Next-to-eikonal/soft corrections in QCD

Stress-testing the Standard Model at the LHC, KITP, Santa Barbara, May 2016

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in collaboration with

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[arXiv 1410.6406, arXiv:1503.05156]

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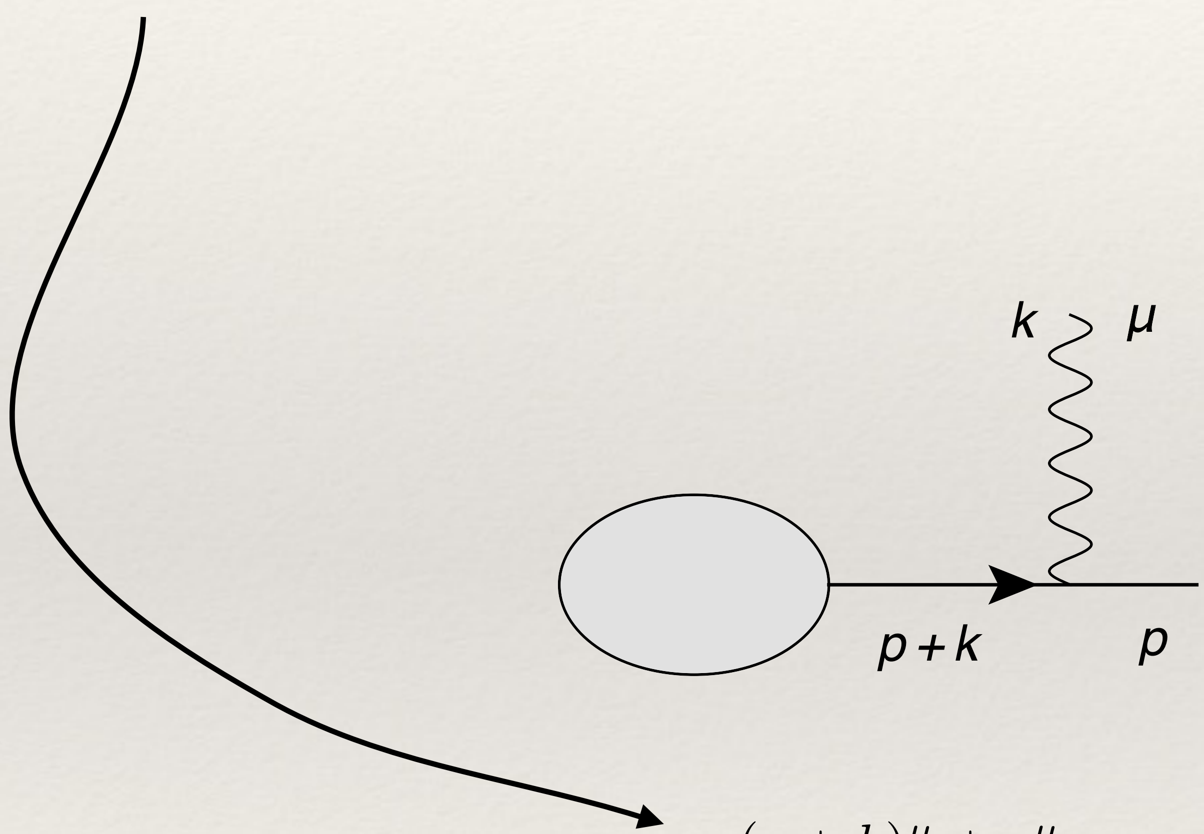
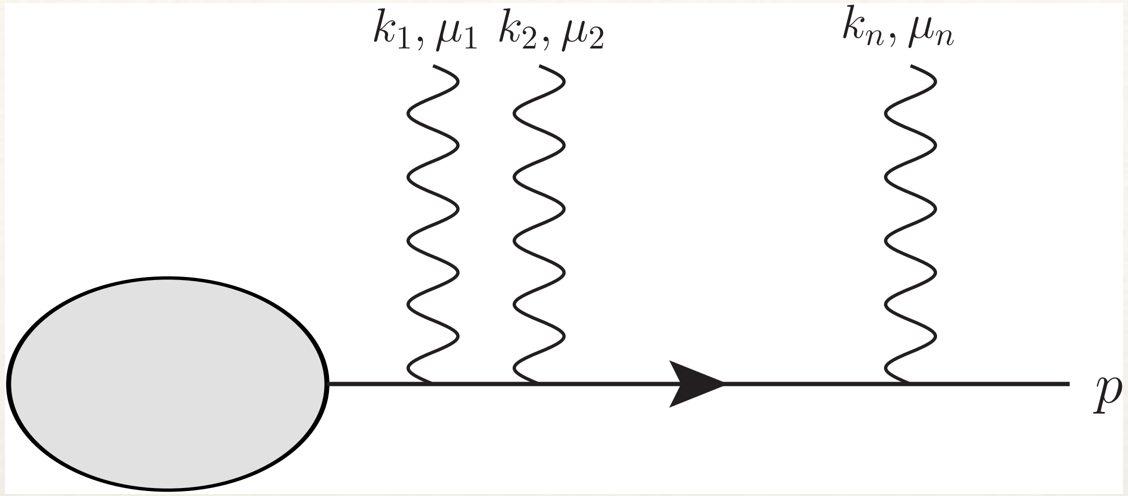
The eikonal approximation in QED

- Consider a charged particle emitting a soft photon
  - Propagator: expand numerator & denominator in soft momentum, keep lowest order
  - Vertex: expand in soft momentum, keep lowest order



$$\frac{(p+k)^\mu + p^\mu}{2p\cdot k + k^2} \longrightarrow \frac{2p^\mu}{2p\cdot k}$$

Eikonal QED



Exact: $$\frac{1}{(p+K_1)^2}(2p+K_2+K_1)^{\mu_1}\dots\frac{1}{(p+K_n)^2}(2p+K_n)^{\mu_n}, \quad K_i=\sum_{m=i}^{n}k_m.$$

Approx: $$\frac{1}{2pK_1}2p^{\mu_1}\dots\frac{1}{2pK_n}2p^{\mu_n}$$

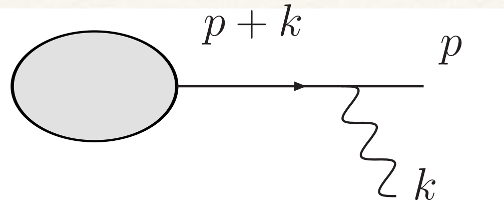
Eikonal identity: $$\frac{1}{p\cdot(k_1+k_2)\,p\cdot k_2}+\frac{1}{p\cdot(k_1+k_2)\,p\cdot k_1}=\frac{1}{p\cdot k_1\,p\cdot k_2}$$

Sum over all perm's: $$\prod_i \frac{p^{\mu_i}}{p\cdot k_i}.$$

Independent, uncorrelated emissions, Poisson process

Eikonal approximation: no dependence on emitter spin

- Emitter spin becomes irrelevant in eikonal approximation
  - Fermion



$$M \frac{i(\not{p}+\not{k})}{(p+k)^2}(-ig_s\gamma^\mu)u(p)$$

  - Approximate, and use Dirac equation $\not{p}u(p) = 0$ Result same as scalar case

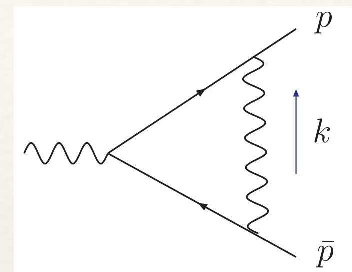
$$g\,(M\,u(p)) \times \frac{p^\mu}{p\cdot k}$$

    - No sign of emitter spin anymore
    - Coupling of photon proportional to p^μ

Eikonal exponentiation

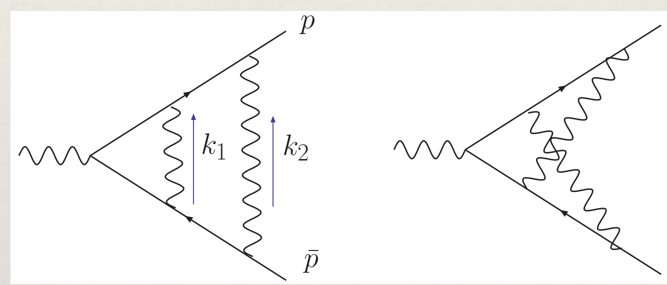
- In the eikonal approximation, interesting patterns emerge

One loop vertex correction, in eikonal approximation



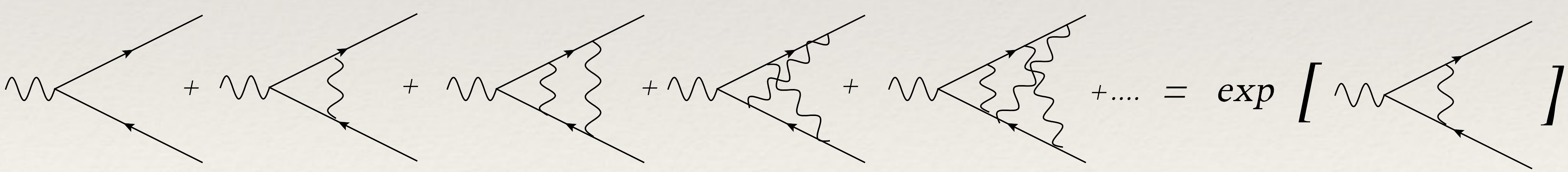
$$\mathcal{A}_0 \int d^n k \frac{1}{k^2} \frac{p \cdot \bar{p}}{(p \cdot k)(\bar{p} \cdot k)}$$

Two loop vertex correction, in eikonal approximation



$$\mathcal{A}_0 \frac{1}{2} \left(\int d^n k \frac{1}{k^2} \frac{p \cdot \bar{p}}{(p \cdot k)(\bar{p} \cdot k)} \right)^2$$

Exponential series!



Yennie, Frautschi, Suura

Exponentiation in QED from path integral

EL, Stavenga, White

Textbook result

$$\text{Sum of all diagrams} = \exp\Big(\text{Connected diagrams}\Big)$$

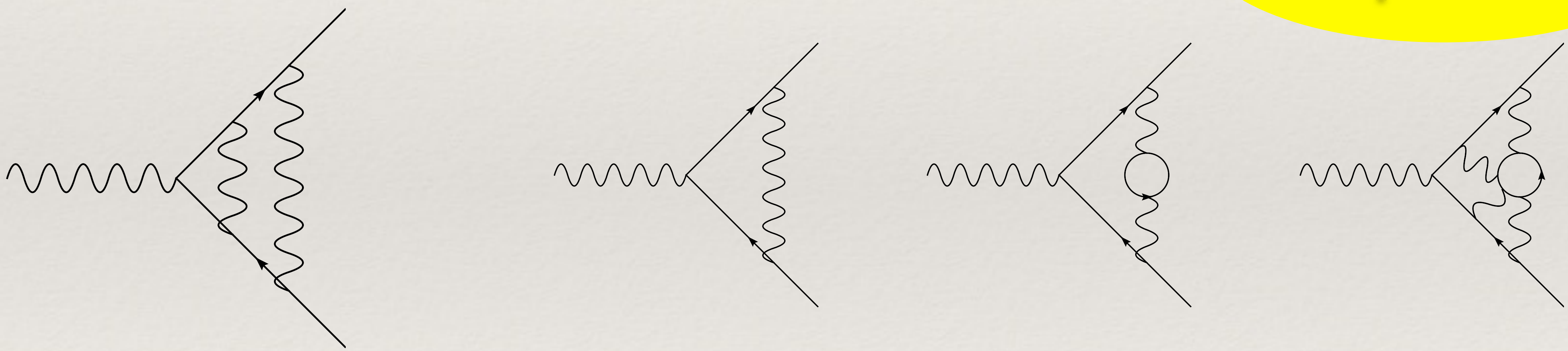
$$f = e^{i\int dt\,(\frac{1}{2}\dot{x}^2 + p\cdot A + ..)}$$

Can write scattering amplitude as nested path integral

$$M(p_1, p_2, \{k\}) = \int \mathcal{D}A_s\,\mathcal{D}x(t)\,H[x]\,f_1[A_s, x(t)]\,f_2[A_s, x(t)]\,e^{iS[A_s]}$$

x(t): path of charged particle

Eikonal vertices: sources for gauge bosons living on lines

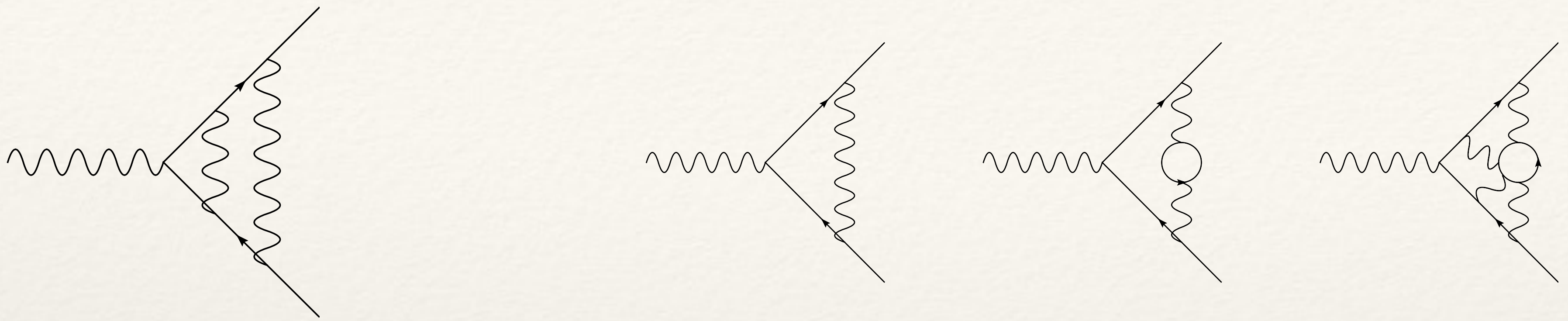


Disconnected

Connected

Exponentiation in QCD from path integral

EL, Stavenga, White



- Not immediately obvious how this could work:
  - Source terms have non-abelian charges
  - External line factors are path-ordered exponentials
- Exponentation still works

$$\sum_D \mathcal{F}_D C_D = \exp\left[\sum_i \bar{C}_i w_i\right]$$

Gatheral; Frenkel, Taylor; Sterman

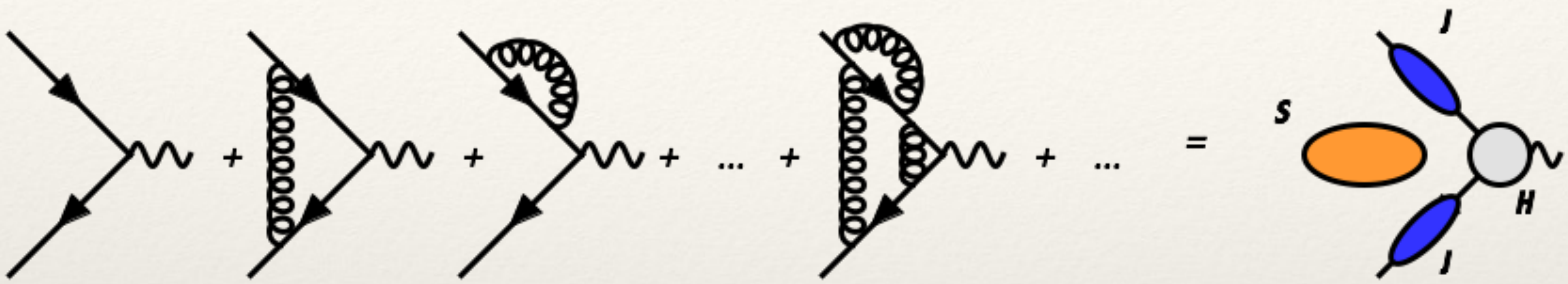
Modified color factors

Webs: subset of diagrams that go into exponent

  - Proof uses replica trick from statistical physics

More than eikonal: resummation for quark form factor

- Consider all corrections to the quark form factor



  - a diagrammatic analysis shows that the sum factorizes into a product of functions:
    - ✓ A soft function "S" (only IR/eikonal modes of loop momenta)
    - ✓ 2 jets functions "J" (collinear modes)
    - ✓ A hard functions "H" (off-shell, hard modes)
- These are also all the virtual diagrams for the Drell-Yan process
- Factorization implies resummation

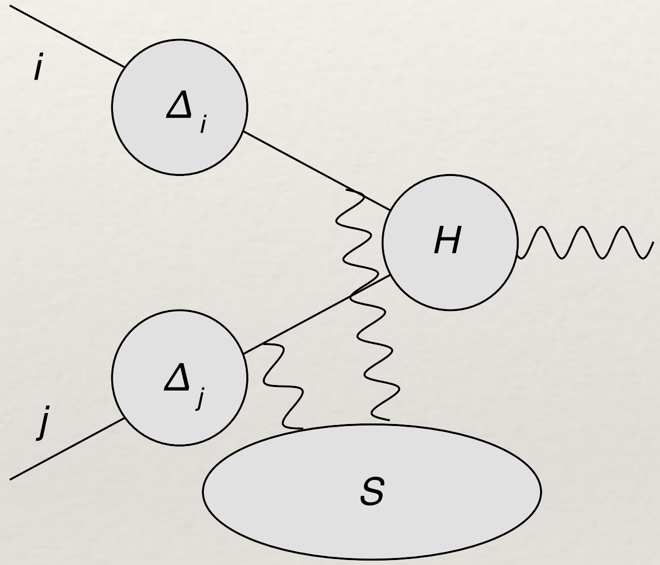
A. Sen; Collins; Magnea, Sterman

Factorization and resummation for Drell-Yan

$$\sigma(N) = \Delta(N, \mu, \xi_1)\Delta(N, \mu, \xi_2)S(N, \mu, \xi_1, \xi_2)H(\mu)$$

- Now with Mellin moment "N" dependence (i.e., with radiation)
- Demand independence of
  - renormalization scale μ
  - gauge dependence parameters $\xi_{1,2}$

$$0 = \mu\frac{d}{d\mu}\sigma(N) = \xi_1\frac{d}{d\xi_1}\sigma(N) = \xi_2\frac{d}{d\xi_2}\sigma(N)$$



  - result: double logarithms in exponent

$$\Delta = \exp[\int \frac{d\mu}{\mu} \int \frac{d\xi}{\xi}..]$$

Contopanagos, EL, Sterman
Forte, Ridolfi

- This can also be done through SCET

Bauer, Fleming, Pirjol, Stewart, Rothstein

Becher, Neubert, Xu; Idilbi, Ji

Generic large x (threshold) behavior

- For Drell-Yan, DIS, Higgs, singular behavior in perturbation theory when x→ 1

$$\delta(1-x) \qquad \left[\frac{\ln^i(1-x)}{1-x}\right]_+ \qquad \ln^i(1-x)$$

  - plus distributions have been organized to all orders (="resummation"), also possible for ln(1-x)?
- After Mellin transform $\quad \text{Constants} \quad \ln^i(N) \quad \frac{\ln^k(N)}{N}$
- "Zurich" method of threshold expansion allows computation (for NNNLO Higgs production)

$$(1-x)^p \ln^q(1-x)$$

  - at least to p=37

Anasthasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger

- Leading NLP logs resum

$$\exp\left[\int_0^1 dz\,(z^{N-1}-1)\frac{1+z^2}{1-z}\int_{\mu_F}^{Q(1-z)}\dots\right]$$

Kraemer, EL, Spira; Catani, De Florian, Grazzini; Kilgore, Harlander

Extended Drell-Yan threshold resummation

EL, Magnea, Stavenga   Gruenberg
Ball, Bonvini, Forte, Marzani, Ridolfi

Ansatz: modified resummed expression

$$\ln\left[\sigma(N)\right] = \mathcal{F}_{\rm DY}\left(\alpha_s(Q^2)\right) + \int_0^1 dz\, z^{N-1}\left\{\frac{1}{1-z}\, D\left[\alpha_s\left(\frac{(1-z)^2Q^2}{z}\right)\right] + 2\int_{Q^2}^{(1-z)^2Q^2/z}\frac{dq^2}{q^2}\, P_s\left[z,\alpha_s(q^2)\right]\right\}_+$$

where

$$P_s^{(n)}(z) = \frac{z}{1-z}A^{(n)} + C_\gamma^{(n)}\ln(1-z) + \overline{D}_\gamma^{(n)}$$

$$\sigma(N) = \sum_{n=0}^{\infty}\left(g^2\right)^n\left[\sum_{m=0}^{2n} a_{nm}\ln^m N + \sum_{m=0}^{2n-1} b_{nm}\frac{\ln^m N}{N}\right] + \mathcal{O}\left(N^{-2}\right)$$

	C_F^2		$C_A C_F$		$n_f C_F$	
b_{23}	4	4	0	0	0	0
b_{22}	$\frac{7}{2}$	4	$\frac{11}{6}$	$\frac{11}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
b_{21}	$8\zeta_2 - \frac{43}{4}$	$8\zeta_2 - 11$	$-\zeta_2 + \frac{239}{36}$	$-\zeta_2 + \frac{133}{18}$	$-\frac{11}{9}$	$-\frac{11}{9}$
b_{20}	$-\frac{1}{2}\zeta_2 - \frac{3}{4}$	$4\zeta_2$	$-\frac{7}{4}\zeta_3 + \frac{275}{216}$	$\frac{7}{4}\zeta_3 + \frac{11}{3}\zeta_2 - \frac{101}{54}$	$-\frac{19}{27}$	$-\frac{2}{3}\zeta_2 + \frac{7}{27}$

Upshot: close, but no cigar..

Back to basics: next-to-eikonal expansion

- Keep 1 term more in k expansion beyond eikonal approximation

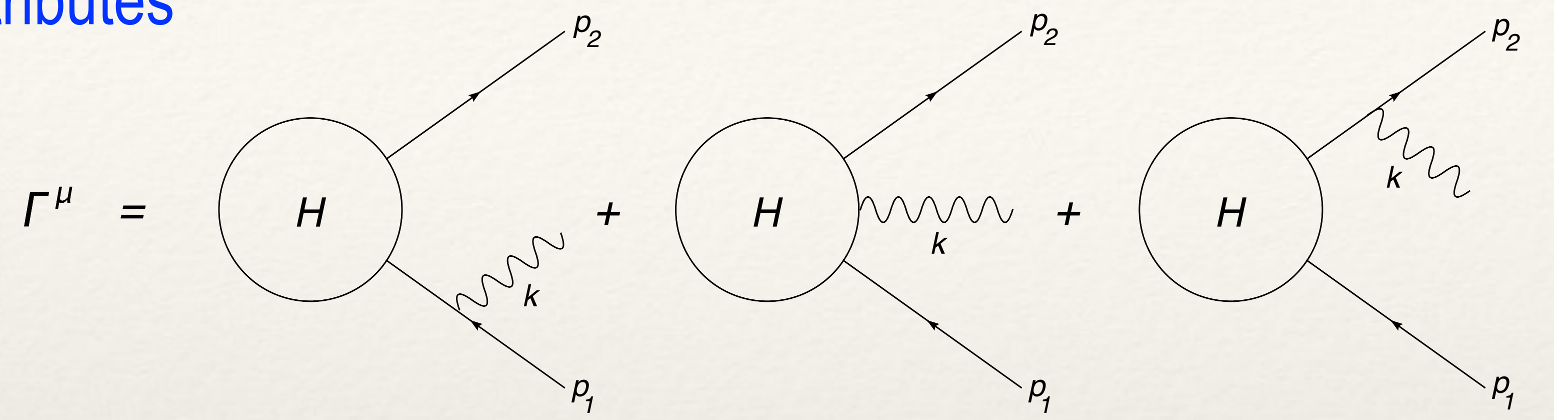
$$\text{scalar}: \quad \frac{2p^\mu + k^\mu}{2p\cdot k + k^2} \longrightarrow \frac{2p^\mu}{2p\cdot k} + \frac{k^\mu}{2p\cdot k} - k^2 \frac{2p^\mu}{(2p\cdot k)^2}$$

$$\text{fermion}: \quad \frac{\not{p} + \not{k}}{2p\cdot k + k^2}\gamma^\mu u(p) \longrightarrow \left[\frac{2p^\mu}{2p\cdot k} + \frac{\not{k}\gamma^\mu}{2p\cdot k} - k^2 \frac{2p^\mu}{(2p\cdot k)^2}\right] u(p)$$

  - Now emitter-spin dependent, and has recoil, decorrelation not obvious
  - Can we still make systematic statements (exponentiation, factorization) about next-to-eikonal/soft corrections?

Inspiration: Low's theorem

- Eikonal: only emission from external lines. At next-to-eikonal/soft order, also 1 "internal" emission contributes



- Low's theorem (for scalars, generalization to spinors by Burnett-Kroll, to massless particles by Del Duca → **LBKD theorem**)
  - ✓ Work to order k, and use Ward identity

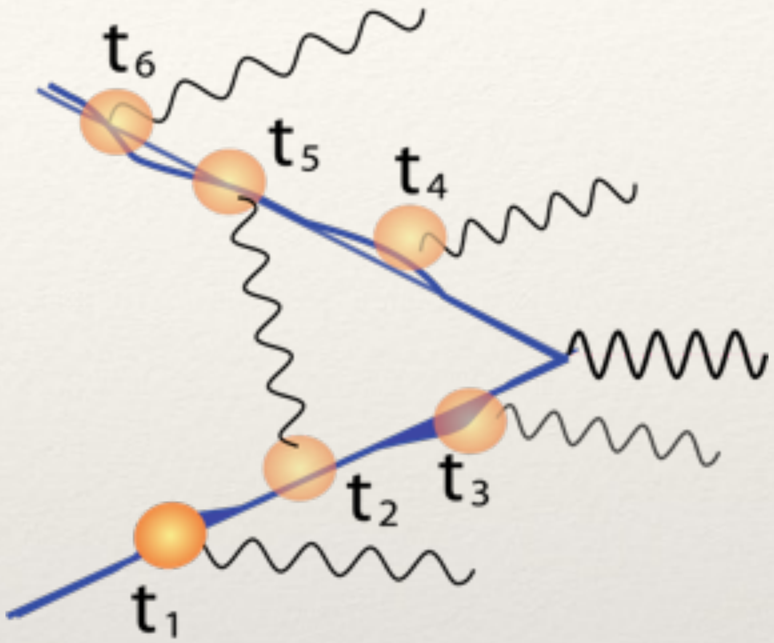
$$\Gamma^\mu = \left[\frac{(2p_1-k)^\mu}{-2p_1\cdot k} + \frac{(2p_2+k)^\mu}{2p_2\cdot k}\right]\Gamma + \left[\frac{p_1^\mu(k\cdot p_2 - k\cdot p_1)}{p_1\cdot k} + \frac{p_2^\mu(k\cdot p_1 - k\cdot p_2)}{p_2\cdot k}\right]\frac{\partial\Gamma}{\partial p_1\cdot p_2}$$

- Non-emitting amplitude still determines the emission to NE accuracy,
  - with a derivative
  - no detailed knowledge of internals needed

Next-to-eikonal exponentiation via path integral

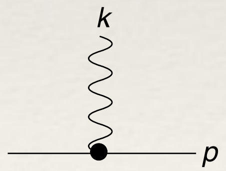
EL, Magnea, Stavenga, White

- Fluctuations around classical path are NE corrections
  - All NE corrections from external lines exponentiate
  - Keep track via scaling variable λ $p^\mu = \lambda n^\mu$

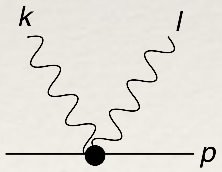


$$f(\infty) = \int_{x(0)=0} \mathcal{D}x \, \exp\left[i \int_0^\infty dt \left(\frac{\lambda}{2}\dot{x}^2 + (n+\dot{x})\cdot A(x_i + nt + x) + \frac{i}{2\lambda}\partial \cdot A(x_i + p_f t + x) \right) \right]$$

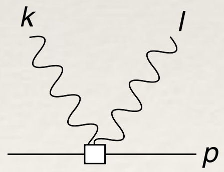
Use 1D field theory to (re)derive NE Feynman rules



$$\frac{k^\mu}{2p\cdot k} - k^2 \frac{p^\mu}{2(p\cdot k)^2}$$



$$+\frac{\eta^{\mu\nu}}{p\cdot(k+l)}$$



$$-\frac{l^\mu p^\nu p\cdot k + k^\nu p^\mu p\cdot l}{p\cdot(k+l)p\cdot k p\cdot l}$$

Exponentiation for NE webs

- Result from 1D path integral is NE Wilson line

EL, Magnea, Stavenga, White

$$\tilde{F}(\beta) = \exp\left[\int \frac{d^d k}{(2\pi)^d}\tilde{A}_\mu(k)\left(-\frac{\beta^\mu}{\beta\cdot k}+\frac{k^\mu}{2\beta\cdot k}-k^2\frac{\beta^\mu}{2(\beta\cdot k)^2}-\frac{ik_\nu\Sigma^{\nu\mu}}{p\cdot k}\right)\right.$$
$$+\int\frac{d^d k}{(2\pi)^d}\int\frac{d^d l}{(2\pi)^d}\tilde{A}_\mu(k)\tilde{A}_\nu(l)\left(\frac{\eta^{\mu\nu}}{2\beta\cdot(k+l)}-\frac{\beta^\nu l^\mu\beta\cdot k+\beta^\mu k^\nu\beta\cdot l}{2(\beta\cdot l)(\beta\cdot k)[\beta\cdot(k+l)]}\right.$$
$$\left.\left.+\frac{(k\cdot l)\beta^\mu\beta^\nu}{2(\beta\cdot l)(\beta\cdot k)[\beta\cdot(k+l)]}-\frac{\Sigma^{\mu\nu}}{2p\cdot k}\right)\right].$$

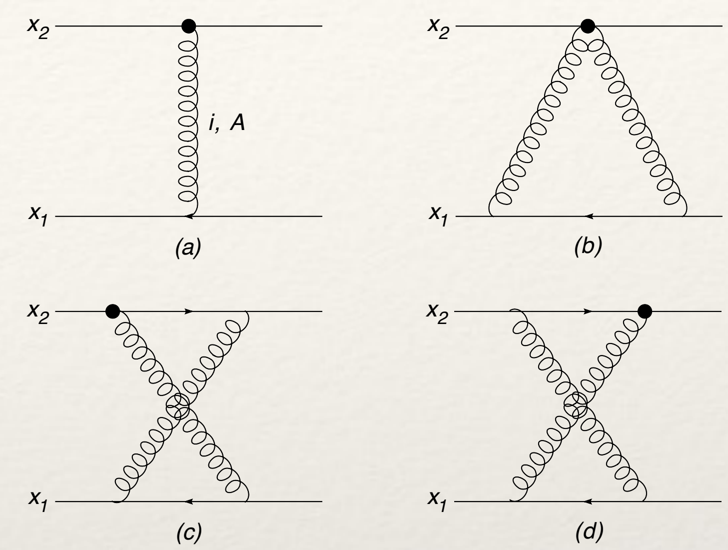
- Exponentation then in terms of NE Webs

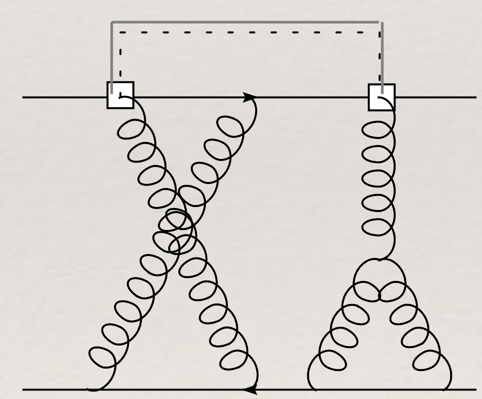
$$\sum C(D)\mathcal{F}(D) = \exp\left[\bar{C}(D)W_{\rm E}(D)+\bar{C}'(D)W_{\rm NE}(D)\right]$$

Next-to-eikonal webs

EL, Magnea, Stavenga, White

- Similar to eikonal webs, with next-to-eikonal vertices
  - Now spin-sensitive
  - New 2-gluon correlations between eikonal webs
- But (NE) webs are not the *only* source of next-to-soft corrections. Also need corrections from
  - hard function
  - collinear loop momenta





Next-to-eikonal logarithms

Vernazza, Bonocore, EL, Magnea, Melville, White

- Approach: understand NE corrections at amplitude level, then construct cross section
- Use NNLO Drell-Yan as stressor to predict NE (=NLP) logs
  - Leading power: known

$$\log^3(1-z)$$

  - Next-to-leading powers?

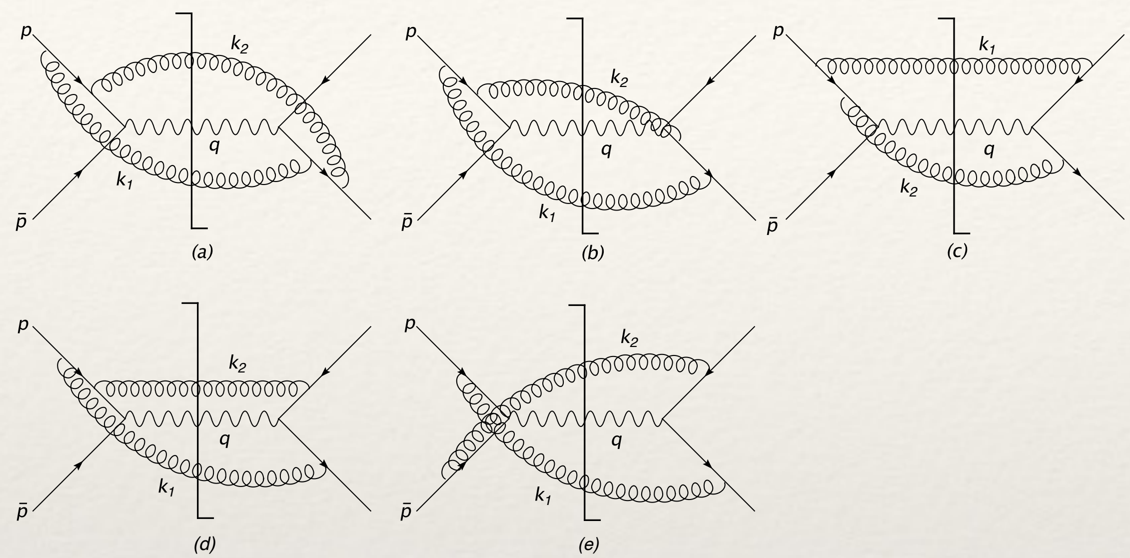
$$\log^i(1-z), \quad i = 2, 1, 0$$

    - Occur in double real emission, and one-real + one-virtual

NE logs in Drell-Yan : double real

EL, Magnea, Stavenga, White

- Check NE Feynman rules for NNLO Drell-Yan RR emission (C_F^2 only)



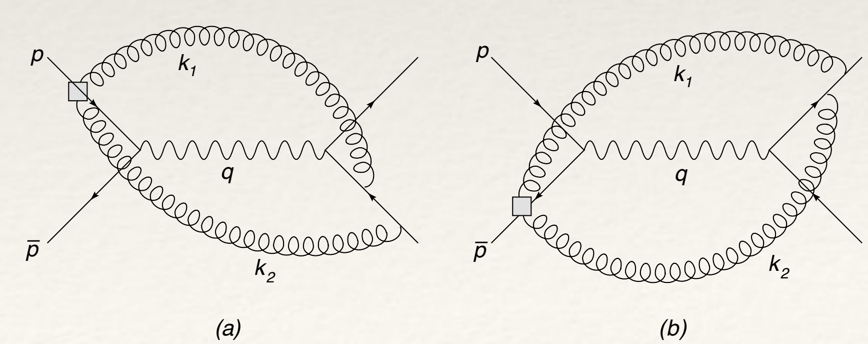
- Result at NE level agrees with exact result

$$K^{(2)}_{\rm NE}(z) = \left(\frac{\alpha_s}{4\pi}C_F\right)^2 \left[-\frac{32}{\epsilon^3}\mathcal{D}_0(z) + \frac{128}{\epsilon^2}\mathcal{D}_1(z) - \frac{128}{\epsilon^2}\log(1-z) - \frac{256}{\epsilon}\mathcal{D}_2(z) + \frac{256}{\epsilon}\log^2(1-z) - \frac{320}{\epsilon}\log(1-z) + \frac{1024}{3}\mathcal{D}_3(z) - \frac{1024}{3}\log^3(1-z) + 640\log^2(1-z)\right],$$

$$\mathcal{D}_i = \left[\frac{\log^i(1-z)}{1-z}\right]_+$$

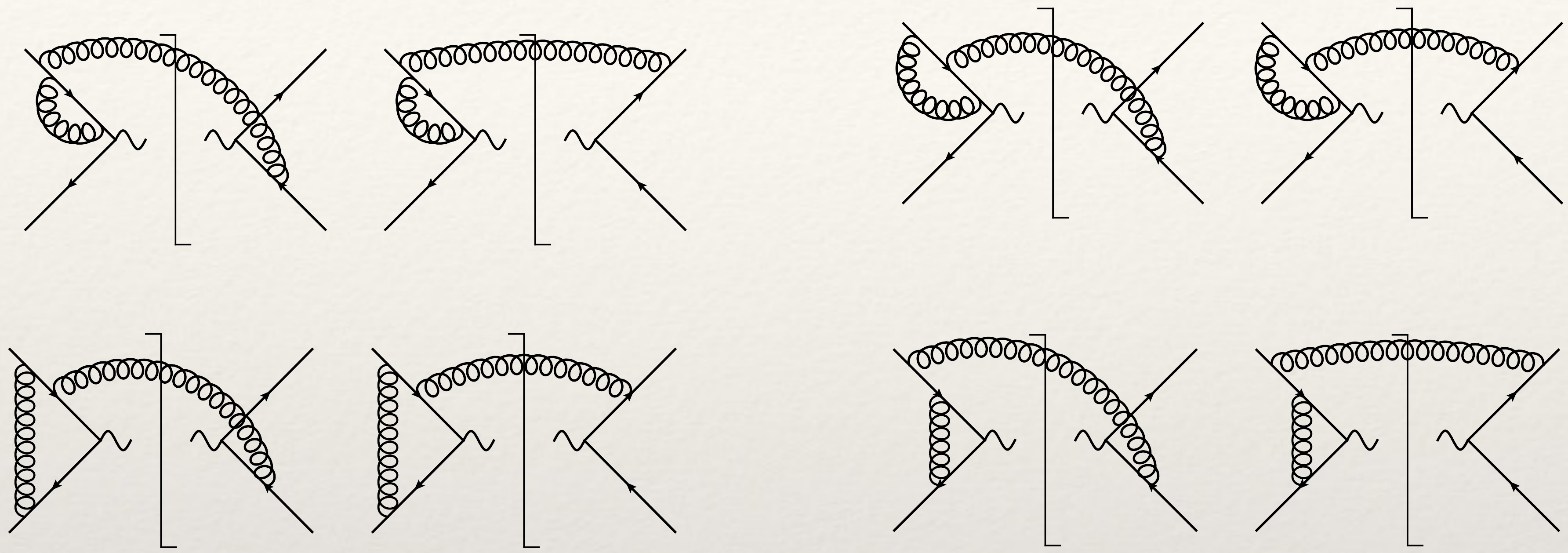
- Special 2-gluon correlation vertex gives zero

$$R^{\mu\nu}(p;k_1,k_2) = -\frac{(p\cdot k_2)p^\mu k_1^\nu + (p\cdot k_1)k_2^\mu p^\nu - (p\cdot k_1)(p\cdot k_2)g^{\mu\nu} - (k_1\cdot k_2)p^\mu p^\nu}{p\cdot(k_1+k_2)}$$



NE logs in Drell-Yan: one real - one virtual

- We must also consider also 1-real plus 1-virtual contributions



- More subtle: virtual momenta are not always (next-to)-soft. We follow two approaches:
  - method of regions
  - factorization

1 Real plus 1 Virtual, exact

- Redid exact calculation, keeping only C_F^2 terms
  - only the full result was known in the literature — Matsuura, van Neerven
  - result, up to constants (dropped higher powers of 1-z)

$$K^{(1)}_{1\mathrm{r},1\mathrm{v}} = \frac{32\mathcal{D}_0 - 32}{\epsilon^3} + \frac{-64\mathcal{D}_1 + 48\mathcal{D}_0 + 64L_1 - 96}{\epsilon^2} + \frac{64\mathcal{D}_2 - 96\mathcal{D}_1 + 128\mathcal{D}_0 - 196 - 64L_1^2 + 208L_1}{\epsilon}$$
$$- \frac{128}{3}\mathcal{D}_3 + 96\mathcal{D}_2 - 256\mathcal{D}_1 + 256\mathcal{D}_0 + \frac{128}{3}L_1^3 - 232L_1^2 + 412L_1 - 408, \quad (4.12)$$

$$\mathcal{D}_i = \left[\frac{\log^i(1-z)}{1-z}\right]_+ \qquad L_1 = \log(1-z)$$

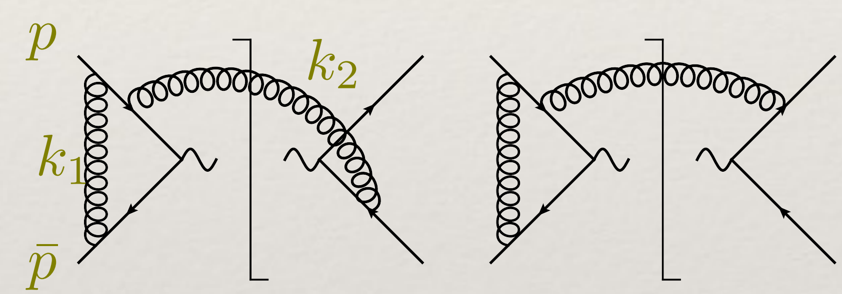
  - “bare” results, no renormalization or factorization counterterms

Method of regions

Bonocore, EL, Magnea, Melville, Vernazza, White

- Method of region approach, extended to next power
  - Should allow treatment of (next-to-)soft and (next-to-)collinear on equal footing
- Instructions: Beneke, Smirnov; Jantzen
  - Divide up k_1 (=loop-momentum) integral into hard, 2 collinear and a soft region, by appropiate scaling

$$\text{Hard}: \quad k_1 \sim \sqrt{\hat{s}}\,(1,1,1)\;; \qquad \text{Soft}: \quad k_1 \sim \sqrt{\hat{s}}\,(\lambda^2,\lambda^2,\lambda^2)\;;$$
$$\text{Collinear}: \quad k_1 \sim \sqrt{\hat{s}}\,(1,\lambda,\lambda^2)\;; \quad \text{Anticollinear}: \quad k_1 \sim \sqrt{\hat{s}}\,(\lambda^2,\lambda,1)\;.$$



  - expand integrand in λ, to leading and next-to-leading order in each region
  - **but then integrate over *all* k_1 anyway**
    - Treat emitted momentum as soft and incoming momenta as hard

$$k_2^\mu = (\lambda^2,\lambda^2,\lambda^2) \qquad p^\mu = \tfrac{1}{2}\sqrt{s}\,n_+^\mu \qquad \bar{p}^\mu = \tfrac{1}{2}\sqrt{s}\,n_-^\mu$$

Collinear(+anti-collinear) region

- Note: terms after loop integral

$$\frac{(-2p\cdot k_2)^{-\epsilon}}{\epsilon}, \qquad \frac{(-2\bar{p}\cdot k_2)^{-\epsilon}}{\epsilon}$$

  - When integrated over k_2, give the right log(1-z) terms
    - expand in ϵ before expanding in k_2!
    - illustrates again breakdown of original LBK theorem

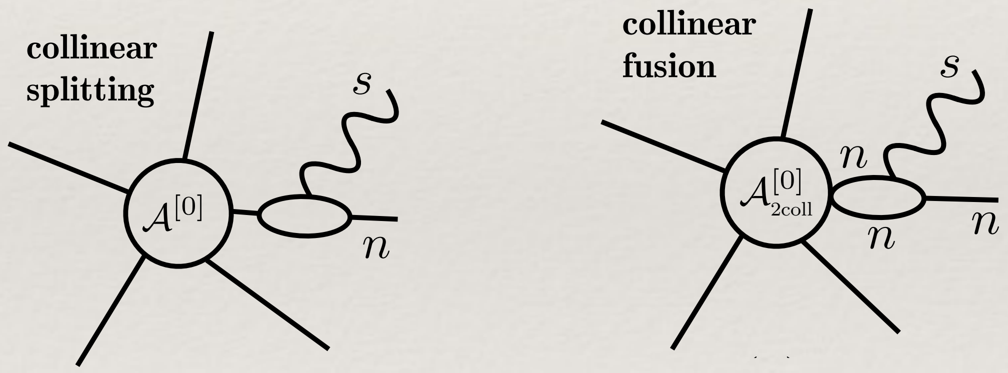
Method of regions upshot

- We find
  - Hard region (expansion in λ^2)
    - ✓ reproduces already all plus-distributions, and some NLP logarithms
  - Soft region (expansion in λ^2)
    - ✓ all integrals are scale-less, hence all zero in dimensional regularization
  - (anti-)collinear regions (expansion in λ)
    - ✓ only give NLP logarithms, once all diagrams in set are summed
- Nice:
  - the full $K^{(1)}_{1r,1v}$ is reproduced, including constants $\longrightarrow$ 4 powers of NLP logs
- **Note:** MoR diagnoses, but has no predictive power
- For this, we need a factorization approach

Next-to-soft in SCET

- Early SCET results beyond leading power in heavy-to-light currents
  - need for multi-pole expansions for appropiate scaling — Beneke, Diehl, Feldmann; Chapovsky
- Analysis of LBKD theorem at one-loop level in SCET
  - very general approach, has collinear splitting and collinear fusion terms

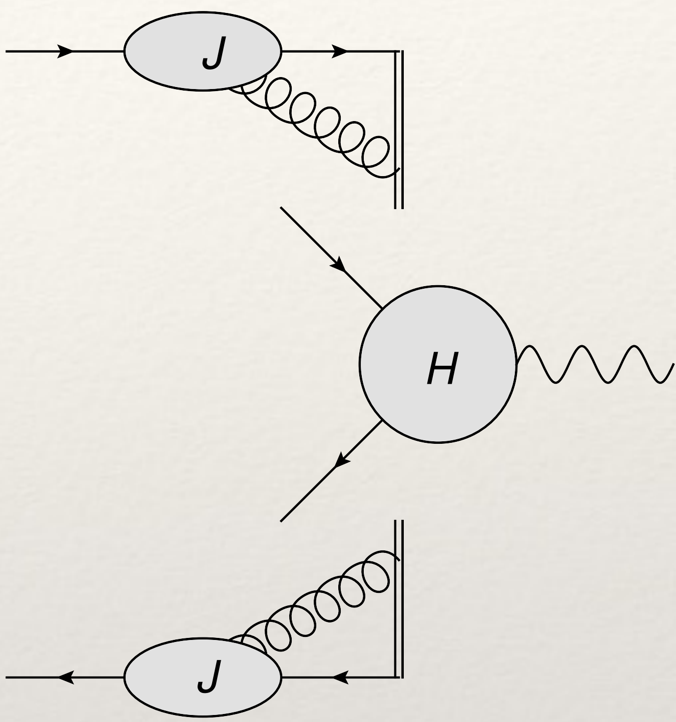
Larkoski, Neill, Stewart



A factorization approach to next-to-soft logarithms

Bonocore, EL, Magnea, Melville, Vernaza, White
arXiv:1503.05156

- Can we *predict* the log(1-z) logarithms?
  - For both we need to factorize the cross section, as earlier
    - ✓ H: both the hard and the soft function
    - ✓ J: incoming jet functions

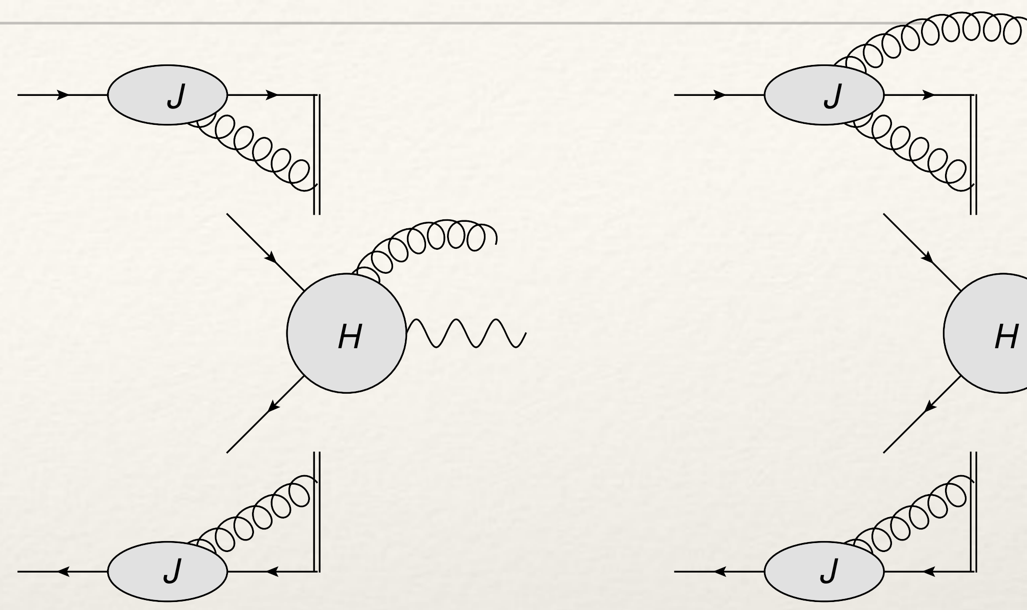


- Now, let every blob radiate!
  - Compute each new "blob + radiation", and put together

Del Duca, 1991

Factorization approach

- Work at amplitude level (C_F^2 terms)
- Emission can occur from either H or J's



$$\mathcal{A}_\mu \, \epsilon^\mu(k) \;=\; \mathcal{A}^J_\mu \, \epsilon^\mu(k) + \mathcal{A}^H_\mu \, \epsilon^\mu(k)$$

- For emission from jet function, define radiative jet function

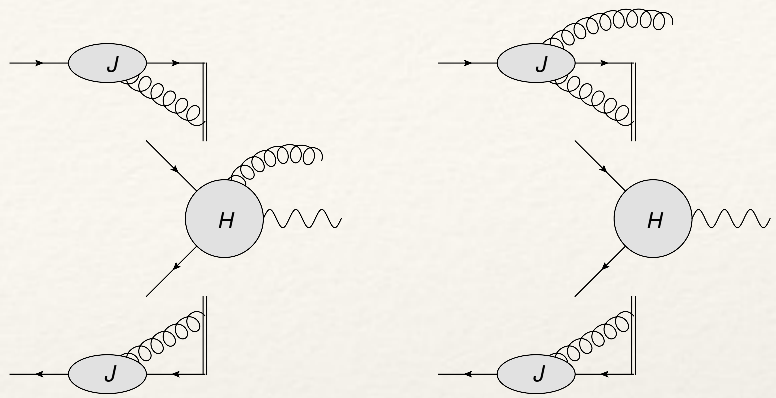
$$J_\mu\,(p,n,k_2)\,u(p) = \left\langle 0 \left| \int d^d y e^{-i(p+k_2)\cdot y} \Phi_n(y,\infty)\psi(y) j_\mu(0) \right| p \right\rangle$$

Ward identities

Del Duca, 1991

- For emission from H, use Ward identity

$$k^\mu \, \mathcal{A}_\mu \; = \; 0 \qquad\qquad k^\mu \, \mathcal{A}^H_\mu \; = \; - \, k^\mu \, \mathcal{A}^J_\mu$$



- For the radiative jet function there is the separate Ward identity

$$k^\mu \, J_\mu \, (\ldots, k, \epsilon) \; = \; q \, J \, (\ldots, \epsilon) \, , \qquad q = \pm 1$$

  - Then hard function emission current via derivative

$$\mathcal{A}^H_\mu (p_i, k) \; = \; \sum_{i=1}^{2} q_i \left(\frac{\partial}{\partial p_i^\mu} H(p_i; p_j, n_j) \right) \prod_{j=1}^{2} J(p_j, n_j)$$

- Split polarization sum of emitted gluon/photon using “K” (leading) and “G” (subleading) projectors

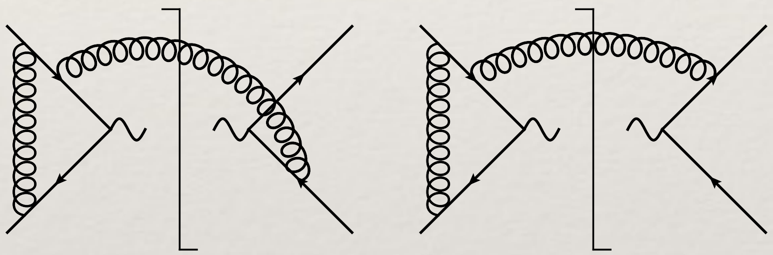
$$\eta^{\mu\nu} \; = \; G^{\mu\nu} + K^{\mu\nu} \, , \qquad K^{\mu\nu}(p; k) \; = \; \frac{(2p - k)^\nu}{2p \cdot k - k^2} \, k^\mu$$

Factorization, main formula

- Upshot: a factorization formula for the emission amplitude (C_F^2 terms)

$$\mathcal{A}^\mu(p_j,k) = \sum_{i=1}^{2}\left[q_i \left(\frac{(2p_i-k)^\mu}{2p_i\cdot k - k^2} + G_i^{\nu\mu}\frac{\partial}{\partial p_i^\nu}\right)\mathcal{A}(p_i;p_j) + \mathcal{H}(p_j,n_j)\,\overline{\mathcal{S}}(\beta_j,n_j)\,G_i^{\nu\mu}\left(J_\nu(p_i,k,n_i) - q_i\frac{\partial}{\partial p_i^\nu}J(p_i,n_i)\right)\prod_{j\neq i}J(p_j,n_j)\right]$$

- Remarks
  - only process dependent terms are H and A
  - J_μ is needed at one-loop level



- We choose $n^\mu = p^\mu$, so $n^2 = 0$. In dimensional regularization we have then

$$J(p_i,n_i) \;=\; 1$$

- Simplification:

$$\mathcal{A}^\mu(p_j,k) \;=\; \sum_{i=1}^{2}\left(q_i\,\frac{(2p_i-k)^\mu}{2p_i\cdot k - k^2} + q_i\,G_i^{\nu\mu}\frac{\partial}{\partial p_i^\nu} + G_i^{\nu\mu}J_\nu(p_i,k)\right)\mathcal{A}(p_i;p_j)$$

External and derivative contributions

- External: straightforward

$$K^{(2)}_{\rm ext}(z) = \left(\frac{\alpha_s}{4\pi}C_F\right)^2 \Bigg\{ \frac{32}{\varepsilon^3}\Big[\mathcal{D}_0(z) - 1\Big] + \frac{8}{\varepsilon^2}\Big[-8\mathcal{D}_1(z) + 6\mathcal{D}_0(z) + 8L(z) - 14\Big]$$
$$+ \frac{16}{\varepsilon}\Big[4\mathcal{D}_2(z) - 6\mathcal{D}_1(z) + 8\mathcal{D}_0(z) - 4L^2(z) + 14L(z) - 14\Big]$$
$$- \frac{128}{3}\mathcal{D}_3(z) + 96\mathcal{D}_2(z) - 256\mathcal{D}_1(z) + 256\mathcal{D}_0(z)$$
$$+ \frac{128}{3}L^3(z) - 224L^2(z) + 448L(z) - 512\Bigg\}. \qquad (5.62)$$

  - Reproduces all LP logs (plus-distributions)
  - Agrees with factorization of eikonal radiation, and NE Feynman rules

- Not through effective Feynman rules, straightforward

$$K^{(2)}_{\partial\mathcal{A}}(z) = \left(\frac{\alpha_s}{4\pi}C_F\right)^2 \Bigg\{ \frac{32}{\varepsilon^2} + \frac{16}{\varepsilon}\Big[-4L(z) + 3\Big] + 64L^2(z) - 96L(z) + 128\Bigg\}.$$

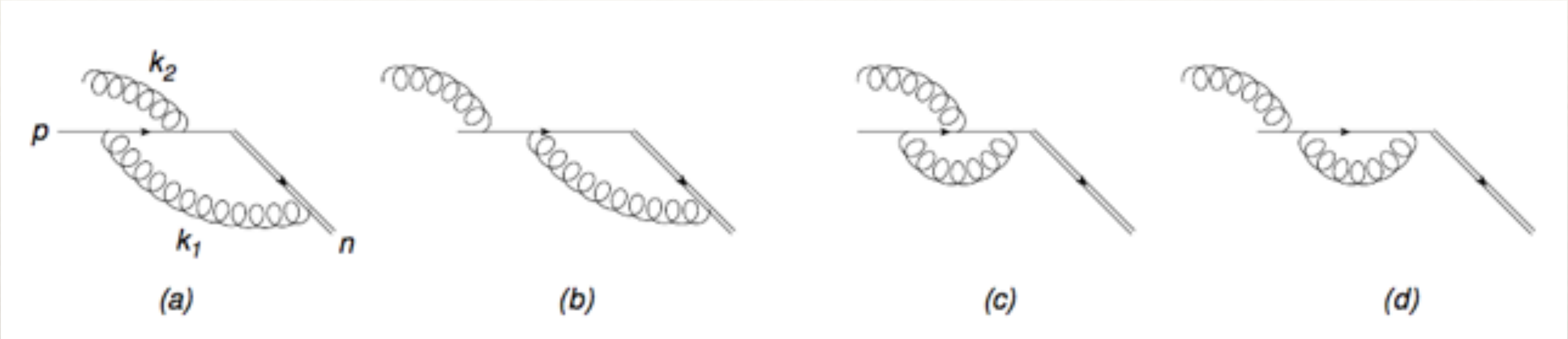
- Sum corresponds precisely to MoR hard region

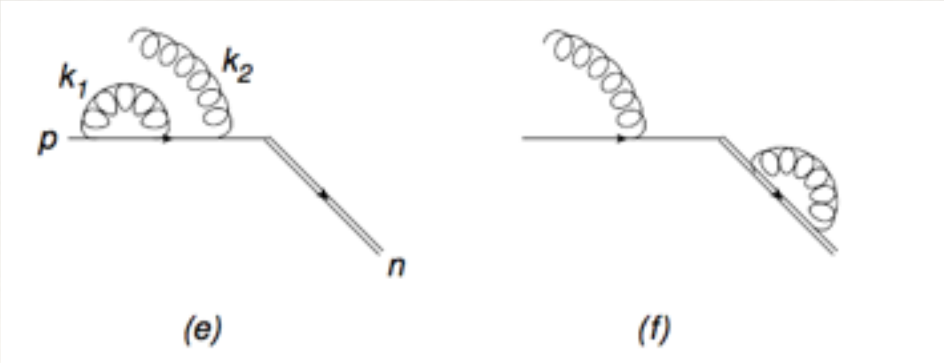
Radiative jet function contribution

- Formal definition

$$J_\mu\left(p,n,k_2\right)u(p)=\left\langle 0\left|\int d^dye^{-i(p+k_2)\cdot y}\Phi_n(y,\infty)\psi(y)j_\mu(0)\right|p\right\rangle$$

- Diagrams:





Radiative jet function contribution

- At one-loop

$$J^{\nu(1)}(p,n,k\,;\epsilon) = (2p\cdot k)^{-\epsilon}\Big[\left(\frac{2}{\epsilon}+4+8\epsilon\right)\left(\frac{n\cdot k}{p\cdot k}\frac{p^\nu}{p\cdot n}-\frac{n^\nu}{p\cdot n}\right)-(1+2\epsilon)\,\frac{\mathrm{i}\,k_\alpha\Sigma^{\alpha\nu}}{p\cdot k} + \left(\frac{1}{\epsilon}-\frac{1}{2}-3\epsilon\right)\frac{k^\nu}{p\cdot k}+(1+3\epsilon)\left(\frac{\gamma^\nu \not{n}}{p\cdot n}-\frac{p^\nu \not{k}\not{n}}{p\cdot k\, p\cdot n}\right)\Big]+\ldots$$

- Occurs with G-tensor: filters spin-dependent part. At lowest order $J^{\nu(0)}$:

$$G^{\nu\mu}\left(-\frac{p_\nu}{p\cdot k_2}+\frac{\not{k}_2\gamma_\nu}{2p\cdot k_2}\right)=\frac{k_{2\,\nu}\,[\gamma^\nu,\gamma^\mu]}{4p\cdot k_2}$$

- One-loop terms breaks next-to-soft theorem. Interestingly it is an eigenstate of $G^{\mu\nu}$

$$G^{\nu\mu}J^{(1)}_\nu(p,n,k) = J^{(1)}_\nu(p,n,k)$$

- Find after phase space (k_2) integral (chosing n=p)

$$K^{(2)}_{\mathrm{radJ}} = \left(\frac{\alpha_s C_F}{4\pi}\right)^2\left[\frac{-16}{\epsilon^2}-\frac{20}{\epsilon}+60\log(1-z)+\frac{48}{\epsilon}\log(1-z)-72\log^2(1-z)-24\right]$$

  - Precise correspondence with collinear region

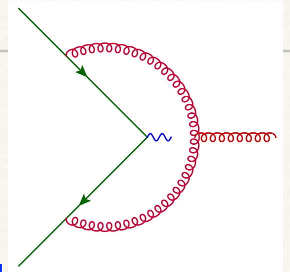
Upshot

- Again: *perfect agreement* with exact NLP result (and of course MoR result), for

$$\log^3(1-z), \quad \log^2(1-z), \quad \log^1(1-z), \quad \log^0(1-z)$$

- So there is strong predictive power for such (threshold) logarithms

Non- abelian terms (prelim)

- New features appear. Diagram  contributes to all regions
  - non-abelian jet contributions when virtual gluon is hard and (anti)-collinear. Scales

$$(2p \cdot k)^{-\varepsilon}, \qquad (2\bar{p} \cdot k)^{-\varepsilon}$$

  - (next-to-soft) contribution when soft. Scale

$$\left(\frac{s\mu^2}{2p \cdot k\, 2\bar{p} \cdot k}\right)^{\varepsilon}$$

  - which suggests using NE webs. Need also extra soft subtractions to avoid double counting. Factorization formula

$$\mathcal{A}^{\mu\,a}(p_j, k)\epsilon_\mu(k) = \epsilon_\mu(k) \sum_{i=1}^{2} \Bigg\{ \left(\frac{1}{2} \overline{\mathcal{S}}^{\mu\,a} + \mathbf{T}_i^a G_i^{\nu\mu} \frac{\partial}{\partial p_i^\nu} \right) \mathcal{A}(\{p_i\})$$
$$+ \left(J^{\mu\,a}(p_i, k, n_i) - \tilde{\mathcal{J}}^{\mu\,a}(\beta_i, k, n_i) - \mathbf{T}_i^a \, G_i^{\nu\mu} \frac{\partial}{\partial p_i^\nu} \frac{J(p_i, n_i)}{\tilde{\mathcal{J}}(\beta_i, n_i)} \right)$$
$$\times \tilde{\mathcal{H}}(p_j, n_j) \, \tilde{\mathcal{S}}(\beta_j, n_j) \prod_{j \neq i} \frac{J(p_j, n_j)}{\tilde{\mathcal{J}}(\beta_j, n_j)} \Bigg\},$$

- Again, four powers of logarithms agree

Summary

- Analyzed next-to-soft corrections in Drell-Yan
- Governed by LBKD theorem; collinear loop momenta important
    - understood through method of regions
    - predictive power through factorization
- Good progress recently, also in SCET approaches (recent workshop in Edinburgh)
- Non-abelian extension soon; resummation: next-to-soon?