Effective Field Theory of Forward Scattering and Factorization Violation

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Working definition of factorization:

A given observable is said to factorize when it can be written as a product of matrix elements that are either calculable in perturbation theory or for which have some reasonable chance of extracting from the data.

$$\langle 0 | O\tilde{O} | 0 \rangle$$ Soft Functions

$$\langle p | O\tilde{O} | p \rangle$$ PDFs

$$\sum_X \langle 0 | O \mid H + X \rangle \langle H + X \mid \tilde{O} | 0 \rangle$$ Frag. Funcs.

Factorization proofs lie at the heart of the science program for any hadronic machine. Yet for a significant number of observables complete proofs do not exist.

Strategy for Proofs

- Determine whats ``modes'' (regions of momentum space) are responsible for the non-analytic structure.
- CSS: Use Ward identities to show decoupling of modes. Contour deformations to eliminate modes.
- EFT (SCET) Write down an action representing each mode with a field. As long as the leading Hamiltonian (including external currents) can be written down as a sum over sectors, then factorization follows. $T_{\mu\nu} = \sum_i T^i_{\mu\nu}$

As a consequence of tensor product nature of Hilbert space factorization follows

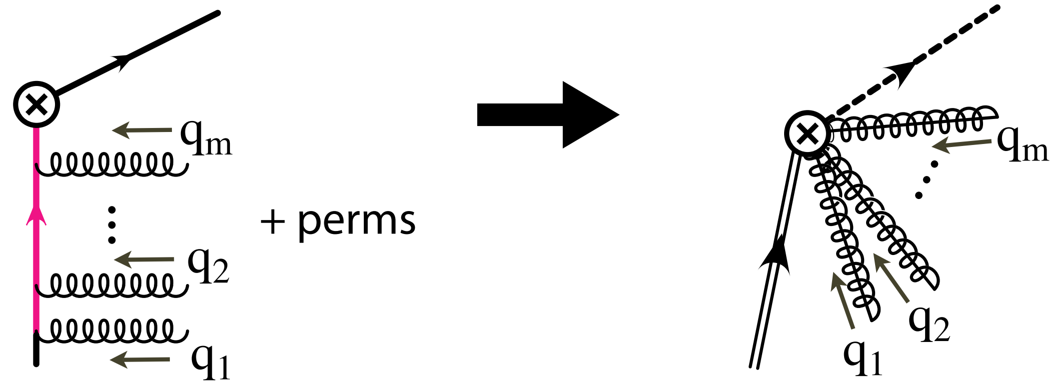
$$\langle\psi_c\psi_s \mid O_sO_c \mid \psi_c\psi_s\rangle = \langle\psi_c \mid O_c \mid \psi_c\rangle\langle\psi_s \mid O_s \mid \psi_s\rangle$$

In general there will be convolution in some number of variables depending upon the choice of observables

Canonical Modes

- Soft $(\lambda, \lambda, \lambda)$ $(\lambda^2, \lambda^2, \lambda^2)$ $\lambda \sim E/Q$
- Collinear $(\lambda^2, 1, \lambda)$ $(1, \lambda^2, \lambda)$
- Hard $(1, 1, 1)$ (Integrate out)

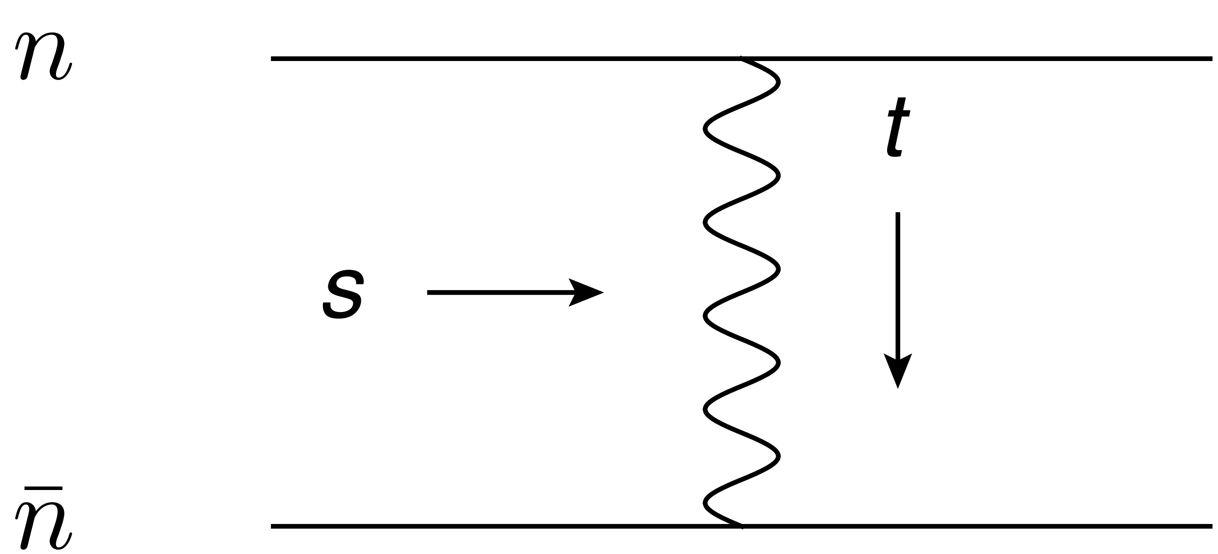
Soft emissions off of collinear lines either throw them off-shell and matching can be done to all orders, or can be eliminated by a field redefinition. In both cases the net effect is to generate a set of Wilson lines, which can be factored out of collinear matrix elements.



$$\bar{\psi}_n W_s, \quad \bar{\psi}_n W_{\bar{n}}$$

Symmetries underlie the factorization of these modes. In SCET **there exists distinct gauge symmetries for soft and collinear modes which facilitate factorization.**

However, there exists another mode in the theory, for which symmetry plays no role, that arise for exceptional external momentum (near forward)



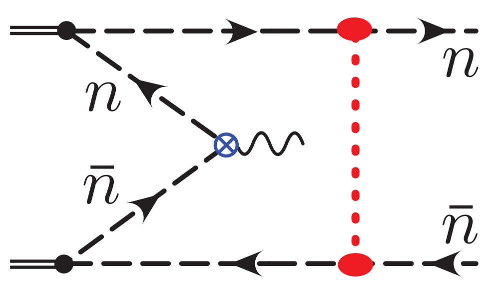
The Glauber mode

$p_g^\mu \sim (\lambda^2, \lambda^2, \lambda)$

$$O_g = \frac{g^2}{\vec{p}_\perp^{\,2}} \left(\bar{\xi}_n \frac{\bar{n}\!\!\!/}{2} \xi_n\right)\left(\xi_{\bar{n}} \frac{n\!\!\!/}{2} \xi_{\bar{n}}\right)$$

The Glauber gluon contribute at leading order in near forward scattering and builds up a coherent shock wave solution. Leading order Glauber contributions threaten factorization.

Primary challenge to factorization theorems



couples n-collinear, n̄-collinear, and soft modes

To prove factorization must either show that they're contributions are subsumed by other modes which factorize, or if not, that they cancel in the observable of interest.

Goal: Write down an EFT which incorporates Glauber interactions into high energy scattering that will allow for a general analysis on their effects on observables

This will abet:

1) Generalize/Simplify factorization proofs.
2) Determine when and at what level Glaubers contribute
3) Calculate systematically when Glaubers do indeed contribute.

Construction: $\lambda \ll 1$ large Q

will do calculations with back-to-back collinear particles for simplicity

mode	fields	p^μ momentum scaling	physical objects	type
n-collinear	$\xi_n,\ A_n^\mu$	$(n\cdot p, \bar{n}\cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	n-collinear "jet"	onshell
$\bar{n}$-collinear	$\xi_{\bar{n}},\ A_{\bar{n}}^\mu$	$(\bar{n}\cdot p, n\cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	$\bar{n}$-collinear "jet"	onshell
soft	$\psi_S,\ A_S^\mu$	$p^\mu \sim Q(\lambda, \lambda, \lambda)$	soft virtual/real radiation	onshell
ultrasoft	$\psi_{us},\ A_{us}^\mu$	$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$	ultrasoft virtual/real radiation	onshell
Glauber	–	$p^\mu \sim Q(\lambda^a, \lambda^b, \lambda),\ a+b>2$ (here $\{a,b\} = \{2,2\},\{2,1\},\{1,2\}$)	forward scattering potential	offshell
hard	–	$p^2 \gtrsim Q^2$	hard scattering	offshell

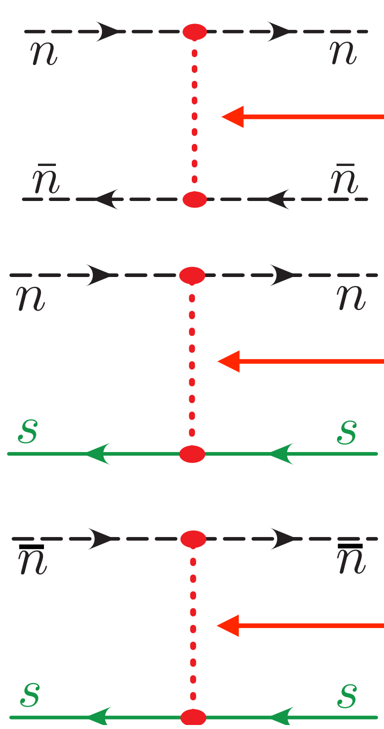
Integrate out

Need 3-types of Glauber momenta:

n-$\bar{n}$ fwd. scattering

n-s fwd. scattering

$\bar{n}$-s fwd. scattering



$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda)$

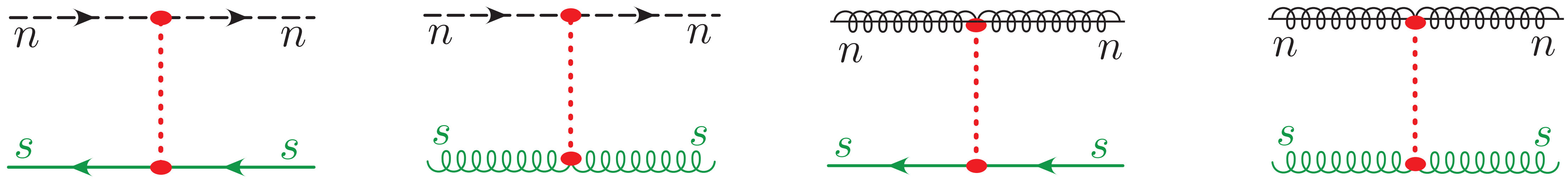
$p^\mu \sim Q(\lambda^2, \lambda, \lambda)$

$p^\mu \sim Q(\lambda, \lambda^2, \lambda)$

- $\dfrac{1}{k_\perp^2}$ potentials
- instantaneous in x^+, x^- (t and z)

n-s fwd. scattering $\quad s \gg t \quad$ integrated out

$\lambda^2 = \frac{t}{s} \ll 1$



determine $\quad \mathcal{O}(\lambda^3): \quad \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B}$ $\quad$ (2 rapidity sectors)

$\lambda^2 \quad \lambda^{-2} \quad \lambda^3$

with bilinear octet operators $\qquad \psi_s^n = S_n^\dagger \psi_s$

$$\mathcal{O}(\lambda^2): \quad \mathcal{O}_n^{qB} = \overline{\chi}_n T^B \frac{\bar{n}\!\!\!/}{2} \chi_n \,,$$

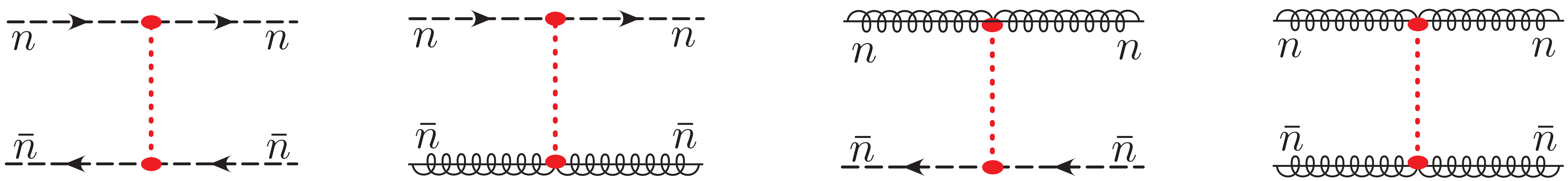
$$\mathcal{O}_n^{gB} = \frac{i}{2} f^{BCD} \mathcal{B}^C_{n\perp\mu} \frac{\bar{n}}{2} \cdot (\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}^{D\mu}_{n\perp}$$

$$\mathcal{O}(\lambda^3): \quad \mathcal{O}_s^{q_n B} = 8\pi\alpha_s \left(\bar{\psi}^n_S T^B \frac{n\!\!\!/}{2} \psi^n_S \right) ,$$

$$\mathcal{O}_s^{g_n B} = 8\pi\alpha_s \left(\frac{i}{2} f^{BCD} \mathcal{B}^{nC}_{S\perp\mu} \frac{n}{2} \cdot (\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}^{nD\mu}_{S\perp} \right)$$

Operators manifestly gauge invariant in all sectors

n-$\bar{n}$ fwd. scattering $\quad s \gg t$



might guess $\quad \mathcal{O}(\lambda^2): \quad \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jB}$

same $\mathcal{O}_n^{iB}$

analogous $\mathcal{O}_{\bar{n}}^{jC}$

actually $\quad \mathcal{O}(\lambda^2): \quad \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jC}$

(3 rapidity sectors)

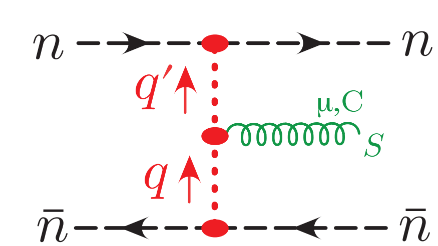
$\lambda^2 \quad \lambda^{-2} \quad \lambda^2 \quad \lambda^{-2} \quad \lambda^2$

same $\mathcal{O}_n^{iB}$

analogous $\mathcal{O}_{\bar{n}}^{jC}$

must allow for soft emission from **between** the rapidity sectors:

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \mathcal{P}_\perp^2 \delta^{BC} + \ldots$$



Soft $\mathcal{O}_s^{BC}$ Operator

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \sum_i C_i O_i^{BC}$$

basis of $\mathcal{O}(\lambda^2)$ operators allowed by symmetries:

$$O_1 = \mathcal{P}_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar n} \mathcal{P}_{\perp\mu}, \qquad O_2 = \mathcal{P}_\perp^\mu \mathcal{S}_{\bar n}^T \mathcal{S}_n \mathcal{P}_{\perp\mu},$$

$$O_3 = \mathcal{P}_\perp \cdot (g\widetilde{\mathcal{B}}_{S\perp}^n)(\mathcal{S}_n^T \mathcal{S}_{\bar n}) + (\mathcal{S}_n^T \mathcal{S}_{\bar n})(g\widetilde{\mathcal{B}}_{S\perp}^{\bar n}) \cdot \mathcal{P}_\perp, \qquad O_4 = \mathcal{P}_\perp \cdot (g\widetilde{\mathcal{B}}_{S\perp}^{\bar n})(\mathcal{S}_{\bar n}^T \mathcal{S}_n) + (\mathcal{S}_{\bar n}^T \mathcal{S}_n)(g\widetilde{\mathcal{B}}_{S\perp}^n) \cdot \mathcal{P}_\perp,$$

$$O_5 = \mathcal{P}_\mu^\perp (\mathcal{S}_n^T \mathcal{S}_{\bar n})(g\widetilde{\mathcal{B}}_{S\perp}^{\bar n\mu}) + (g\widetilde{\mathcal{B}}_{S\perp}^{n\mu})(\mathcal{S}_n^T \mathcal{S}_{\bar n})\mathcal{P}_\mu^\perp, \qquad O_6 = \mathcal{P}_\mu^\perp (\mathcal{S}_{\bar n}^T \mathcal{S}_n)(g\widetilde{\mathcal{B}}_{S\perp}^{n\mu}) + (g\widetilde{\mathcal{B}}_{S\perp}^{\bar n\mu})(\mathcal{S}_{\bar n}^T \mathcal{S}_n)\mathcal{P}_\mu^\perp,$$

$$O_7 = (g\widetilde{\mathcal{B}}_{S\perp}^{n\mu})\mathcal{S}_n^T \mathcal{S}_{\bar n}(g\widetilde{\mathcal{B}}_{S\perp\mu}^{\bar n}), \qquad O_8 = (g\widetilde{\mathcal{B}}_{S\perp}^{\bar n\mu})\mathcal{S}_{\bar n}^T \mathcal{S}_n(g\widetilde{\mathcal{B}}_{S\perp\mu}^{n}),$$

$$O_9 = \mathcal{S}_n^T n_\mu \bar n_\nu (ig\widetilde{G}_s^{\mu\nu})\mathcal{S}_{\bar n}, \qquad O_{10} = \mathcal{S}_{\bar n}^T n_\mu \bar n_\nu (ig\widetilde{G}_s^{\mu\nu})\mathcal{S}_n,$$

octet Wilson line

octet reps

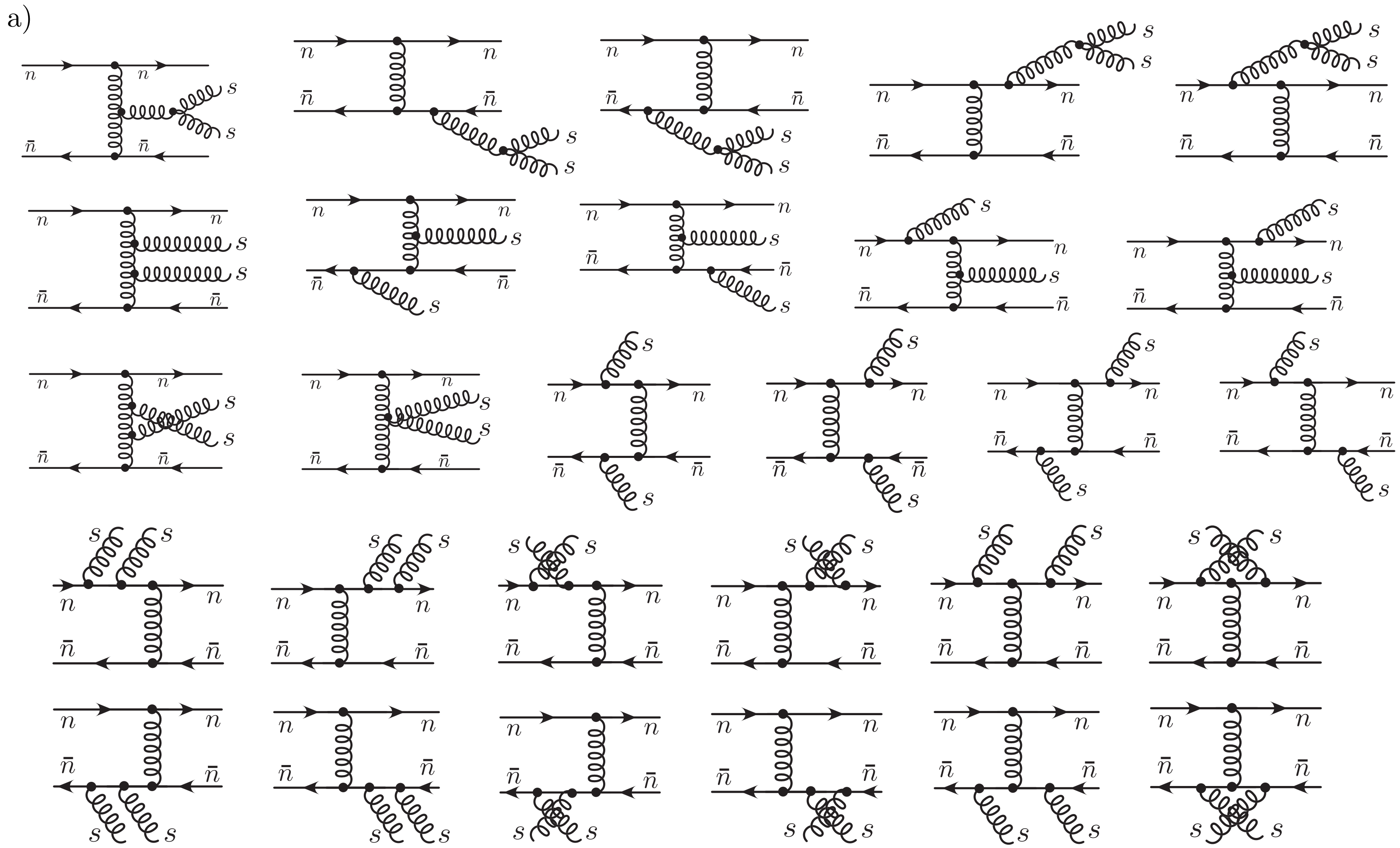
Restricted by: Hermiticity $O_i^\dagger\big|_{n\leftrightarrow\bar n} = O_i$, one $\mathcal{S}_n$, one $\mathcal{S}_{\bar n}$

operator identities: eg. $[\mathcal{P}_\perp^\mu(\mathcal{S}_n^T\mathcal{S}_{\bar n})] = -g\widetilde{\mathcal{B}}_{S\perp}^{n\mu}(\mathcal{S}_n^T\mathcal{S}_{\bar n}) + (\mathcal{S}_n^T\mathcal{S}_{\bar n})g\widetilde{\mathcal{B}}_{S\perp}^{\bar n\mu}$

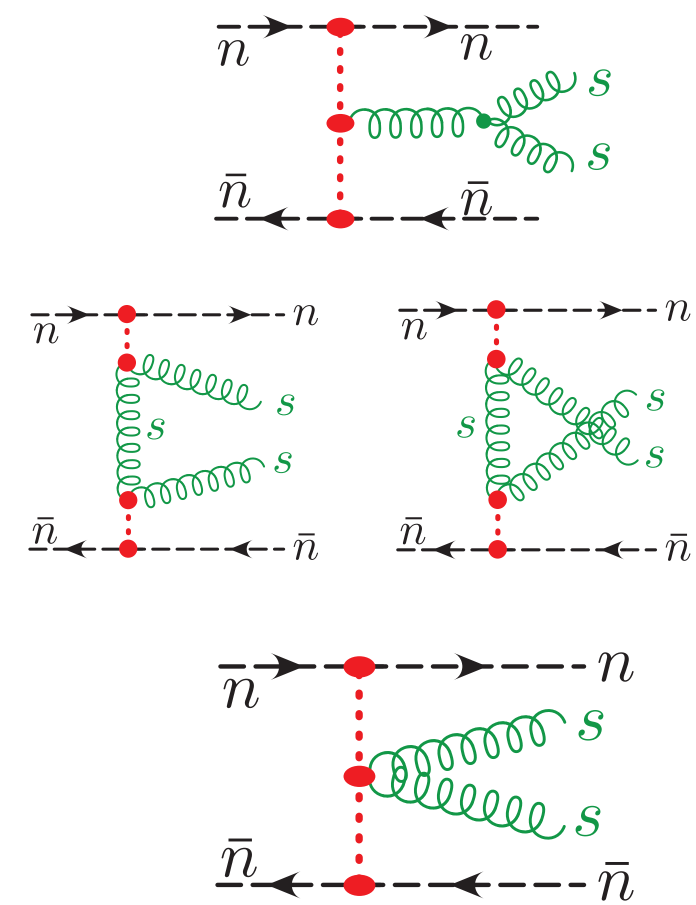
Tree level matching at two gluons fixes all coefficients

Two Soft Gluons:

QCD



EFT



new vertex

Find:

$$\boxed{C_2 = C_4 = C_5 = C_6 = C_8 = C_{10} = 0},$$

$$\boxed{C_1 = -C_3 = -C_7 = +1\,, \qquad C_9 = -\frac{1}{2}}$$

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \Big\{ \mathcal{P}_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar n} \mathcal{P}_{\perp\mu} - \mathcal{P}_\mu^\perp g\widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar n} - \mathcal{S}_n^T \mathcal{S}_{\bar n} g\widetilde{\mathcal{B}}_{S\perp}^{\bar n\mu} \mathcal{P}_\mu^\perp - g\widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar n} g\widetilde{\mathcal{B}}_{S\perp\mu}^{\bar n} - \frac{n_\mu \bar n_\nu}{2} \mathcal{S}_n^T ig\widetilde{G}_s^{\mu\nu} \mathcal{S}_{\bar n} \Big\}^{BC}.$$

Form is unique to all loops since there are no hard α_s corrections to this matching (more later)

Full Leading Power Glauber Lagrangian:

$$\mathcal{L}_G^{\mathrm{II}(0)} = \sum_{n,\bar{n}} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jC} + \sum_{n} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B}$$

(3 rapidity sectors)

(2 rapidity sectors)

↑ sum pairwise on all collinears

↑ sum on all collinears

- Interactions with more sectors is given by T-products
- No Wilson coefficient ie. no new structures at loop level. [more later]

This EFT has multiple uses, e.g. for small x physics. For the purpose of understanding factorization, here we will use it to determine the role of the Glauber mode in hard scattering processes.

Note: At loop level ZB subtraction crucial for matching, insure no double counting. This is despite the fact that the matching is exact.

eg. 1-loop SCET_{II} graphs:

$$S = \tilde{S} - S^{(G)}$$

naive soft graph → $\tilde{S}$

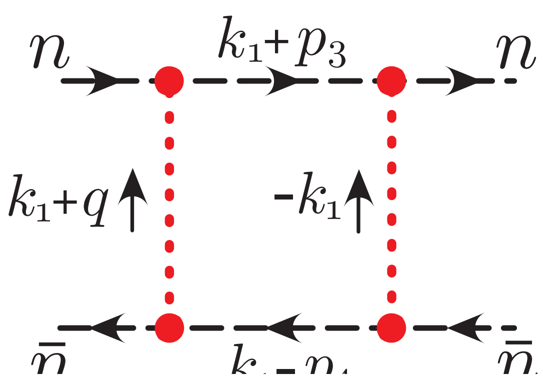
glauber limit of soft graph → $S^{(G)}$

$$C_n = \tilde{C}_n - C_n^{(S)} - C_n^{(G)} + C_n^{(GS)}$$

naive collinear graph → $\tilde{C}_n$

glauber limit of collinear graph → $C_n^{(G)}$

When do expect Glaubersto play a central role? Forward Scattering



$$\sim \int d^d k \frac{1}{(k_+ + \Delta_3(k_\perp)} \frac{1}{(k_- + \Delta_4(k_\perp)} \frac{1}{k_\perp^2} \frac{1}{(k+q)_\perp^2}$$

Rapidity divergences necessitate the introduction of an additional regulator beyond dim. reg.

$$I_{\rm Gbox} = \int \frac{{d\!\!{}^-}^{d-2}k_\perp \; {d\!\!{}^-}k^0 \; {d\!\!{}^-}k^z \; |k^z|^{-2\eta} \, (\nu/2)^{2\eta}}{(\vec{k}_\perp^{\,2})(\vec{k}_\perp + \vec{q}_\perp)^2 \Big(k^+ - \Delta_1(k_\perp) + i0\Big)\Big(-k^- - \Delta_2(k_\perp) + i0\Big)}$$

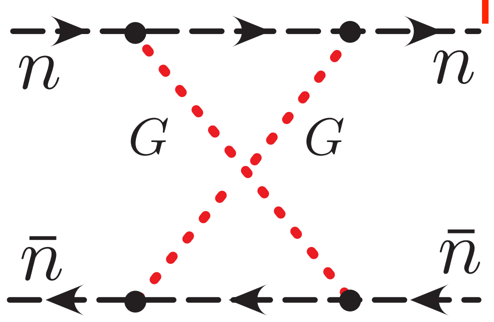
$$= -i \int \frac{{d\!\!{}^-}^{d-2}k_\perp {d\!\!{}^-}k^z \; |k^z|^{-2\eta} \, (\nu/2)^{2\eta}}{(\vec{k}_\perp^{\,2})(\vec{k}_\perp + \vec{q}_\perp)^2 (-2k^z - \Delta + i0)}$$

$$= \frac{-i}{4\pi} \int \frac{{d\!\!{}^-}^{d-2}k_\perp}{(\vec{k}_\perp^{\,2})(\vec{k}_\perp + \vec{q}_\perp)^2} \Big[(\nu/2)^{2\eta} \, (-2i\pi) \csc(2\pi\eta) \sin(\pi\eta) \, (i\Delta)^{-2\eta}\Big]$$

$$= \Big(\frac{-i}{4\pi}\Big) \int \frac{{d\!\!{}^-}^{d-2}k_\perp}{(\vec{k}_\perp^{\,2})(\vec{k}_\perp + \vec{q}_\perp)^2} \Big[- i\pi + \mathcal{O}(\eta)\Big]$$

Effectively Eikonal

Furthermore cross box vanishes



$$I_{\rm Gcbox} = \int \frac{đ^{d-2}k_\perp \; đk^0 \; đk^z \; |k^z|^{-2\eta}\,(\nu/2)^{2\eta}}{(\vec{k}_\perp^{\,2})(\vec{k}_\perp + \vec{q}_\perp)^2 \Big(k^+ - \Delta_1(k_\perp) + i0\Big)\Big(k^- - \Delta_2(k_\perp) + i0\Big)}$$

$$= 0$$

k^0 poles on same side

We can resum the full series in impact parameter space

$$\phi(b_\perp) = C_F\, g^2(\mu) \int \frac{đ^2 q_\perp}{\vec{q}_\perp^{\,2} + m^2}\, e^{i\vec{q}_\perp\cdot\vec{b}_\perp} = -2C_F\,\alpha_s(\mu)\,\ln\left(\frac{|\vec{b}_\perp| m e^{\gamma_E}}{2}\right)$$

$$G(q_\perp) = (2\pi)^2\delta^2(q_\perp) + \int d^2b_\perp\, e^{-i\vec{q}_\perp\cdot\vec{b}_\perp}\big(e^{i\phi(b_\perp)} - 1\big)$$

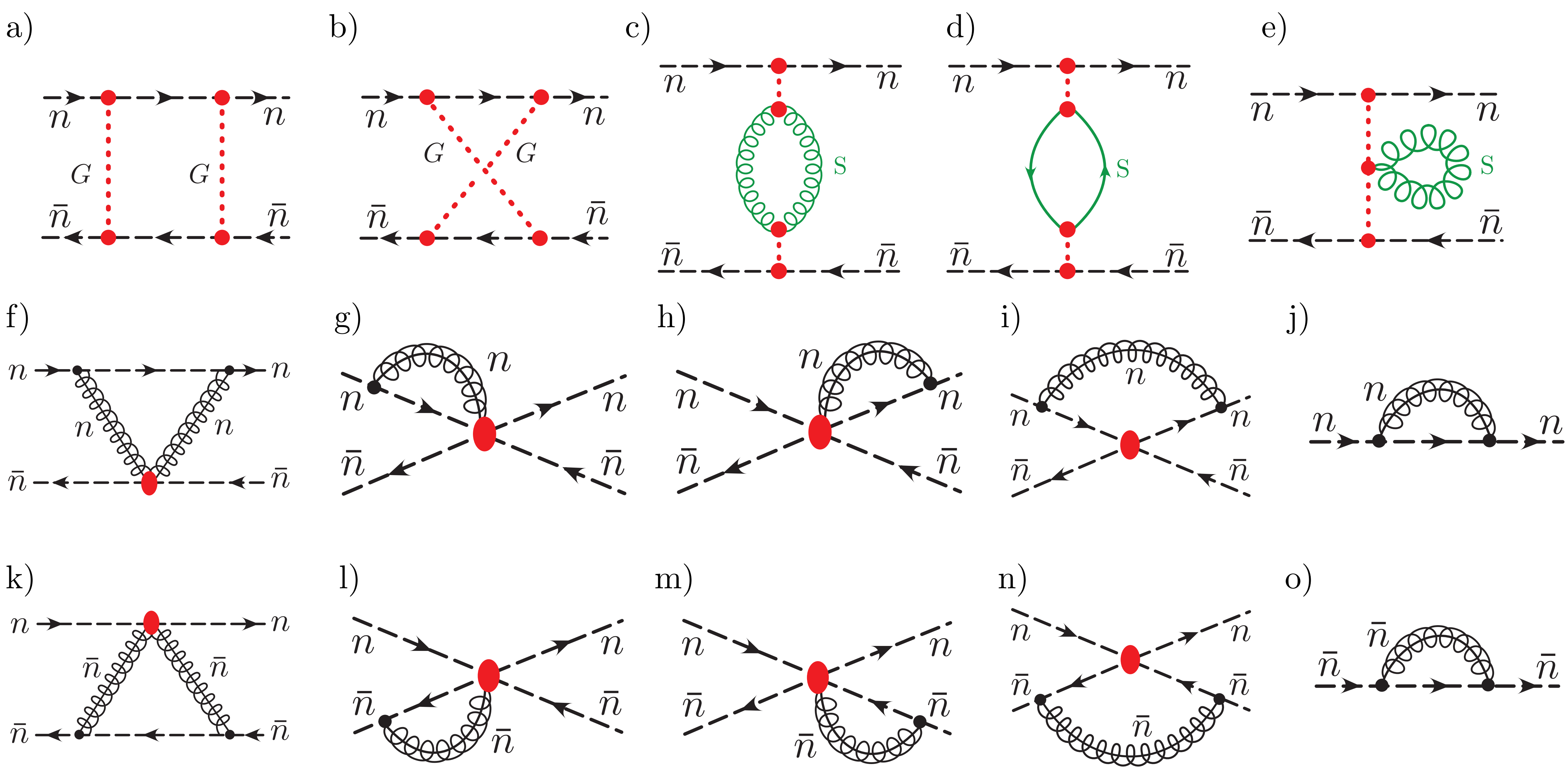
$$= (2\pi)^2\delta^2(q_\perp) + \frac{i4\pi C_F\alpha_s(\mu)}{(-t)}\,\frac{\Gamma\big(1 - iC_F\alpha_s(\mu)\big)}{\Gamma\big(1 + iC_F\alpha_s(\mu)\big)}\left(\frac{-t}{m^2 e^{2\gamma_E}}\right)^{iC_F\alpha_s(\mu)}$$

$$= (2\pi)^2\delta^2(q_\perp) + \frac{i4\pi C_F\alpha_s(\mu)}{(-t)}\; e^{i\delta(t,\alpha_s)}\,,$$

$$\delta(t,\alpha_s) = C_F\alpha_s(\mu)\ln\left(\frac{-t}{m^2}\right) + 2\sum_{k=1}^{\infty}\frac{(-1)^k\zeta_{2k+1}}{2k+1}\big(C_F\alpha_s(\mu)\big)^{2k+1}\,.$$

This result holds even if we place insert in greens functions, as long as additional loop does not need rapidity regularization, will use this property later on.

What about NA-Corrections?



Soft graphs dress Glauber kernel lead to running of coupling

Therefore at least part of collinear must also exponentiate, but could be a remainder.

Role of Glaubers in Hard Matching

Empirically Glaubers are not needed in hard matching (active only), but from the point of view of EFT (formally), they should be included as they are part of the theory.

$$J_\Gamma = (\bar{\xi}_n W_n) S_n^\dagger \Gamma S_{\bar{n}} (W_{\bar{n}}^\dagger \xi_{\bar{n}})$$

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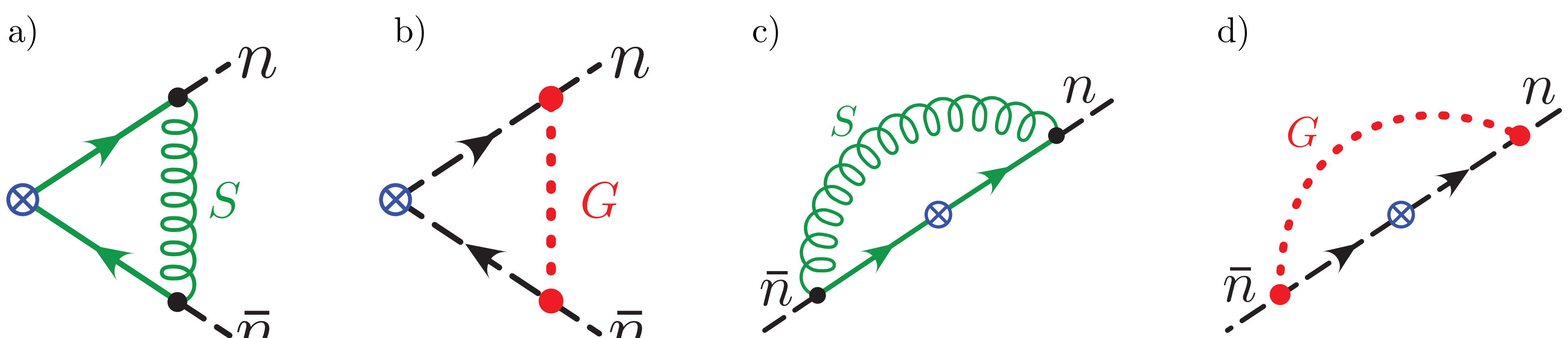
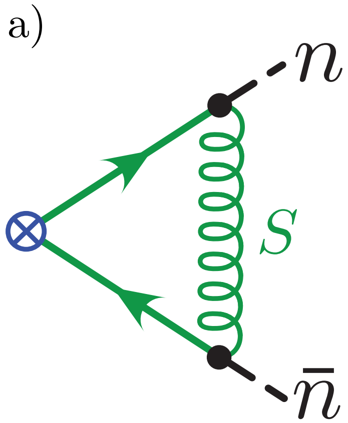


BUT: If we include Glaubers we must make sure to subtract the Zero Bin from soft contribution

Once we include Glaubers, the direction of the soft Wilson line (in or out to infinity) becomes irrelevant. Glaubers measure whether FF is space like or timelike

a)



$$\tilde{S} = \mathcal{S}_\Gamma \frac{C_F \alpha_s}{2\pi}\Big[\frac{-2h(\epsilon,\mu^2/m^2)}{\eta} + \ln\frac{\mu^2}{-\nu^2 - i0}\Big(\frac{1}{\epsilon} + \ln\frac{\mu^2}{m^2}\Big) + \frac{1}{\epsilon^2} - \frac{1}{2}\ln^2\frac{\mu^2}{m^2} - \frac{\pi^2}{12}\Big]$$

G-zero bin

$$S^{(G)} = \mathcal{S}_\Gamma \frac{C_F \alpha_s}{2\pi}\Big[(i\pi)\Big(\frac{1}{\epsilon} + \ln\frac{\mu^2}{m^2}\Big)\Big].$$

Wilson lines direction is inherited from $\tilde{S}$

$$G = \mathcal{S}_\Gamma \frac{C_F \alpha_s}{2\pi}\Big[(i\pi)\Big(\frac{1}{\epsilon} + \ln\frac{\mu^2}{m^2}\Big)\Big].$$

Glauber vanishes for space-like, direction of soft Wilson line flips

$$\tilde{S} - S^{(G)} = Real$$

Alternatively, we can absorb this Glauber contribution into the soft Wilson line IF we choose the direction properly

$$G - S^{(G)} = 0$$

Does this correspondence hold for the most general set of matrix elements?

Persistance of Glauber/Soft Zero Bin Correspondence

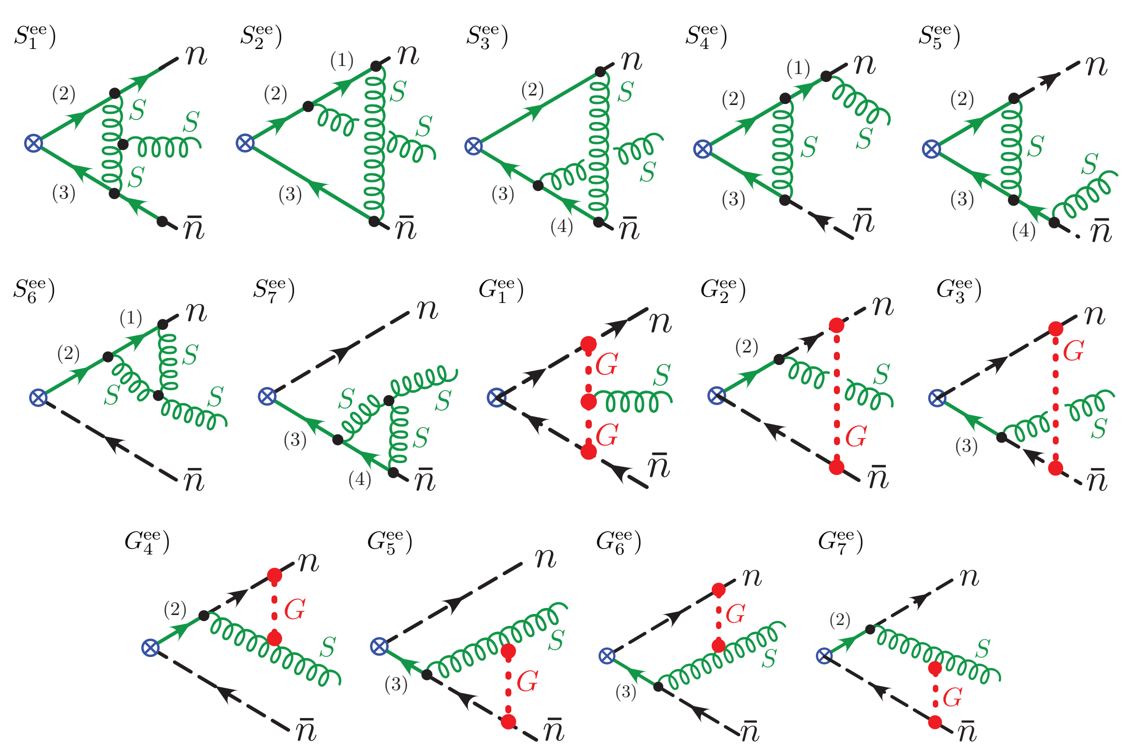
Allow for Soft radiation

General form of two loop soft current for three different kinematic situations (pp,ep,e+e-)

$$iA_i^{\rm chan} = (i\pi)\frac{g^3}{\pi}T^A\Big(\frac{n^\mu}{n\cdot k} - \frac{\bar n^\nu}{\bar n\cdot k}\Big)\Big[-\frac{1}{2}\,a_i^{\rm chan}\,C_A\,n\cdot k\,\bar n\cdot k\,I_\perp^{(1)}(k_\perp) + b_i^{\rm chan}\,C_F I_\perp^{(0)} - \frac{1}{2}\,c_i^{\rm chan}\,C_A I_\perp^{(0)}\Big].$$

$$I_\perp^{(0)} = \int \frac{d\!\!{}^-{}^{d-2}\ell_\perp\,(\iota^\epsilon\mu^{2\epsilon})}{\vec\ell_\perp^{\,2}}\,, \qquad I_\perp^{(1)}(k_\perp) = \int \frac{d\!\!{}^-{}^{d-2}\ell_\perp\,(\iota^\epsilon\mu^{2\epsilon})^2}{\vec\ell_\perp^{\,2}\,(\vec\ell_\perp+\vec k_\perp)^2}\,,$$

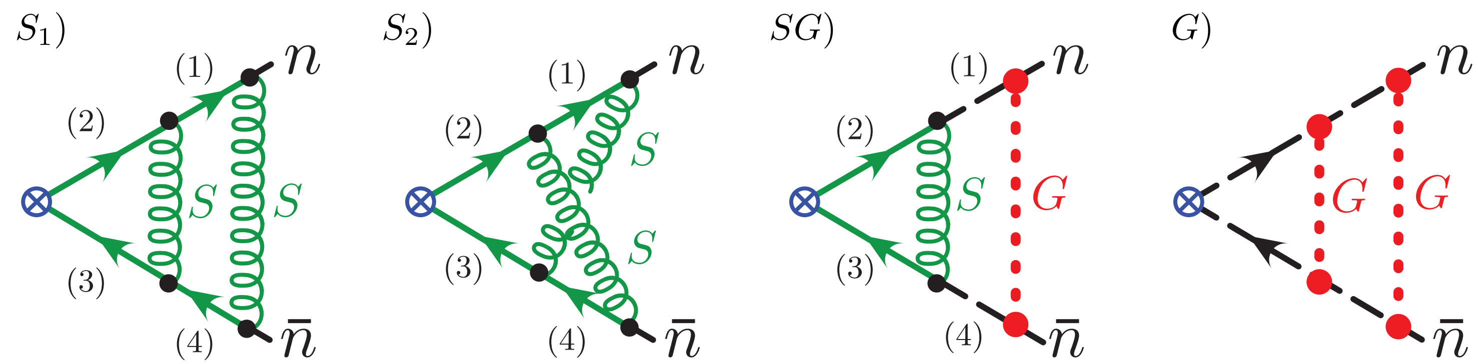
e.g. e+e-



Considering all possible subtractions of soft lines leads to the same conclusion

		G_1	G_{2+3}	G_{4+5}	G_6	G_7	$S_1^{(2,3)}$	$S_1^{(23)}$	$S_{2+3}^{(13,24)}$	$S_{2+3}^{(23)}$	$S_{2+3}^{(1,..,4)}$	$S_{4+5}^{(23)}$	$S_{4+5}^{(2,3)}$	$S_{6+7}^{(1,4)}$
$C_A I_\perp^{(1)}$:	a_i^{ee}	$-\frac{1}{4}$	0	0	$+\frac{1}{4}$	$+\frac{1}{4}$	$+\frac{1}{2}$	$-\frac{1}{4}$	0	0	0	0	0	0
	a_i^{ep}	0	0	0	$+\frac{1}{4}$	0	$+\frac{1}{4}$	0	0	0	0	0	0	0
	a_i^{pp}	$-\frac{1}{4}$	0	0	0	0	0	$-\frac{1}{4}$	0	0	0	0	0	0
$C_F I_\perp^{(0)}$:	b_i^{ee}	0	$-\frac{1}{2}$	0	0	0	0	0	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	0
	b_i^{ep}	0	0	0	0	0	0	0	0	0	$-\frac{1}{2}{}^*$	0	$+\frac{1}{2}{}^*$	0
	b_i^{pp}	0	$-\frac{1}{2}$	0	0	0	0	0	$-\frac{1}{2}$	$+\frac{1}{2}$	0	$-\frac{1}{2}$	0	0
$C_A I_\perp^{(0)}$:	c_i^{ee}	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$+\frac{1}{4}$	0	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0	$+\frac{1}{4}$
	c_i^{ep}	0	0	0	0	0	$+\frac{1}{4}{}^*$	0	0	0	$-\frac{1}{2}{}^*$	0	0	$+\frac{1}{4}{}^*$
	c_i^{pp}	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	0	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0

We may also consider two loop virtuals which now will include effects of Liptov vertex



Abelian

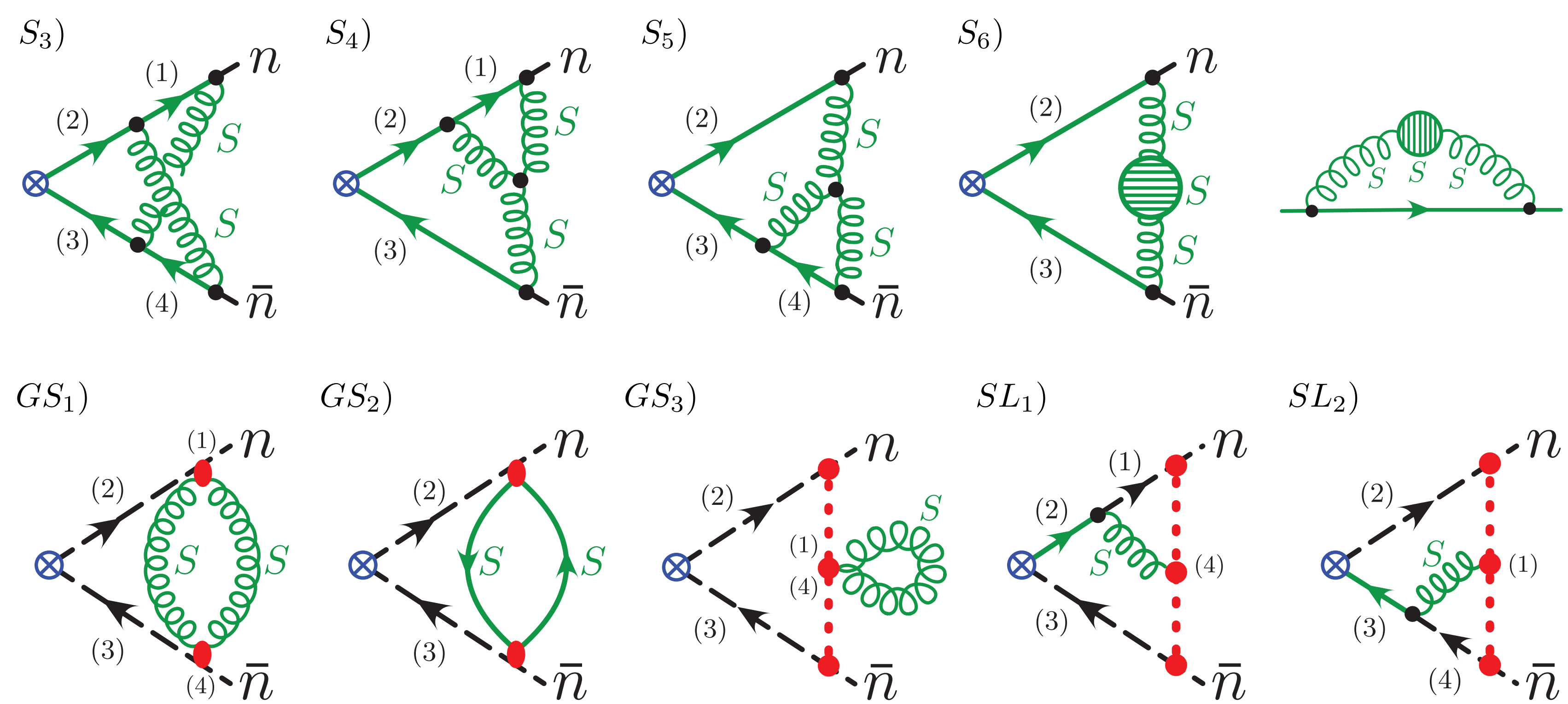
$$S_1 + S_2 + SG + G = \tilde{S}_1 + \tilde{S}_2 \,,$$

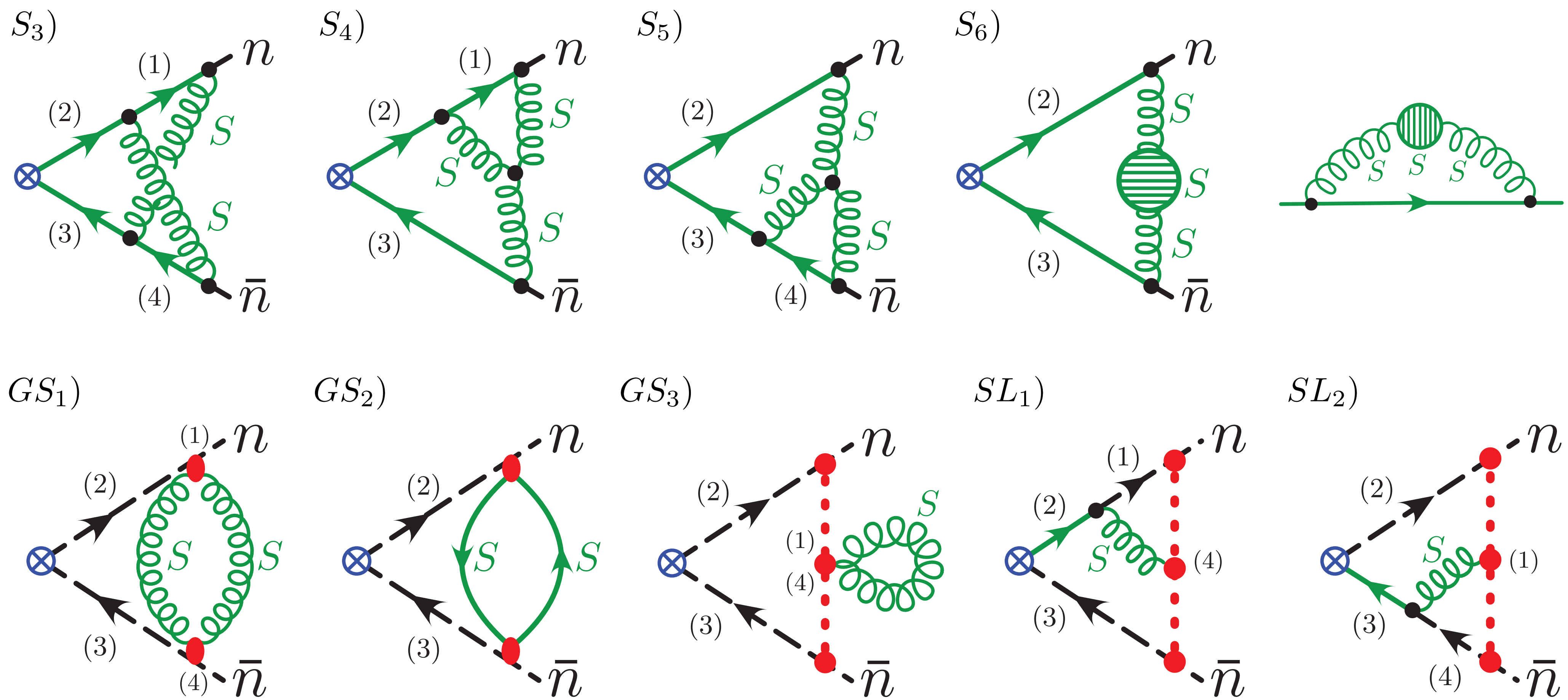
naive soft graph

i.e. soft subtractions= Glaubers

subtracted

non-Abelian





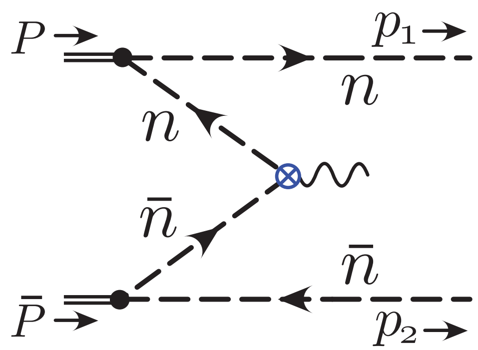
As before we find that the Glaubers sit inside the softs

$$S_3 + S_4 + S_5 + S_6 + GS_1 + GS_2 + GS_3 + LS_1 + LS_2 = \tilde{S}_3 + \tilde{S}_4 + \tilde{S}_5 + \tilde{S}_6\,.$$

Subtracted softs

Naive softs

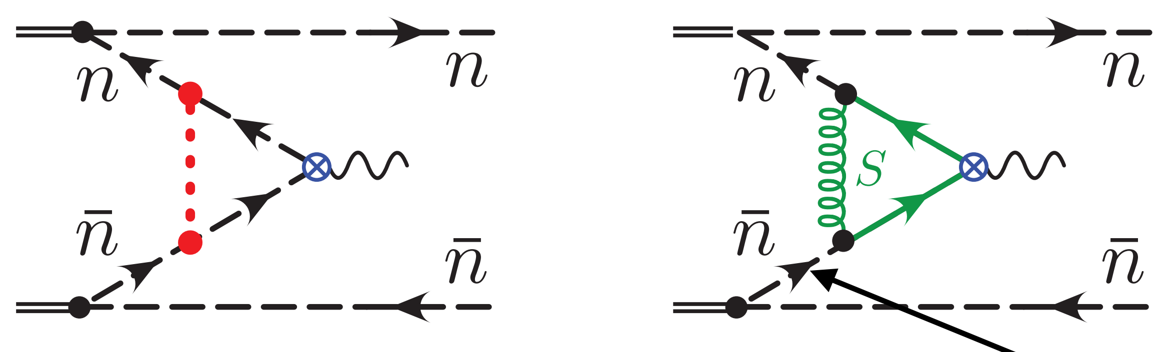
So it seems that Glaubers are absorbable into Wilson line when we consider partonic scattering, what about hadronic?



$$= \mathcal{S}^\gamma \, \frac{i\,\bar n\cdot(p_1 - P)}{(P-p_1)^2}\, \frac{i\,n\cdot(\bar P - p_2)}{(\bar P - p_2)^2}$$

$$= \mathcal{S}^\gamma \left[\frac{1}{\vec p_{1\perp}^{\,2}}\,\frac{1}{\vec p_{2\perp}^{\,2}}\right]\left[\frac{\bar n\cdot p_1\,\bar n\cdot(P-p_1)}{\bar n\cdot P}\,\frac{n\cdot p_2\,n\cdot(\bar P - p_2)}{n\cdot \bar P}\right]$$

$$\equiv S^\gamma\, E(p_{1\perp}, p_{2\perp}),$$



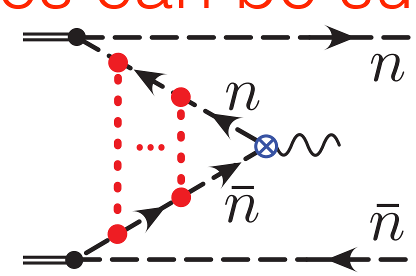
a) b)

only difference from previous calculation is that line is not longer on shell. Just shifts Δ which we know from Glauber loop computation does not effect result.

So again Glauber absorbable into S Wilson line

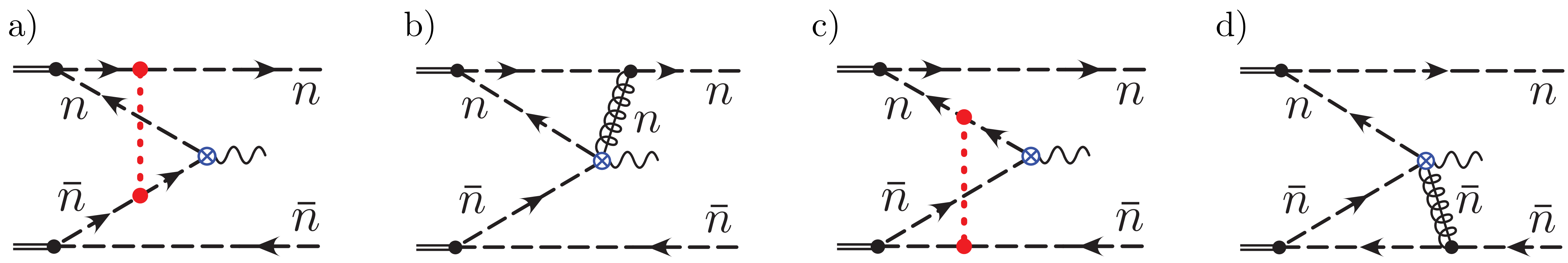
Moreover, the entire series can be summed to generate a phase

$\sum_{\#\text{rungs}}$



$$= \; S^\gamma E(p_{1\perp}, p_{2\perp})\, e^{i\phi(0)/2}\,.$$

What About Active Spectator?



Glauber subtraction of collinear

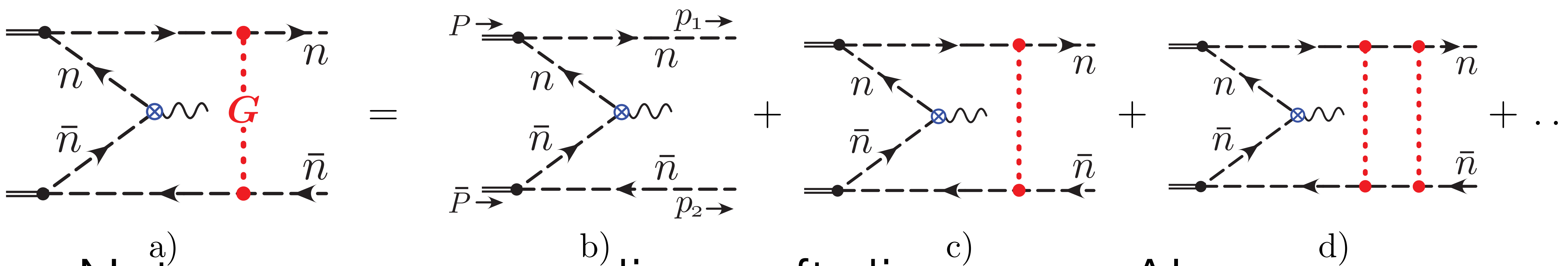
$$C_n^{(G)}(\text{Fig.}35b) = 2S^\gamma \frac{n\cdot(\bar{P}-p_2)}{(\bar{P}-p_2)^2} \int đ^d k \frac{G^0(k_\perp)\, |\bar{n}\cdot k|^{-\eta}\nu^\eta}{[k^- + i0][-k^+ - \Delta_1 + i0][k^+ - \Delta_1' + i0]} \,.$$

$$\text{Fig.}35a = 2S^\gamma \frac{n\cdot(\bar{P}-p_2)}{(\bar{P}-p_2)^2} \int đ^d k \frac{G^0(k_\perp)|2k^z|^{-\eta}\nu^\eta}{[k^- - \Delta_2 + i0][-k^+ - \Delta_1 + i0][k^+ - \Delta_1' + i0]} \,,$$

Rapidity regulator makes differences irrelevant

Glauber Effect absorbed into Collinear Wilson line.
These Glaubers are also benign

Spectator-Spectator



Note: no corresponding soft diagrams. Also fermions don't eikonalize

Sum over all boxes

$$\mathcal{S}^\gamma i^4 \int đ^d k \, \frac{-2G(k_\perp)\,(-1)^2}{[k^+ - \Delta_1' + i0][-k^+ - \Delta_1 + i0][k^- - \Delta_2 + i0][-k^- - \Delta_2' + i0]}$$

$$= \mathcal{S}^\gamma \int đ^{d-2} k_\perp \, G(k_\perp)\, E(p_{1\perp} + k_\perp, p_{2\perp} - k_\perp)\,.$$

$$= \mathcal{S}^\gamma \int d^{d-2} b_\perp \, e^{-i\Delta\vec{p}_\perp \cdot \vec{b}_\perp} \, \tilde{E}'(b_\perp, q_\perp)\, e^{i\phi(b_\perp)}.$$

Exponentiates in impact parameter space

$$q_\perp = (-p_{2\perp} - p_{1\perp}) \qquad \Delta p = (p_{2\perp} - p_{1\perp})/2$$

When does this phase cancel?

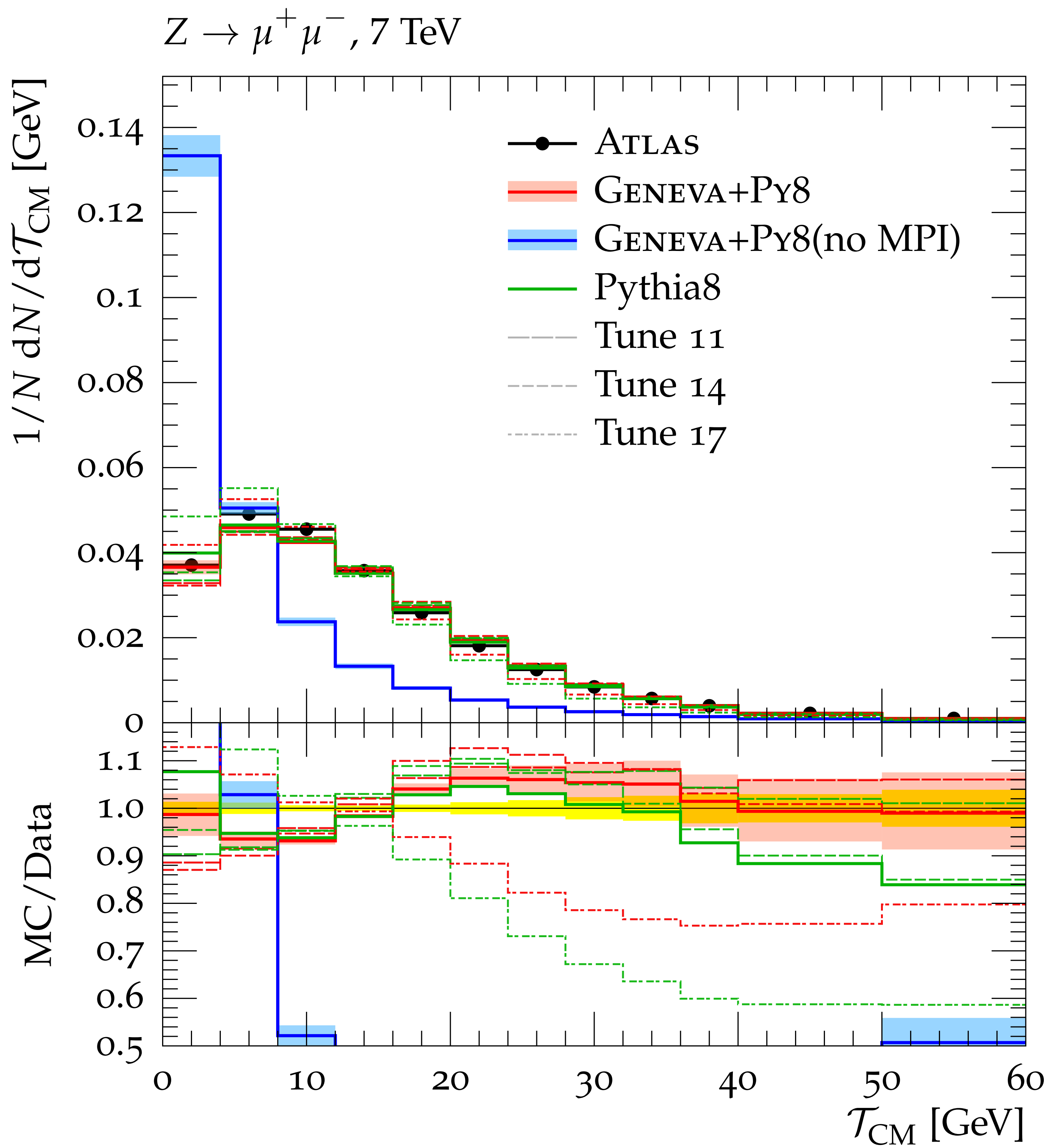
$$\int \!\!d\!\!\!{}^{-}{}^{d-2}\Delta p_\perp \left|\mathcal{A}_{SS}(\Delta p_\perp, q_\perp)\right|^2 \tag{337}$$
$$= |\mathcal{S}^\gamma|^2 \int \!\!d\!\!\!{}^{-}{}^{d-2}\Delta p_\perp \int d^{d-2}b_\perp \, d^{d-2}b'_\perp \, e^{i\Delta\vec{p}_\perp\cdot(\vec{b}'_\perp - \vec{b}_\perp)} \, \tilde{E}'(b_\perp, q_\perp)\tilde{E}'^\dagger(b'_\perp, q_\perp) \, e^{i\phi(b_\perp) - i\phi(b'_\perp)}$$
$$= |\mathcal{S}^\gamma|^2 \int d^{d-2}b_\perp \left|\tilde{E}'(b_\perp, q_\perp)\right|^2$$

As long as we integrate over full range of $\Delta p_\perp$

e.g. Beam thrust (Gaunt, Zeng),
factorization violated at $O(\alpha^4)$

However, given our working definition of factorable, one would say that this rate is still factorable since one can still make a prediction in terms of PDF's.

However, there is more going on, data disagrees with these predictions by an amount of order one.



(Alioli, Bauer, Guns, Tackmann)
(to appear)

Beam Thrust:

$$\tau_B = \frac{1}{Q} \sum_k |\, p_{k\perp} \,|\, e^{|\eta_k - Y|}$$

Conclusions

Set up systematic EFT to address the question of
Glauber Gluons (Completes SCET)

- In purely active parton interactions the Glauber is responsible for the direction of Wilson lines (relevant for possible non-universality of matrix elements.

- In spectator interactions, there are no corresponding soft graphs. Glaubers have their own life, burden of proof on user. Sufficient criteria for NON-cancellation is integration over transverse momentum difference.

Other uses not discussed

- Systematics of Reggeization
- Small x physics, BFKL resummations