Top pair production at NNLO

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Content of the talk

- What will be discussed: calculation of top-pair x-section at NNLO
- Why? What can we learn from this?
  - ✓ Switching gears: moving beyond "testing QCD" :
    - ➢ Study top physics with *reliable, high precision* in order to scrutinize the SM and search for bSM
    - ➢ M. Peskin: "bSM hides beneath top"
  - ✓ Measure precisely, to the extend possible, all top quark related parameters
  - ✓ Search for deviations from SM through high precision analyses (percent-level precision is plausible both experimentally and theoretically)
  - ✓ Top production is a process that challenges our ability to describe complex hadron collider processes (and helps develop new capabilities)
    - ➢ Developed resummation techniques
    - ➢ IR subtraction scheme — "STRIPPER" Czakon `10
    - ➢ Amplitude calculations — Baernreuter, Czakon, Fiedler `08 – 13; Gehrmann et al `09-`13
- Some technical details regarding calculation available as backup slides

In this talk I'll focus exclusively on the total inclusive x-section:

NOTE: differential distributions are well understood at NLO.
The total x-section is the first step into NNLO.
Approximations to differential NNLO exist

Kidonakis
Ahrens, Ferroglia, Neubert, Pecjak, Yang `10-`12
Ferroglia, Pecjak, Yang`13

$$\sigma_{\rm tot} = \sum_{i,j} \int_0^{\beta_{\rm max}} d\beta\, \Phi_{ij}(\beta, \mu_F^2)\, \hat{\sigma}_{ij}(\beta, m^2, \mu_F^2, \mu_R^2)$$

Partonic fluxes
(derived from PDF's) →

$$\Phi_{ij}(\beta, \mu_F^2) = \frac{2\beta}{1-\beta^2}\, \mathcal{L}_{ij}\left(\frac{1-\beta_{\rm max}^2}{1-\beta^2}, \mu_F^2\right)$$

$$\mathcal{L}_{ij}(x, \mu_F^2) = x\,(f_i \otimes f_j)\,(x, \mu_F^2)$$

Partonic x-section
(perturbative) →

$$\hat{\sigma}_{ij}(\beta) = \frac{\alpha_S^2}{m^2}\left(\sigma_{ij}^{(0)} + \alpha_S \sigma_{ij}^{(1)} + \alpha_S^2 \sigma_{ij}^{(2)} + \mathcal{O}(\alpha_S^3)\right)$$

The partonic x-section depends on a single variable

$$\beta = \sqrt{1-\rho}, \text{ with } \rho \equiv 4m^2/s.$$

$$0 < \rho \leq 1$$

- ✓ Point β = 0 (absolute threshold)
- ✓ Point β = 1 (high energy limit, i.e. m=0)

Calculation of the total inclusive x-section tT @ NNLO during the last year

- Published qQ → tt +X — Bärnreuther, Czakon, Mitov `12
- Published all fermionic reactions (qq,qq',qQ') — Czakon, Mitov `12
- Published gq — Czakon, Mitov `12
- Published gg — Czakon, Fiedler, Mitov '13

Now the top pair total x-section is known exactly at NNLO in QCD

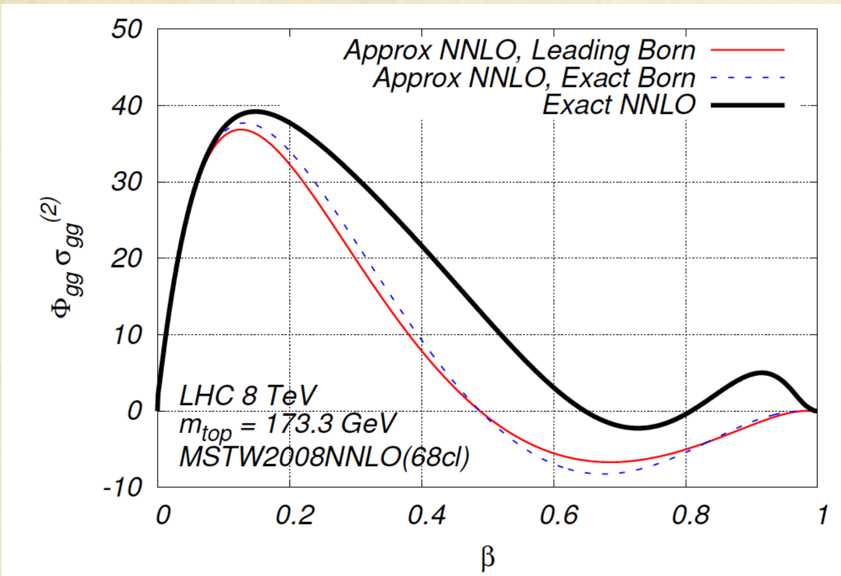
No approximations of any kind

- First hadron collider calculation at NNLO with more than 2 colored partons.
- First NNLO hadron collider calculation with massive fermions.

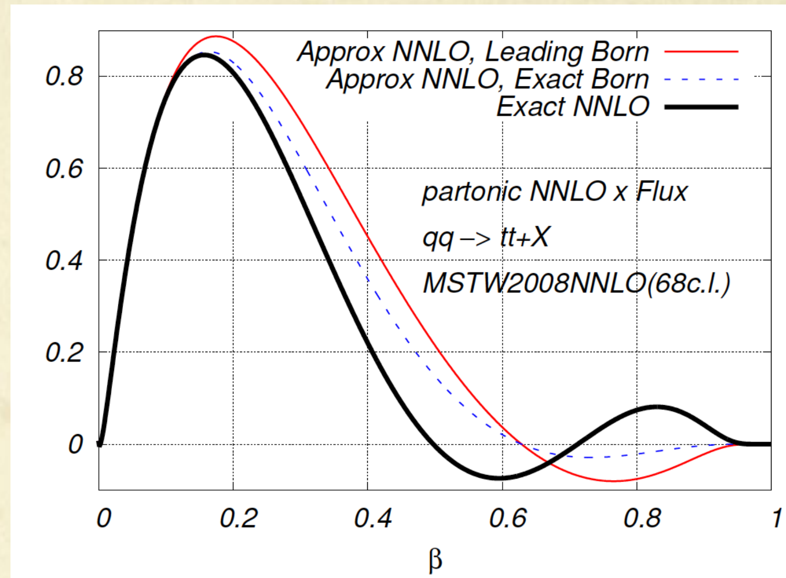
Parton level results

Partonic NNLO cross-sections, convoluted with LHC/Tevatron partonic fluxes

Czakon, Fiedler, Mitov '13



Bärnreuther, Czakon, Mitov `12



The exact NNLO allows for a critical examination of approximate NNLO approaches

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Results @ parton level: gg -> ttbar +X

Notable features:

- ✓ Small numerical errors
- ✓ Agrees with limits

Partonic cross-section through NNLO:

$$\sigma_{ij}\left(\beta, \frac{\mu^2}{m^2}\right) = \frac{\alpha_S^2}{m^2}\left\{\sigma_{ij}^{(0)} + \alpha_S\left[\sigma_{ij}^{(1)} + L\,\sigma_{ij}^{(1,1)}\right] + \alpha_S^2\left[\sigma_{ij}^{(2)} + L\,\sigma_{ij}^{(2,1)} + L^2\sigma_{ij}^{(2,2)}\right] + \mathcal{O}(\alpha_S^3)\right\},$$

The NNLO term:

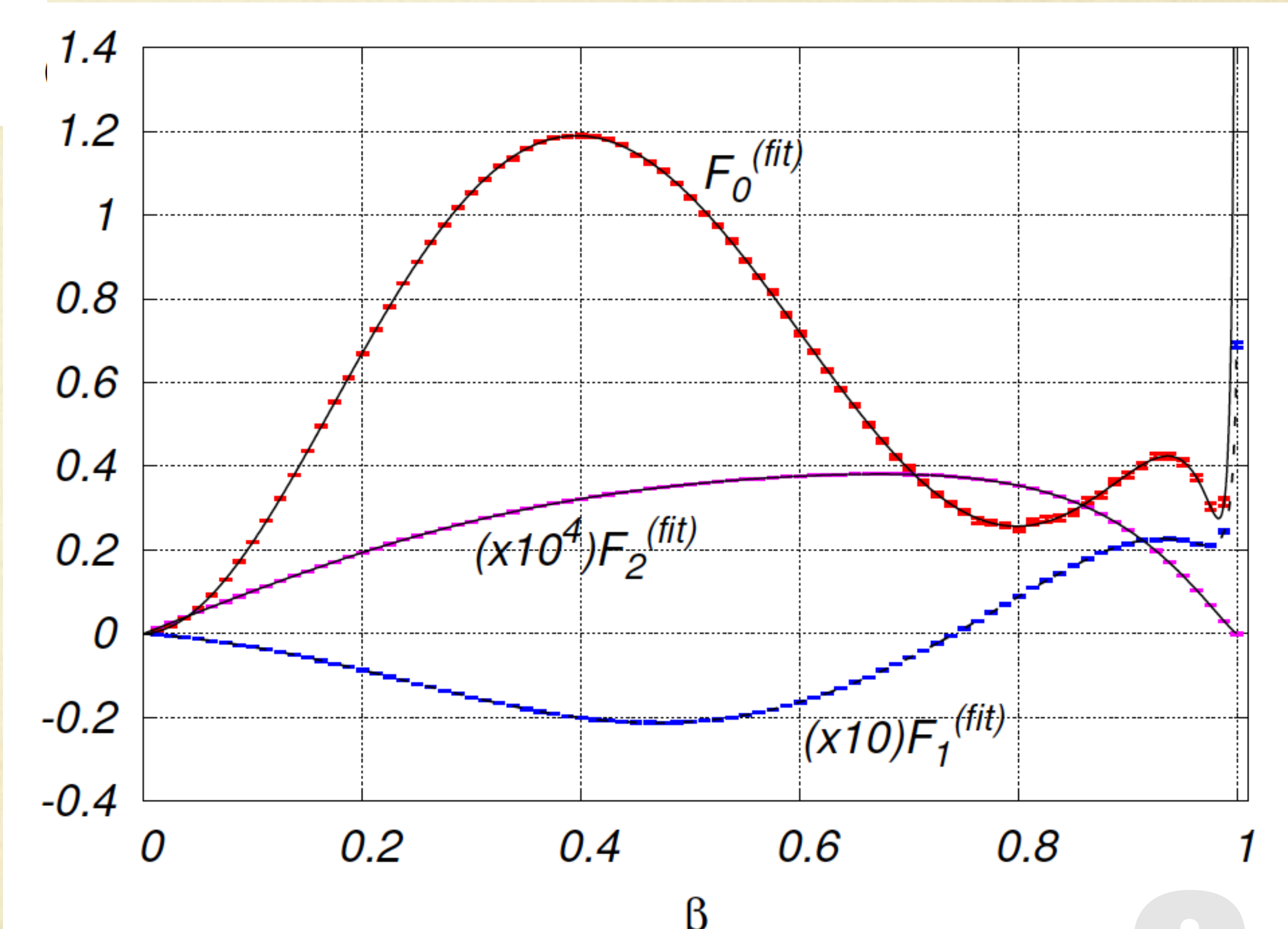
$$\sigma_{gg}^{(2)}(\beta) = F_0(\beta) + F_1(\beta)N_L + F_2(\beta)N_L^2$$

Numeric

$$F_i \equiv F_i^{(\beta)} + F_i^{(\rm fit)},\ \ i = 0, 1, 2$$

The known threshold approximation

Beneke, Czakon, Falgari, Mitov, Schwinn `09



Czakon, Fiedler, Mitov '13

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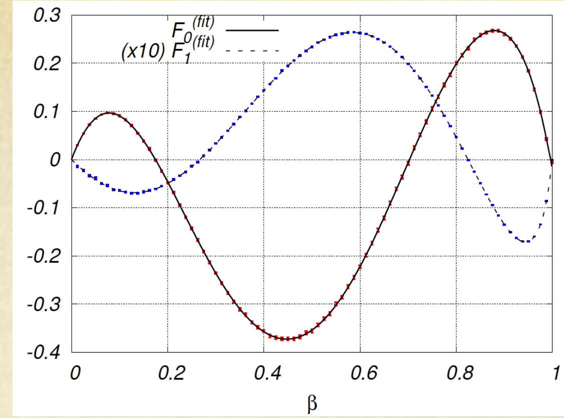
Results @ parton level:
The all-fermionic reactions

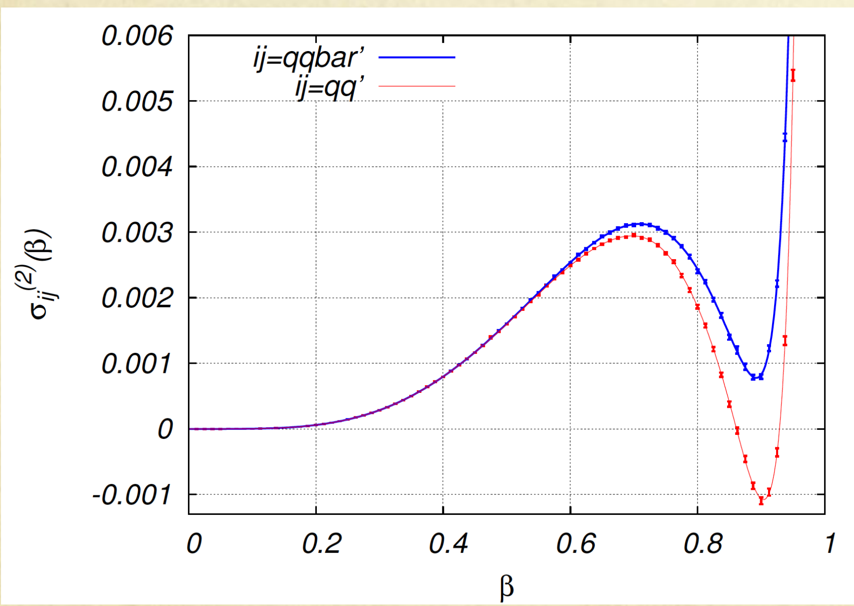
$$
\begin{aligned}
q\bar{q} &\to t\bar{t} + q\bar{q}\big|_{\mathrm{NS}}\,,\\
q\bar{q}' &\to t\bar{t} + q\bar{q}'\,,\\
qq' &\to t\bar{t} + qq'\,,\\
qq &\to t\bar{t} + qq\,.
\end{aligned}
$$

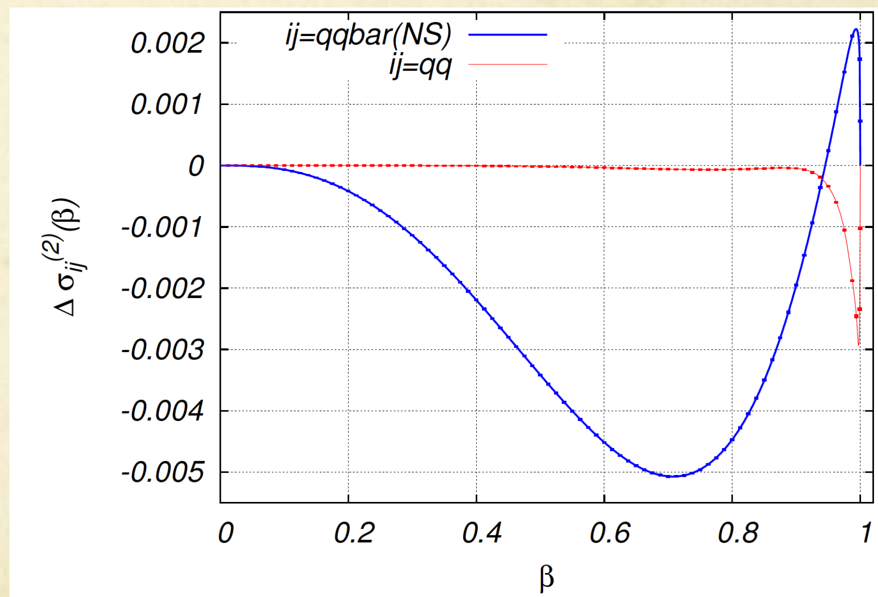
Czakon, Mitov '12



P. Bärnreuther et al arXiv:1204.5201

These partonic cross-sections are very small.
Compare to the ones involving qqbar!

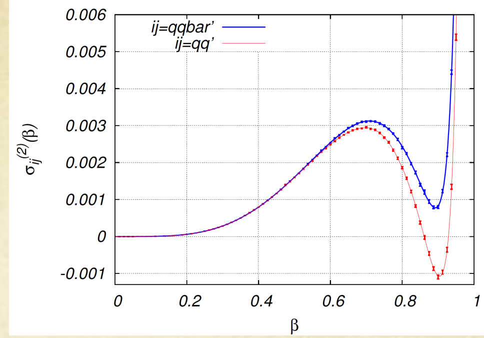


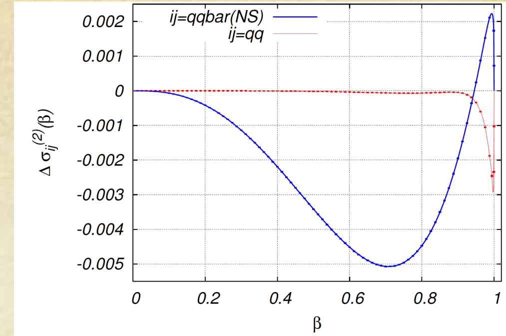


✧ Had to compute up to beta=0.9999 to get the high-energy behavior right.

Results @ parton level:
The all-fermionic reactions

$$
\begin{aligned}
q\bar{q} &\to t\bar{t} + q\bar{q}\big|_{\rm NS}\,,\\
q\bar{q}' &\to t\bar{t} + q\bar{q}'\,,\\
qq' &\to t\bar{t} + qq'\,,\\
qq &\to t\bar{t} + qq\,.
\end{aligned}
$$





The interesting feature: high-energy logarithmic rise:

$$
\sigma^{(2)}_{f_1 f_2 \to t\bar{t} f_1 f_2}\Big|_{\rho\to 0} \approx c_1 \ln(\rho) + c_0 + \mathcal{O}(\rho) \qquad \rho = \frac{4m_t^2}{s}
$$

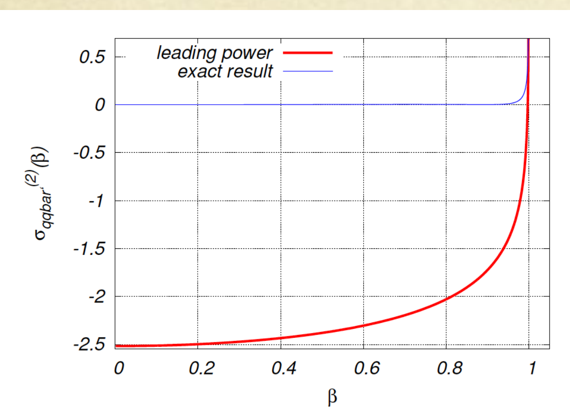
$$
c_1 = -0.4768323995789214
$$

Known analytically

Ball, Ellis `01

$$
c_0 \text{ (from Eqs. (6.3, 6.4))} = \begin{cases} -2.5173 & \text{from} \quad \sigma^{(2)}_{q\bar{q}'} \\ -2.5186 & \text{from} \quad \sigma^{(2)}_{qq'} \end{cases}
$$

- Direct extraction from the fits.
  5% uncertainty. Czakon, Mitov '12
- Agrees with independent prediction.
  50% uncertainty. Moch, Uwer, Vogt '12



High-energy expansion non-convergent.

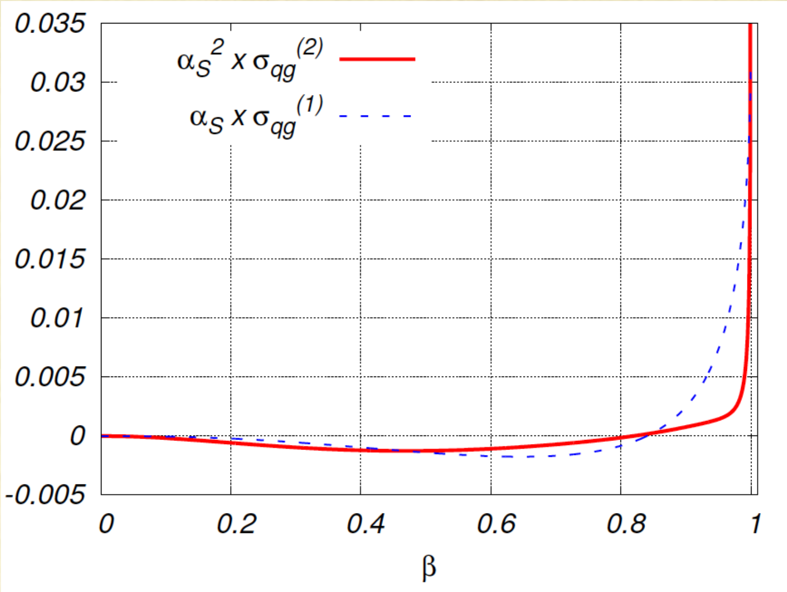
Applies only to the high-energy limit.

	Tevatron	LHC 7 TeV	LHC 8 TeV	LHC 14 TeV
$\Delta\sigma_{q\bar{q},(\rm NS)}$ [pb]	-0.0020	-0.0097	-0.0124	-0.0299
$\sigma_{q\bar{q},(\rm NS)}$ [pb]	-0.0009	-0.0001	0.0021	0.0464
$\sigma_{\rm all}$ [pb]	0.0003	0.0970	0.1504	0.7885
$\sigma_{\rm tot}$ [pb]	7.0056	154.779	220.761	852.177

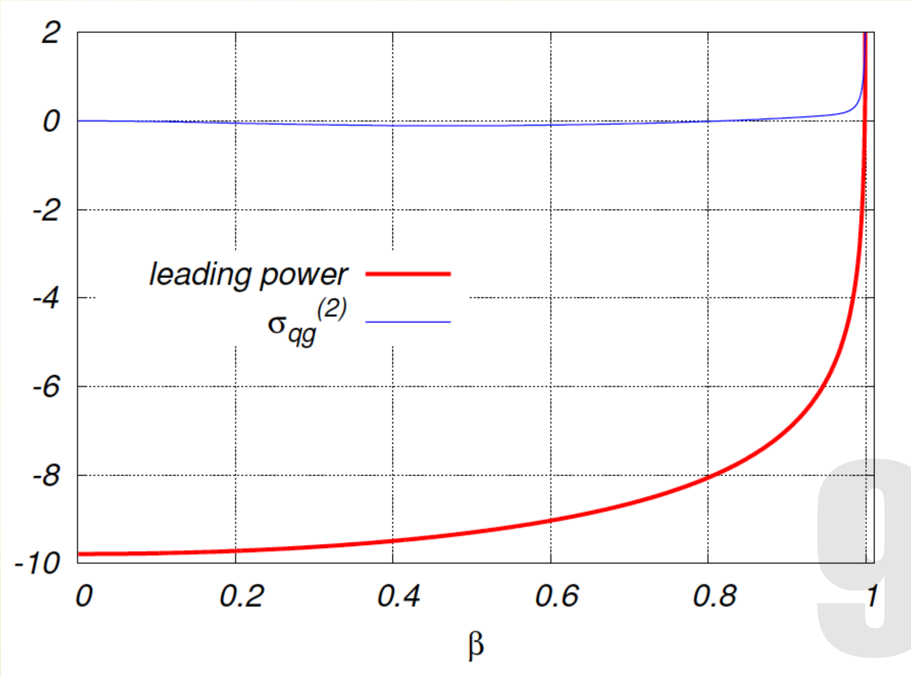
Czakon, Mitov '12

Results @ parton level:qg -> ttbar +X

Czakon, Mitov `12



- ✓ Correction about -1% (Tev and LHC).
- ✓ Notable decrease of scale dependence at LHC.
- ✓ NNLO large compared to NLO.



- ✓ High-energy log-limit correct

Ball, Ellis `01

- ✓ Agree for the constant with

Moch, Uwer, Vogt '12

- ✓ The limit itself plays no Pheno role

Checking the high-energy limit approximation

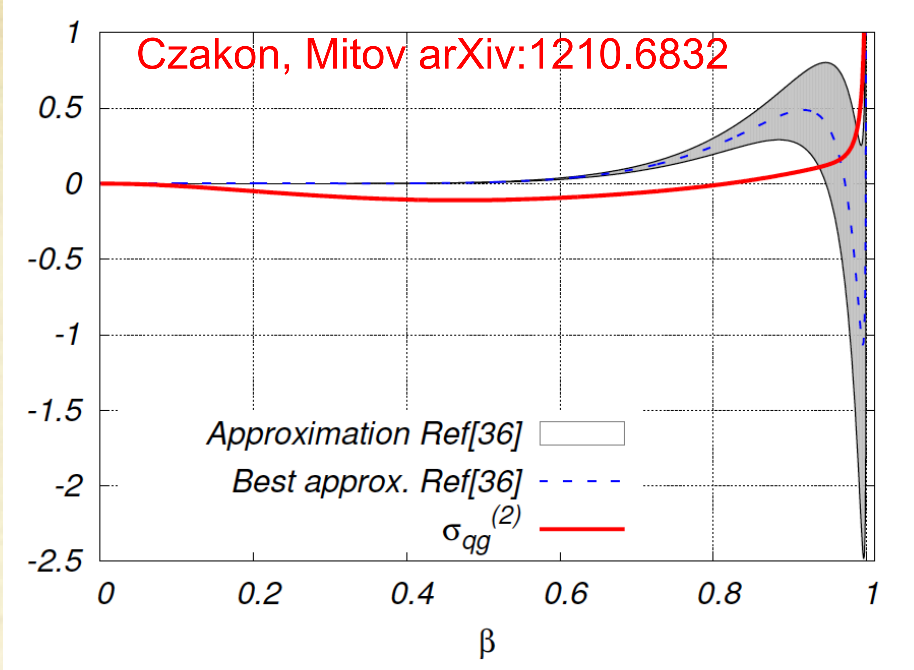
- ✓ It was suggested to use the high-energy limit of the X-section to predict it everywhere:

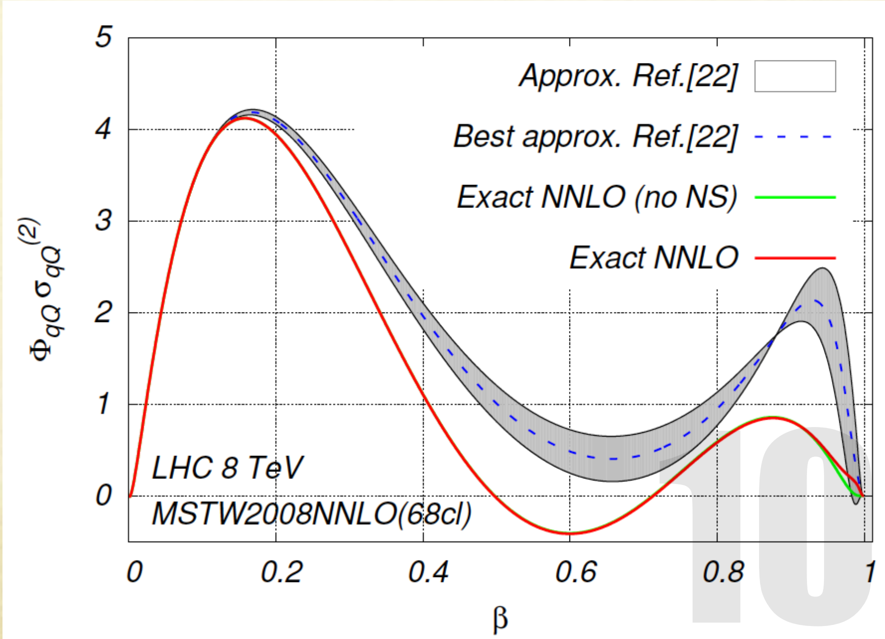
Moch, Uwer, Vogt ’12

- ✓ MUV approximation dramatically deviates from the exact gq NNLO result

- ✓ Leads to large difference for the x-section O(5%) from gq alone !

- ✓ Similar deviation for qq->tT+X (flux included)





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Precision phenomenological applications

Prediction at NNLO+ resummation (NNLL)

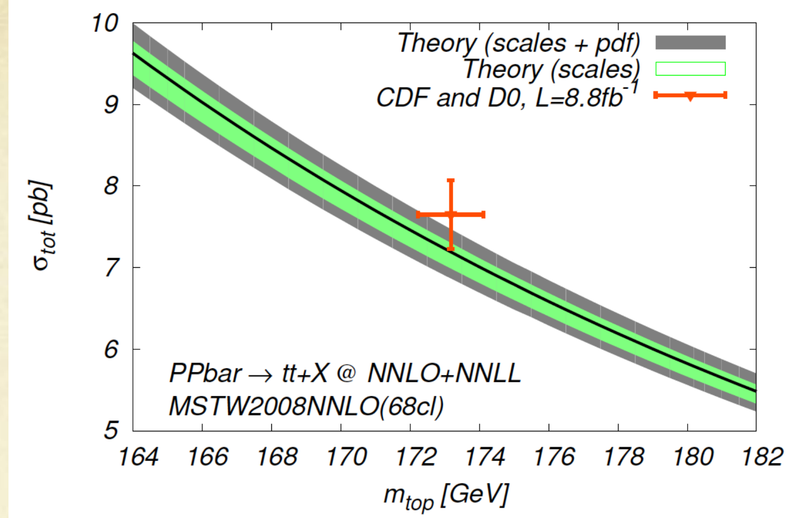
Collider	σ_{tot} [pb]	scales [pb]	pdf [pb]
Tevatron	7.164	$+0.110(1.5\%)$ $-0.200(2.8\%)$	$+0.169(2.4\%)$ $-0.122(1.7\%)$
LHC 7 TeV	172.0	$+4.4(2.6\%)$ $-5.8(3.4\%)$	$+4.7(2.7\%)$ $-4.8(2.8\%)$
LHC 8 TeV	245.8	$+6.2(2.5\%)$ $-8.4(3.4\%)$	$+6.2(2.5\%)$ $-6.4(2.6\%)$
LHC 14 TeV	953.6	$+22.7(2.4\%)$ $-33.9(3.6\%)$	$+16.2(1.7\%)$ $-17.8(1.9\%)$

Pure NNLO

Collider	σ_{tot} [pb]	scales [pb]	pdf [pb]
Tevatron	7.009	$+0.259(3.7\%)$ $-0.374(5.3\%)$	$+0.169(2.4\%)$ $-0.121(1.7\%)$
LHC 7 TeV	167.0	$+6.7(4.0\%)$ $-10.7(6.4\%)$	$+4.6(2.8\%)$ $-4.7(2.8\%)$
LHC 8 TeV	239.1	$+9.2(3.9\%)$ $-14.8(6.2\%)$	$+6.1(2.5\%)$ $-6.2(2.6\%)$
LHC 14 TeV	933.0	$+31.8(3.4\%)$ $-51.0(5.5\%)$	$+16.1(1.7\%)$ $-17.6(1.9\%)$

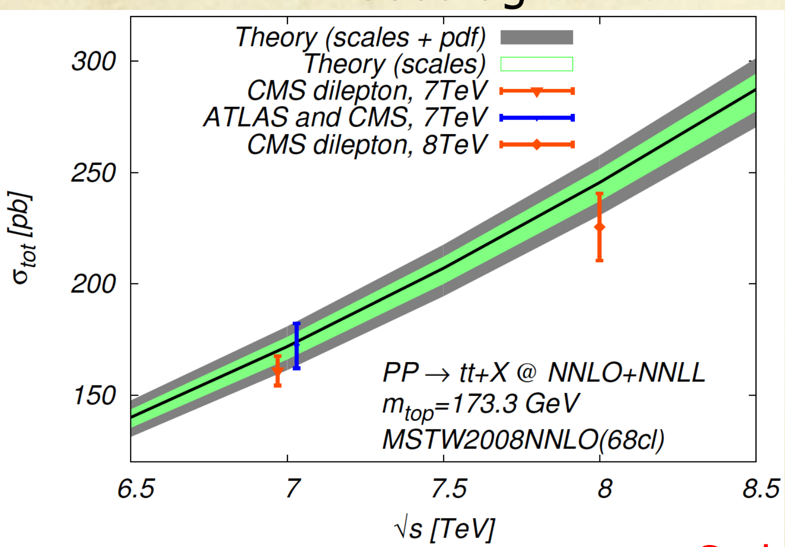
Czakon, Fiedler, Mitov '13

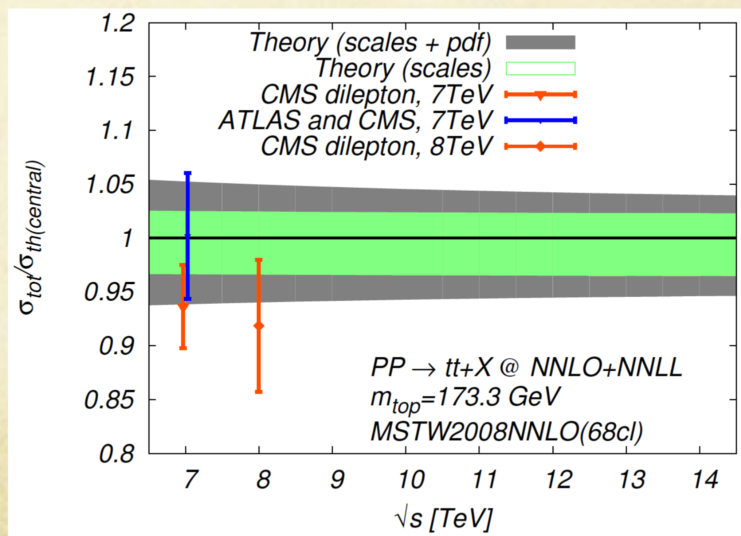
Good agreement with Tevatron measurements



- ✓ Independent F/R scales
- ✓ MSTW2008NNLO
- ✓ mt=173.3

Good agreement with LHC measurements

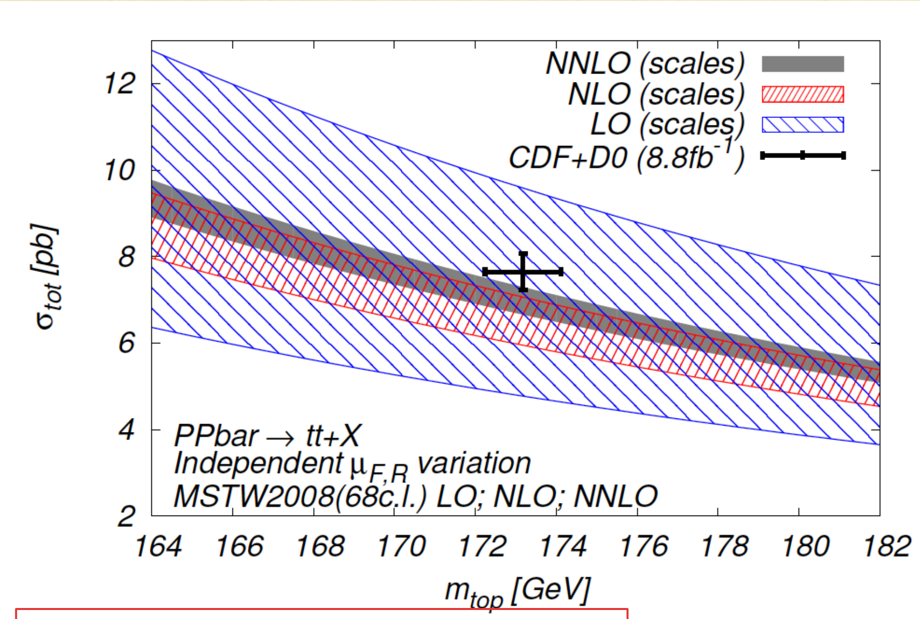




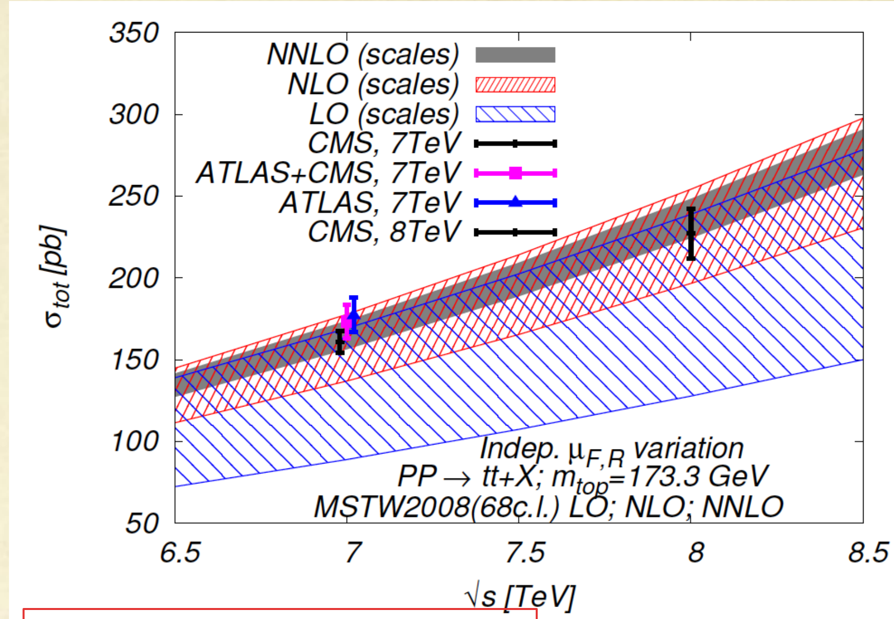
Czakon, Fiedler, Mitov '13

Good perturbative convergence

✓ Independent F/R scales variation



Scale variation @ Tevatron



Scale variation @ LHC

✓ Good overlap of various orders (LO, NLO, NNLO).

✓ Suggests the (restricted) independent scale variation is a good estimate of missing higher order terms!

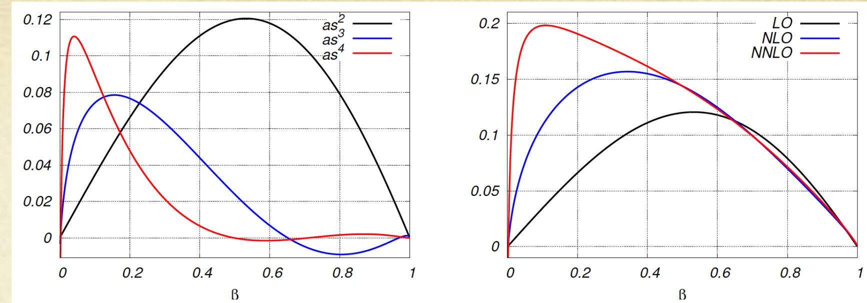
This is very important: good control over the perturbative corrections justifies less-conservative overall error estimate, i.e. more predictive theory (see next 2 slides).

For more detailed comparison, including soft-gluon resummation, see arXiv 1305.3892

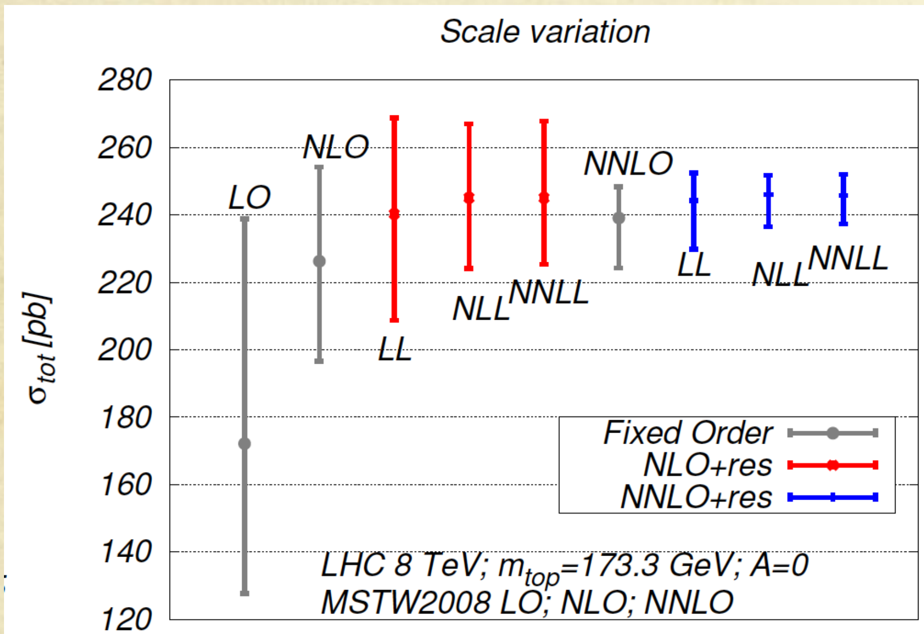
Quantifying soft-gluon resummation

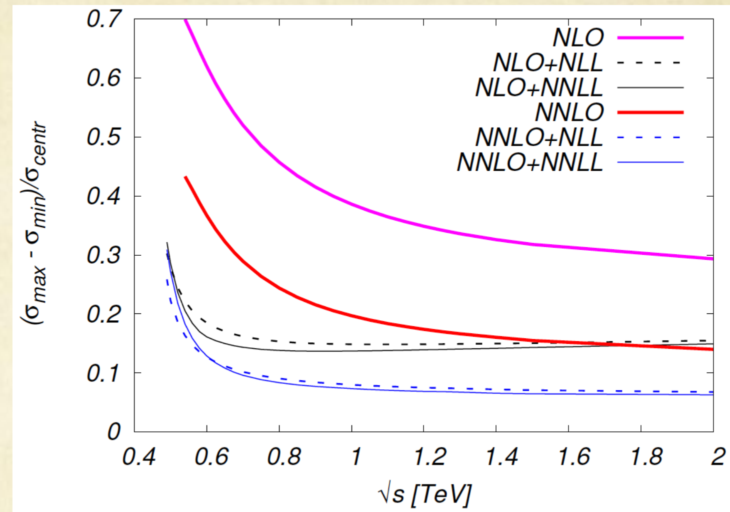
Partonic x-section's growth
close to threshold (qq reaction):

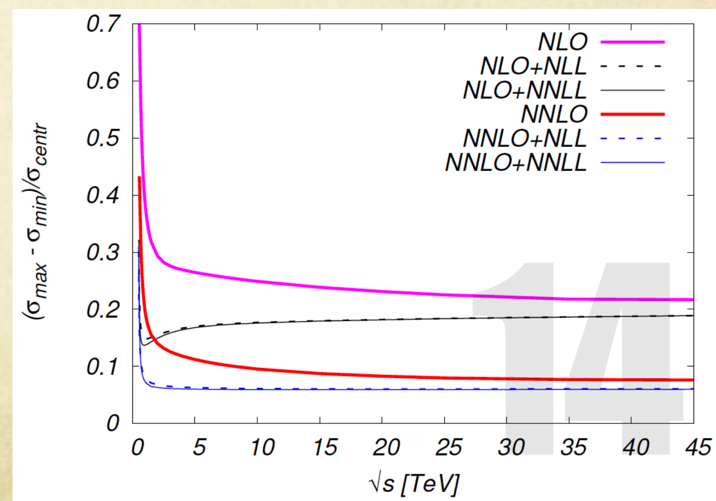
The expansion there is not converging
Resummation needed



$$\hat{\sigma}(\beta) = \frac{\alpha_S^2}{m^2}\left(\sigma^{(0)} + \alpha_S\sigma^{(1)} + \alpha_S^2\sigma^{(2)} + \dots\right) \equiv \frac{\alpha_S^2}{m^2}\left(f_{\alpha_S^2} + f_{\alpha_S^3} + f_{\alpha_S^4} + \dots\right)$$







The resummed results are better, as expected.

Update of: Cacciari, Czakon, Mangano, Mitov, Nason '11

LHC: general features at NNLO+NNLL

Czakon, Fiedler, Mitov '13

Czakon, Mangano, Mitov, Rojo '13

✓ We have reached a point of saturation: uncertainties due to

- ✓ scales (i.e. missing yet-higher order corrections) ~ 3%
- ✓ pdf (at 68%cl) ~ 2-3%
- ✓ α_S (parametric) ~ 1.5%
- ✓ m_{top} (parametric) ~ 3%

→ All are of similar size!

✓ Soft gluon resummation makes a difference: scale uncertainty 5% → 3%

✓ The total uncertainty tends to decrease when increasing the LHC energy

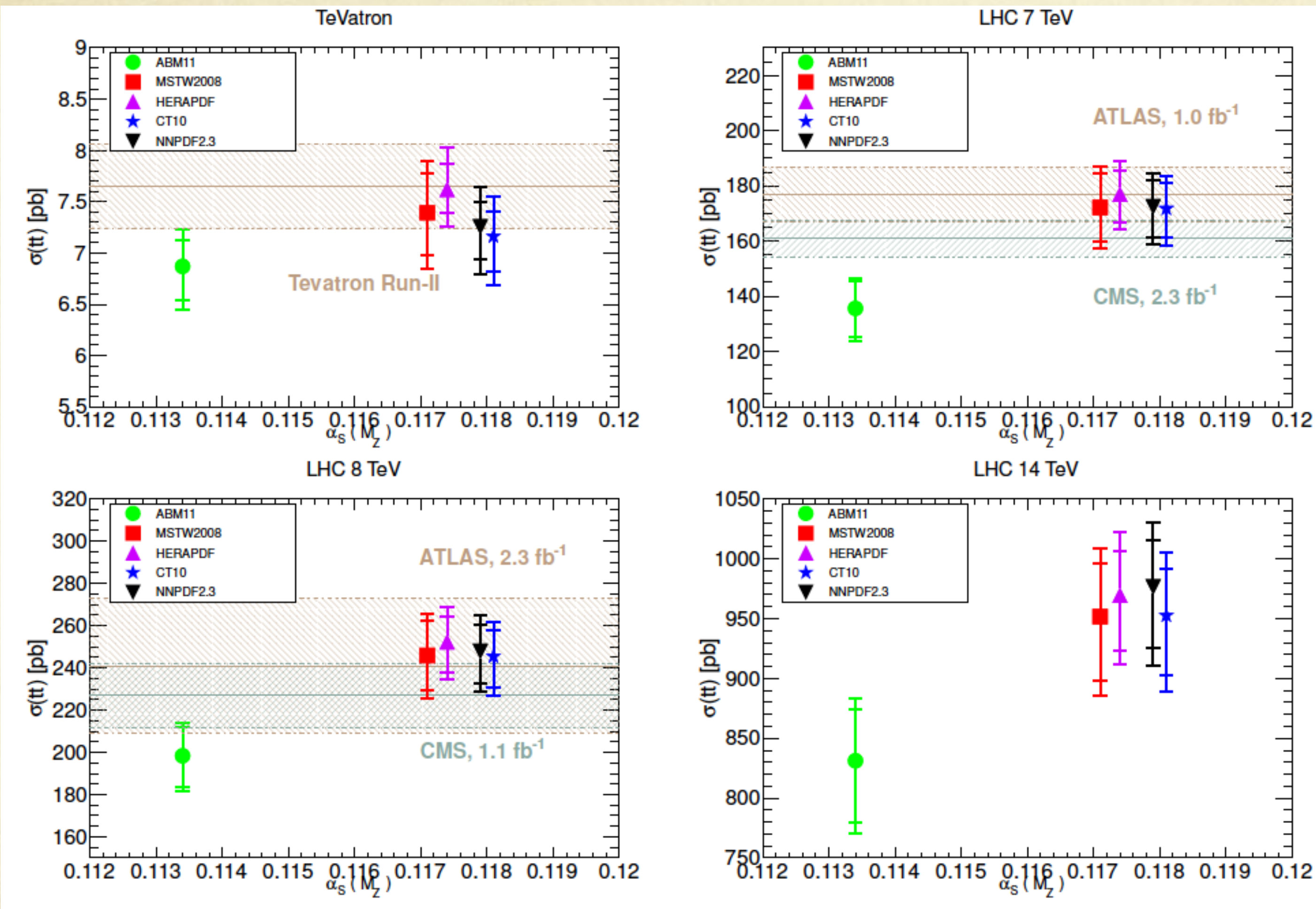
Application to PDF's

Czakon, Mangano, Mitov, Rojo '13

How existing pdf sets fare when compared to existing data?

Most conservative theory uncertainty:

Scales + pdf + α_S + mtop



Excellent agreement between almost all pdf sets

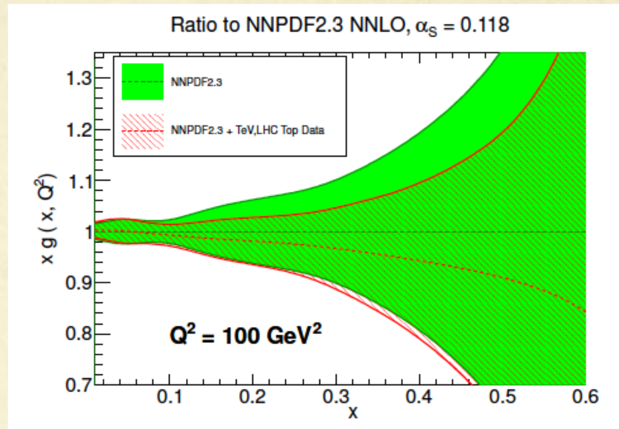
Application to PDF's

✓ tT offers for the first time a direct NNLO handle to the gluon pdf (at hadron colliders)

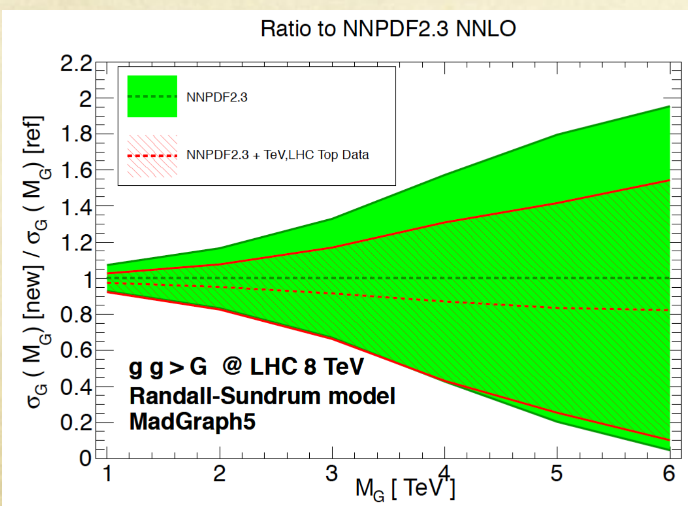
✓ Implications to many processes at the LHC: Higgs and bSM production at large masses

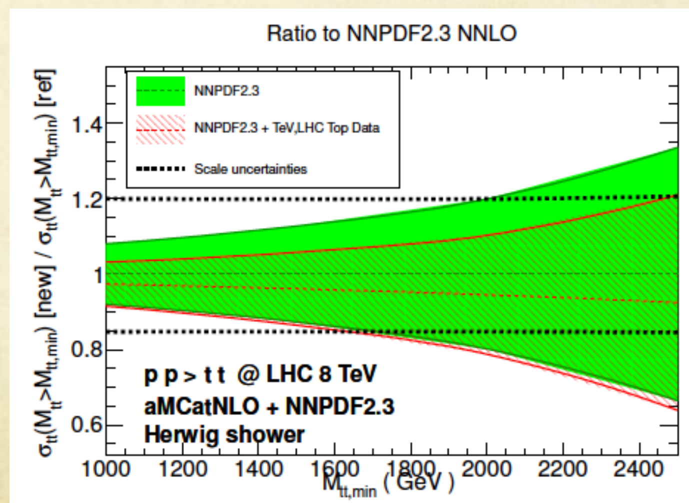
One can use the 5 available (Tevatron/LHC) data-points to improve gluon pdf

"Old" and "new" gluon pdf at large x:



… and PDF uncertainty due to "old" vs. "new" gluon pdf: Czakon, Mangano, Mitov, Rojo '13





Application to bSM searches: stealthy stop

- ✓ Scenario: stop → top + missing energy
  - ✓ m_stop small: just above the top mass.
  - ✓ Stop mass < 225 GeV is allowed by current data
  - ✓ Usual wisdom: the stop signal hides in the top background

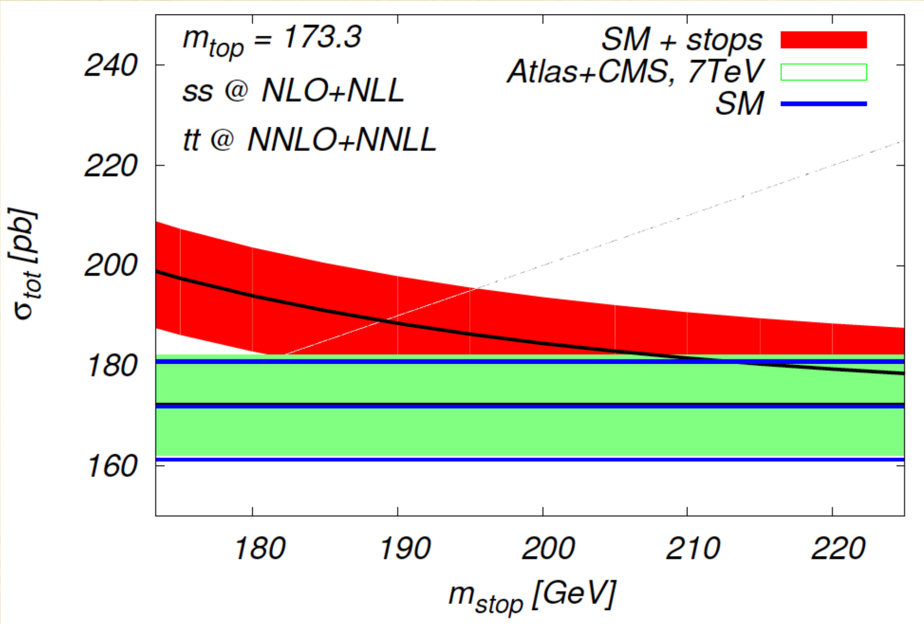
- ✓ The idea: use the top x-section to derive a bound on the stop mass. Assumptions:
  - ✓ Same experimental signature as pure tops
  - ✓ the measured x-section is a sum of top + stop
  - ✓ Use precise predictions for stop production @ NLO+NLL

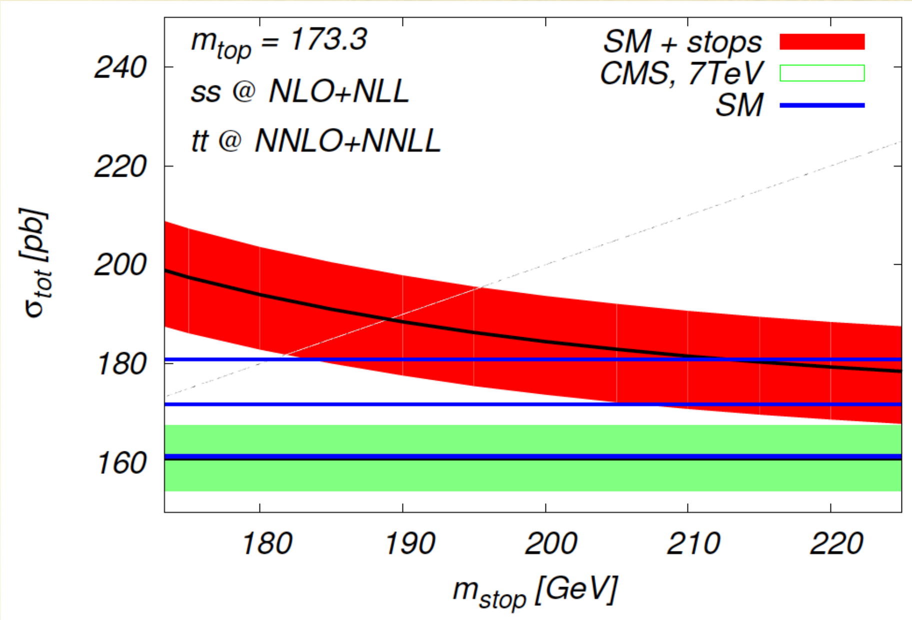
Krämer, Kulesza, van der Leeuw, Mangano, Padhi, Plehn, Portell `12

  - ✓ Total theory uncertainty: add SM and SUSY ones in quadrature.

Applications to the bSM searches: stealth stop

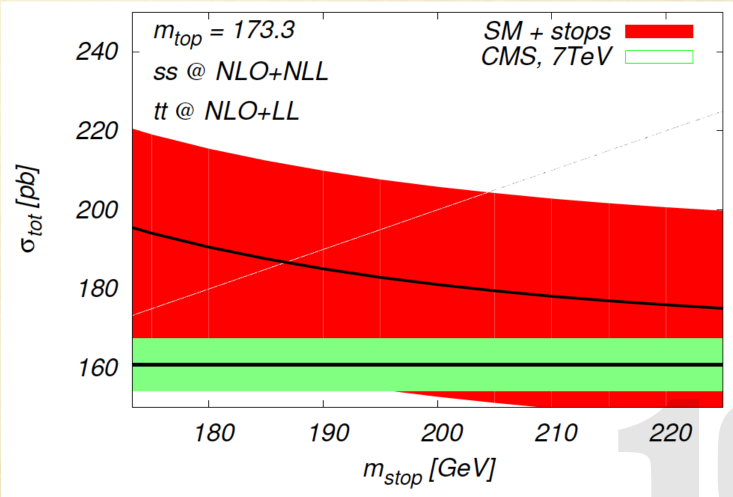
✓ Predictions Preliminary





Wonder why limits were not imposed before?

Here is the result with "NLO+shower" accuracy :



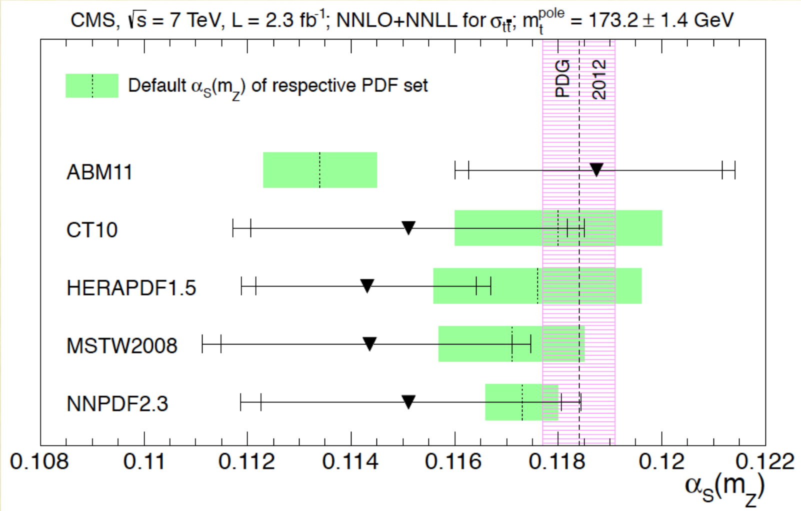
Improved NNLO accuracy makes all the difference

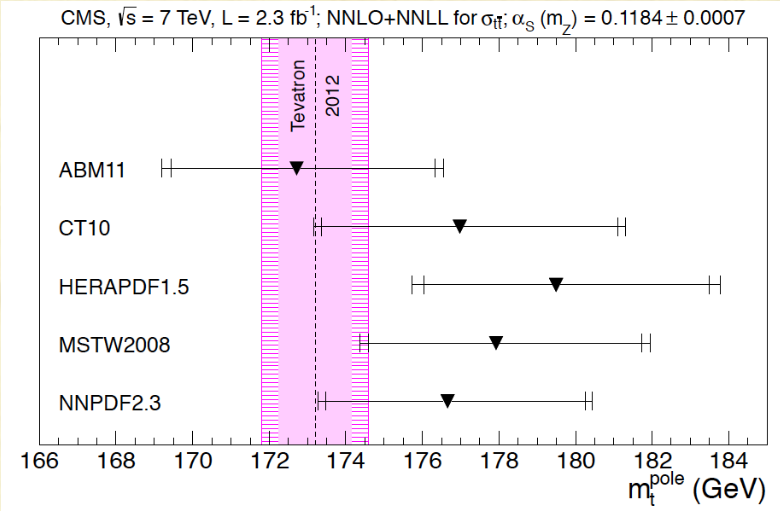
Currently refining the analysis (with Czakon, Papucci, Ruderman, Weiler)

Precision applications: α_S extraction

CMS Collaboration arXiv:1307.1907

- ✓ First experimental analysis at full NNLO+NNLL accuracy! It allows:
  - ✓ Extraction of α_S or m_{TOP} .
  - ✓ Self-consistency test of SM at the few % level.





- ✓ Four of the PDF sets return self-consistent results
- ✓ Recommendation: update the m_{TOP} extraction plot with the default value for α_S for each pdf set (better consistency that will change the plot)

Summary and Conclusions

- Total x-section for tT production now known in full NNLO
- Result of a number of theoretical innovations
- Small scale uncertainty (2.2% Tevatron, 3% LHC). Similar to uncertainties from pdf, α_S, M_{top}
- Important phenomenology
  - Constrain and improve PDF's
  - Searches for new physics
  - Very high-precision test of SM (given exp is already at 5% !). Good agreement.

Future tasks

- This is the beginning of a new stage in precision phenomenology
  - Differential top production, with decays (NWA)
  - H+1jet was already computed (expect related Z,W+jet) at NNLO

    Boughezal, Caola, Melnikov, Petriello, Schulze ‘13
  - Full dijet @ NNLO will become available too

    Gehrmann-De Ridder, Gehrmann, Glover, Pires ‘13
  - WW, etc.

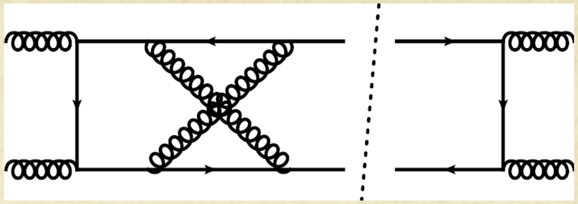
Towards 2-loop amplitudes: Gehrmann, Tancredi, Weihs ` 13

BACKUP SLIDES

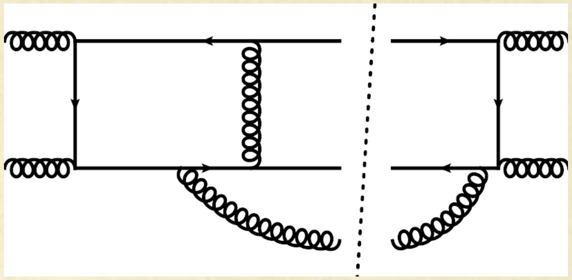
Calculation of the top-pair x-section at NNLO

What's needed for NNLO?

There are 3 principle contributions:

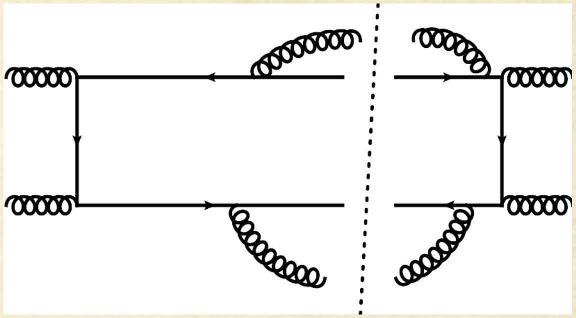


✓ 2-loop virtual corrections (V-V)



✓ 1-loop virtual with one extra parton (R-V)

✓ 2 extra emitted partons at tree level (R-R)



And 2 secondary contributions:

✓ Collinear subtraction for the initial state

Known, in principle. Done numerically.

✓ One-loop squared amplitudes (analytic)

May be avoided?

Korner, Merebashvili, Rogal `07
Anastasiou, Mert-Aybot `08

Weinzierl `11

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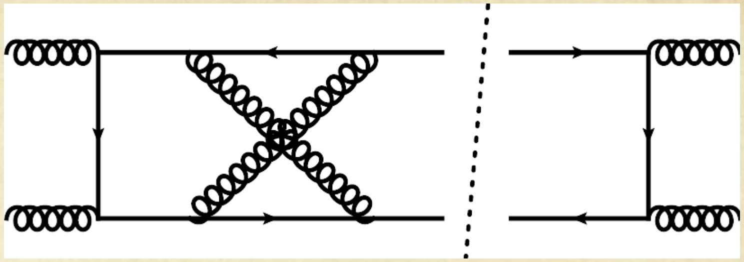
What's needed for NNLO? V-V

Required is the two-loop amplitude gg → QQ.



✓ Computed numerically

Bärnreuther, Czakon, Fiedler, to appear

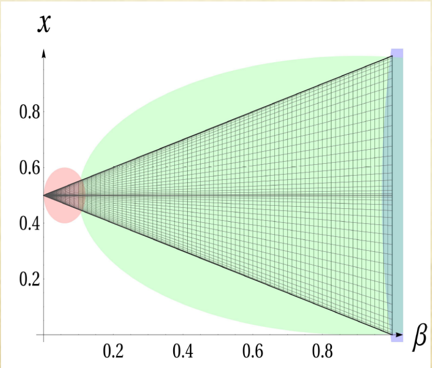
✓ (method similar to qq → QQ)

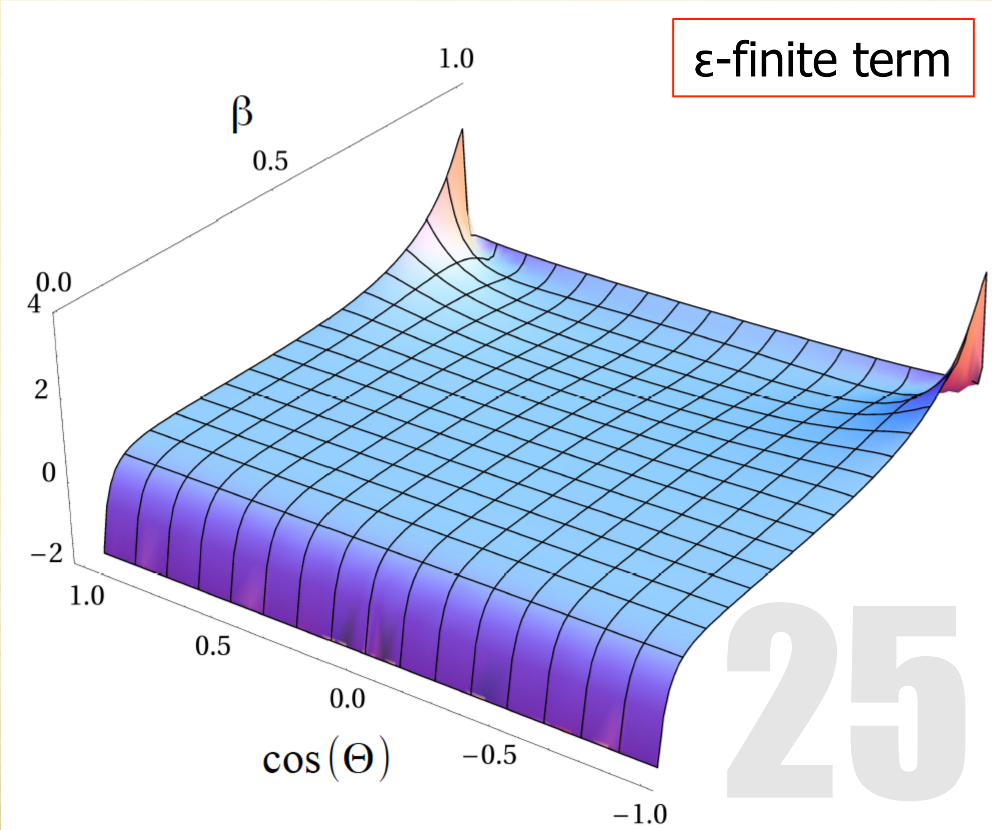
Czakon `07

System of 422 masters of 2 variables

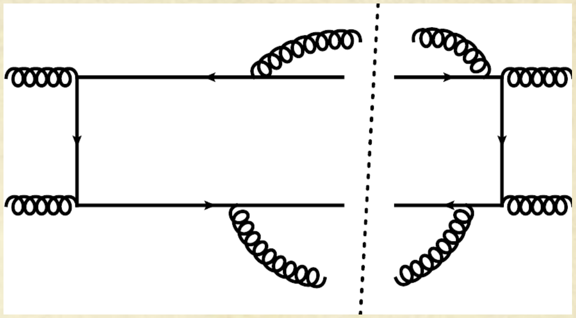
$$x \equiv \frac{m^2 - \hat{t}}{\hat{s}} = \frac{1}{2}(1 - \beta\cos(\Theta))$$

Integrated numerically





What's needed for NNLO? R-R

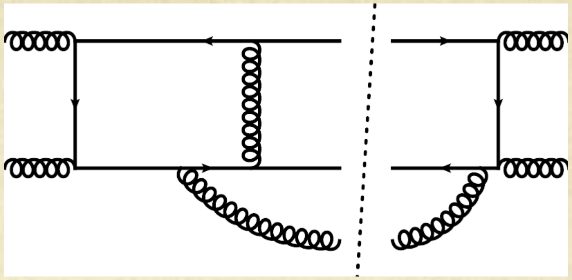


- ✓ A wonderful result By M. Czakon

Czakon `10-11

- ✓ The method is general (also to other processes, differential kinematics, etc).
- ✓ Explicit contribution to the total cross-section given.
- ✓ Just been verified in an extremely non-trivial problem.

What's needed for NNLO? R-V



✓ Counterterms all known (i.e. all singular limits)

Bern, Del Duca, Kilgore, Schmidt '98-99
Catani, Grazzini '00
Bierenbaum, Czakon, Mitov '11

The finite piece of the one loop amplitude computed with a private code of Stefan Dittmaier.

Extremely fast code!

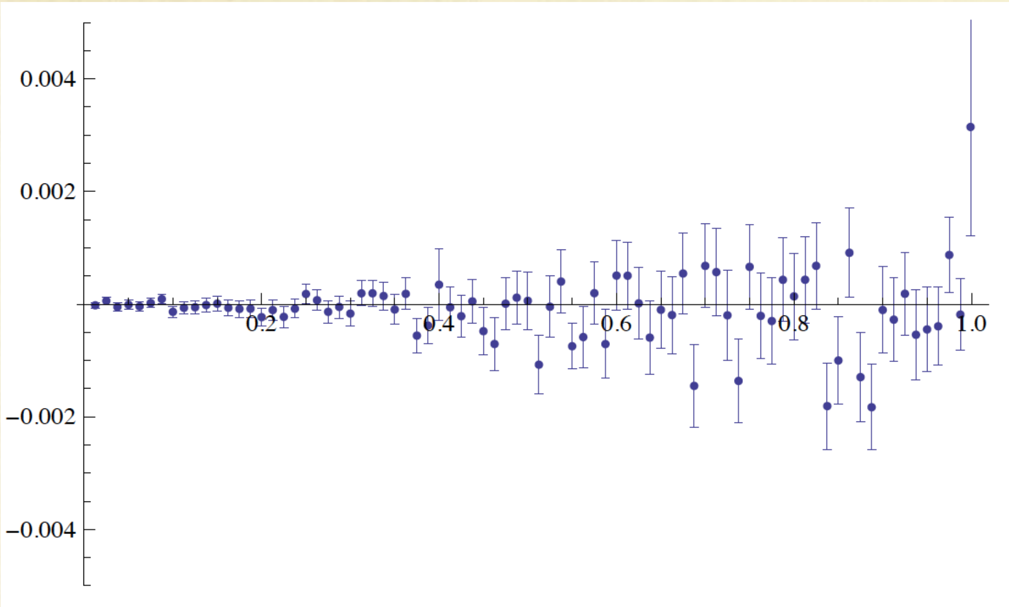
A great help!

Many thanks!

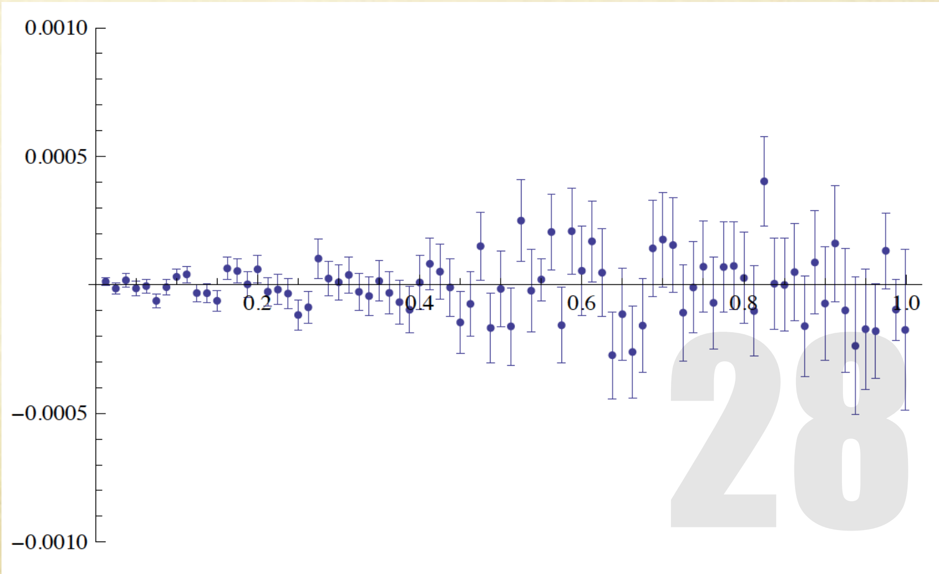
A note on the calculation

✓ Many details about the calculation were discussed in the talk by F. Caola

✓ Will only show the cancellation of the deepest singularity 1/ε in gg-> tt:



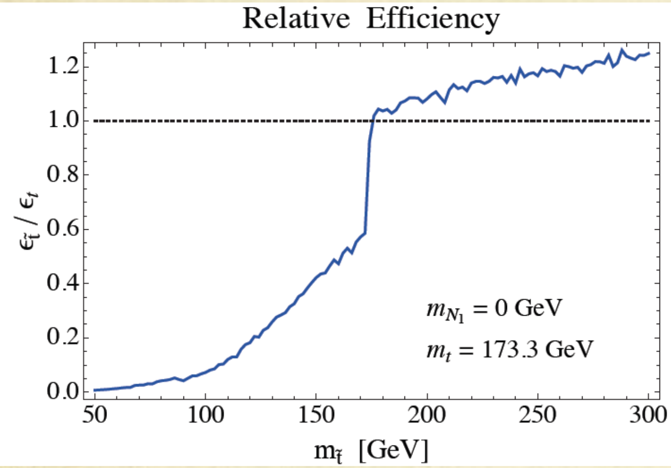
✓ And for $1/\varepsilon^2$ in gg-> tt:

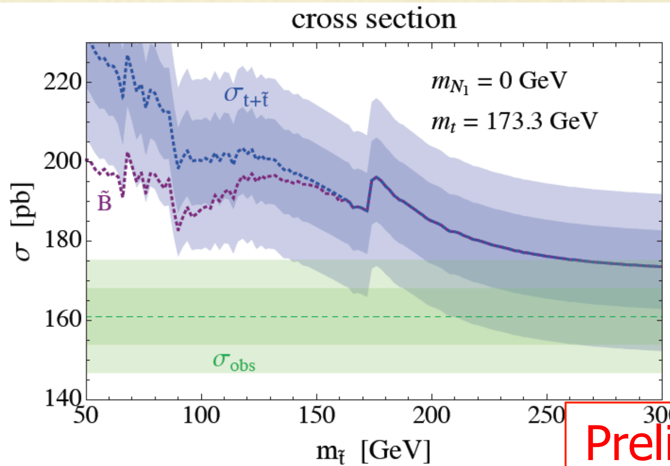


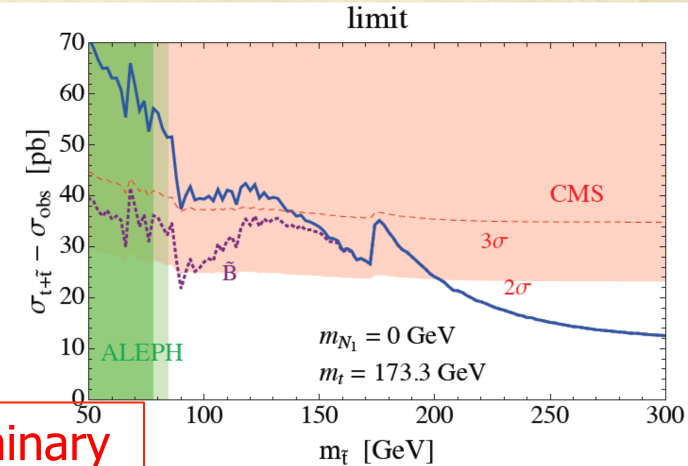
Stealthy Stops

Applications to bSM searches: stealth stop

Currently refining the analysis (with Czakon, Papucci, Ruderman, Weiler)

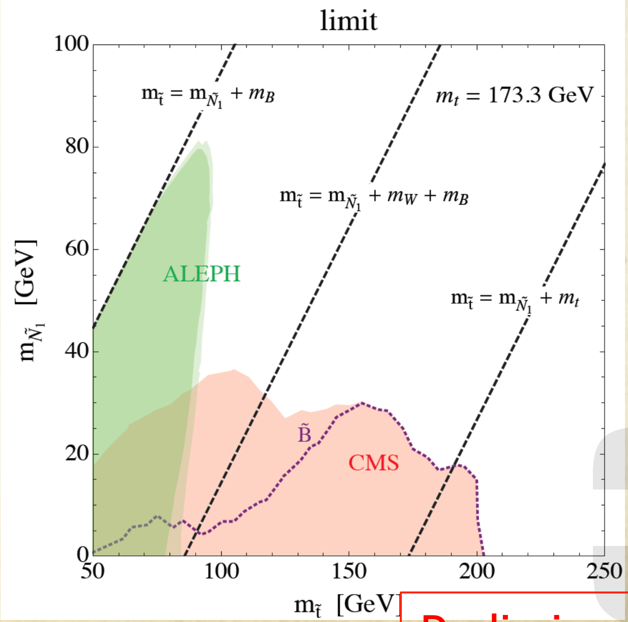






Preliminary

For the 7 TeV CMS dilepton (cut- and-count) measurement



Preliminary

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