G_2 Gauge Theory

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collaboration with

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eff. Polyakov Loop dynamics
Phys. Rev. **D80** (2009) 065028

Casimir Scaling/string breaking
Phys. Rev. **D83** (2011) 016001

phase diagram YMH
Phys. Rev. **D83** (2011) 114502

phase diagram with fermions
Phys. Rev. **D86** (2012) 111901

phase diagram, spectroscopy
Phys. Rev. **D89** (2014) 056007

- rank G_2 =rank SU(3) $=$ 2, dimension $=$ 14
- SU(3) subgroup of G_2
- $G_2/SU(3) \sim S^7 \to$ efficient parametrization
- G_2 subgroup of SO(7)
- smallest (simply connected) Lie group with trivial center
- representations are real
- fundamental representations $\{7\},\ \{14\}$ (= adjoint)
  7 quarks instead of 3 (cp. GUTS)
- can be broken to SU(3) with scalars in $\{7\}$

- singlet representation → colorless states

$$\{7\}\otimes\{7\} = \{1\}\oplus\{7\}\oplus\{14\}\oplus\{27\}$$
$$\{7\}\otimes\{7\}\otimes\{7\} = \{1\}\oplus 4\cdot\{7\}\oplus 2\cdot\{14\}\oplus\dots$$
$$\{14\}\otimes\{14\} = \{1\}\oplus\{14\}\oplus\{27\}\oplus\dots,$$
$$\{14\}\otimes\{14\}\otimes\{14\} = \{1\}\oplus\{7\}\oplus 5\cdot\{14\}\oplus\dots,$$
$$\{7\}\otimes\{14\}\otimes\{14\}\otimes\{14\} = \{1\}\oplus\dots$$

- branching $G_2 \to SU(3)$

fermions: $\{7\} \longrightarrow \{3\}\oplus\{\bar{3}\}\oplus\{1\},$

gauge bosons: $\{14\} \longrightarrow \{8\}\oplus\{3\}\oplus\{\bar{3}\}.$

- quark gluon plasma at high T (asymptotically free)
- confining at low T
- trivial center:

  what about confinement models based on center?
- SU(3) as subgroup: $G_2 \stackrel{SSB}{\longrightarrow}$ SU(3)
- colorless baryons and mesons, different to SU(2)
- no sign problem for any N_f:

  $\rightarrow$ simulations at finite T and finite μ
- phase-transition at finite T and n_B

  $\rightarrow$ relevant degrees of freedom under extreme conditions?

  $\rightarrow$ proposals/speculations on exotic phases of cold dense matter

- 1st order confinement/deconfinement PT — Hollands, Minkowski, Pepe, Wiese 2003; Cossu, D'Elia, di Giacomo, Lucini, Pica 2007
- Casimir scaling — Greensite, Langfeld, Olejnik, Reinhardt, Tok 2007; Liptak, Olejnik 2008
- Effective Polyakov loop dynamics — Wellegehausen, AW, Wozar 2009
- chiral restoration in quenched theory at same T_c — Danzer, Gattringer, Maas 2009
- G_2 monopoles (Shnir), semiclassical dyon picture — (Diakonov, Petrov 2010), ...
- string breaking, gluelump spectrum — Wellegehausen, AW, Wozar 2011
- SSB $G_2 \to SU(3)$ — Wellegehausen, AW, Wozar 2012
- topological properties, instantons — Maas, Olejnik, Ilgenfritz 2013
- G_2 with adjoint Weyl on $S^1 \times \mathbb{R}^3$, small L, m — Poppitz, Schäfer, Ünsal 2013
- equation of state, $\langle T^\mu_\mu \rangle_T \propto T^2$ — Bruno, Caselle, Panero, Pellegrini 2015
- ongoing: G_2 with dynamical fermions, $\mu, T > 0$ — Maas, Smekal, Wellegehausen, AW

approximate
order parameter:
Polyakov loop:

$$\chi_7 = \mathrm{tr}\,\mathcal{P}$$
$$\chi_{14} = \mathrm{tr}\,\mathcal{P}_{\mathrm{adj}}$$

strong coupling,
mean field
$\rightarrow$ effective models

$$S_{\mathrm{eff}}[\mathcal{P}]$$

$\rightarrow$ approximate
phase portrait

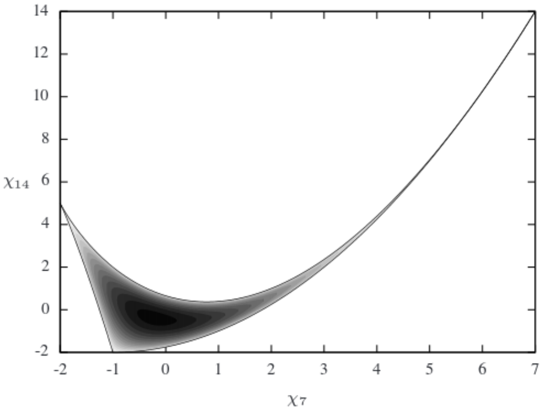
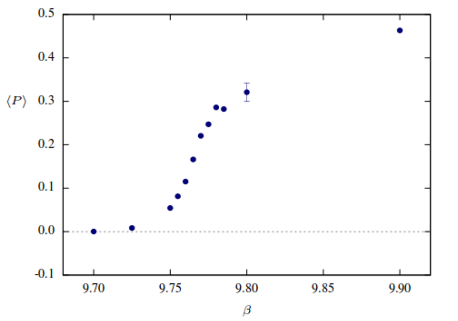
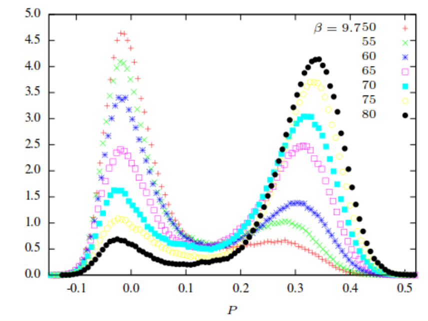


FIG. 2: Fundamental domain of G_2. Darker regions indicate a bigger Haar measure.

- simulations: first order PT
- local HMC, moderate $16^3 \times 6$ lattice



rapid change with β



histogram in vicinity of $\beta_c \approx 9.765$

- static quark-antiquark potential

$$\langle P(\boldsymbol{x})P^\dagger(\boldsymbol{y})\rangle_\beta = e^{-\beta V(R)}$$

- heavy quarks in representation $\mathcal{R}$:

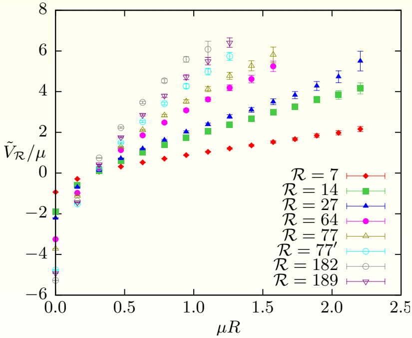
$$V_\mathcal{R}(R) = \gamma_\mathcal{R} - \frac{\alpha_\mathcal{R}}{R} + \sigma_\mathcal{R} R$$

- $e^{-\beta V(R)}$ varies over ≈ 50 orders of magnitude until string breaks
- exponential error reduction Lüscher-Weisz multilevel algorithm
- Casimir scaling hypothesis for string tensions:

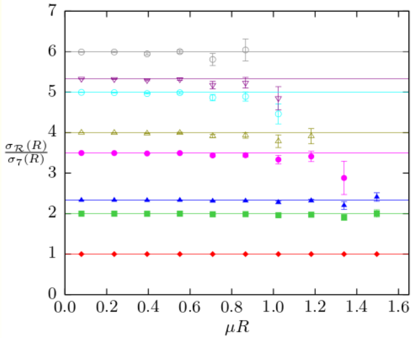
$$\frac{\sigma_\mathcal{R}}{c_\mathcal{R}} = \frac{\sigma_{\mathcal{R}'}}{c_{\mathcal{R}'}}$$

from ratios of Wilson(Polyakov) loops

- static potential for heavy quarks in 8 smallest G_2 representations
- multilevel, $N = 28$, $\beta = 40$
- no sign of Casimir scaling violation within
  4% for (7,...,189) in 3d, 5% for (7,...,64) in 4d
- continuum string tension (Nair et al.), Lüscher term (CFT)



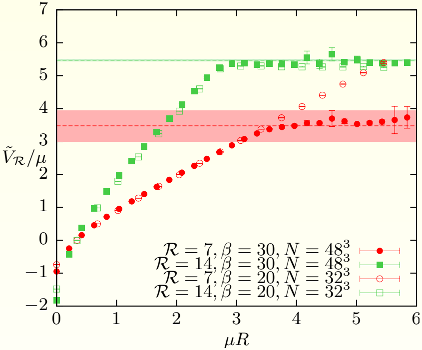
potential ($\mu^2 = \sigma_7$)



(scaled) string tensions

Liptak, Olejnik (smearing), Wellegehausen, AW., Wozar

- $N = 48, \ \beta = 30$
- near string breaking scale: Casimir scaling within 2.5%
- beyond: static $\mathcal{R}$-quarks screened by gluons
- decay products: gluelumps
- cont. limit for V_7 not reached



$$\exp(-m_{\mathcal{R}}T) \propto \left\langle \bigotimes_{n=1}^{N(\mathcal{R})} F_{\mu\nu}(y)_{\mathcal{R},a}\mathcal{R}(U_{yx})_{ab} \bigotimes_{n=1}^{N(\mathcal{R})} F_{\mu\nu}(x)_{\mathcal{R},b} \right\rangle$$

Wellegehausen, AW., Wozar (2011)

string breaking in SU(2): Philipsen, Wittig; Stephenson; de Forcrand, Phillipsen; Pepe, Wiese

G_2 Higgs Model

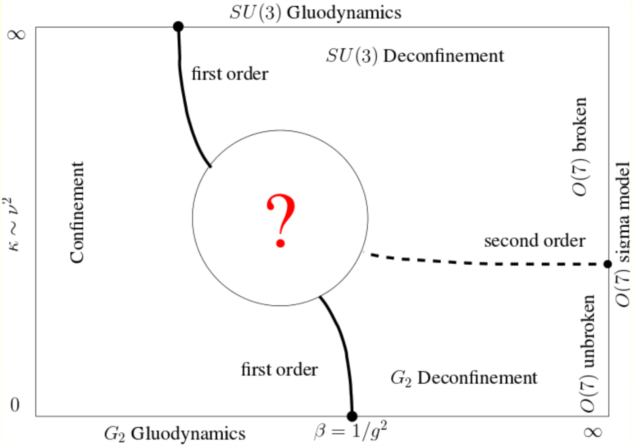
- lattice action with normalized Higgs $\varphi = (\varphi_1, \dots, \varphi)^T$ in $\{7\}$

$$S_{\mathrm{YMH}}[\mathcal{U}, \varphi] = -\beta \sum_{\square} \left(1 - \frac{1}{7} \operatorname{tr} \mathcal{U}_{\square}\right) - \kappa \sum \varphi_x^T \mathcal{U}_{x,\mu} \varphi_{x+\mu}$$

- SSB-pattern for $v = \langle \varphi \rangle \neq 0$:
- $\{14\} \longrightarrow \{8\} \oplus \{3\} \oplus \{\bar{3}\}$
  $\{8\}$: SU(3) gluons
  $\{3\} + \{\bar{3}\}$: massive Vector bosons
- scalars $7 \to 1$

- $\kappa = 0$: pure G_2 gauge theory:
  known 1st order deconfinement transition
- $\kappa = \infty$: 6 vector bosons decouple, pure $SU(3)$
  well-known 1st order deconfinement transition
- Is there 1st order transition line connecting two gauge theories?
- observables/histograms (from up to 5×10^5 configurations)
  on grid in (β, κ)-plane ($\beta = 5 \ldots 10$, $\kappa = 0 \ldots 10^4$) $\Longrightarrow$

Phase diagram of G_2 YMH theory

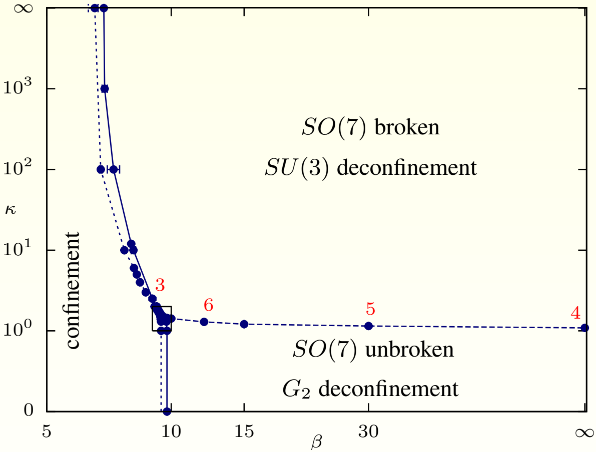


Osterwalder, Seiler; Fradkin, Shenker does not apply

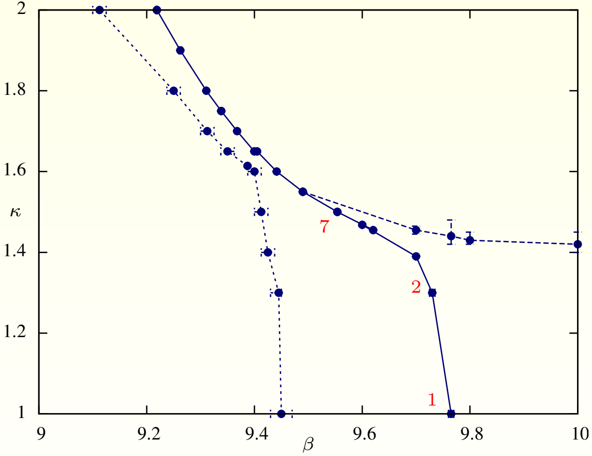
- LHMC, $N_s \leq 24$
- study of Higgs transition line (beware of bulk transition!)
- average plaquette action, Higgs action and Polyakov loop
- susceptibilities (and higher derivatives), finite size analysis
- large β: line of second order PT $O(7) \to O(6)$
- line of first order PT $G_2 \to SU(3)$ with (very) small gap
- (would be) triple point

$$\beta_{\text{crit}} = 9.55(5) \quad , \quad \kappa_{\text{crit}} = 1.50(4)$$

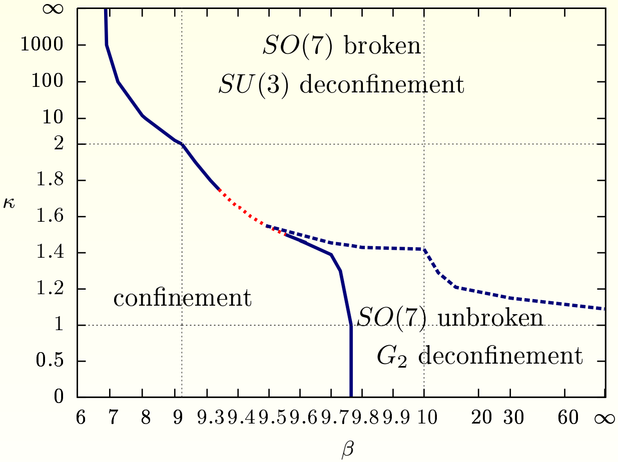
- first order (almost?) line hits second order line



$16^3 \times 6$-lattice



$N_s \leq 24$

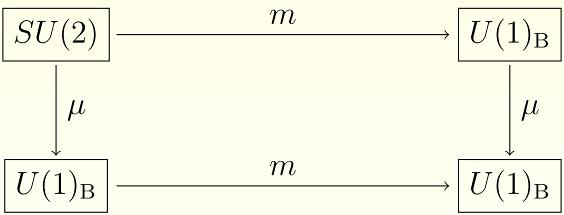


Summarizing the phase diagram of G_2 YMH-theory

Wellegehausen, Wozar, AW

- det $D(U, \mu, m] \geq 0$
  $\Rightarrow$ simulations at finite T and μ
- phases characterized by:
  chiral symmetry, baryon-density, Polyakov loop, condensates, ...
- interpretation of phase transitions and condensates
  $\Rightarrow$ particle spectrum (T=0)
- glueballs
- mesons
- baryons: $n_q = 1, 2, 3$

[a] collaboration with Axel Maas, Lorenz von Smekal und Bjoern Wellegehausen



- enlarged chiral symmetry (Pauli-Gürsey)
- well-defined baryon number
- Goldstone bosons

$$d(0^{++}) \sim \bar{\psi}^C \gamma_5 \psi + \bar{\psi} \gamma_5 \psi^C \quad \text{and} \quad d(0^{+-}) \sim \bar{\psi}^C \gamma_5 \psi - \bar{\psi} \gamma_5 \psi^C$$

- massive state

$$f(0^{++}) \sim \bar{\psi} \psi$$

- Wilson fermions, Symanzik improved gauge action
- RHMC-algorithm
- $\beta = 0.96,\ \kappa = 0.159$ fixed (light ensemble)

Spectroscopy ensemble

16×8^3 lattice
Proton mass $m_N = 938\,\mathrm{MeV}$
Diquark mass $m_{d(0^+)} = 247\,\mathrm{MeV}$
Lattice spacing $a = 0.343\,\mathrm{fm} \sim (575\,\mathrm{MeV})^{-1}$

Finite temperature and density

Lattice size: $N_t \times 8^3$ with $N_t = 2 \dots 16$
$T = 36 \dots 287$ MeV (9 values)
$\mu = 0 \dots 354$ MeV (60 values below lattice saturation)

Name	Operator	Pos.	Spin	Colour	Flavour	J	P
η	$\bar{u}\gamma_5 u$	S	A	S	S	0	-
f	$\bar{u}u$	S	A	S	S	0	+
ω	$\bar{u}\gamma_\mu u$	S	S	S	A	1	-
h	$\bar{u}\gamma_5\gamma_\mu u$	S	S	S	A	1	+
π	$\bar{u}\gamma_5 d$	S	A	S	S	0	-
a	$\bar{u}d$	S	A	S	S	0	+
ρ	$\bar{u}\gamma_\mu d$	S	S	S	A	1	-
b	$\bar{u}\gamma_5\gamma_\mu d$	S	S	S	A	1	+

$$\{7\} \otimes \{7\} = \{1\} \oplus \ldots$$

- unquenched
- partially quenched in one-flavour simulations

Name	Operator	Pos.	Spin	Col.	Flav.	J	P
Hybrid	$\epsilon_{abcdefg} u^a F^p_{\mu\nu} F^q_{\mu\nu} F^r_{\mu\nu} T^{bc}_p T^{de}_q T^{fg}_r$	S	S	A	S	1/2	$\pm$
$\tilde{\Delta}$	$T^{abc}(\bar{u}_a \gamma_\mu u_b) u_c$	S	S	A	S	3/2	$\pm$
$\tilde{N}$	$T^{abc}(\bar{u}_a \gamma_5 d_b) u_c$	S	A	A	A	1/2	$\pm$

$$\{7\} \otimes \{7\} \otimes \{7\} = \{1\} \oplus \dots$$
$$\{7\} \otimes \{14\} \otimes \{14\} \otimes \{14\} = \{1\} \oplus \dots$$

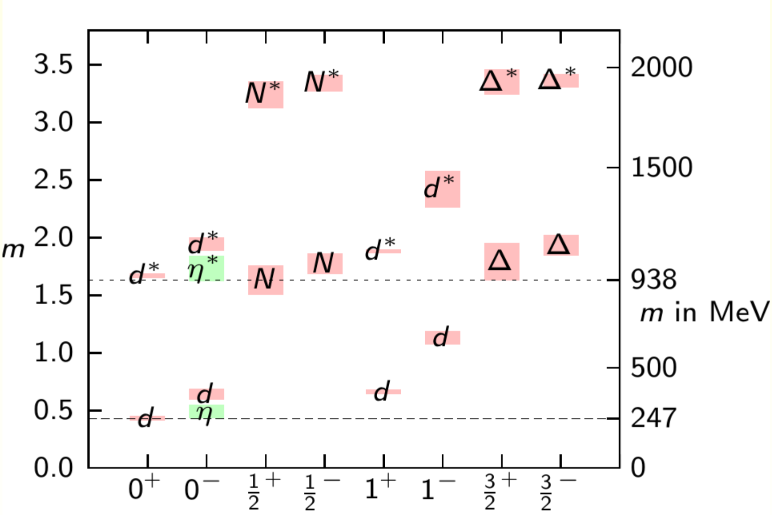
Name	Operator	Pos.	Spin	Colour	Flavour	J	P	C
$d(0^{++})$	$\bar{u}^C\gamma_5 u + \bar{u}\gamma_5 u^C$	S	A	S	S	0	+	+
$d(0^{+-})$	$\bar{u}^C\gamma_5 u - \bar{u}\gamma_5 u^C$	S	A	S	S	0	+	-
$d(0^{-+})$	$\bar{u}^C u + \bar{u} u^C$	S	A	S	S	0	-	+
$d(0^{--})$	$\bar{u}^C u - \bar{u} u^C$	S	A	S	S	0	-	-
$d(1^{++})$	$\bar{u}^C\gamma_\mu d + \bar{u}\gamma_\mu d^C$	S	S	S	A	1	+	+
$d(1^{+-})$	$\bar{u}^C\gamma_\mu d - \bar{u}\gamma_\mu d^C$	S	S	S	A	1	+	-
$d(1^{-+})$	$\bar{u}^C\gamma_5\gamma_\mu d + \bar{u}\gamma_5\gamma_\mu d^C$	S	S	S	A	1	-	+
$d(1^{--})$	$\bar{u}^C\gamma_5\gamma_\mu d - \bar{u}\gamma_5\gamma_\mu d^C$	S	S	S	A	1	-	-

$$\{7\} \otimes \{7\} = \{1\} \oplus \dots$$

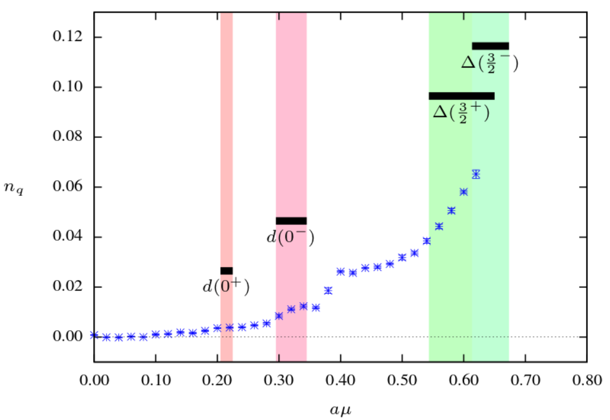
Name	Operator	Pos.	Spin	Colour	Flavour	J	P
Δ	$T^{abc}(\bar{u}_a^C \gamma_\mu u_b) u_c$	S	S	A	S	3/2	$\pm$
N	$T^{abc}(\bar{u}_a^C \gamma_5 d_b) u_c$	S	A	A	A	1/2	$\pm$

$$\{7\} \otimes \{7\} \otimes \{7\} = \{1\} \oplus \ldots$$

- diquark masses are degenerate (cc)
- $m_\eta - m_{\mathrm{diquark}} =$ disconnected contributions
- no sources for diquarks needed
- N_f complex-valued pseudo-fermions plus RHMC



Wellegehausen, Maas, Smekal, AW (2013)

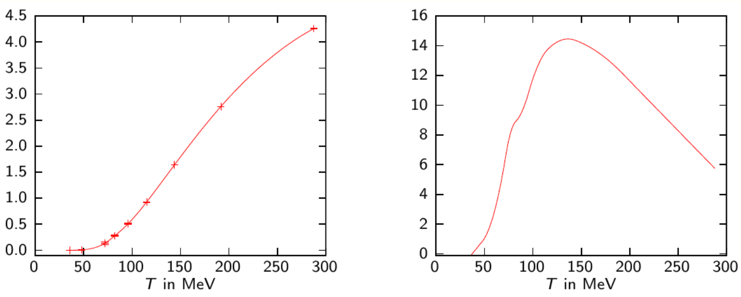


- silver blaze
- diquark BE condensation $m_{d(0^+)} = 247$ MeV
- plateaus visible for larger n_q
- three transitions
- hadronic phase at $m_\Delta \approx m_N$

Wellegehausen, Maas, Smekal, AW (2013)

- low density: in accordance with silver blaze
- two small jumps at diquark thresholds
  $\Rightarrow$ two (probably) second order PT?
- two plateaus after thresholds
- Bose-condensates of diquarks?
- one (probably) first order PT at $\approx \Delta$ threshold
- hadronic phase for higher n_q (under investigation)
- $a\mu \gtrsim 1$: lattice artifacts, e.g. saturation effects

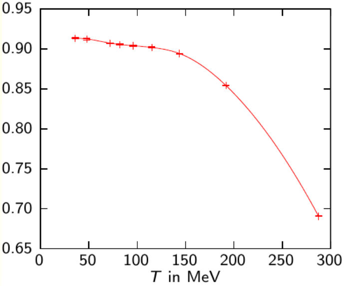
Polyakov loop $\langle P\rangle$ and susceptibility $\frac{\partial\langle P\rangle}{\partial T}$ at $\mu = 0$

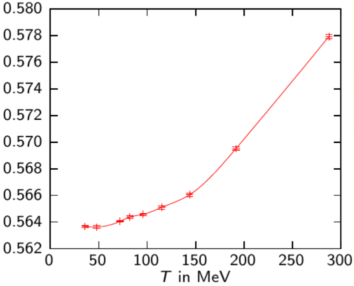


Deconfinement transition at $T_c(\mu = 0) \approx 137\,\mathrm{MeV}$
(cubic splines, no thermalization at 80 MeV)

Loewe cluster, 15 000

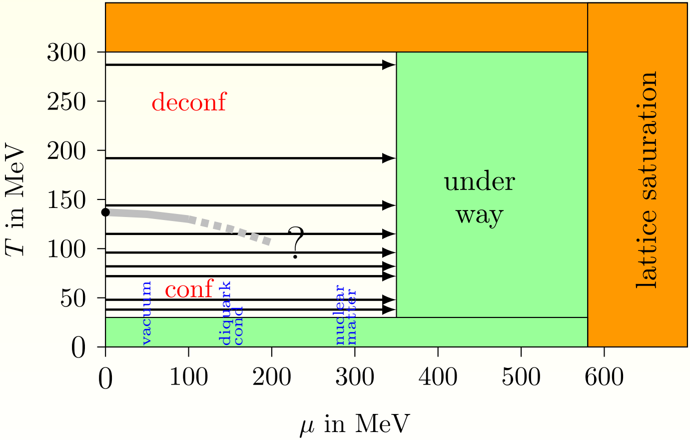
chiral condensate $\langle\Sigma\rangle - \Sigma_{\mathrm{SB,Latt}}$ and plaquette $\langle Plaq\rangle$ at $\mu = 0$



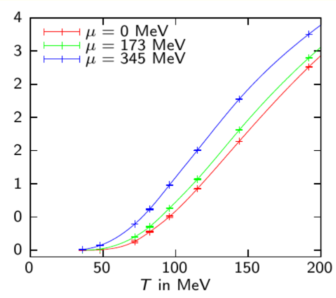


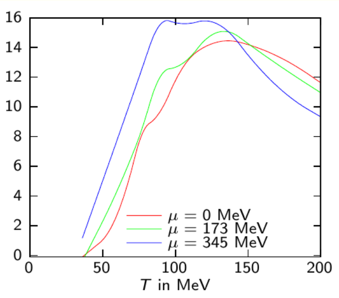
still not clear whether deconfinement
and chiral transition temperature agree

- temperatures: $T = 287, 192, 144, 115, 96, 82, 72, 48, 36$ MeV
- chemical potential: 60 μ-values between 0 and 354 MeV for each T



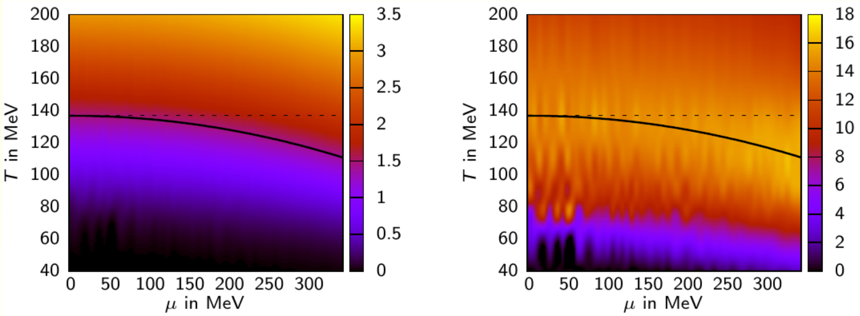
Polyakov loop $\langle P\rangle$ and susceptibility $\frac{\partial\langle P\rangle}{\partial T}$





deconfinement transition shifts to smaller T for larger μ
peak of susceptibility increases

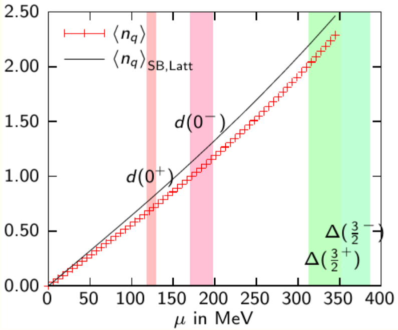
Polyakov loop $\langle P\rangle$ and susceptibility $\frac{\partial\langle P\rangle}{\partial T}$ (60 μ's)

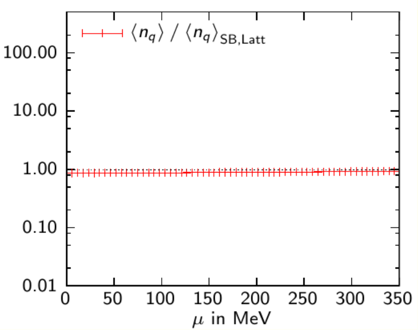


deconfinement transition shifts to smaller T for larger μ
peak of susceptibility increases, consistent with $T_c'(\mu)\big|_{\mu=0} = 0$

$$\frac{T_c(\mu)}{T_c(0)} \sim 1 + 0.001(8)\frac{\mu}{T_c(0)} - 0.031(6)\left(\frac{\mu}{T_c(0)}\right)^2$$

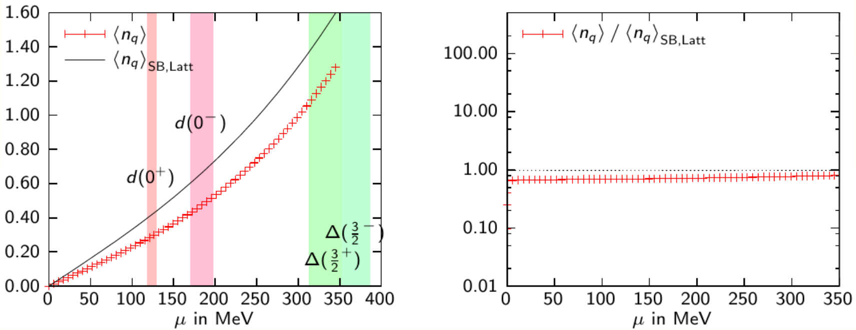
quark number density





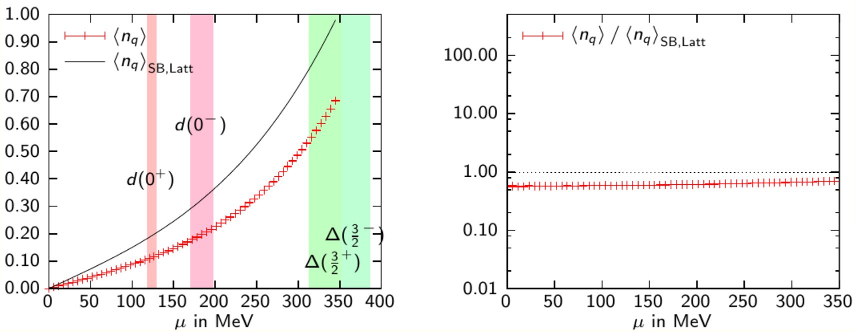
$T = 287\,\text{MeV}$

quark number density



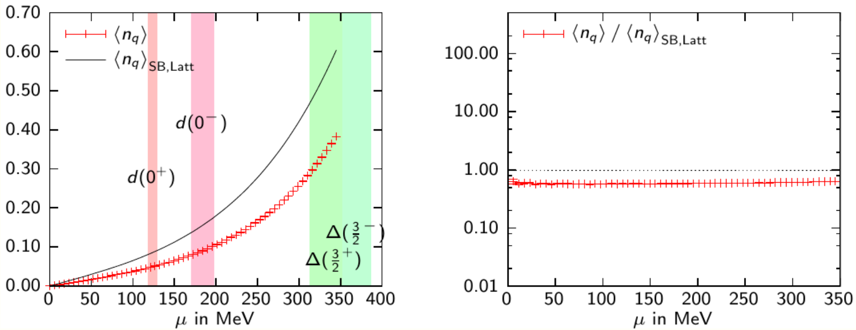
$T = 192\,\text{MeV}$

quark number density



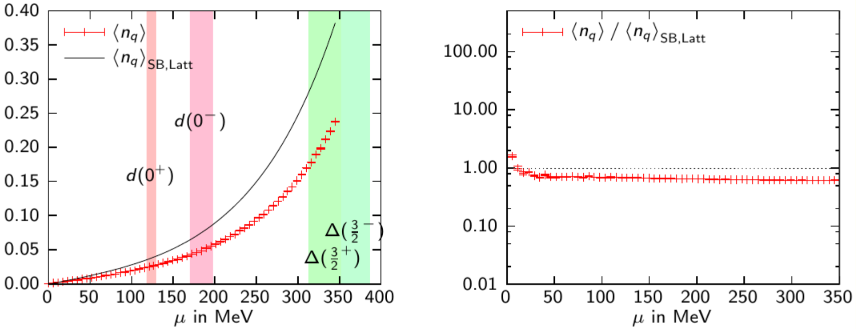
$T = 144\,\text{MeV}$

quark number density



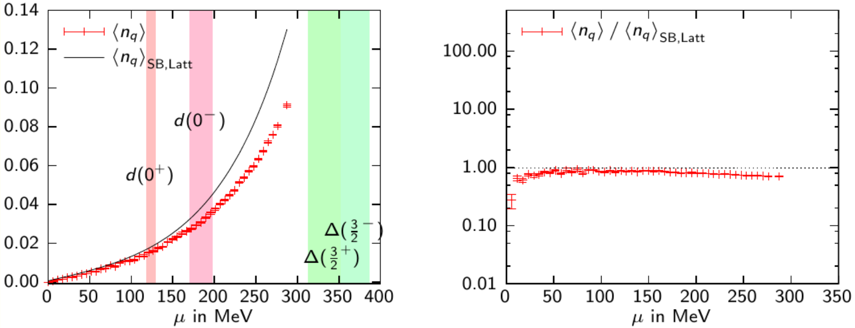
$T = 115\,\text{MeV}$, $\mu_{\text{deconf}} = 317\,\text{MeV}$

quark number density



$T = 96\,\text{MeV}$, threshold effects?

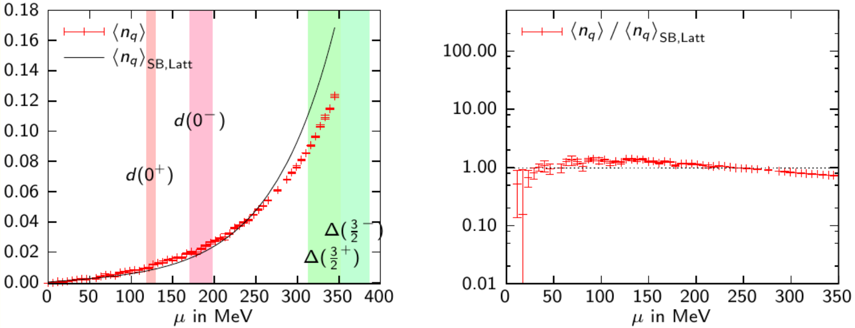
quark number density



$T = 82\,\mathrm{MeV}$

estimater problem for small μ when $\langle n_{\mathrm{B}} \rangle \approx 10^{-3} - 10^{-4}$

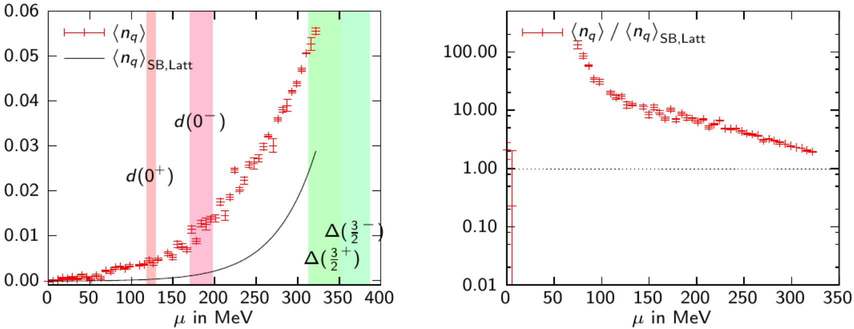
quark number density



$T = 72\,\mathrm{MeV}$

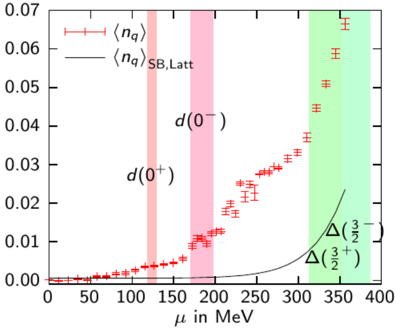
$\langle n_{\rm B} \rangle$ may jump at thresholds

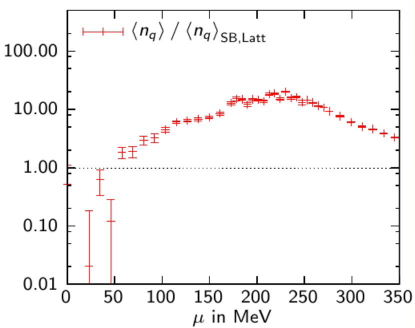
quark number density



$T = 48\,\text{MeV}$

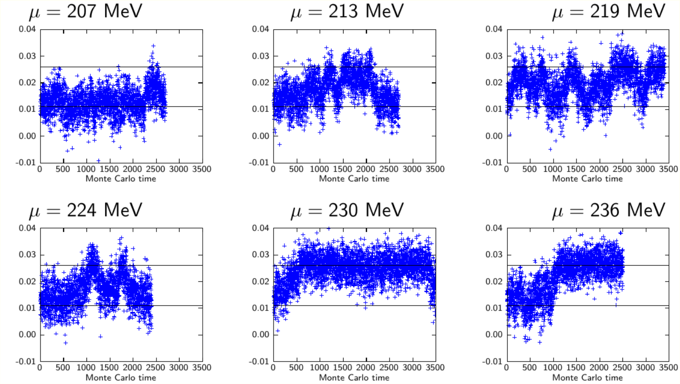
quark number density





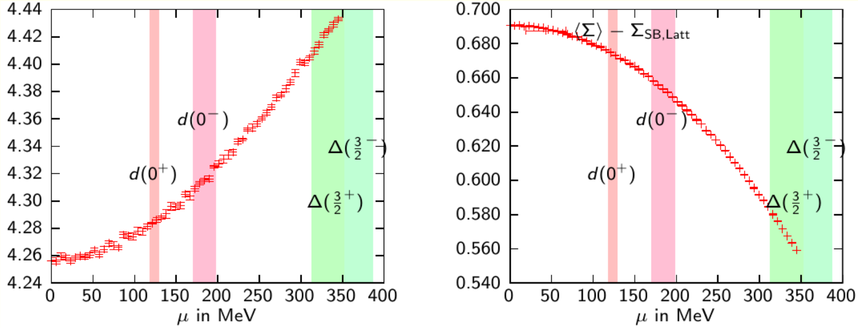
$T = 36\,\text{MeV}$

quark number density at $T = 36\,\text{MeV}$



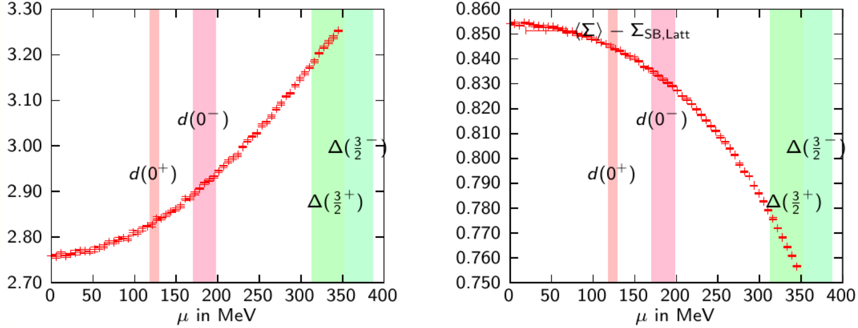
first order nuclear matter transition at $\mu_c \approx 219\,\text{MeV}$?
10 times estimated $\rightarrow$ wide bands
simulation expensive near μ_c (small m_d)

Polyakov loop and chiral condensate



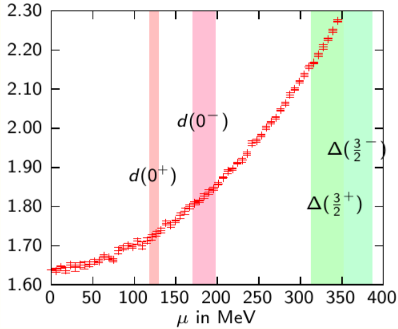
$T = 287\,\mathrm{MeV}$

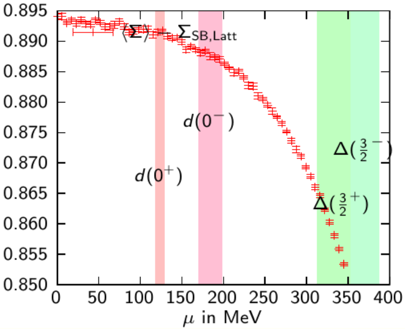
Polyakov loop and chiral condensate



$T = 192\,\text{MeV}$

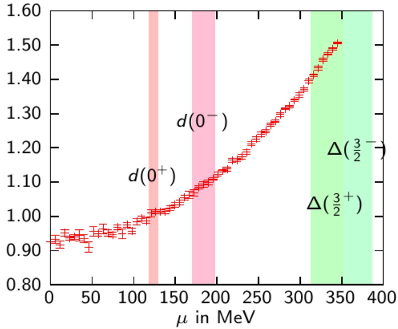
Polyakov loop and chiral condensate

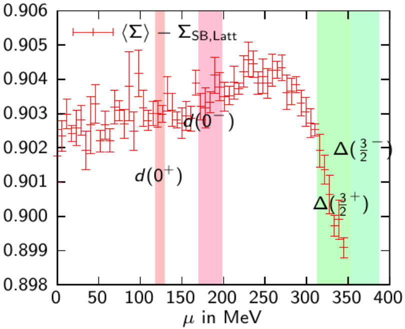




$T = 144\,\mathrm{MeV}$

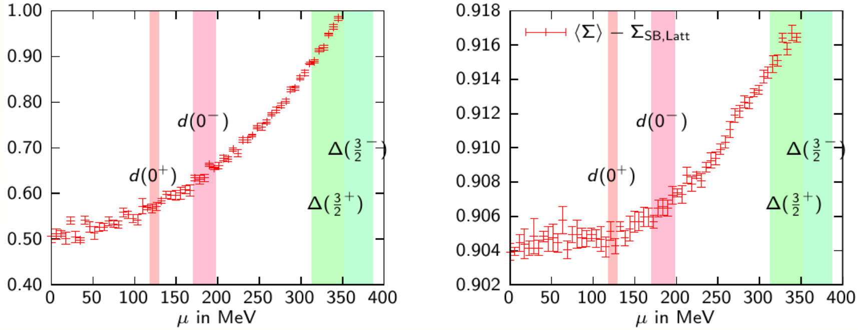
Polyakov loop and chiral condensate





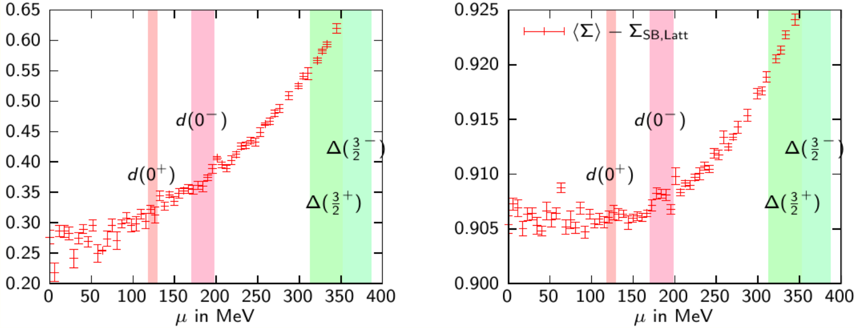
$T = 115\,\mathrm{MeV}$, $\mu_{\rm deconf} = 317\,\mathrm{MeV}$
(extrapolation of χ_P)

Polyakov loop and chiral condensate



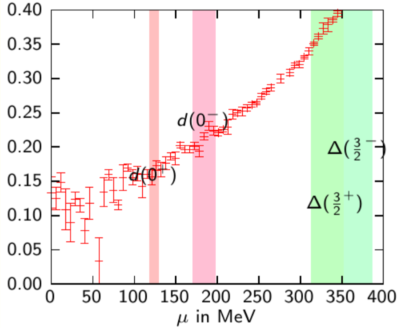
$T = 96\,\mathrm{MeV}$

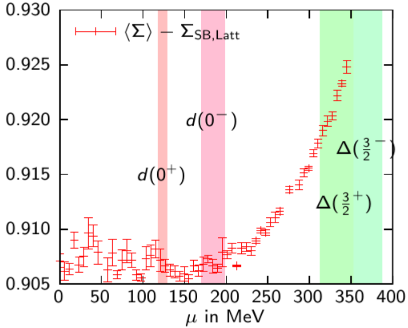
Polyakov loop and chiral condensate



$T = 82\,\text{MeV}$

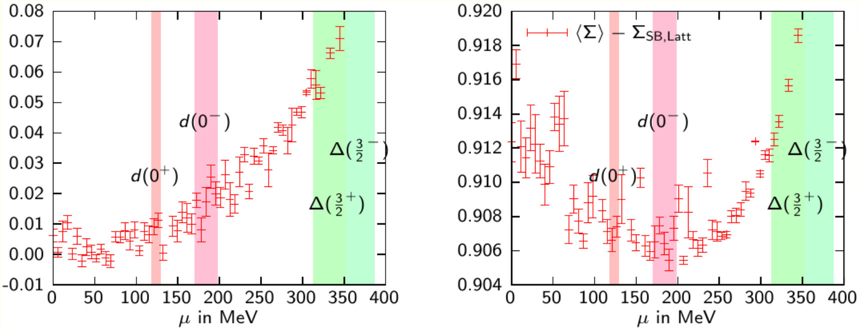
Polyakov loop and chiral condensate





$T = 72\,\mathrm{MeV}$

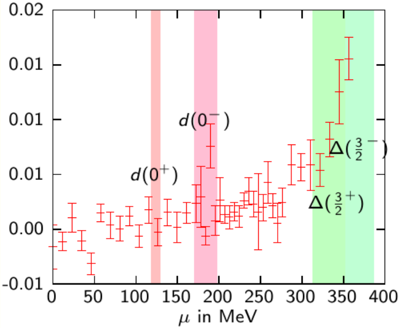
Polyakov loop and chiral condensate

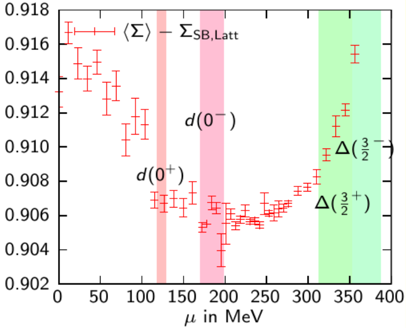


$T = 48\,\text{MeV}$

sign of di-quark condensation
chiral condendate $\rightarrow$ diquark condensate

Polyakov loop and chiral condensate





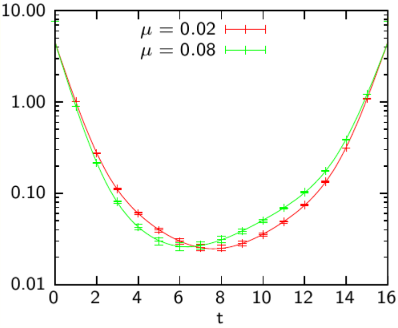
$T = 36\,\text{MeV}$

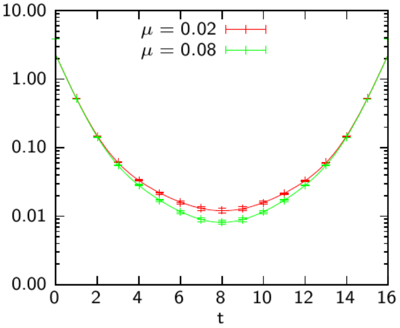
number of estimaters?

Lattice correlation function for operator with quark number n_q

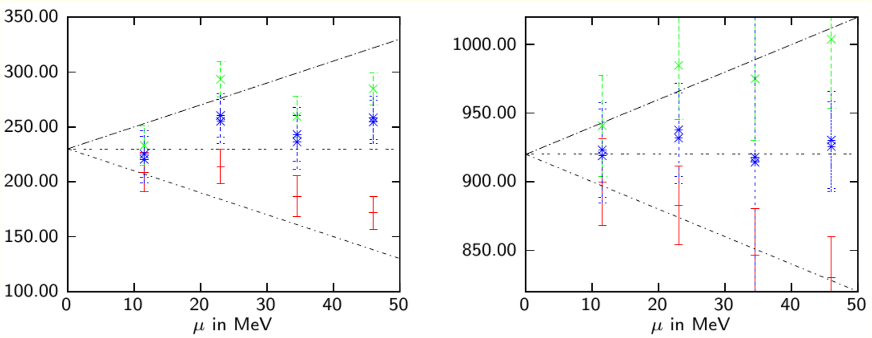
$$C(\mu, n_q) \sim a \exp^{-(m(\mu)+n_q\mu)t} + b \exp^{(m(\mu)-n_q\mu)t}$$

$d(0^+)$ and π correlation functions fitted with 4 exponentials for ground and excited states



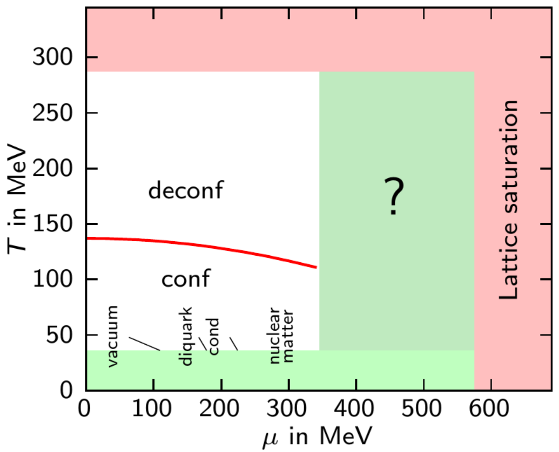


ground state and first excited state masses of $d(0^+)$ and π



below the silver blaze transition the masses do not depend on μ

G_2-QCD phase diagram with $V = (2.7\,\text{fm})^3$ and $m_{d(0^+)} = 247\,\text{MeV}$



- finite size effects?
- evidence for first order nuclear matter transition
- chiral symmetry restoration ($\to N_f = 2$ with pions)?
- deconfinement transition at lower temperatures / critical endpoint?

- hybrid spectroscopy (difficult)
- spectroscopy at finite density (larger μ)
- diquark condensation
- $N_f = 2$ phase diagram
- equations of state

A. Maas, L. von Smekal, B. H. Wellegehausen and A. Wipf, *The phase diagram of a gauge theory with fermionic baryons*, arXiv:1203.5653 [hep-lat], 2012.

A. Maas, L. von Smekal, B. H. Wellegehausen and A. Wipf, *Hadron masses and baryonic scales in G2-QCD at finite density*, arXiv:1312.5579 [hep-lat], 2014.

- G_2 QCD is a useful laboratory
- interpolation G_2-QCD $\rightarrow$ QCD with Higgs-mechanism possible
- full phase diagram in principle accessible to simulations
- follow first order(?) transition line (critical end point?)
- break G_2-QCD $\rightarrow$ QCD with quarks via Higgs-mechanism
- deformation vs. sign problem?
- testbed for model building
- testbed for alternative approaches (expansions, renormalization group)?
- heavy ensemble vs. strong coupling / hopping parameter expansion