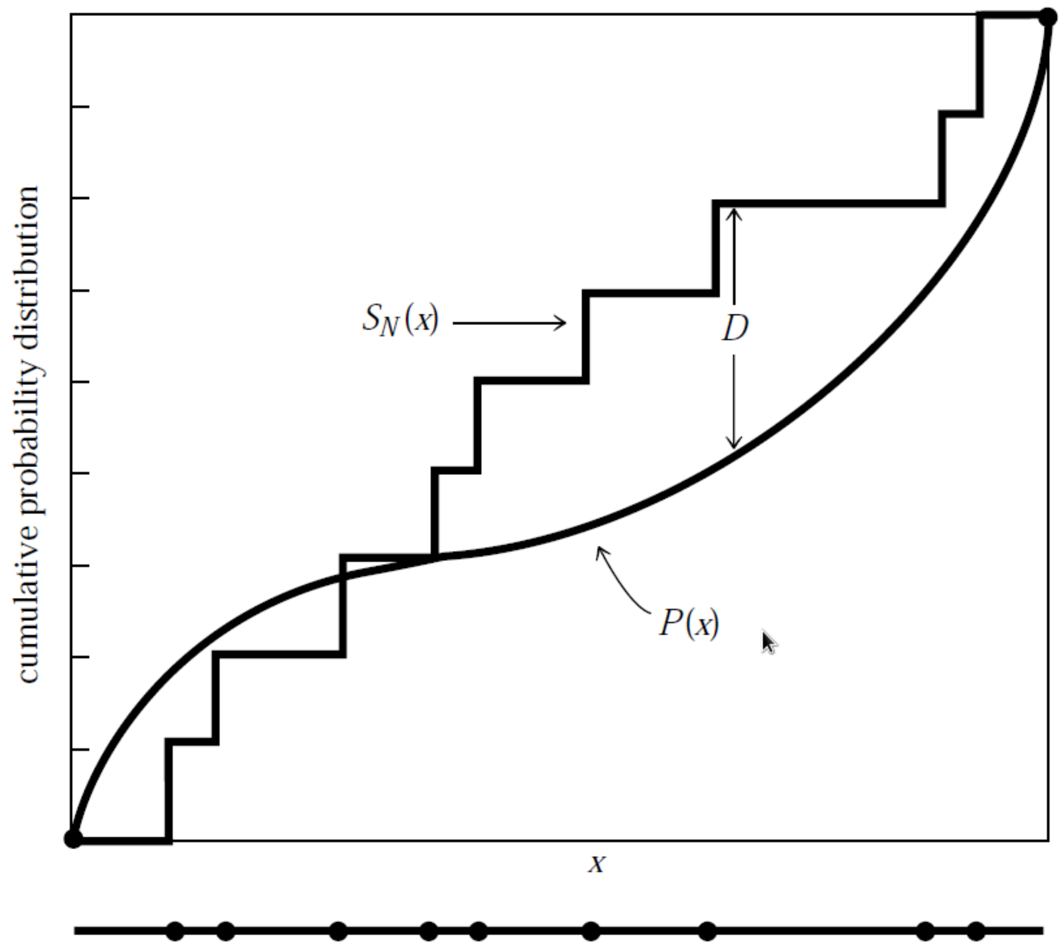
Let $S_N(x)$ be measured cumulative probability distribution from N samples

Did it come from drawing N samples from a given distribution with cumulative probability distribution $P(x)$?

Need to quantify difference between $S_N(x)$ and $P(x)$

$$D = max|S_N(x) - P(x)|$$

Kolmogorov-Smirnoff test



Kolmogorov-Smirnoff test

$$D = max|S_N(x) - P(x)|$$

The probability p_{KS} that largest deviation is actually D after drawing N samples from $P(x)$ is:

$$p_{KS} = Q\left(D\left(\sqrt{N} + 0.12 + 0.11/\sqrt{N}\right)\right)$$

where $Q(\lambda) = 1 - \vartheta_4\left(e^{-2\lambda^2}\right)$

Valid for large N, i.e. $N > 4$

Systematic uncertainty 1: interpolation orders

Summary

Apply Kolmogorov-Smirnoff test on q-values (compare with uniform distribution)

Demand $p_{KS} > 30\%$

Systematic uncertainty 1: interpolation orders

Consider (almost) all possible interpolation orders

$L/a = 8, 12, 16, 18, 24$ we let $n = 3, 4, 5$

$L/a = 20, 30, 36$ we let $n = 4, 5$

Total: $3^5 \cdot 2^3 = 1944$ interpolations using 5 lattice spacings (i.e. all volumes)

Allow only those that have $p_{KS} > 30\%$

SSC : 240 and WSC : 306

Systematic uncertainty 2: continuum extrapolation

Using all 5 or only 4 (dropping roughest) steps, i.e. lattice spacings

Previous page was for 5-point extrapolations

With 4-point extrapolations, drop 8 $\rightarrow$ 12:

Total: $3^4 \cdot 2^3 = 648$ interpolations

SSC : 240 and WSC : 249 with $p_{KS} > 30\%$

Systematic uncertainty 2: continuum extrapolation

Still need to average all the allowed continuum fits

Options:

- weight by AIC: $\sim \exp(-\chi^2/2 - p)$
- weight by q-values
- no weight

$\longrightarrow$ weighted histogram, read off central 68%

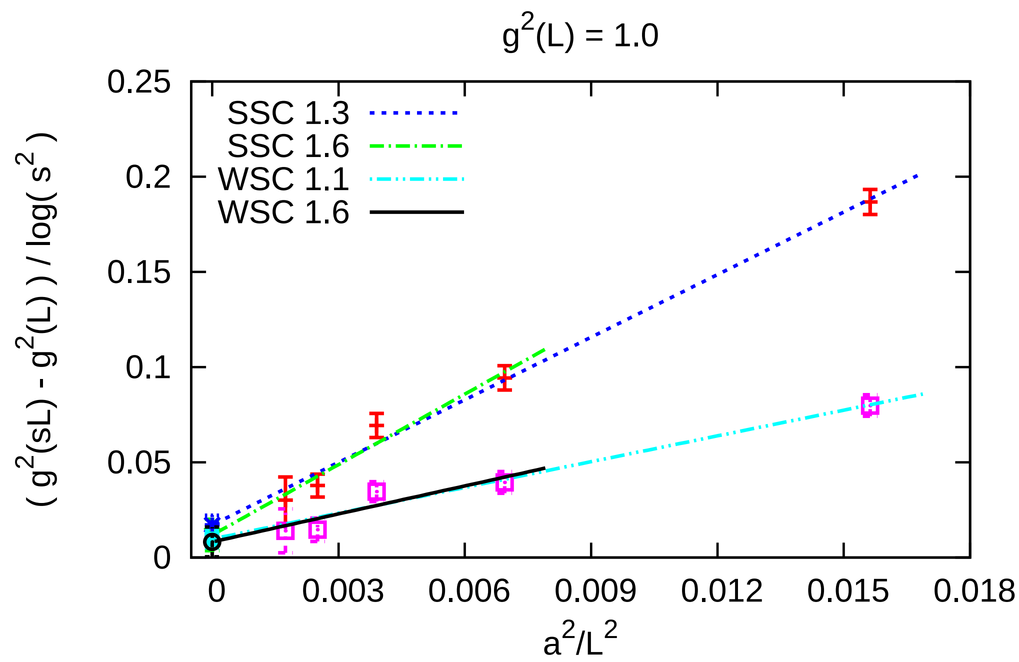
We have our estimate of the systematic error! (add it in quadrature)

Which option from above doesn't matter, all consistent, in practice choose AIC

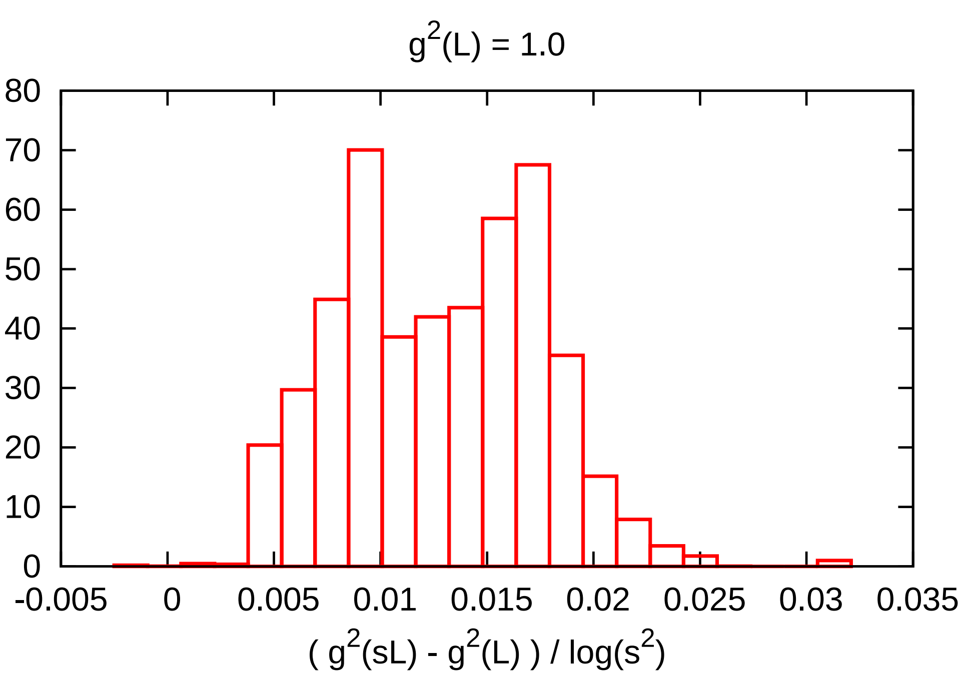
Repeat all this for each $g^2 = 1.0,\ 2.0,\ 3.0,\ 4.0,\ 5.0,\ 6.0$

Histograms and representative example of actual extrapolations for both WSC and SSC

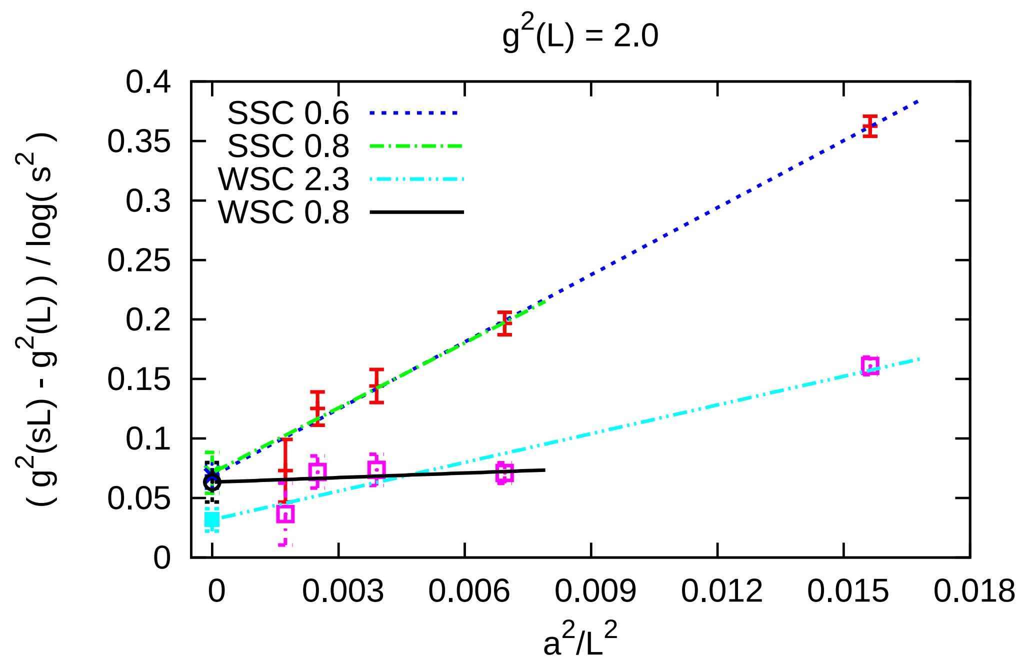
χ^2/dof of each fit in legend

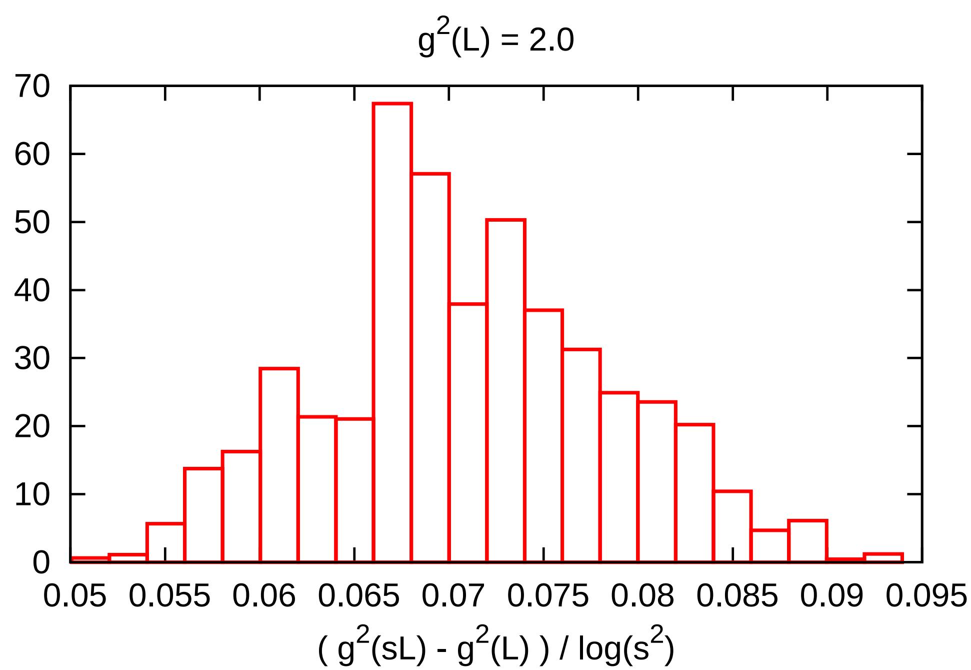


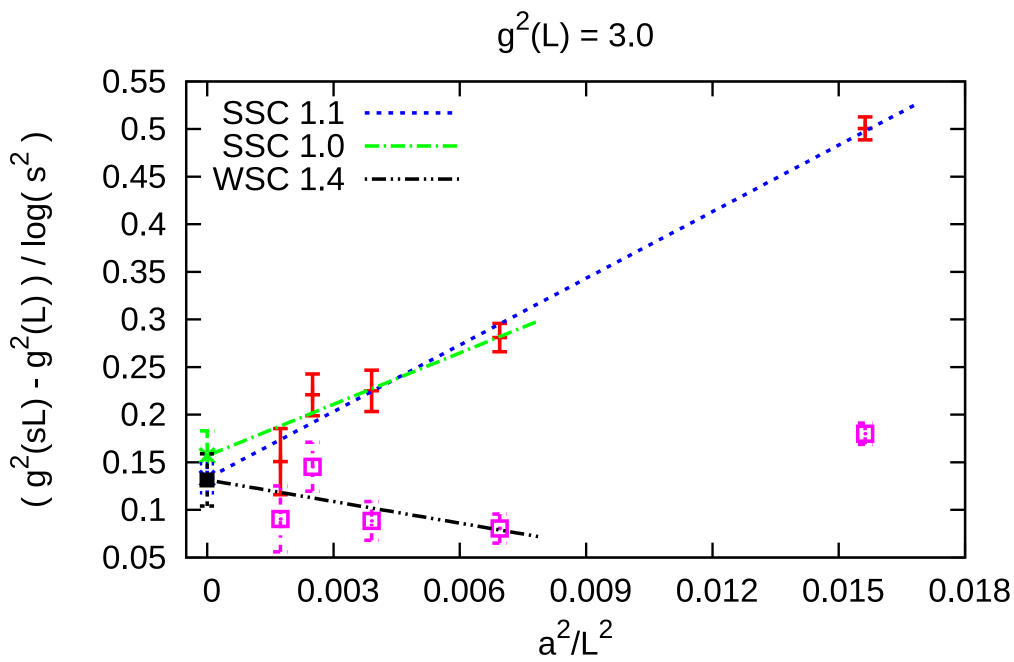
g^2(L) = 1.0
SSC 1.3
SSC 1.6
WSC 1.1
WSC 1.6
(g^2(sL) - g^2(L)) / log(s^2)
a^2/L^2
0
0.05
0.1
0.15
0.2
0.25
0.003
0.006
0.009
0.012
0.015
0.018

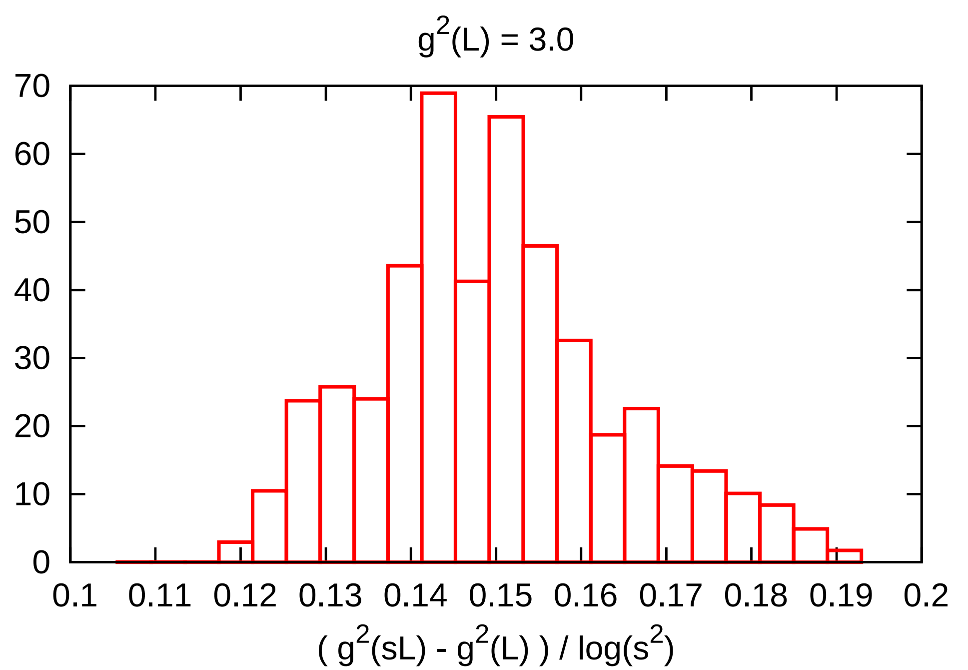


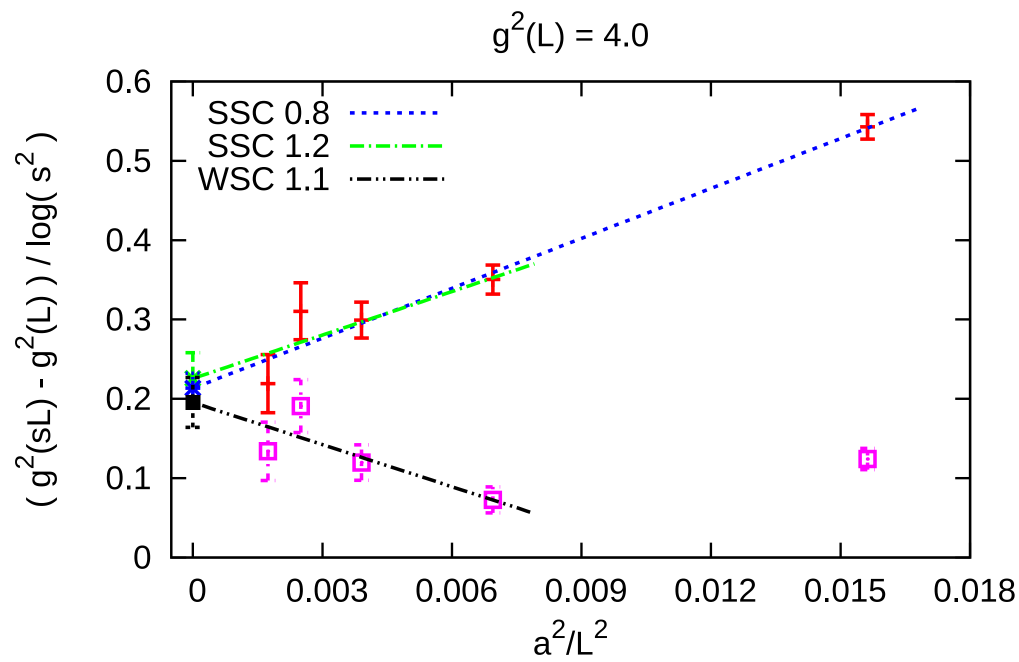
g^2(L) = 1.0
(g^2(sL) - g^2(L)) / log(s^2)
0
10
20
30
40
50
60
70
80
-0.005
0.005
0.01
0.015
0.02
0.025
0.03
0.035


g^2(L) = 2.0
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
(g^2(sL) - g^2(L)) / log(s^2)
SSC 0.6
SSC 0.8
WSC 2.3
WSC 0.8
0
0.003
0.006
0.009
0.012
0.015
0.018
a^2/L^2

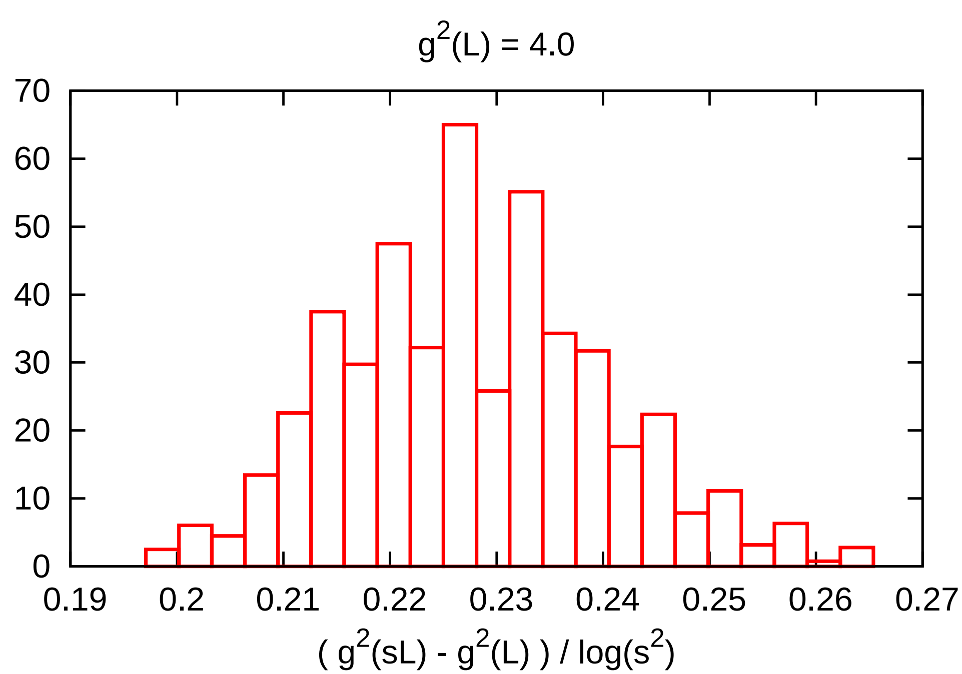

g^2(L) = 2.0
70
60
50
40
30
20
10
0
0.05
0.055
0.06
0.065
0.07
0.075
0.08
0.085
0.09
0.095
(g^2(sL) - g^2(L)) / log(s^2)


g2(L) = 3.0
SSC 1.1
SSC 1.0
WSC 1.4
(g2(sL) - g2(L)) / log(s2)
a2/L2
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.003
0.006
0.009
0.012
0.015
0.018

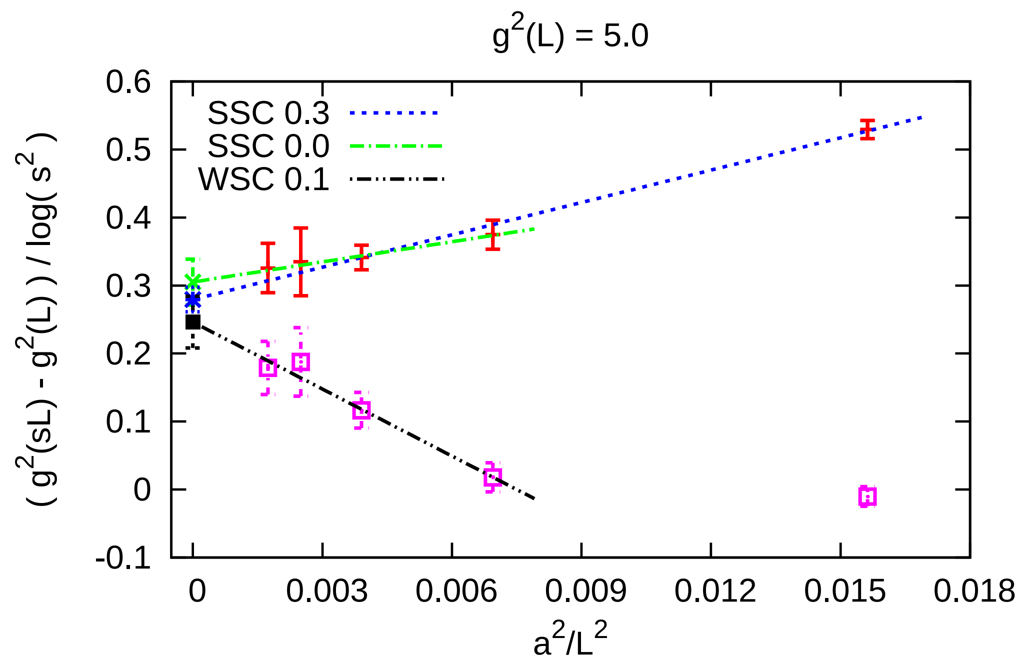

g2(L) = 3.0
70
60
50
40
30
20
10
0
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
0.2
(g2(sL) - g2(L)) / log(s2)



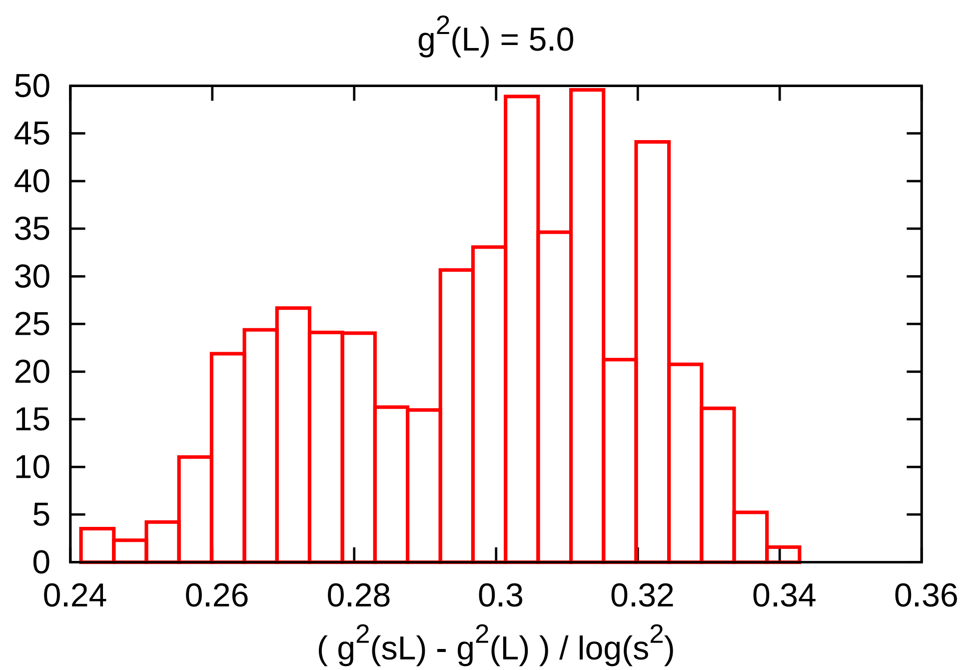
g2(L) = 4.0
SSC 0.8
SSC 1.2
WSC 1.1
(g2(sL) - g2(L)) / log(s2)
a2/L2



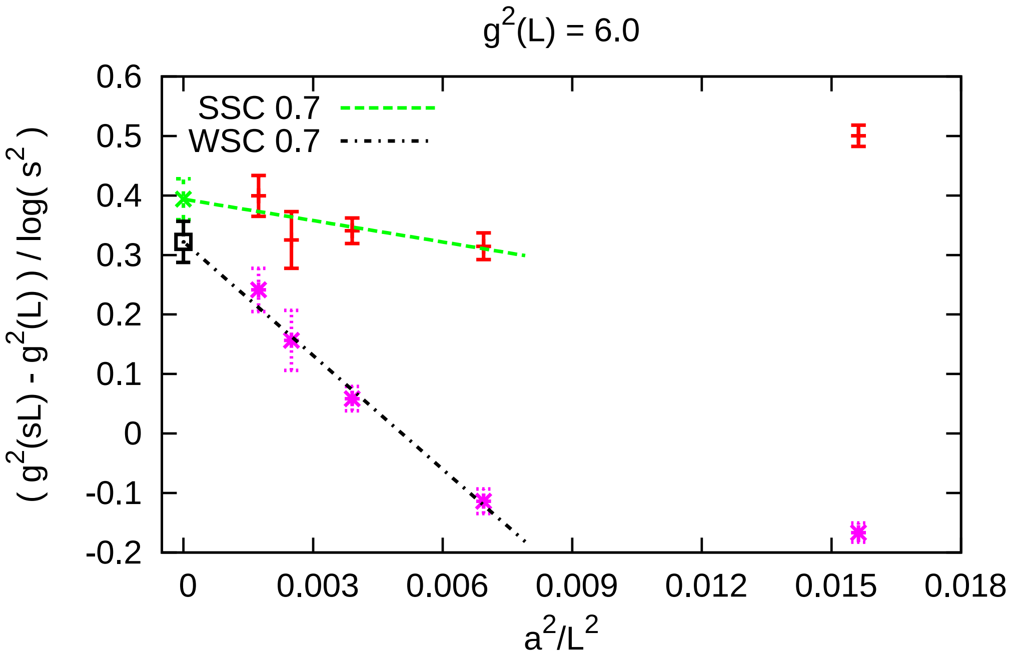
g2(L) = 4.0
(g2(sL) - g2(L)) / log(s2)



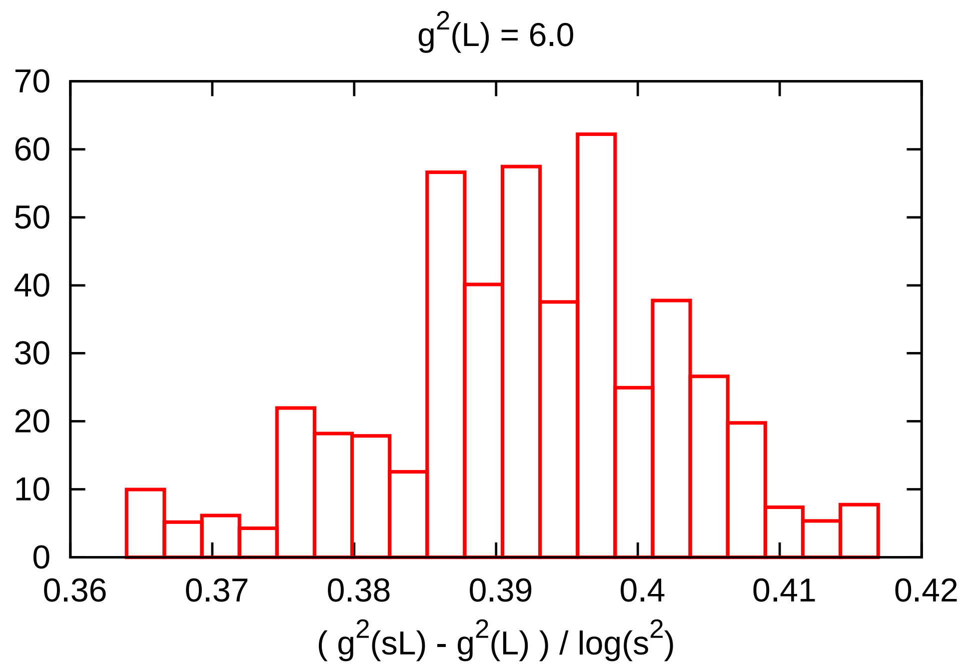
g^2(L) = 5.0
SSC 0.3
SSC 0.0
WSC 0.1
(g^2(sL) - g^2(L)) / log(s^2)
a^2/L^2



g^2(L) = 5.0
(g^2(sL) - g^2(L)) / log(s^2)



g^2(L) = 6.0
SSC 0.7
WSC 0.7
(g^2(sL) - g^2(L)) / log(s^2)
a^2/L^2
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0
0.003
0.006
0.009
0.012
0.015
0.018



g^2(L) = 6.0
(g^2(sL) - g^2(L)) / log(s^2)
70
60
50
40
30
20
10
0
0.36
0.37
0.38
0.39
0.4
0.41
0.42

Notes

Agreement between continuum WSC and SSC, consistency check

5 points in scaling region: $g^2_{WSC} < 2.5$ and $g^2_{SSC} < 5.5$

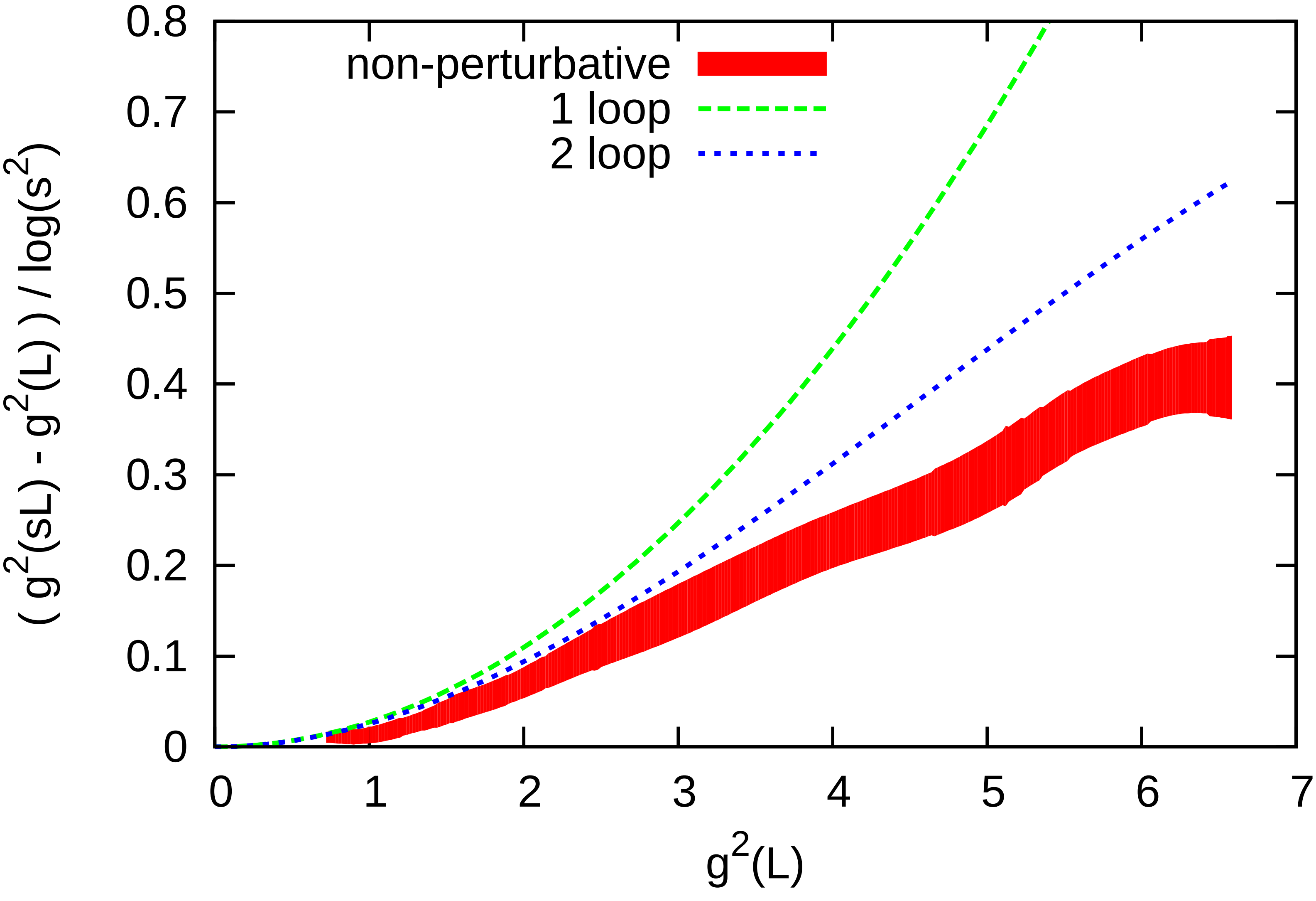
SSC scales better, final result for SSC

What happened to the WSC "fix points" at finite lattice volumes?

They didn't survive the continuum limit $\rightarrow$ lattice artifacts

Only remnant: some of the points in the continuum extrapolation was from negative values, but the continuum result is positive

Final result from SSC



Conclusions and lessons learned

- In the range $0 < g^2 < 6.5$ no sign of β-function turning back

- This range includes 3-loop and 4-loop MSbar fixed points $g_*^2 = 6.28$ and $g_*^2 = 5.73$

- Probably they are perturbative artifacts, similarly to large 2-loop fixed point, $g_*^2 = 10.58$

- Agreement with Schwinger-Dyson resummation (chiral symmetry breaking happens before reaching would-be fixed point)

- Consistency with our previous work on chiral dynamics and mass spectrum

Conclusions and lessons learned

- Continuum limit extremely important and control of related uncertainty

- May lead to qualitative change in behavior (finite lattice volume "fix point" disappears in continuum)

- Extremely important to consider large volumes

- Extremely important to consider several discretizations

Backup slides