Mass Determinations in Decay Chains with Missing Energy

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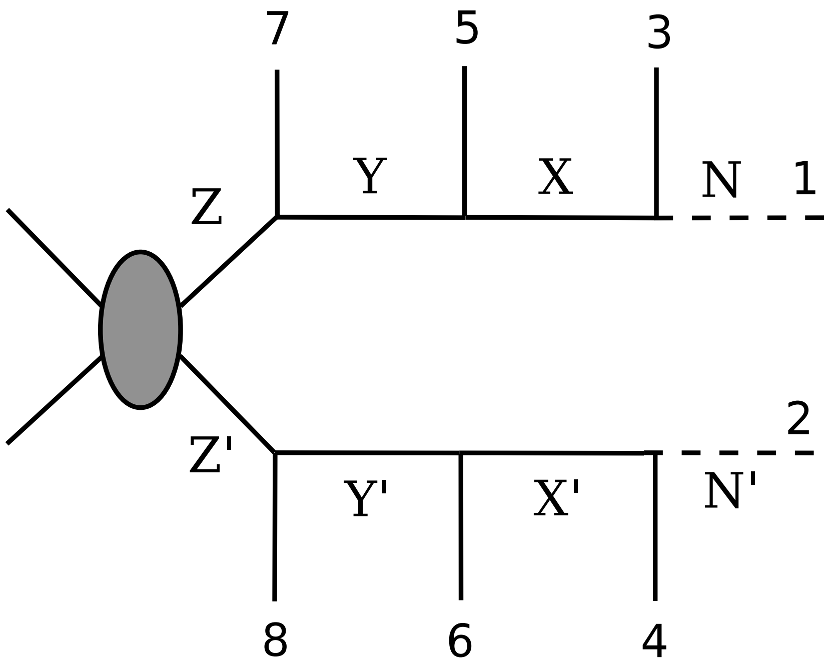
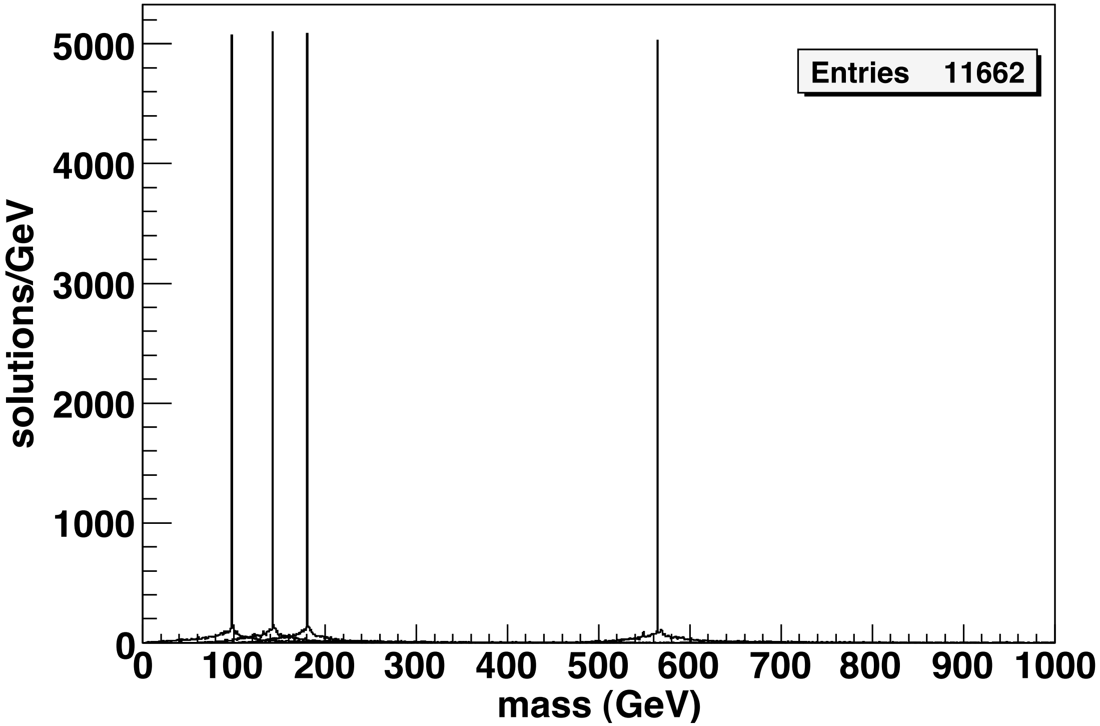
Case 1



An ideal example

$\tilde{q}\tilde{q} \to q\tilde{\chi}_2^0 q\tilde{\chi}_2^0 \to q\tilde{l}lq\tilde{l}l \to q\tilde{\chi}_1^0 llq\tilde{\chi}_1^0 ll$
SPS1a, masses: (97.4, 142.5, 180.3, 564.8) GeV

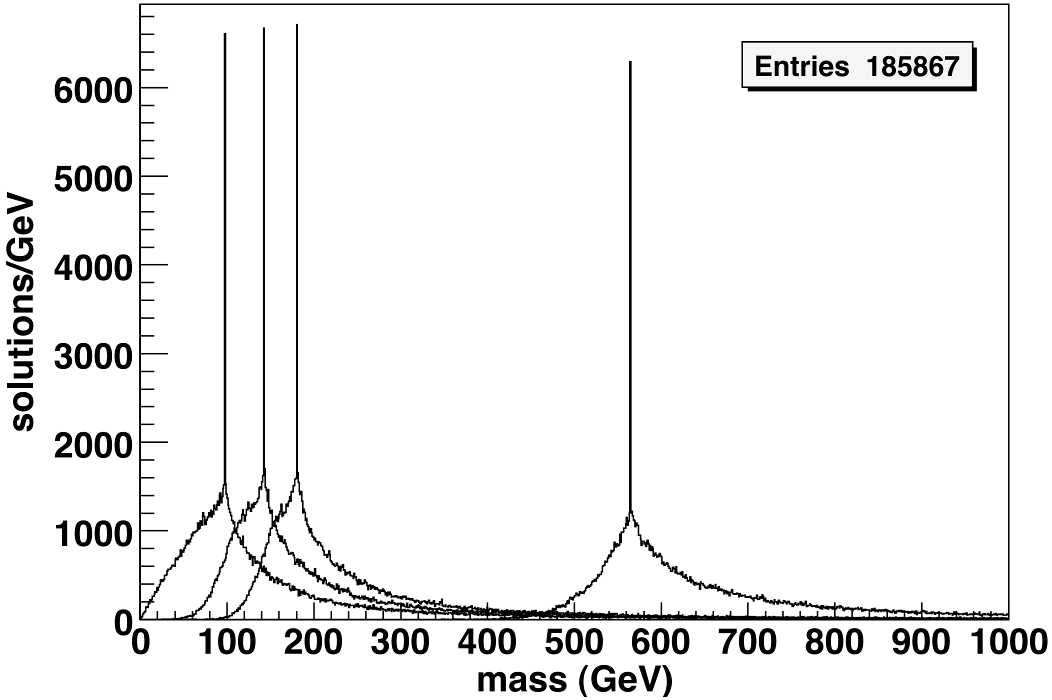


2 solutions per pair on average.

Realistic case

Wrong combinations

- One event, 8 combinations for $2\mu 2e$ channel, 16 for 4μ or $4e$ channel.
- A pair of events, 64, 128 or 256 combinations.



16 times more solutions.

Other issues

PYTHIA+ATLFAST

- Finite width, 5GeV, 20MeV, 200 MeV for $\widetilde{q}_L$, $\widetilde{\chi}_2^0$ and $\widetilde{\ell}_R$.
- Flavor splitting $m_{\widetilde{d}_L} - m_{\widetilde{u}_L} \sim 6 GeV$.
- Initial/final state radiation. Use a p_T cut to get rid of soft jets.
- Extra jet from $\widetilde{g} \rightarrow q\widetilde{q}_L$.
  $m_{\widetilde{g}} - m_{\widetilde{q}_L} = 40\ \mathrm{GeV}$, $m_{\widetilde{q}_L} - m_{\widetilde{\chi}_2^0} = 380\ \mathrm{GeV}$.
  $\rightarrow$ Select two jets with highest p_T.
- Experimental resolutions simulated by ATLFAST.

Background events

The SM background can be ignored by requiring large missing p_T+ 4 isolated leptons+2 enegetic jets.
SUSY background:

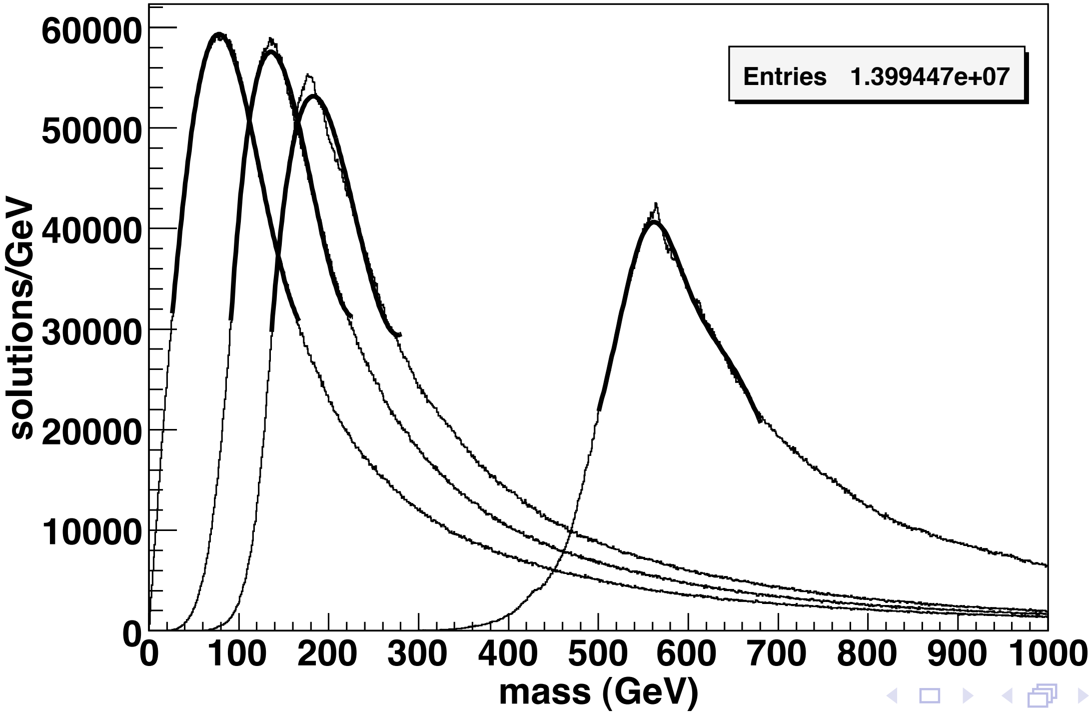
- $\widetilde{q}_L \rightarrow q\widetilde{\chi}_2^0 \rightarrow q\tau\widetilde{\tau} \rightarrow q\tau\tau\widetilde{\chi}_1^0$, both τ's decay letonically. $BR(\widetilde{\chi}_2^0 \rightarrow \tau\widetilde{\tau}) \sim 14\ BR(\widetilde{\chi}_2^0 \rightarrow \mu\widetilde{\mu})$. Require matching flavors and charges; lepton p_T cut.
- $\tilde{b}$'s have very different mass, $m_{\tilde{b}_1} \sim 520 GeV$, so $\tilde{b}$ pair production must be taken as a background. → Require: no b-jet.
- Electroweak processes: $\widetilde{\chi}_2^0 + \widetilde{\chi}_2^0$, $\widetilde{\chi}_2^0 + \widetilde{g}$. A jet p_T cut helps to reduce these backgrounds.

Realistic solution distributions

Cuts:

1. 4 isolated leptons with $p_T > 10\ \mathrm{GeV}$, $|\eta| < 2.5$, consistent flavors and charges.
2. No b-jet, $\geq$ 2 jets with $p_T > 100\ \ \mathrm{GeV}$, $|\eta| < 2.5$. Take 2 highest-p_T jets as partiles 7 and 8.
3. $p_{Tmiss} > 50\ \mathrm{GeV}$.

About 1000 events ($\sim$ 700 signals) after cuts for 300 fb^{-1}.



Fit the masses

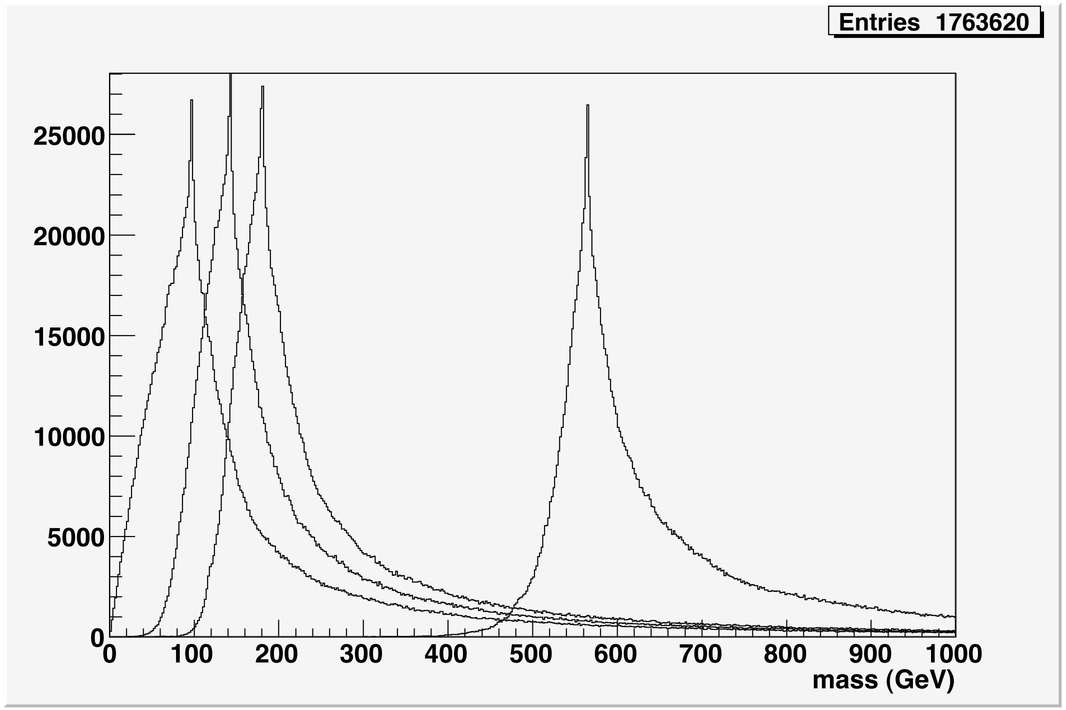
Fitting each curve using a sum of a Gaussian and a quadratic polynomial and take the peak positions as the estimated masses, we get $\{77.8, 135.6, 182.7, 562.0\}$ GeV.
Averaging over 10 different data sets:

$$\begin{aligned} m_N &= 76.7 \pm 1.4 \,\mathrm{GeV}, \quad & m_X &= 135.4 \pm 1.5 \,\mathrm{GeV}, \\ m_Y &= 182.2 \pm 1.8 \,\mathrm{GeV}, \quad & m_Z &= 564.4 \pm 2.5 \,\mathrm{GeV}. \end{aligned}$$

The statistical errors are very small, but the masses are biased.

The combinatoric background

Plot solutions from wrong combinations only, without smearing.



The peaks are at the true masses. But the curves are asymmetric. After smearing, the peak positions move around to yield biases.

Some model-independent techniques
I. Cut off “bad” combinations

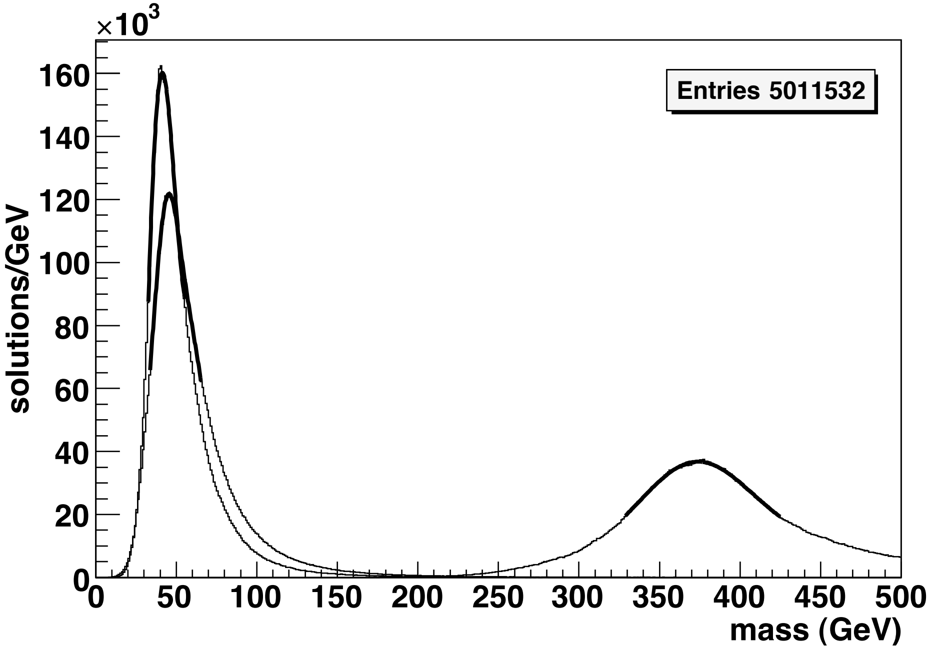
- For the ideal case, the correct combination of one event can always pair with any other event and yield solutions. So the number of events that pair with this combination is maximized as $N_{evt} - 1$.
- After smearing, this is no longer true, but the correct combinations still have statistically larger number of events to pair.
- We cut on this number so that we have about 4 combinations per event left (originally 11).

II. Number of solutions weighting.

A pair with many solutions enters with a large weight, although at most one of the solutions can be the true masses.
→ Treat each pair equally, weight the solutions by $1/n$, n=number of solutions for the pair.

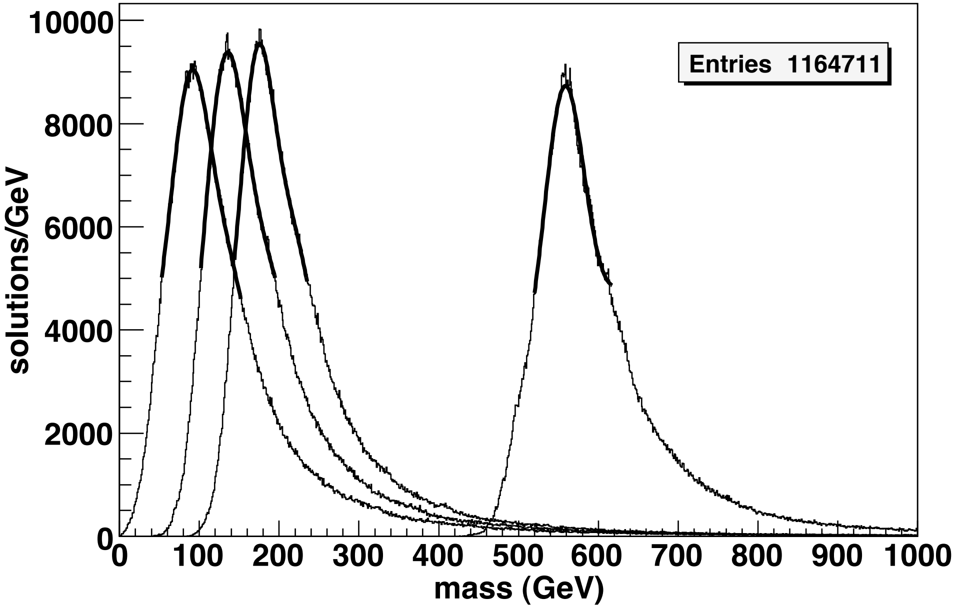
III. Cut on mass difference

Some solutions may have one or more, but not all four masses to be close to the true masses. Remove these solutions by a mass window cut.



Require all three mass differences to be within the mass window defined by 0.7× peak height.

Mass peaks with smaller biases
SPS1a

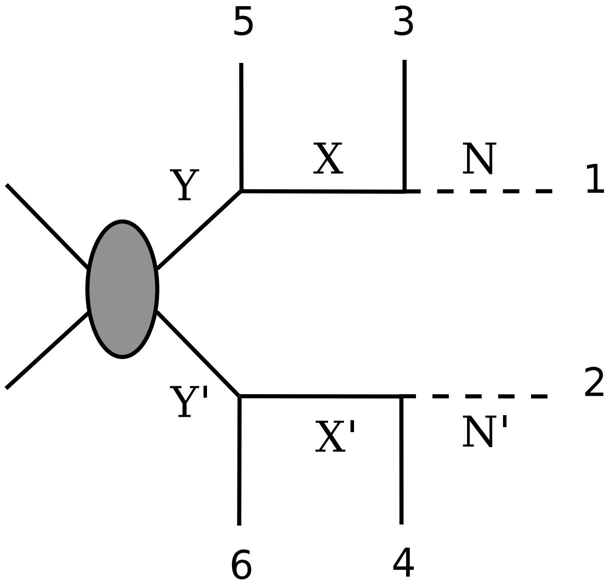


10 sets:

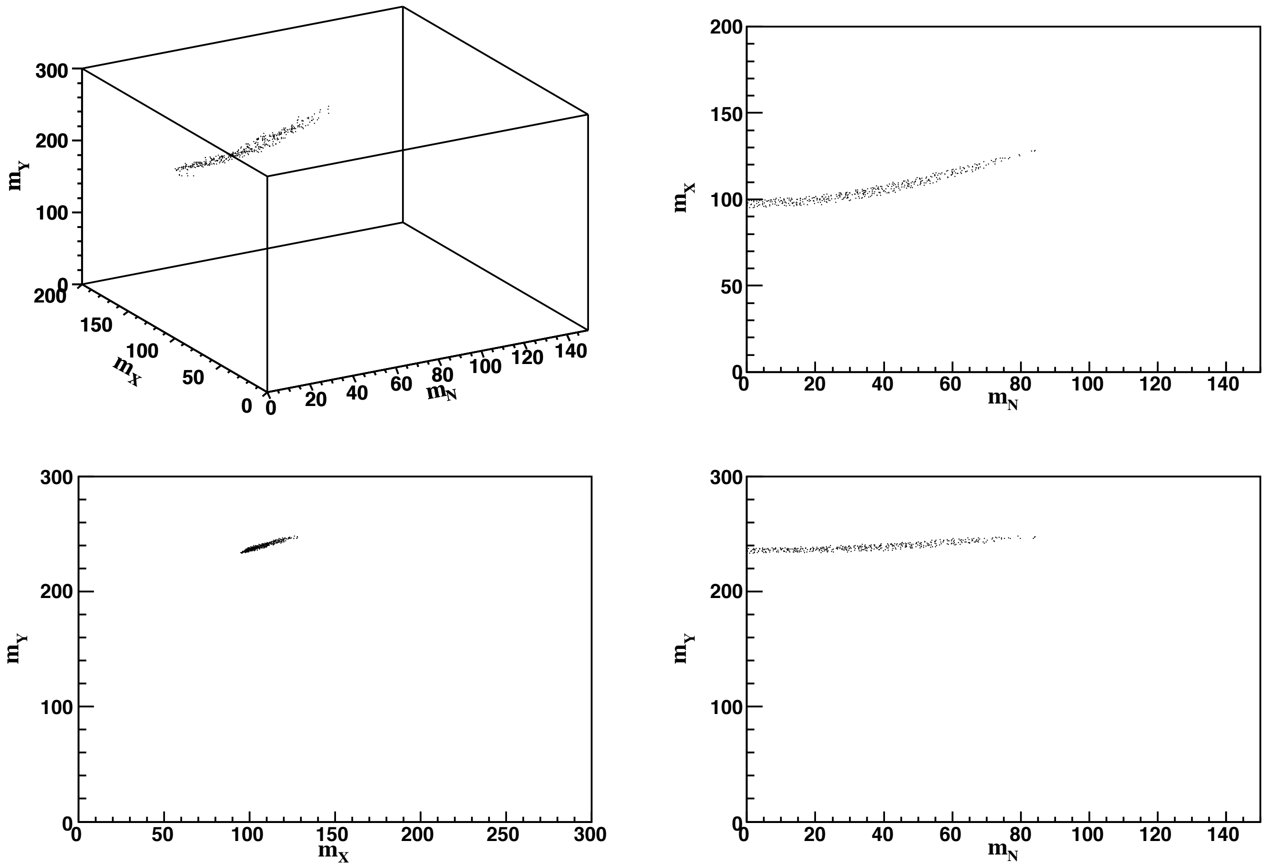
$$
\begin{aligned}
m_N &= 94.1 \pm 2.8\,\mathrm{GeV}, \quad & m_X &= 138.8 \pm 2.8\,\mathrm{GeV}, \\
m_Y &= 179.0 \pm 3.0\,\mathrm{GeV}, \quad & m_Z &= 561.5 \pm 4.1\,\mathrm{GeV}.
\end{aligned}
$$

Compare: { 97.4, 142.5, 180.3, 564.8 } GeV

Case 2



Consistent region–the ideal case



No smearing, correct combination, (246.6, 128.4, 85.3) GeV, 500 events.

Why the endpoint?

We found that the correct masses are located at the endpoint of the allowed region. This is true in general.

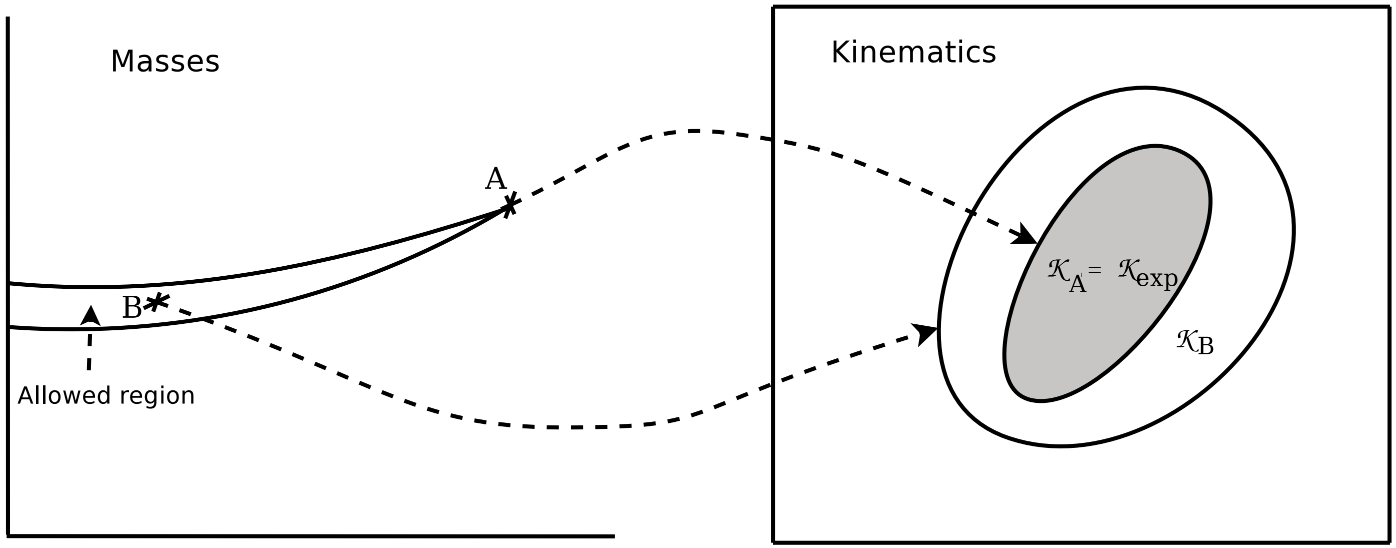
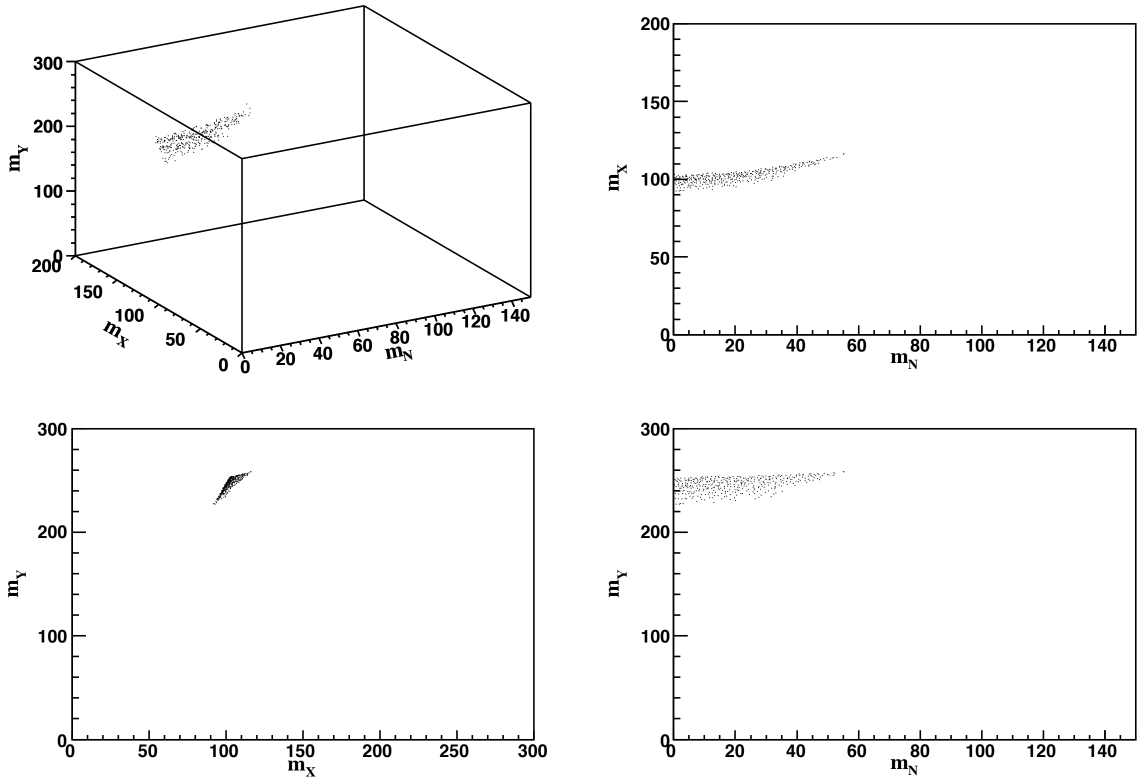


Figure: Map between mass space and kinematic space. The input masses, point *A*, produces a kinematic region that coincides with the experimental region: $\mathcal{K}_A = \mathcal{K}_{exp}$. A point *B* inside the allowed mass region produces a larger kinematic region: $\mathcal{K}_B \supset \mathcal{K}_{exp}$.

The effect of smearing and wrong combinatorics

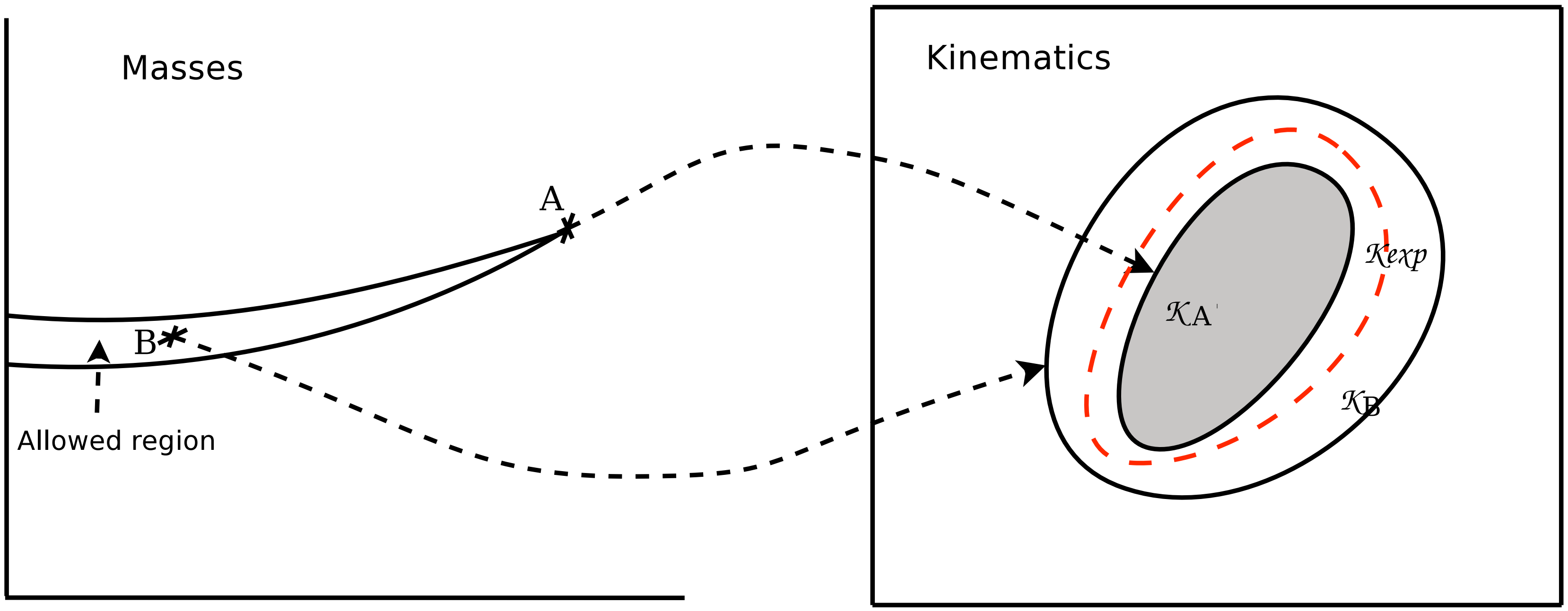
Smeared with ATLFAST, all combinations, (246.6, 128.4, 85.3) GeV, 500 events. 4μ channel.



- Larger region at low m_N–wrong combinations.
- True masses endpoint disappeared!

The effect of smearing and wrong combinatorics

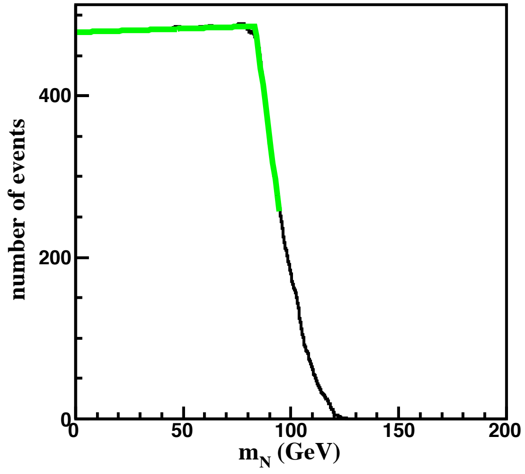
The endpoint corresponding to the true masses disappeared.
With more events, the entire allowed region could disappear.



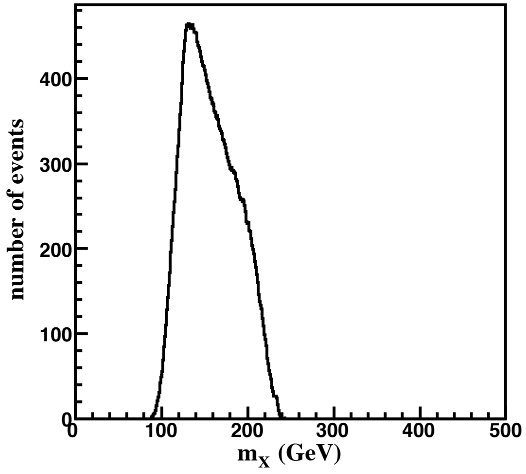
The “turning” points

- One can not read the masses on the tip of the allowed region.
- One can not maximize the number of solvable events either–it will favor the low m_N region.
- Look for the turning points by fixing two of the three masses, where the number of solvable events starts to change rapidly.

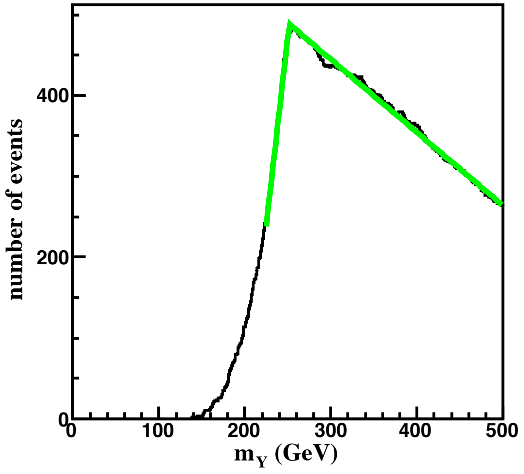
(a) m_N fit



(b) m_X fit



(c) m_Y fit



- In order to determine where the truning point is located, fit (a) and (c) to two straight lines. The peak in (b) is eminent, so identify it as the turning point.

One-dimensional recursive fits

Starting from some random masses satisfying $m_N < m_X < m_Y$, apply one-dimensional recursive fits in the order m_N, m_X, m_Y with the other two masses fixed. Update the masses after each fit.

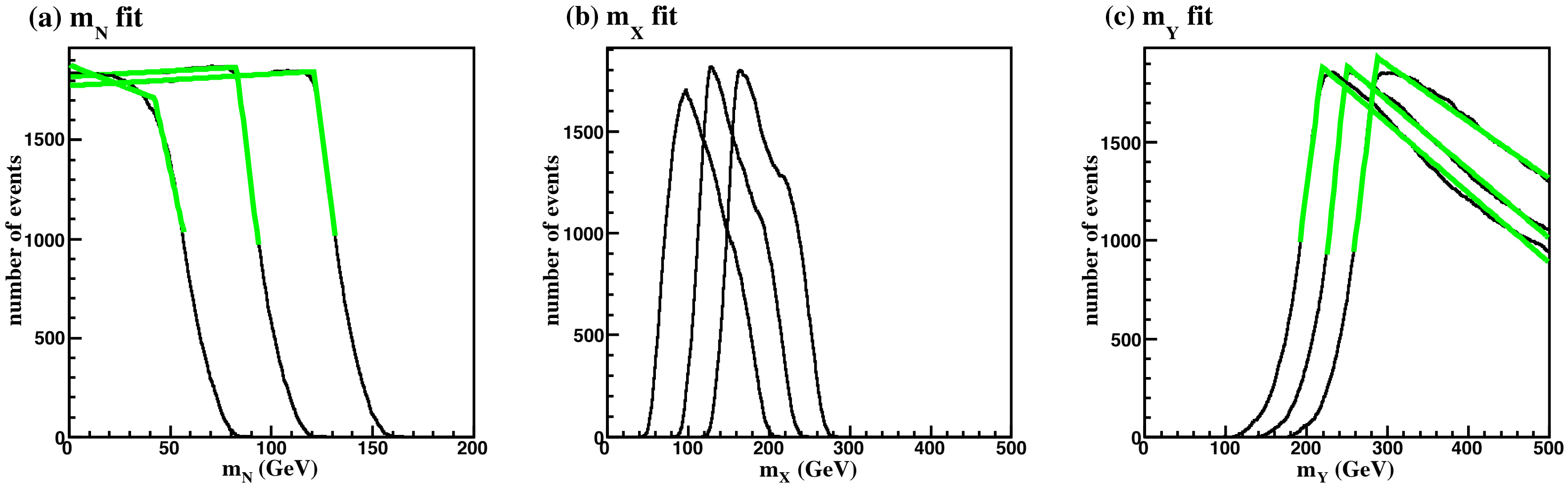


Figure: After cuts $|\eta|_\mu < 2.5, p_{T\mu} > 10$ GeV, $\not{p}_T > 50$ GeV

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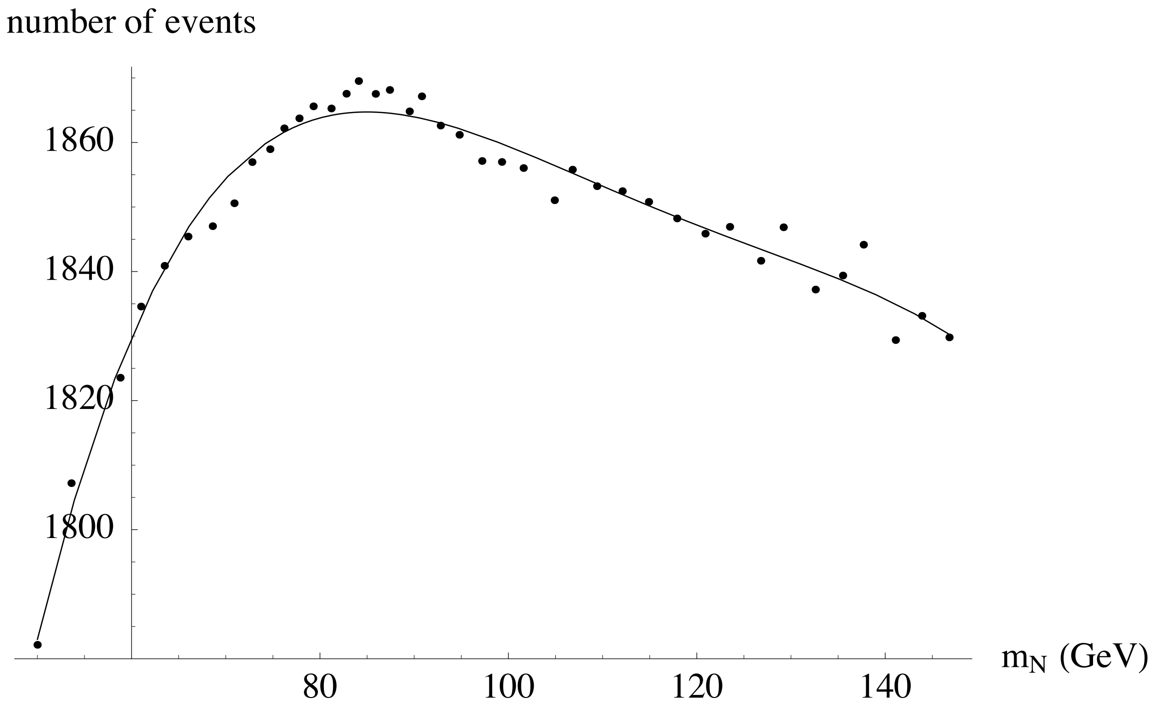
The masses go up, but the fits in general do not converge. However, the number of events at the "turning" points are maximized around the correct mass.

Number of events at the turning points



- Record the number of events at the turning points after each fit of m_N. Fit the plot to a polynomial and take the peak position as the estimation for m_N.
- Do a few one-dimensional fits for m_X and m_Y with fixed m_N until they are stablized.

- The above plot corresponds to about 1900 events. The masses are estimated as (252.2, 130.4, 85.0) GeV, compare (246.6, 128.4, 85.3) GeV.
- Average ove 10 sets:

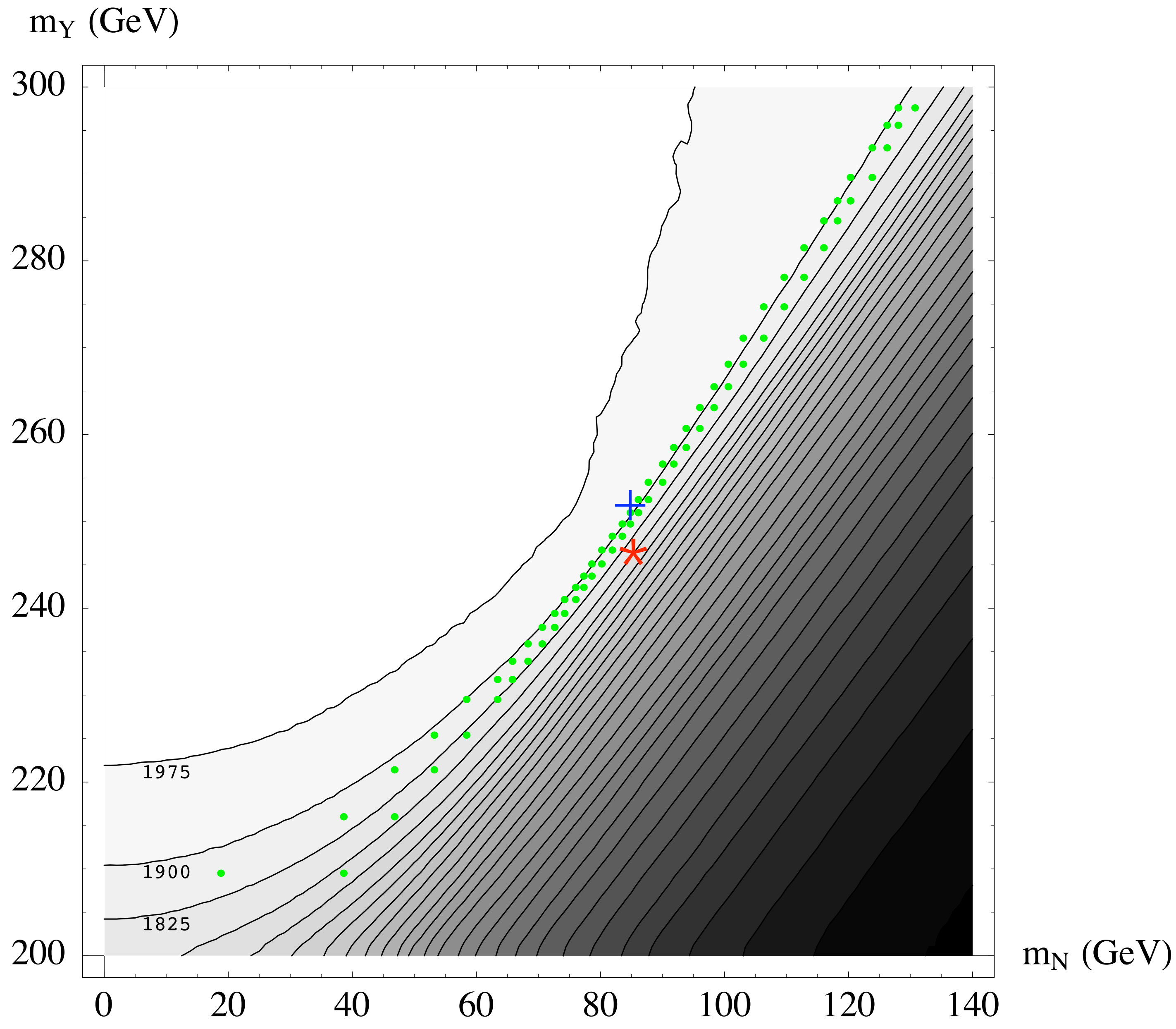
$$\{252.2 \pm 4.3 \text{ GeV}, 130.4 \pm 4.3 \text{ GeV}, 86.2 \pm 4.3 \text{ GeV}\}.$$

- The mass differences are determined better:

$$m_Y - m_X = 119.8 \pm 1.0 \text{ GeV}, \quad m_X - m_N = 46.4 \pm 0.7 \text{ GeV}$$

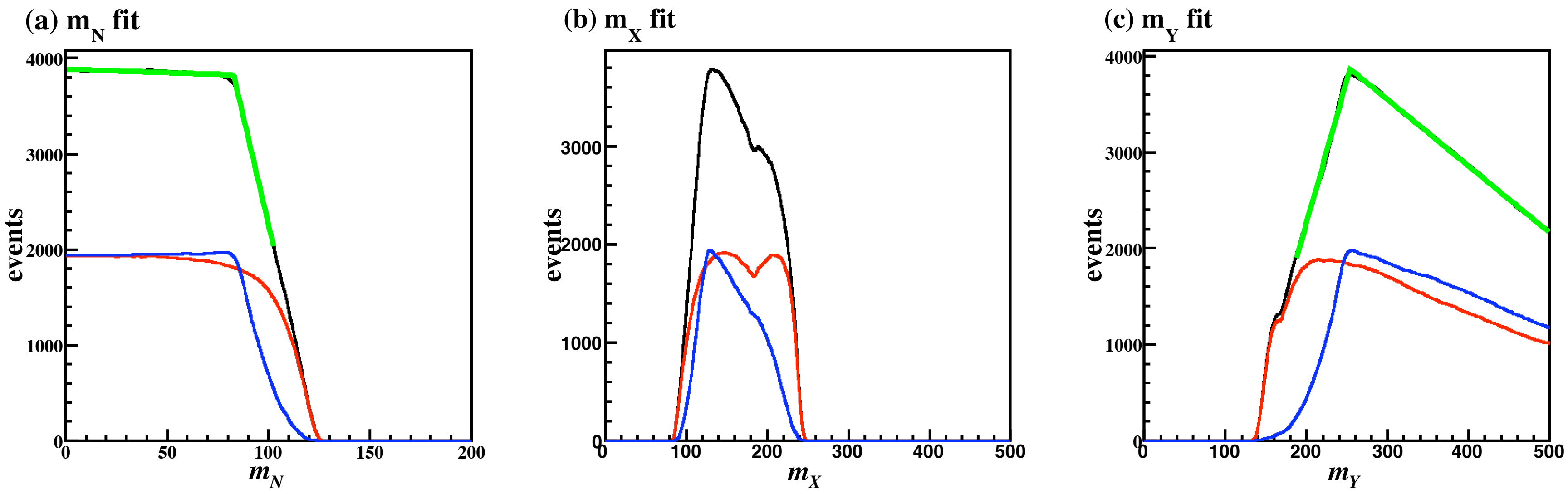
- Remove the biases by comparing with Monte Carlo.

Contour plot for our recursive fitting procedure



More Realistic Case

Backgrounds can also deteriorate the mass determination.



Equal number of signal and background events

Signal: $(m_Y, m_X, m_N) = (246.6,\ 128.4,\ 85.3)$ GeV

Background: $t\bar{t}$, with 2 μ's coming from W decays and 2 μ's coming b's (without isolation cuts).

Blue=signals Red=backgrounds

Black=signals+backgrounds Green= fits

More Realistic Case