

Convergence and scatter of CDM halo density profiles

with Ben Moore, Joachim Stadel (Uni Zürich),
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method: n-body re-simulations of CDM halos

refined initial conditions (GRAFICS, Bertschinger 2001)

evolve from $z \sim 50$ to present with PKDGRAV (Stadel 2001)

small sample sizes, results depend on halo selection

no hydro; no stars, no galaxies ...

much higher resolution

(relative to uniform resolution CDM cubes and halos with hydro)

ideal for convergence tests

no hydro; no recipes for sub-grid physics, no free parameters, no overcooling, ...

our latest run: “Via Lactea”

a Milky Way halo simulated with over 200 million particles

➤

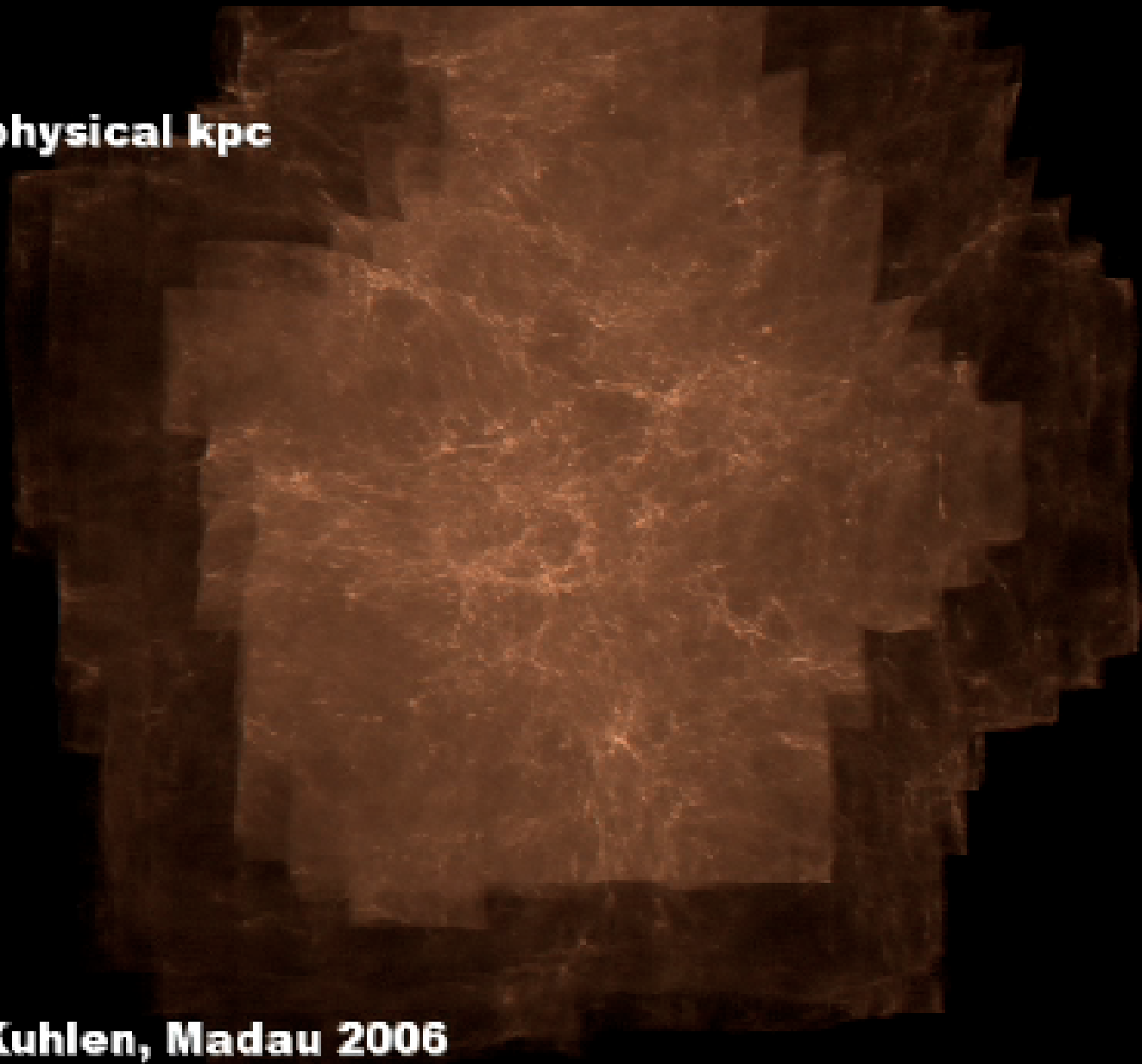
- largest DM simulation to date at these scales.
320,000 cpu-hours on NASA's Project Columbia supercomputer.



- 213,217,920 high resolution particles, embedded in a periodic 90 Mpc box sampled at lower resolution to account for tidal field.
- WMAP (year 3) cosmology:
 $\Omega_m=0.238$, $\Omega_L=0.762$, $H_0=73$ km/s/Mpc, $n_s=0.951$, $\sigma_8=0.74$.
- force resolution: 90 parsec
- mass resolution: $20,900 M_\odot$

$z=11.9$

800 x 600 physical kpc



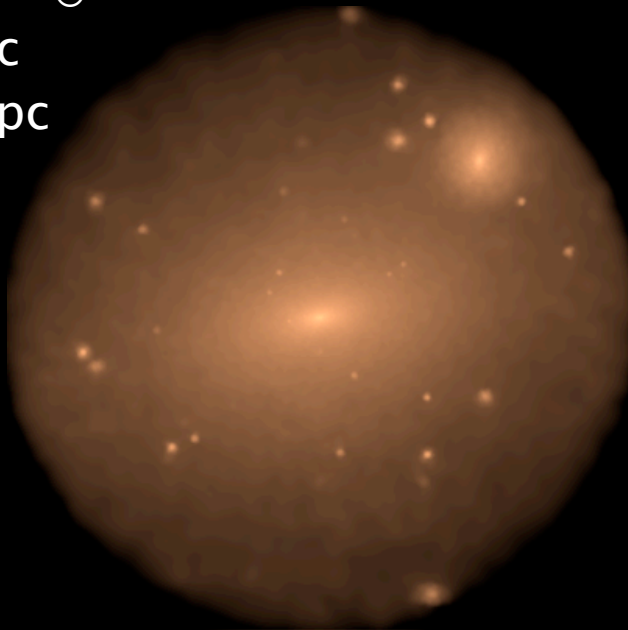
Diemand, Kuhlen, Madau 2006

Sub-Subhalos in all well resolved subhalos

$M_{\text{sub}} = 9.8 \cdot 10^9 M_{\odot}$

$r_{\text{tidal}} = 40.1 \text{ kpc}$

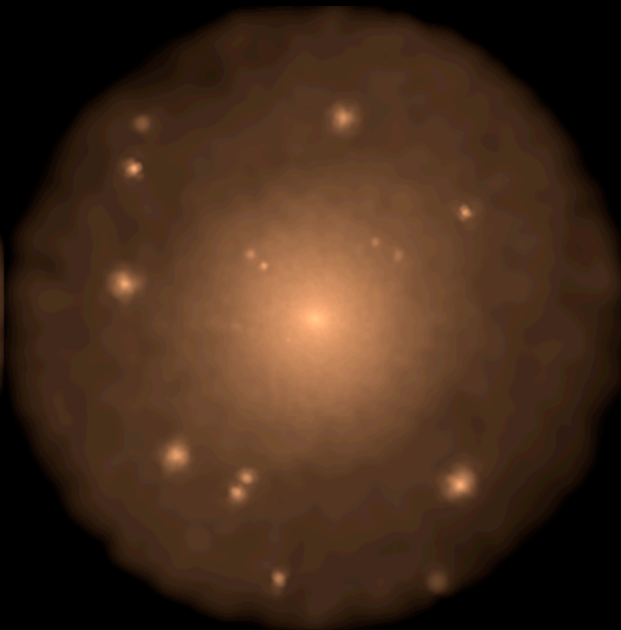
$D_{\text{center}} = 345 \text{ kpc}$



$M_{\text{sub}} = 3.7 \cdot 10^9 M_{\odot}$

$r_{\text{tidal}} = 33.4 \text{ kpc}$

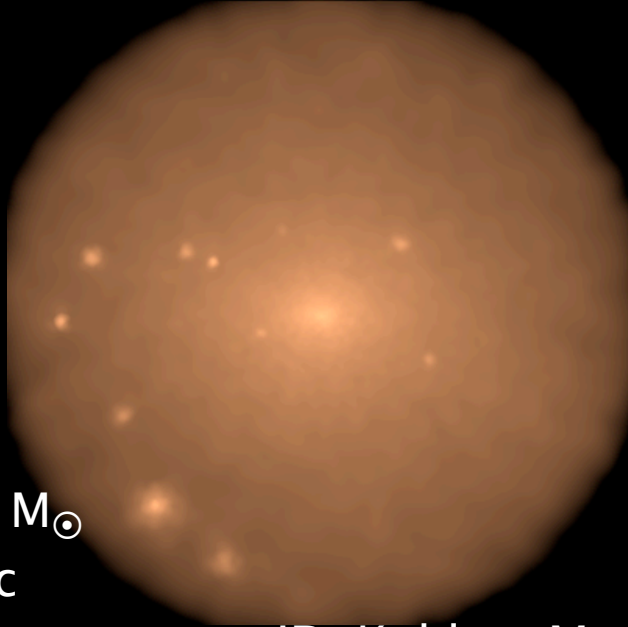
$D_{\text{center}} = 374 \text{ kpc}$



$M_{\text{sub}} = 2.4 \cdot 10^9 M_{\odot}$

$r_{\text{tidal}} = 14.7 \text{ kpc}$

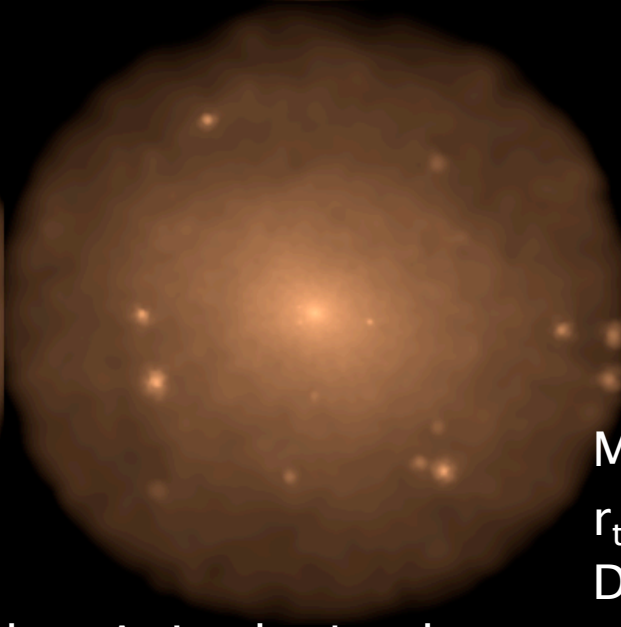
$D_{\text{center}} = 185 \text{ kpc}$



$M_{\text{sub}} = 3.0 \cdot 10^9 M_{\odot}$

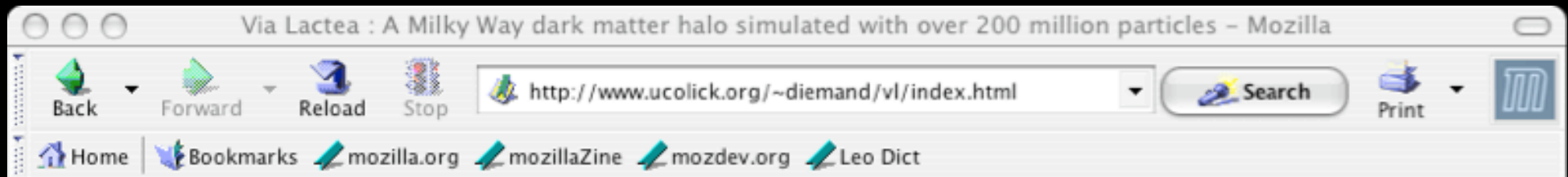
$r_{\text{tidal}} = 28.0 \text{ kpc}$

$D_{\text{center}} = 280 \text{ kpc}$



JD, Kuhlen, Madau, ApJ submitted

www.ucolick.org/~diemand/vl



via lactea

A Milky Way dark matter halo simulated with 234 million particles on NASA's [Project Columbia](#) supercomputer

[main](#)

[movies](#)

[images](#)

[publications](#)

data (subhalo properties, histories etc. will be available soon. Please contact diemand 'at' ucolick.org with requests and suggestions)

Simulation description

The simulation was performed with PKDGRAV (principal author [Joachim Stadel](#); Stadel, J. 2001, PhD thesis, U. Washington) and employed multiple mass particle grid initial conditions generated with the [GRAFICS](#) package (Bertschinger, E. 2001, ApJSS, 137, 1). The high resolution region is embedded within a periodic box of comoving size 90 Mpc, which is sampled at lower resolution to account for the large scale tidal forces.

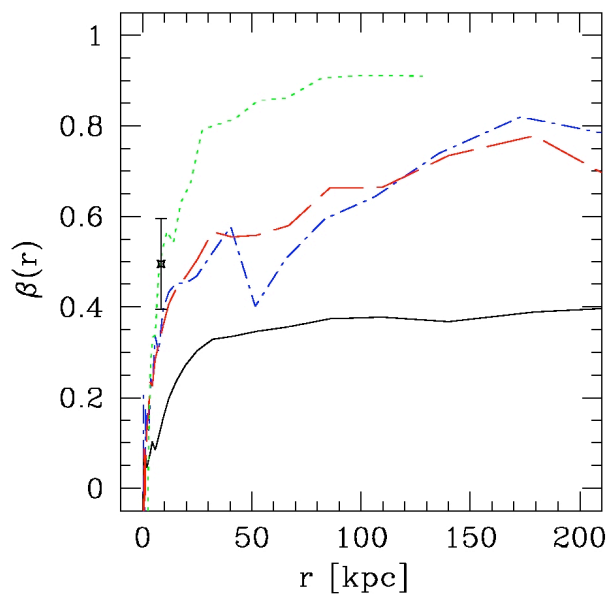
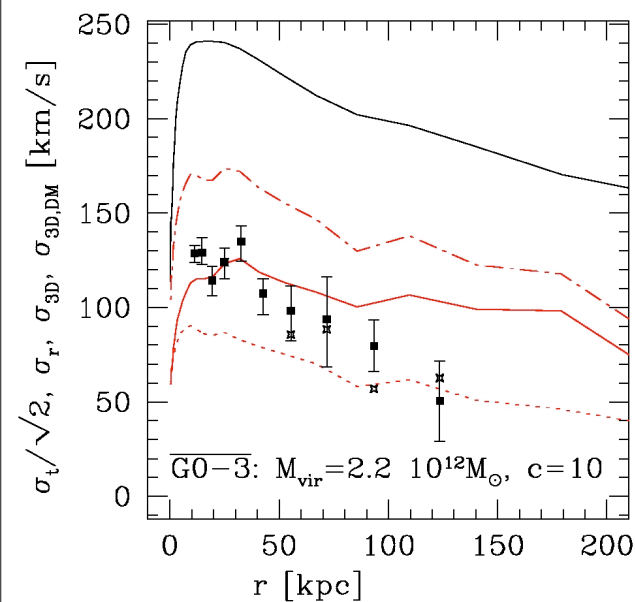
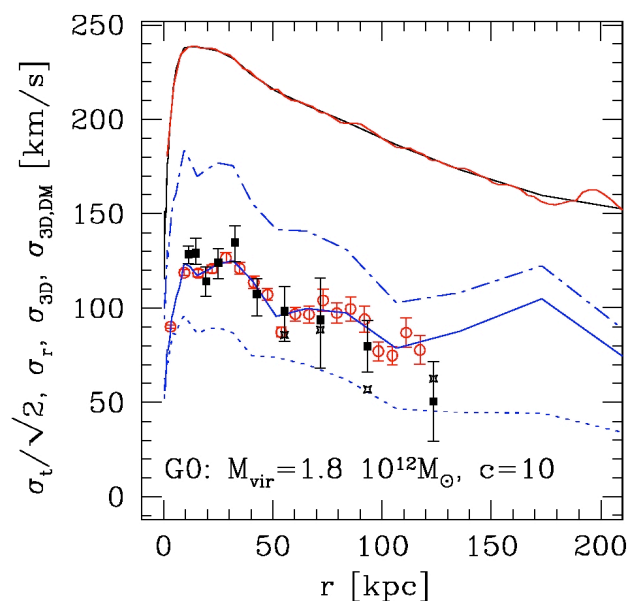
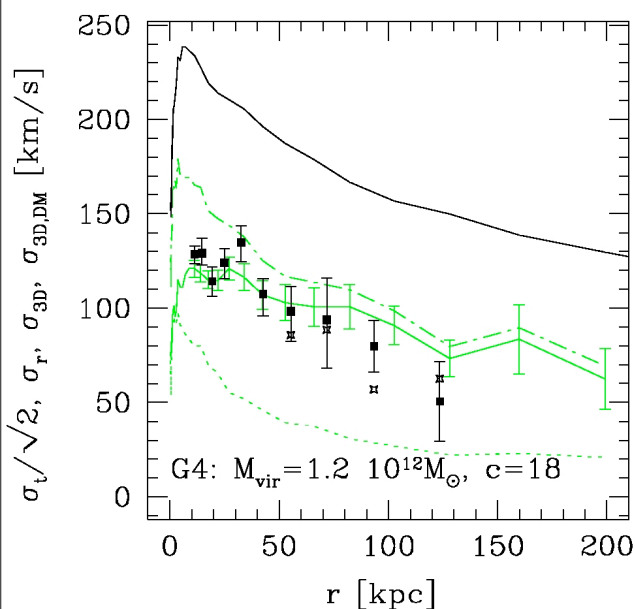
We adopt the best-fit cosmological parameters from the [WMAP](#) three-year data release ([Spergel, D. N. et al. 2006](#)): $\Omega_M = 0.238$, $\Omega_\Lambda = 0.762$, $H_0 = 73$ km/s/Mpc, $n = 0.951$, and $\sigma_8 = 0.74$.

The high resolution region was centered on a isolated halo that had no major merger after $z = 1.7$, making it a suitable host for a Milky Way-like disk galaxy. Its mass at $z = 0$ is $M_{vir} = 1.77e12$ solar masses within a radius of $R_{vir} = 389$ kpc (M_{vir} is the radius within which the enclosed average density is 200 times the mean matter value).

The "Via Lactea" run features adaptive time-steps as short as the age of the universe divided by 200'000, which is about 68'500 years, a force resolution of 90 pc and particle masses of 20'900 solar masses.

Last Updated: September 15, 2006, by J. Diemand

Milky Way halo mass form stellar halo radial velocities?



cosmological stellar halo kinematics fit the observations well

The outer halo and therefore the virial mass are not well constrained

low M_{vir} / high c
high M_{vir} / low c
both possible

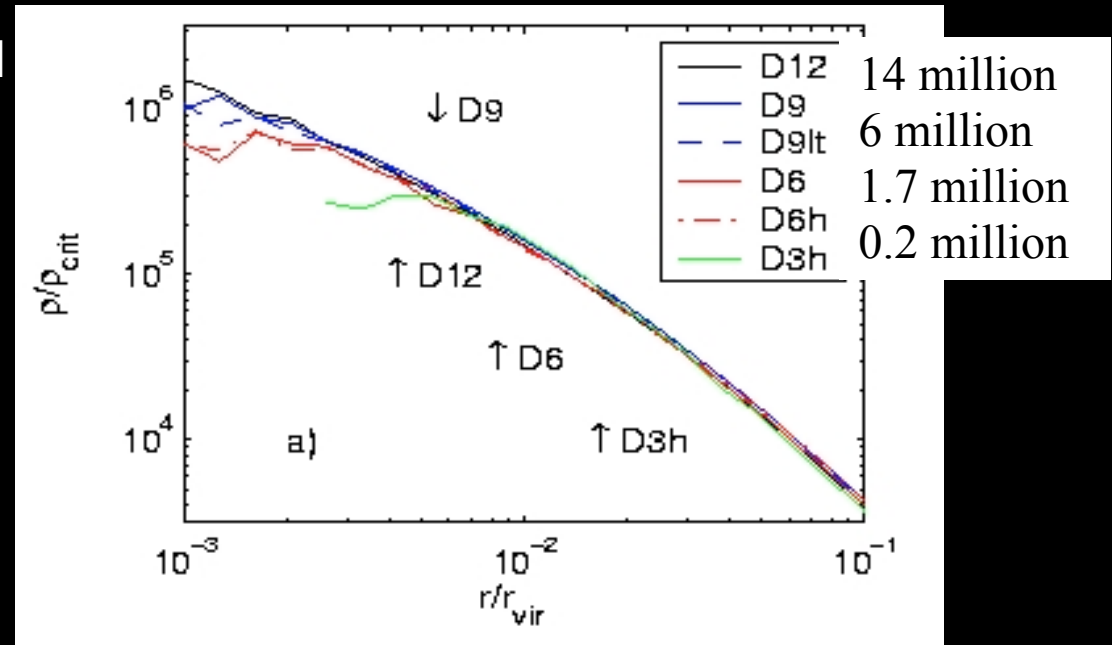
$\beta(r)$ follows relates to tracer profile slope as in Hansen&Moore, 2004

JD, Madau, Moore 2005

numerical convergence of density profiles

eg. Moore et al 1998, Klypin et al 2001, Power et al 2003, Fukushige et al 2004, JD et al 2004

- 1) convergence radius ~ 3 force softening lengths
- 2) Numerical flattening due to two body relaxation:
slow convergence, $r \sim N^{-1/3}$
1 million to resolve 1% of R_{virial}
1000 to resolve 10%



JD, Moore, Stadel, MNRAS, 2004, 353, 624

numerical convergence of density profiles

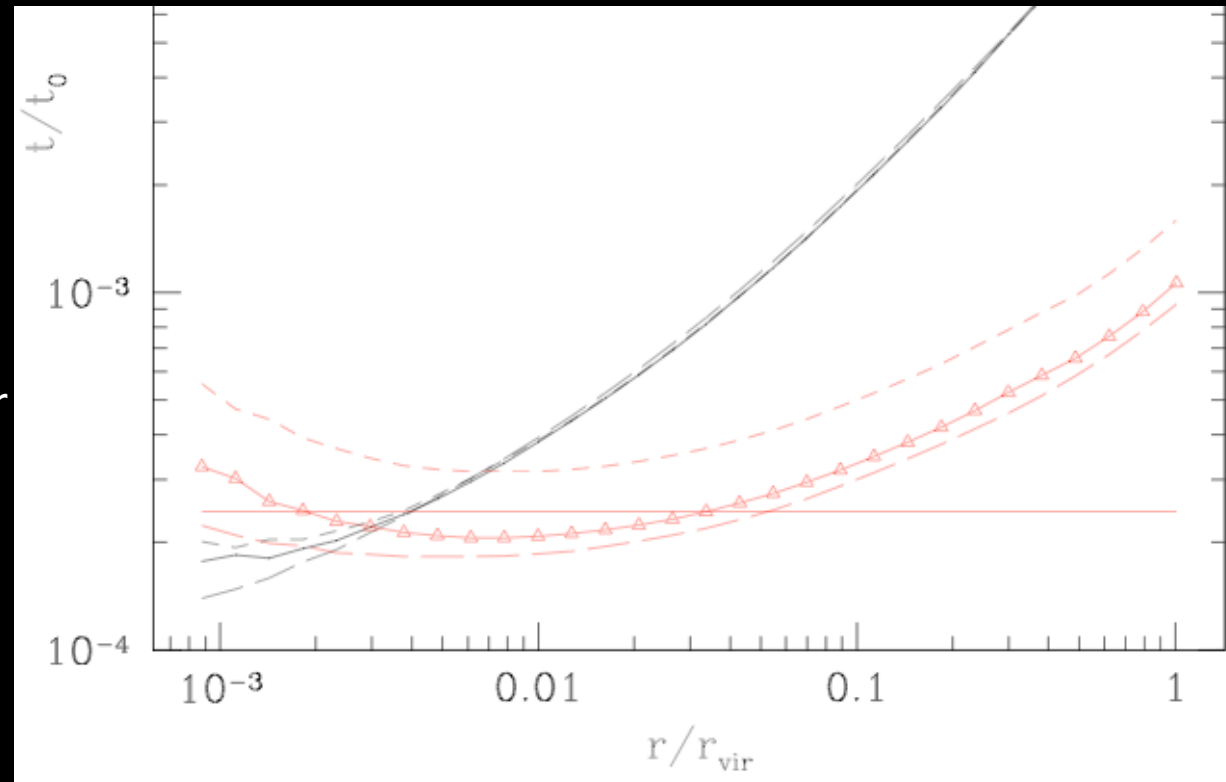
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- 1) convergence radius ~ 3 force softening lengths
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1 million to resolve 1% of R_{virial}
1000 to resolve 10%
- 3) about 15 time-steps per local dynamical time

Note: the empirical criterion

$$\eta \sqrt{\epsilon(z)/a} \quad \text{scales much slower}$$

with radius than the local dynamical time



numerical convergence of density profiles

eg. Moore et al 1998, Klypin et al 2001, Power et al 2003, Fukushige et al 2004, JD et al 2004

- 1) convergence radius is limited to at least 2 to 3 force softening lengths
- 2) Numerical flattening due to two body relaxation:
slow convergence, $r \sim N^{-1/3}$
1 million to resolve 1% of R_{virial}
1000 to resolve 10%
- 3) about 15 time-steps per
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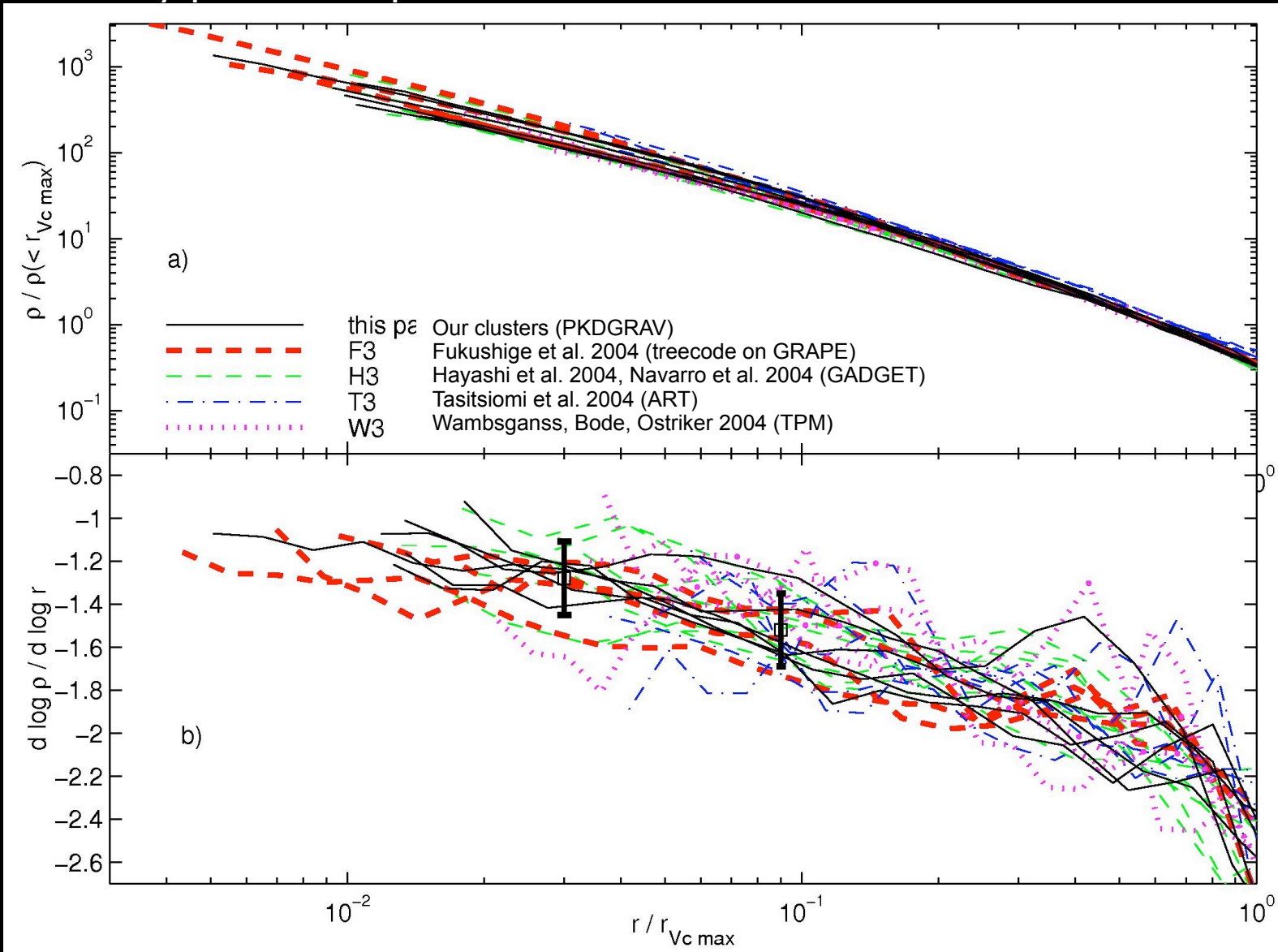
with radius than the local
dynamical time

- caution:
- a) effects of finite force, mass and time resolution are not independent
 - b) different codes might have different requirements
 - c) “convergence radius” is the largest of the the radii above
at r_{conv} the error in local density should be $< 10\%$
larger errors in M_{encl} , v_c , slopes. subhalo abundance, shape, ...

scatter in CDM cluster density profiles

eg. Fukushige et al 2004, Navarro et al 2004, JD et al 2004

CDM density profiles are close to universal (e.g. NFW), but individual halo density profile shapes have scatter:



JD, Moore, Stadel,
MNRAS, 2004

scatter in CDM cluster density profiles

	$1\%r_{\text{vir}}$	$3\%r_{\text{vir}}$	$3\%r_{\text{Vcmax}}$	$9\%r_{\text{Vcmax}}$
<i>A9</i>	1.22	1.36	1.24	1.64
<i>B9</i>	1.33	1.43	1.21	1.63
<i>C9</i>	1.24	1.21	1.25	1.26
<i>D12</i>	1.28	1.54	1.32	1.58
<i>E9</i>	1.31	1.44	1.41	1.62
<i>F9cm</i>	1.19	1.47	1.22	1.43
a) A-F	1.26 ± 0.05	1.41 ± 0.11	1.28 ± 0.08	1.53 ± 0.15
b) F03	1.25 ± 0.05	1.52 ± 0.06	1.33 ± 0.15	1.54 ± 0.15
c) H03	1.18 ± 0.13	1.38 ± 0.14	1.23 ± 0.17	1.50 ± 0.14
d) T03	1.50 ± 0.14	1.79 ± 0.07	—	1.56 ± 0.12
e) W03	1.11 ± 0.04	1.41 ± 0.13	—	1.35 ± 0.06
avg. (a-e)	1.26	1.50	—	1.49
avg. (a-c)	1.23	1.44	1.28	1.52
NFW			1.12	1.32
Moore et al.			1.54	1.65

JD, Moore, Stadel,
MNRAS, 2004, 353, 624

why are profiles nearly universal? what causes the scatter?

fitting functions

2 parameter functions (only two 'scaling' parameters):

NFW

$$\rho = \frac{\rho_s}{x(1+x)^2}$$

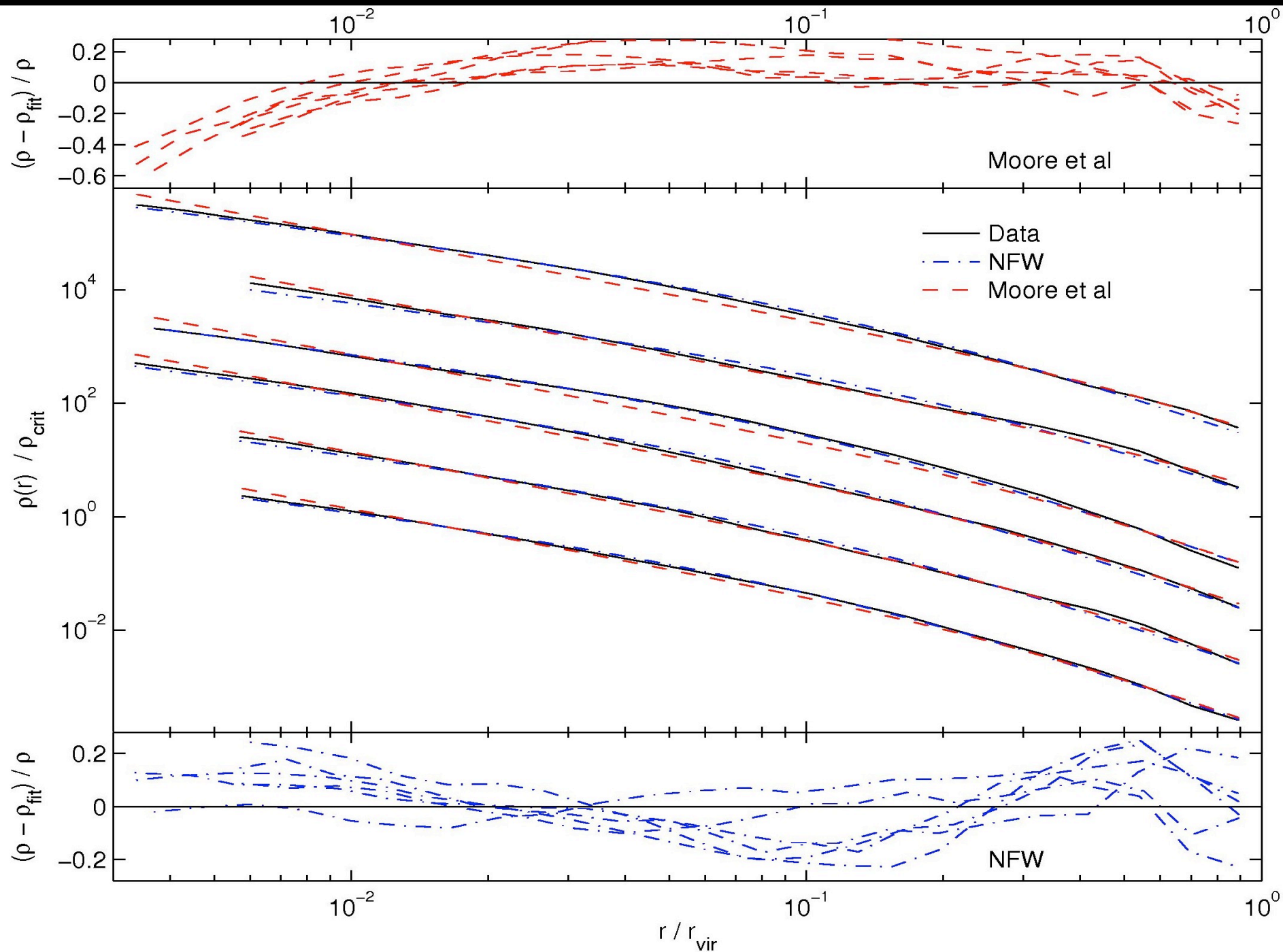
Moore et al 1999

$$\rho = \frac{\rho_s}{x^{1.5}(1+x)^{1.5}}$$

$$x = r/r_s$$

2 parameter functions (only two 'scaling' parameters):

JD, Moore, Stadel,
MNRAS, 2004



more fitting functions

2 parameter functions (only two 'scaling' parameters):

NFW

$$\rho = \frac{\rho_s}{x(1+x)^2}$$

$$x = r/r_s$$

Moore et al 1999

$$\rho = \frac{\rho_s}{x^{1.5}(1+x)^{1.5}}$$

3 parameter functions (one additional 'profile shape' parameter):

gamma model (cusp)

JD, Moore, Stadel, 2004

$$\rho_G(r) = \frac{\rho_s}{(r/r_s)^\gamma (1 + (r/r_s)^\alpha)^{(\beta-\gamma)/\alpha}}$$

$$\alpha = 1, \beta = 3$$

Sersic/Einasto (core)

Navarro etal 2004

Merrit etal 2005/2006

$$\rho(r) = \rho_e \exp \left\{ -d_n \left[(r/r_e)^{1/n} - 1 \right] \right\}$$

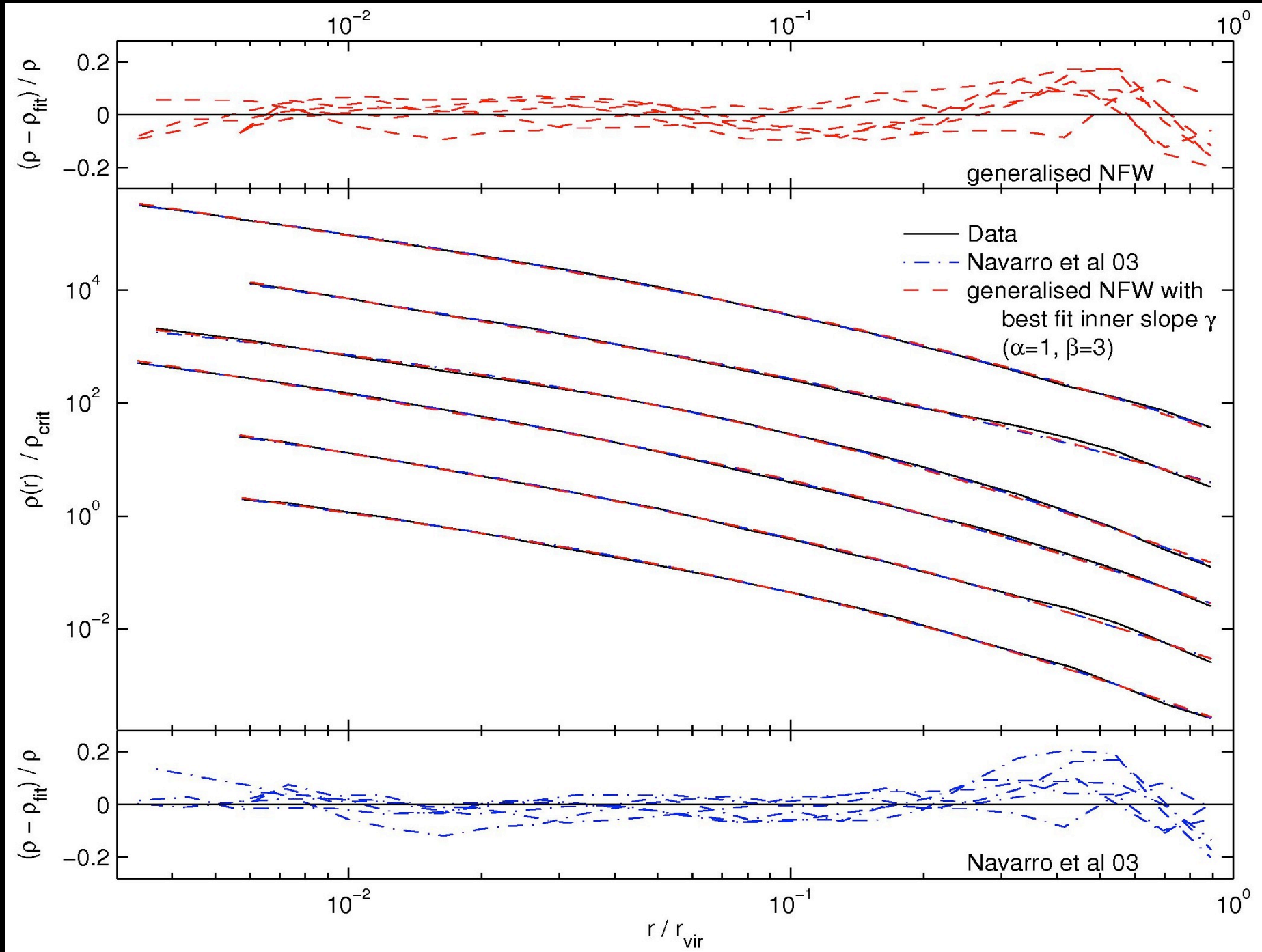
Prugniel-Simien (deprojected Sersic)

Merritt, Navarro, Ludlow, Jenkins, 2005

Merritt, Graham, Moore, JD, Terzic, 2006

Graham etal 2006

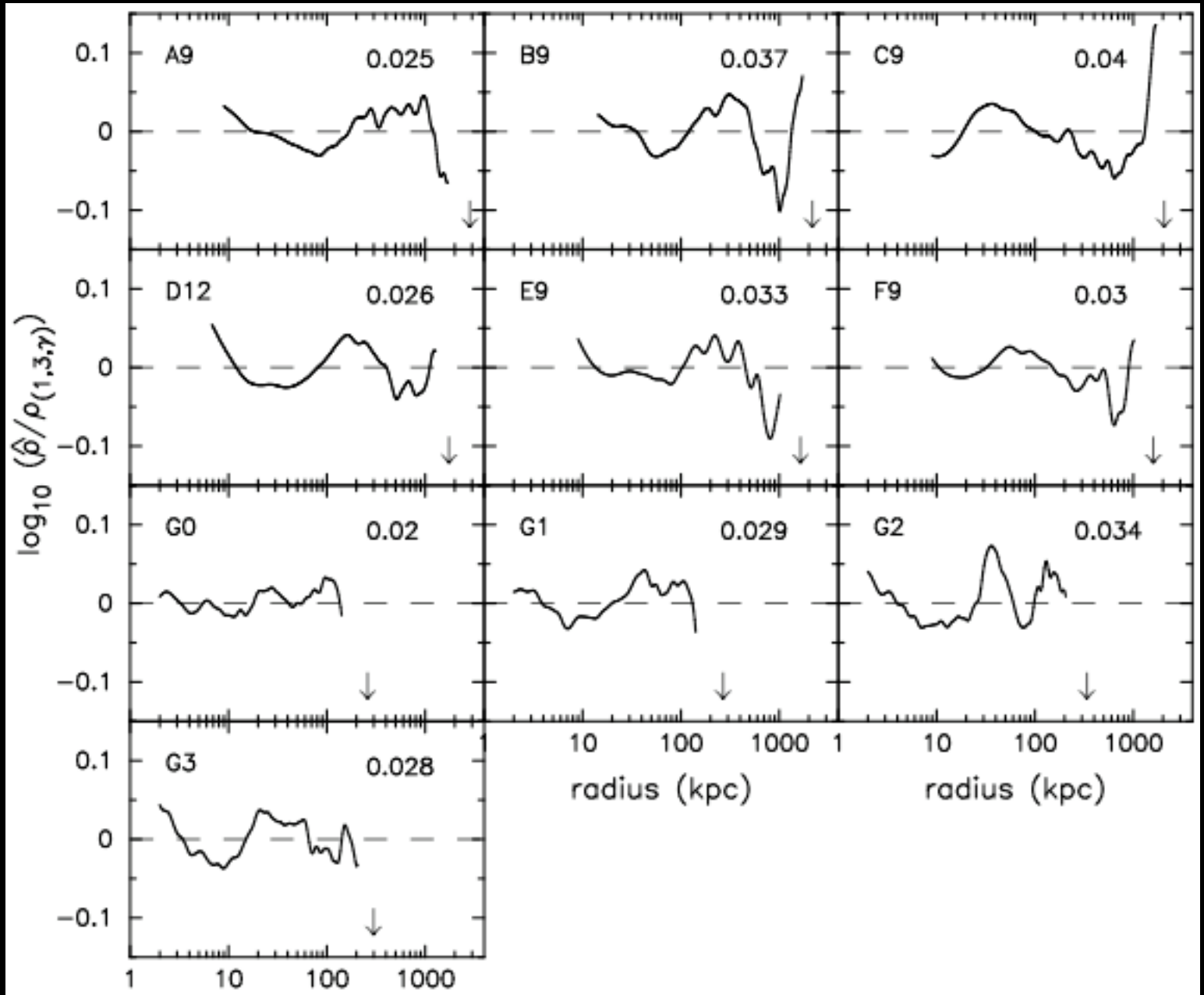
$$\rho(r) = \rho' \left(\frac{r}{R_e} \right)^{-p} \exp \left[-b (r/R_e)^{1/n} \right]$$



3 parameter functions (one additional 'profile shape' parameter):

gamma-model

fitted to
non-parametric
density profiles



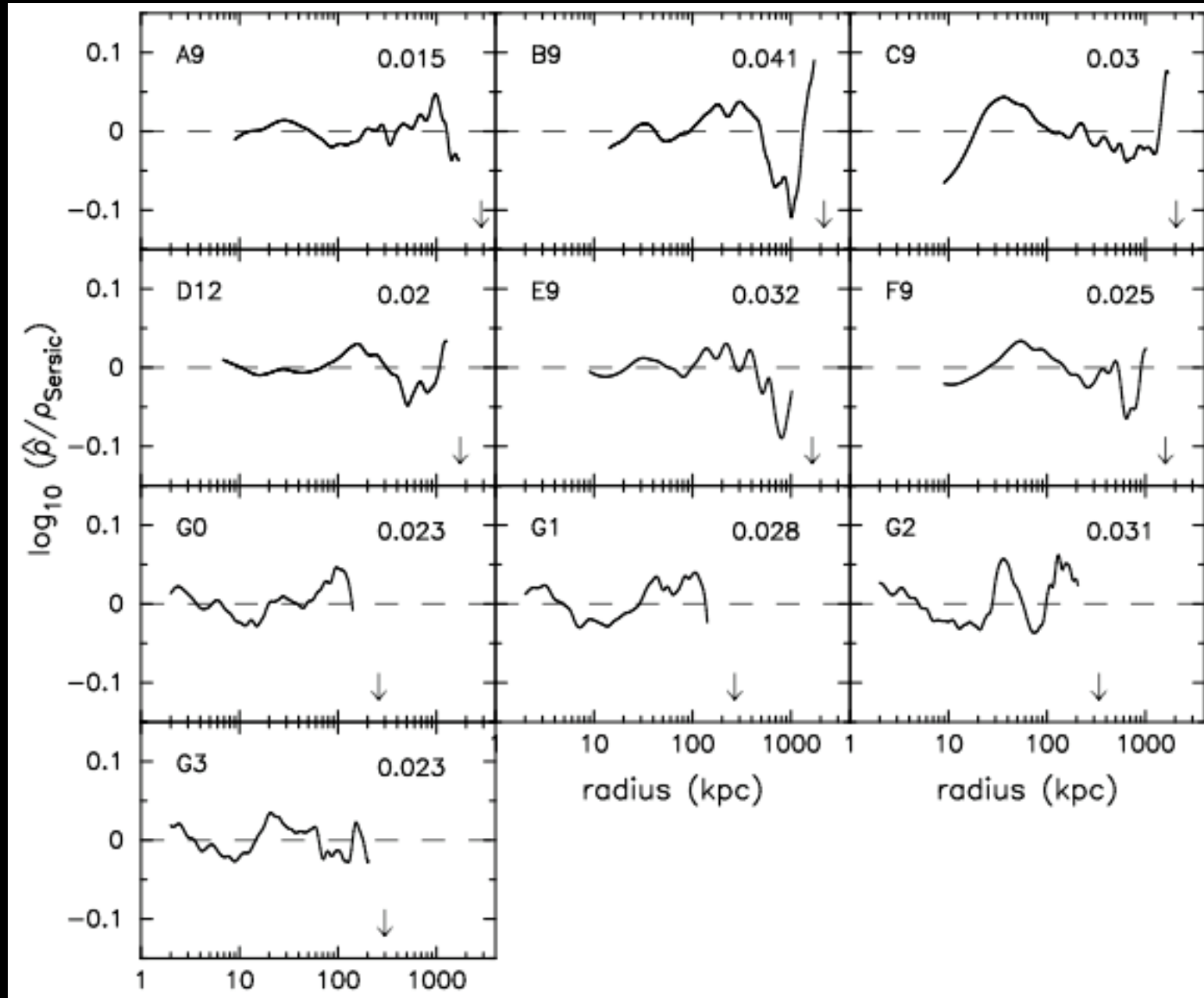
3 parameter functions (one additional 'profile shape' parameter):

Sersic-model

rms deviations
are often
smaller than
for the
gamma-model

both have largest
deviations in the
outer halo

which one fits the
inner halo better?



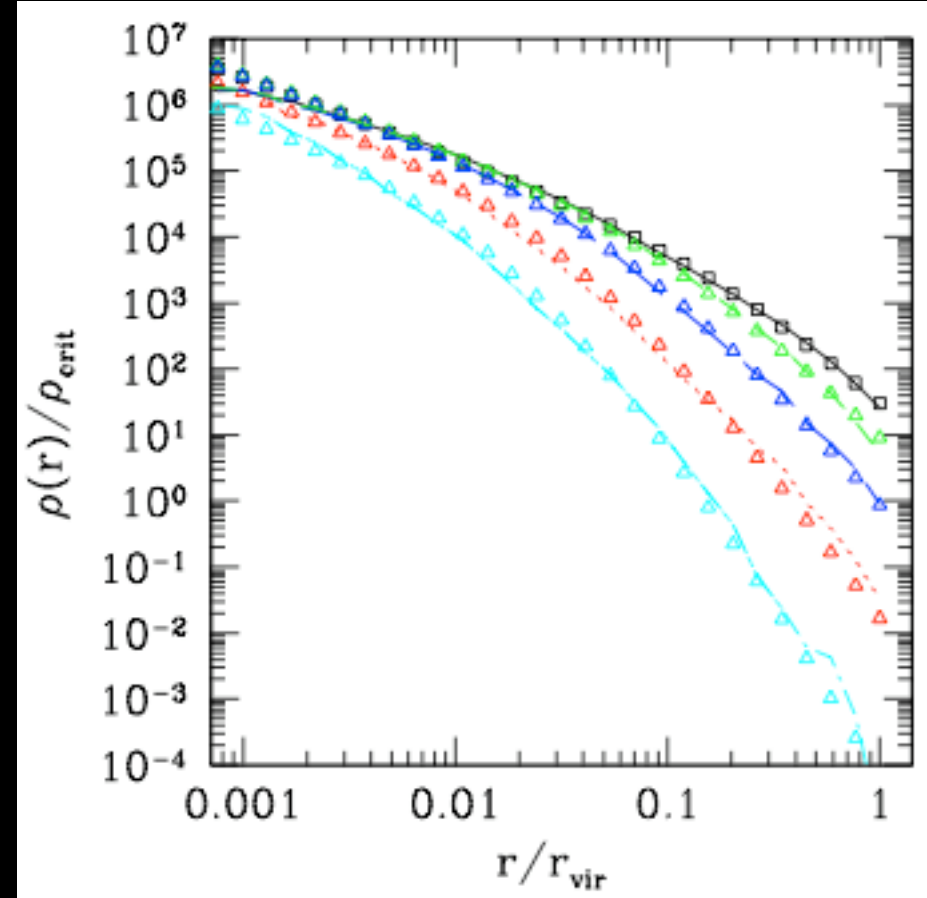
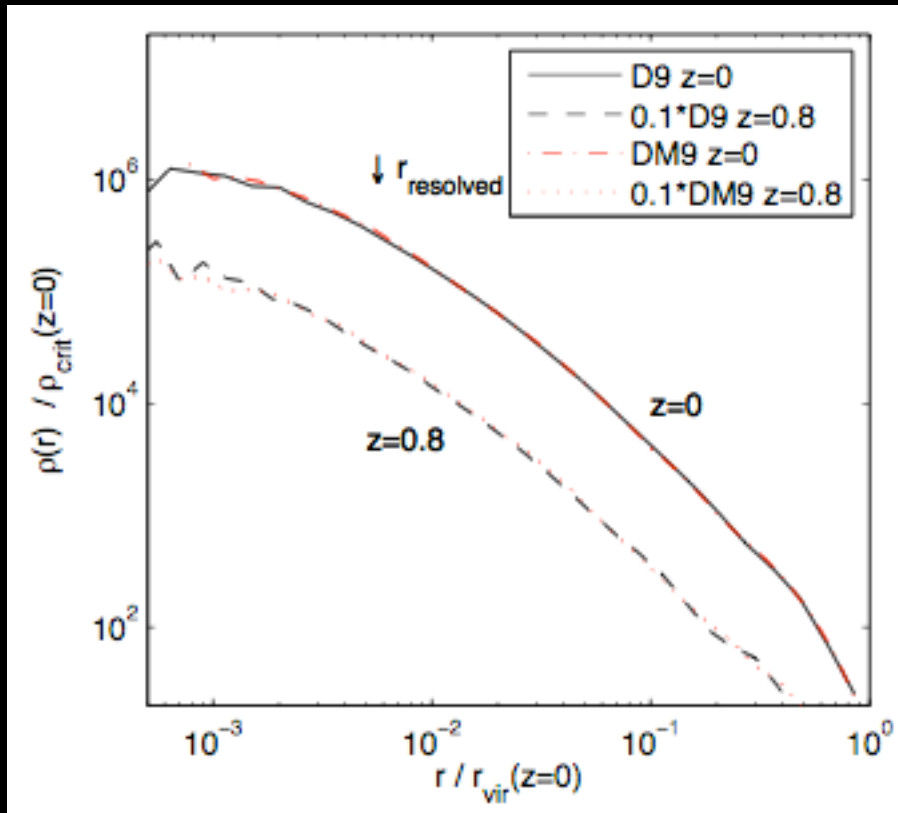
resolving the very inner profile

JD, Zemp, Moore, Stadel, Carollo,
MNRAS, 2005, 364, 665

Multi-mass technique:

inner profile is dominated by material
from rare > 2 sigma peaks
(JD, Madau, Moore 2005)

sufficient to sample these regions at the
highest mass resolution



same density profiles with both
 $N_{\text{vir}} = 6$ million and $N_{\text{vir, effective}} = 6$ million
the later takes 10 times less CPU time

resolving the very inner profile

JD, Zemp, Moore, Stadel, Carollo,
MNRAS, 2005, 364, 665

physical time-steps:

the empirical $\Delta t_i < \eta \sqrt{\epsilon/a_i}$, $\eta=0.25$ is no longer sufficient

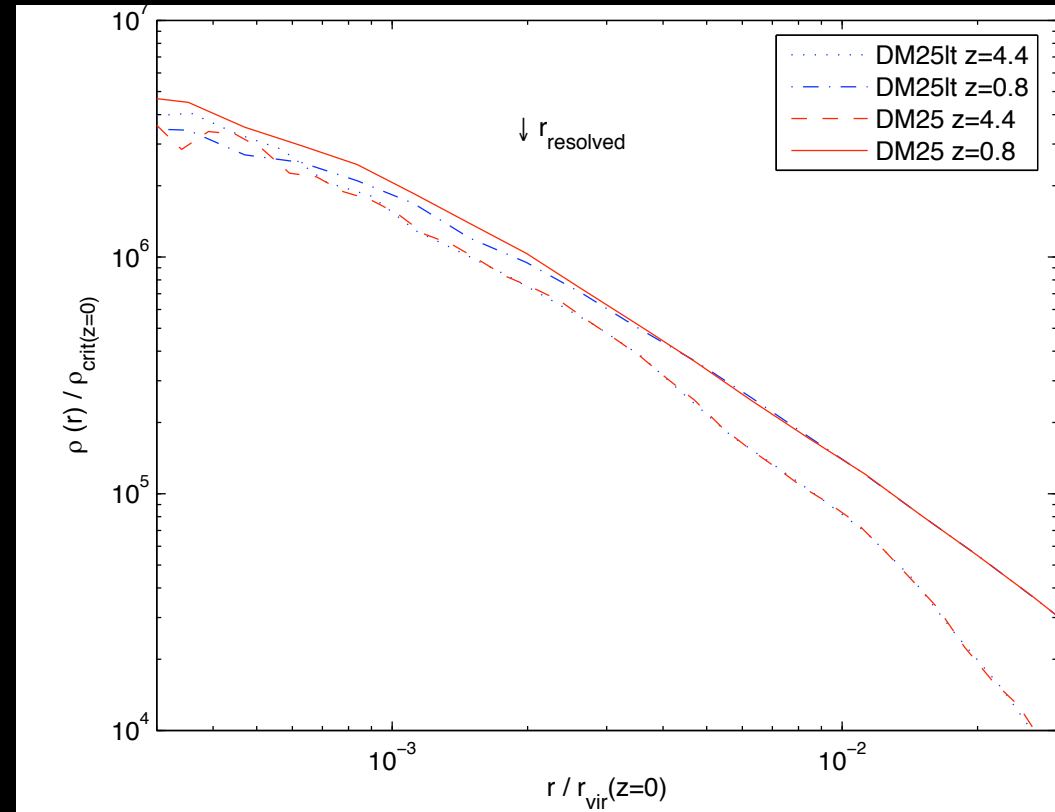
using $\Delta t < \min(\eta \sqrt{\epsilon/a_i}, \eta/4 \sqrt{G\rho_i})$ instead

this ensures step are at least 12 times smaller than the local dynamical time

$$1/\sqrt{G\rho(< r_i)}$$

but increases CPU time by a factor of two

recently Zemp, Stadel, Moore, Carollo (2006) have implemented a more efficient algorithm which scales with the local dynamical time everywhere.



resolving the very inner profile

Via Lactea run:
great for substructure, but
not for very inner profile:

resolved scale is set by the

$$\Delta t_i < \eta \sqrt{\epsilon/a_i}$$

$\eta=0.2$ time-step,
not by mass or force resolution

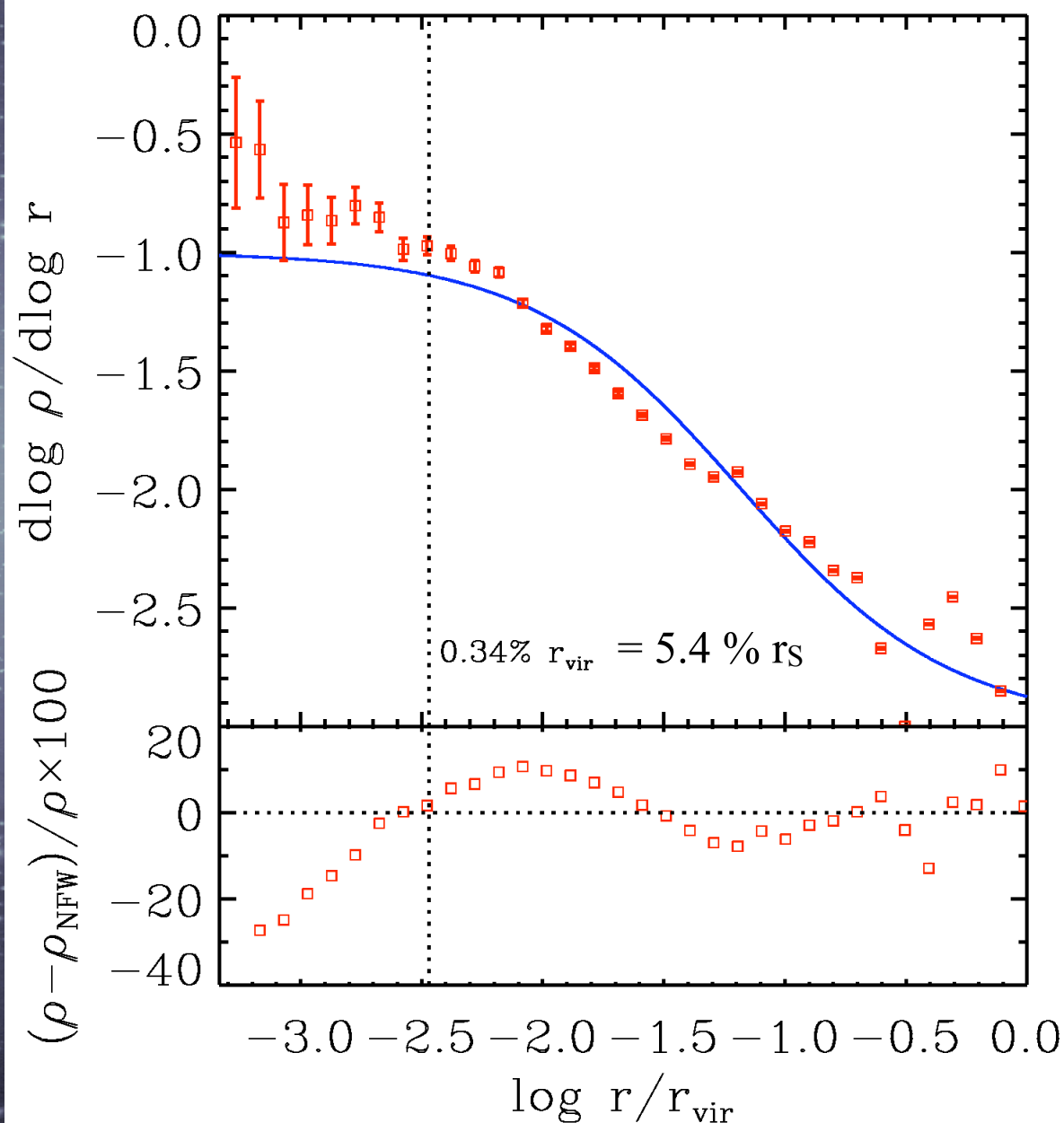
NFW fit with $c=15.8$ is passable

denser than NFW around $0.01 r_{\text{vir}}$,
but shallower below $0.8\% r_{\text{vir}}$

Caution:

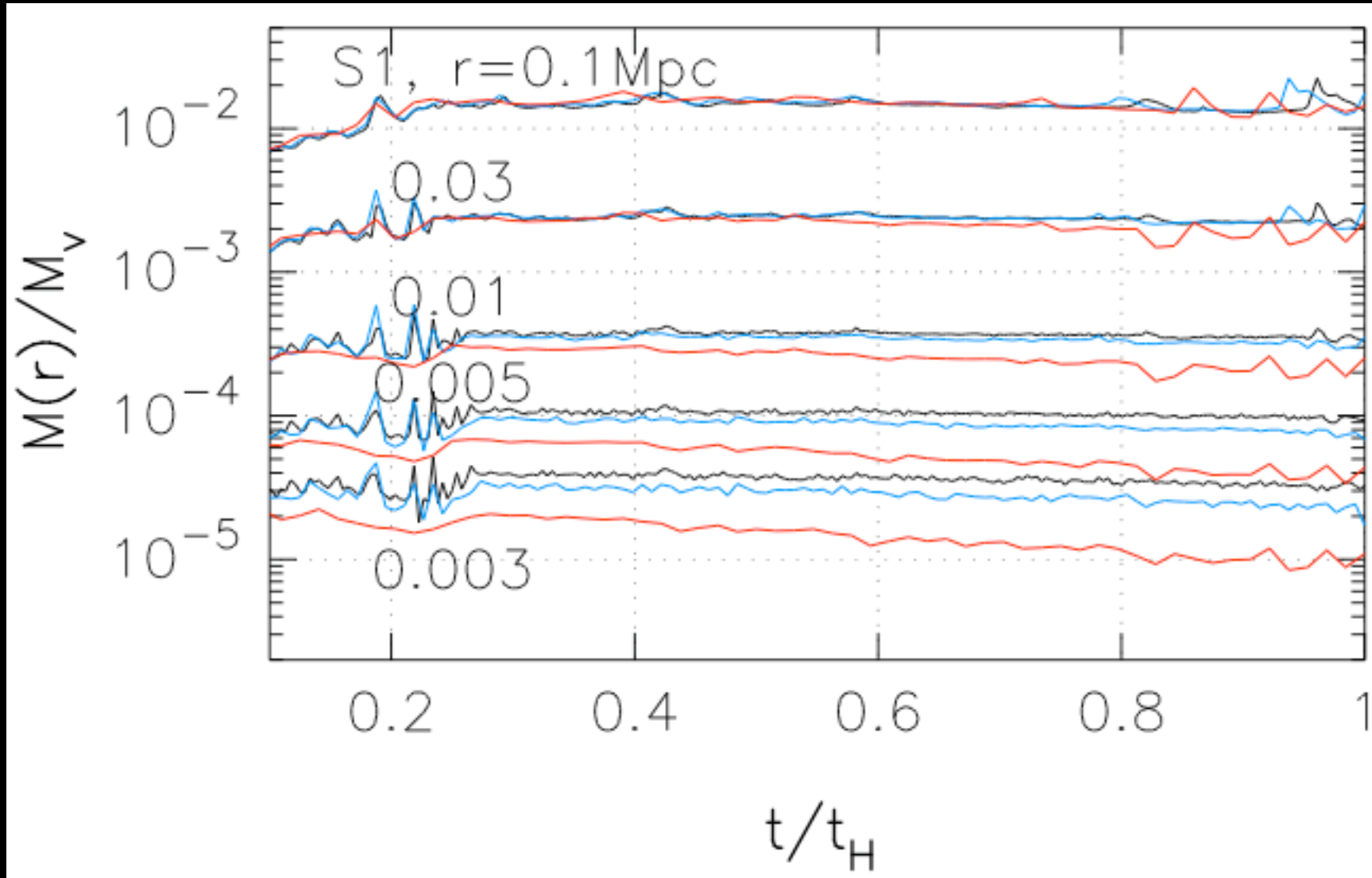
$r_{\text{converged}} = 0.34\% r_{\text{vir}}$
was estimated from cluster scale
convergence tests, requirements
for galaxy halos might differ

JD, Kuhlen, Madau, ApJ submitted



resolving the very inner profile

inner halo is assembled early:



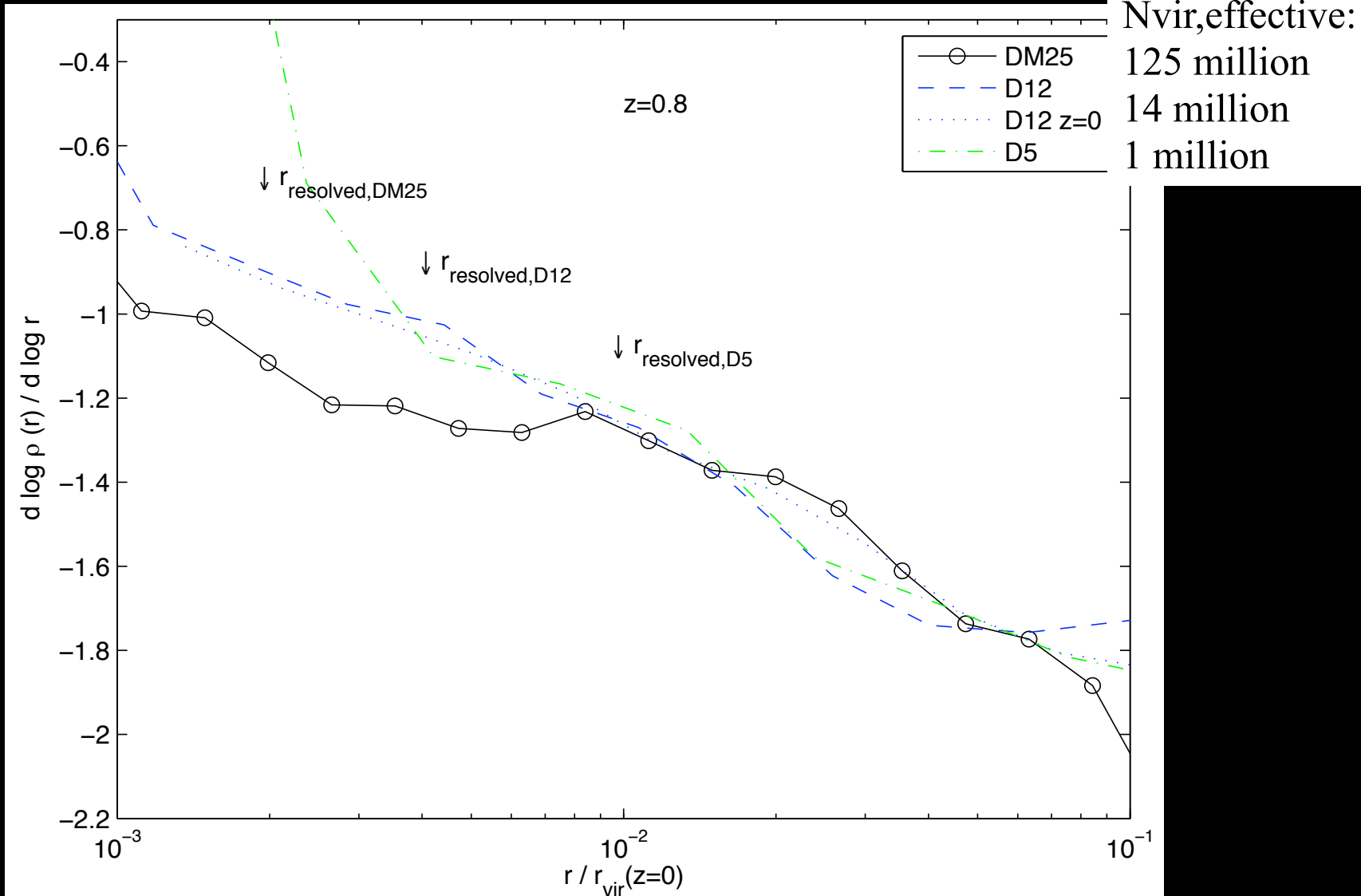
from Fukushige, Kawai, Makino, ApJ, 2004

resolving the very inner profile

JD, Zemp, Moore, Stadel, Carollo,
MNRAS, 2005, 364, 665

steeper slopes with increasing mass resolution:

the “D” cluster had an inner slope near the mean of our 6 cluster sample



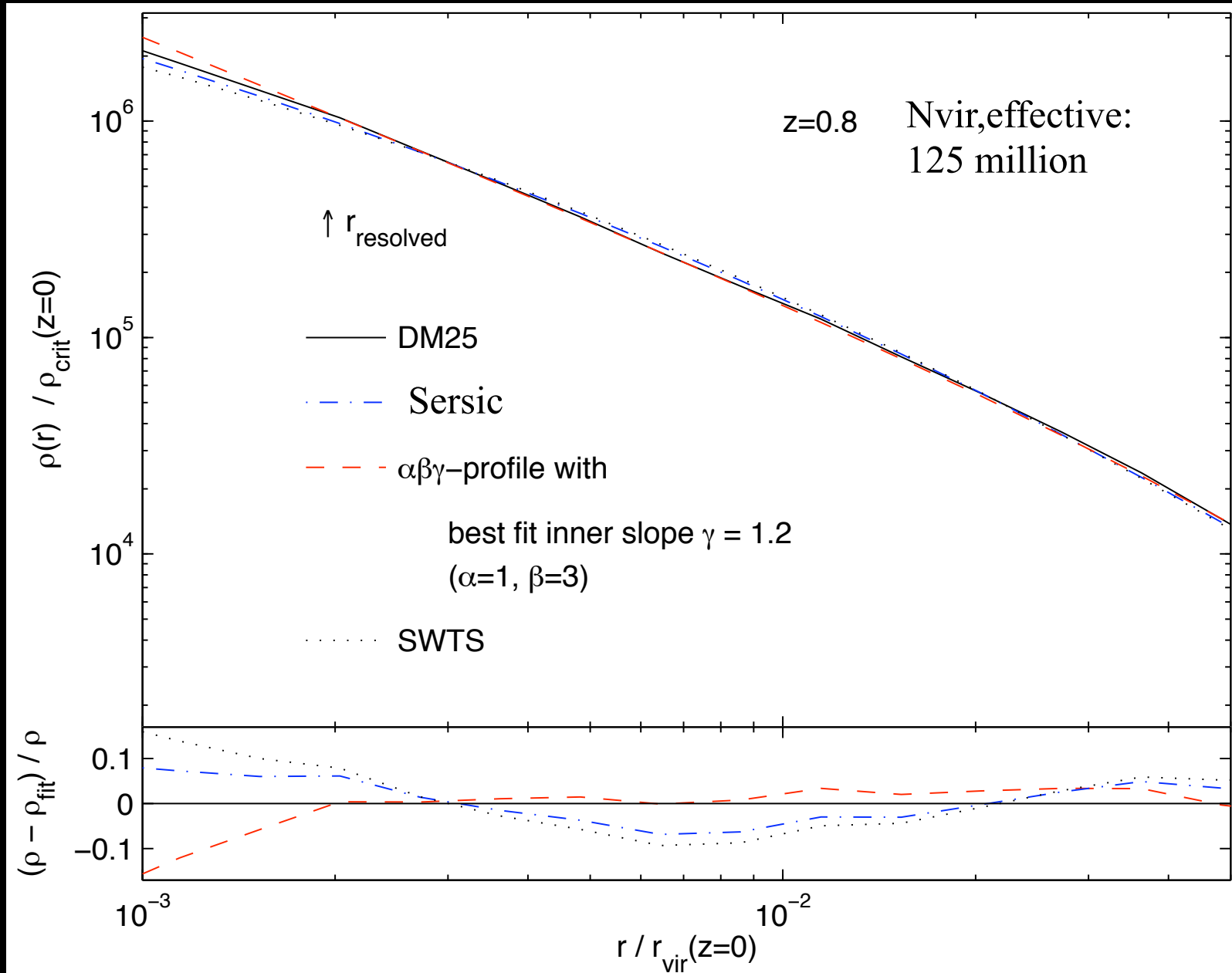
resolving the very inner profile

JD, Zemp, Moore, Stadel, Carollo,
MNRAS, 2005, 364, 665

3 parameter fitting functions

Sersic fit tends to underestimate the very inner densities

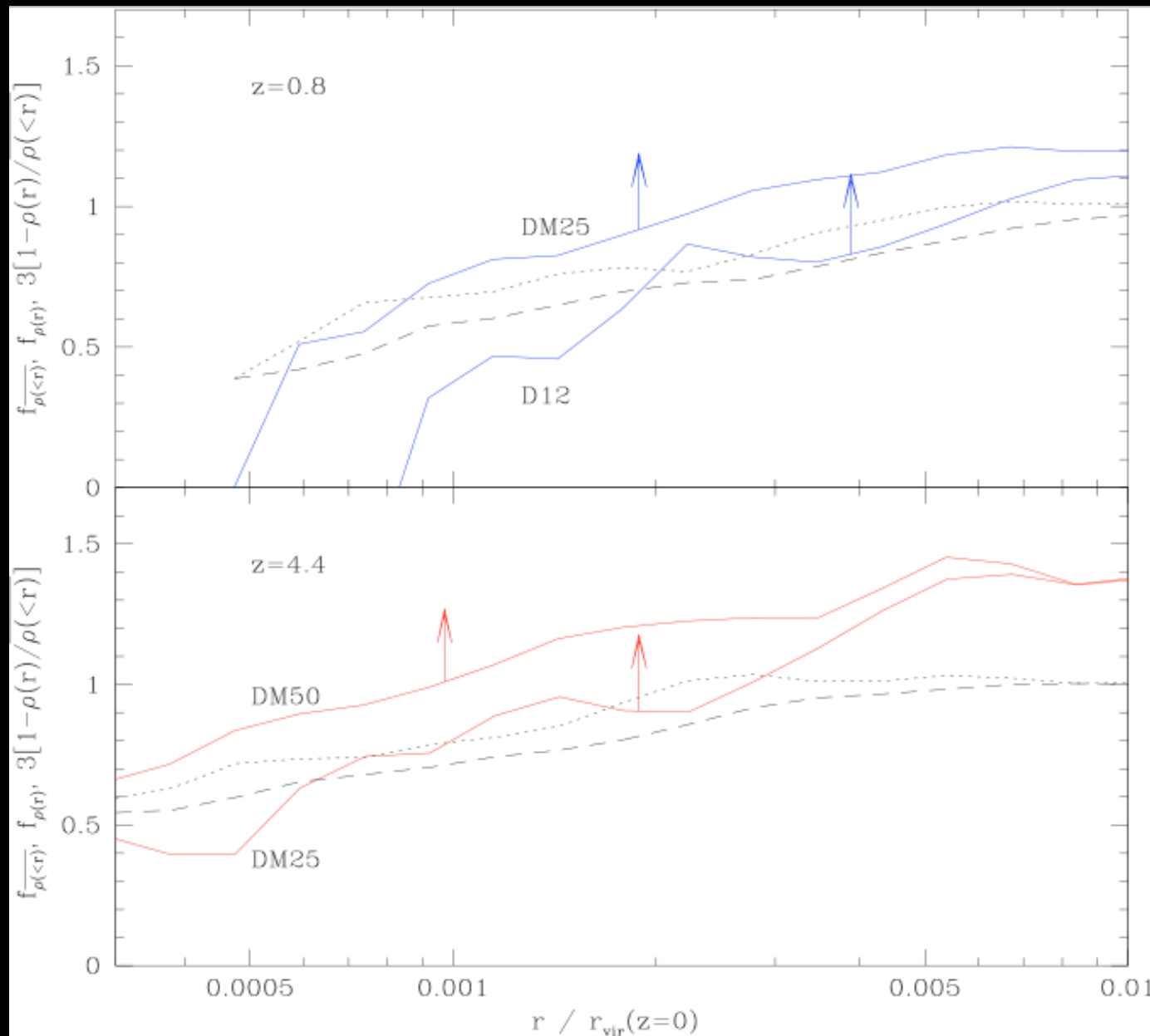
even within R_{resolved} , where the simulated densities are probably too low



resolving the very inner profile

JD, Zemp, Moore, Stadel, Carollo,
MNRAS, 2005, 364, 665

using enclosed mass to estimate the maximum inner slope?



enclosed mass converges
slower (further out) than
the local density

estimate biased low

Conclusions

- CDM density profile shapes are not exactly universal:
 - inner slopes at a give fraction of the scale radius have about 0.2 rms halo to halo scatter
 - outer slopes (near R_{vir}) are very noisy
- most halos are denser than NFW at $0.01 R_{\text{vir}}$, but not as dense as the Moore et al 1999 fits
- CDM cluster profiles resolved with around 20 million particles can be fitted equally well with a cuspy gamma-model and with the cored Sersic function
- the one halo resolved with substantially higher mass, force and time-resolution is consistent with a -1.2 cusp. its inner halo is denser than the best fit Sersic-model