Ideas behind parton showers 1984-2016

Davison E. Soper
University of Oregon

KITP, Santa Barbara
May 2016

Factorization

- For an observable F we have

$$\sigma(F) = \sum_{a,b} \int_0^1 d\eta_{\rm a} \int_0^1 d\eta_{\rm b}\ f_{a/A}(\eta_{\rm a},\mu^2) f_{b/A}(\eta_{\rm b},\mu^2) \times \hat\sigma(a,b,\eta_{\rm a},\eta_{\rm b},F,\mu^2) + \mathcal{O}(1\ {\rm GeV}^2/Q^2(F))$$

- μ^2 is an adjustable factorization scale.

- $Q^2(F)$ is a hard scale corresponding to the observable F.

- Errors are power suppressed when $Q^2(F)$ is large.

The observable

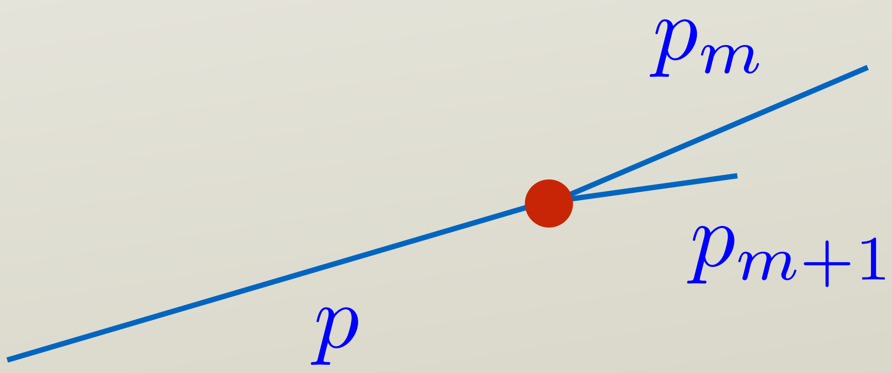
$$\hat{\sigma}(a, b, \eta_{\rm a}, \eta_{\rm b}, F, \mu^2) = \sum_m \frac{1}{m!} \int dy_1 \prod_{j=2}^{m} \int dp_{\perp,j}\, dy_j\, d\phi_j$$
$$\times \frac{d\hat{\sigma}}{dy_1\;\, dp_{\perp,2}\, dy_2\, d\phi_2 \cdots dp_{\perp,m}\, dy_m\, d\phi_m}$$
$$\times F_m(p_1, p_2, \ldots p_m)$$

- F defines, for instance, three jets with given $P_\perp$ values.
- By adding flavor indices, we could describe leptons, photons.
- We can choose $F_m(p_1, p_2, \ldots, p_m)$ to be symmetric under interchange of its arguments.

Infrared safety

- For our discussion, F needs to be infrared safe.
- We can be (a little) more precise by saying that F is infrared safe at scale $Q^2(F)$.
- For partons m and $m+1$ becoming collinear,

$$
\begin{aligned}
p_m &\to zp \\
p_{m+1} &\to (1-z)p
\end{aligned}
$$



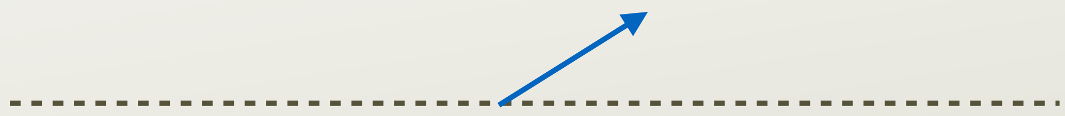
when they are sufficiently collinear,

$$(p_m + p_{m+1})^2 < Q^2(F)$$

we ask that combining the partons leaves F unchanged:

$$F_{m+1}(p_1, \ldots, p_{m-1}, p_m, p_{m+1}) \approx F_m(p_1, \ldots, p_{m-1}, p)$$

- Also when one parton is becoming aligned to the beam axis



$$\boldsymbol{p}^2_{m+1,\perp} < Q^2(F)$$

we ask that leaving it out leaves F unchanged:

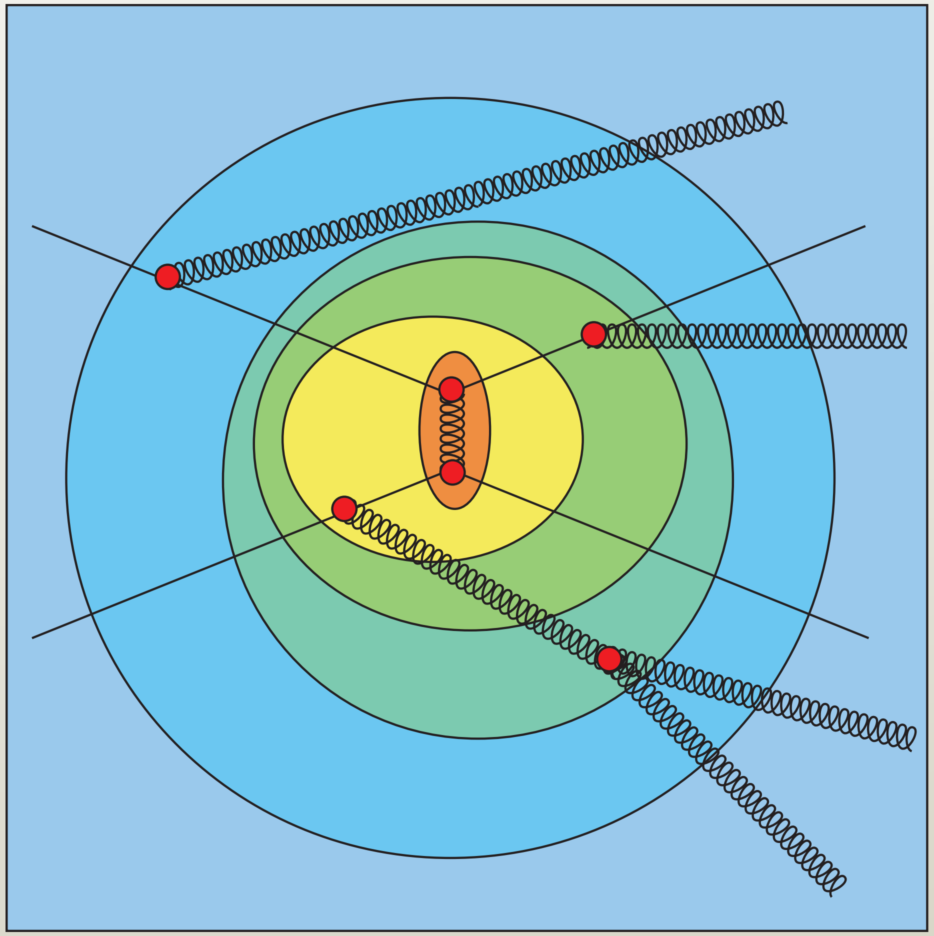
$$F_{m+1}(p_1, \ldots, p_{m-1}, p_m, p_{m+1}) \approx F_m(p_1, \ldots, p_{m-1}, p_m)$$

Pythia (1985)

- Torbjörn Sjöstrand proposed starting at the hardest interaction.

- Then one generates parton splittings that are softer and softer.

- For initial state splittings, this means going backwards in time.

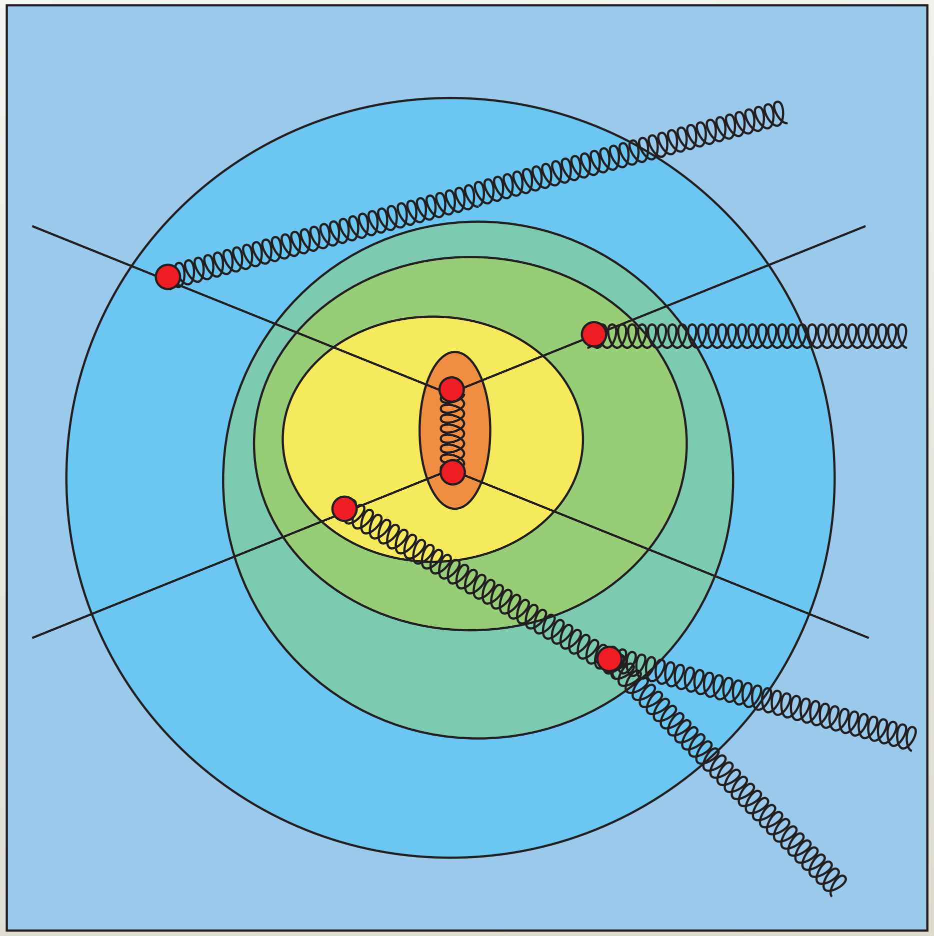


- In 1985, this was quite counterintuitive.

- In 2015, it is standard.

- This makes shower evolution into a renormalization group equation.

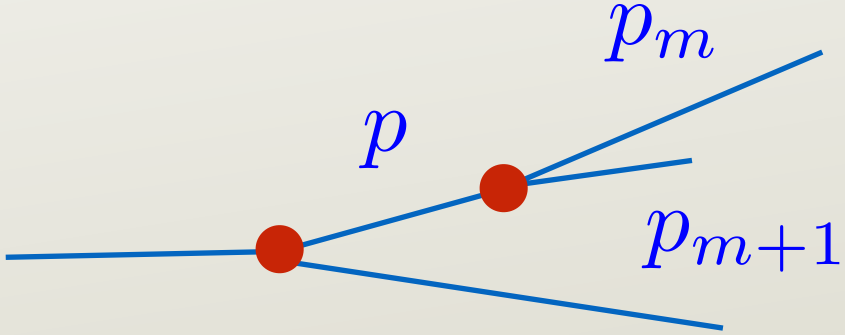
Relation to factorization



- Suppose that we stop the shower at scale Q_1^2 and measure an observable F with $Q_1^2 \lesssim Q^2(F)$.
- Then we continue the shower and measure F again.
- Since the later splittings have $Q^2 < Q_1^2 \lesssim Q^2(F)$, they are unresolvable by F.
- So $\sigma(F)$ is unchanged.

The perturbative expansion

$$\sigma(F) = \sum_{a,b} \int_0^1 d\eta_{\rm a} \int_0^1 d\eta_{\rm b}\ f_{a/A}(\eta_{\rm a},\mu^2) f_{b/A}(\eta_{\rm b},\mu^2)$$
$$\times\ \hat\sigma(a,b,\eta_{\rm a},\eta_{\rm b},F,\mu^2)$$
$$+\ \mathcal{O}(1\ {\rm GeV}^2/Q^2(F))$$



- The function $\hat\sigma(a,b,\eta_{\rm a},\eta_{\rm b},F,\mu^2)$ has a perturbative expansion:

$$\hat\sigma(a,b,\eta_{\rm a},\eta_{\rm b},F,\mu^2) = \sum_n \alpha_{\rm s}(\mu^2)^n\ \hat\sigma_n(a,b,\eta_{\rm a},\eta_{\rm b},F,\mu^2)$$

- However, a parton shower does not evaluate this exactly.
- Rather, each splitting is approximated as being very collinear or very soft compared to the hardness of the previous splitting.

NLO matching (2002-2004)

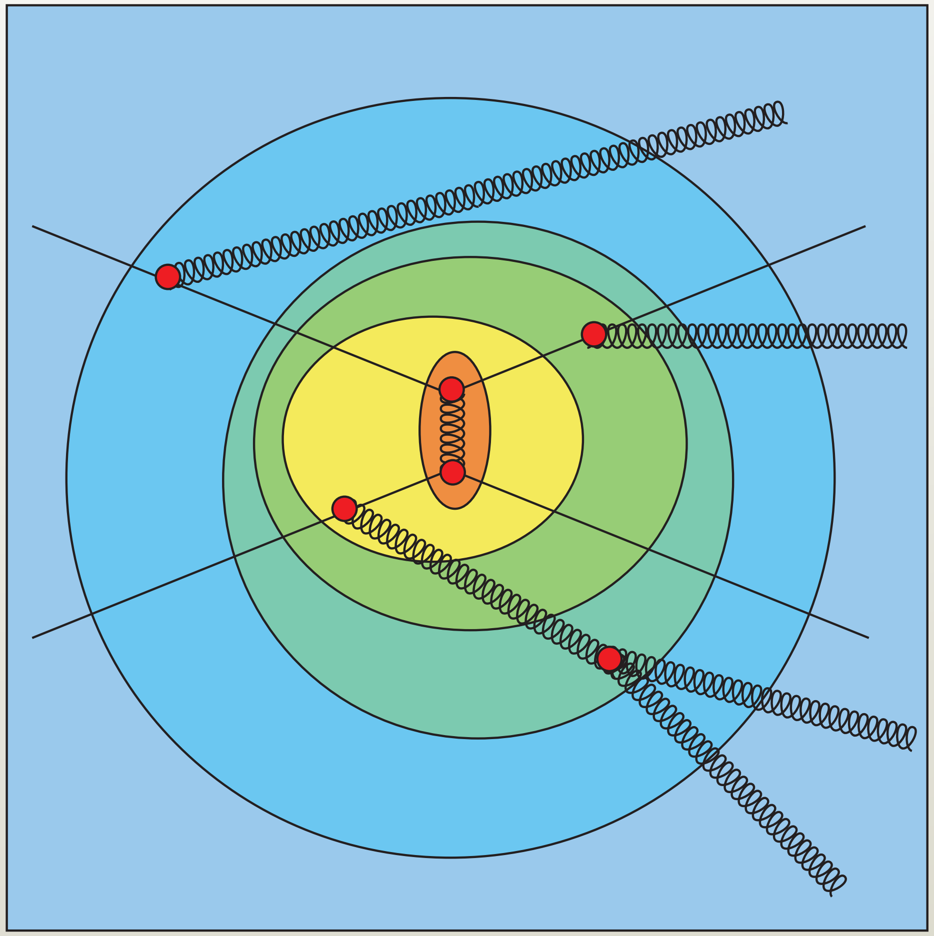
NNLO matching, to 2016

See talks of Höche and Baur

- The hardest scattering is LO order only.

- The hardest splitting gives an approximate NLO correction.

- We can correct this to give NLO exactly plus some yet higher order corrections.

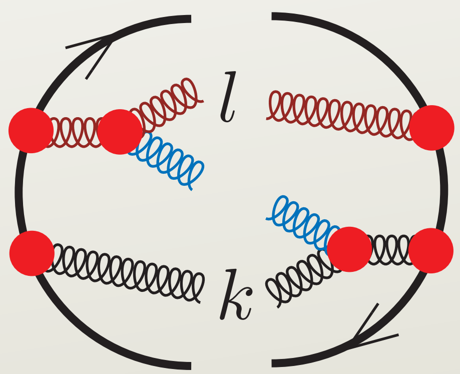


- Then running the simple shower further does not affect the result for $\sigma(F)$ for a large $Q^2(F)$ jet cross section.

- This is the basis of NLO matching schemes.
  - MC@NLO (Frixione, Webber)
  - POWHEG (Nason)

Why wasn't Pythia perfect?

- There is quantum interference between soft gluon emission from parton l and gluon emission from parton k.



- The interference is destructive when $\theta > \theta_{lk}$.

- So radiation from the l-k "dipole" is limited to $\theta \lesssim \theta_{lk}$.

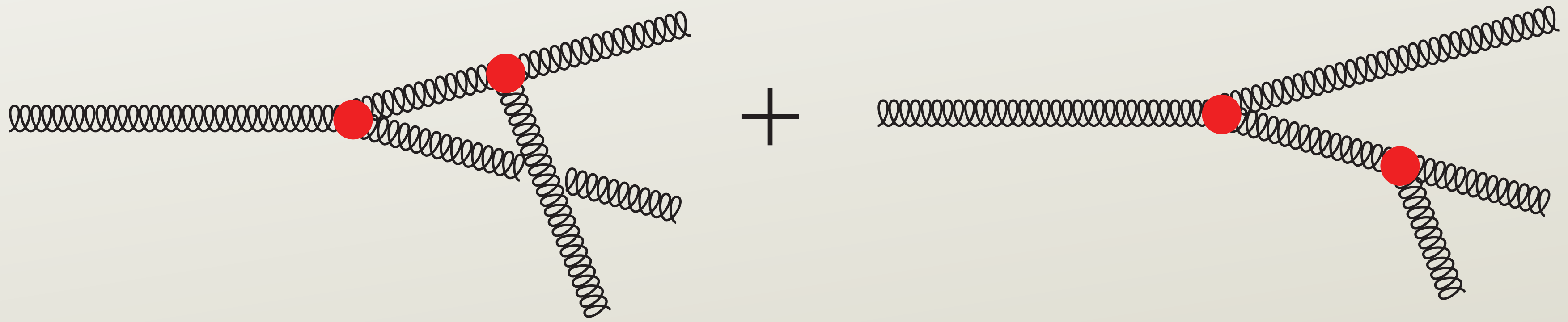
- In (old) PYTHIA, the only limit was $\theta \lesssim 1$.

- Thus soft, wide angle radiation was completely wrong.

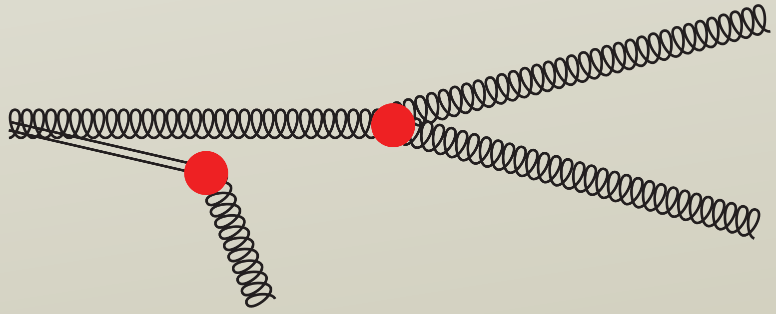
How Herwig fixed this (1984)

- Suppose that a gluon splits into two almost collinear gluons.

- Then each daughter radiates a soft, wide angle gluon.

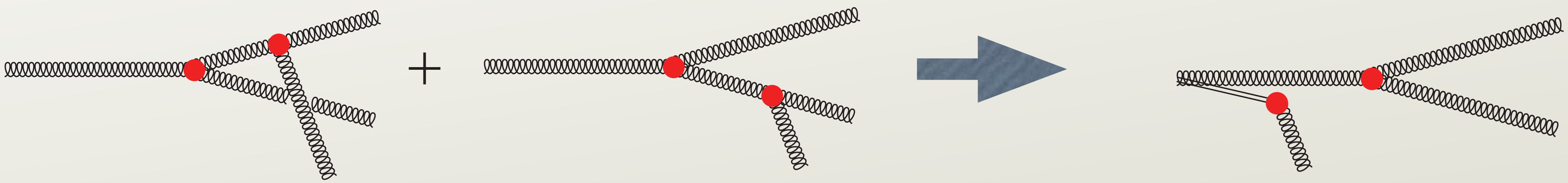


- This is as if the soft gluon were emitted from the mother.



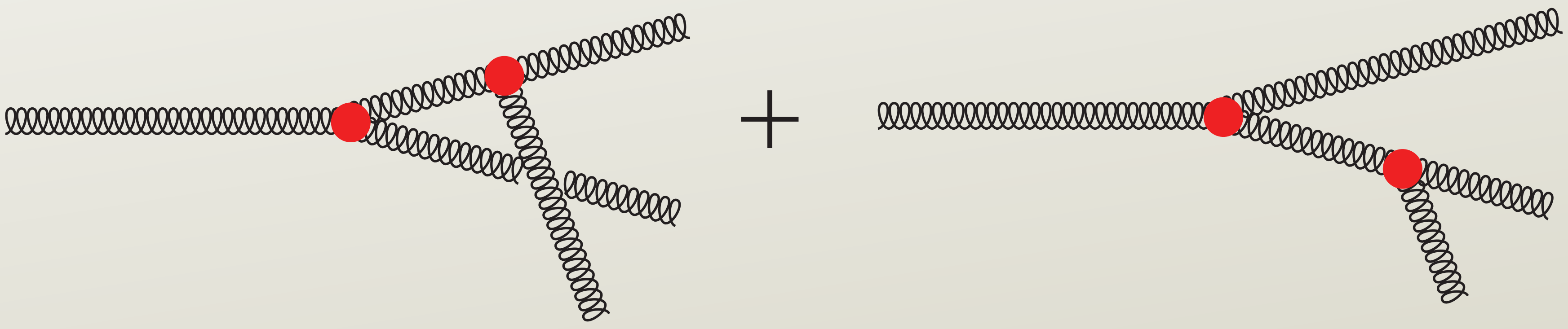
- Or, rather, to an on-shell approximation to the mother.

Implementing color coherence



- Webber and Marchesini (1984) showed how to implement this in an event generator.
- This became the basis of Herwig (Webber, 1984).
- Put the wide angle splittings first.
- This involves an approximation for the azimuthal angle distributions.

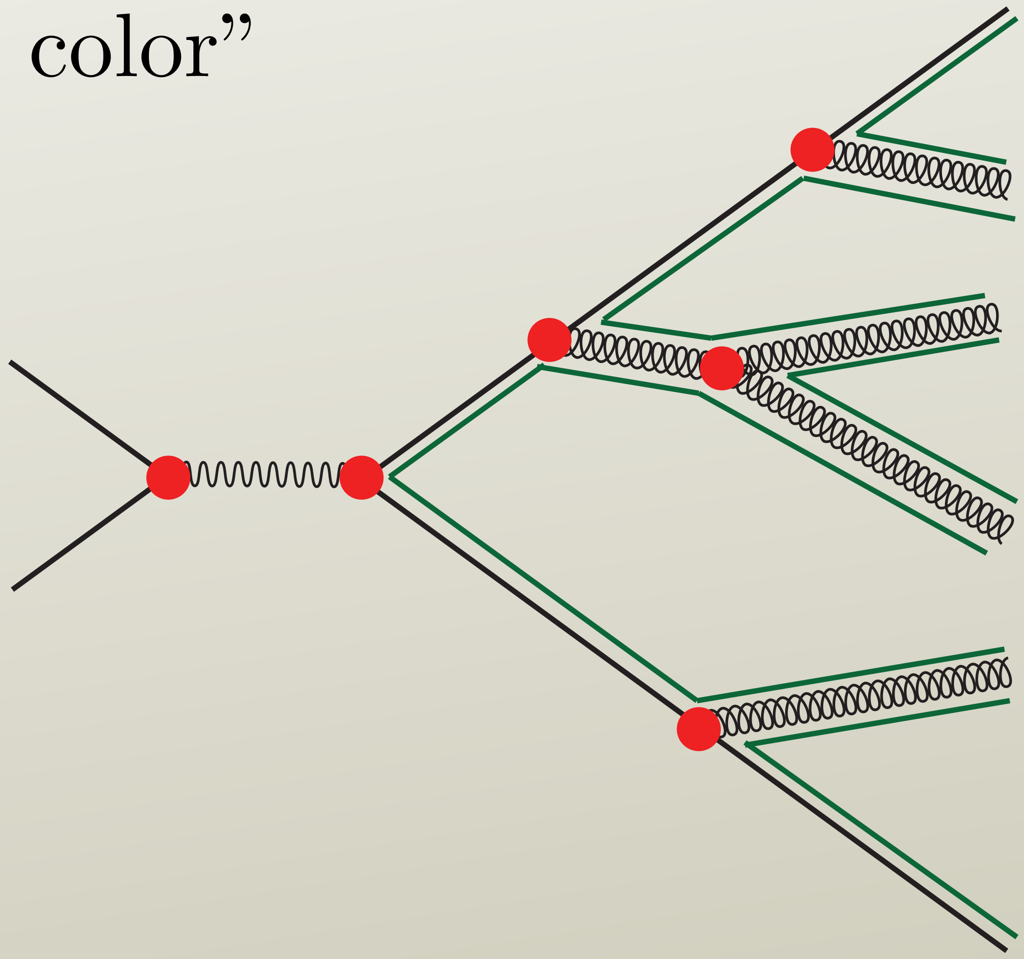
What about Pythia?



- Early Pythia just imposed a cut on angles.
- This roughly simulates the coherence effect.

Color

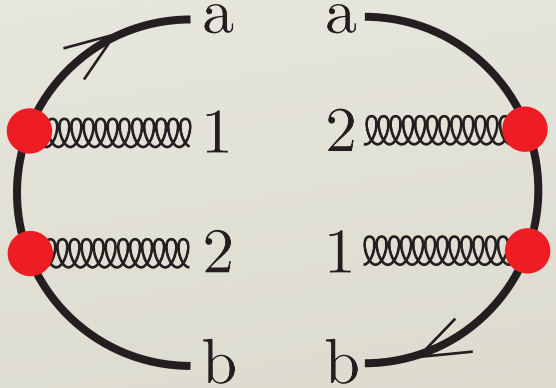
- Parton shower event generators track color.
- Mostly they use the "leading color" approximation.
- Gluons carry color $\mathbf{3} \times \bar{\mathbf{3}}$ rather than $\mathbf{8}$.
- Corrections are order $1/N_c^2$ ($N_c = 3$).



Doing better with color

- A parton shower should track the color density matrix,

$$\sum_{\{c\}_m,\{c'\}_m} \rho(\{c\}_m,\{c'\}_m)\ \big|\{c\}_m\big\rangle\big\langle\{c'\}_m\big|$$



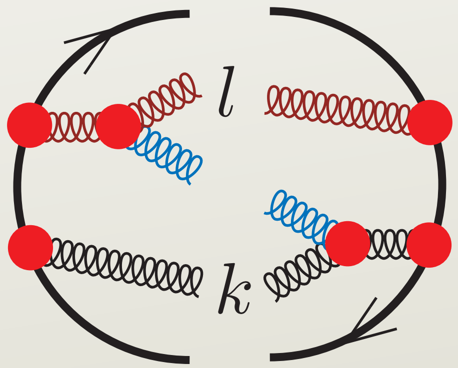
$\big|\{c\}_m\big\rangle$ $\big\langle\{c'\}_m\big|$

- But this gives exponentials of large matrices.

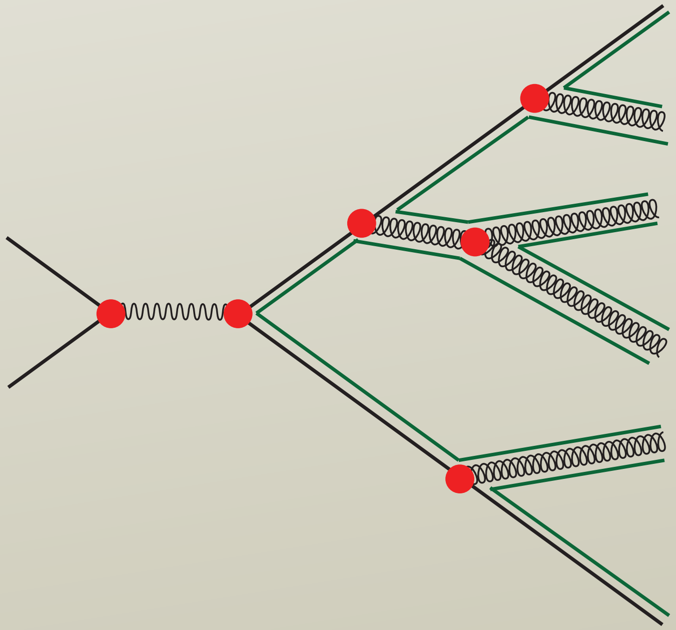
- So implementing full color in a parton shower is an unsolved problem.

- DEDUCTOR (Nagy-Soper 2014) has an improved color treatment, "LC+."

Color and dipoles



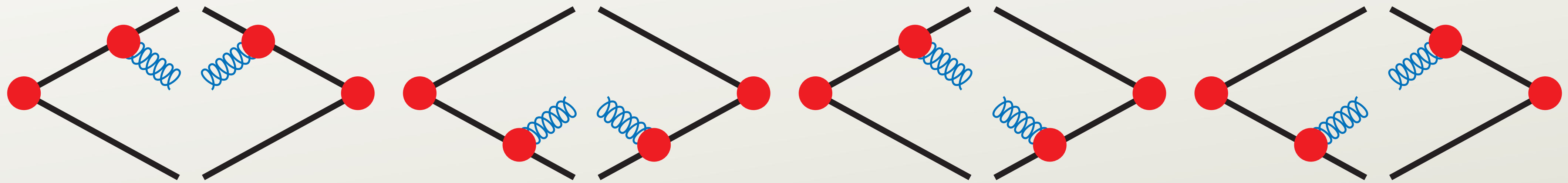
- A gluon line has two ends.
- So we can always consider it to be radiated by a dipole.
- In general, we need color matrices, $\boldsymbol{T}_l^a\ \boldsymbol{T}_k^a$.



- In the leading color approximation, we consider only pairs of partons that are color connected.
- Then we have just C_{F} or C_{A} instead of matrices.

Ariadne (1988, 1992)

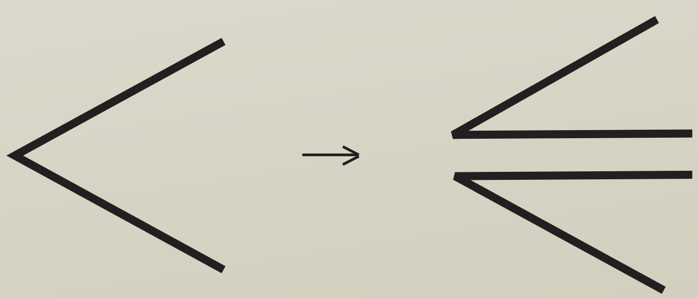
- For gluon emission from a (leading color) dipole, there are four possible graphs.



- We can combine all four into one.

- Use the approximation that the emitted gluon is soft or collinear to one of the constituent partons.

- Then one dipole splits to two dipoles.



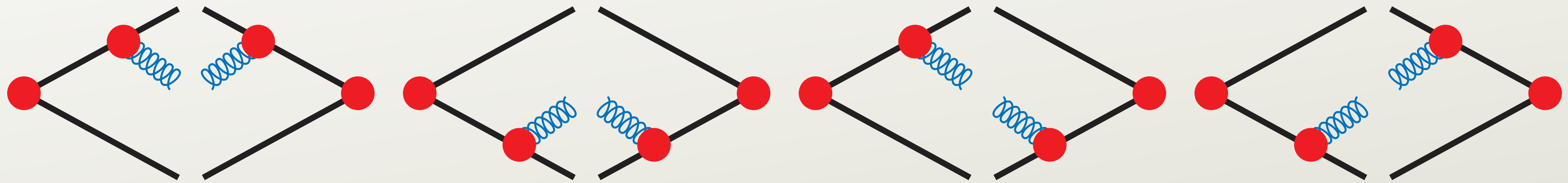
- That is, two partons split to three partons.

- Splittings can be organized by decreasing hardness.

- This was proposed by Gustafson and Petersson (1988).
- It was implemented as ARIADNE by Lönnblad (1992).
- I like to call this the dipole antenna picture.
- Note that it nicely captures quantum interference (at leading color).
- This works well for final state splittings, but not so well for splittings with an initial state parton.
- Winter and Krauss (2008) devised a reasonable extension for initial state partons.
- Giele, Kosower, Skands implemented a dipole antenna shower in VINCIA (2008).
- Ritzmann, Kosower and Skands extended VINCIA to cover initial state dipoles (2013).

Partitioned dipoles

- For emission of a soft gluon with momentum q from a dipole with parton momenta p_l, p_k, there are four possible graphs.



- The sum is the soft eikonal factor

$$\psi_{lk}^{\text{dipole}} = \frac{2\, p_l \cdot p_k}{q \cdot p_l \;\, q \cdot p_k}$$

- Multiply this by $1 = A'_{lk} + A'_{kl}$ where (for example)

$$A'_{lk} = \frac{q \cdot p_k \;\, Q \cdot p_l}{q \cdot p_k \;\, Q \cdot p_l + q \cdot p_l \;\, Q \cdot p_k}$$

and Q is the total final state momentum after the splitting.

- This partitions the dipole radiation into two terms.

- The first of the two terms is

$$\psi_{lk}^{\rm dipole} A'_{lk} = \frac{2\, p_l \cdot p_k}{\cancel{q \cdot p_k}\; q \cdot p_l} \; \frac{\cancel{q \cdot p_k}\; Q \cdot p_l}{q \cdot p_k\; Q \cdot p_l + q \cdot p_l\; Q \cdot p_k}$$

- This has a collinear singularity when q is collinear with p_l.
- It has no collinear singularity when q is collinear with p_k.
- We associate this term with emission from parton l with parton k as helper.
- The other term describes emission from parton k with parton l as helper.
- Thus each emission has a definite emitter.
- But we keep the quantum interference.

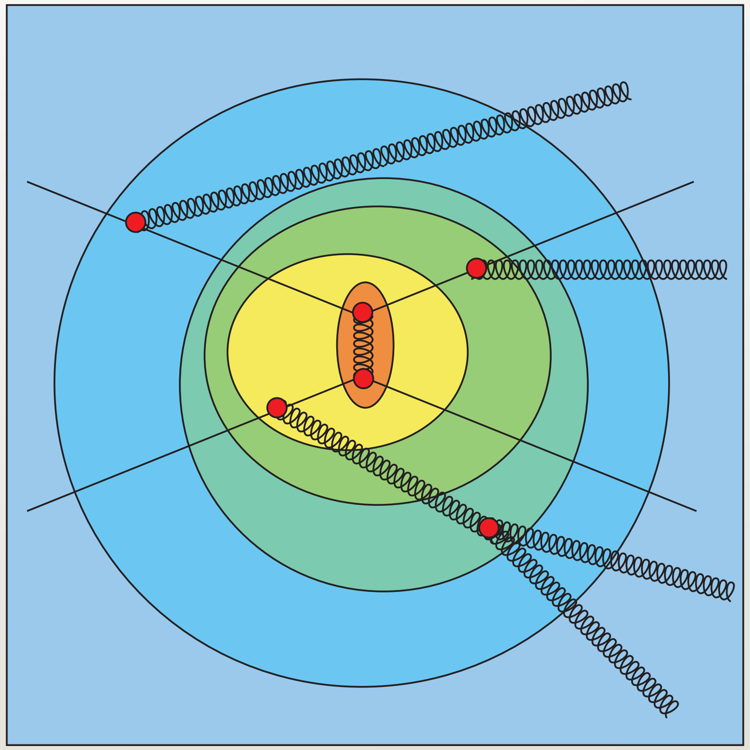
Partitioned dipole showers

- Deductor is a partitioned dipole shower.

- Pythia-8 is similar to a partitioned dipole shower for the final state.
  - But not for initial state emissions.

Partitioned dipole showers Catani-Seymour style

- The splitting functions of a properly formulated shower capture the collinear and soft gluon singularities of QCD.



- So $full - shower$ has the singularities removed.
- So the shower splitting functions can serve as the subtractions in an NLO calculation.
- Also, the subtraction terms for an NLO calculation can serve as the splitting functions for a shower.
- Catani and Seymour (1997) created a subtraction scheme based on dipoles for doing NLO calculations.
- There are some advantages to using this subtraction scheme to define splitting functions of a shower (Nagy-Soper, 2006).

Catani-Seymour dipole showers

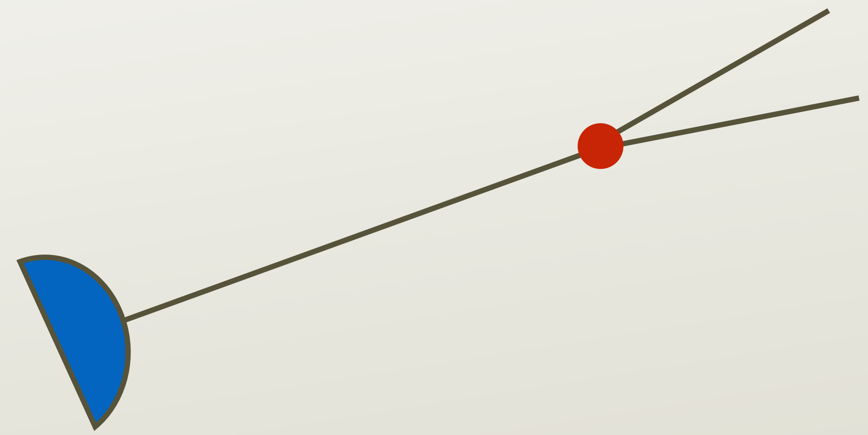
- There are small variations among these.

1. Dinsdale, Ternick and Weinzierl (2007).
2. Schumann and Krauss (2008) (default in Sherpa).
3. Plätzer and Gieseke (2011, 2012) (available in Herwig).
4. Höche and Prestel (2015) (available in Sherpa and Pythia).

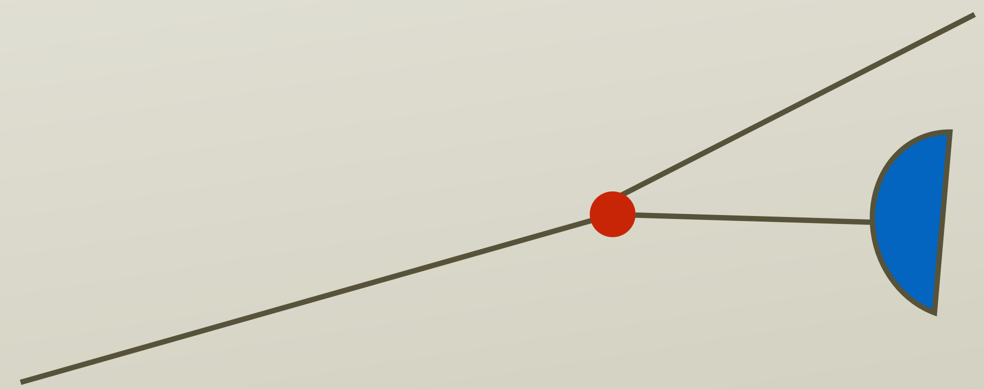
Choices in partitioned dipole showers

Momentum mapping

- In a final state splitting, the mother parton was on-shell.
- Afterwards, we see that mother parton is off-shell.



- In an initial state splitting, the mother parton had zero $\boldsymbol{p}_{\perp}$.
- Afterwards, we see that the mother parton must have non-zero $\boldsymbol{p}_{\perp}$



- To make this work, some other partons must pay a momentum tax.

- In Deductor, all of the other final state partons pay according to their momentum wealth.

- In the Catani-Seymour scheme, this also applies for an IS splitting with an IS spectator.

- Otherwise in the Catani-Seymour scheme, a *single parton* pays the momentum tax: the dipole partner parton.

but

- Plätzer and Gieseke take the momentum from all final state particles for *all* initial state splittings.

- For the $\boldsymbol{p}_\perp$ distribution in the Drell-Yan process, this allows the vector boson to recoil against all initial state radiation.

The partitioning function

- DEDUCTOR uses

$$A'_{lk} = \frac{q\cdot p_k\; Q\cdot p_l}{q\cdot p_k\; Q\cdot p_l + q\cdot p_l\; Q\cdot p_k}$$

In the $\vec{Q} = 0$ frame, this is a function only of the directions of $\vec{q}$, $\vec{p}_l$ and $\vec{p}_k$.

- The Catani-Seymour dipole subtraction scheme uses

$$A'_{lk} = \frac{q\cdot p_k}{q\cdot p_k + q\cdot p_l}$$

This is simple.

Splitting functions

- The splitting functions have to match QCD in the soft and collinear collinear limits.

- This implies that the splitting functions approach the DGLAP kernels $P_{ab}(z)$ in the collinear limit.

- Away from the soft and collinear collinear limits there are no sure guidelines.

- Catani and Seymour have a simple choice.

Evolution variable

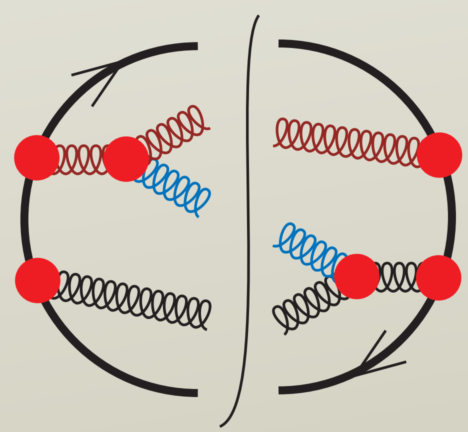
- One needs a hardness variable to order splittings from hardest to softest.
- The hardness variable needs to vanish for an exactly collinear splitting and for emission of a zero momentum parton.
- $\boldsymbol{k}_\perp^2$ is the most popular choice.
- Usually $\boldsymbol{k}_\perp$ is defined in the rest frame of a dipole.
- DEDUCTOR uses q^2/E where q^2 is the virtuality and E is the energy of the mother parton as measured in a fixed frame.
- To my knowledge, no choice is demonstrably best.

The Sudakov factor

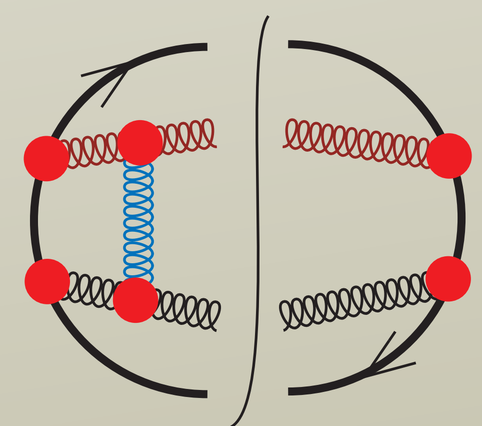
- Let $\big|\rho(t)\big)$ represent the probability distribution of parton variables after the shower has run for time t.
- $\big|\rho(t)\big)$ is a density matrix in color space.
- That is, $\big|\rho(t)\big)$ is the state of the system as described by quantum statistical mechanics.
- Evolution:

$$\frac{d}{dt}\big|\rho(t)\big) = [\mathcal{H}_I(t) - \mathcal{S}(t)]\big|\rho(t)\big)$$

$\mathcal{H}_I(t) =$ real emissions.



$\mathcal{S}(t) =$ parton evolution + virtual graphs.



- Define the Sudakov or "no splitting" operator:

$$\mathcal{N}_{\mathcal{S}}(\tau, t_0) = \mathbb{T} \exp\left[-\int_{t_0}^{\tau} d\tau' \; \mathcal{S}(\tau') \right]$$

- Then the solution of the evolution equation is

$$\big|\rho(t)\big) = \mathcal{N}_{\mathcal{S}}(t, t_0)\big|\rho(t_0)\big) + \int_{t_0}^{t} d\tau \; \mathcal{N}_{\mathcal{S}}(t, \tau)\mathcal{H}_I(\tau)\mathcal{N}_{\mathcal{S}}(\tau, t_0)\big|\rho(t_0)\big)$$

$$+ \int_{t_0}^{t} d\tau_2 \int_{t_0}^{\tau_2} d\tau_1 \; \mathcal{N}_{\mathcal{S}}(t, \tau_2)\mathcal{H}_I(\tau_2)\mathcal{N}_{\mathcal{S}}(\tau_2, \tau_1)\mathcal{H}_I(\tau_1)\mathcal{N}_{\mathcal{S}}(\tau_1, t_0)\big|\rho(t_0)\big)$$

$$+ \cdots$$

- Beware:

$$\frac{d}{dt}\big|\rho(t)\big) = [\mathcal{H}_I(t) - \mathcal{S}(t)]\big|\rho(t)\big)$$

does not conserve the Born-level cross section under shower evolution:

$$\big(1\big|[\mathcal{H}_I(t) - \mathcal{S}(t)] \neq 0$$

totally inclusive measurement

- Thus people substitute

$$\frac{d}{dt}\big|\rho(t)\big) = [\mathcal{H}_I(t) - \mathcal{V}(t)]\big|\rho(t)\big)$$

where

$$\big(1\big|[\mathcal{H}_I(t) - \mathcal{V}(t)] = 0$$

- Nagy and I argue that it is better to use $\mathcal{S}(t)$.

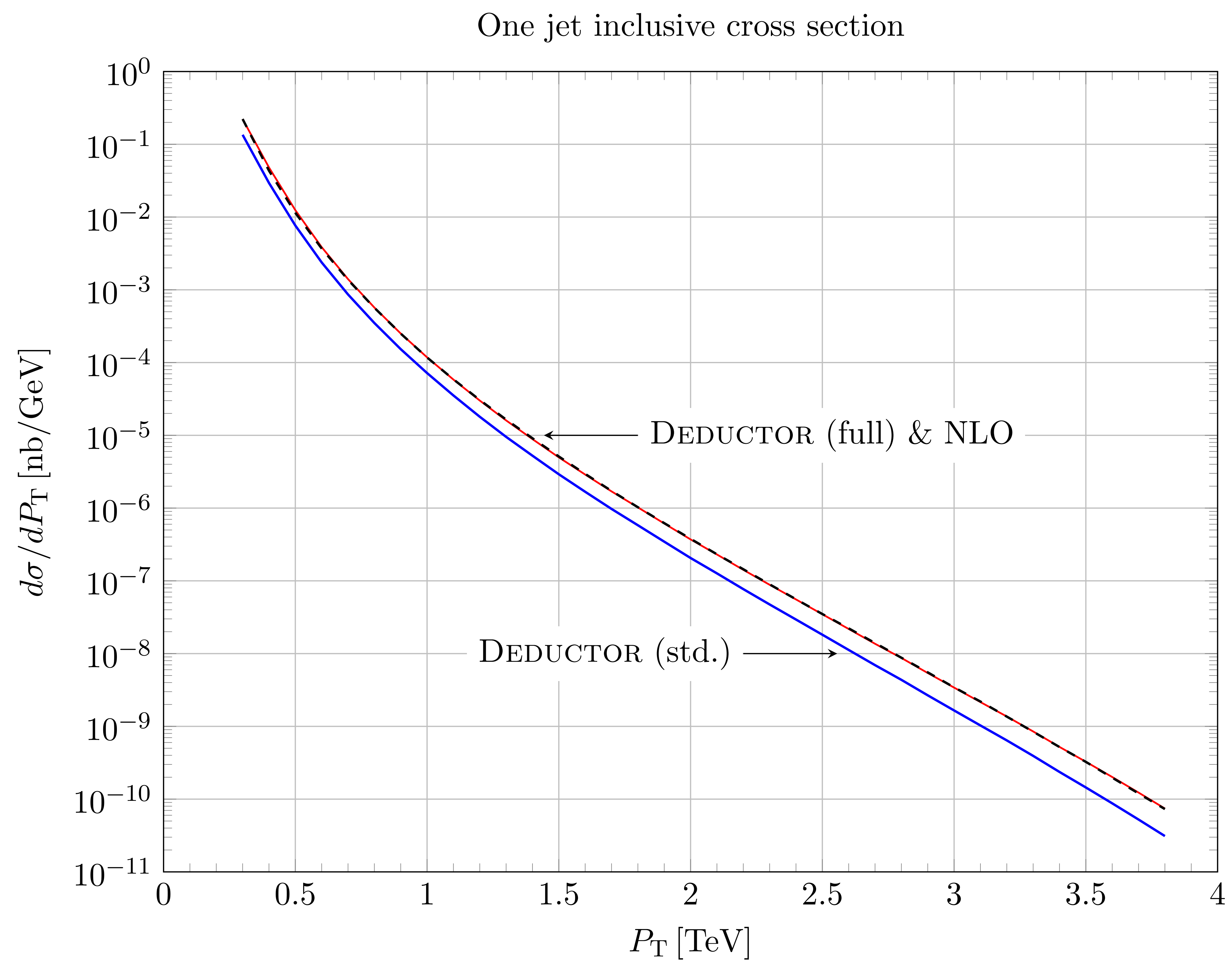
- Then the parton shower sums “threshold logarithms.”

- Example: one jet inclusive cross section $d\sigma/(dP_{\mathrm{T}})$ with

  DEDUCTOR(std.) with $\mathcal{V}$

  DEDUCTOR(full) with $\mathcal{S}$
  - includes also a factor for redefinition of parton distribution functions

  NLO


One jet inclusive cross section
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
$d\sigma/dP_T$ [nb/GeV]
0
0.5
1
1.5
2
2.5
3
3.5
4
P_T [TeV]
DEDUCTOR (full) & NLO
DEDUCTOR (std.)

Conclusions

- There has been considerable development of parton shower algorithms since the beginning, but especially in the past fifteen years.
- The essential physics input is factorization and quantum interference.
- There are choices that are not fixed by this input.
- Partons carry quantum spin, but I have skipped a discussion of spin issues.
- Partons carry quantum color, which I have discussed.
- Implementing full color is an outstanding problem.
- One can sum (approximately) threshold logarithms.

There is more to understand

- What is the relation of parton showers to summing large logarithms?

  - Can parton showers account for rapidity logarithms, as in HIGH ENERGY JETS (Andersen and Smillie, 2010)?

- What would one mean by a parton shower algorithm with the splitting functions defined beyond order α_s.