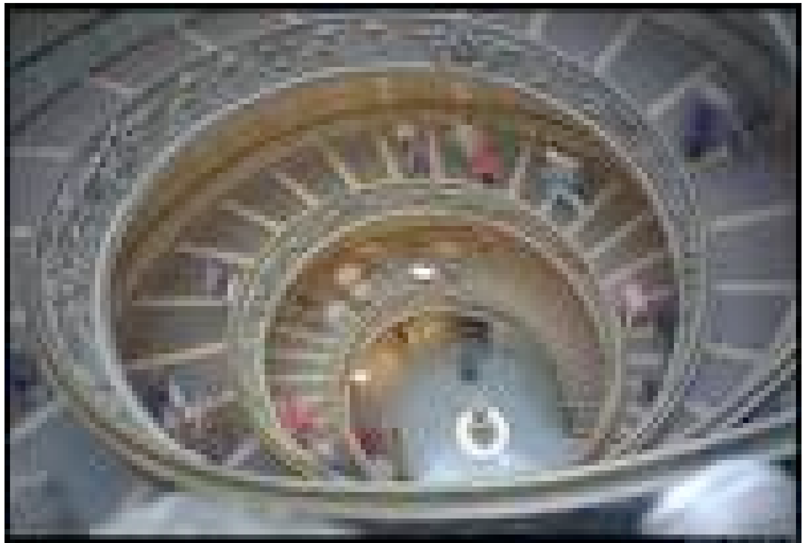
# A Parton Shower based on Exact Phase Space



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KITP LHC Workshop
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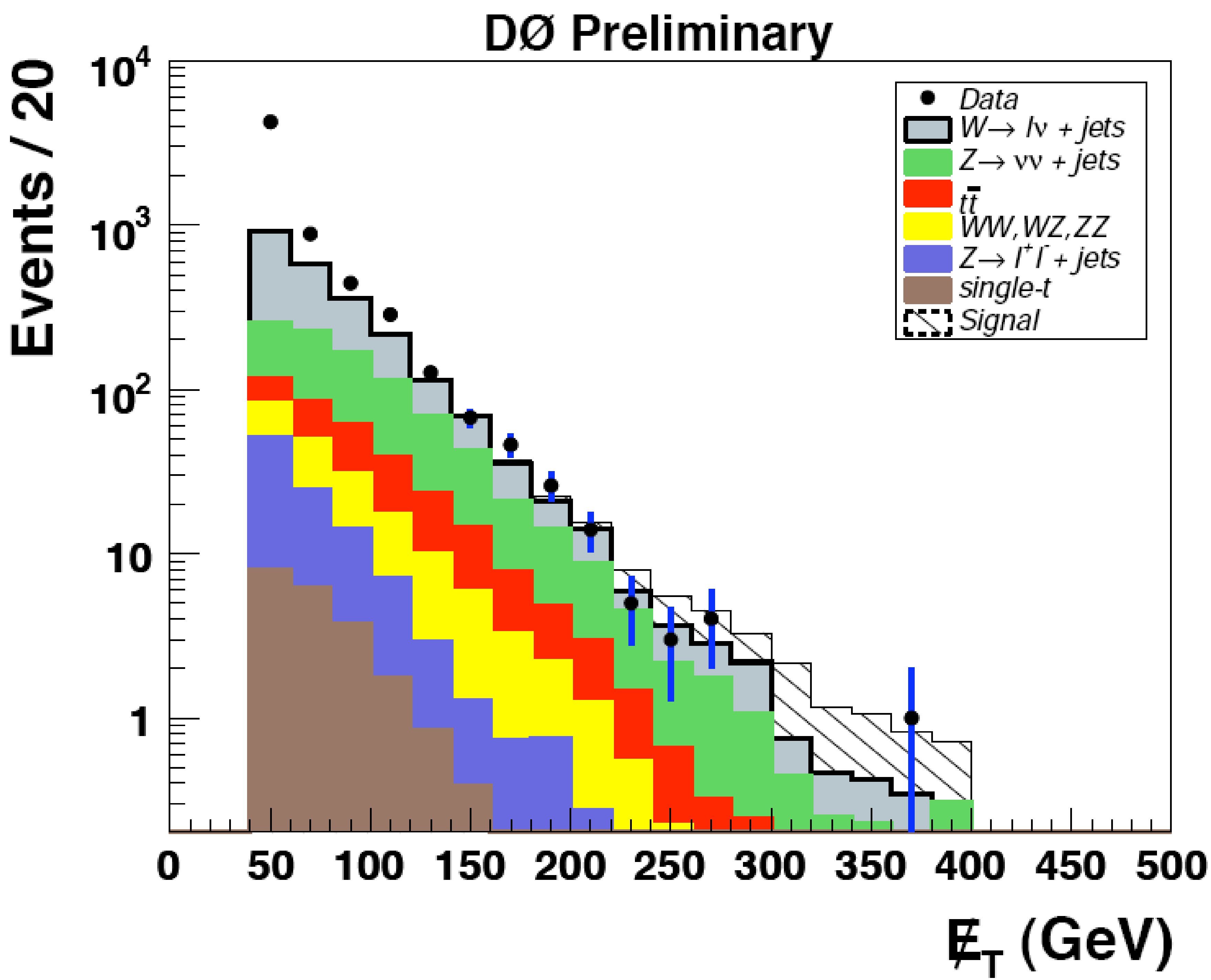
Our ability to understand physics at the LHC could well be limited by our understanding of the shapes of Standard Model backgrounds.

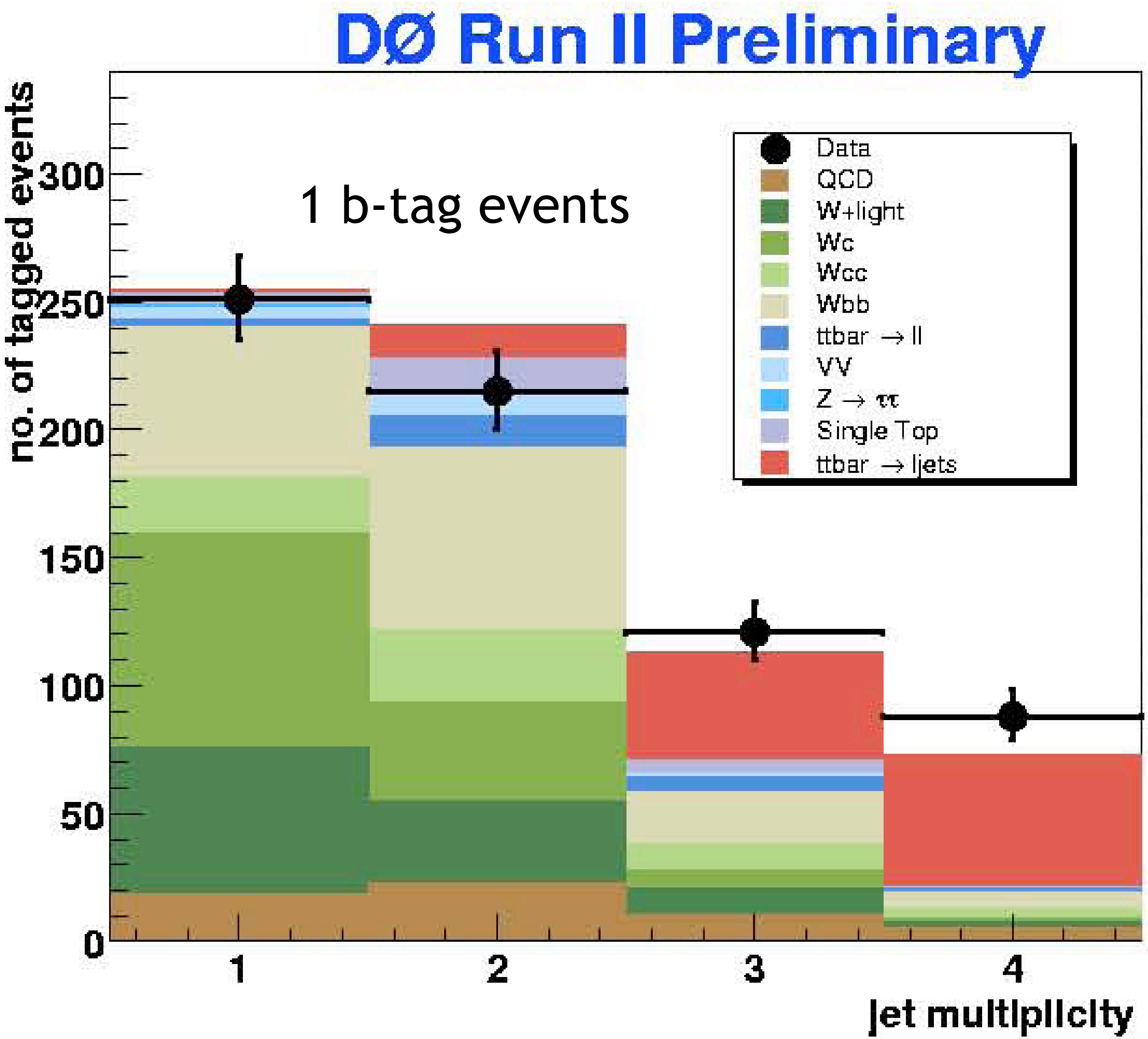
New physics appears at large values of

missing energy, HT, number of jets, ...

These are precisely the regions treated poorly by standard parton shower algorithms.

So it no wonder that many people are interesting in methods for merging partons showers at low pT with exact QCD calculations for the hardest jets.


DØ Preliminary
Events / 20
$10^4$
$10^3$
$10^2$
10
1
0
50
100
150
200
250
300
350
400
450
500
$\not\!\!E_T$ (GeV)
Data
$W \rightarrow l\nu$ + jets
$Z \rightarrow \nu\nu$ + jets
$t\bar{t}$
WW,WZ,ZZ
$Z \rightarrow l^+l^-$ + jets
single-t
Signal


DØ Run II Preliminary
1 b-tag events
no. of tagged events
0
50
100
150
200
250
300
1
2
3
4
jet multiplicity
Data
QCD
W+light
Wc
Wcc
Wbb
ttbar → ll
VV
Z → ττ
Single Top
ttbar → ljets

There are many approaches to this problem with somewhat different ambitions:

correction of high-order matrix element calculations for consistency with subsequent parton showering

ALPGEN, MADEVENT, SHERPA, HELAC
Catani-Krauss-Kuhn-Webber

correction of parton showers to incorporate exact 1-loop calculations

MC@NLO Frixione, Webber, Nason

improvement of parton shower algorithms using QCD resummation in the soft and collinear regions

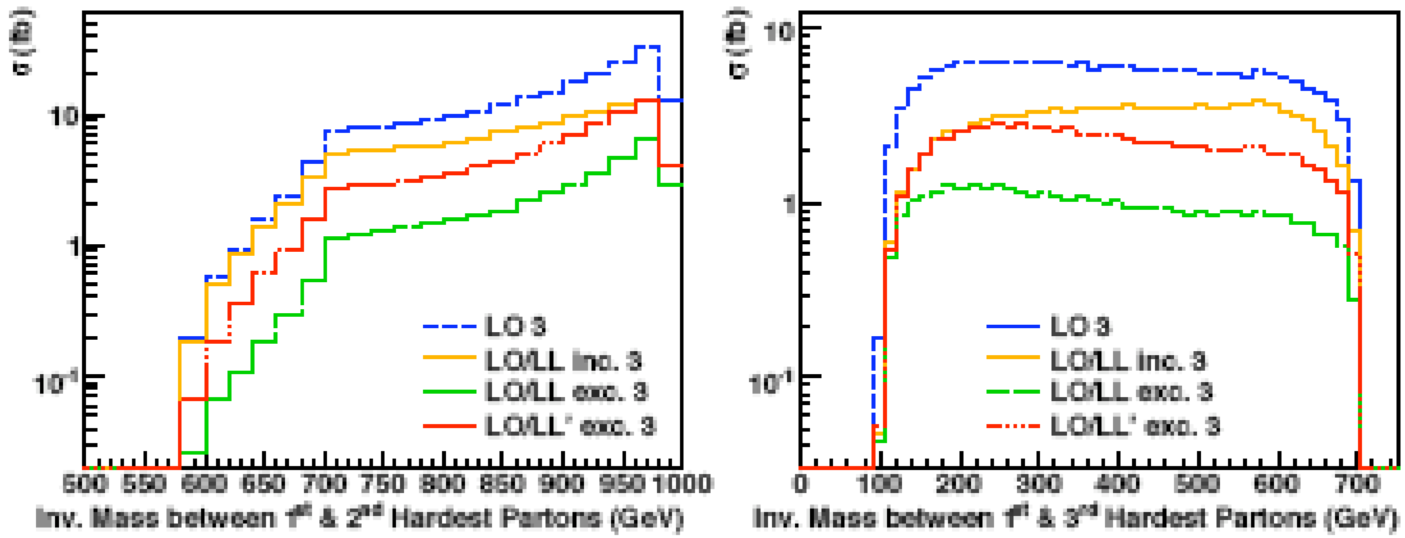
Becher + Schwartz

Here is the more specific problem that I am interested in:

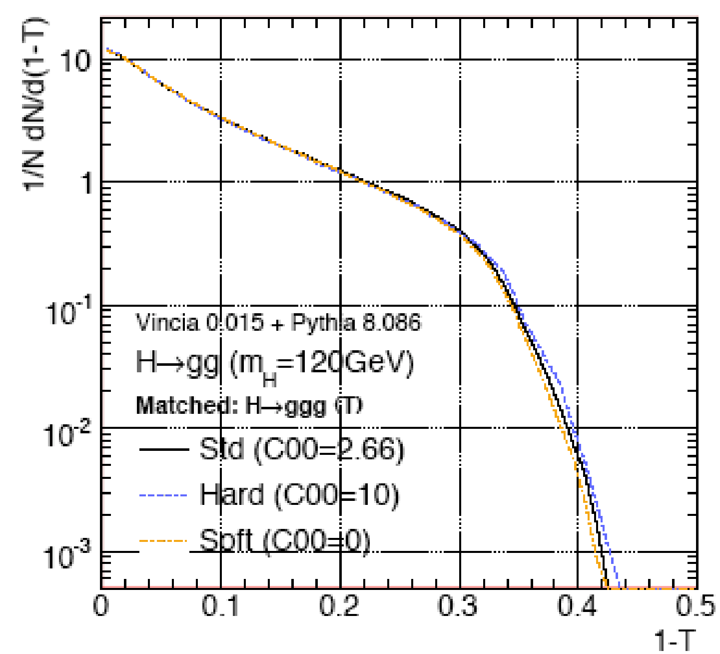
Can one write a parton shower algorithm that can systematically incorporate QCD tree amplitudes,

so that n-jet emission automatically has the shape that those amplitudes predict?

Bauer, Tackmann, and Thaler have attacked this same problem, and are much further along: (GenEVa)




Giele, Kosower, and Skands also have a new, more systematic approach to parton showers (VINCIA).



I will describe the approach that John Conley, Tommer Wizansky, and I have been developing.

Our model is built on the answers to three questions:

1. How can we rapidly produce exact QCD amplitudes ?

2. These amplitudes refer to points in n-particle phase space. How do we parametrize exact phase space so that it looks like a parton shower ?

3. The use of these amplitudes requires reweighting and acceptance/rejection in a parton shower. How is this done ?

In this talk, I will discuss the shower in $h^0 \to ng$ .

To generate tree amplitudes, we use the Britto-Cachazo-Feng recursion formula for on-shell amplitudes:

$$i\mathcal{M}(1\cdots n) \quad = \quad \sum_{splits} i\mathcal{M}(b+1\,\cdots\,\hat{i}\,\cdots\,a-1\;-\hat{Q}) \cdot \frac{1}{s_{a\cdots b}} \cdot i\mathcal{M}(a\,\cdots\,\hat{j}\,\cdots b\,\hat{Q})$$

We are content with amplitudes at the leading order in Nc.

We use the BCF formula to recursively break amplitudes down (numerically, on the fly) to the MHV result (Dixon, Glover, Khoze)

$$i\mathcal{M}(h^0 \to g_1^+ g_2^+ \cdots g_n^+) = i g^{n-2} C \frac{m_h^4}{\langle 12\rangle\langle 23\rangle\cdots\langle N1\rangle}$$

and the conjugate, with all g(-).

Duhr, Hoche, and Maltoni and Dinsdale, Ternick, and Weinzierl (DTW) have investigated how to do this. The latter group has written an especially fast code for multigluon amplitudes. Both groups conclude that, if you are sophisticated, BCF recursion has no advantage over the more venerable Berends-Giele recursion. However, BCF recursion can be implemented to give fast computations with a very simple code.

Our code is not as fast as Weinzierl's, but we can compute h-> 8 g amplitudes in 0.1 msec. That is quite fast enough.

Some fun C++ programming is involved. In our approach, we use as the basic object a C++ class called a bispinor. This holds 8 complex numbers

$$\begin{aligned} p^{\pm} &= p^0 \pm p^1 \quad q^{\pm} = p^2 \pm i p^3 \\ p\rangle &= u_R(p) \quad p] = u_L(p) \end{aligned}$$

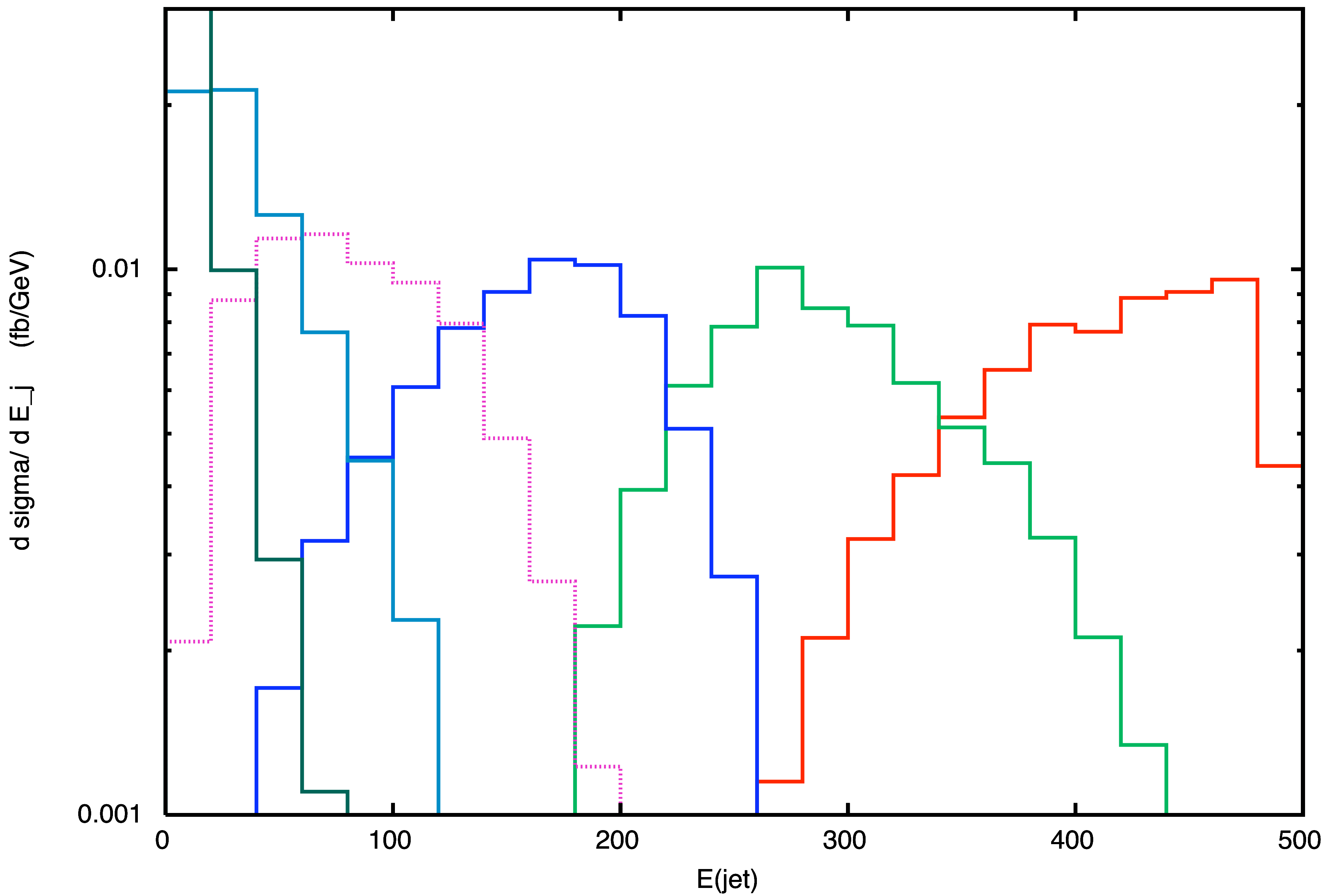
and implements methods that derive the vector components from the spinor components and vice versa.

It is important not to lose phase information in converting vectors to bispinors. Having obtained $u_R^1$ with some phase, we use

$$u_L^1 = \frac{q^-}{u_R^1} \quad u_L^2 = \frac{p^+}{u_R^1} \quad u_R^2 = \frac{q^+}{p^+} u_R^1$$

Then the composite object $p\rangle[p$ preserves the original phases.

DTW emphasize the importance of memory management. So, actually, we replace the bispinor class by a cone class that holds all of the bispinors needed to compute the desired amplitude.

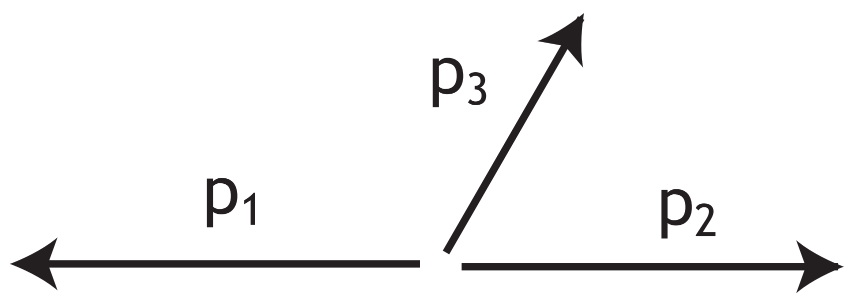


d sigma/ d E_j (fb/GeV)
0.01
0.001
0
100
200
300
400
500
E(jet)

Once we have the amplitudes, we need to integrate them over phase space. To do this, we need to efficiently generate multiparticle phase space, enhanced in the region where the QCD denominators are large.

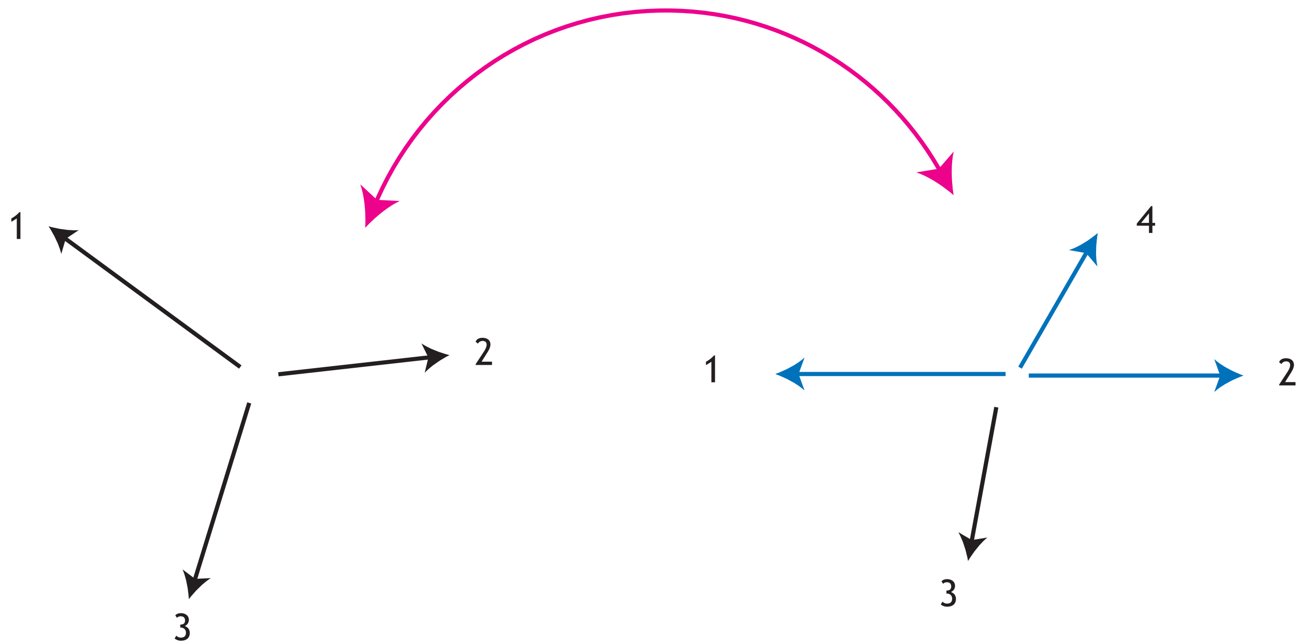
An effective trick has been introduced by Draggiotis, van Hameren, and Kleiss as the basis of their SARGE algorithm

Start with two back-to-back lightlike vectors. Add a third lightlike vector $p_3 = \xi_1 p_1 + \xi_2 p_2 + p_\perp$




Then boost and rescale to the original CM frame and energy.

To add the fourth vector, pick two neighbors, boost these back-to-back, add a vector as before, and then boost the entire system back to the CM frame.




Effectively, the entire event recoils when a new vector is added.

The logarthmic integral over the parameters reproduces massless phase space

$$\int \frac{d^3 p_3}{(2\pi)^2 2p_3} \frac{2p_1 \cdot p_2}{2p_1 \cdot p_3 \, 2p_3 \cdot p_2} = \frac{1}{(4\pi)^2} \int \frac{d\xi_1}{\xi_1} \int \frac{d\xi_2}{\xi_2} \int \frac{d\phi}{2\pi}$$

Applying this operation repeatedly, we build up phase space with all of the QCD denominators of the color-ordered amplitude for emission of final-state radiation.

$$\int d\Pi_n \frac{1}{2p_1 \cdot p_2 \, 2p_2 \cdot p_3 \, \cdots 2p_n \cdot p_1}$$
$$= \frac{1}{8\pi Q^4} \prod_i \left[ \frac{1}{(4\pi)^2} \int \frac{d\xi_{1i}}{\xi_{1i}} \int \frac{d\xi_{2i}}{\xi_{2i}} \int \frac{d\phi}{2\pi} \right]$$

This is an exact formula for massless phase space with QCD denominators, but only if we integrate over every point in phase space exactly once.

Draggiotis, van Hameren, and Kleiss suggested adding the vectors 1, 2, 3 in fixed (color) order. This requires very large values for the $\xi_i$ to reproduce some phase space configurations.

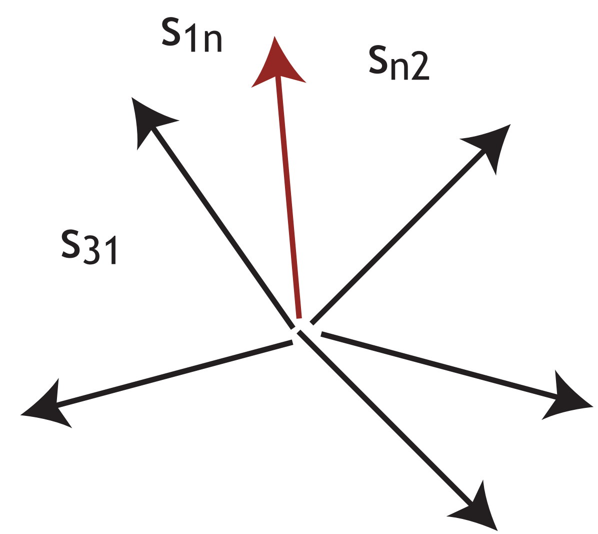
An alternative approach is to choose arbitrarily at each step one interval in which to insert a new vector. We call the set of such choices a chamber. It is then necessary to define the limits of each chamber so that the full set of chambers tiles phase space.

Here is a useful definition of a chamber:

Let the nth vector be inserted between 1 and 2. Then allow all values of $\xi_1, \xi_2, \phi$ such that

$s_{1n}$ is the smallest invariant mass of two neighbors, and $s_{n2} < s_{13}$

Reversing the inequality defines a second chamber with n between 1 and 2.



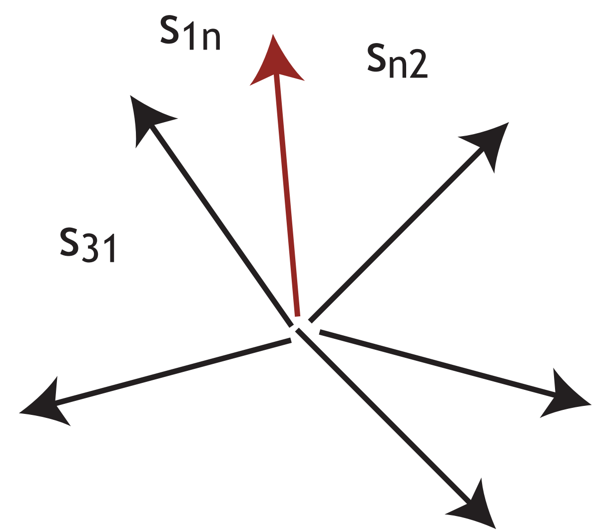

These prescriptions put reasonable upper limits on the $\xi_{1j}$ integrals.

The ordering of virtualities $s_{ij}$ is similar to the ordering in a parton shower. In fact, we can identify $s_{ij}$ with the evolution variable of a parton shower.

We look at the emission in the chamber

between 1 and 2, on the side of 1

as an emission from the gluon 1
in the antenna (in the sense of VINCIA)
of gluons 1 and 2.




At each stage in the shower, we choose an antenna and an emission side at random.

The correspondence to Altarelli-Parisi is

$$(1-z) = \frac{1}{(1+\xi_1+\xi_2)}$$

and

$$\int \frac{d\xi_2}{\xi_2} \int \frac{d\xi_1}{\xi_1} \approx \int \frac{dQ^2}{Q^2} \int \frac{dz}{z(1-z)}$$

We also choose definite values of the gluon helicities.

We can exactly solve for emissions with the measure

$$\int \frac{d\xi_2}{\xi_2} \int \frac{d\xi_1}{\xi_1} \int \frac{d\phi}{2\pi} \cdot \frac{3\alpha_s(\xi_2 s_{12})}{2\pi}$$

with the leading-log formula for $\alpha_s(Q^2)$

The gluon splitting functions are:

$$\begin{aligned} P(g_+ \to g_+ g_+) &= \frac{1}{z(1-z)} \\ P(g_+ \to g_- g_+) &= \frac{(1-z)^4}{z(1-z)} \\ P(g_+ \to g_+ g_-) &= \frac{z^4}{z(1-z)} \end{aligned}$$

We implement the numerators by hit-or-miss: accept weighted events if $w(x) > \mathrm{ran}()$ .

Actually, it is subtle to do hit-or-miss in a parton shower. We need a criterion to reject an emission without stopping the whole shower. (This is needed anyway in our technique, since we define the boundaries of chambers by assigning zero weight when the inequalities are violated.)

Write the emission probability as

$$dP = dx\, f(x) \quad x = \log(s/Q^2)$$

or, better, for the first emission after x=0:

$$S(x) = \int_0^{x_m} dx\, f(x) \quad dP = e^{-S(x)} dx\, f(x)$$

Write $f(x) = f_0(x) w(x)$ where $f_0(x)$ gives a problem we can solve exactly. It is not correct to stop the whole chain of emissions unless $w(x) > \mathrm{ran}()$ .

Here is a solution. In the Monte Carlo preparation stage, collect data to model

$$S(x) = S_0(x) \cdot C(x)$$

where C(x) is a polynomial. We crudely use $C(x) = A + Bx$ but with a different A and B for each helicity choice and each successive emission.

Then zeroth-order events are chosen according to the probabilities

$$S'(x) = S_0(x) \cdot C'(x) + f_0(x) \cdot C(x)$$

The new weight is $w(x)/(S'(x)/f_0(x))$. This can be > 1. So choose N to be greater than this number. Then accept an event if

$$w(x)/N(S'(x)/f_0(x)) > \mathrm{ran}()$$

Allow N tries, then terminate the parton shower. If an event is accepted, go back and get the next emission.

For a simple parton shower, we choose for w(x) the numerators of the splitting functions:

$$1\ ,\quad \frac{1}{(1+\xi_1+\xi_2)^4}\ ,\quad \frac{\xi_2^4}{(1+\xi_1+\xi_2)^4}$$

times zero if the chosen point violates the chamber inequalities.

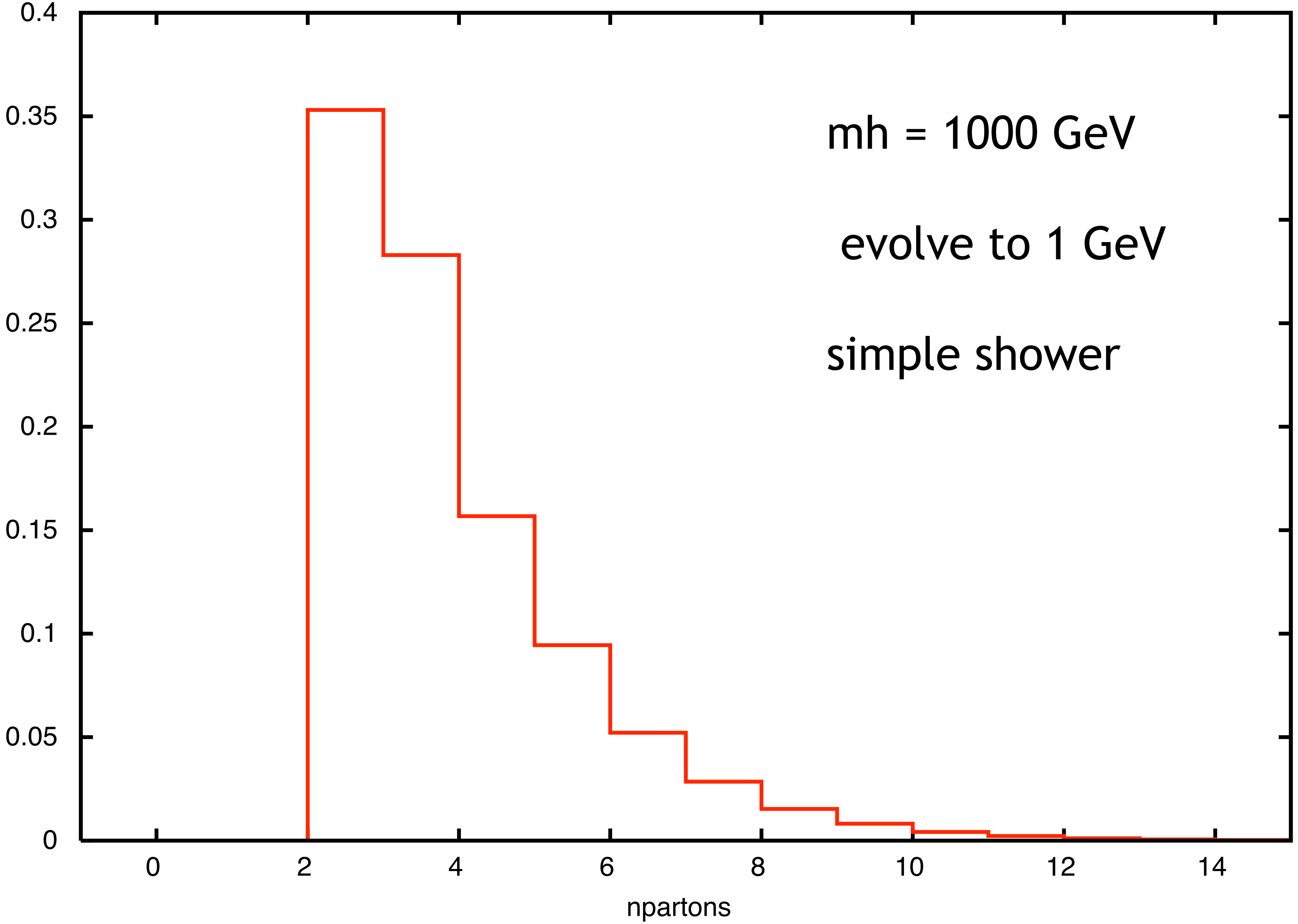
To reweight a shower to exact matrix elements, replace the above by

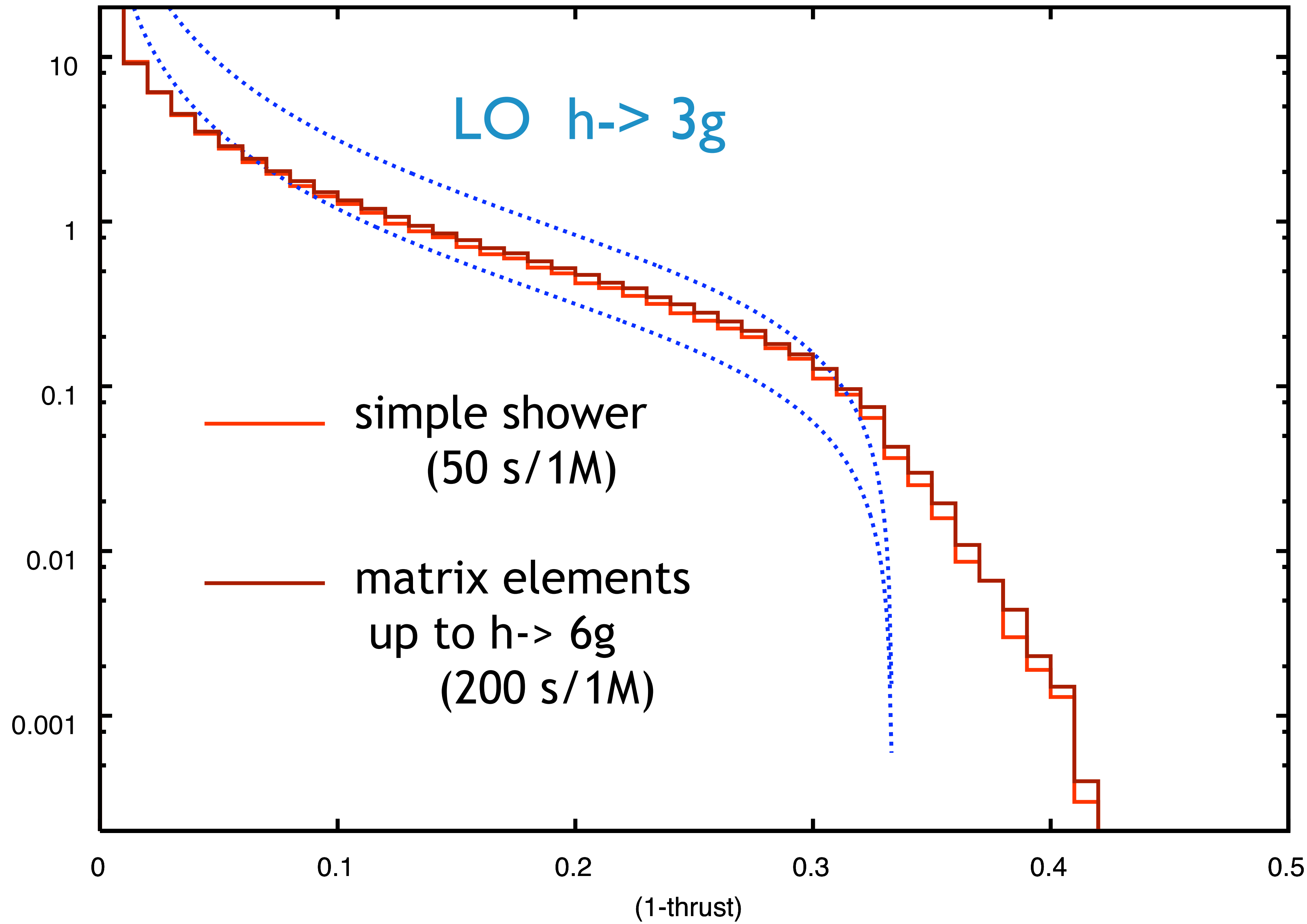
$$|(\mathcal{M}(h^0 \to ng)/g^{n-2}C) \cdot \langle 12\rangle\langle 23\rangle \cdots \langle N1\rangle|^2$$

or, rather, by the ratios of these factors for successive levels. The two prescriptions agree for h-> 3 g, so the simple shower is exact at this level. It is quite accurate at higher levels.

Finally, here are some results of the program.

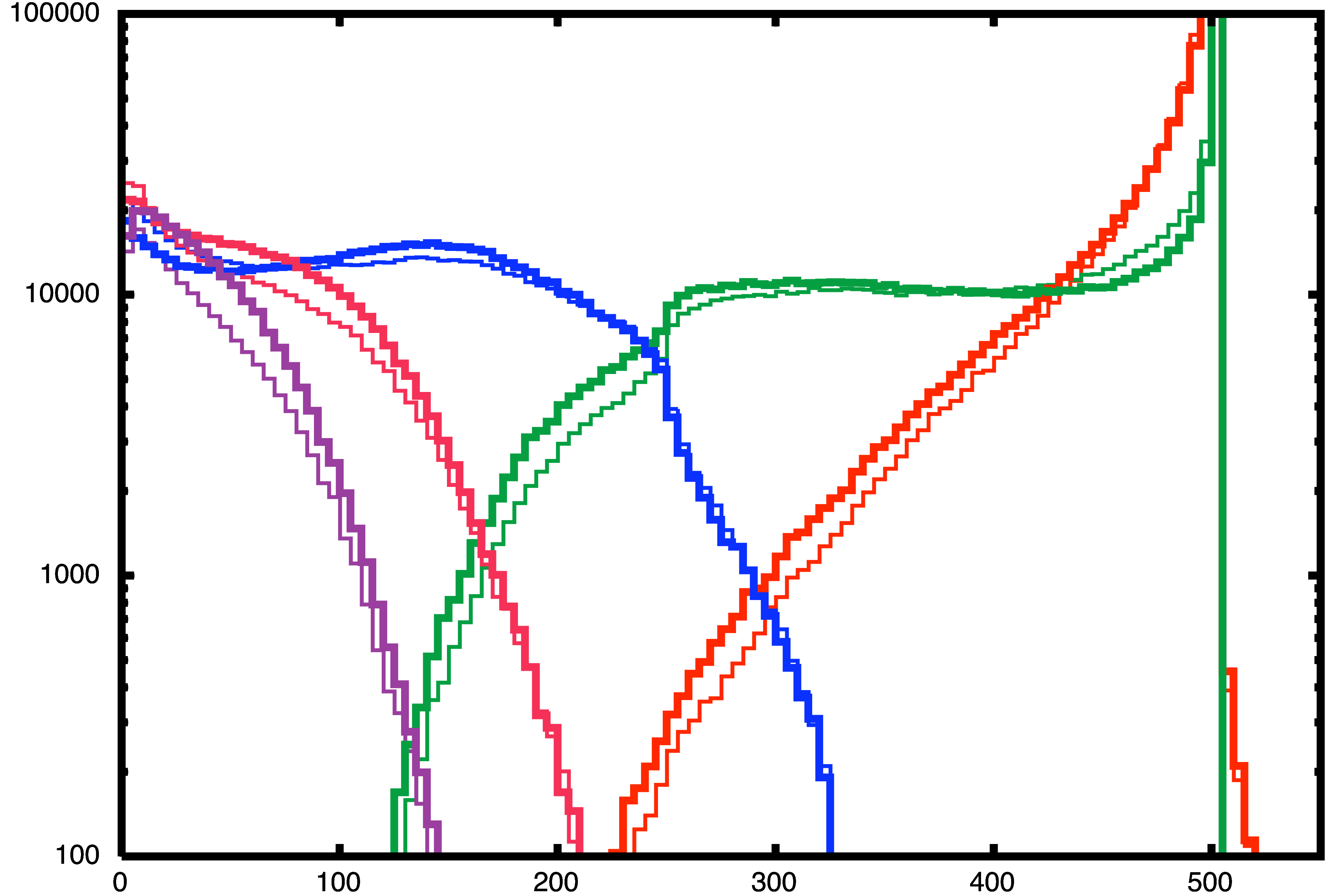
These are very preliminary (generated last night).


0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
2
4
6
8
10
12
14
npartons
mh = 1000 GeV
evolve to 1 GeV
simple shower



LO h-> 3g
simple shower
(50 s/1M)
matrix elements
up to h-> 6g
(200 s/1M)
10
1
0.1
0.01
0.001
0
0.1
0.2
0.3
0.4
0.5
(1-thrust)

energies of 5 hardest clusters w. ycut = 0.0001 (10 GeV)




light - simple shower; heavy - shower w. h->6g matrix elements

## Conclusions:

This is a proof of principle for a new way to incorporate exact matrix elements into a parton shower. The numerical results shown here are very preliminary.

This method generalizes to initial-state radiation with all massless emissions. The generalization to processes in which massive particles are emitted is nontrivial and not yet worked out.

Still, there is promise that this might be an interesting tool for modeling multijet QCD processes.