4+8 flavors: a model for BSM dynamics

Oliver Witzel
Higgs Centre for Theoretical Physics

THE UNIVERSITY of EDINBURGH

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[JETP 120 (2015) 3, 423] [PoS Lattice2014 254] [CCP proceedings 2014]

(a detailed paper is in preparation)

motivation

Ingredients

- Dynamical model arising from strong interactions
- Higgs boson emerging from these new interactions
- 3 Goldstone bosons ($W^{\pm}$, Z)
- Light 0^{++} state well separated from other hadrons
- Other states experimentally to be observed
  - $\rightarrow$ Maybe 2 TeV vector state (ϱ)

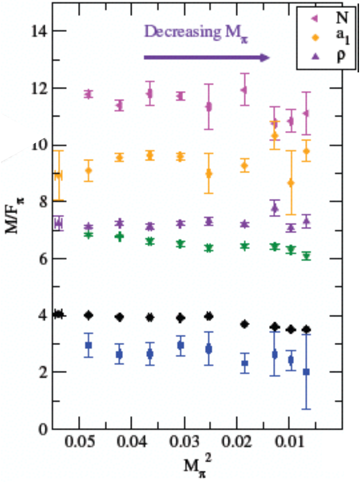
Candidates

- 2 flavor sextet model [see Talk by Julius Kuti (LatHC)]
  $\rightarrow$ Maybe conformal? [see Talks by Yuzhi Liu and Daniel Nogradi]
- 8 flavor in the fundamental representation
  [see Talks by George Flemming (LSD) and Hiroshi Ohki (LatKMI)]
  $\rightarrow$ Maybe conformal? [see Talk by Elisabetta Pallante]
- and probably more ...

- These theories have several common features
  but explore with large efforts only a very specific model

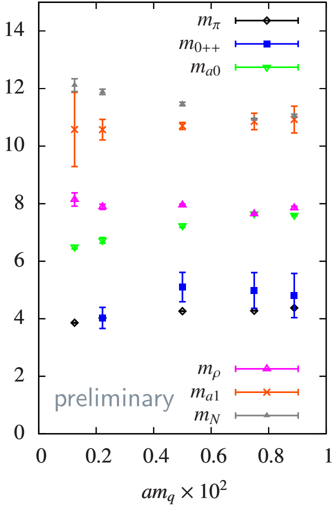
Common features

N_f=2 sextet



[J. Kuti Lattice 2015]

N_f=8 fundamental



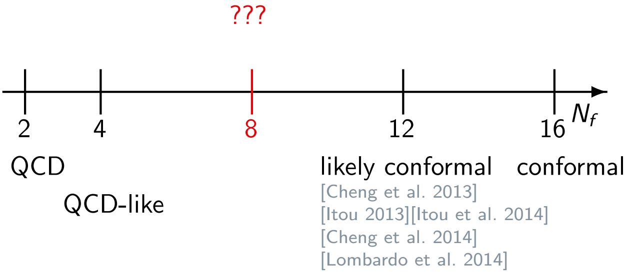
[LSD in preparation]

- Light 0^{++} $M_\pi \lesssim M_{0^{++}} < \frac{M_\varrho}{2M_\pi}$
- $M_\varrho / F_\pi \sim 8$
- Spectrum of $N_f = 2$ sextet and $N_f = 8$ fundamental exhibits similarities

plot courtesy by Anna Hasenfratz and Evan Weinberg

A more general model to study near conformal behavior

- SU(3) gauge theories with N_f fundamental fermions



- Staggered fermions come in multiplicities of 4 (no rooting)
  $\Rightarrow$ 4, 8, 12, 16 are preferred N_f

Our model: 4+8 flavors

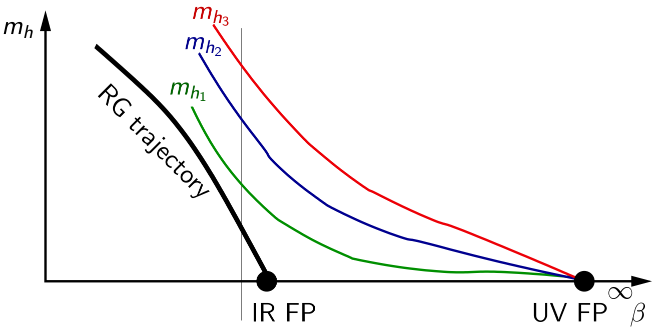
- SU(3) gauge theory with 4 light and 8 heavy flavors
- General model to study near conformal behavior
  - → phenomenologically more viable: 2+10, 2+8, or 2+6 flavors
- Light quark mass m_ℓ will be chirally extrapolated
- Heavy quark mass m_h is additional free, continuous parameter
- Sufficiently well known limits
  - → $m_h \to \infty$: 4-flavors
  - → $m_h \to m_\ell$: 12-flavors
- Has a continuum limit
  - ⇒ Something interesting must happen
  - ⇒ We can tune to be near the conformal window

in collaboration with

Richard Brower, Anna Hasenfratz, Claudio Rebbi and Evan Weinberg

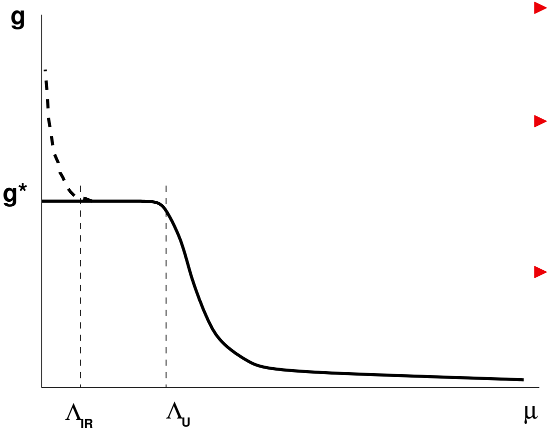
Continuum limit in 4+8 flavors

- We have 3 parameters: β, m_ℓ, and m_h
- First we take the chiral limit, i.e. $m_\ell \to 0$ and only 2 parameters remain
- Now we take the continuum limit by sending *together* $\beta \to \infty$ and $m_h \to 0$



- In practice this may be a challenging tuning exercise

Expected dynamics in the $m_\ell = 0$ limit



Sketch: [Del Debbio and Zwicky 2010]

- Similar to mass-deconformed conformal 12-flavor system
- 4-flavor infrared dynamics (dashed line) different to QCD-like 4-flavors
- Walking regime is driven by the IRFP:
  ⇒ hyper scaling in m_h for all hadrons for $m_\ell \to 0$
  ⇒ ratios of hadron masses constant w.r.t. m_h (if $m_\ell \to 0$)
  → maybe the 0^{++} is an exception

Lattice setup

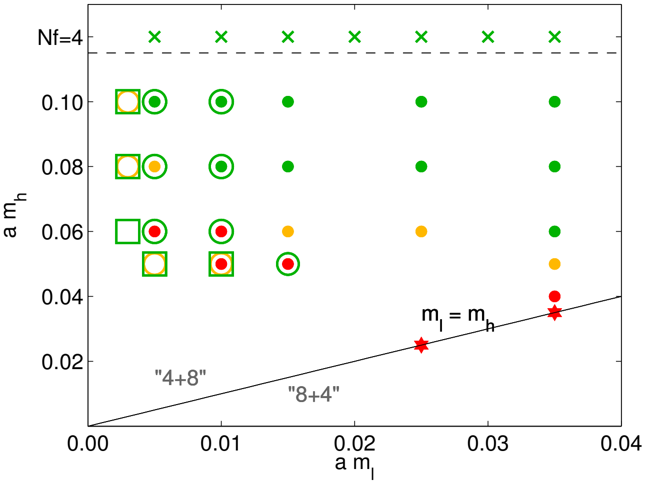
- Setup
  - SU(3) gauge group
  - Fundamental adjoint gauge action with $\beta_a = -\beta/4$ [Cheng et al. 2013][Cheng et al. 2014]
  - nHYP smeared staggered Fermions [Hasenfratz et al. 2007]
  - Most simulations/measurements performed with FUEL [J. Osborn]
- Goals
  - Explore near conformal or conformal dynamics
  - Compute the iso-singlet 0^{++}
- References

  [JETP 120 (2015) 3, 423] [PoS Lattice2014 254] [CCP proceedings 2014]

  (a detailed paper is in preparation)

simulations

Performed simulations



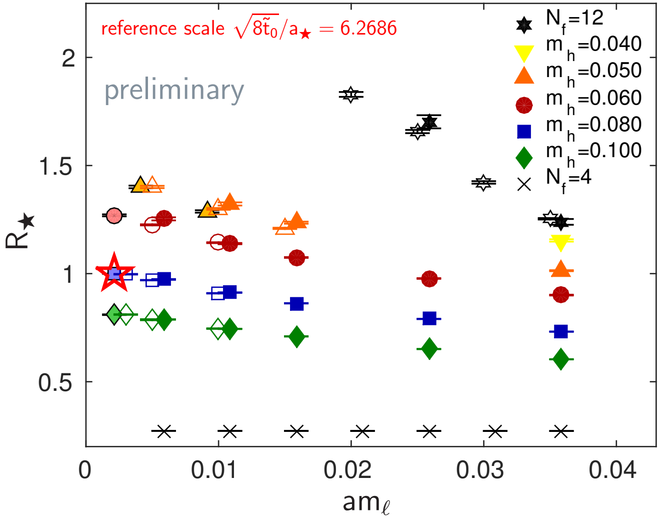
- Symbols indicate volumes and colors finite volume effects

□: $48^3 \times 96$ or $36^3 \times 64$

○: $32^3 \times 64$

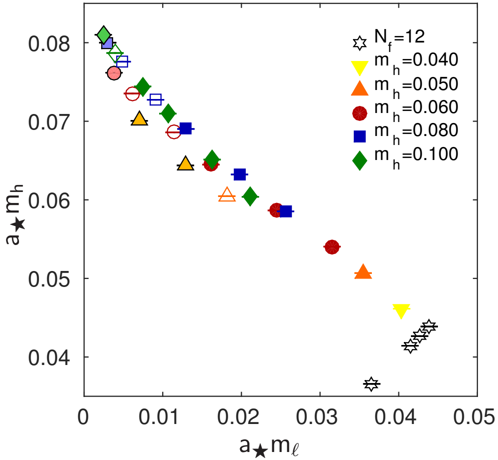
●: $24^3 \times 48$

Ratio of lattice scales $R_\star = \left[\sqrt{8t_0}/a\right] / \left[\sqrt{8t_0}/a\right]_{\text{ref}}$



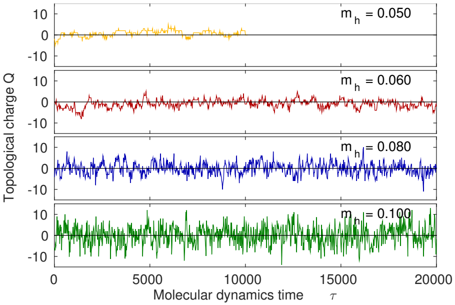
- Wilson flow scale $\sqrt{8t_0}$
- 4, 4+8, and 12 flavors at $\beta = 4.0$
- Data on 24^3, 36^3, and 48^3 volumes are shown with small horizontal offset
- Small FV effects in the gauge sector

Input masses in $a_\star$ units

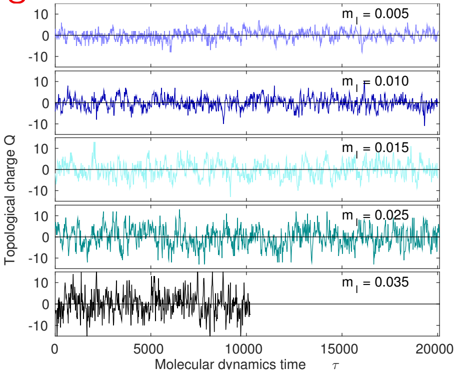


- 12 flavor data fall on a diagonal line pointing to the origin
- In the $m_\ell \to 0$ limit hyper-scaling suggests a small spread of $a_\star m_h$ for our choices of m_h
- Good idea(s) still needed to explain $a_\star m_\ell$ dependence

History of the topological charge



- $m_\ell = 0.010$ fixed



- $m_h = 0.080$ fixed

- Frequency of tunneling slows down and amplitude of oscillations reduces
  - $\rightarrow$ when reducing m_h for fixed m_ℓ
  - $\rightarrow$ when reducing m_ℓ for fixed m_h (milder effect)

running coupling

Running coupling form gradient flow

- Gradient flow defines the renormalized coupling [Narayanan and Neuberger 2006] [Lüscher 2010]

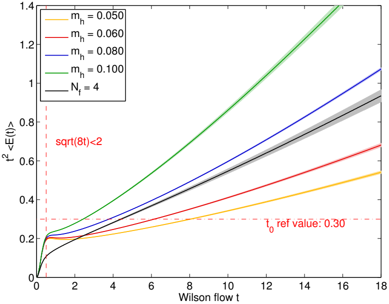
$$g_{GF}^2(\mu = 1/\sqrt{8t}) = t^2 \langle E(t) \rangle / \mathcal{N}$$

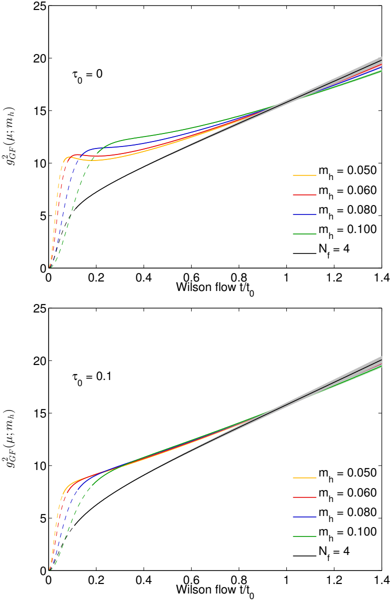
  t: flow time; $E(t)$ energy density
- g_{GF}^2 is used for scale setting

$$g_{GF}^2(t = t_0) = 0.3/\mathcal{N} \qquad (\text{“}t_0\text{-scale”})$$

- Can determine renormalized running coupling on large enough volumes and large enough flow times in the continuum limit

From $t^2\langle E(t)\rangle$ to the running coupling



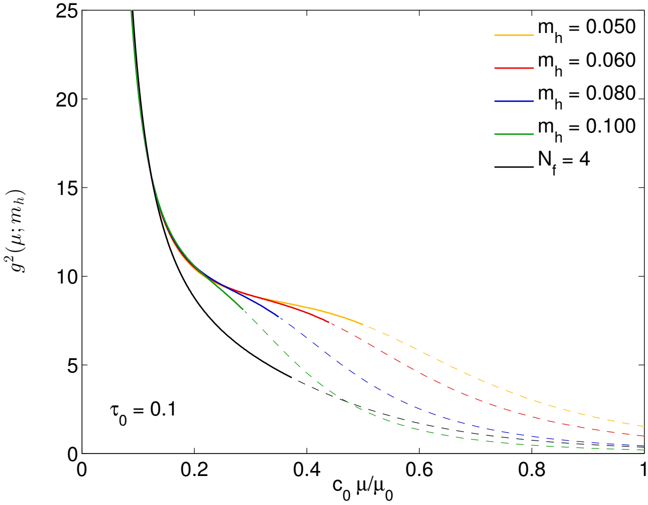


- Compute $t^2\langle E(t)\rangle$ as function of the flow time t
- Normalize by $\mathcal{N}$ and t_0
- Remove $O(a^2)$ errors by tau-shift

$$g^2_{GF}(\mu) = \langle t^2 E(t + \tau_0 a^2)\rangle / \mathcal{N}$$

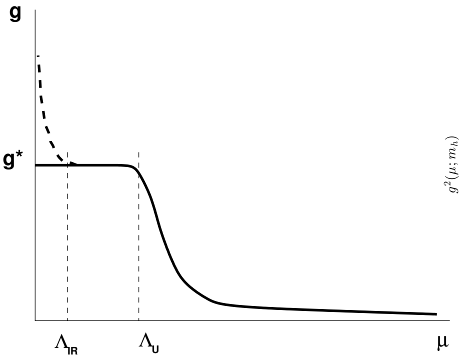
- Invert: $\mu = 1/t$

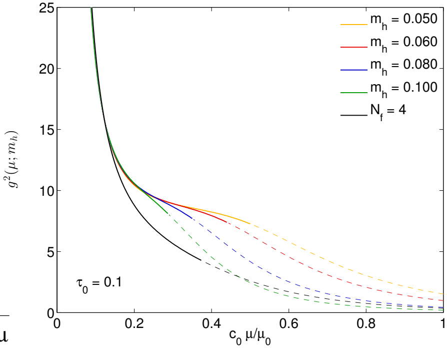
Running coupling form gradient flow: 4+8 flavors



- Extrapolated to $m_\ell = 0$
- $N_f = 4$ shows fast running
- "Shoulder" increases for smaller m_h $\Rightarrow$ walking
- Walking range is tuned as function of m_h
- Data with error bars!

Expectation vs. numerical data (at $m_\ell = 0$)





Sketch: [Del Debbio and Zwicky 2010]

anomalous dimension

Anomalous dimension

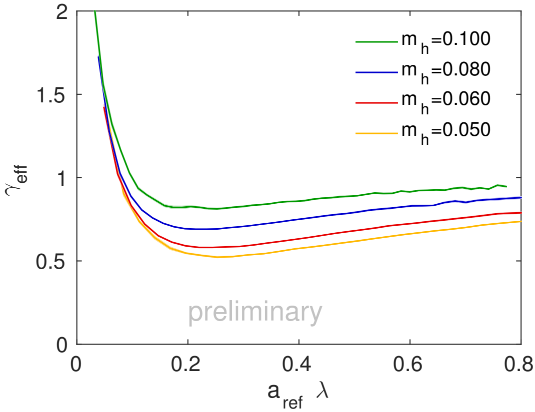
- We can predict a scale dependent anomalous dimension $\gamma_{\text{eff}}(\mu)$ form the mode number of the Dirac operator

$$\mu(\lambda) \propto \lambda^{4/(\gamma_{\text{eff}}(\lambda)+1)} \quad \text{with} \quad \lambda \propto \mu$$

→ For large $\mu \sim \lambda$: $\gamma_{\text{eff}}(\mu)$ matches perturbative value

→ At $\lambda = 0$: $\gamma_{\text{eff}}(\mu)$ matches universal IRFP, if the system is conformal; meaningless once chiral symmetry breaks

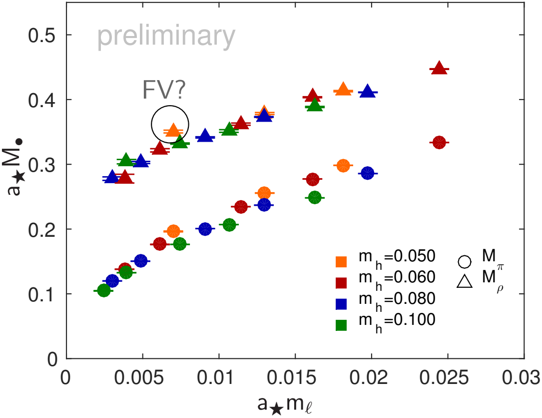
Scale dependent anomalous dimension $\gamma_{\text{eff}}(\mu)$



- Anomalous dimension is not large but still $O(1)$ and can persist
- For $m_h \to 0$ it approaches the value corresponding to the 12 flavor IRFP $\gamma_{\text{IRFP}}^{\text{12f}} = 0.235(15)$

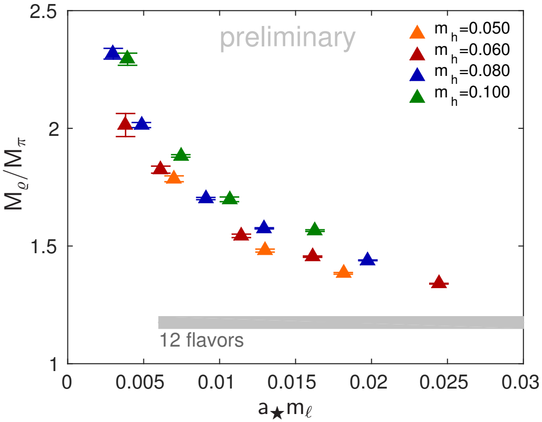
spectrum

Connected spectrum: M_π and M_ϱ



- Rescaling m_ℓ, M_π and M_ϱ by $a_\star$
- M_π and M_ϱ more or less degenerate for different m_h
- M_ϱ has noticeable downward curvature

Are we chirally broken?



- In 4 flavors (QCD-like) we know the ratio diverges
- In 12 flavors an almost constant ratio is observed [Cheng at al. 2014]

— as expected for conformal systems

Disconnected spectrum: the 0^{++} scalar

Numerical measurement on the lattice

- 6 U(1) sources with dilution on each time slice, color and even/odd spatially
- Variance reduced $\langle\overline{\psi}\psi\rangle$

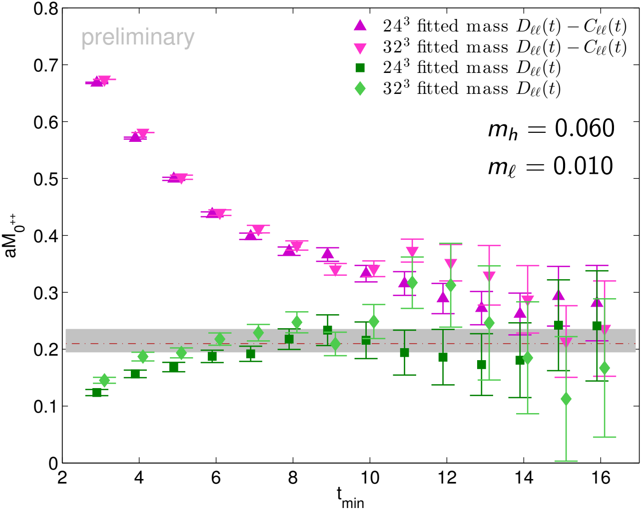
Analysis strategy

- Correlated fit to both parity states (staggered)
- Vacuum subtraction introduces very large uncertainties
- Advantageous to fit additional constant

$$C(t) = c_{0^{++}} \cosh\left(M_{0^{++}}\left(\frac{N_T}{2} - t\right)\right) + c_{\pi_{sc}}(-1)^t \cosh\left(M_{\pi_{sc}}\left(\frac{N_T}{2} - t\right)\right) + \nu$$

- Equivalent to fitting the finite difference: $C(t+1) - C(t)$

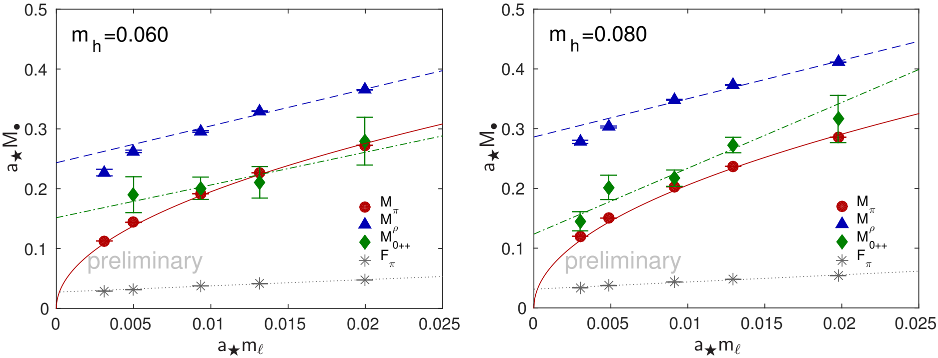
Comparison of $D_{\ell\ell}$ and $D_{\ell\ell} - C_{\ell\ell}$



- For $t \to \infty$: $D_{\ell\ell}$ and $D_{\ell\ell} - C_{\ell\ell}$ should agree (up to mixing effects)
- Compare fits with different t_{min} and $t_{\text{max}} = N_T/2$
- Compare results for two volumes

$\Rightarrow$ Consistent results!

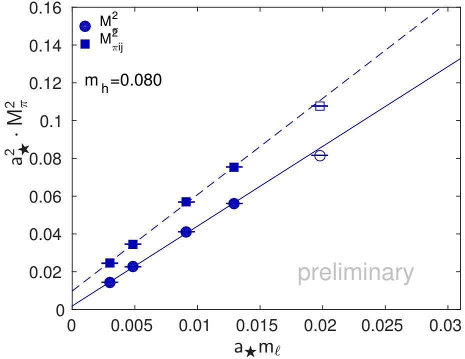
F_π, M_π, M_ρ, and $M_{0^{++}}$ for $m_h = 0.060$ and $m_h = 0.080$



- $m_\ell = 0.003$: $F_\pi L = 0.027 \cdot 48 = 1.3$
- $m_\ell = 0.003$: $F_\pi L = 0.034 \cdot 36 = 1.2$
- Lines solely to guide the eye!

Pion taste splitting

- Taste splitting is artifact of staggered fermions
- In QCD modern, smeared staggered actions show small taste splitting effects
- Taste splitting is typically constant w.r.t. m_ℓ



- Taste splitting increases for larger m_ℓ when reducing m_h

remarks

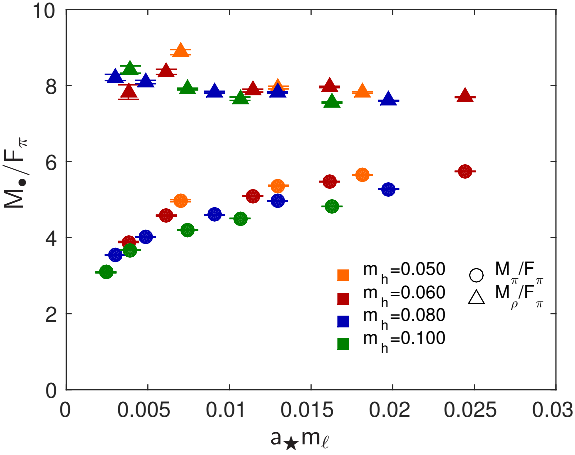
Concluding remarks

- A great model to explore near conformal dynamics by varying the continuous parameter m_h
- Limiting cases of 4 and 12 flavors help to understand what is happening
- 4+8 is chosen for convenience of unrooted staggered fermions; investigating 2+10, 2+8, or 2+6 flavors with e.g. Wilson fermions is highly intersting

- Non-QCD like features
  - → Running coupling develops a "shoulder"
  - → Chiral behavior can be tuned with m_h
  - → Curvature of M_ϱ
  - → Non-constant taste splitting
  - → The 0^{++} is light: $M_{0^{++}} < M_\varrho, 2M_\pi$

Like in many other theories we find $M_\varrho/F_\pi \sim 8$

$\Rightarrow$ setting the scale with $F_\pi = 242$ GeV results in a ~ 2 TeV vector state!



appendix

LSD: 8 fundamental flavors

Lattice Strong Dynamics Collaboration

Xiao-Yong Jin
James Osborn

Joe Kiskis

Graham Kribs

Ethan Neil
Sergey Syritsyn

Richard Brower
Claudio Rebbi
Evan Weinberg

Oliver Witzel

David Schaich

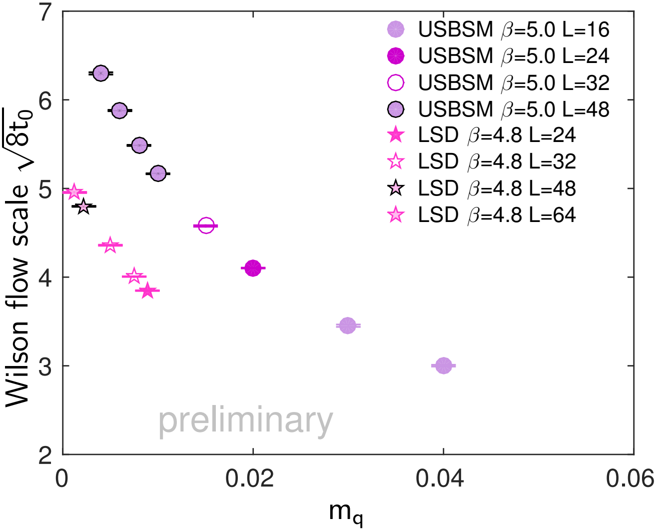
Meifeg Lin

Ethan Neil

Evan Berkowitz
Mike Buchoff
Enrico Rinaldi
Chris Schroeder
Pavlos Vranas

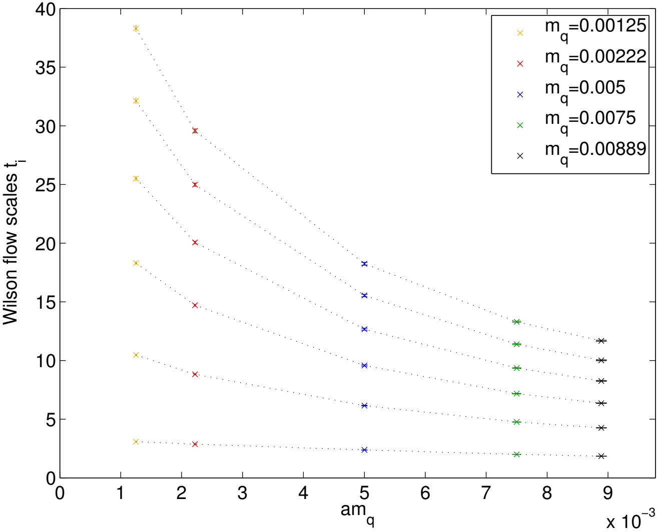
Tom Applequist
George Flemming

Lattice scales: 8 flavor



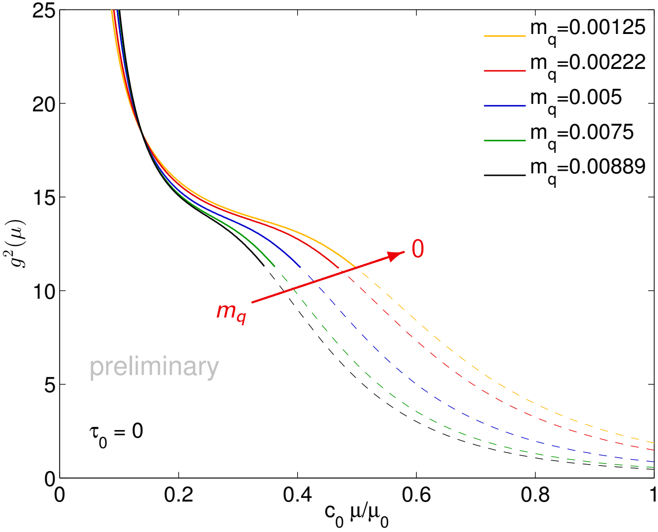
- Wilson flow scale $\sqrt{8t_0}$
- 8 flavors at $\beta = 4.8$ and $\beta = 5.0$
- USBSM:
  [Schaich, PoS Lattice2013 072]

Running coupling form gradient flow: 8 flavors

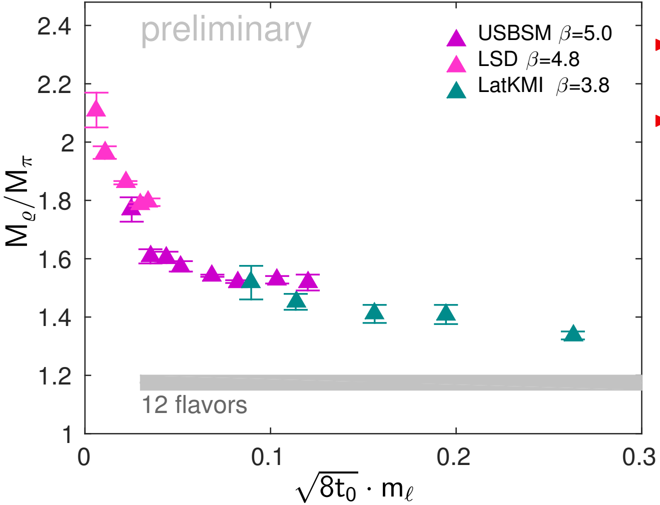


- $\beta = 4.8$
- Chiral extrapolation is not feasible:
  $\rightarrow$ non-linear chiral extrapolation
  $\rightarrow$ no comparable data at different β
- Linear extrapolation observed in QCD and derived in χPT
  [Bär and Goltermann 2014]

Running coupling form gradient flow: 8 flavors

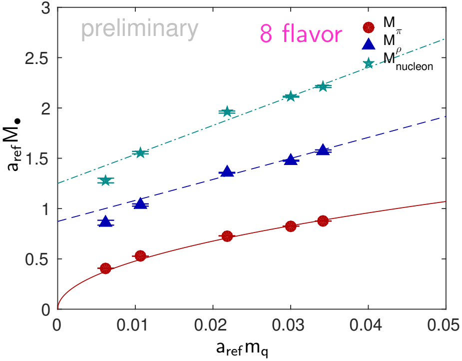


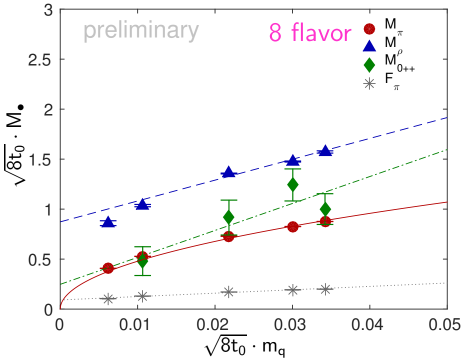
- Déjà vu?
- Is this walking?
- "Shoulder" extends as we lower m_ℓ
- Gauge dynamics causes the "shoulder": slow evolution
- Fermions trigger the fast rise
- What consequences arise form the two separate regimes?



- LatKMI data:
  [Y. Aoki et al. 2014]
- USBSM data:
  [Schaich, PoS Lattice2013 072]

F_π, M_π, M_ρ, M_{nucleon} and $M_{0^{++}}$ for 8 flavors





- $m_\ell = 0.00222$: $F_\pi L = 0.027 \cdot 48 = 1.3$
- Connected spectrum not too happy with "naive assumptions for fit"

Pion taste splitting