Mass generation without symmetry breaking: staggered quarks

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Kitaev-Wen-Xu from staggered fermions ...?

- CMT models predict $N_f = 16$ Majorana fermions can be gapped without generating a bilinear condensate via specially chosen four fermi interactions
- Revisit old DWF constructions of lattice chiral gauge theories and try to decouple mirror fermions ...?
- Try to understand how this works in Euclidean lattice path integrals. Connection to staggered fermions ? Structure of 4 fermi term ...
- Start: Chandrasekharan and Ayyer, 1410.6474. Plus some additional work ...

Model

Take *reduced staggered fermions* equipped with additional $SO(4)$ symmetry.

$$
\begin{aligned}
S &= \sum_x \sum_\mu \eta_\mu(x)\psi^a(x)\Delta_\mu^{ab}\psi^b(x) - \\
&\frac{1}{8}G^2 \sum_x \epsilon_{abcd}\psi^a(x)\psi^b(x)\psi^c(x)\psi^d(x)
\end{aligned}
$$

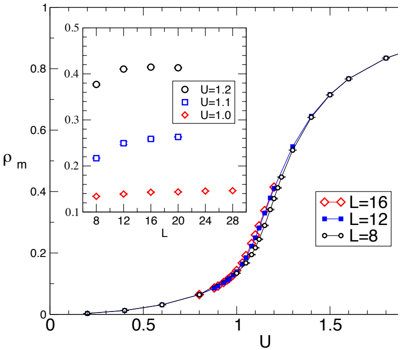
Single component fermions resulting from spin diagonalization of naive fermions. Truncate so ψ,$\overline{\psi}$ defined only on even/odd parity sites ...

2×4 Dirac fermions in naive continuum limit

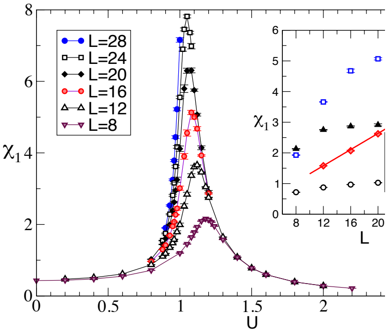
No $SO(4)$ invariant bilinear possible

Numerical results in 3D

1410.6474 (note: $SU(4)$ not $SO(4)$ symmetry.)



$< \psi^1\psi^2\psi^3\psi^4 >$



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Auxiliary fields

Initial results obtained using worm algorithm.

To attempt (R)HMC simulation need to replace four fermi term by Yukawa. Note: pfaffian positive for $SO(4)$

Notice:

$$\epsilon^{abcd}\psi^a\psi^b\psi^c\psi^d = \left(\psi^a\psi^b + \frac{1}{2}\epsilon^{abcd}\psi^c\psi^d\right)^2$$

Introduce antisymmetric self-dual auxiliary scalar ϕ_+^{ab}

$$S_4 = G\psi^a\psi^b\phi_+^{ab} + \frac{1}{2}(\phi_+^{ab})^2$$

Note: $SO(4) = SO(3) \times SO(3)$ with ϕ_+^{ab} transforming as fundamental under one of the $SO(3)$'s

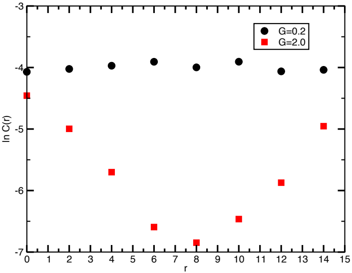
Dynamics

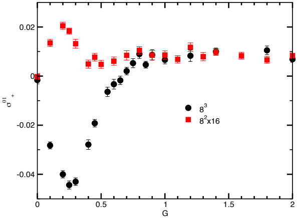
- For large G expect auxiliary can develop vev - four fermion condensate. Vacuum manifold is S^2.
- Boundary of system is also S^2. Hence topologically stable configurations possible (hedgehogs).
- Hedgehogs suppressed (action diverges linearly with size) but neutral combinations $H\overline{H}$ possible.

Conjecture that vacuum populated by condensate of such objects – restores $SO(4)$ while ensuring that fermions propagating in such a background acquire a mass

Numerical results from (R)HMC: 3D

$\ln < \psi^a(0)\psi^a(r) >$





Order parameter σ_+ Add symmetry breaking field $m = 0.1$.

Four dimensions ? General questions ...

In 4D mapping is $S^3 \to S^2$. Topological fields are Hopf defects.

Summary:

- Model has same number of fermions as CMT constructions. Similar four fermion interaction. Key feature: no symmetric fermion bilinear possible. Explicit connection to CMT constructions ?
- 3D See dynamical mass generation and no symmetry breaking. Continuous phase transition with non Heisenberg critical exponents $\eta \sim 0.8 - 0.9$

- Does this survive to 4D/5D ?
- Can one gauge $SO(4)$ and dispense with four fermi ?
- Can this mechanism be used to revisit the problem of constructing chiral lattice gauge theories ?

Backup: Hedgehogs and Hopf defects

$$\phi^a_{\text{hedgehog}} = vf(r)\hat{x}_a$$

where $\hat{x}_a = x_a/r$ and

$$f(r) \to 0 \quad r \to 0$$
$$f(r) \to 1 \quad r \to \infty$$

$$\phi_{\text{hopf}} = vf(r) \begin{pmatrix} 2\left(\hat{x}_1\hat{x}_3 + \hat{x}_2\hat{x}_4\right) \\ 2\left(\hat{x}_2\hat{x}_3 - \hat{x}_1\hat{x}_4\right) \\ \hat{x}_1^2 + \hat{x}_2^2 - \hat{x}_3^2 - \hat{x}_4^2 \end{pmatrix} \tag{1}$$

In both cases $\phi^a\phi^a = v^2$ as $r \to \infty$.