Insensitive Unification of Gauge Couplings, Muon g-2 and Higgs decays

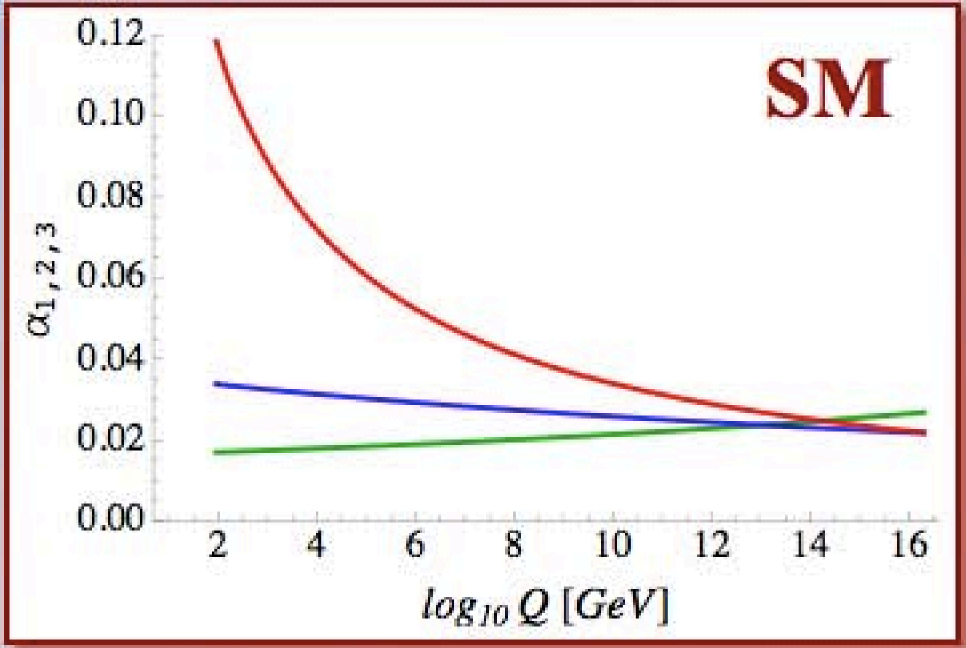
Radovan Dermisek

Indiana University, Bloomington

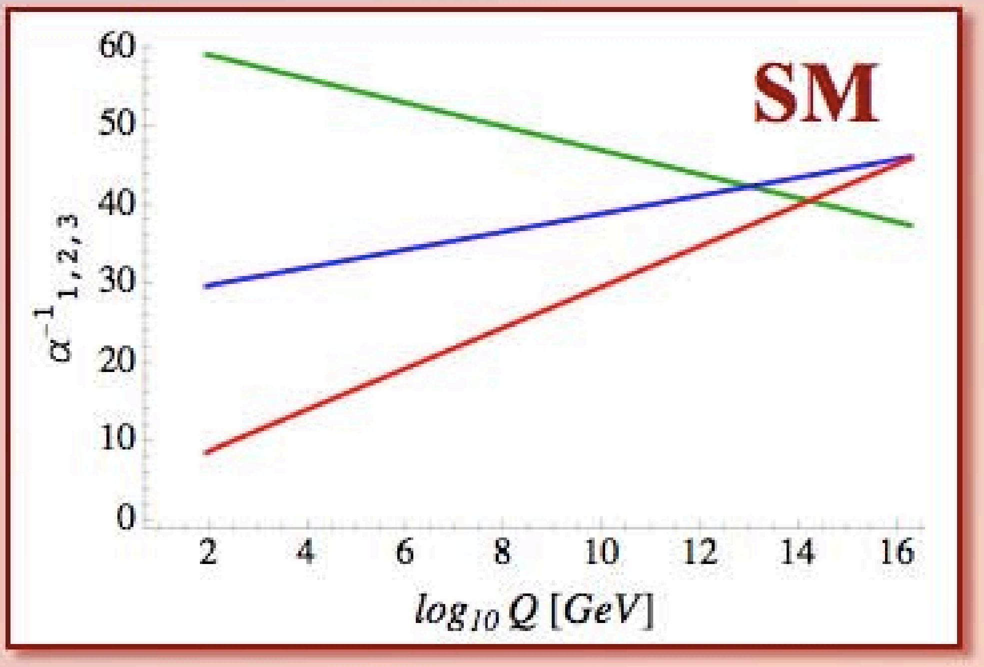
R.D., arXiv:1204.6533 [hep-ph], arXiv:1212.3035 [hep-ph]

R.D. and A. Raval, arXiv:1305.3522 [hep-ph]

Gauge couplings in the standard model



$\alpha_3(M_Z)_{exp} = 0.1184$
$\alpha_2(M_Z)_{exp} = 0.03380$
$\alpha_1(M_Z)_{exp} = 0.01695$
$\alpha_{EM}(M_Z) = 1/127.916$
$\sin^2\theta_W = 0.2313$



RGEs:

$$\frac{d\alpha_i}{dt} = \beta(\alpha_i) = \frac{\alpha_i^2}{2\pi} b_i \qquad t = \ln Q/Q_0$$

$$b_i = (41/10,\ -19/6,\ -7) \qquad b_i = \left(\frac{1}{10} + \frac{4}{3}n_g,\ -\frac{43}{6} + \frac{4}{3}n_g,\ -11 + \frac{4}{3}n_g\right)$$

sensitivity

$$\frac{\delta\alpha_3(M_Z)}{\alpha_3(M_Z)} = \frac{\alpha_3(M_Z)}{\alpha_G}\frac{\delta\alpha_G}{\alpha_G}$$

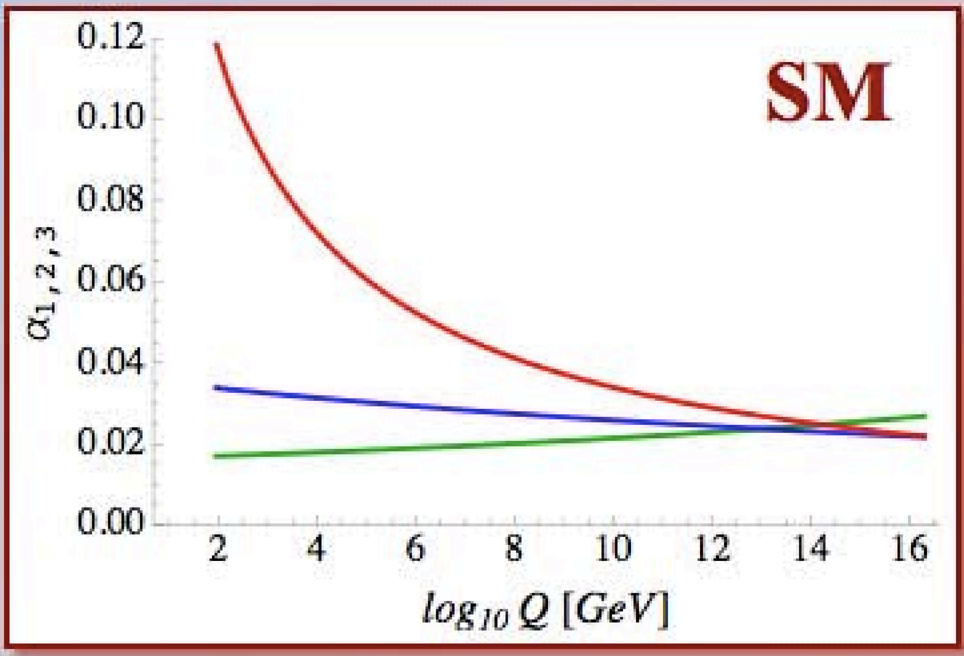
~4

solution:

$$\alpha_i^{-1}(M_Z) = \frac{b_i}{2\pi}\ln\frac{M_G}{M_Z} + \alpha_i^{-1}(M_G)$$

SM

SM+3VF



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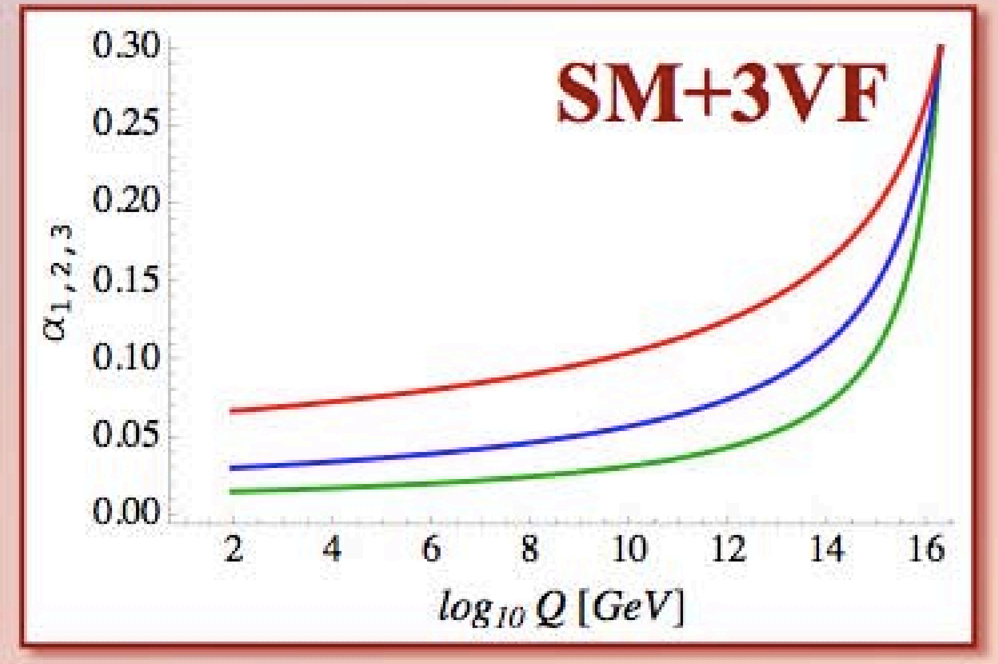

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$$b_i = (121/10,\ 29/6,\ +1)$$

sensitivity

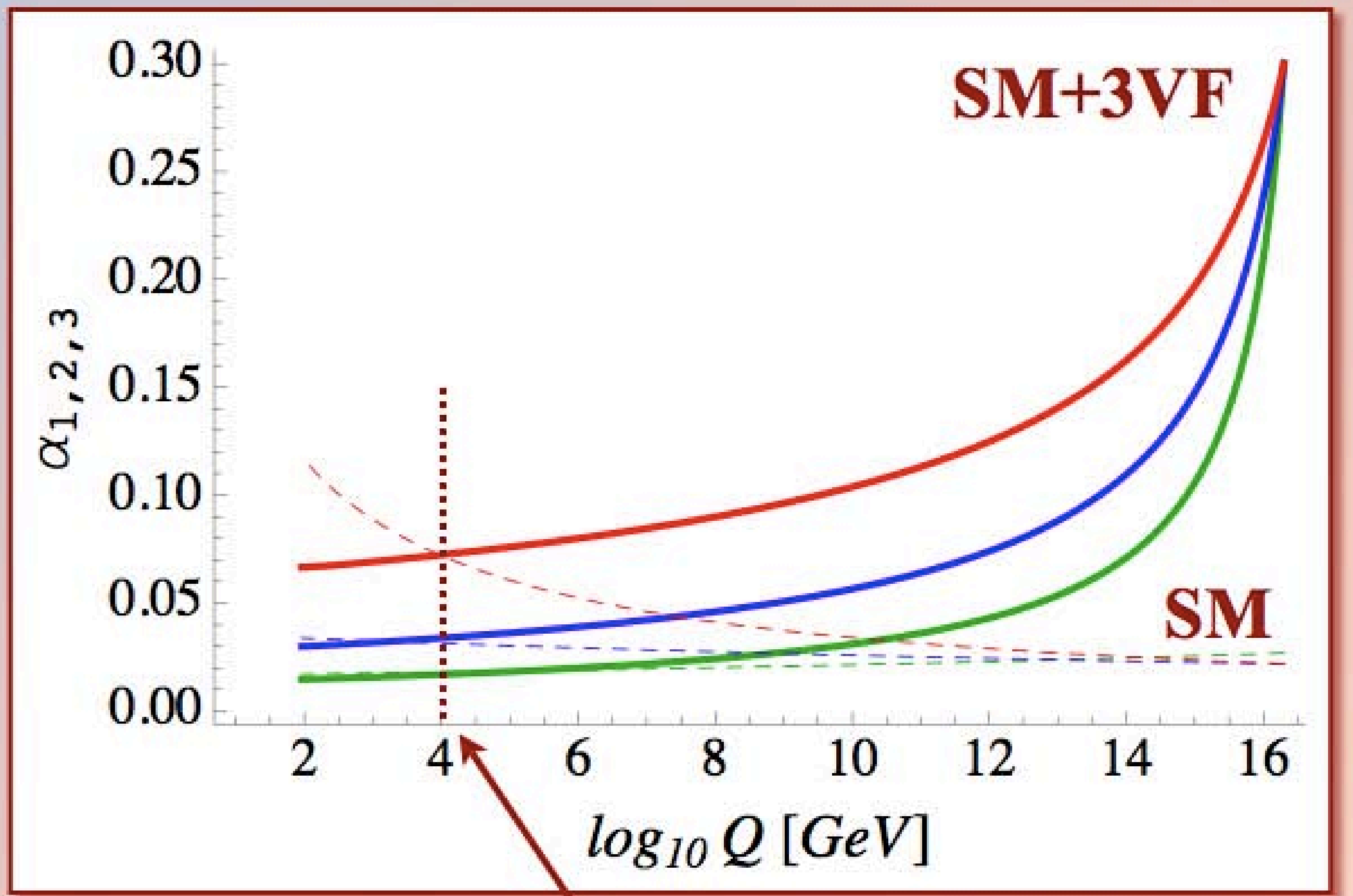
$$\frac{\delta\alpha_3(M_Z)}{\alpha_3(M_Z)} = \frac{\alpha_3(M_Z)}{\alpha_G}\frac{\delta\alpha_G}{\alpha_G}$$

~4

solution:

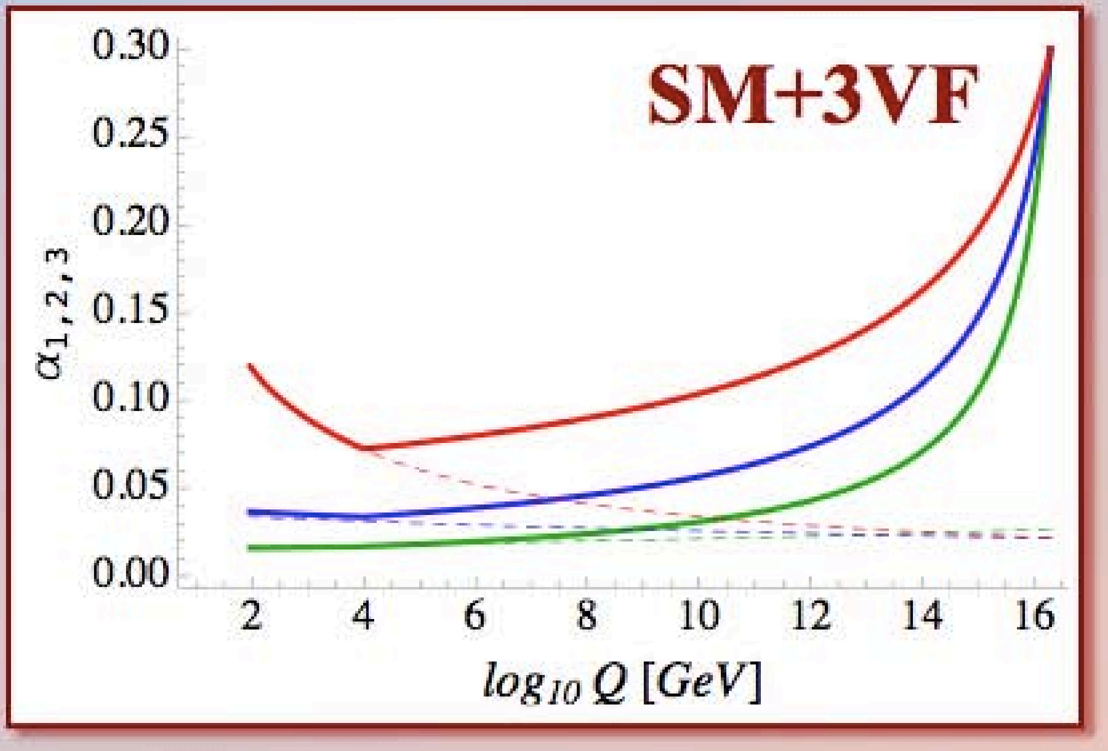
$$\alpha_i^{-1}(M_Z) = \frac{b_i}{2\pi}\ln\frac{M_G}{M_Z} + \alpha_i^{-1}(M_G)$$

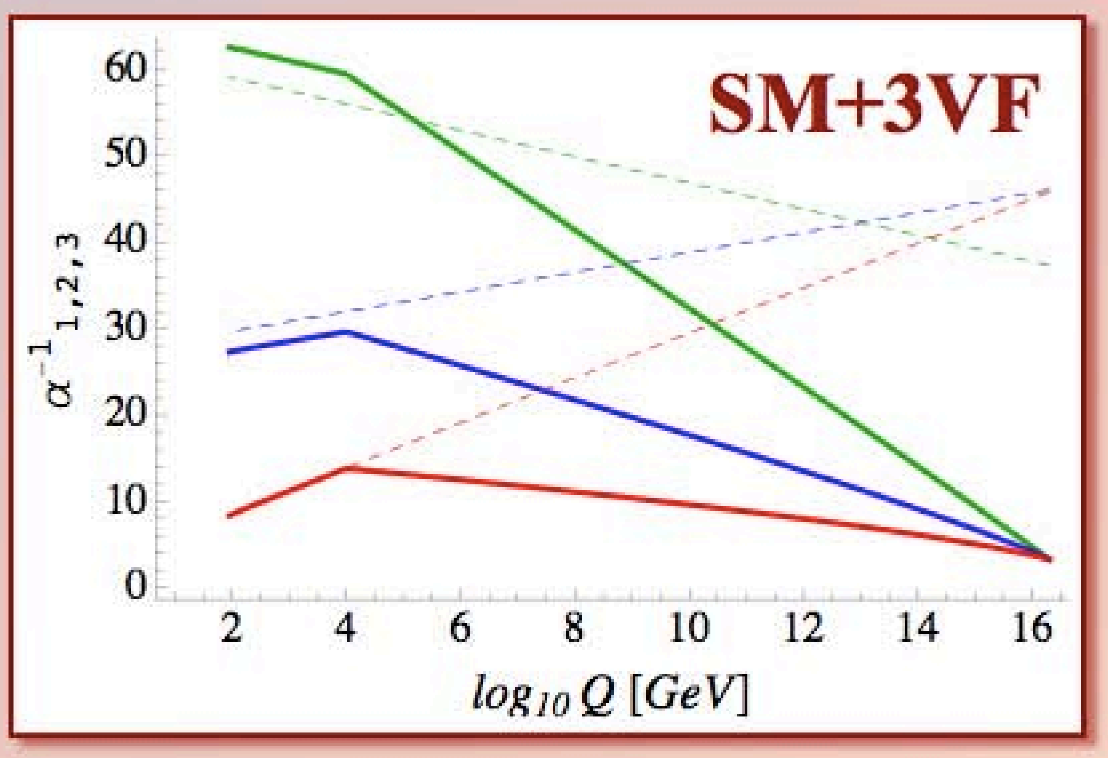
SM + 3 vectorlike families



Is this a threshold effect?

SM + 3 vectorlike families at 10 TeV





Exp. values of gauge couplings reproduced within 8%

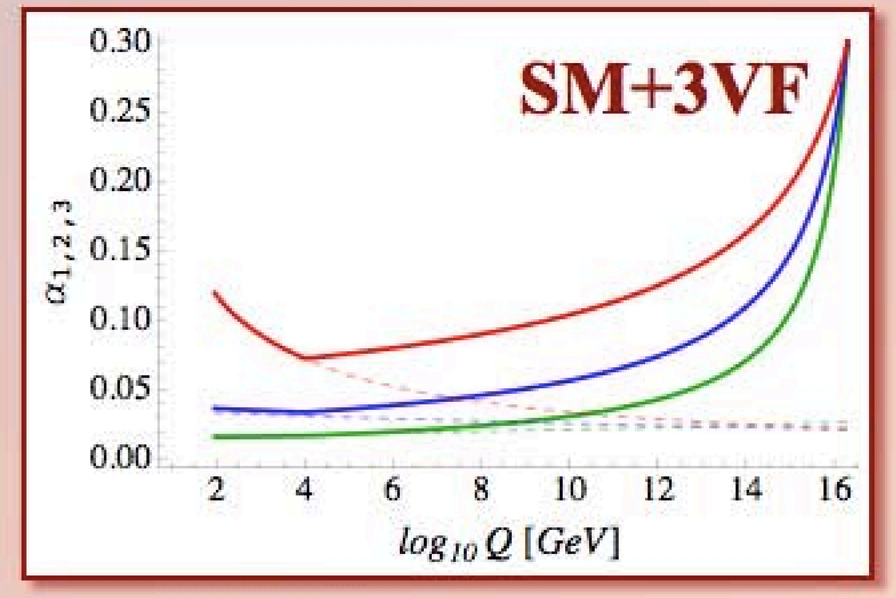
the only relevant parameters are M_G and M_{VF}
(the best fit: gauge couplings within 6%)

Predictions and Sensitivity

RGEs:

$$\frac{d\alpha_i}{dt} = \beta(\alpha_i) = \frac{\alpha_i^2}{2\pi} b_i$$

$$b_i = (121/10,\ 29/6,\ +1)$$



solution at 1-loop (good approximation for i=1,2):

$$\alpha_i^{-1}(M_Z) = \frac{b_i}{2\pi} \ln \frac{M_G}{M_Z} + \alpha_G^{-1} - T_i$$

$$T_i = \frac{1}{2\pi} \sum_f b_i^f \ln \frac{M_f}{M_Z}$$

~ 60, 30 ~ 3 ~ 6 (for common mass 10 TeV)

neglecting threshold effect and α_G :

$$\frac{\alpha_i(M_Z)}{\alpha_j(M_Z)} \simeq \frac{b_j}{b_i}$$

$$\alpha' = \frac{3}{5}\alpha_1, \quad b' = \frac{5}{3} b_1$$

Parameter free prediction:

$$\sin^2\theta_W \equiv \frac{\alpha'}{\alpha_2 + \alpha'} = \frac{b_2}{b_2 + b'} = 0.193$$

$$\alpha_{EM} = \alpha_2 \sin^2\theta_W$$

Maiani, Parisi, and Petronzio (1978)

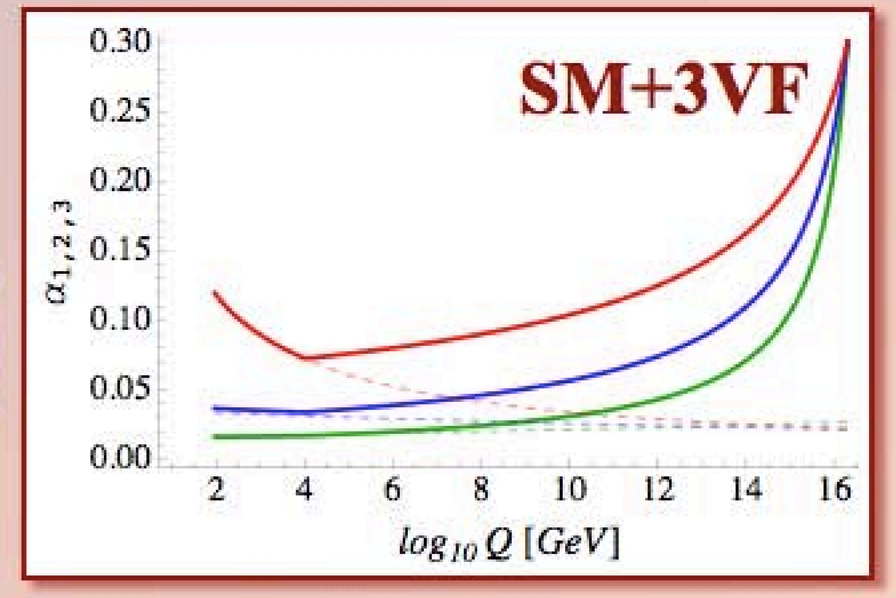
Predictions and Sensitivity

RGEs:

$$\frac{d\alpha_i}{dt} = \beta(\alpha_i) = \frac{\alpha_i^2}{2\pi} b_i + \frac{\alpha_i^3}{8\pi^2} B_i + \dots$$

$$b_i = (121/10,\ 29/6,\ +1)$$

$$B_3 = -102 + (76/3)n_g = 126$$



neglecting 1-loop **(good approximation for i=3):**

$$\alpha_3^{-1}(M_Z) \simeq \sqrt{\frac{B_3}{4\pi^2}\ln\frac{M_G}{M_Z} + \alpha_G^{-2}} - T_i$$

$$T_i = \frac{1}{2\pi}\sum_f b_i^f \ln\frac{M_f}{M_Z}$$

~8 ~100 ~9 ~6 (for common mass 10 TeV)

Parameter free prediction:

(neglecting threshold effect and α_G)

$$\frac{\alpha_3^2(M_Z)}{\alpha_{EM}(M_Z)} \simeq 2\pi\frac{b_2 + b'}{B_3}$$

predicts $\alpha_3 = 0.099$

1-loop contribution can be added:

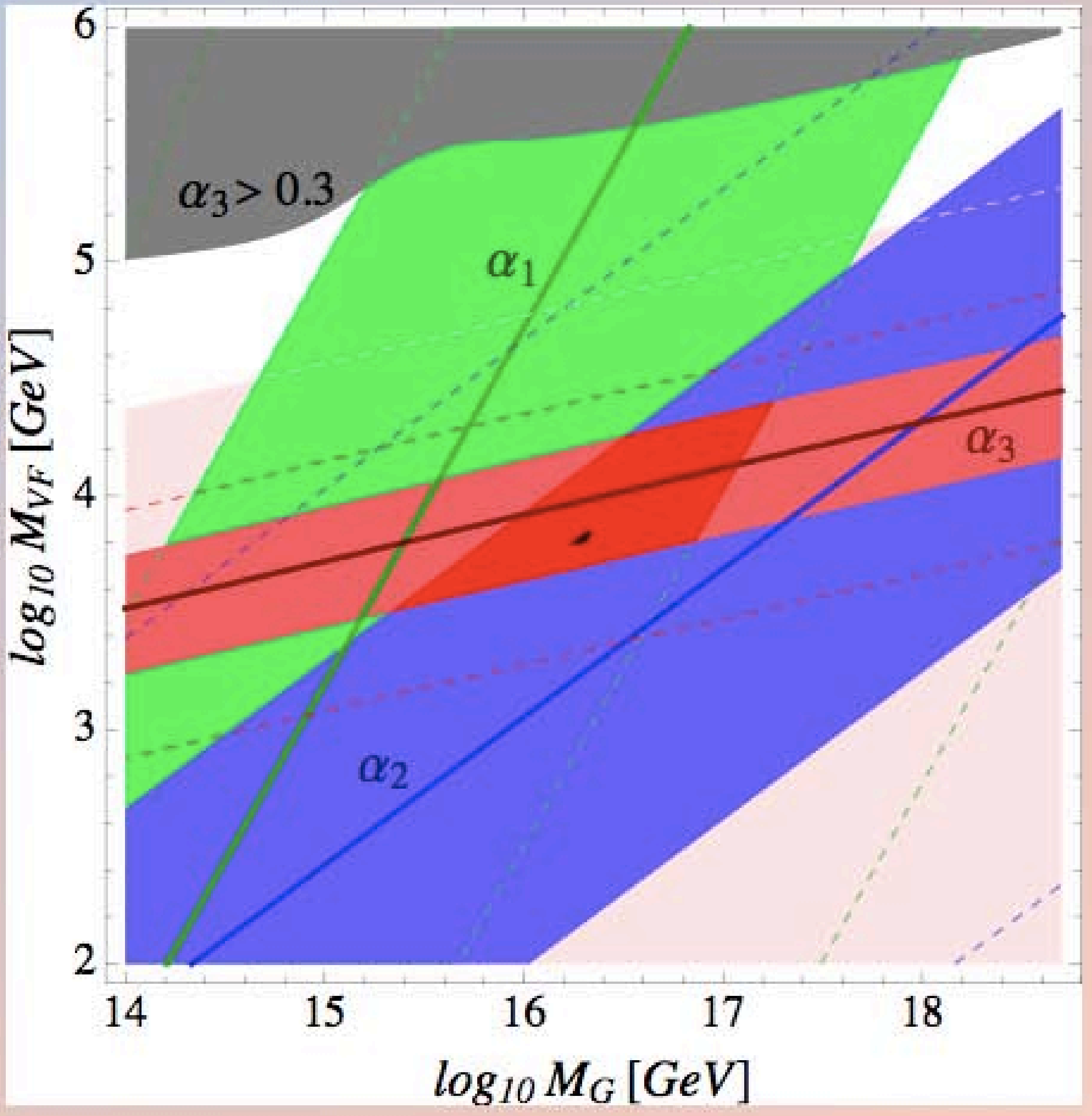
$$\alpha_3(M_Z) \to \frac{\alpha_3(M_Z)}{1 + \frac{1}{3}\frac{\epsilon}{\alpha_3(M_Z)} - \frac{1}{12}\left(\frac{\epsilon}{\alpha_3(M_Z)}\right)^2 + \dots}$$

$$\epsilon = 4\pi b_3/B_3$$

predicts $\alpha_3 = 0.073$

Ranges of M_G , and M_{VF}

$\alpha_G = 0.3$ (for larger values results very similar)



+20%

+10%

central value of $\alpha_2(M_Z)$

-10%

-20%

● the best fit (all three couplings within 6%)

Gauge couplings at M_Z within 10% in a large range of parameters!

Realistic example

$\alpha_3(M_Z)_{exp} = 0.1184$
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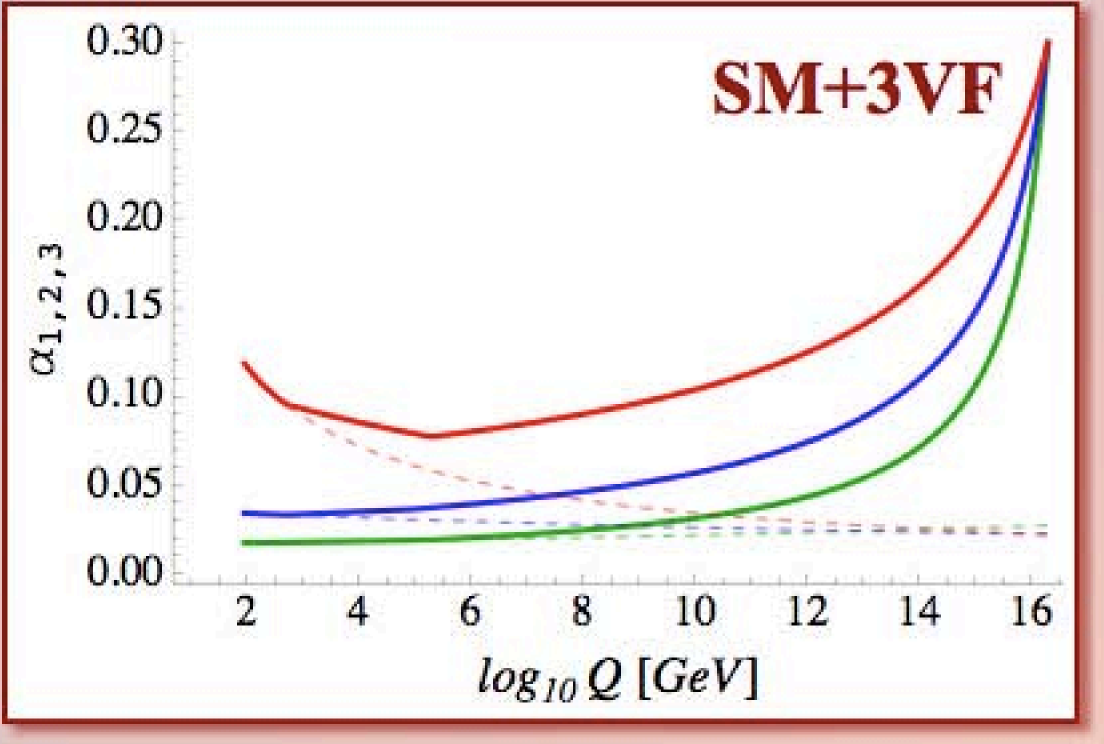
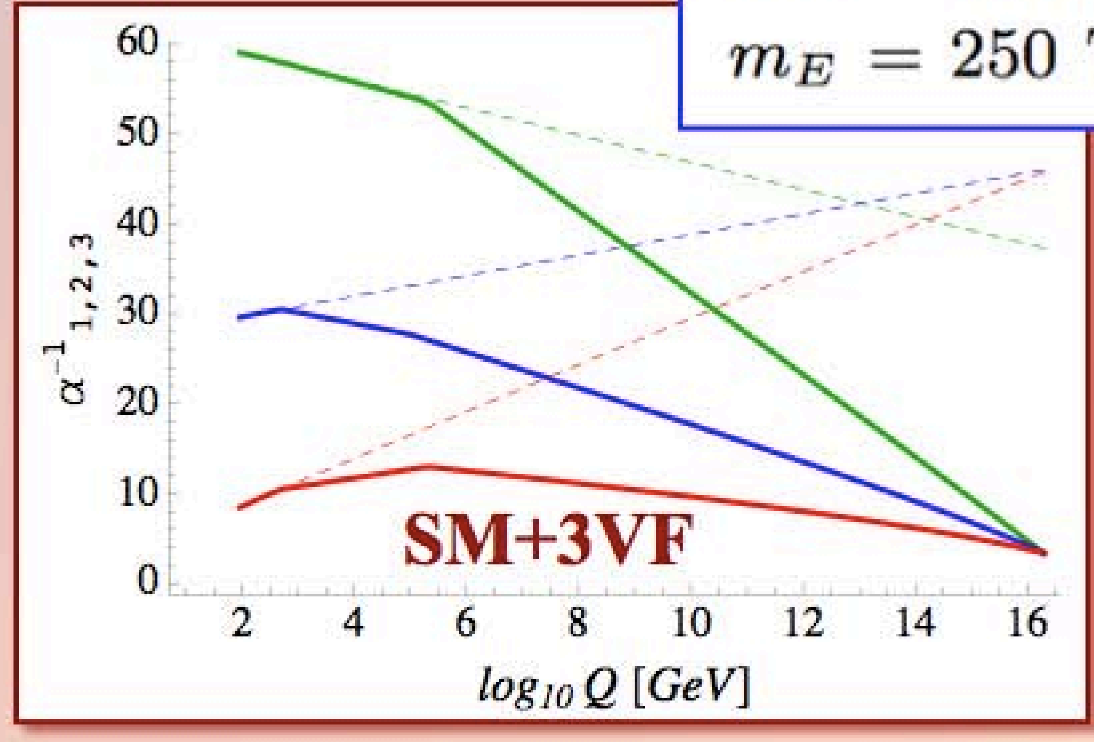
Gauge couplings reproduced (within fractions of exp. uncertainties) for :

4 sig. figures

2 sig. figures

$m_Q = 500$ GeV
$m_L = 95$ TeV
$m_U = 220$ TeV
$m_D = 180$ TeV
$m_E = 250$ TeV

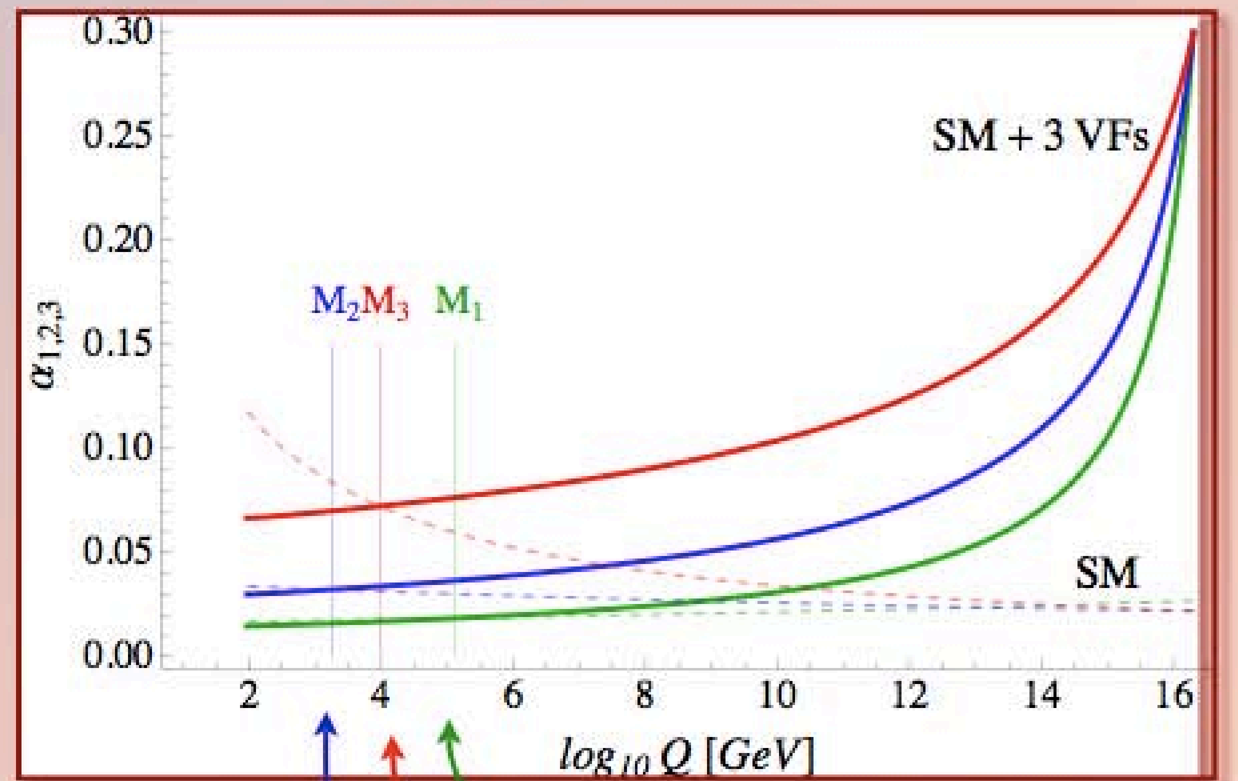




Many possible solutions!

Classifying solutions - mass rules

To get gauge coupling unification, weighted sum of logs of masses of particles charged under given symmetry must be as if all particles had mass that corresponds to the crossing scale of RG evolutions of gauge couplings in the SM and SM+3VFs:



$$\frac{1}{2\pi}\sum_{i=1}^{3}\left(b_3^Q \ln\frac{M_{Qi}}{M_Z} + b_3^U \ln\frac{M_{Ui}}{M_Z} + b_3^D \ln\frac{M_{Di}}{M_Z}\right) = \frac{4}{\pi}\ln\frac{M_3}{M_Z}$$

$$\frac{1}{2\pi}\sum_{i=1}^{3}\left(b_2^Q \ln\frac{M_{Qi}}{M_Z} + b_2^L \ln\frac{M_{Li}}{M_Z}\right) = \frac{4}{\pi}\ln\frac{M_2}{M_Z}$$

$$\frac{1}{2\pi}\sum_{i=1}^{3}\left(b_1^Q \ln\frac{M_{Qi}}{M_Z} + b_1^U \ln\frac{M_{Ui}}{M_Z} + b_1^D \ln\frac{M_{Di}}{M_Z} + b_1^L \ln\frac{M_{Li}}{M_Z} + b_1^E \ln\frac{M_{Ei}}{M_Z}\right) = \frac{4}{\pi}\ln\frac{M_1}{M_Z}$$

$$T_i = \frac{1}{2\pi}\sum_f b_i^f \ln\frac{M_f}{M_Z}$$

Classifying solutions - mass rules

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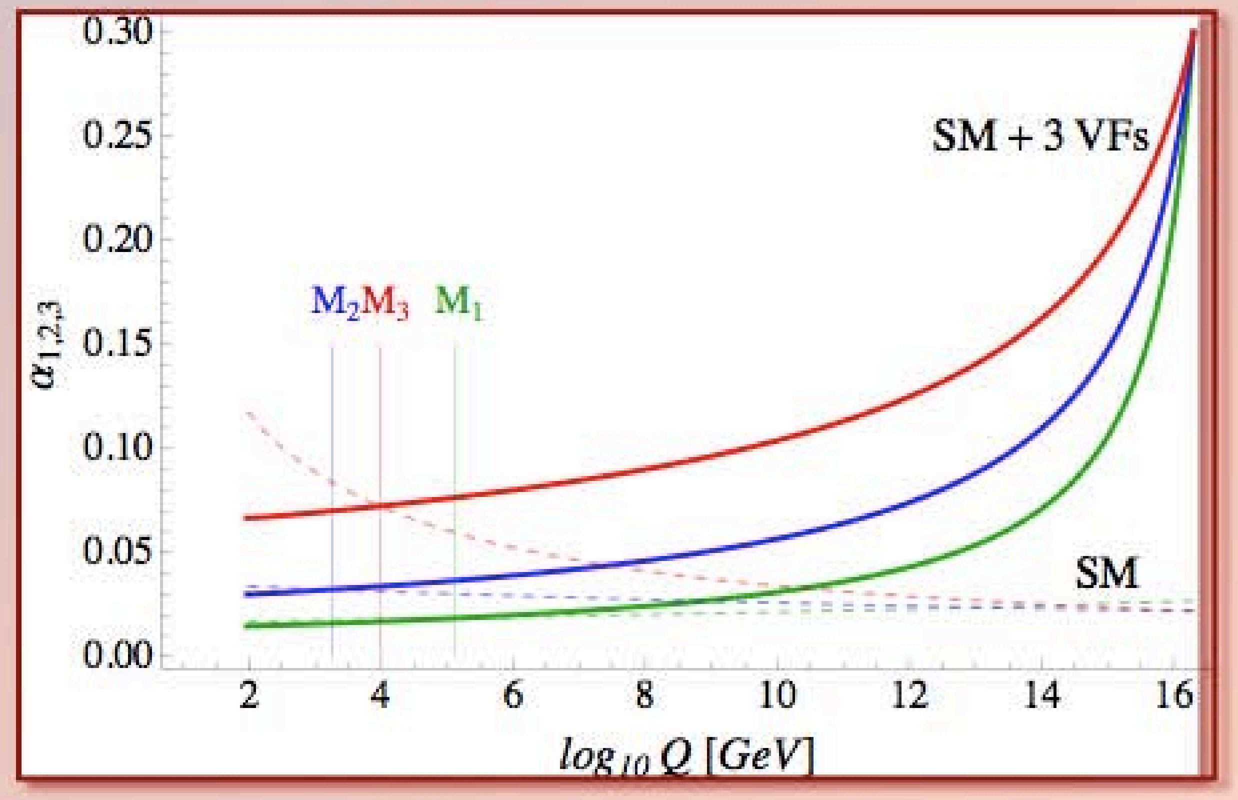


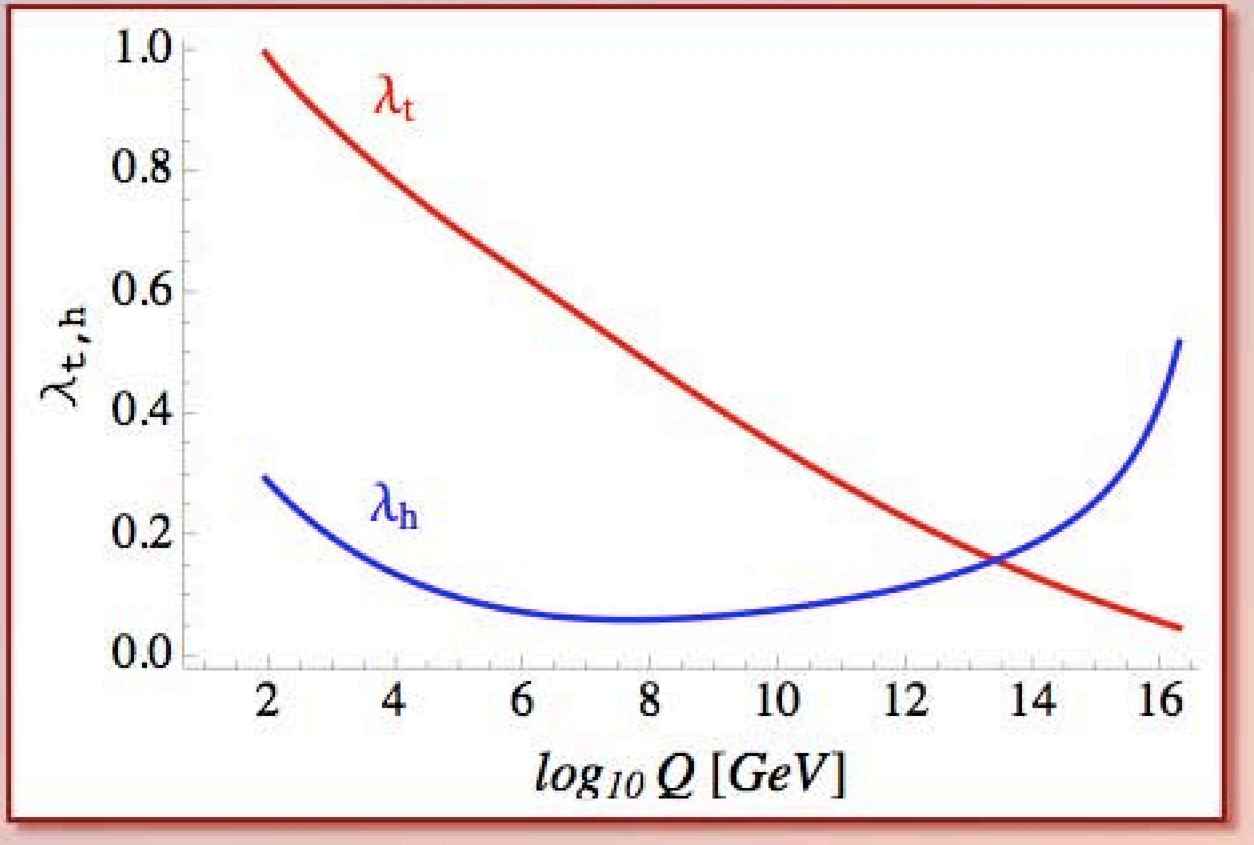
$$M_3^4 = M_Q^2 M_U M_D,$$

$$M_2^4 = M_Q^3 M_L,$$

$$M_1^{20} = M_Q M_U^8 M_D^2 M_L^3 M_E^6.$$

$$M_F \equiv (M_{F_1} M_{F_2} \ldots M_{F_N})^{1/N}$$

Top Yukawa and Higgs quartic couplings



$m_H = 125$ GeV

(different textures for fermion masses compared to usual GUTs)

- Electroweak minimum is stable!

What else are extra VFs good for?

- **muon g-2**

K. Kannike, M. Raidal, D.M. Straub and A. Strumia, 1111.2551 [hep-ph]
R.D. and A. Raval, 1305.3522 [hep-ph]

$$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = 2.7 \pm 0.80 \times 10^{-9}$$

- **anomalies in Z-pole observables $\mathbf{A_{FB}^{b}}$ and $\mathbf{A_e}$**

D. Choudhury, T.M.P. Tait and C.E.M. Wagner, hep-ph/0109097
R.D., S.G. Kim and A. Raval, 1105.0773 [hep-ph], 1201.0315 [hep-ph]
B. Batell, S. Gori and L.T. Wang 1209.6382 [hep-ph]

- **$h \to \gamma\gamma$, ...**

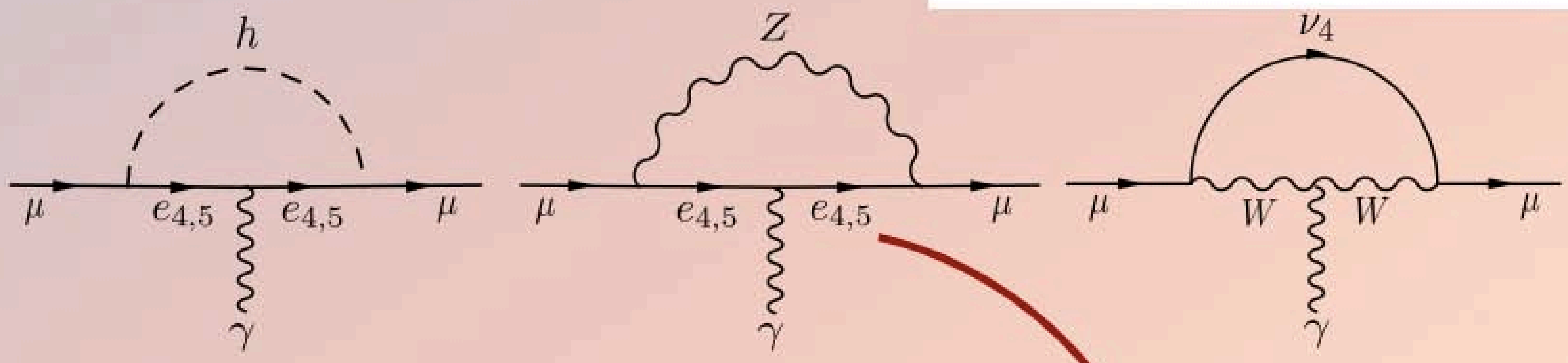
A. Joglekar, P. Schwaller and C.E.M. Wagner, 1207.4235 [hep-ph]
N. Arkani-Hamed, K. Blum, R.T. D'Agnolo and J. Fan, 1207.4482 [hep-ph]
L.G. Almeida, E. Bertuzzo, P.A.N. Machado and R.Z. Funchal, 1207.5254 [hep-ph]
J. Kearney, A. Pierce and N. Weiner, 1207.7062 [hep-ph]

Muon g-2

K. Kannike, M. Raidal, D.M. Straub and A. Strumia, 1111.2551 [hep-ph]
R.D. and A. Raval, 1305.3522 [hep-ph]

$$\mathcal{L} \supset -\bar{l}_{Li} y_{ij} e_{Rj} H - \bar{l}_{Li} \lambda_i^E E_R H - \bar{L}_L \lambda_j^L e_{Rj} H - \lambda \bar{L}_L E_R H - \bar{\lambda} H^\dagger \bar{E}_L L_R$$
$$- M_L \bar{L}_L L_R - M_E \bar{E}_L E_R + h.c.,$$

$$(\bar{e}_{Li}, \bar{L}_L^-, \bar{E}_L) \begin{pmatrix} y_{ij} v & 0 & \lambda_i^E v \\ \lambda_j^L v & M_L & \lambda v \\ 0 & \bar{\lambda} v & M_E \end{pmatrix} \begin{pmatrix} e_{Rj} \\ L_R^- \\ E_R \end{pmatrix}$$



$$\delta a_\mu^Z = -\frac{g^2 m_\mu}{8\pi^2 M_W^2} \left\{ (g_L^2 + g_R^2) m_\mu F_Z(x) + g_L g_R m_f G_Z(x) \right\}$$

$$x = (m_f / M_Z)^2$$

Interesting insight can be obtained by integrating out vectorlike leptons:

$$\mathcal{L} \supset -\bar{l}_{Li} y_{ij} e_{Rj} H - \bar{l}_{Li} \lambda_i^E E_R H - \bar{L}_L \lambda_j^L e_{Rj} H - \lambda \bar{L}_L E_R H - \bar{\lambda} H^\dagger \bar{E}_L L_R$$
$$- M_L \bar{L}_L L_R - M_E \bar{E}_L E_R + h.c.,$$

$$\lambda_E v, \lambda_L v, \bar{\lambda} v, \lambda v \ll M_E, M_L$$

Effective lagrangian is given by:

$$\mathcal{L}_{eff} \supset -\bar{\mu}_L \left(y_\mu + \frac{\lambda^L \bar{\lambda} \lambda^E}{M_L M_E} H H^\dagger \right) \mu_R H + h.c. \longrightarrow - \left(m_\mu^H + m_\mu^{LE} \right) \bar{\mu}_L \mu_R + h.c.$$

Contribution to the muon g-2 can be written as:

contribution to the muon mass and muon g-2 is correlated!

$$\Delta a_\mu \simeq c \frac{m_\mu m_\mu^{LE}}{(4\pi v)^2} \simeq 0.85\, c \frac{m_\mu^{LE}}{m_\mu} \Delta a_\mu^{exp}$$

$$c = -1$$

$$M_E \simeq M_L \gg M_Z$$

K. Kannike, M. Raidal, D.M. Straub and A. Strumia, 1111.2551 [hep-ph]

$$\mathcal{L}_{eff} \supset -\bar{\mu}_L \left(y_\mu + \frac{\lambda^L \bar{\lambda} \lambda^E}{M_L M_E} H H^\dagger \right) \mu_R H + h.c. \longrightarrow - \left(m_\mu^H + m_\mu^{LE} \right) \bar{\mu}_L \mu_R + h.c.$$

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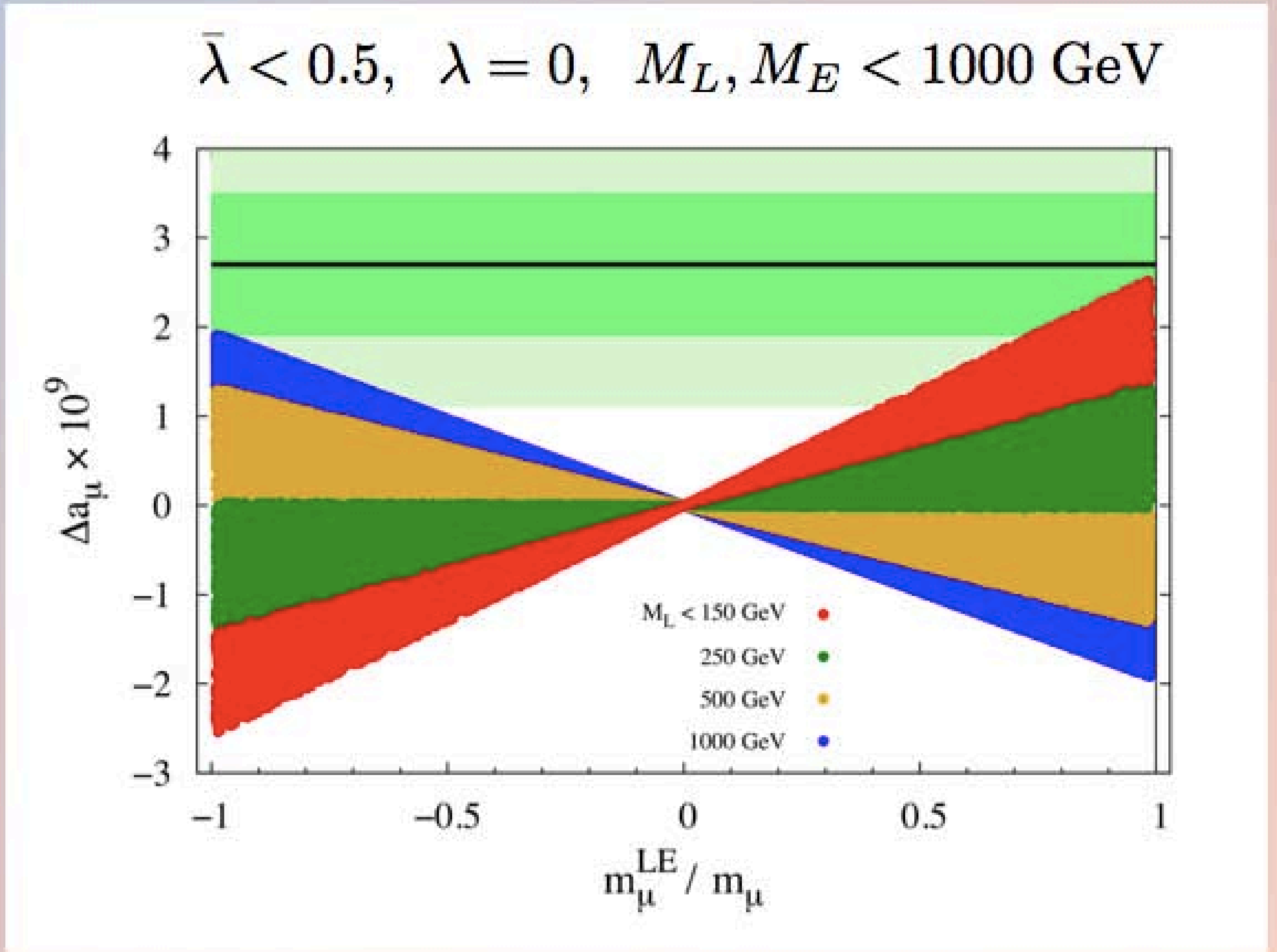
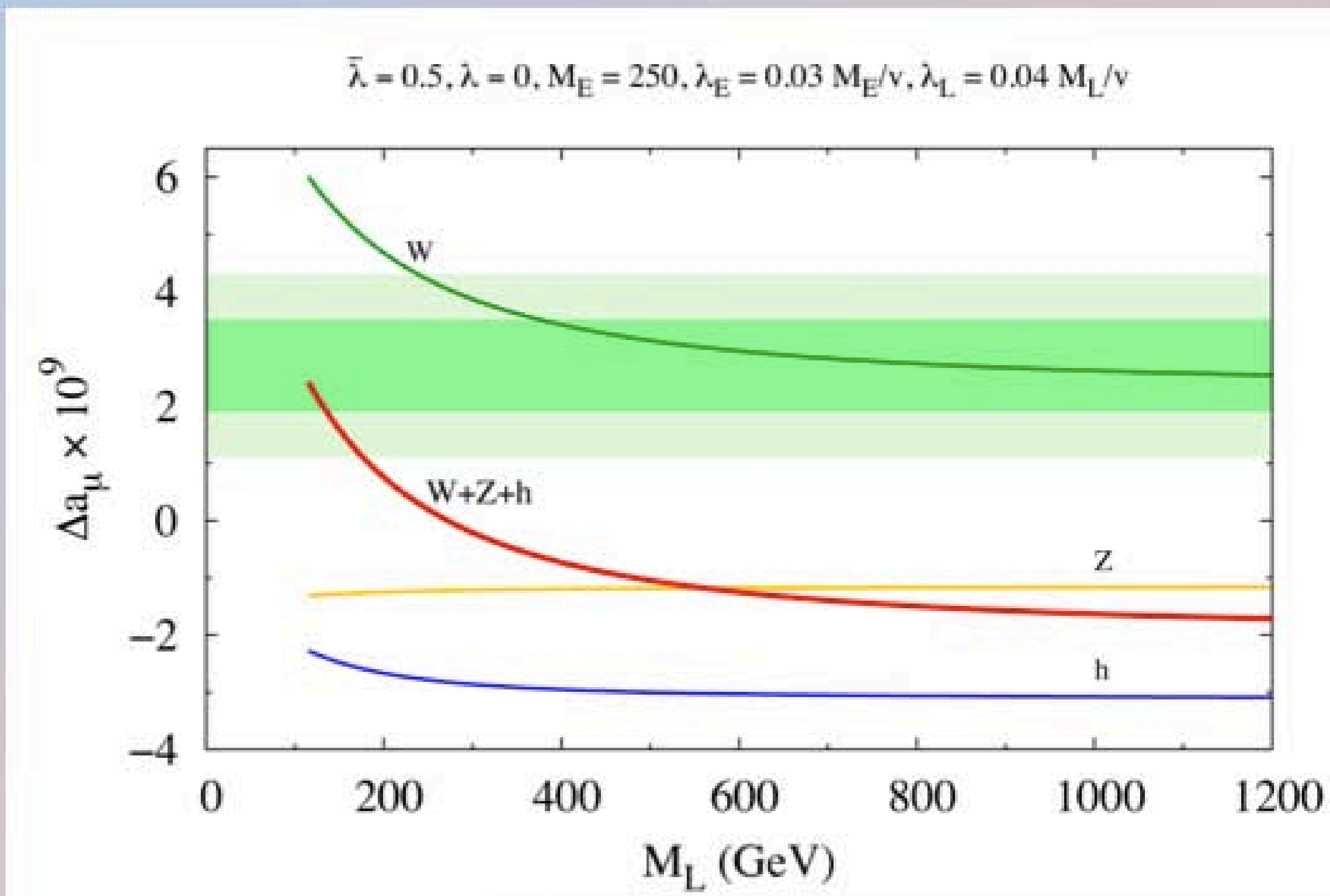
Random scan:

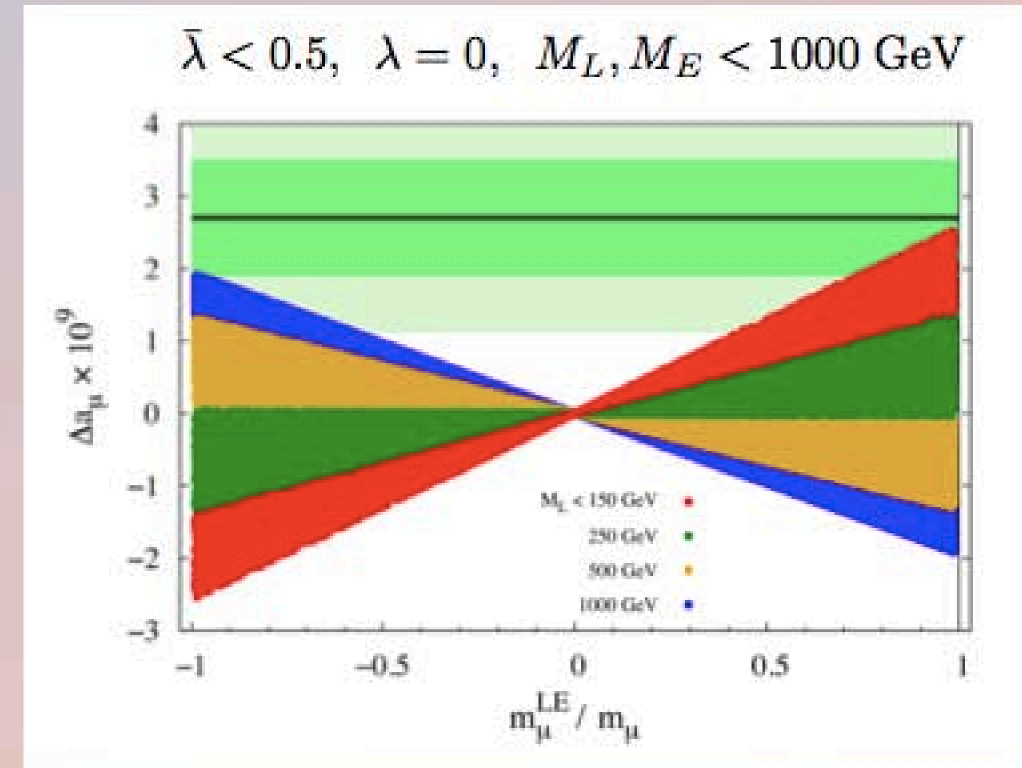


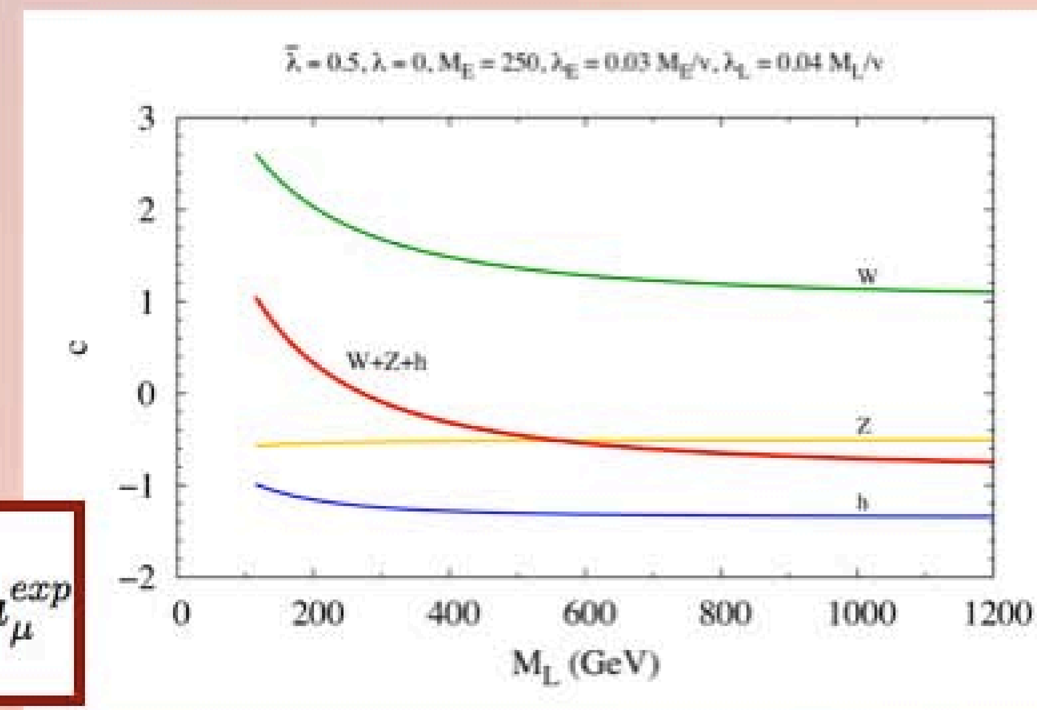
Constraints from precision EW data:

$$\frac{\lambda_E v}{M_E} < 0.03, \qquad \frac{\lambda_L v}{M_L} < 0.04$$

$$c = f(M_L)$$







$$\Delta a_\mu \simeq c\,\frac{m_\mu m_\mu^{LE}}{(4\pi v)^2} \simeq 0.85\,c\,\frac{m_\mu^{LE}}{m_\mu}\,\Delta a_\mu^{exp}$$

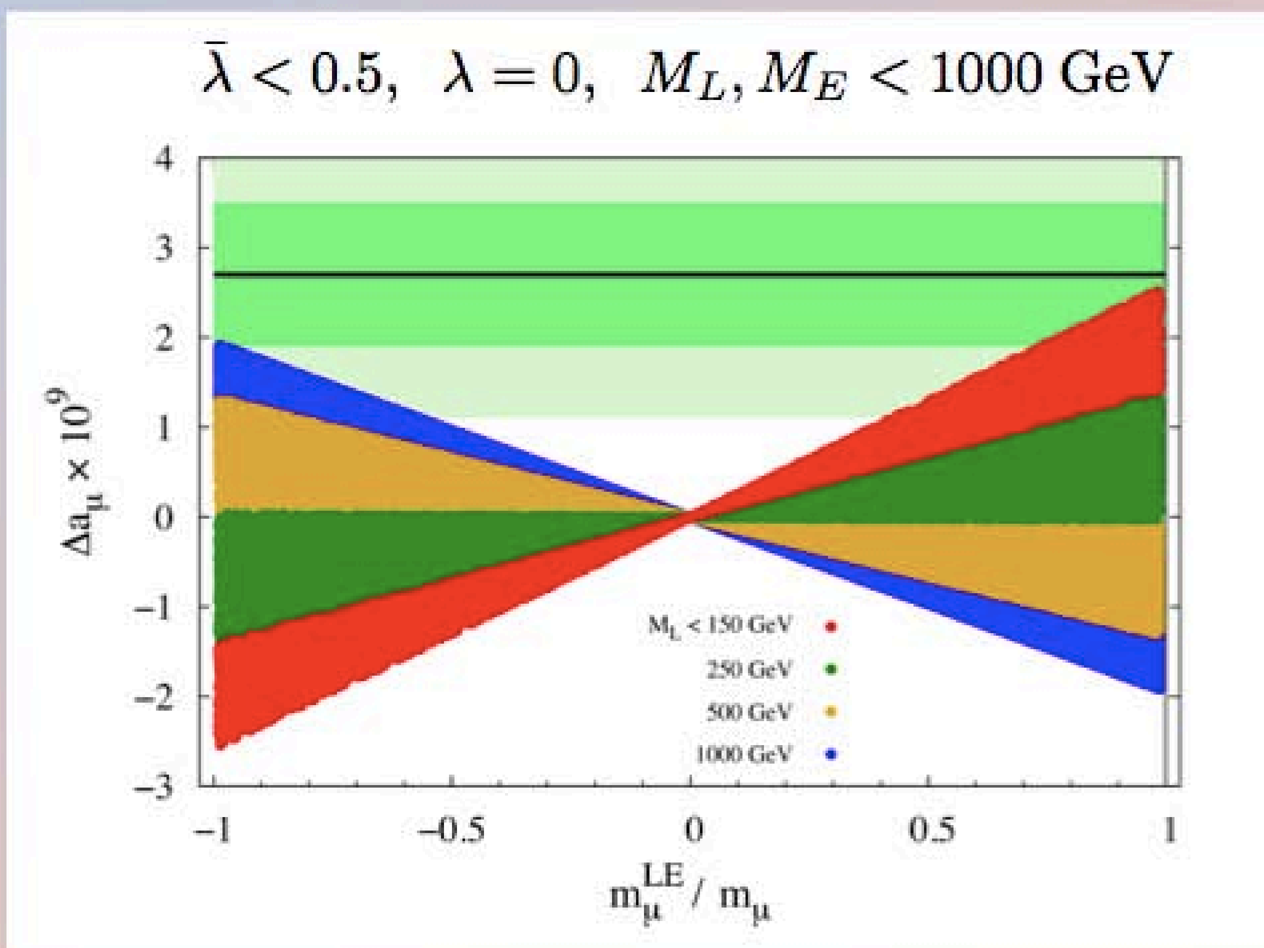
The dependence originates from the G functions:

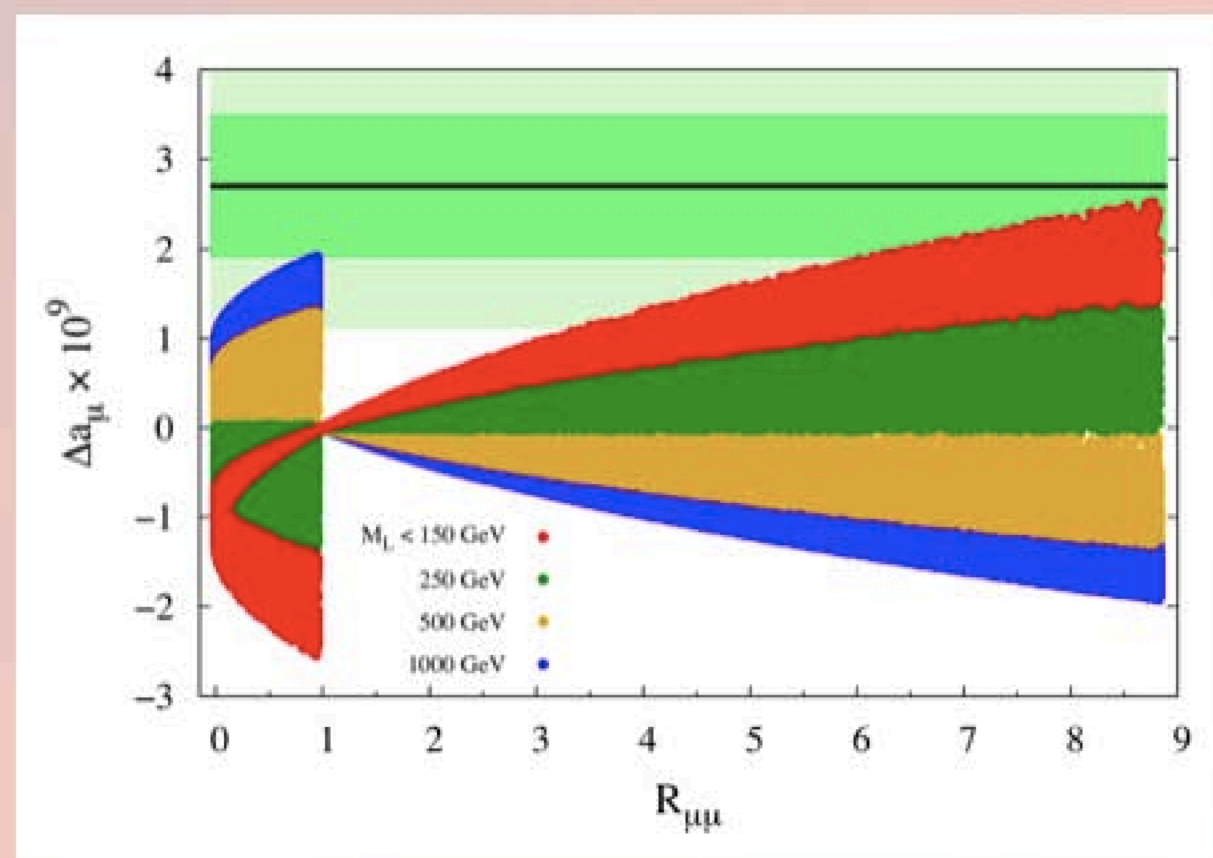
$$\delta a_\mu^Z = -\frac{g^2 m_\mu}{8\pi^2 M_W^2}\left\{(g_L^2+g_R^2)m_\mu F_Z(x) + g_L g_R m_f G_Z(x)\right\}$$

$$x = (m_f/M_Z)^2$$

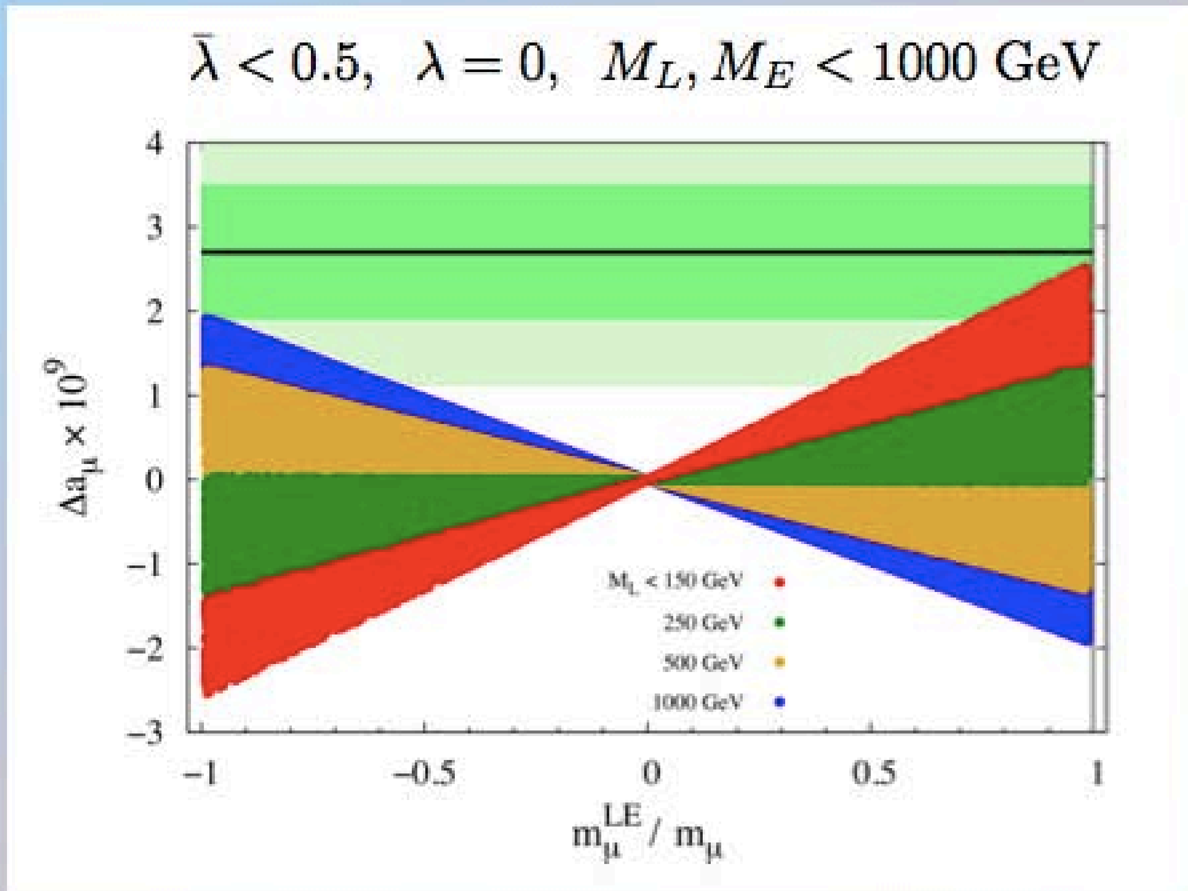
$h \to \mu\mu$

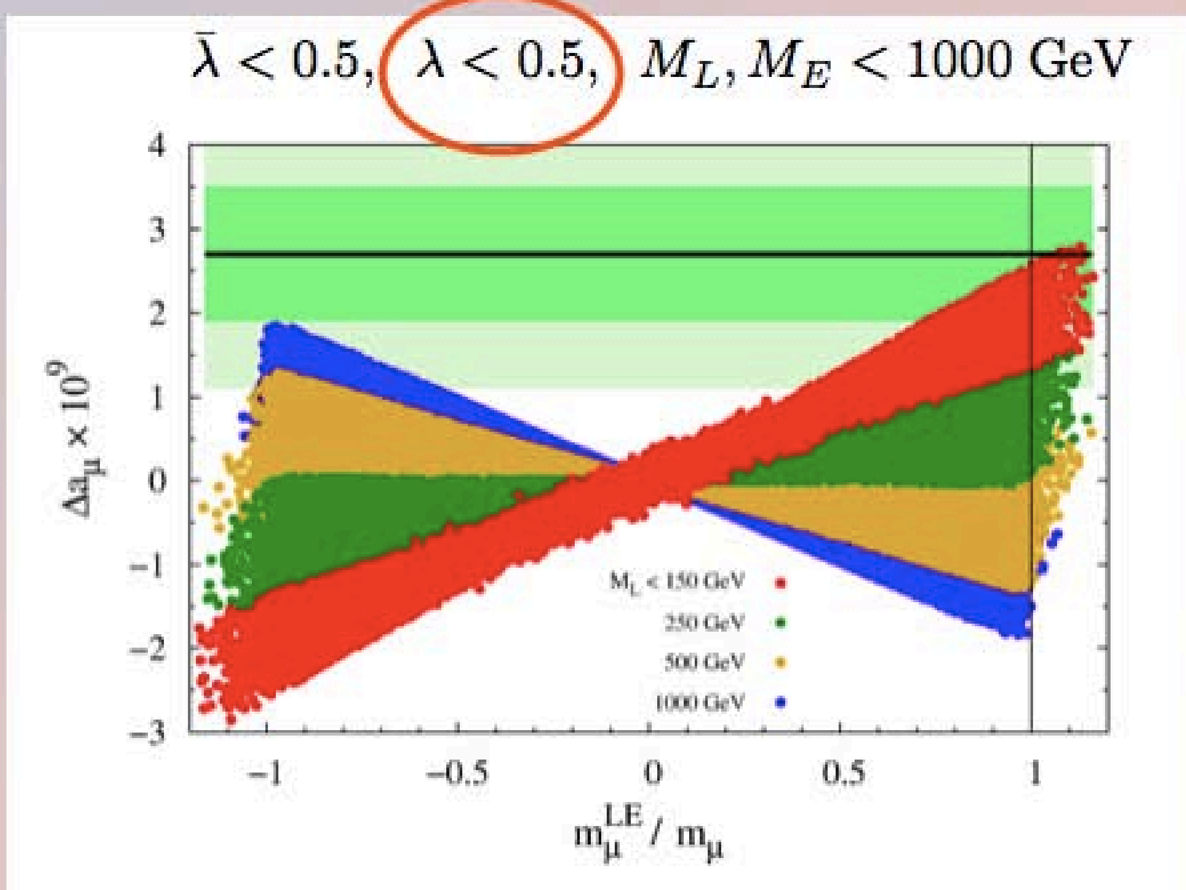
$$\mathcal{L}_{eff} \supset -\bar{\mu}_L \left(y_\mu + \frac{\lambda^L \bar{\lambda} \lambda^E}{M_L M_E} H H^\dagger \right) \mu_R H + h.c. \longrightarrow -\left(m_\mu^H + m_\mu^{LE}\right) \bar{\mu}_L \mu_R + h.c.$$





$$R_{\mu\mu} = \frac{\Gamma(h \to \mu^+\mu^-)}{\Gamma(h \to \mu^+\mu^-)_{SM}}$$



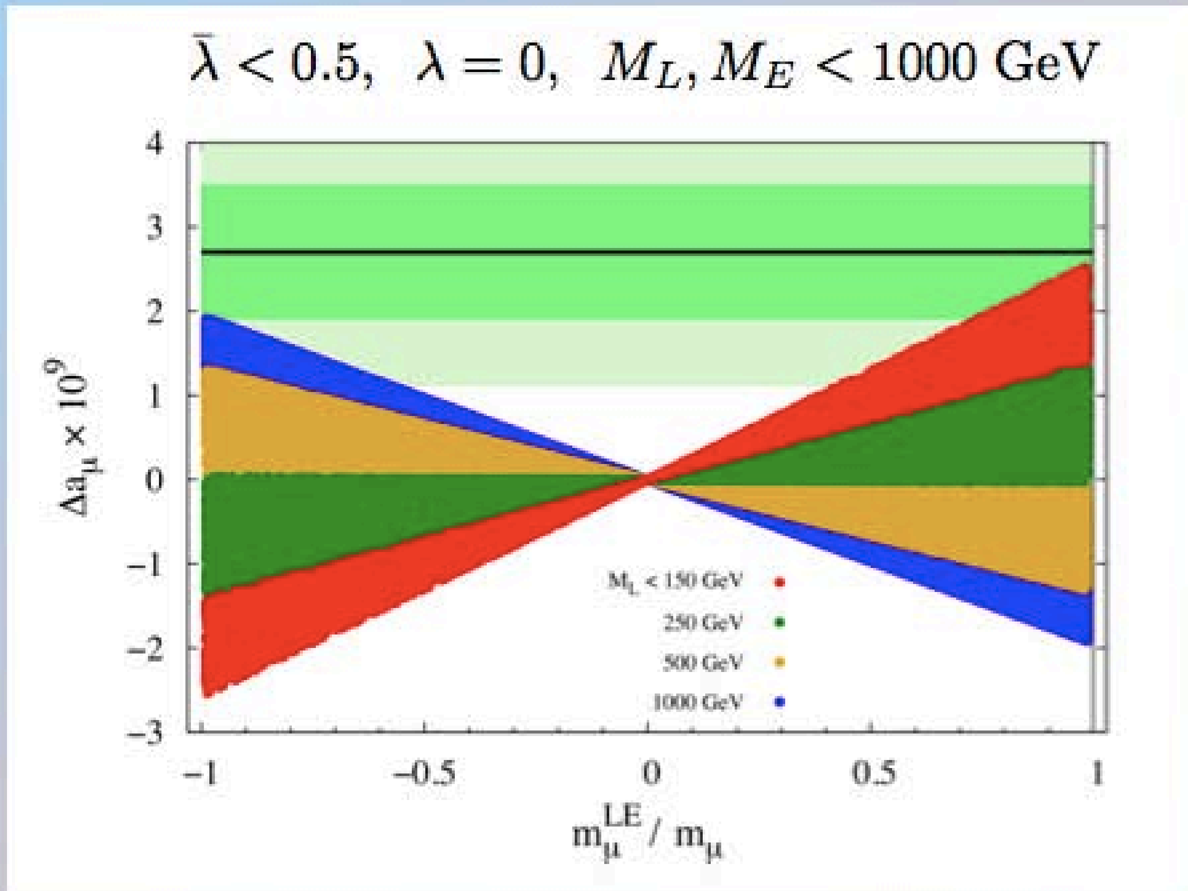


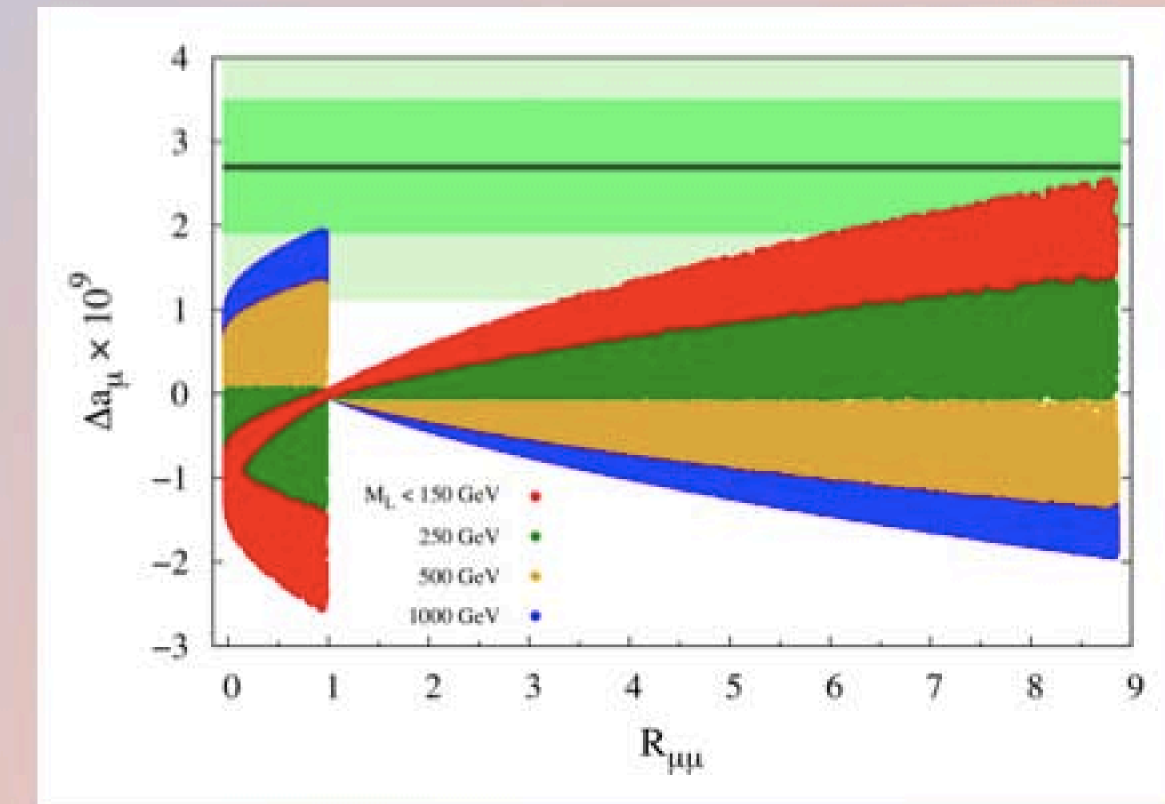
$$\mathcal{L} \supset -\bar{l}_{Li} y_{ij} e_{Rj} H - \bar{l}_{Li} \lambda_i^E E_R H - \bar{L}_L \lambda_j^L e_{Rj} H - \lambda \bar{L}_L E_R H - \bar{\lambda} H^\dagger \bar{E}_L L_R - M_L \bar{L}_L L_R - M_E \bar{E}_L E_R + h.c.,$$

$$(\bar{e}_{Li}, \bar{L}_L^-, \bar{E}_L) \begin{pmatrix} y_{ij} v & 0 & \lambda_i^E v \\ \lambda_j^L v & M_L & \lambda v \\ 0 & \bar{\lambda} v & M_E \end{pmatrix} \begin{pmatrix} e_{Rj} \\ L_R^- \\ E_R \end{pmatrix}$$

$$\mathcal{L}_{eff} \supset -\bar{\mu}_L \left(y_\mu + \frac{\lambda^L \bar{\lambda} \lambda^E}{M_L M_E} H H^\dagger \right) \mu_R H + h.c. \longrightarrow -\left(m_\mu^H + m_\mu^{LE}\right) \bar{\mu}_L \mu_R + h.c.$$

$$\Delta a_\mu \simeq c \frac{m_\mu m_\mu^{LE}}{(4\pi v)^2} \simeq 0.85\, c \frac{m_\mu^{LE}}{m_\mu} \Delta a_\mu^{exp}$$


$\bar{\lambda} < 0.5, \quad \lambda = 0, \quad M_L, M_E < 1000$ GeV
$\Delta a_\mu \times 10^9$
m_μ^{LE} / m_μ
M_L < 150 GeV
250 GeV
500 GeV
1000 GeV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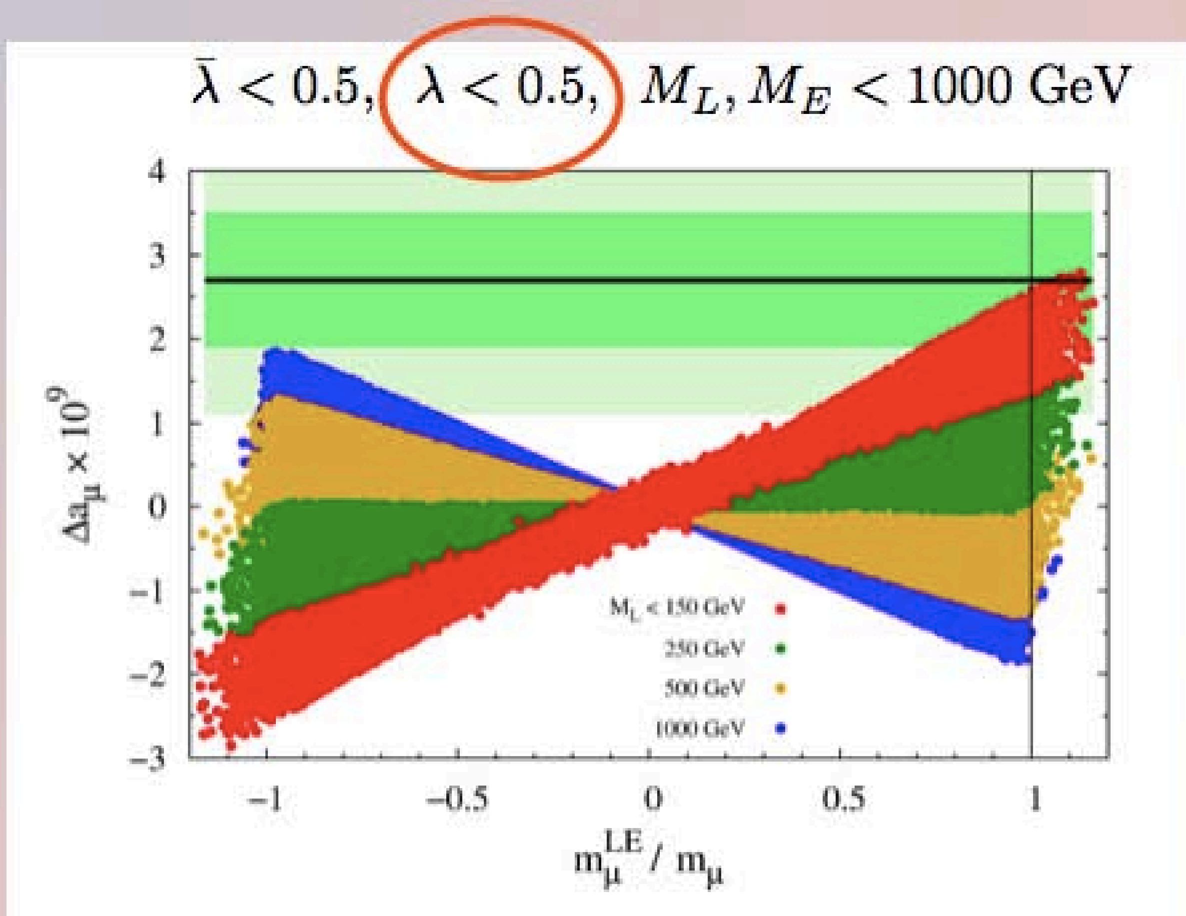
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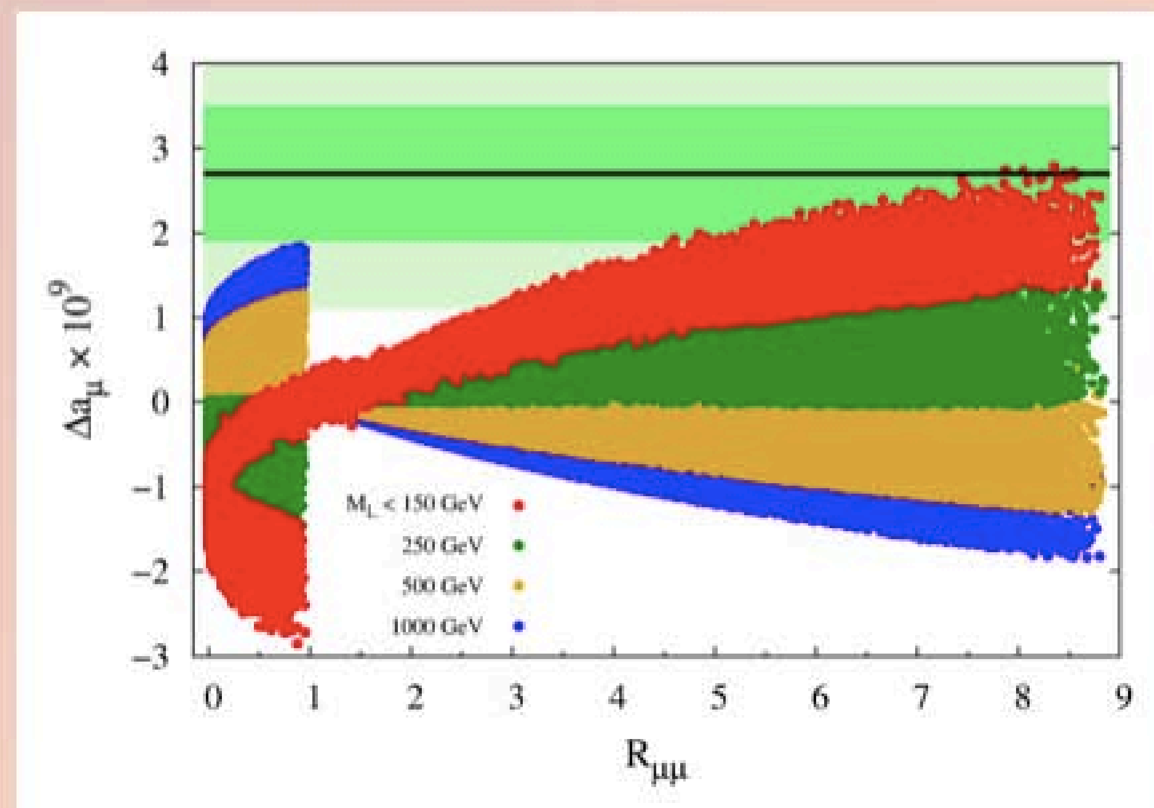
$\Delta a_\mu \times 10^9$
$R_{\mu\mu}$
M_L < 150 GeV
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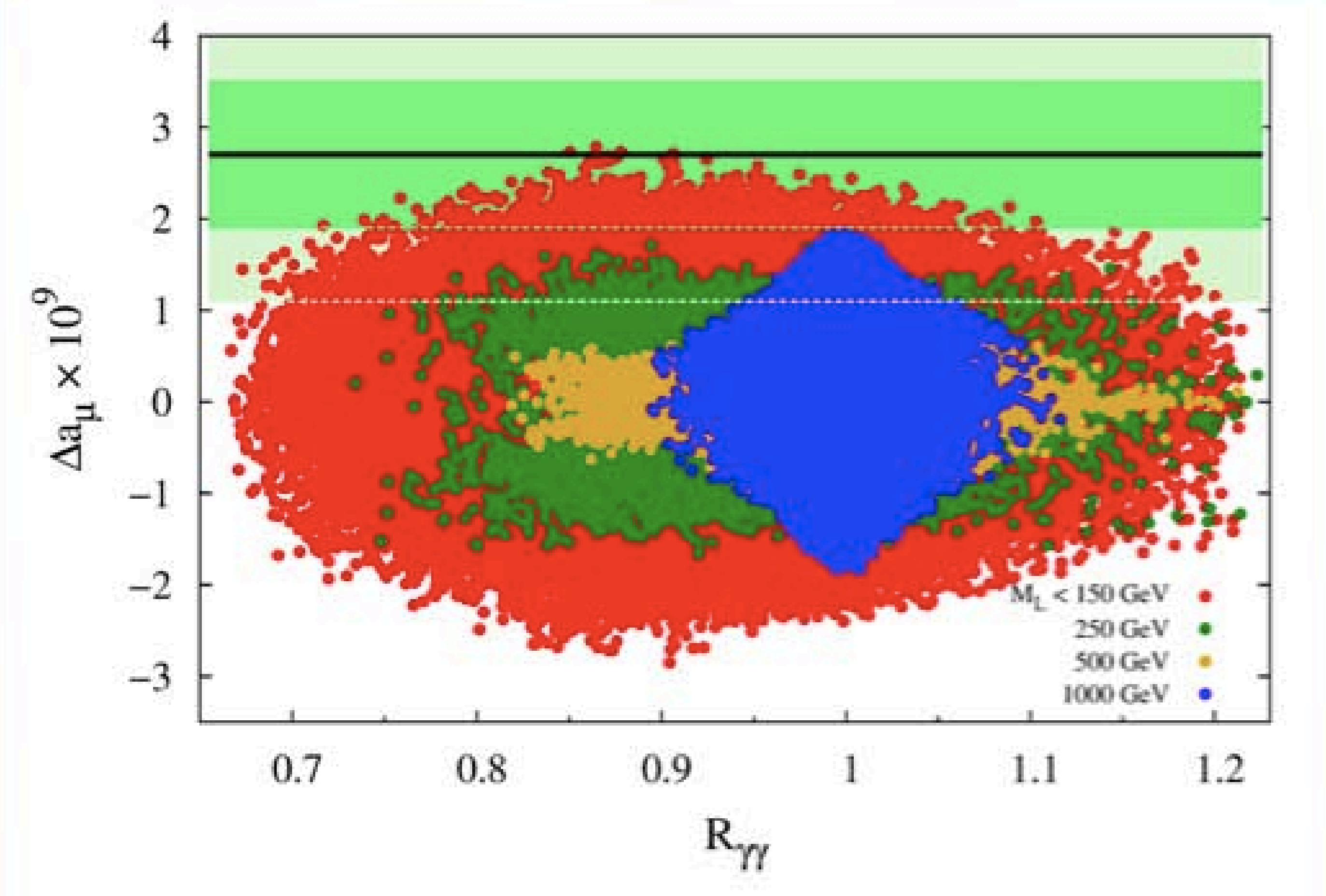
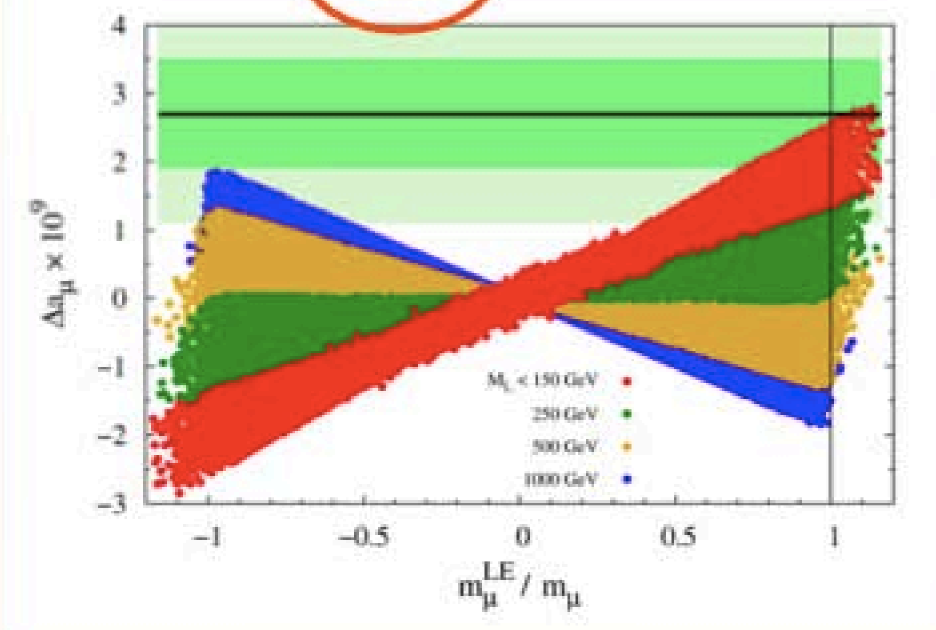
h → γγ

$$R_{\gamma\gamma} = \frac{\Gamma(h \to \gamma\gamma)}{\Gamma(h \to \gamma\gamma)_{SM}}$$



$$\text{amplitude} \propto -\frac{\bar\lambda\lambda v^2}{M_L M_E - \bar\lambda\lambda v^2}$$

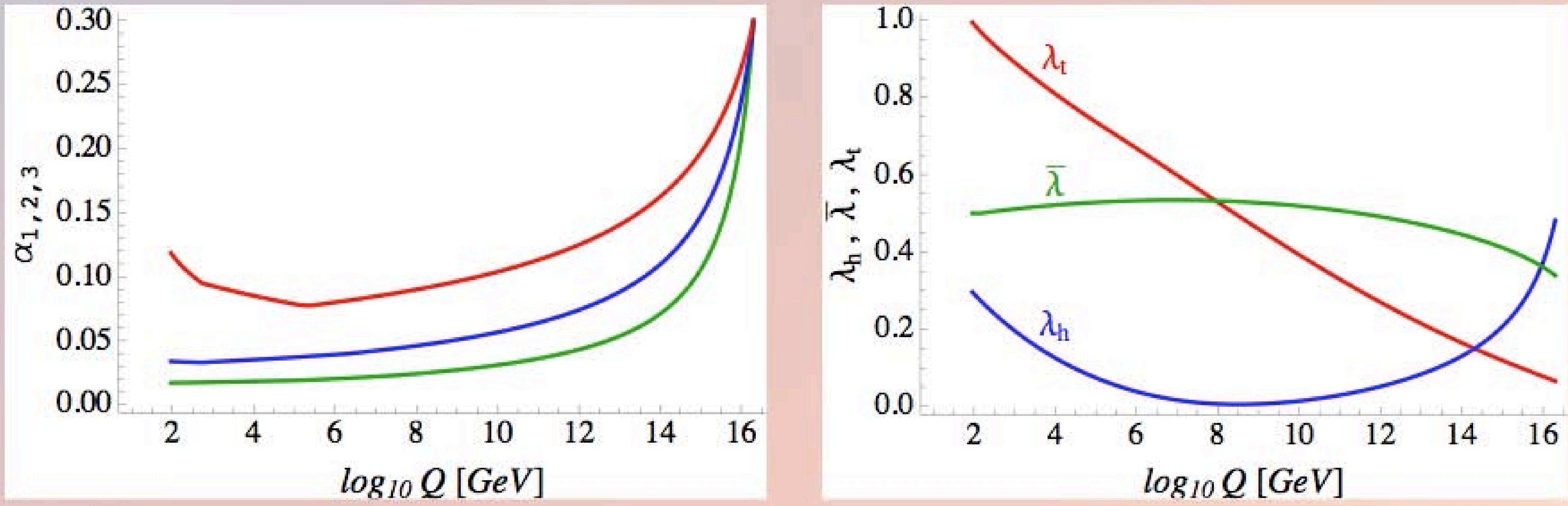
$\bar\lambda < 0.5$, $\lambda < 0.5$, $M_L, M_E < 1000$ GeV



$$(\bar e_{Li}, \bar L^-_L, \bar E_L) \begin{pmatrix} y_{ij}v & 0 & \lambda^E_i v \\ \lambda^L_j v & M_L & \lambda v \\ 0 & \bar\lambda v & M_E \end{pmatrix} \begin{pmatrix} e_{Rj} \\ L^-_R \\ E_R \end{pmatrix}$$

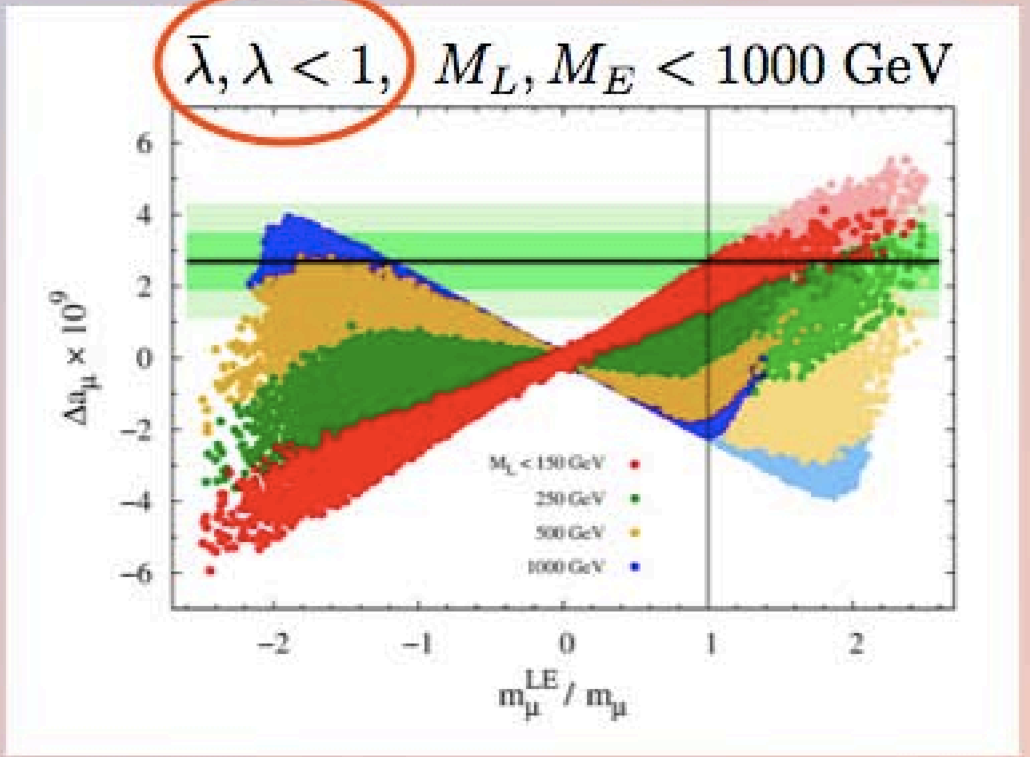
UV completion with 3VFs

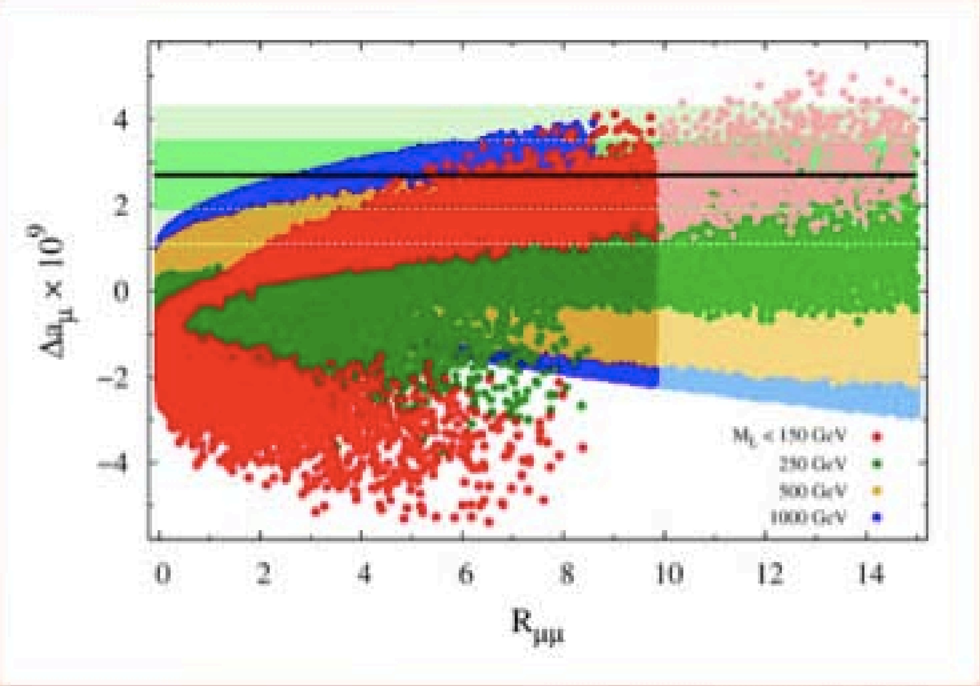
All the features of insensitive unification, including the stability of the EW minimum of the Higgs potential, with Yukawa couplings of the size needed for the explanation of the muon g-2 anomaly:

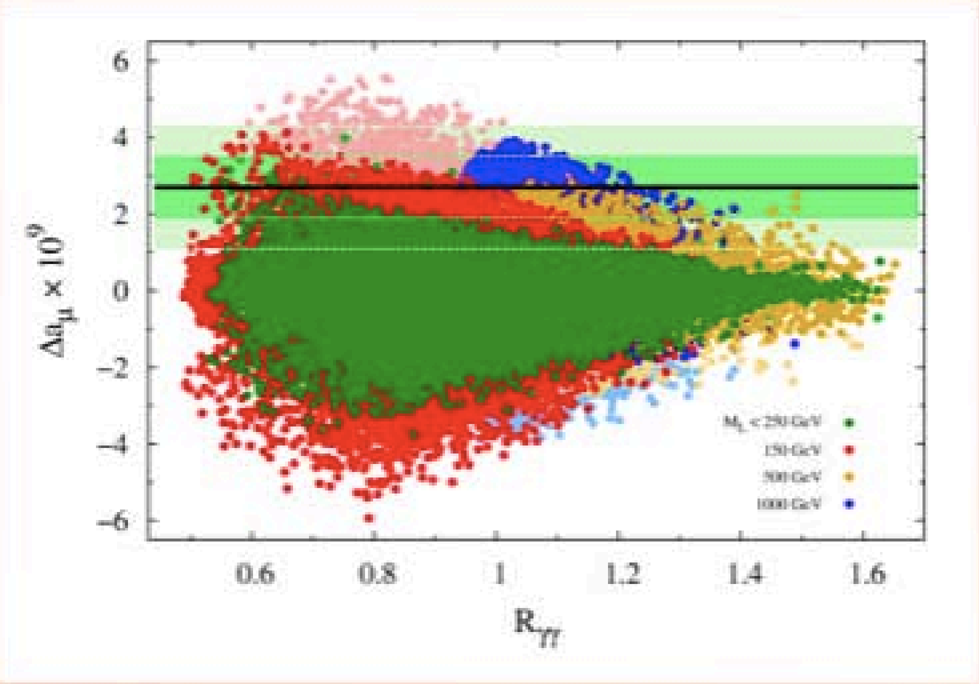


$M_{L_1} = M_{E_1} = 150$ GeV

Allowing larger Yukawa couplings







Examples of constraints from LHC:

- **Type III see-saw:**

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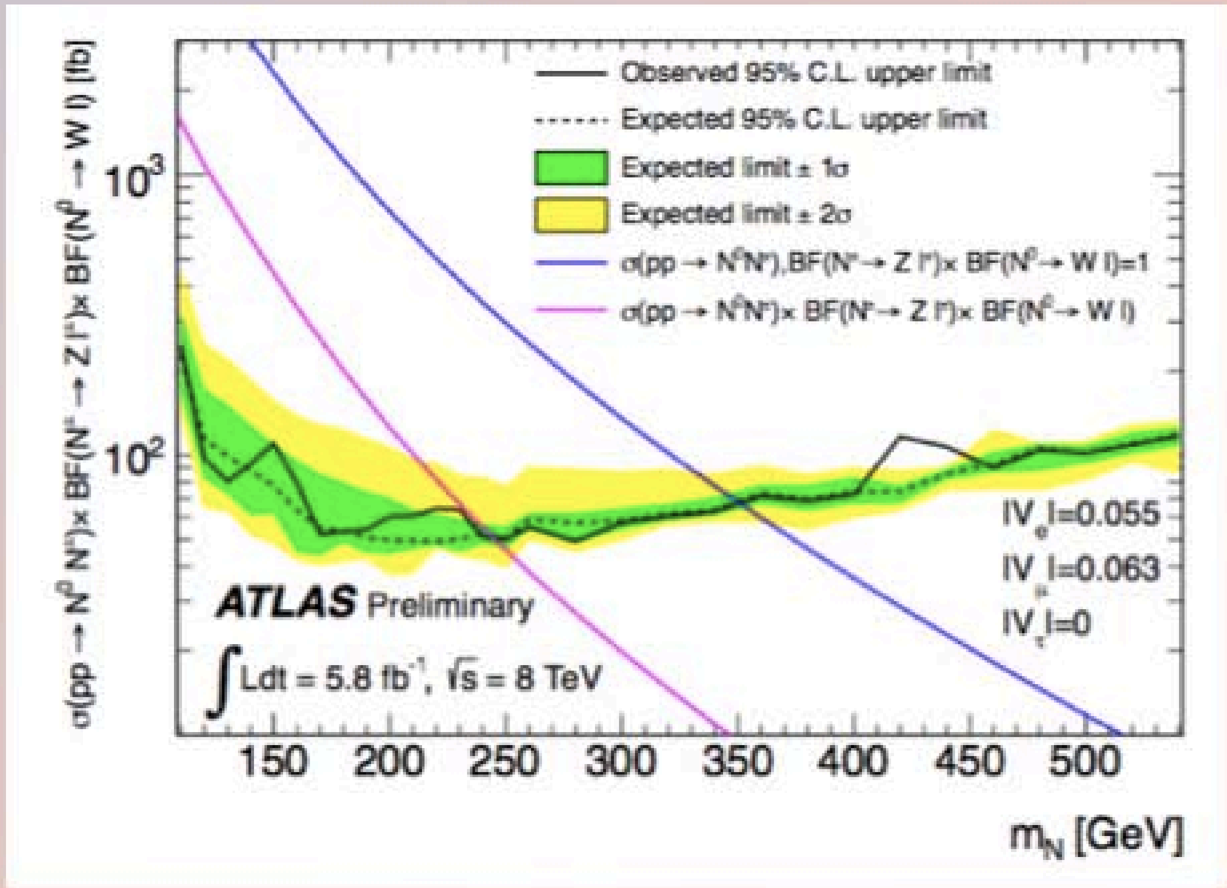

- **multilepton events**

CMS, arXiv:1204.5341

Conclusions

- 3 (or more) pairs of vectorlike families allow for insensitive unification of gauge couplings

  predictive

- resurrects simple non-supersymmetric GUTs (proton decay)

  the GUT scale is adjustable and could be identified with the string or Planck scale

- the electroweak minimum is stable all the way to the GUT scale

- some of the extra fermions might be within the reach of the LHC

  and modify phenomenology of the SM: small flavor violation from mixing through Yukawa couplings, contributions in loops, ...