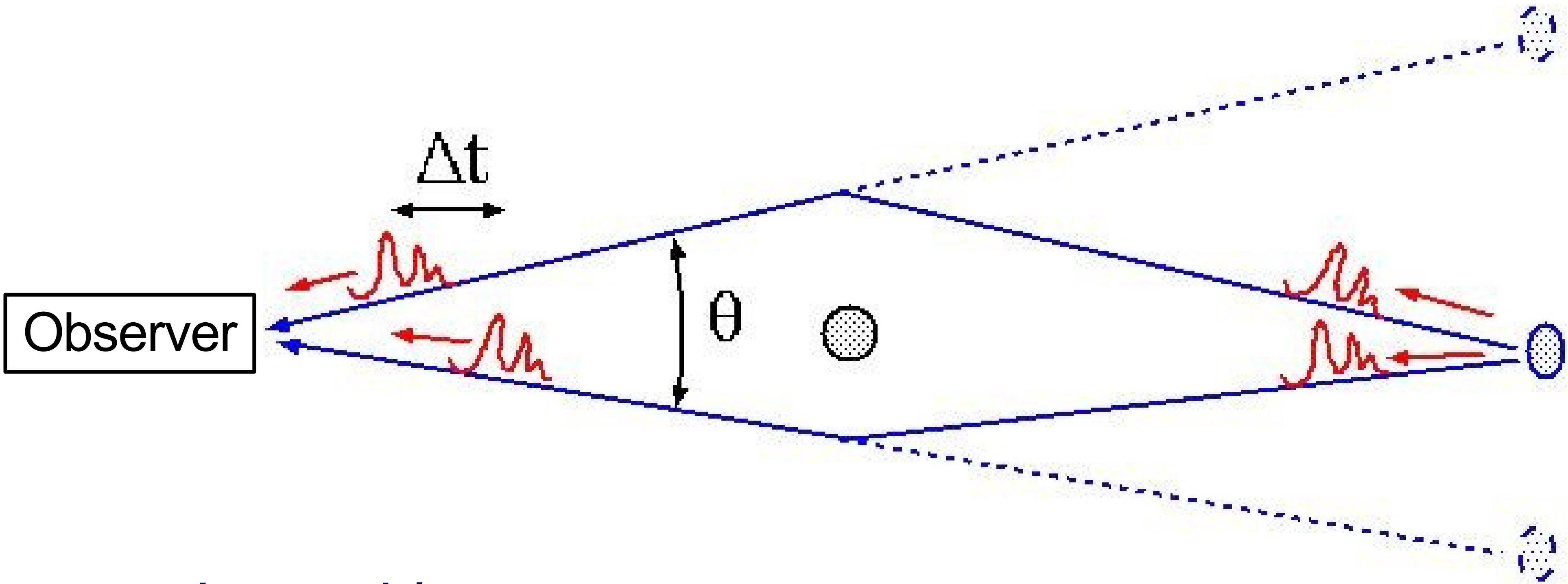
A Statistical Approach to Gravitational Lens Time Delays

(based on astro-ph/0609694)

Masamune Oguri (KIPAC, Stanford)

Time delay



observable

assume

$$\Delta t = H_0^{-1} F(z_{lens}, z_{src}, \theta_1, \theta_2, \text{mass model})$$

→ Direct determination of the Hubble constant H_0

(Refsdal 1964)

Degeneracy with mass models

For a lens potential $\varphi = r^{\alpha}$

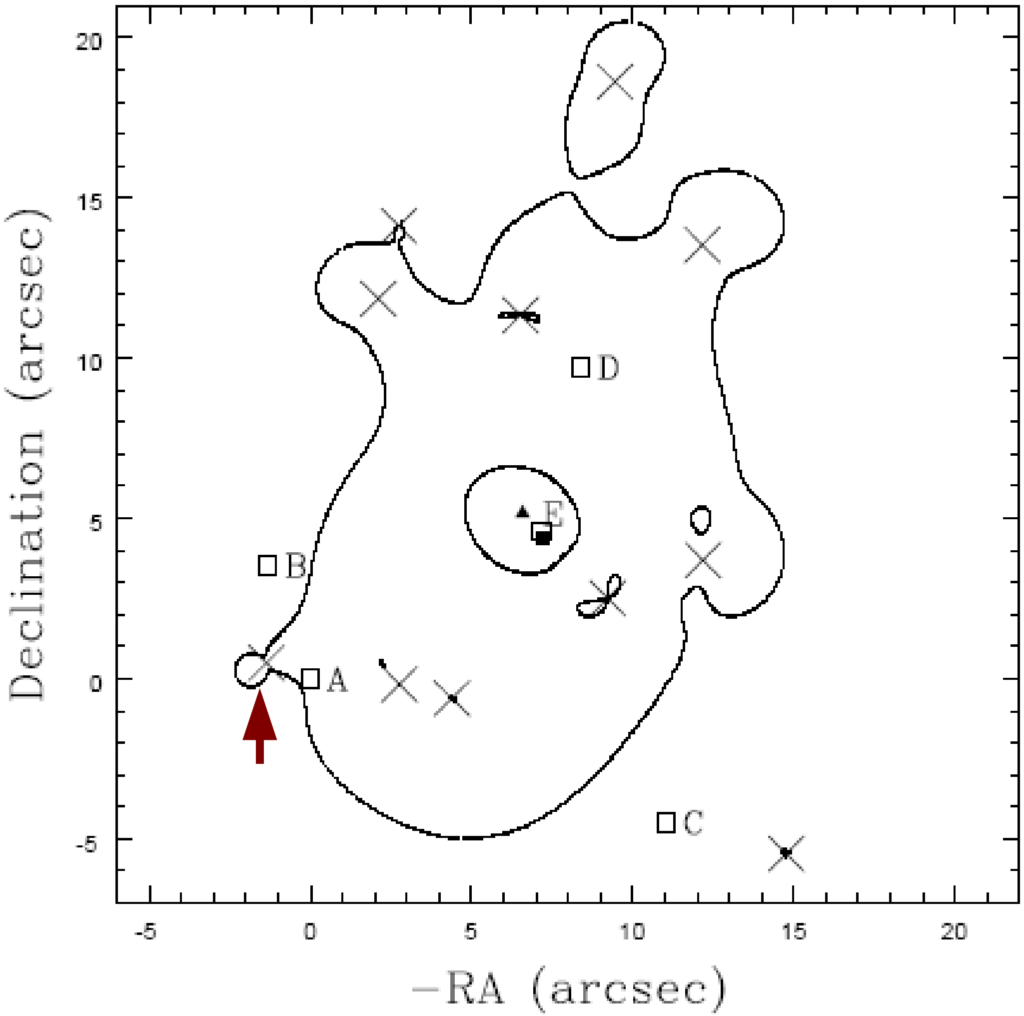
$$\Delta t \approx (2-\alpha)\Delta t_{SIS}$$

$H_0 \leftrightarrow$ radial profile !

(e.g., Refsdal & Surdej 1994; Witt et al. 1995; ...)

Note: The magnification factor can break the degeneracy
... from strong lensing of SNeIa (Oguri & Kawano 2003)
... from the FP of lensed galaxies (Bertin & Lombardi 2006)

Alpha is not the whole story...



The time delay btw AB of SDSS J1004+4112 appears to be higher than what simple models predict

".... The models failed because they neglected the perturbations from cluster member galaxies"
(Fohlmeister et al. 2006)

Even small perturbations could be important!

Questions

- How do time delays depend on mass models?
- How does the model dependence change with image configuration?

A statistical approach

Strategy

1. Define a "reduced time delay" that can be a measure of how time delays differ from those in simple isothermal mass models

2. Characterize configuration of image pairs by using two dimensionless quantities

3. See reduced time delays for various lens potentials as a function of image configuration

4. Input a distribution of realistic lens potentials to derive a conditional PDF of time delays, p(delay|image config.)

Reduced time delay

Define the following quantity

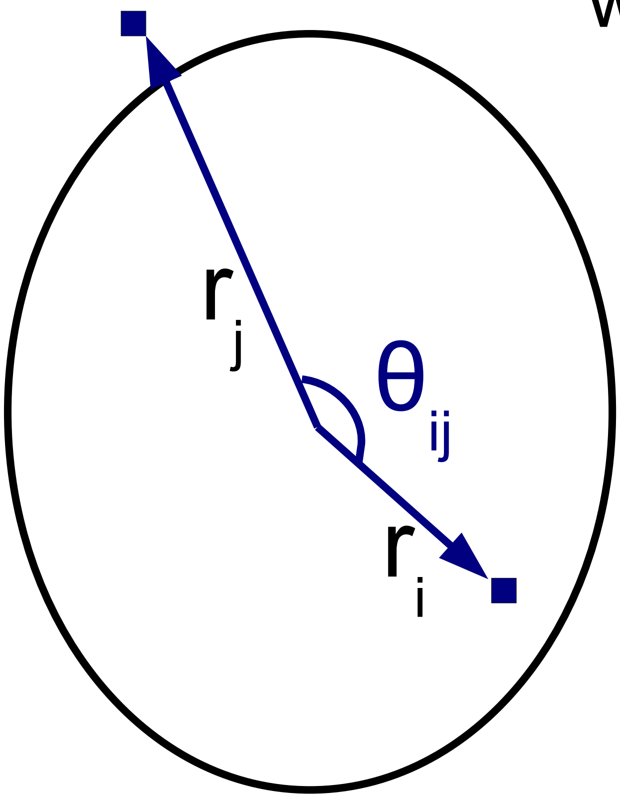
$$\Xi \equiv \left|\frac{\Delta t_{ij}}{r_j^2 - r_i^2}\right| \frac{2c}{1+z_l} \frac{D_{\rm ls}}{D_{\rm ol} D_{\rm os}} = \left|\frac{(\mathbf{x}_i - \mathbf{u})^2 - (\mathbf{x}_j - \mathbf{u})^2 - 2\phi(\mathbf{x}_i) + 2\phi(\mathbf{x}_j)}{r_j^2 - r_i^2}\right|$$

observables+cosmology ≈lens potential

- Ξ=1 if the lens potential is isothermal (φ=rF(θ)), but can be Ξ≠1 for more general lens potentials (see Witt et al. 2000)
  → measure of the "complexity" of lens potentials

- Can be computed for each time delay by assuming the value of the Hubble constant
  → relates observed delay to mass models

Image configuration



We characterize image pairs with the following two parameters

1. *Asymmetry*

$$R_{ij} = \left| \frac{r_j - r_i}{r_j + r_i} \right|$$

2. Opening angle

$$\theta_{ij} = \cos^{-1}\left(\frac{\vec{r}_j \cdot \vec{r}_i}{r_j \, r_i} \right)$$

dimensionless & model-independent!

Analytic examination

Consider multipole expansion of a lens potential

$$\phi(\mathbf{x}) = \frac{c_n}{\beta} R_{\rm Ein}^{2-\beta} r^{\beta} \cos n(\theta - \theta_n)$$

Then the reduce time delay becomes

$$\Xi = \left| 1 + c_n \left(\frac{2R_{\rm Ein}}{r_j + r_i} \right)^{2-\beta} X(R_{ij}, \theta_{ij}) \cos(n\theta_n - \delta) \right|$$

$$\tan\delta \equiv \frac{r_j^{\beta} \sin n\theta_j - r_i^{\beta} \sin n\theta_i}{r_j^{\beta} \cos n\theta_j - r_i^{\beta} \cos n\theta_i}$$

$$X(R_{ij}, \theta_{ij}) \equiv \frac{1-\beta}{2\beta} \frac{1}{R_{ij}} \left[(1+R_{ij})^{2\beta} + (1-R_{ij})^{2\beta} - 2(1-R_{ij}^2)^{\beta} \cos n\theta_{ij} \right]^{1/2}$$

- $\Xi \rightarrow 1$ by averaging many lenses for any perturbations (if the position angle is random)

Analytic examination (contd.)

Limiting behaviors of X

$$X(R_{ij} \to 0, \theta_{ij} \to 0) \approx (1-\beta)\left[1 + \left(\frac{n\theta_{ij}}{2\beta R_{ij}}\right)^2\right]^{1/2}$$

$$X(R_{ij} \to 1, \theta_{ij}) \approx \frac{2^{\beta-1}(1-\beta)}{\beta}$$

- Divergence occurs at the symmetric limit (R→0)
→ time delays are very sensitive to small perturbations

Monte-Carlo simulation

Derive a realistic PDF of reduced time delays using a Monte-Carlo simulation

1. Generate a lens potential from assumed distributions of lens potentials

2. Put many sources in the source plane and compute reduced time delays between image pairs

3. Repeat this for many lens potentials to construct the PDF (with a weight of $\mu^{\beta-1}$)

Note: The PDF is weighted by biased cross sections

Input potentials

- Primary lens galaxy: power-law ellipsoid + m=3,4 terms

$$\phi_{\rm G}(\mathbf{x}) = \frac{1}{\alpha} R_{\rm Ein}^{2-\alpha} r^{\alpha} G(\theta)$$

α=1.0±0.15, e=0.3±0.16

(LSD, SLACS, ...)

$$\phi_{\rm M}(\mathbf{x}) = \frac{1}{\alpha} R_{\rm Ein}^{2-\alpha} r^{\alpha} \sum_m (1-m^2) A_m \cos m(\theta - \theta_m)$$

A_m=0±0.01

- External perturbation: shear + third order term

$$\phi_{\rm E2}(\mathbf{x}) = -\frac{\gamma}{2} r^2 \cos 2(\theta - \theta_\gamma)$$

γ=0.05±0.2dex

$$\phi_{\rm E3}(\mathbf{x}) = \frac{\sigma}{4} R_{\rm Ein}^{-1} r^3 \left[\cos(\theta - \theta_\sigma) - \cos 3(\theta - \theta_\sigma)\right]$$

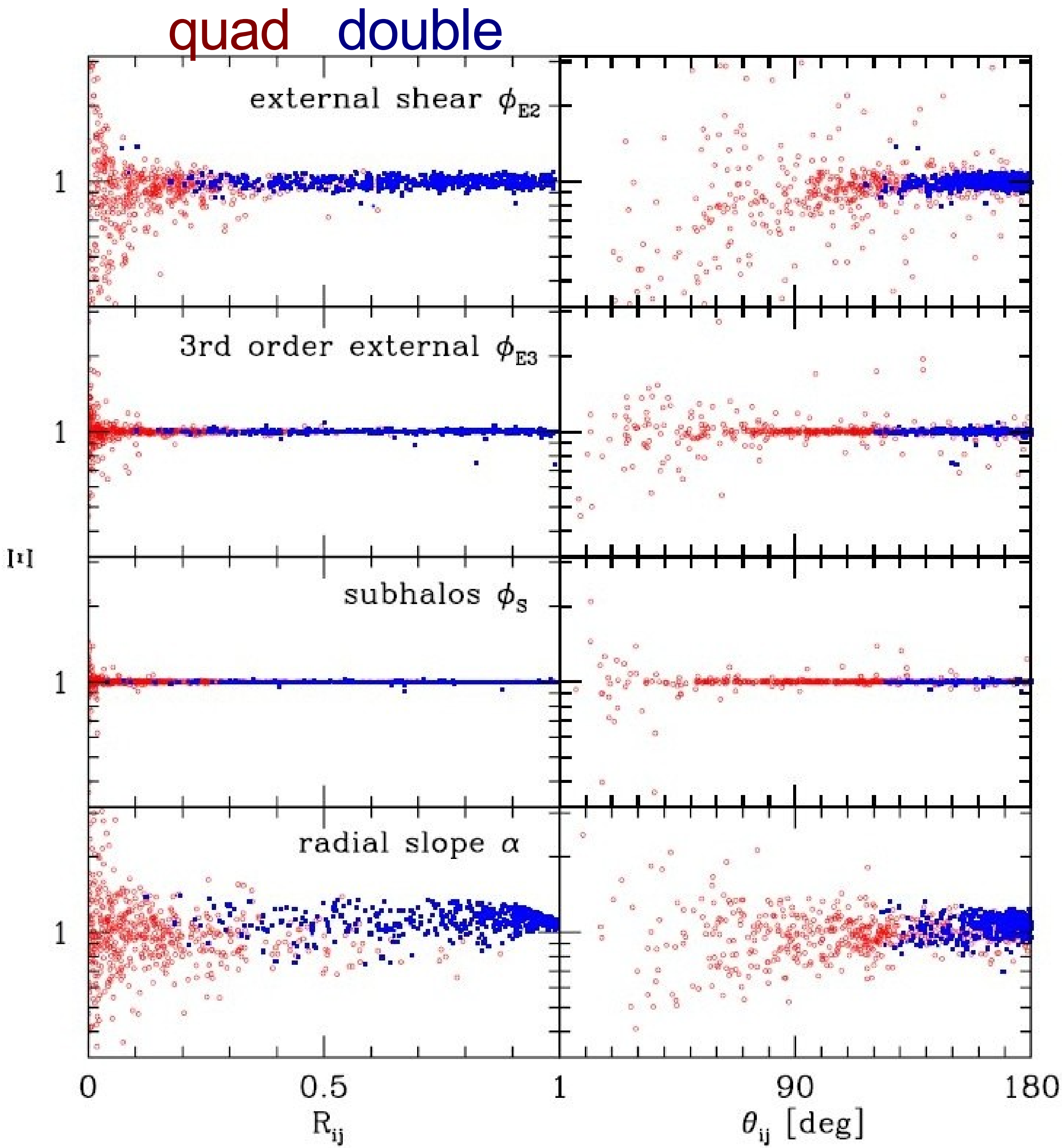
$\sigma=\gamma^2$±0.2dex

- Substructure: truncated SISs

$$\phi_{\rm S}(\mathbf{x}) = \sum_k \phi_{{\rm PJ},k}(\mathbf{x} - \mathbf{x}_{{\rm sub},k}) - \frac{1}{2} r^2 \bar{\kappa}_{\rm sub}$$

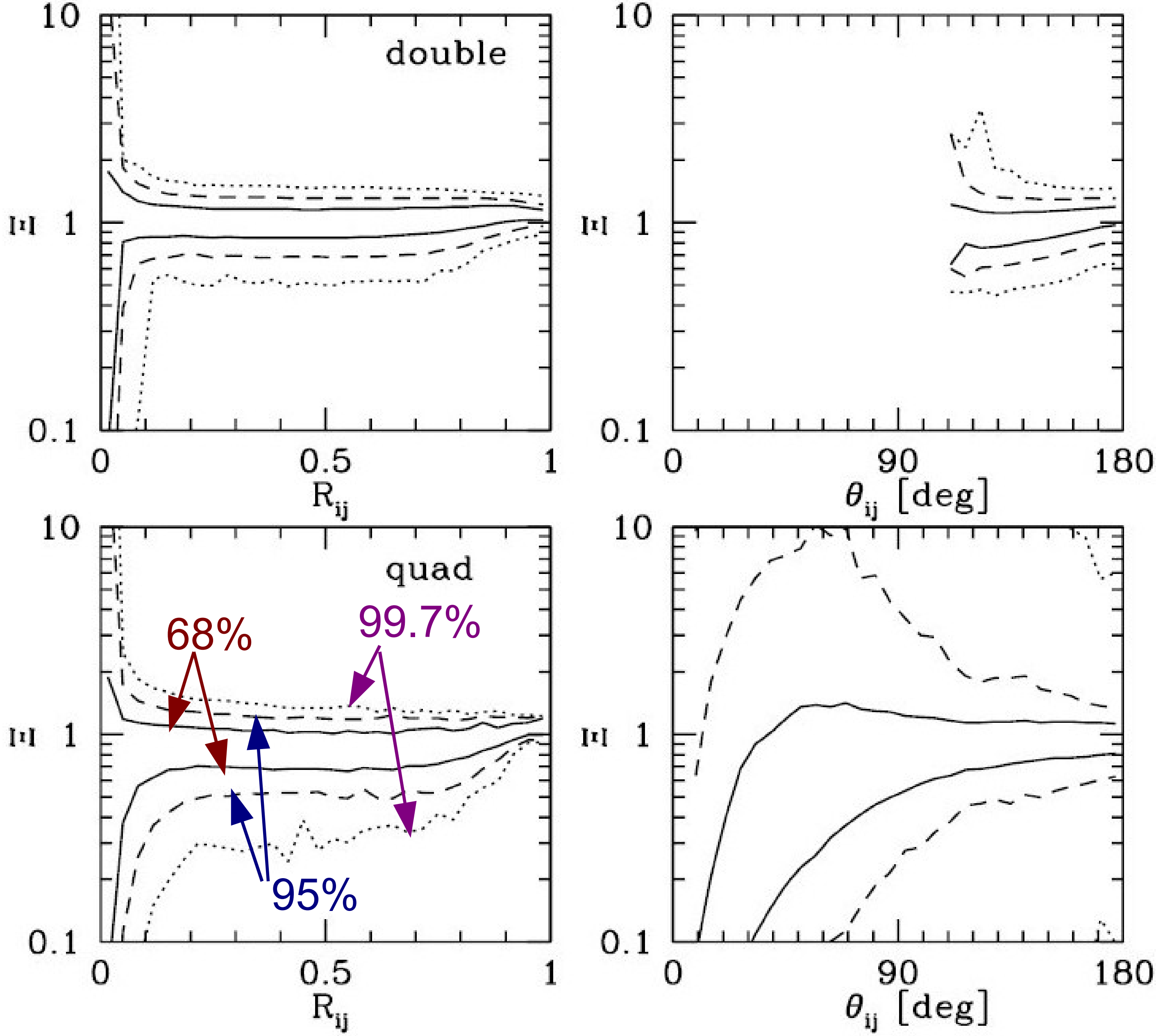
≈ CDM subhalos

Model dependence



- At R→0 any potentials can change Ξ drastically
- The scatter is caused mainly by external shear and radial slope, but 3rd external and subhalos can also cause notable changes
- Compared with the radial slope, other potentials have larger dependence on image configuration

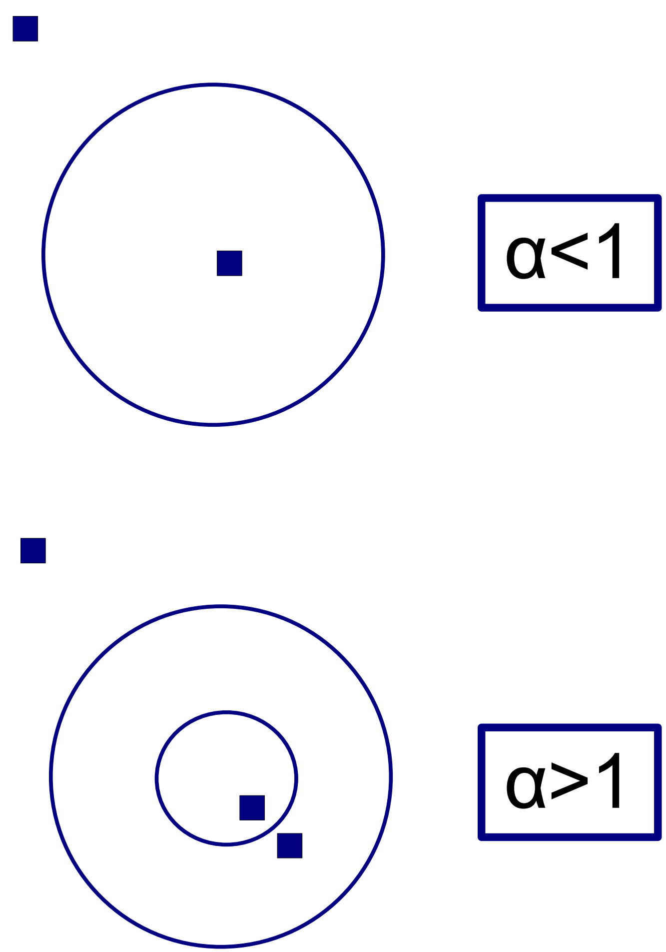
Time delays for realistic potentials



- Conditional prob. for quad and double lenses
- Larger scatter for more symmetric or smaller opening angle image pairs
- Quad has larger scatter
- Systematic shifts from unity at R_{ij}~1 and θ_{ij}~0

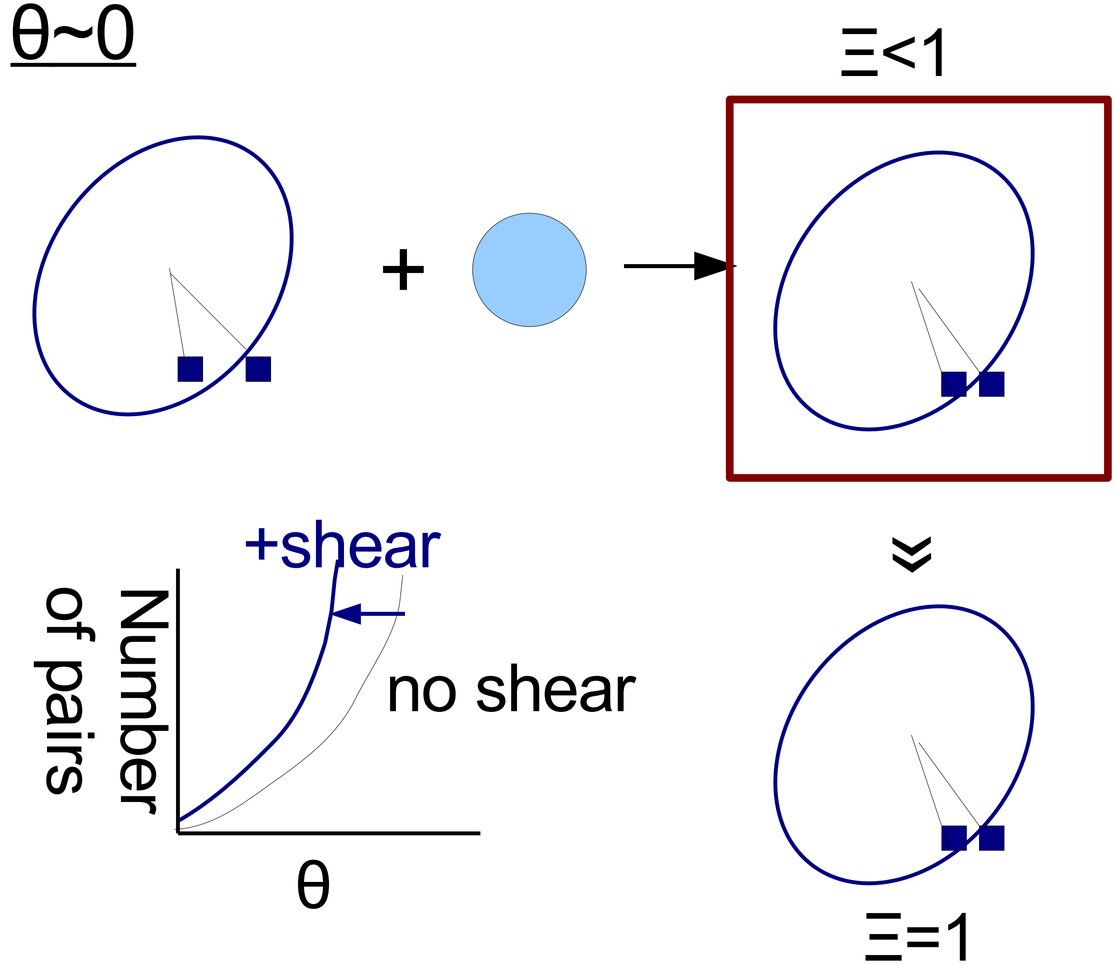
Understanding systematic shifts

R~1



For α>1, R has R_{max}<1 that is corresponds to inner crit curve

θ~0



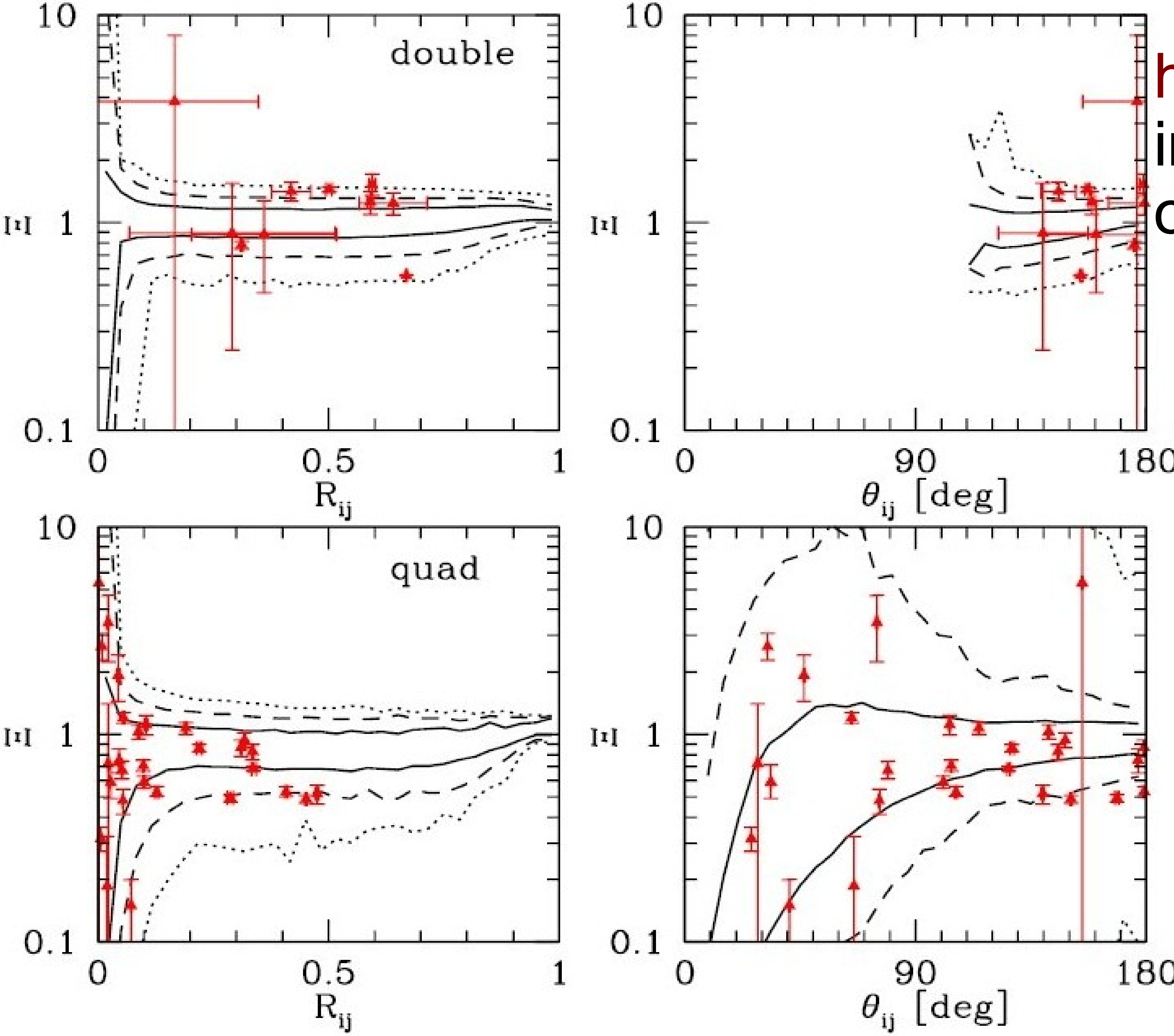
For a close image pair, shear direction is not random but more likely to be parallel to the segment connecting two images

Comparing with observed delays

Compare the PDFs with observed time delays

Lens Name	N_{img}	z_s	z_l	Images	R_{ij}	θ_{ij} [deg]	Δt [days]	$\Xi(h = 0.73)^a$	References
B0218+357	2	0.944	0.685	AB	0.167 ± 0.181	176.5 ± 21.3	10.5 ± 0.2	3.835 ± 4.155	1, 2, 3, 4, 5
HE0435−1223	4	1.689	0.455	AD	0.099 ± 0.002	103.9 ± 0.2	$14.4^{+0.9}_{-0.8}$	$0.704^{+0.046}_{-0.041}$	6, 7, 8
				AB	0.053 ± 0.001	79.3 ± 0.2	$8.0^{+0.8}_{-0.7}$	$0.671^{+0.069}_{-0.060}$	
				AC	0.002 ± 0.002	155.1 ± 0.2	$2.1^{+0.7}_{-0.8}$	$5.360^{+6.070}_{-6.150}$	
				BD	0.046 ± 0.002	176.8 ± 0.3	6.4 ± 0.8^e	0.750 ± 0.100	
				CD	0.100 ± 0.002	100.9 ± 0.3	12.3 ± 0.8^e	0.590 ± 0.040	
				BC	0.054 ± 0.001	75.9 ± 0.3	5.9 ± 0.8^e	0.479 ± 0.066	
RX J0911+0551	4	2.800	0.769	A1B	0.452 ± 0.003	150.8 ± 0.4	143.0 ± 6.0	0.488 ± 0.021	9, 10, 11, 12
				A2B	0.408 ± 0.003	179.3 ± 0.5	149.0 ± 8.0	0.530 ± 0.029	
				A3B	0.476 ± 0.003	139.6 ± 0.5	154.0 ± 16.0	0.516 ± 0.054	
SBS0909+532	2	1.377	0.830	AB	0.291 ± 0.223	139.7 ± 17.6	$45.0^{+5.5}_{-0.5}$	$0.891^{+0.657}_{-0.646}$	4, 13, 14, 15
FBQ0951+2635	2	1.246	0.24^b	AB	0.591 ± 0.007	158.8 ± 1.2	16.0 ± 2.0	1.256 ± 0.157	12, 16, 17
Q0957+561	2	1.413	0.36	AB	0.669 ± 0.002	154.5 ± 0.5	417.0 ± 1.5	0.558 ± 0.002	18, 19, 20, 21
SDSS J1004+4112	5	1.734	0.68	AB^c	0.006 ± 0.001	25.9 ± 0.1	38.4 ± 1.0	0.315 ± 0.041	22, 23, 24, 25
HE1104−1805	2	2.319	0.729	AB	0.312 ± 0.002	175.5 ± 0.3	161.0 ± 7.0	0.780 ± 0.034	4, 26, 27, 28
PG1115+080	4	1.735	0.310	$A1B^d$	0.105 ± 0.003	103.4 ± 0.4	11.7 ± 1.2	1.115 ± 0.120	29, 30, 31, 32, 33
				BC	0.191 ± 0.003	114.7 ± 0.3	25.0 ± 1.6	1.069 ± 0.070	
				$A1C^d$	0.068 ± 0.003	141.9 ± 0.3	13.3 ± 1.0	1.032 ± 0.083	
RX J1131−1231	4	0.658	0.295	AB	0.008 ± 0.001	32.3 ± 0.1	$12.0^{+1.5}_{-1.3}$	$2.660^{+0.411}_{-0.376}$	34, 35
				AC	0.028 ± 0.000	33.7 ± 0.0	$9.6^{+2.0}_{-1.6}$	$0.589^{+0.123}_{-0.098}$	
				BC	0.020 ± 0.001	66.0 ± 0.1	2.2 ± 1.6	0.167 ± 0.136	
				AD	0.311 ± 0.002	179.2 ± 0.1	87.0 ± 8.0	0.863 ± 0.079	
				BD	0.318 ± 0.002	148.5 ± 0.2	99.0 ± 8.0^e	0.940 ± 0.076	
				CD	0.335 ± 0.002	145.5 ± 0.1	96.6 ± 8.0^e	0.825 ± 0.068	
B1422+231	4	3.620	0.337	AB	0.023 ± 0.002	28.4 ± 0.2	1.5 ± 1.4	0.726 ± 0.680	32, 36, 37
				AC	0.022 ± 0.003	74.9 ± 0.3	7.6 ± 2.5	3.461 ± 1.216	
				BC	0.045 ± 0.003	46.5 ± 0.3	8.2 ± 2.0	1.924 ± 0.482	
SBS1520+530	2	1.855	0.717	AB	0.501 ± 0.007	157.1 ± 0.8	130.0 ± 3.0	1.444 ± 0.037	38, 39, 40
B1600+434	2	1.589	0.414	AB	0.640 ± 0.073	179.4 ± 14.2	51.0 ± 2.0	1.242 ± 0.150	41, 42, 43, 44
B1608+656	4	1.394	0.630	AB	0.127 ± 0.004	105.9 ± 0.4	$31.5^{+2.0}_{-1.0}$	$0.525^{+0.037}_{-0.023}$	45, 46, 47, 48, 49
				BC	0.056 ± 0.002	65.1 ± 0.3	36.0 ± 1.5	1.202 ± 0.072	
				BD	0.338 ± 0.005	126.4 ± 0.6	$77.0^{+2.0}_{-1.0}$	$0.681^{+0.020}_{-0.013}$	
				AC	0.072 ± 0.002	40.8 ± 0.2	4.5 ± 1.5^e	0.150 ± 0.050	
				AD	0.220 ± 0.006	127.8 ± 0.7	45.5 ± 1.5^e	0.858 ± 0.036	
				CD	0.287 ± 0.006	168.5 ± 0.7	41.0 ± 1.5^e	0.494 ± 0.021	
SDSS J1650+4251	2	1.547	0.577	AB	0.420 ± 0.043	145.8 ± 6.8	49.5 ± 1.9	1.415 ± 0.149	50, 51
PKS1830−211	2	2.507	0.89	AB	0.360 ± 0.157	160.5 ± 20.3	$26.0^{+4.0}_{-5.0}$	$0.874^{+0.401}_{-0.414}$	4, 52, 53, 54, 55
HE2149−2745	2	2.033	0.495	AB	0.594 ± 0.007	178.9 ± 1.5	103.0 ± 12.0	1.531 ± 0.179	12, 56, 57

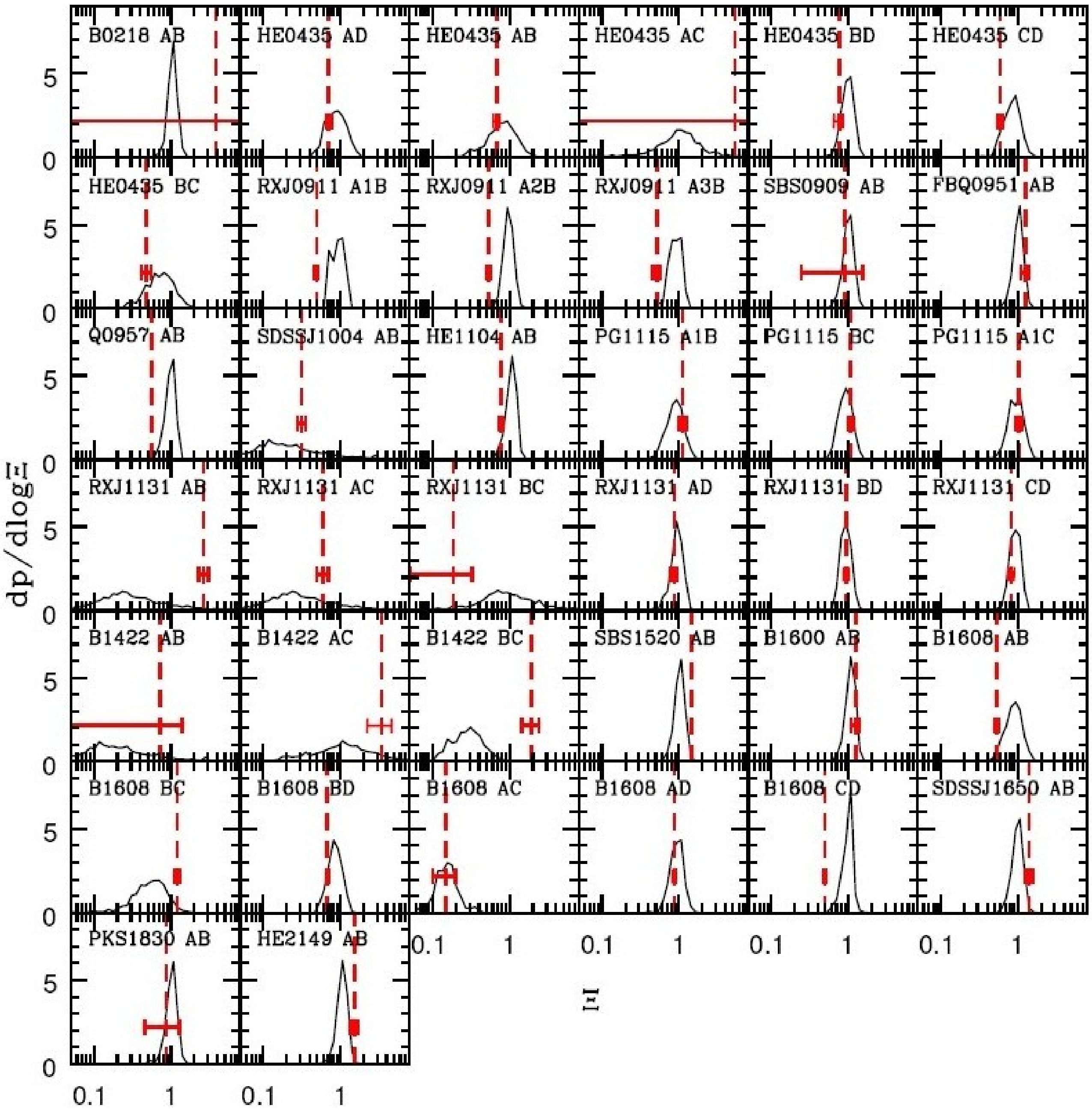
p(Ξ|R) and p(Ξ|θ)



h=0.73 is assumed in computing Ξ for observed delays

- Ξ_{obs} on average agree with the PDFs from sim
- R and θ dependence also seen

p(Ξ|R,θ)



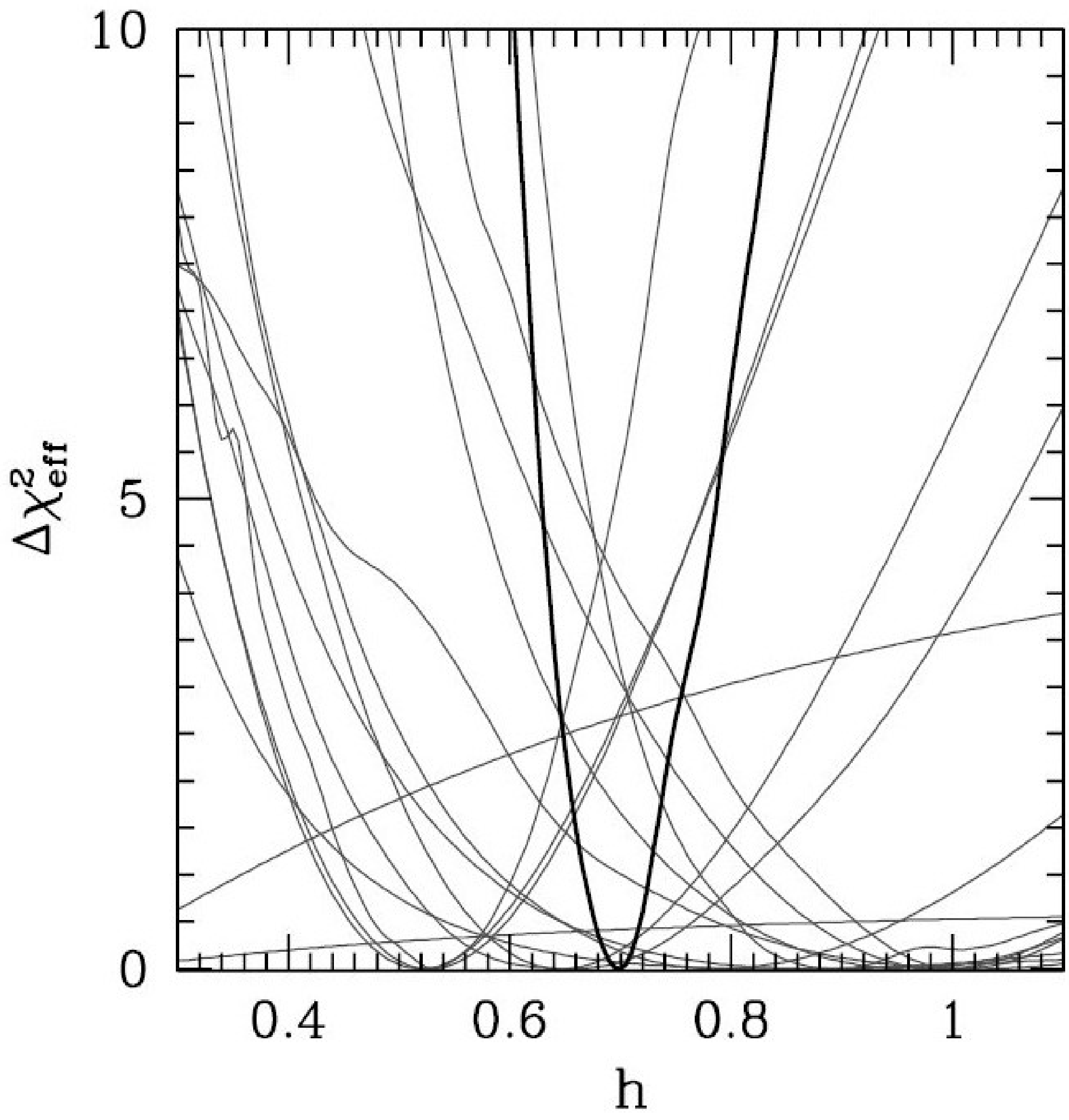
More directly we should compare the PDF for fixed R and θ...

Implications for H_0

Idea: by comparing Ξ_{obs} for different h with the simulated PDF we can constrain H_0

- We use all observed delays except SDSSJ1004 (cluster, center uncertain) and FBQ0951 (z_l uncertain)
- Correct environmental convergence for Q0957 (κ=0.26±0.08) and RXJ0911 (κ=0.30±0.04)

Result



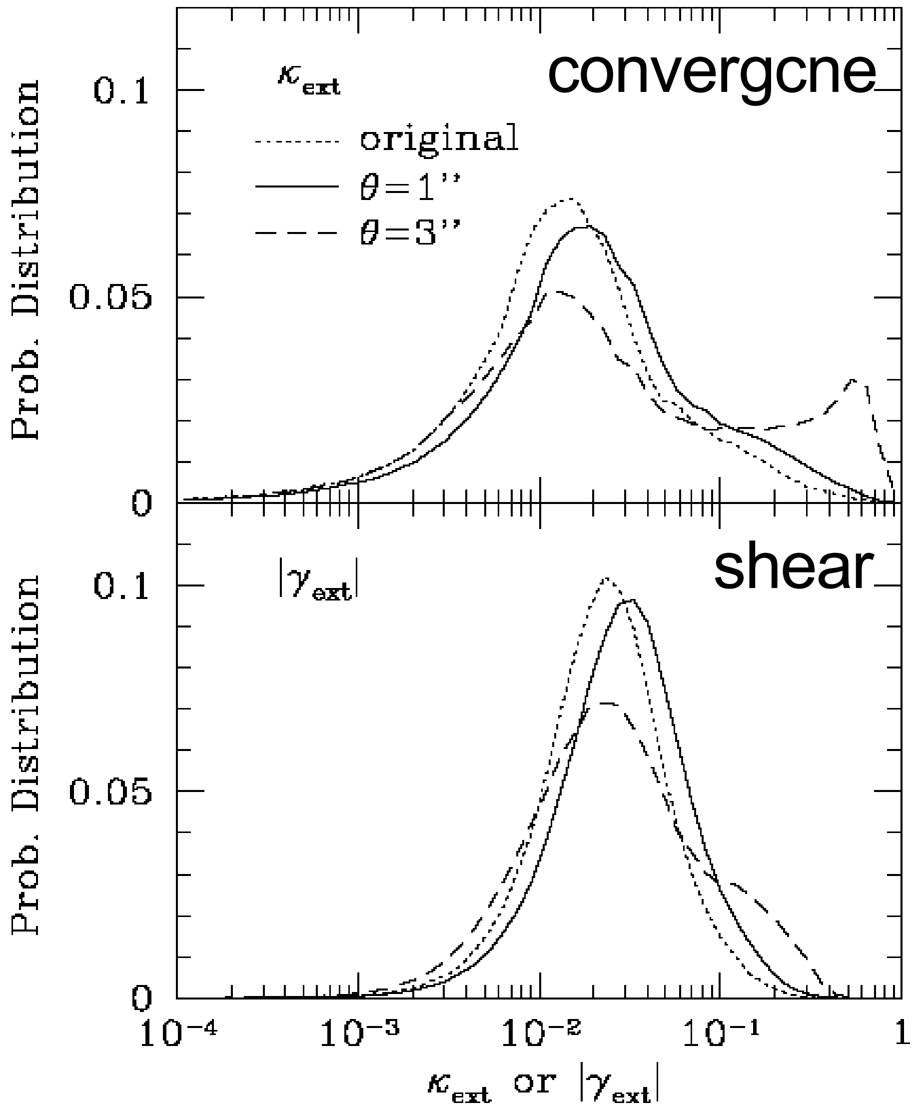
By combining 15 systems,
h=0.70±0.03 (68%)

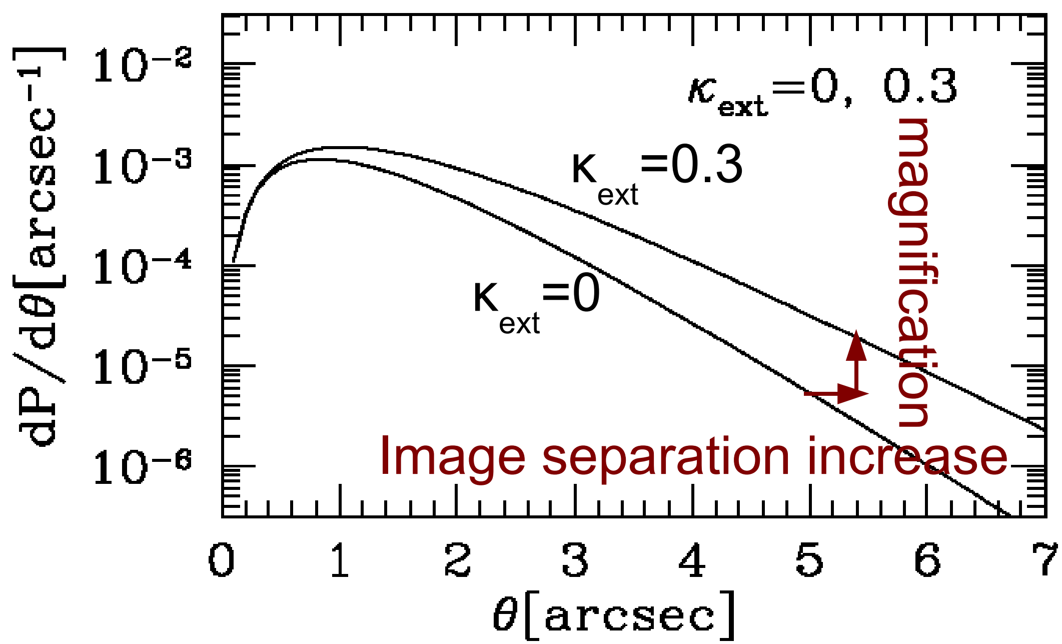
Systematic effects

- The sample is heterogeneous (many of time delay quasars are *extreme*)
- Input distributions? ($H_0 \approx (2-\bar{\alpha})^{-1} H_0(\bar{\alpha}=1)$)
- Lens galaxy environment?

Lens galaxy environments

Oguri, Keeton & Dalal, MNRAS 364(2005)1451





- Lens galaxies have biased environment distributions because of env. convergence
- The bias is a strong function of image separation

Larger separation lenses are more affected by environments

Conclusion

Using a new statistical approach we examined the dependence of time delays on various lens potentials as a function of image configuration

- Time delays are (statistically) less dependent on models
  ... if it is more asymmetric
  ... if it has larger opening angle
  ... if it is double rather than quad

Conclusion: Go ahead and monitor typical ("boring") lenses!

(Controversial?) Conclusion

- 15 time delay quasars constrain H_0 to be:

  H_0=70±3 km/s/Mpc (68%)

- Or you can assume the Hubble constant to derive the nearly isothermal lens profile from observed delays (if you prefer...)

- Importance to construct clean statistical samples
  - future surveys like LSST will do a good job