

Antenna Subtraction and Its Recent Applications

James Currie

based on work with or by

Chen, Gehrman, Gehrman-De Ridder, Glover, Huss, Morgan, Niehues, Pires, Wells



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MC@NNLO

Coming Up

- Higher order corrections
- Antenna Subtraction
- Technicalities and possible improvements
- Recent applications of the method
- Looking to the future

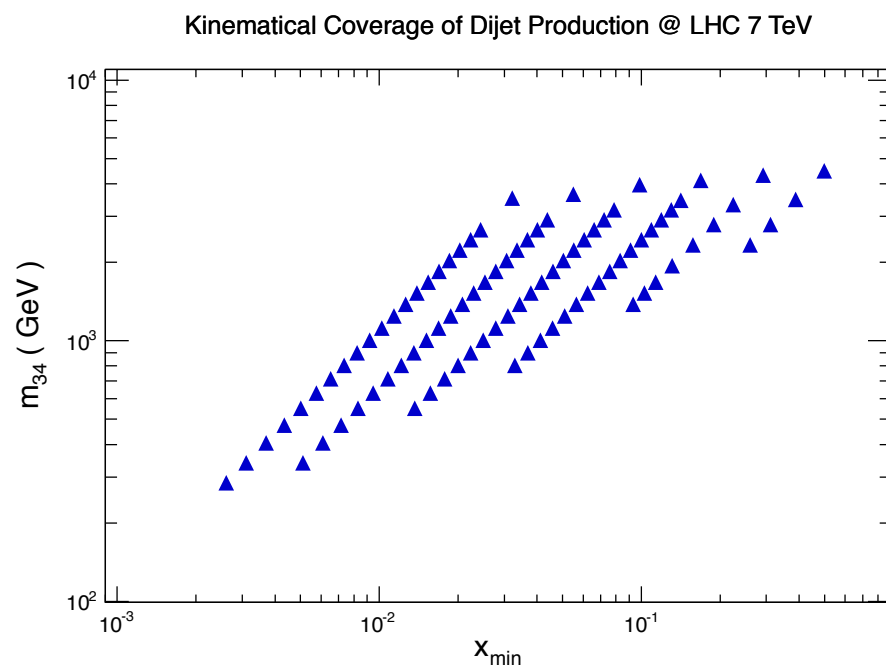
Motivation for doing higher order calculations



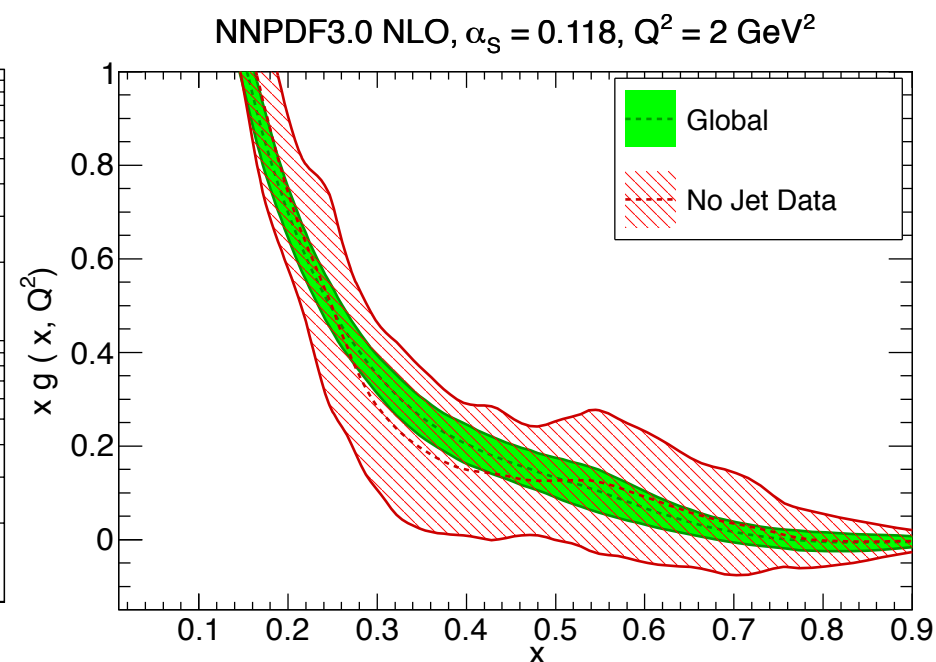
Constraining PDFs

LHC is mainly a gluon collider but gluon PDF is not well known:

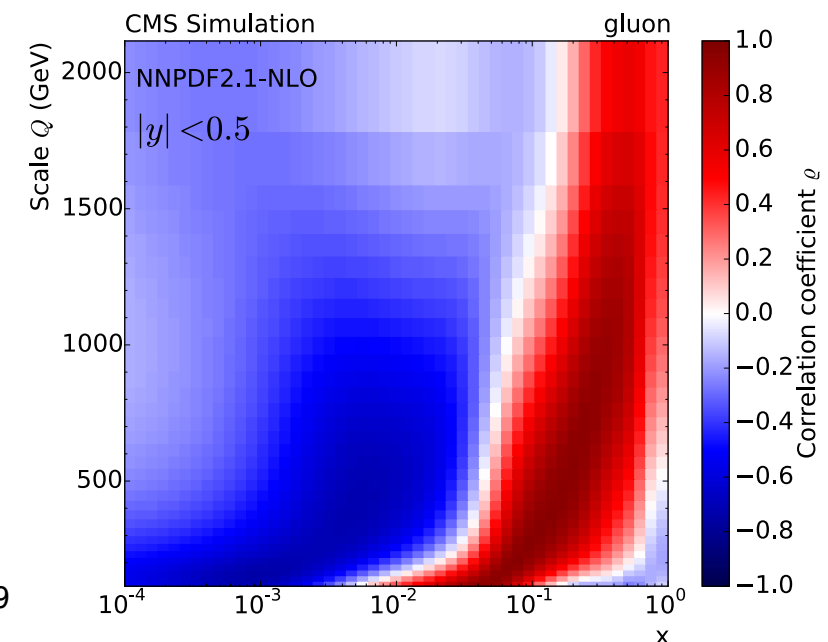
- LHC jets probe a wide range of x
- gluon PDF directly sensitive to jet data, especially at large x
- would like to consistently include NNLO jet data in NNLO PDF fits without using kinematically limited approximations



Rojo hep-ph [1410.7728]

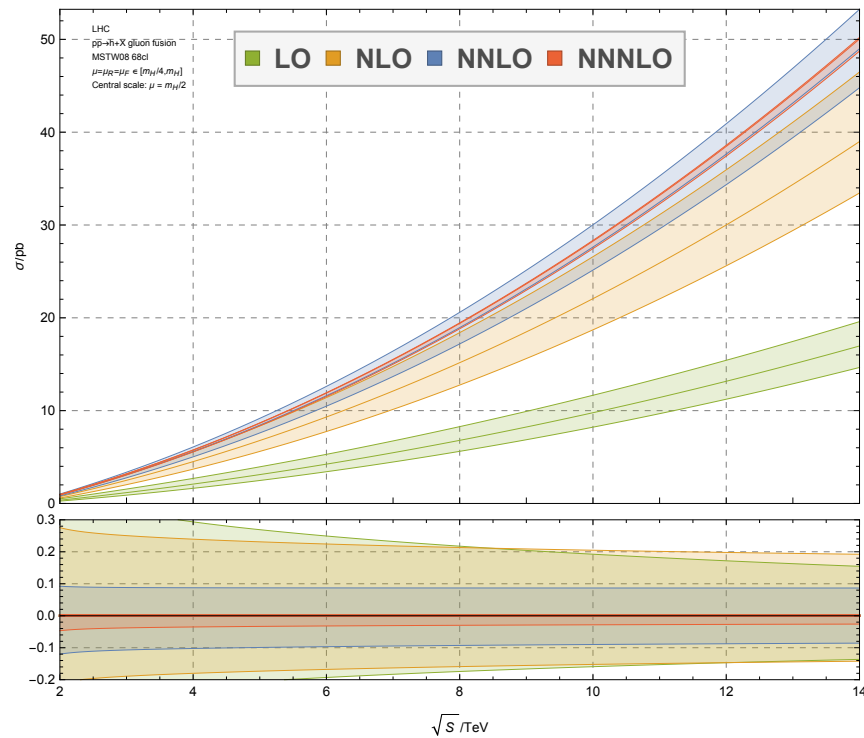


NNPDF collaboration hep-ph [1410.8849]

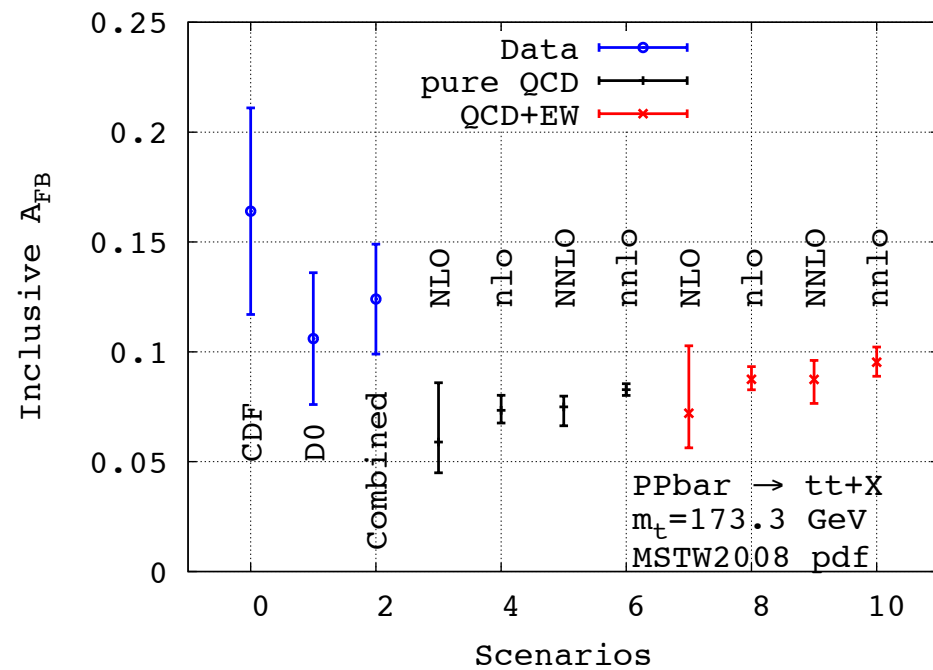


Rojo hep-ph [1410.7728]

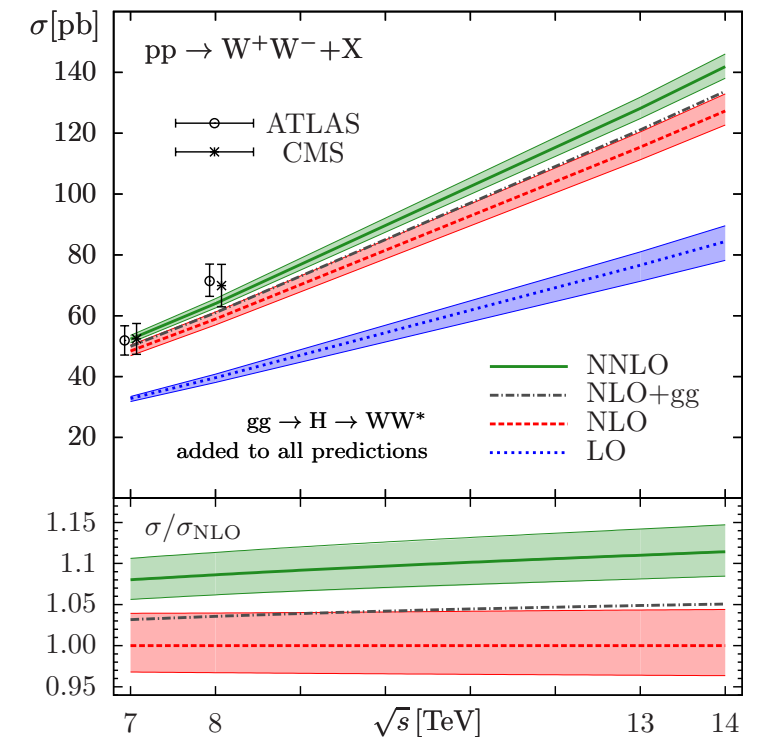
Discrepancies with data



Anastasiou, Duhr, Dulat, Herzog,
Mistlberger



Czakon, Fiedler, Mitov

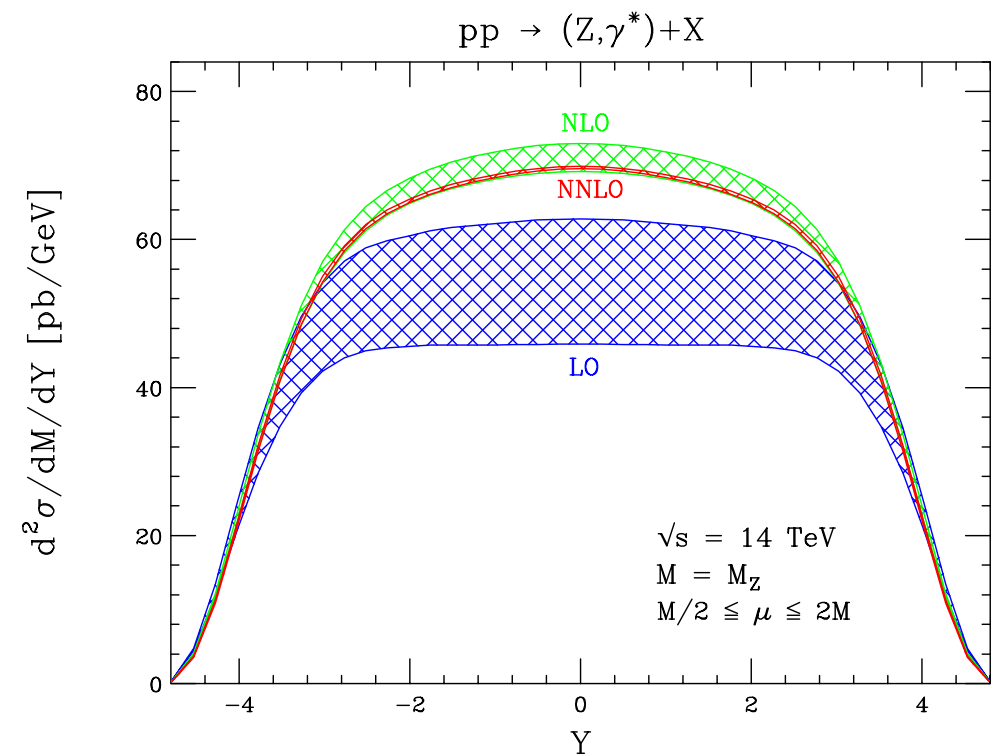


Gehrmann, Grazzini, Kallweit,
Maierhöfer, von Manteuffel,
Pozzorini, Ravlev, Tancredi

No BSM discovered yet... but plenty of BNLO

Theoretical Uncertainties

- Scale uncertainty one of the main obstacles, especially in jet veto and exclusive searches
- NNLO contains all features of calculation
 - initial-state radiation
 - non-trivial jet algorithm
 - all partonic channels
 - non-trivial physical scales

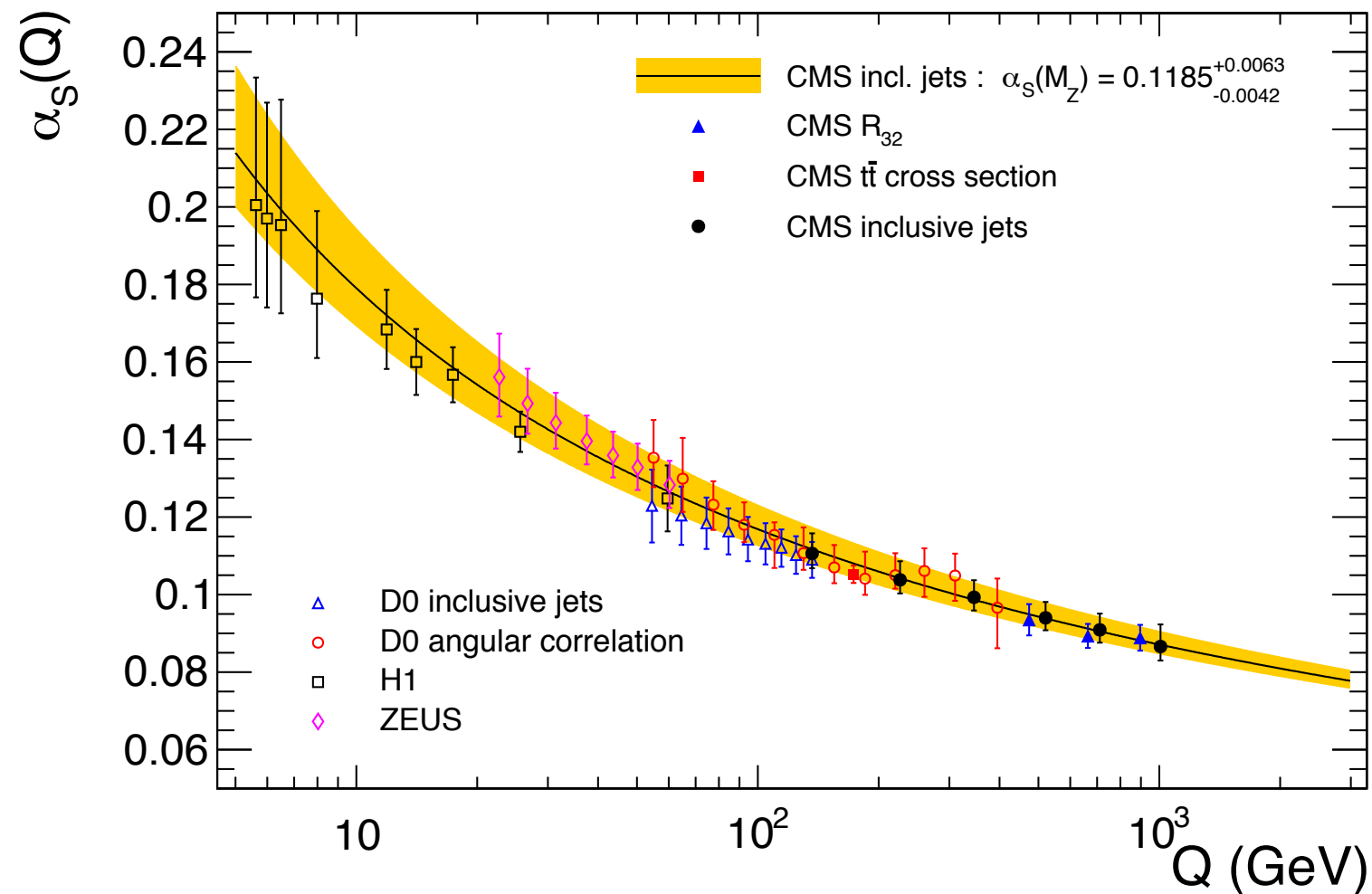


Anastasiou, Dixon, Melnikov, Petriello '04

Strong Coupling

Can use the single inclusive jet cross section to determine [CMS-PAS-SMP-12-028]:

- $\alpha_s(M_Z)$ and running coupling from single experiment



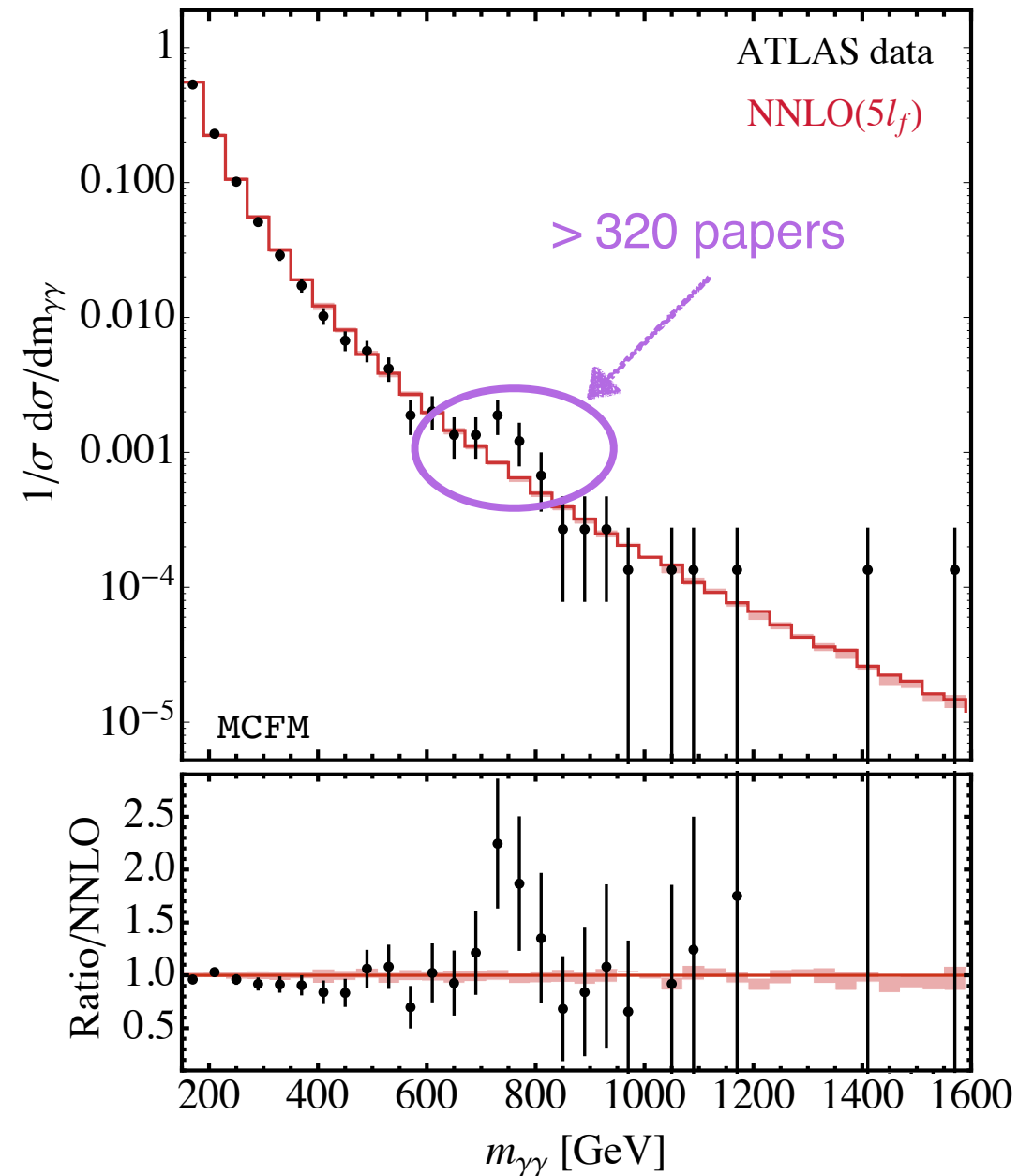
- very satisfying test of QCD and the LHC

CMS hep-ex [1410.6765]

- model independent probe of new physics

New Physics

- Bump hunting often uses data driven methods
- extrapolation to region with little data
- can fit rate and compare with precise SM prediction
- fit works well... bump survives until at least ICHEP



Campbell, Ellis, Li, Williams:
1603.02663

Methods at NLO

Main problem at NLO is extracting singularities...
many ways to do this:

- Dipole subtraction [Catani, Seymour '96]
- FKS subtraction [Frixione, Kunszt, Signer '95]
- Sector decomposition [Hepp '67; Binoth, Heinrich '00]
- Phase space slicing [Giele, Glover '91]

Methods at NNLO

Main problem at NNLO is disentangling singularities

Most methods basically a generalization of NLO:

- Antenna subtraction [Kosower '03; Gehrmann, Gehrmann-De Ridder, Glover '05]
- CoLorFul subtraction [Dei Duca, Somogyi, Trocsanyi '06] (dipoles)
- Sector-improved residue subtraction [Czakon '10] (FKS+sectors)
- q_T and N-Jettiness subtraction [Catani, Grazzini '07; Gaunt, Stahlhofen, Tackmann, Walsh '15; Boughezal, Focke, Liu, Petriello '15] (slicing)

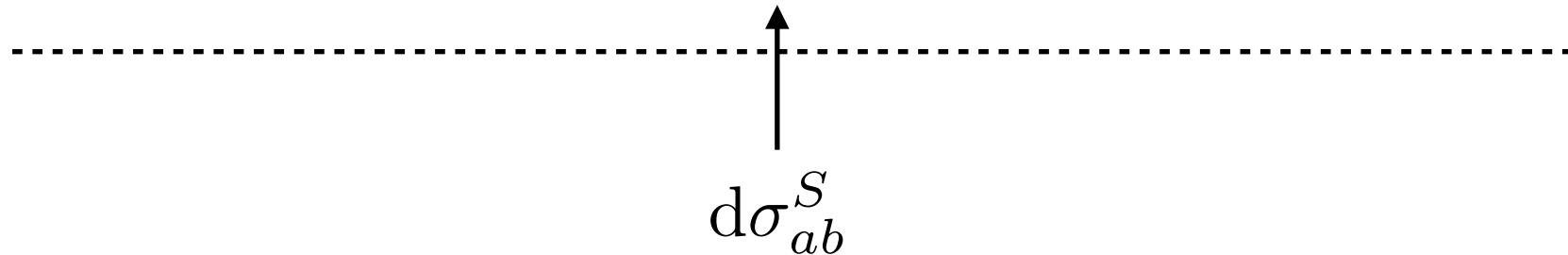
(not an exhaustive list)

Subtraction at NLO

No overlapping singularities so very simple structure

$$\begin{aligned} d\sigma_{ab,NLO} = & \int_{\Phi_{m+1}} \left[d\sigma_{ab}^R - d\sigma_{ab}^S \right] \\ & + \int_{\Phi_m} \left[d\sigma_{ab}^V - d\sigma_{ab}^T \right] \end{aligned}$$

$$d\sigma_{ab}^T = - \int_1 d\sigma_{ab}^S + d\sigma_{ab}^{MF}$$

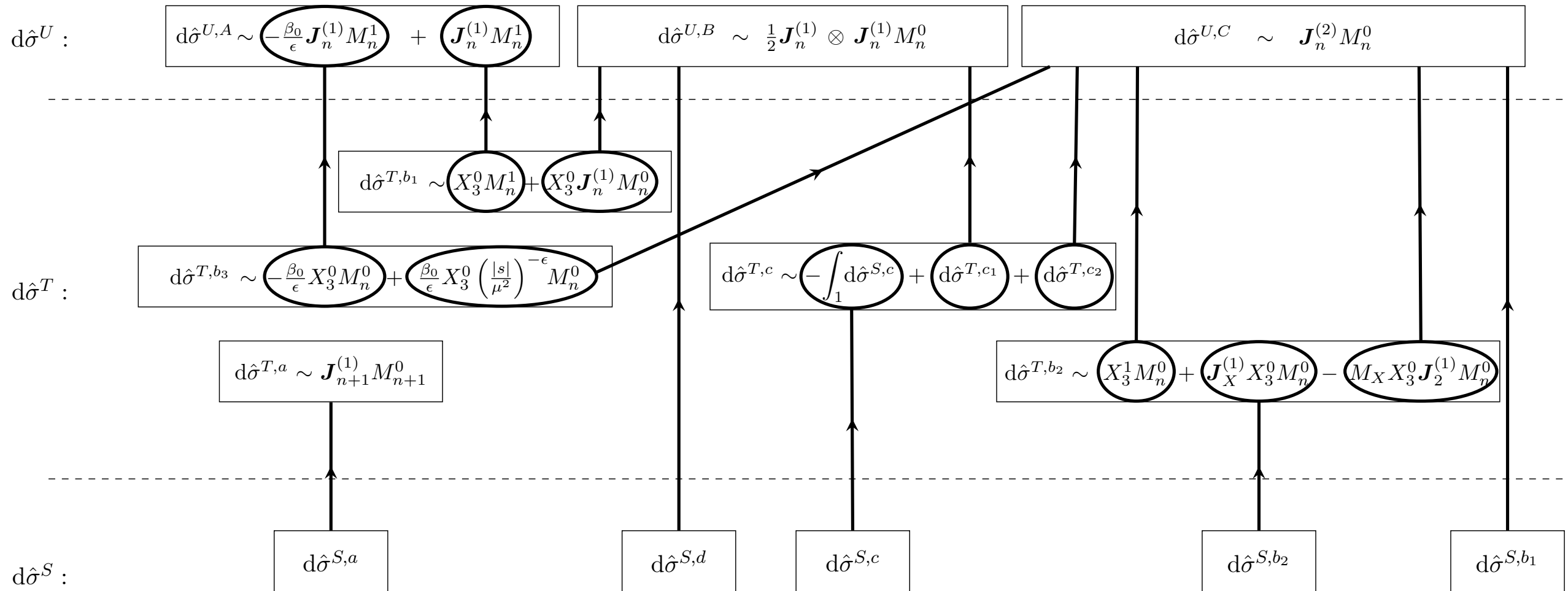


Subtraction at NNLO

At NNLO more terms to regulate

$$\begin{aligned} d\sigma_{ab,NNLO} = & \int_{\Phi_{m+2}} \left[d\sigma_{ab,NNLO}^{RR} - d\sigma_{ab,NNLO}^S \right] \\ & + \int_{\Phi_{m+1}} \left[d\sigma_{ab,NNLO}^{RV} - d\sigma_{ab,NNLO}^T \right] \\ & + \int_{\Phi_m} \left[d\sigma_{ab,NNLO}^{VV} - d\sigma_{ab,NNLO}^U \right] \end{aligned}$$

But real problem is dealing with intricate overlapping singularities



Antenna Subtraction

Basic idea:



construct a counterterm that mimics the matrix element
in all singular regions of phase space

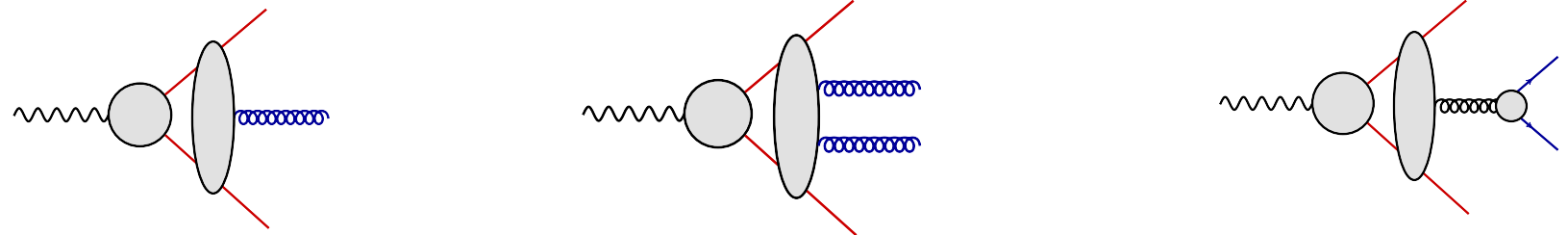
Antennae

Antenna functions built from matrix elements:

$$X_3^0(i, j, k) \sim \frac{|\mathcal{M}_3^0(i, j, k)|^2}{|\mathcal{M}_2^0(I, K)|^2}, \quad X_4^0(i, j, k, l) \sim \frac{|\mathcal{M}_4^0(i, j, k, l)|^2}{|\mathcal{M}_2^0(I, L)|^2}$$

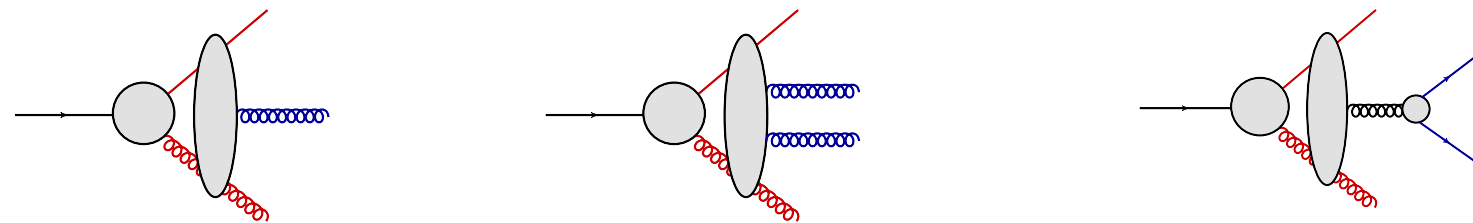
Quark-antiquark:

$$\gamma^* \rightarrow q\bar{q} + \dots$$



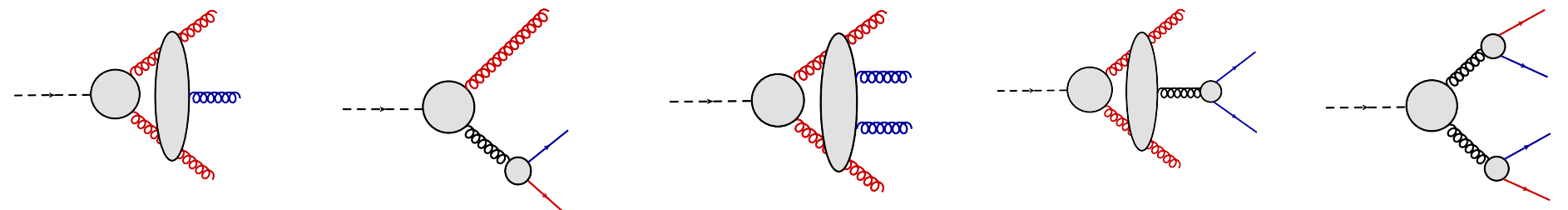
Quark-gluon:

$$\bar{\chi}^0 \rightarrow \tilde{g}g + \dots$$

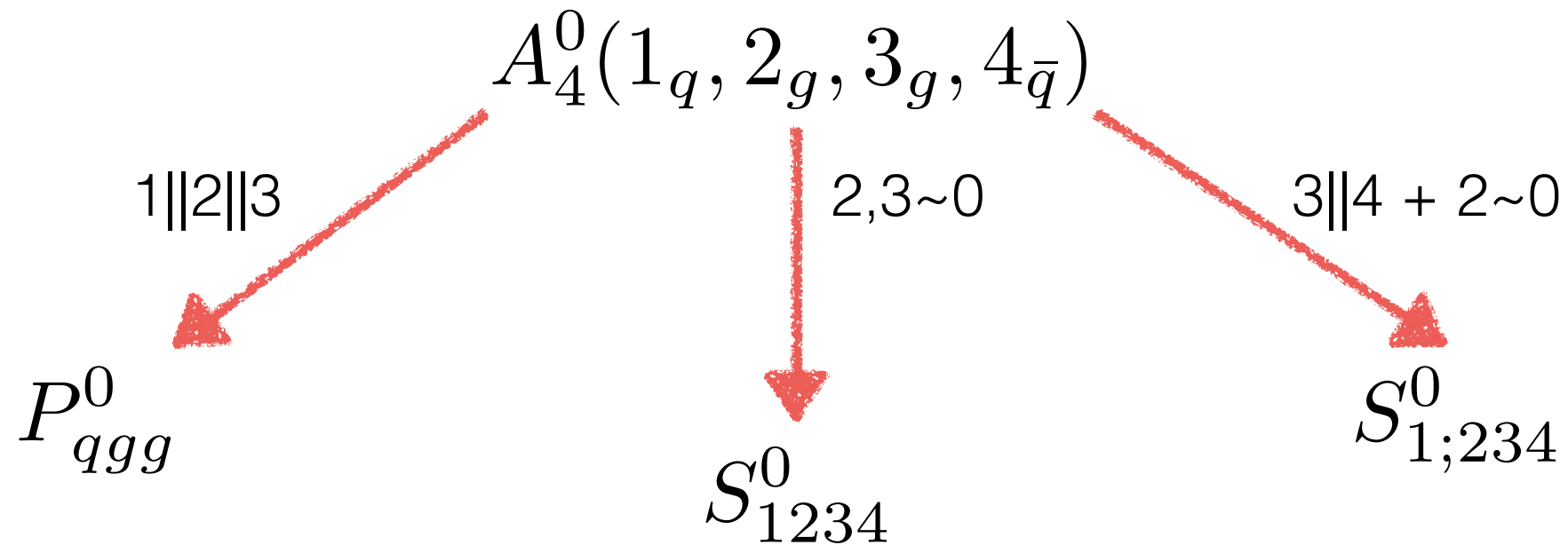


Gluon-gluon:

$$H \rightarrow gg + \dots$$



Antenna mimics all singularities of QCD



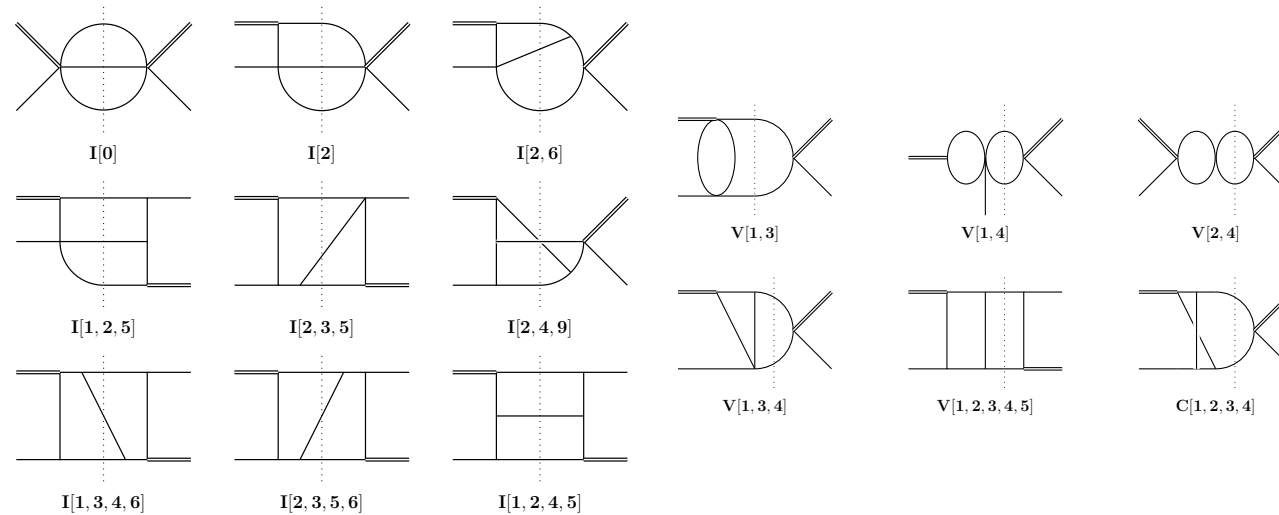
Phase space map smoothly interpolates momenta for reduced matrix element between limits

$$\widetilde{(123)} = xp_1 + r_1p_2 + r_2p_3 + zp_4$$

$$\widetilde{(234)} = (1-x)p_1 + (1-r_1)p_2 + (1-r_2)p_3 + (1-z)p_4$$

Integrating the Antennae

- Relate phase space integrals to multiloop integrals via optical theorem
- apply well developed techniques IBP, LI to masters

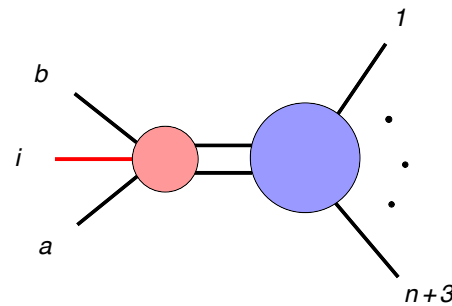


- all antennae in all crossings now successfully integrated:
 - Final-Final [Gehrman, Gehrmann-De Ridder, Glover '04, '05]
 - Initial-Final [Daleo, Gehrmann-De Ridder, Gehrmann, Luisoni '10]
 - Initial-Initial [Gehrmann, Monni '11; Boughezal, Gehrmann-De Ridder, Ritzmann '11; Gehrmann, Ritzmann '12]

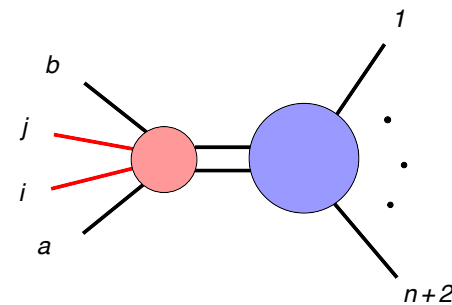
Double Real

Subtraction term constructed to remove:

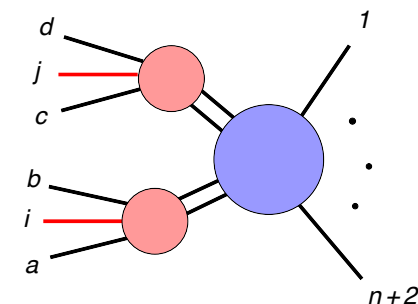
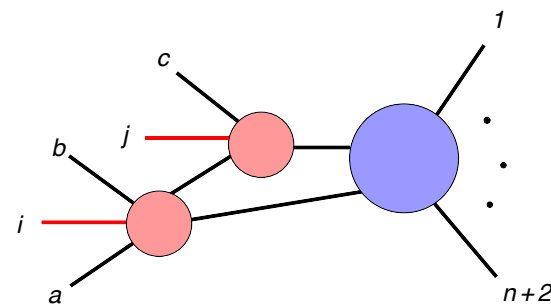
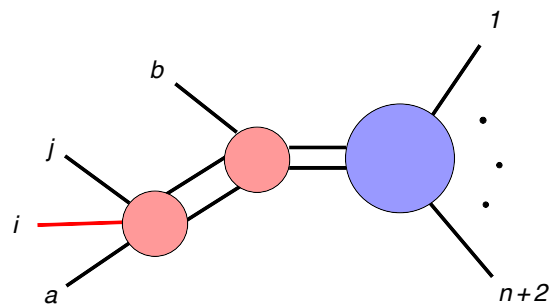
- single unresolved



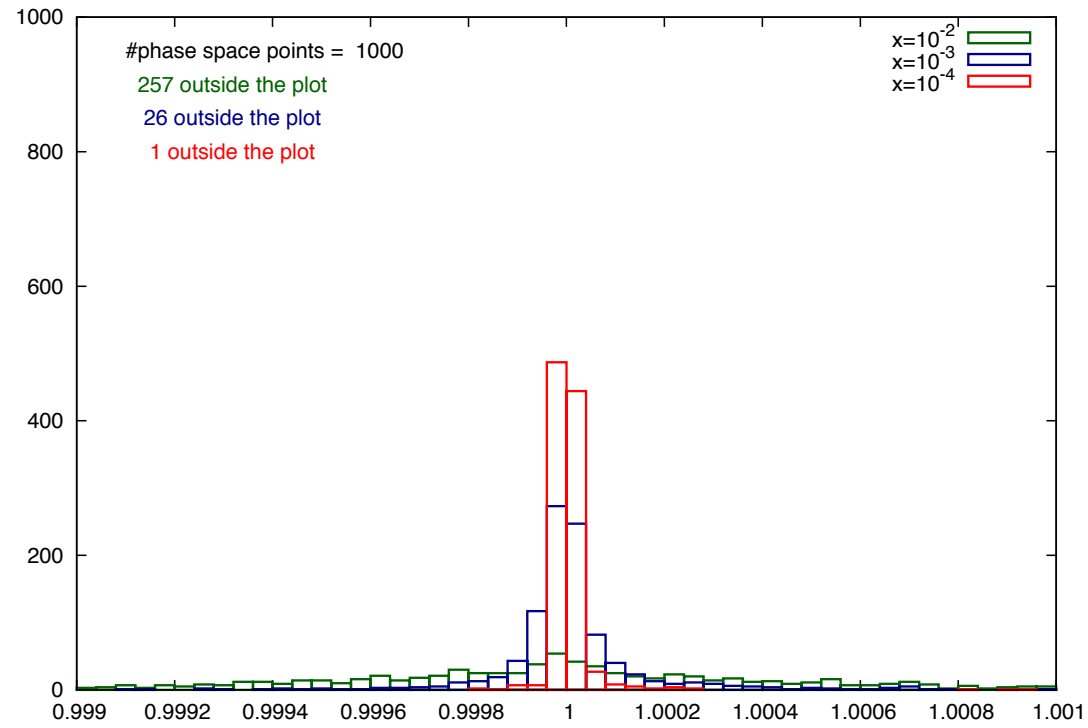
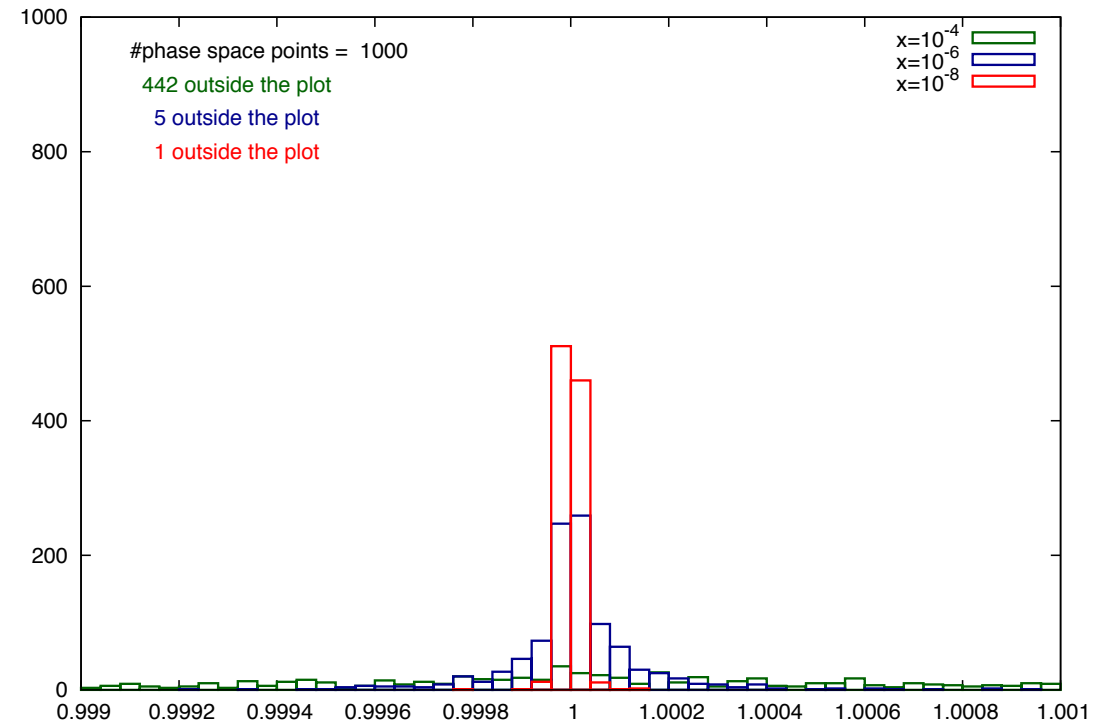
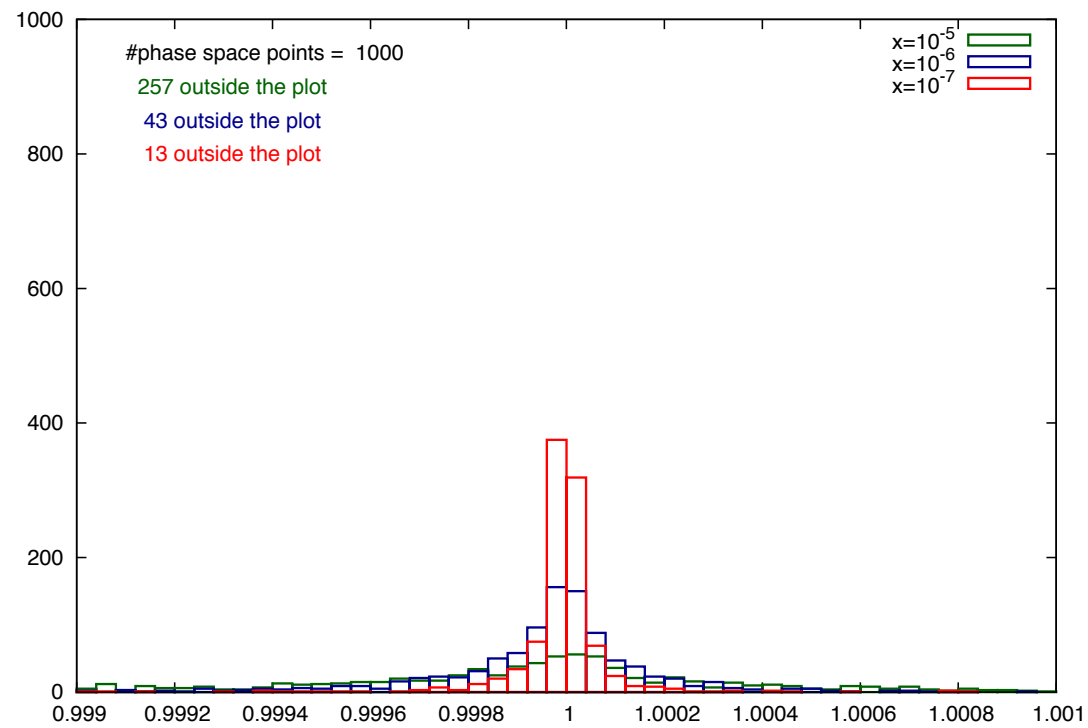
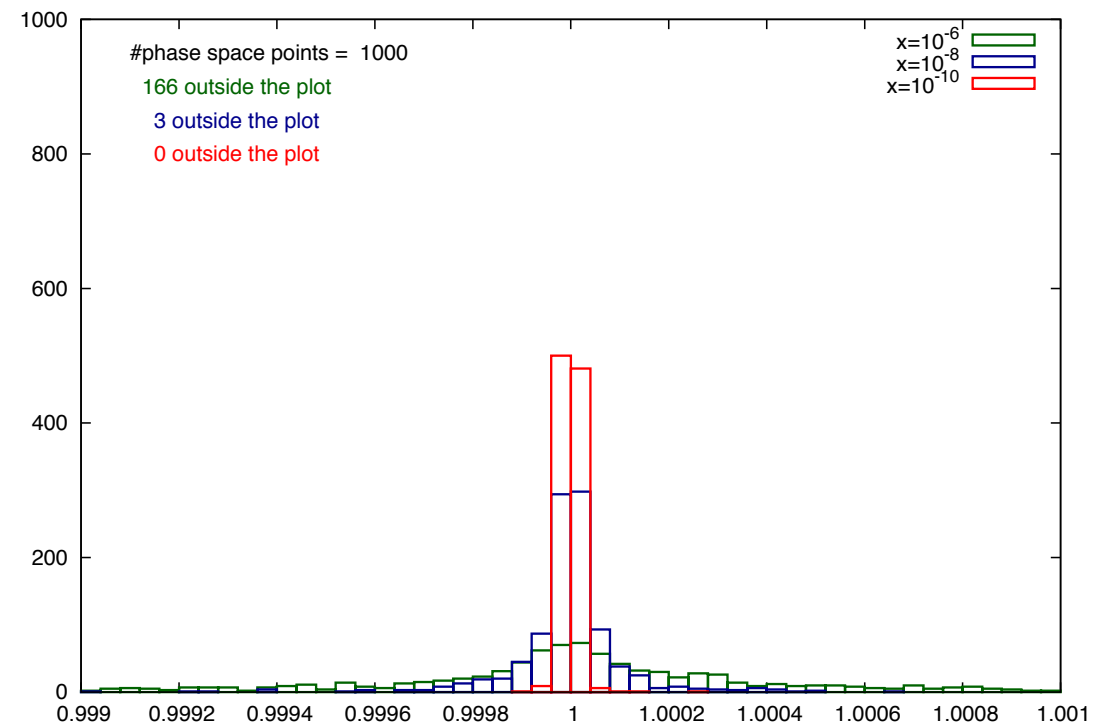
- colour connected double unresolved



- over-subtraction in single and double unresolved limits



7.Double soft

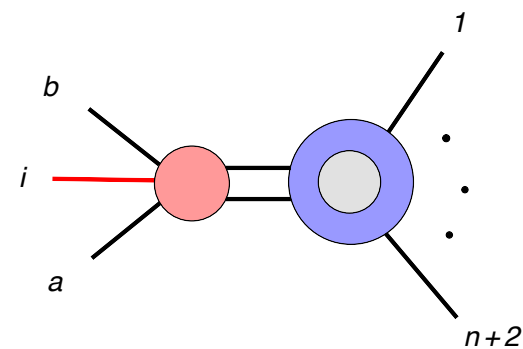
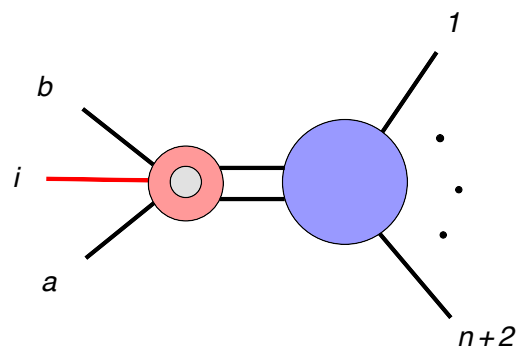
11.IF triple collinear $g|g|g$ 20.Soft + FF collinear $q|g$ 14.IF-FF double collinear $q|g-g|g$ 

Real Virtual

1. Analytic pole cancellation against 1-loop matrix element

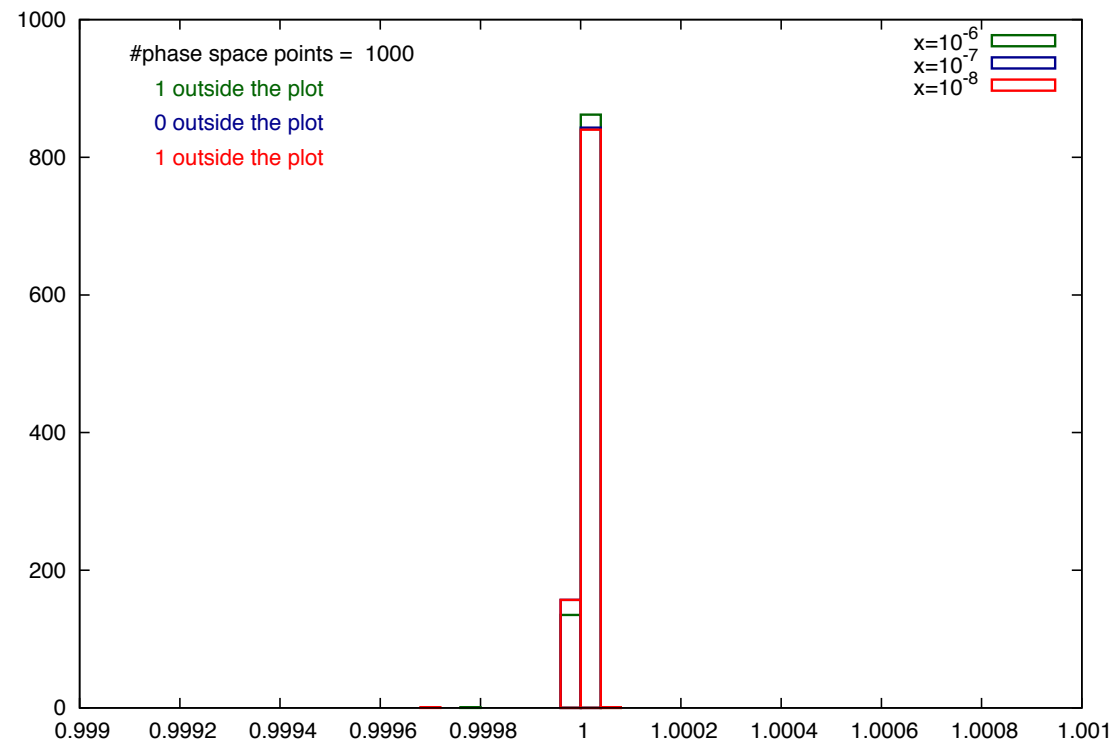
$$2\text{Re}\langle \mathcal{M}_{n+3}^0 | \mathcal{M}_{n+3}^1 \rangle + \mathbf{J}_{n+3}^{(1)}(1, \dots, n+3; \epsilon) \langle \mathcal{M}_{n+3}^0 | \mathcal{M}_{n+3}^0 \rangle = \mathcal{O}(\epsilon^0)$$

2. Only single unresolved limits

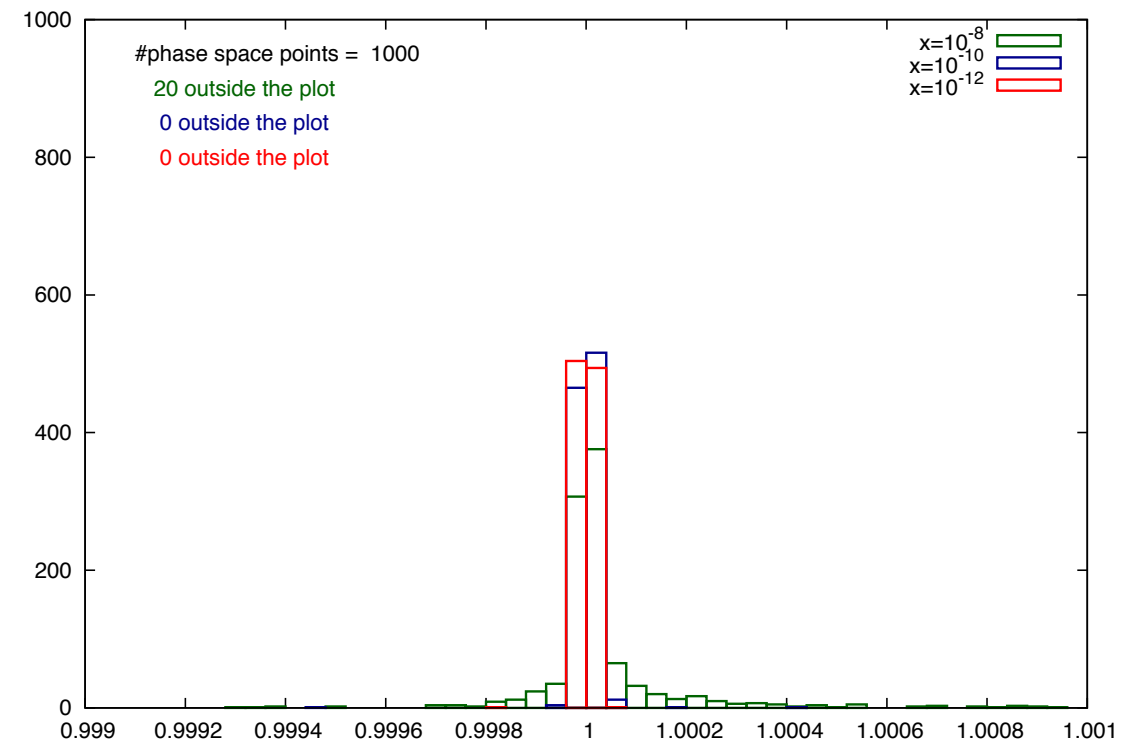


Single unresolved of (1) and poles of (2) also subtracted

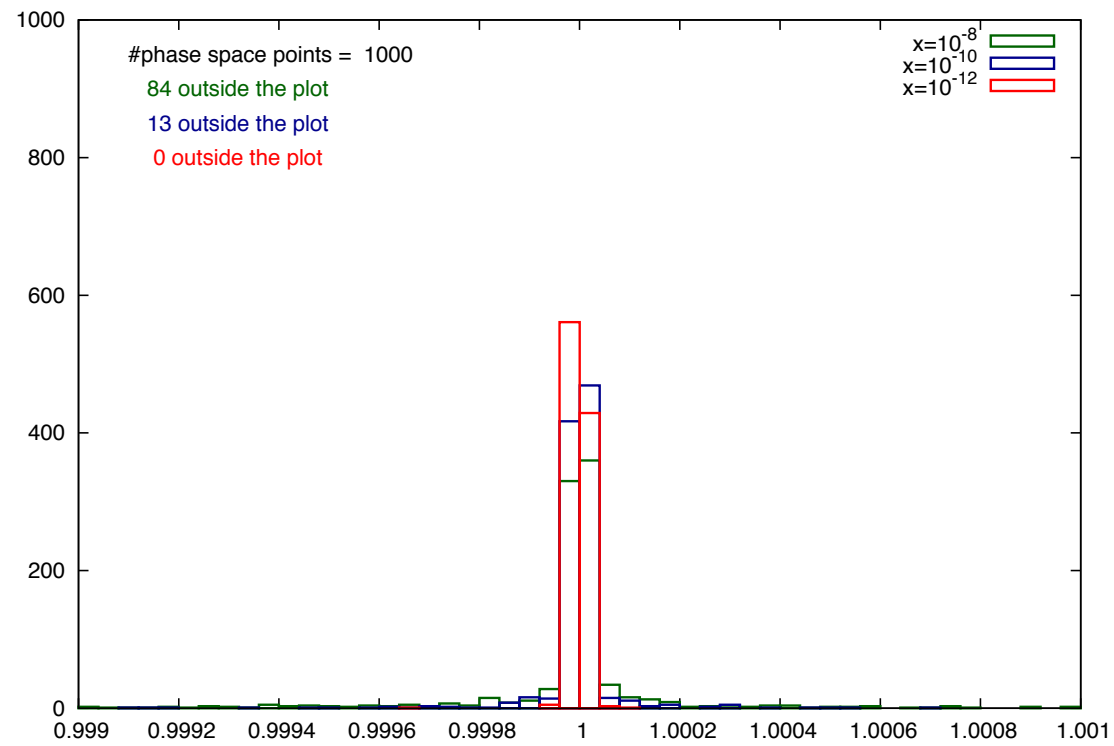
Single soft - 3



Single collinear - 3/4



Single collinear - 1/3



Double Virtual

Analytic pole cancellation against 2-loop and (1-loop)² matrix element

```
James@Jamess-MacBook-Pro-4:~/hepforge/maple/process/jet$ form autoA4g2XU.frm  
FORM 4.1 (Mar 13 2014) 64-bits Run: Wed May 4 18:13:43 2016
```

```
#-
```

```
poles = 0;
```

```
19.40 sec out of 20.04 sec
```

```
James@Jamess-MacBook-Pro-4:~/hepforge/maple/process/jet$ form autoA4g2YU.frm  
FORM 4.1 (Mar 13 2014) 64-bits Run: Wed May 4 18:14:10 2016
```

```
#-
```

```
poles = 0;
```

```
7.43 sec out of 7.50 sec
```

```
James@Jamess-MacBook-Pro-4:~/hepforge/maple/process/jet$ form autoAh4g2XU.frm  
FORM 4.1 (Mar 13 2014) 64-bits Run: Wed May 4 18:14:42 2016
```

```
#-
```

```
poles = 0;
```

```
8.83 sec out of 8.88 sec
```

```
James@Jamess-MacBook-Pro-4:~/hepforge/maple/process/jet$ form autoAh4g2YU.frm  
FORM 4.1 (Mar 13 2014) 64-bits Run: Wed May 4 18:14:55 2016
```

```
#-
```

```
poles = 0;
```

```
5.37 sec out of 5.41 sec
```

```
James@Jamess-MacBook-Pro-4:~/hepforge/maple/process/jet$ form autoqgB2g2XU.frm  
FORM 4.1 (Mar 13 2014) 64-bits Run: Wed May 4 18:15:25 2016
```

```
#-
```

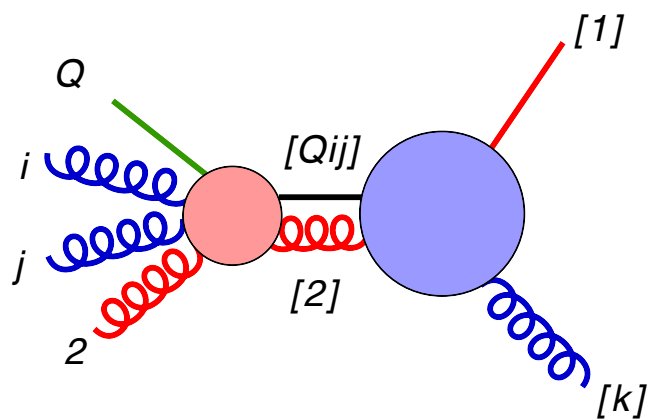
```
poles = 0;
```

Disentangling initial-final collinear limits

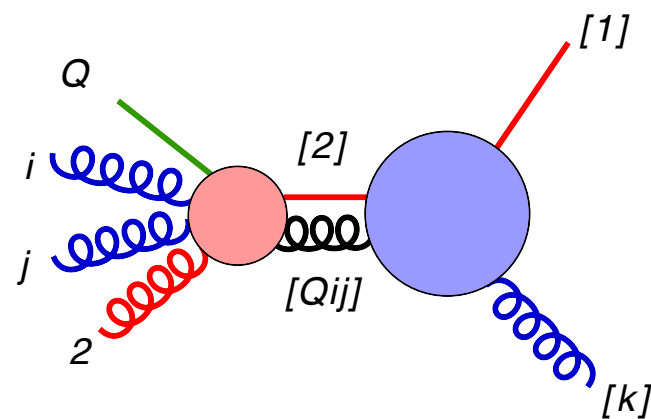
Additional complication with hadronic initial states

- phase space map smoothly interpolates between all limits
- smooth mapping between limits broken by reduced matrix element crossings

$$D_4^0(Q, i, j, 2) M_4^0(1, k, \bar{2}, (\widetilde{ijQ}))$$



$$D_4^0(Q, i, j, 2) M_4^0(1, k, (\widetilde{ijQ}), \bar{2})$$



Can always be disentangled with the appropriate combination of antennae

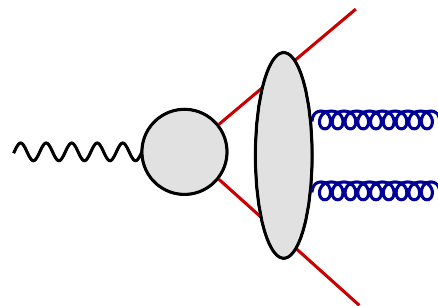
Better Antennae?

Computation time is limited mainly by the subtraction term

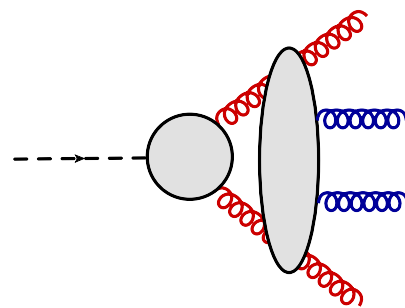
- many calls to libraries and PS maps due to many terms
- many (most) terms in the subtraction are to cancel spurious limits

Worth investigating if it is possible to define antennae with more focussed properties

- cf. quark-antiquark antenna (no spurious limits)



- with gluon-gluon antenna (unresolved “hard” radiators)



Colour Space

Antennae are colour stripped objects:

- contain all singularities
- analytic integration is the hard part

Overall singularity structure best organised in colour space:

- can dress colour stripped functions with charges
- algorithmic approach \rightarrow automation along the lines of Catani-Seymour

$$X_3^0(i, j, k) \langle \mathcal{M}_n^0 | \mathcal{M}_n^0 \rangle \rightarrow X_3^0(i, j, k) \langle \mathcal{M}_n^0 | \mathbf{T}_{(\tilde{i}j)} \cdot \mathbf{T}_{(\tilde{j}k)} | \mathcal{M}_n^0 \rangle$$

Recent Applications

- $pp \rightarrow 2J$ (gluons only) [Gehrmann, Gehrmann-De Ridder, Glover, Pires '12]
- $pp \rightarrow H+J$ [Chen, Gehrmann, Glover, Jaquier '14, '16]
- $pp \rightarrow Z+J$ [Gehrmann-De Ridder, Gehrmann, Glover, Huss, Morgan '15; '16]
- $ep \rightarrow (2+1)J$ [Currie, Gehrmann, Niehus 1605.XXXX]
- $pp \rightarrow 2J$ (all channels) [Currie, Glover, Pires 160Z.XXXX]

H+J

Phenomenologically interesting:

- dominated by scale uncertainty
- necessary for accurate differential properties of Higgs

3 independent calculations exist:

$$\sigma_{antenna}^{NNLO} = 9.44^{+0.59}_{-0.85} \text{ fb}$$

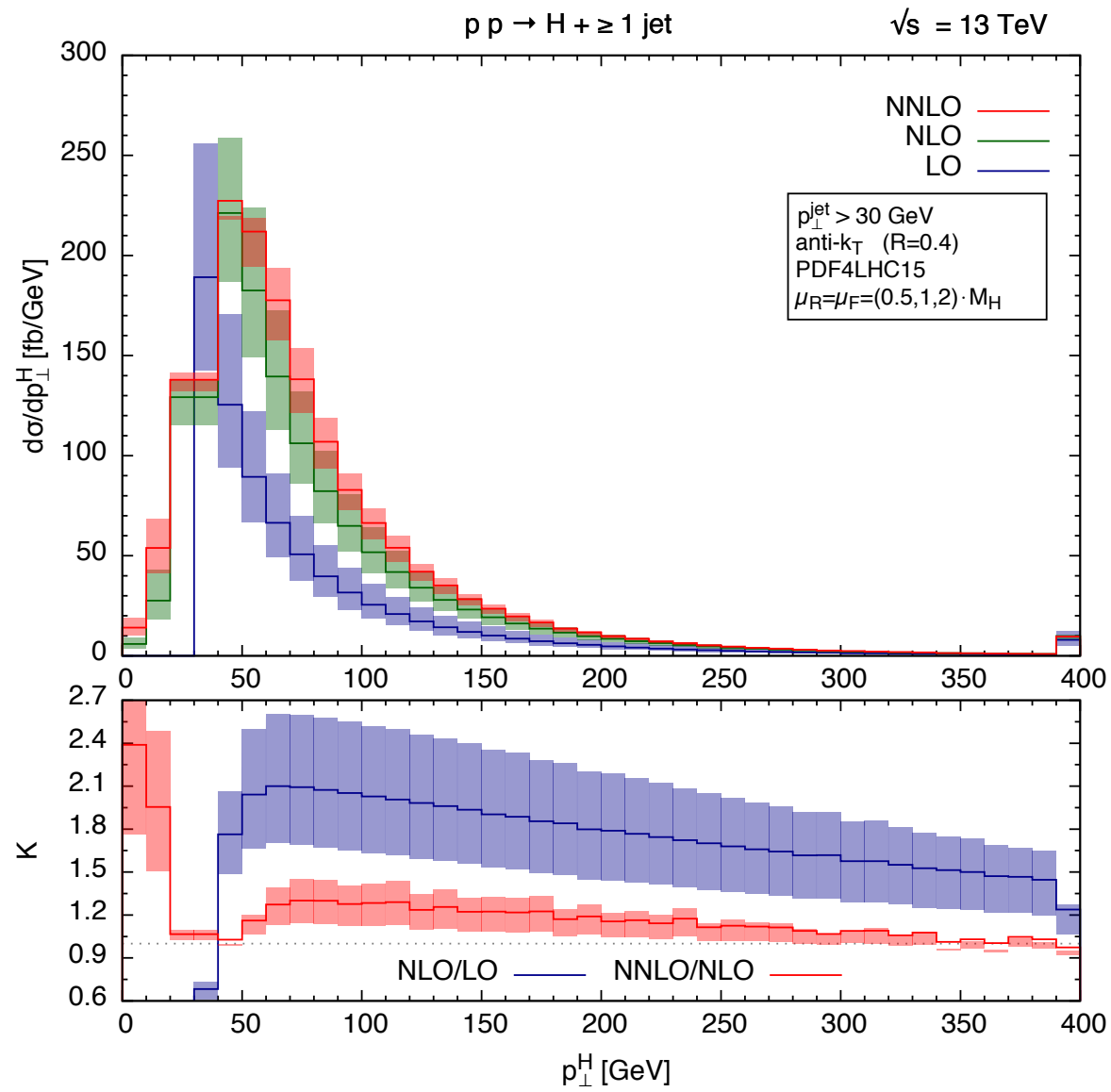
- Antennae [Chen, Gehrmann, Glover, Jaquier '14, '16] $\sigma_{sectors}^{NNLO} = 9.45^{+0.58}_{-0.82} \text{ fb}$

- Sectors [Boughezal, Caola, Melnikov, Petriello, Schulze '13; Caola, Melnikov, Shultz '15]

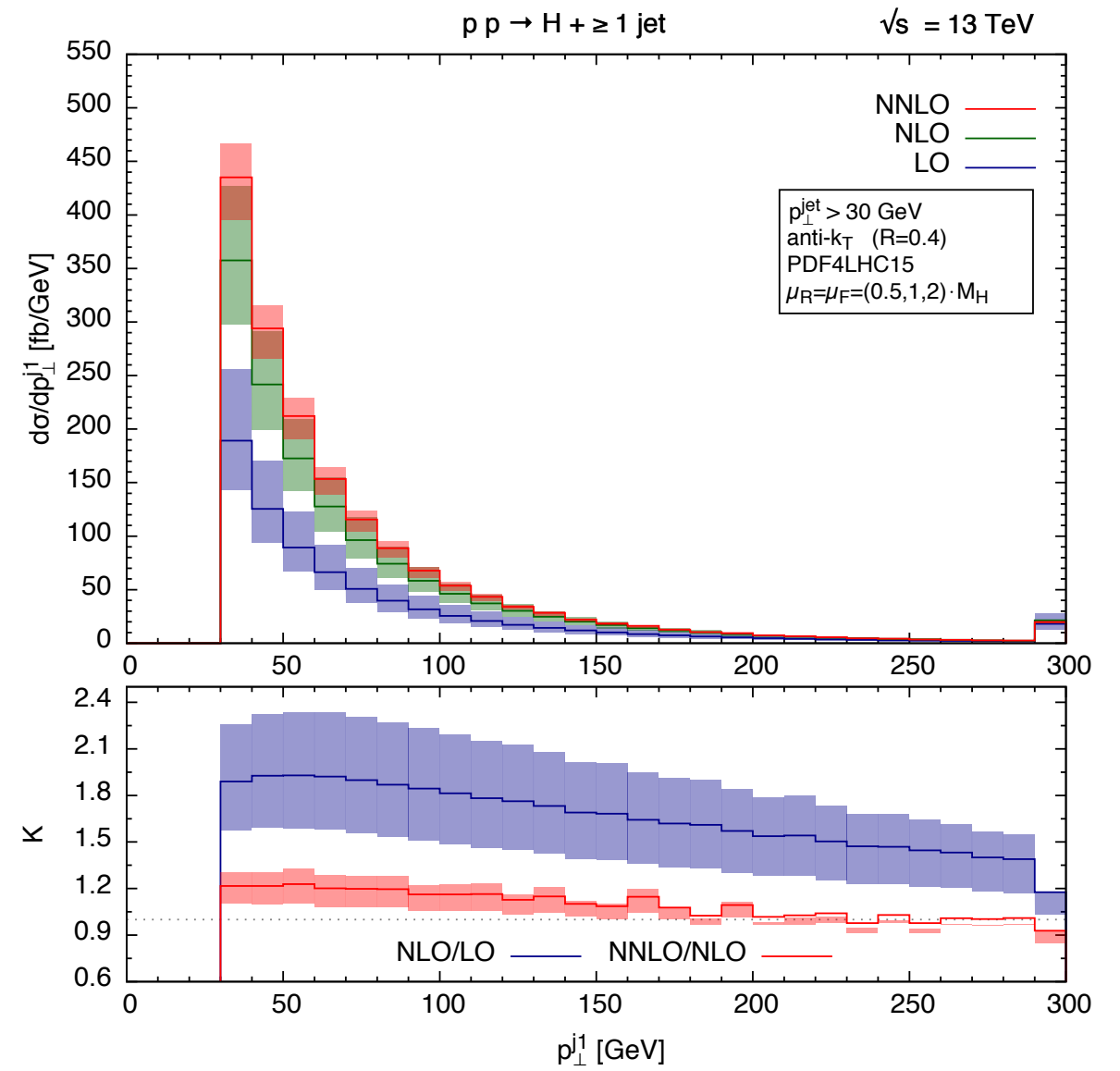
- N-Jettiness [Boughezal, Focke, Giele, Liu, Petriello '15]

Provides testbed for comparison between main methods for the first time!

p_T

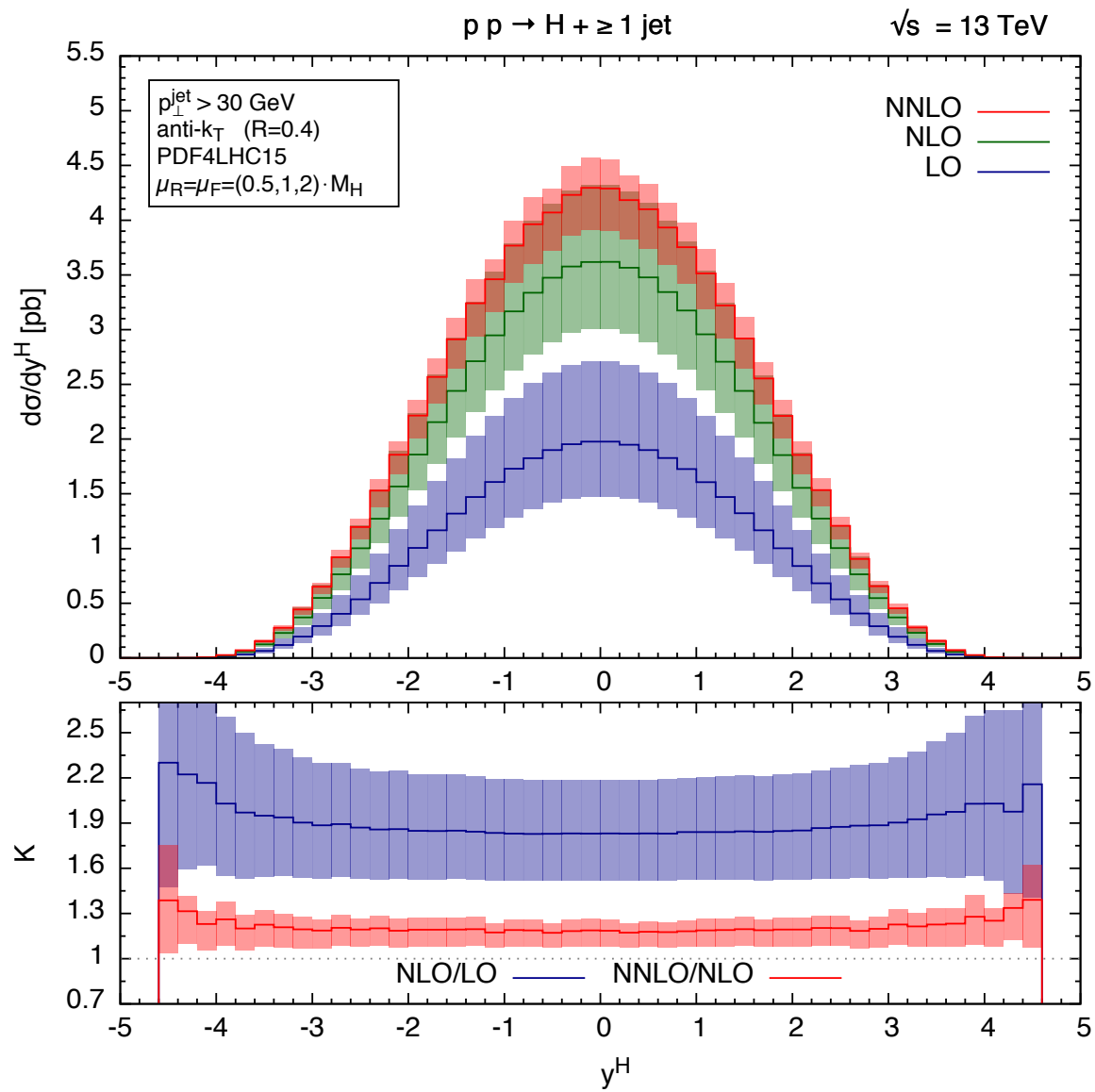


Higgs

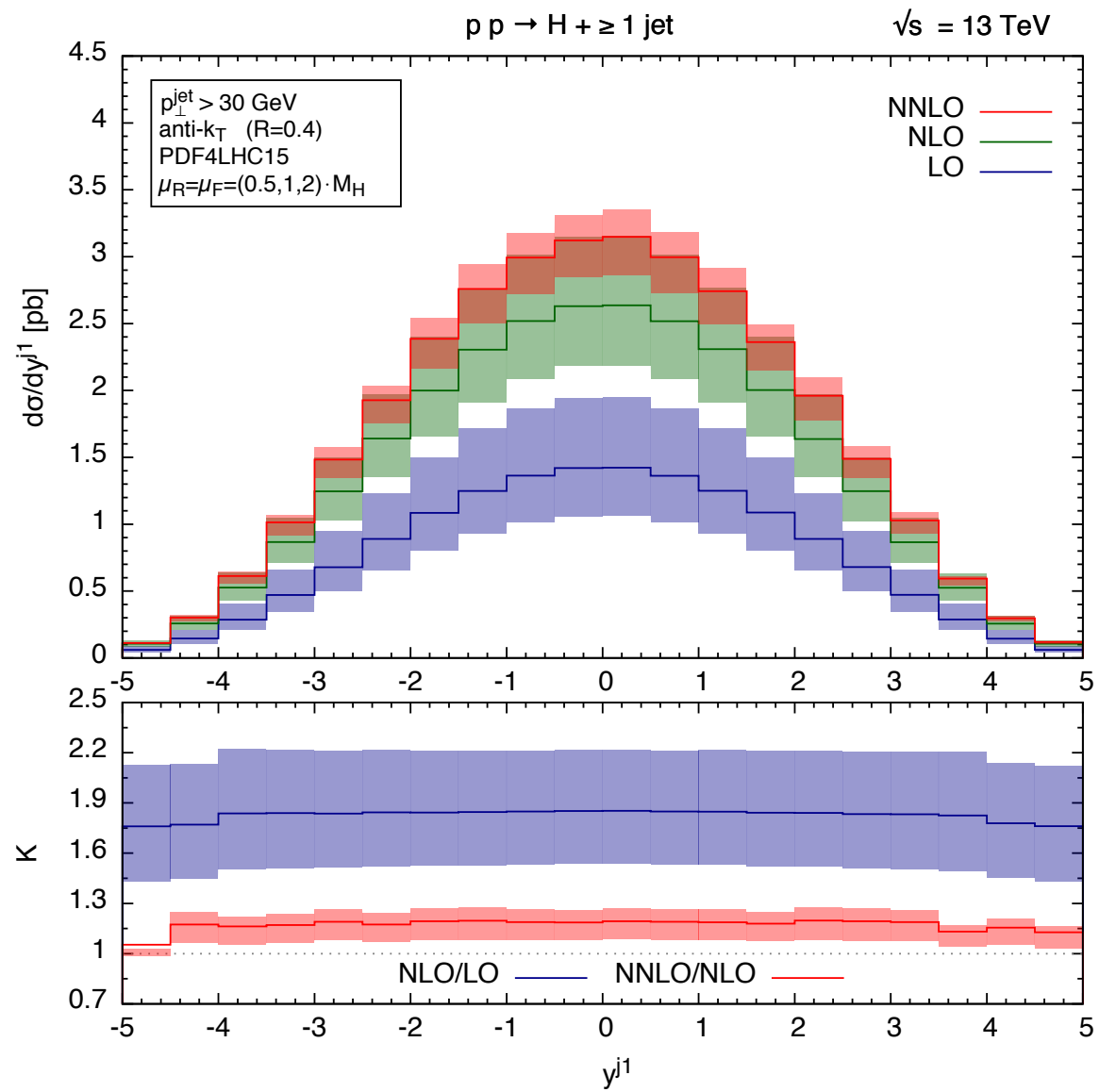


Jet

Rapidity

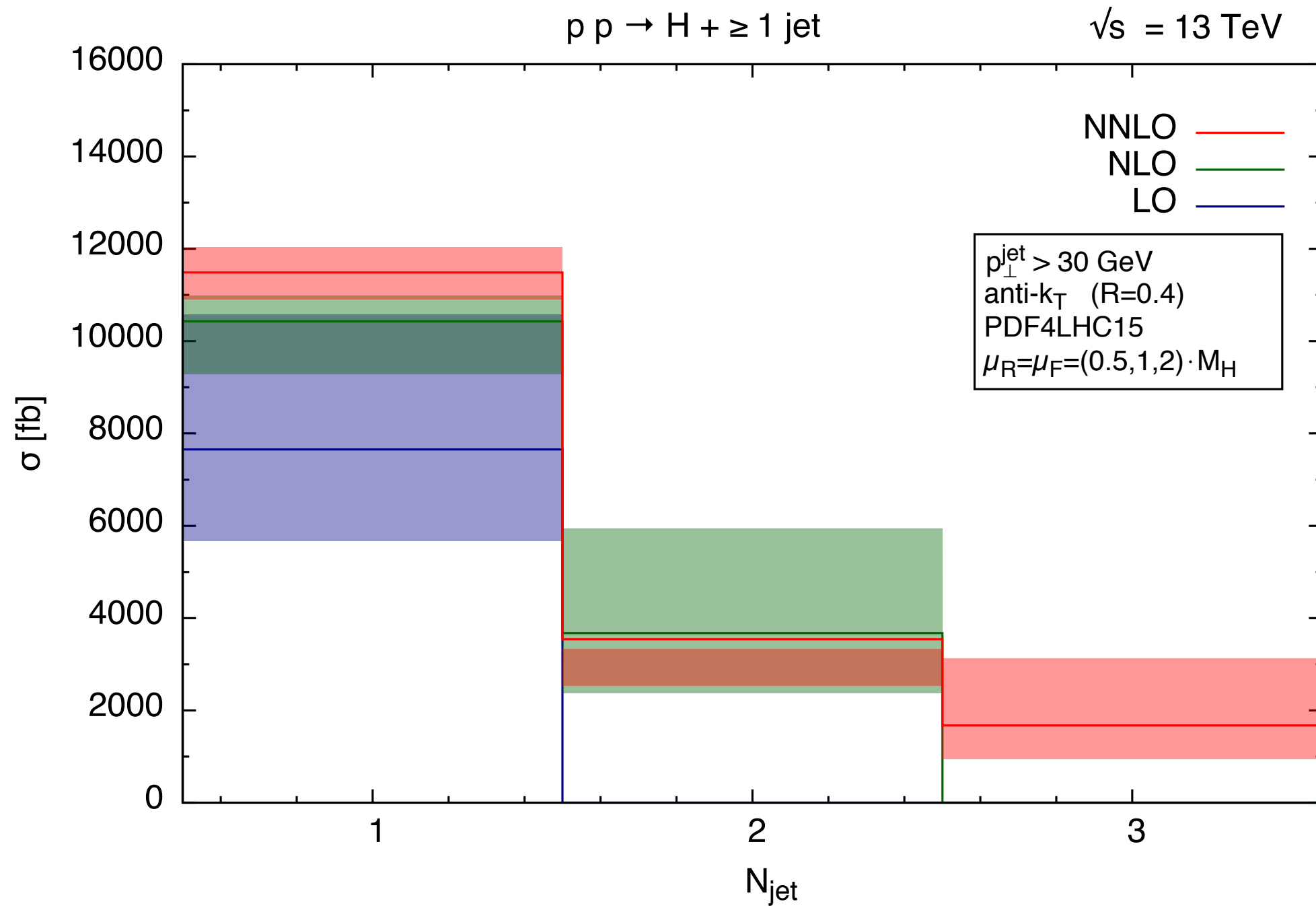


Higgs



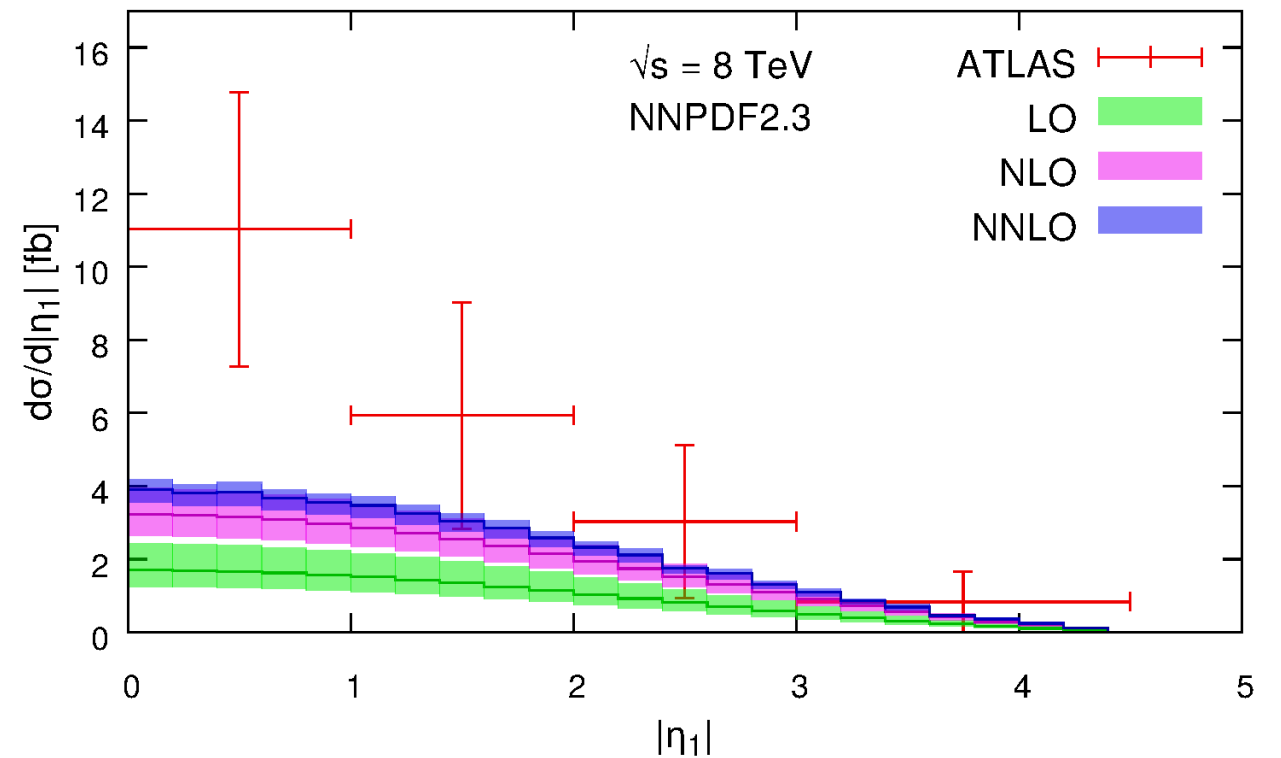
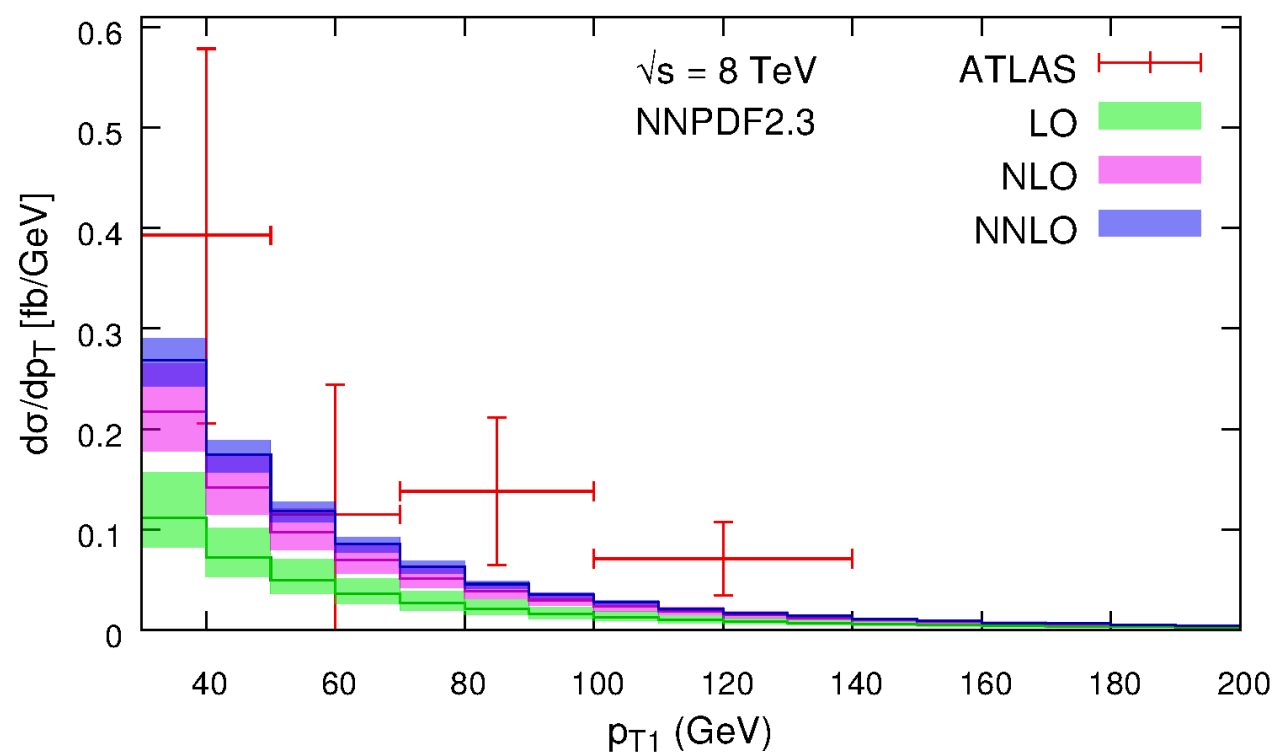
Jet

Exclusive



Data

- H+J NNLO prediction still undershoots data (Atlas)
- errors are reasonably large
- finite mass effects probably small $\sim 2\text{-}3\%$ @NLO [Harlander, Neumann, Ozeren, Wiesemann '12]



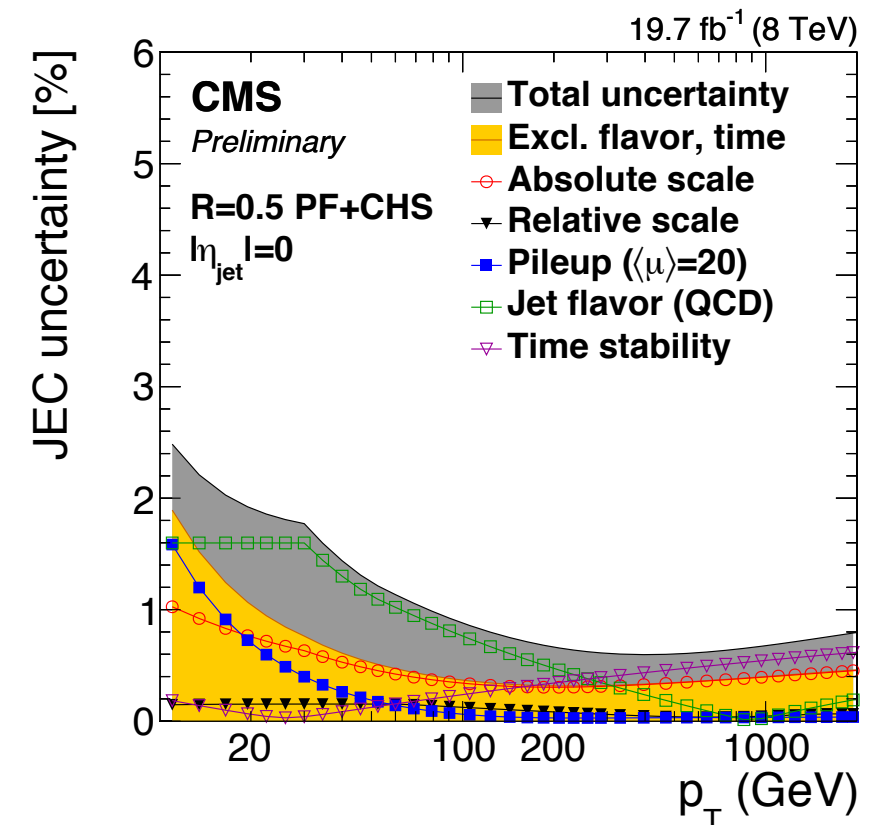
Z+J

Phenomenologically interesting signal:

- clean leptonic signature
- good handle on Jet Energy Scale
- inclusive Z p_T large discrepancy with data

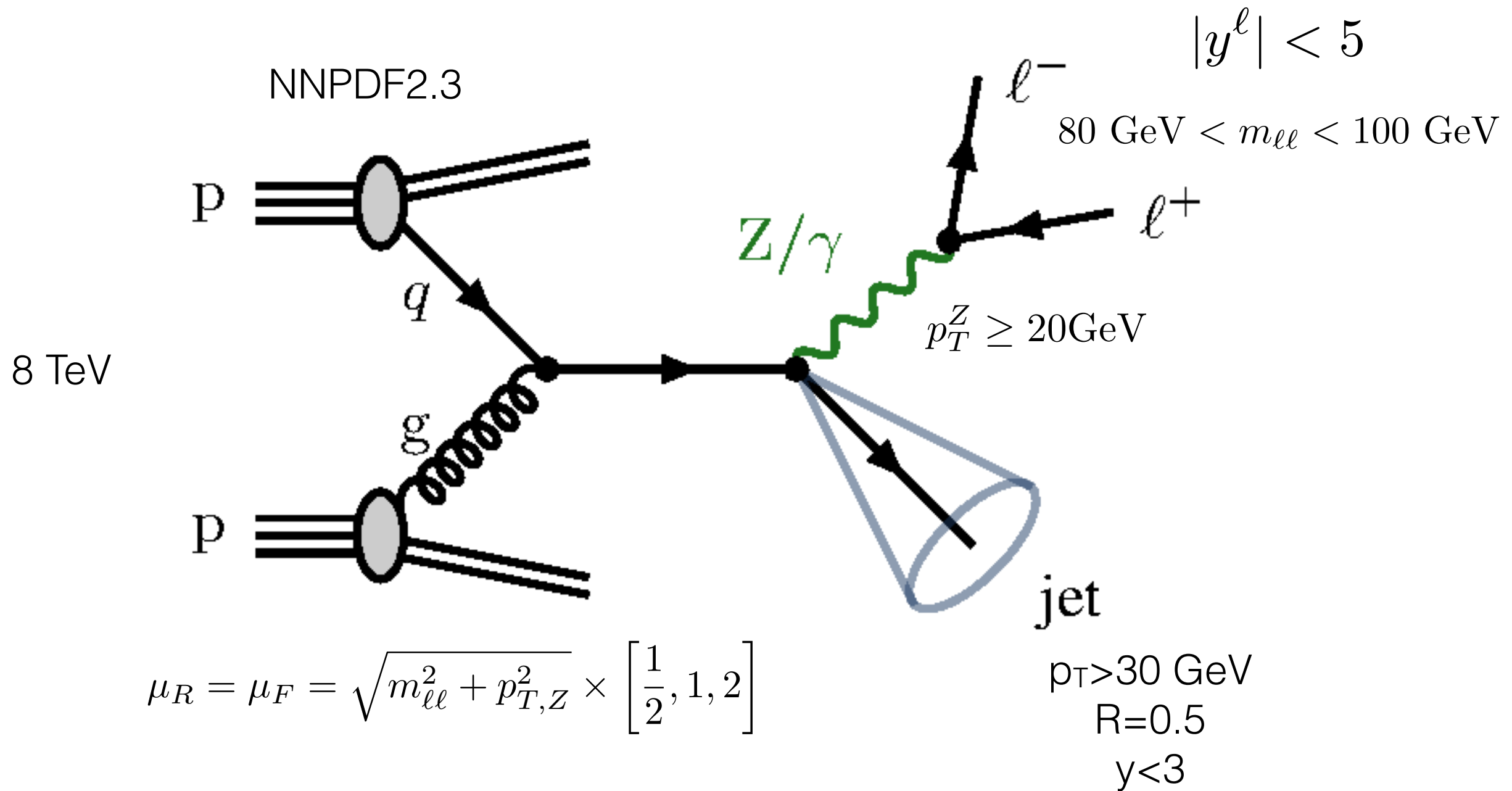
Theoretically interesting calculation:

- significant NLO K-factors and scale uncertainty
- completely independent implementation using N-Jettiness [Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello.'15; Boughezal, Liu, Petriello.'16]
- another opportunity to compare methods and give reliable results for comparison with data



CMS-DP-2015/044

Setup- $Z+\geq 1J$

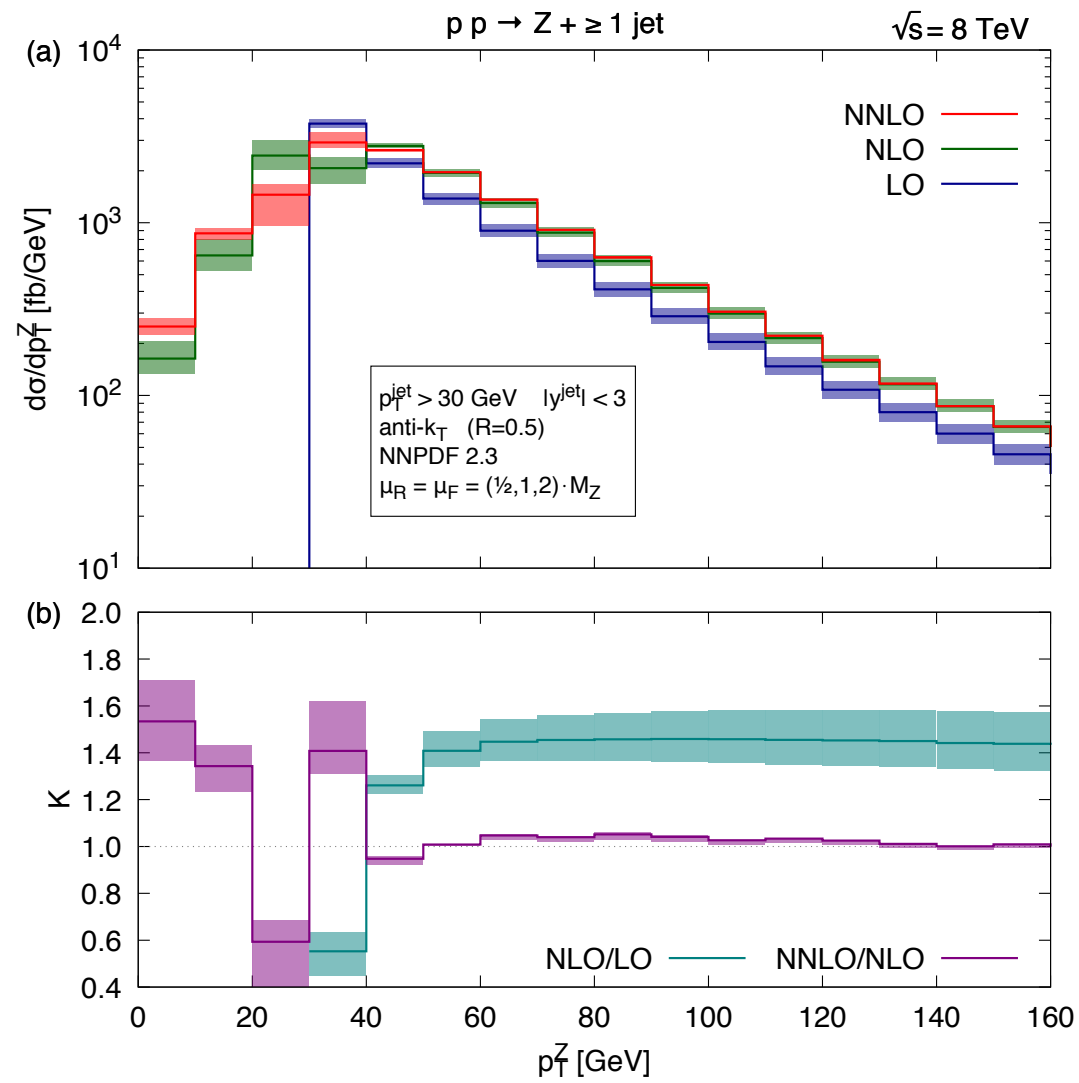


Total cross section

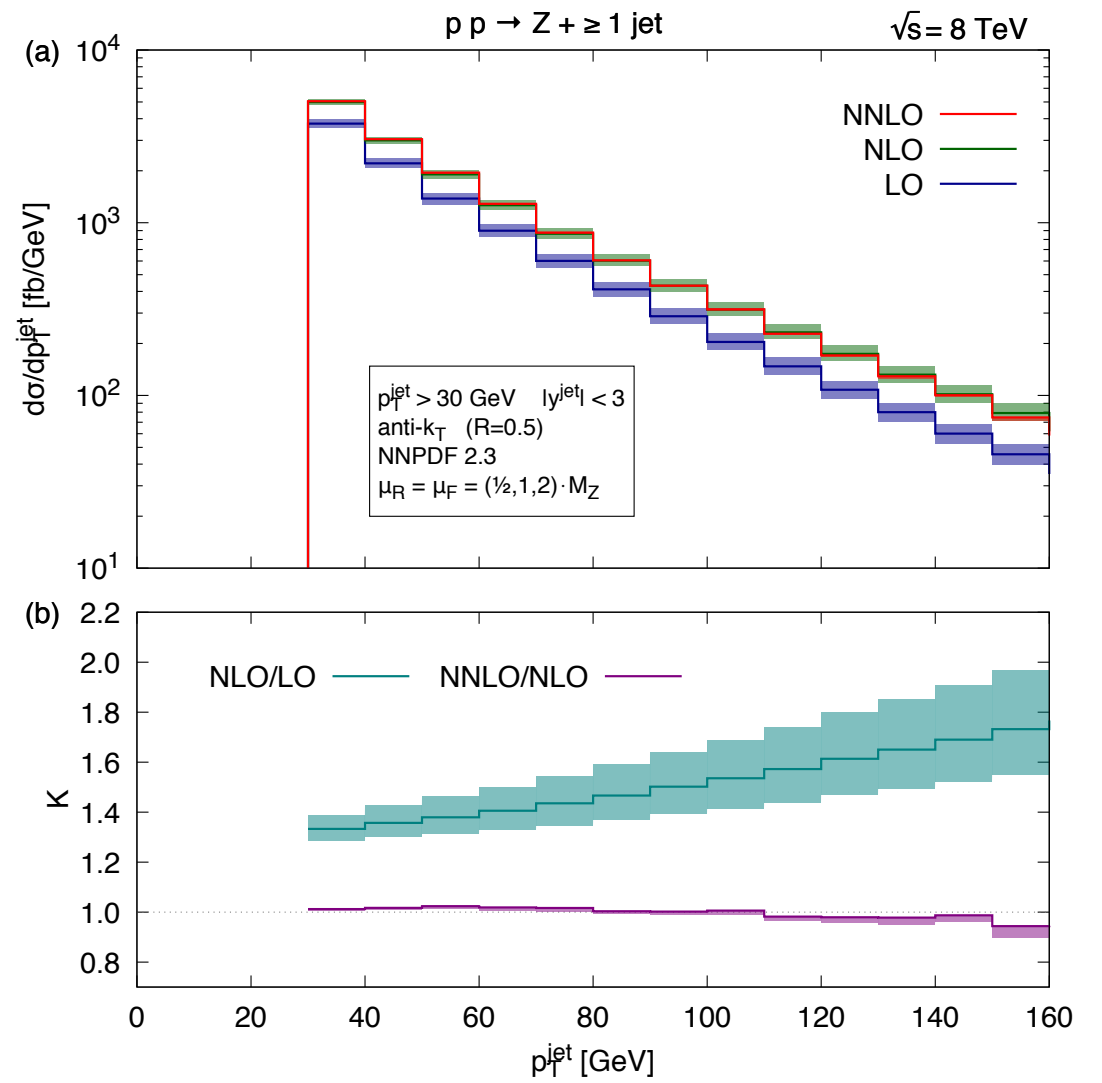
$$\begin{aligned}\sigma_{LO} &= 103.6_{-7.5}^{+7.7} \text{ pb} \\ \sigma_{NLO} &= 144.4_{-7.2}^{+9.0} \text{ pb} && +40\% \\ \sigma_{NNLO} &= 145.8_{-1.2}^{+0.0} \text{ pb} && +1\%\end{aligned}$$

- All channels included from NLO onwards
- NNLO stabilizes perturbative series
- scale uncertainty significantly reduced

p_T

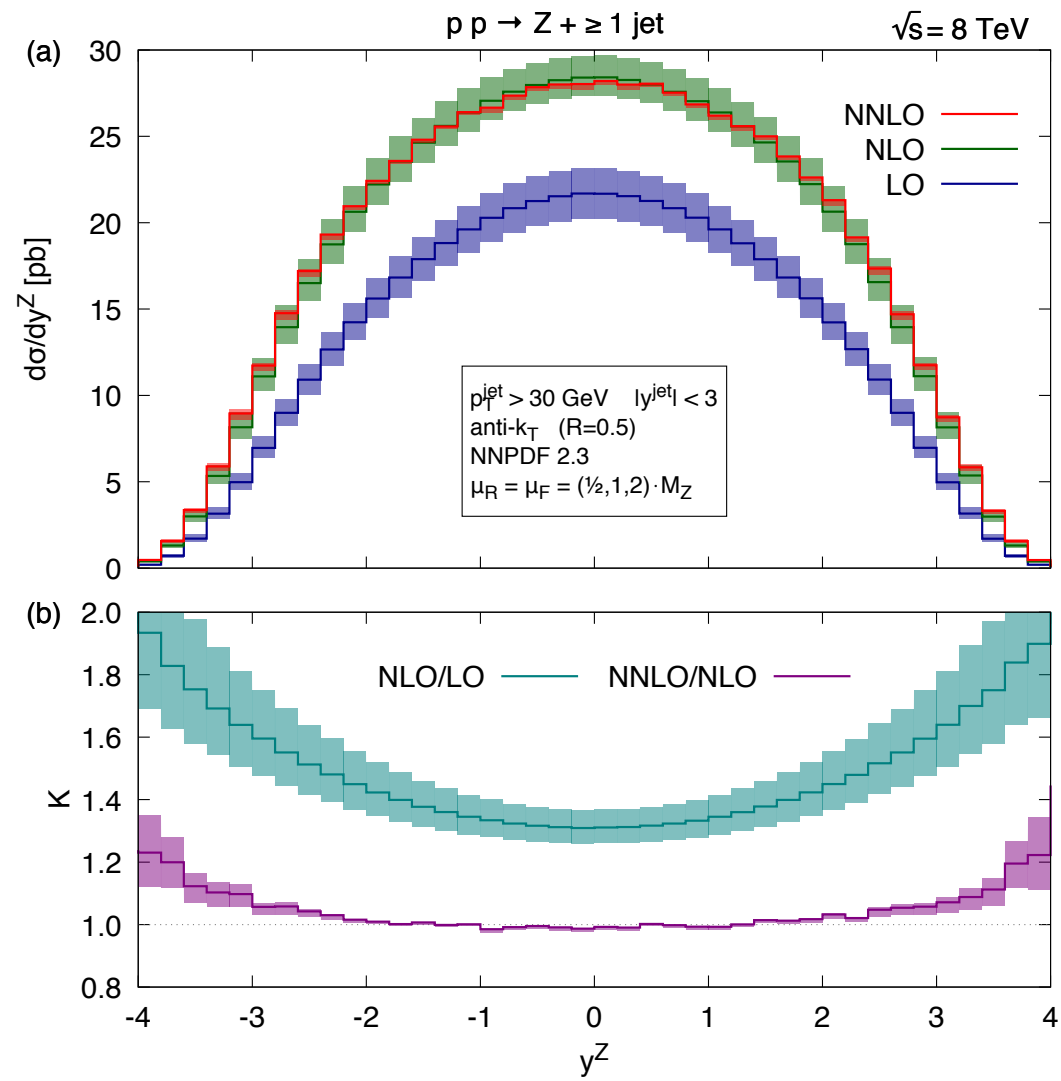


Z

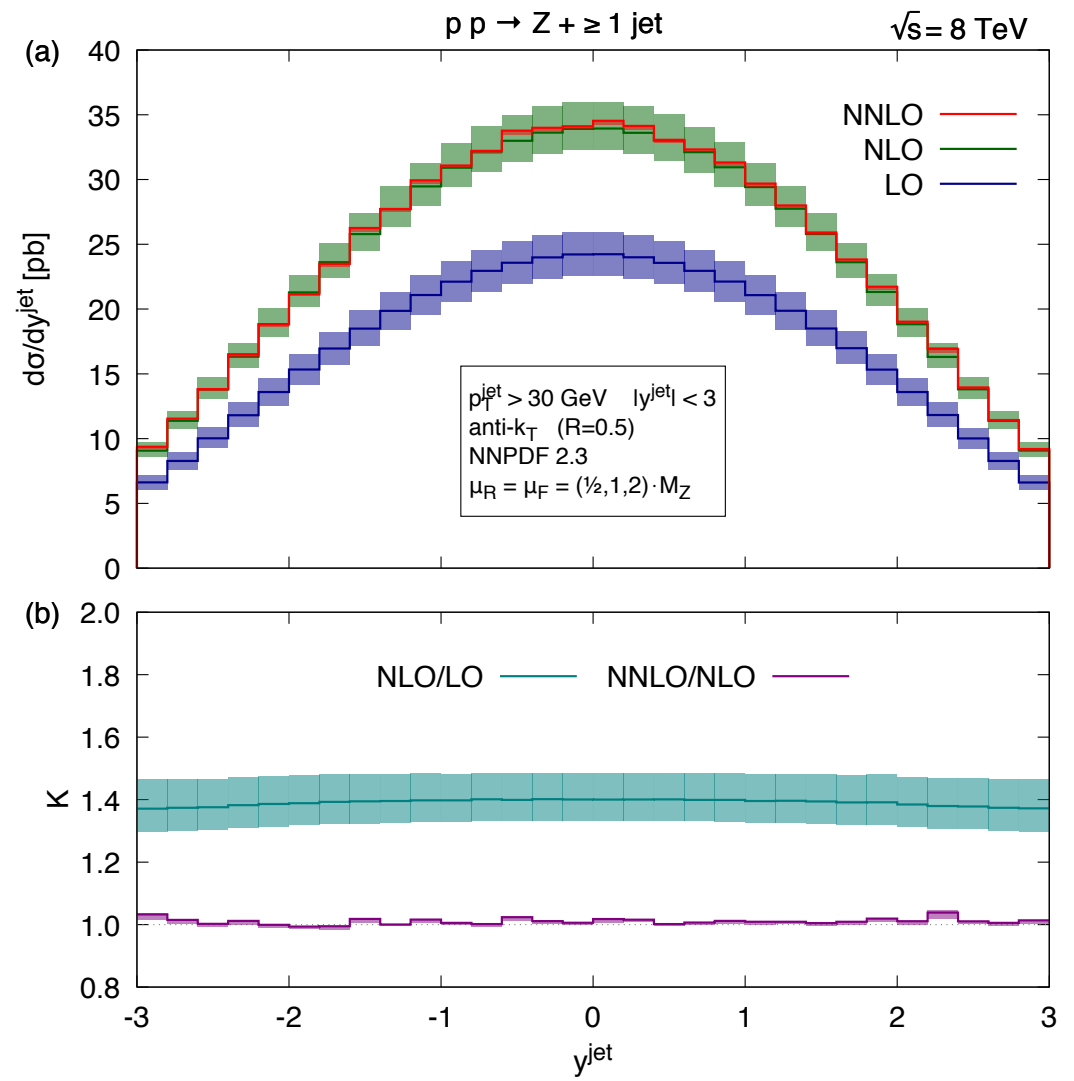


Jet

Rapidity



Z

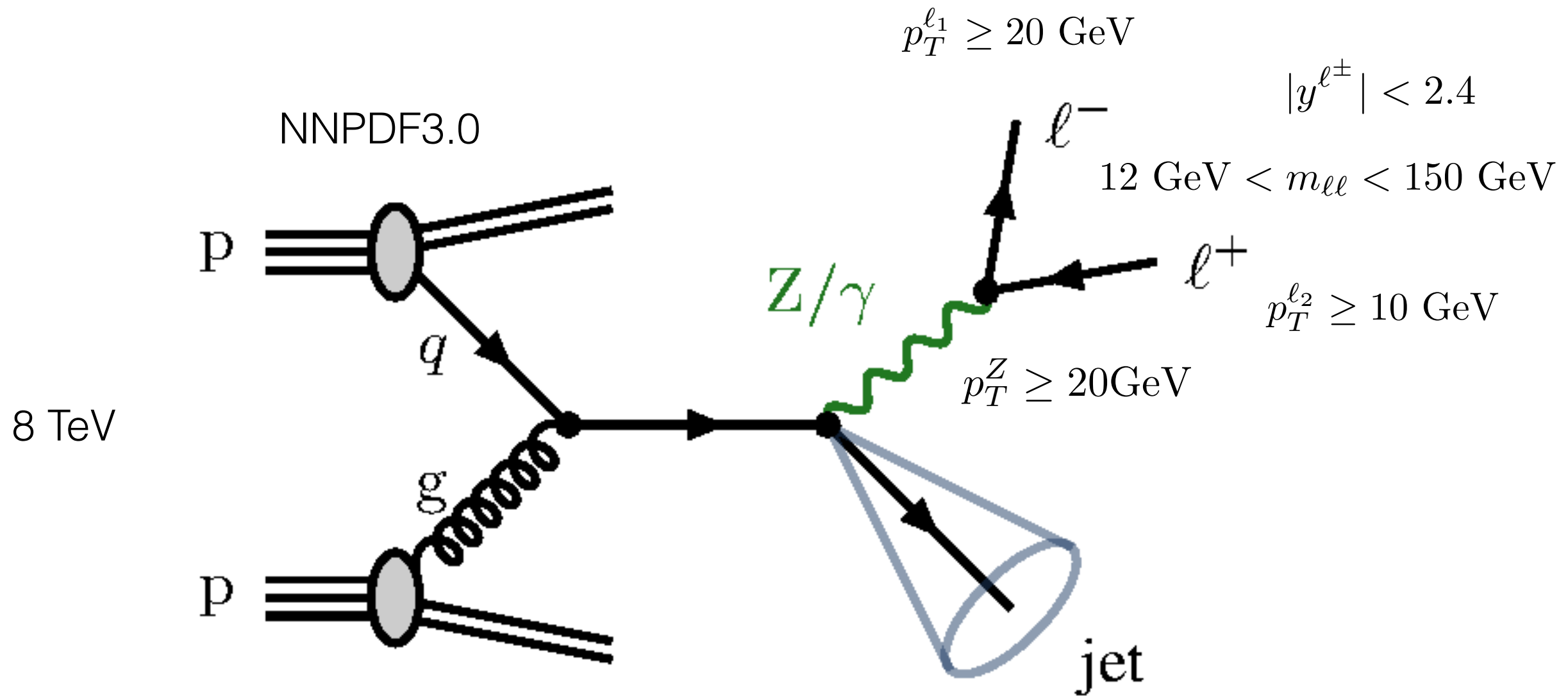


Jet

NNLO dramatically improves Z+J prediction:

- total cross section central value stabilized
- NNLO/NLO K-factors mostly flat
- scale uncertainty significantly reduced
- NNLO/NLO K-factors $\approx 5\%$ for distributions

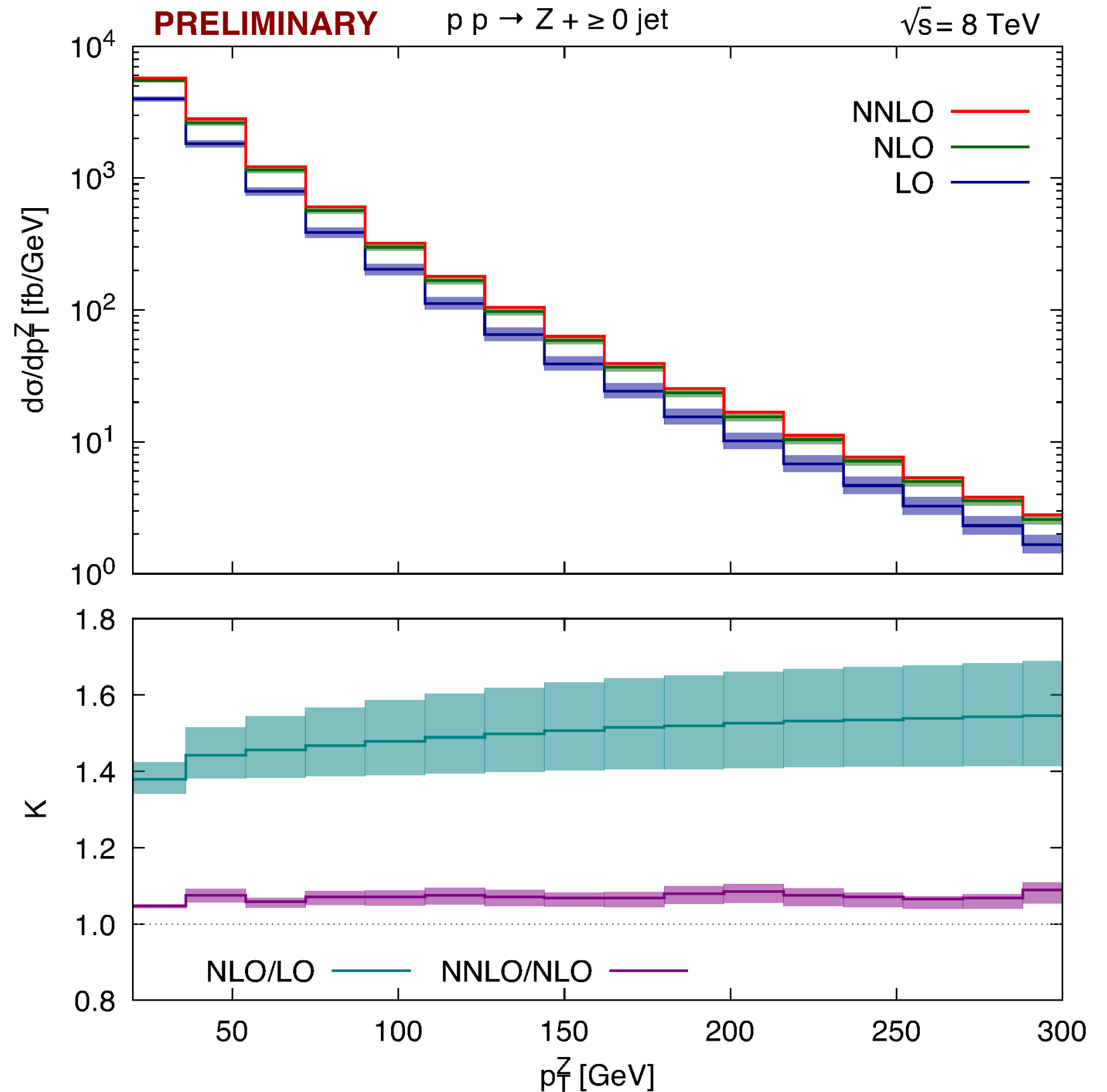
Setup-inclusive Z



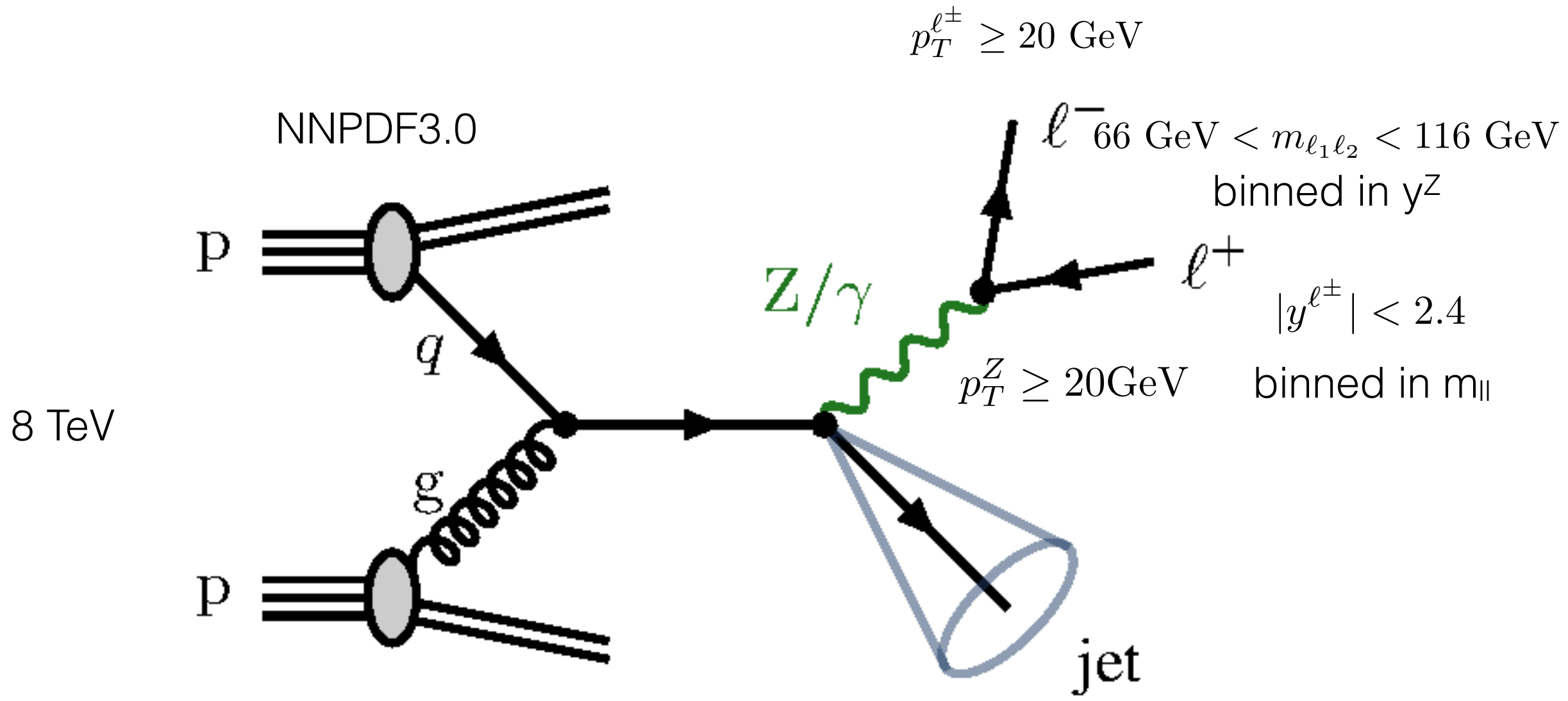
$$\mu_R = \mu_F = \sqrt{m_{\ell\ell}^2 + p_{T,Z}^2} \times \left[\frac{1}{2}, 1, 2 \right]$$

Fully inclusive in
QCD radiation

- large NLO corrections
40-60%
- moderate NNLO corrections
5-10%
- significantly improved scale uncertainty
- essentially flat K-factor



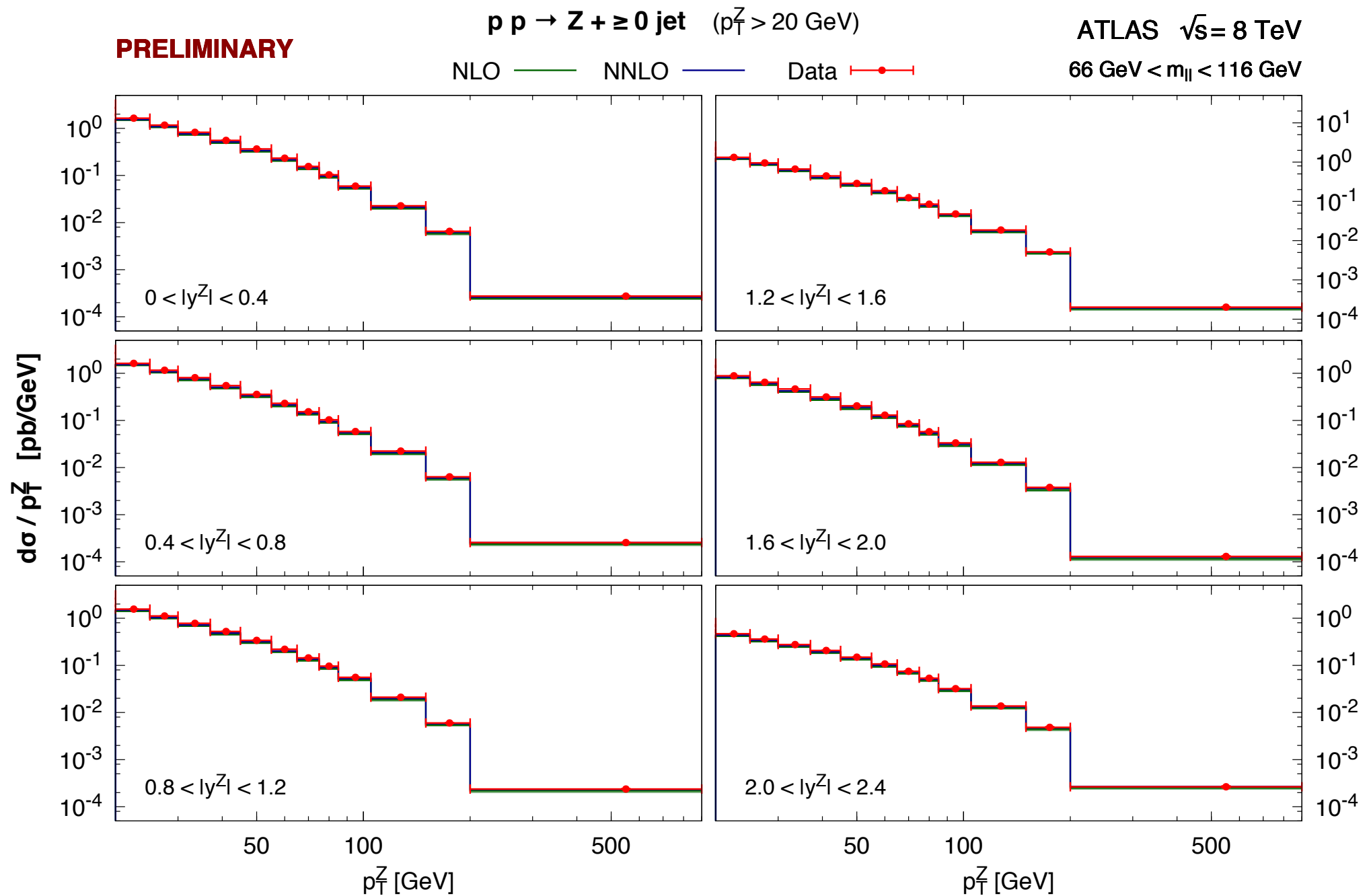
Setup-inclusive Z-ATLAS cuts



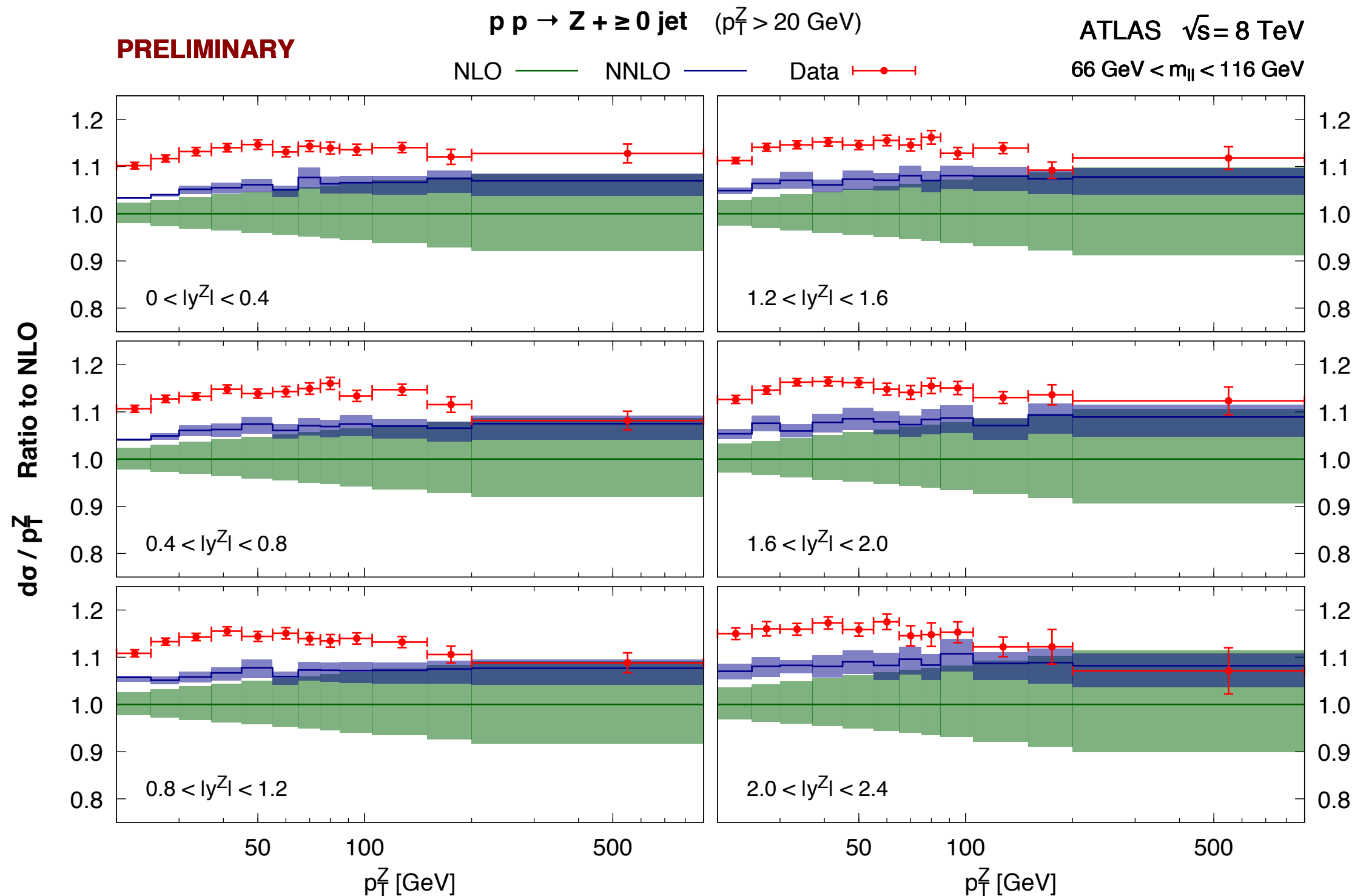
$$\mu_R = \mu_F = \sqrt{m_{\ell\ell}^2 + p_{T,Z}^2} \times \left[\frac{1}{2}, 1, 2 \right]$$

Fully inclusive in
QCD radiation

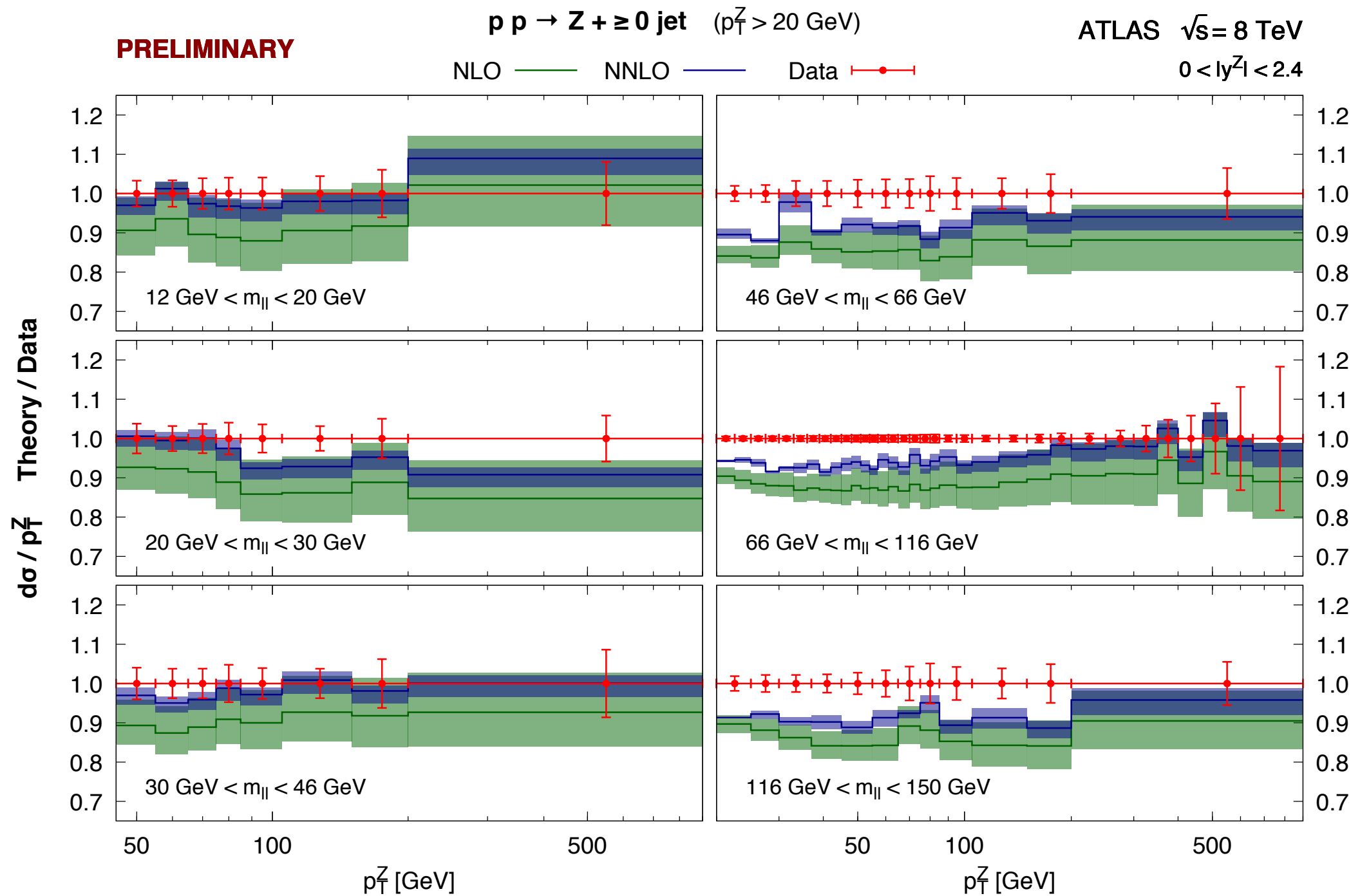
$d\sigma/dp_T^Z$ binned in y^Z



Ratio to NLO

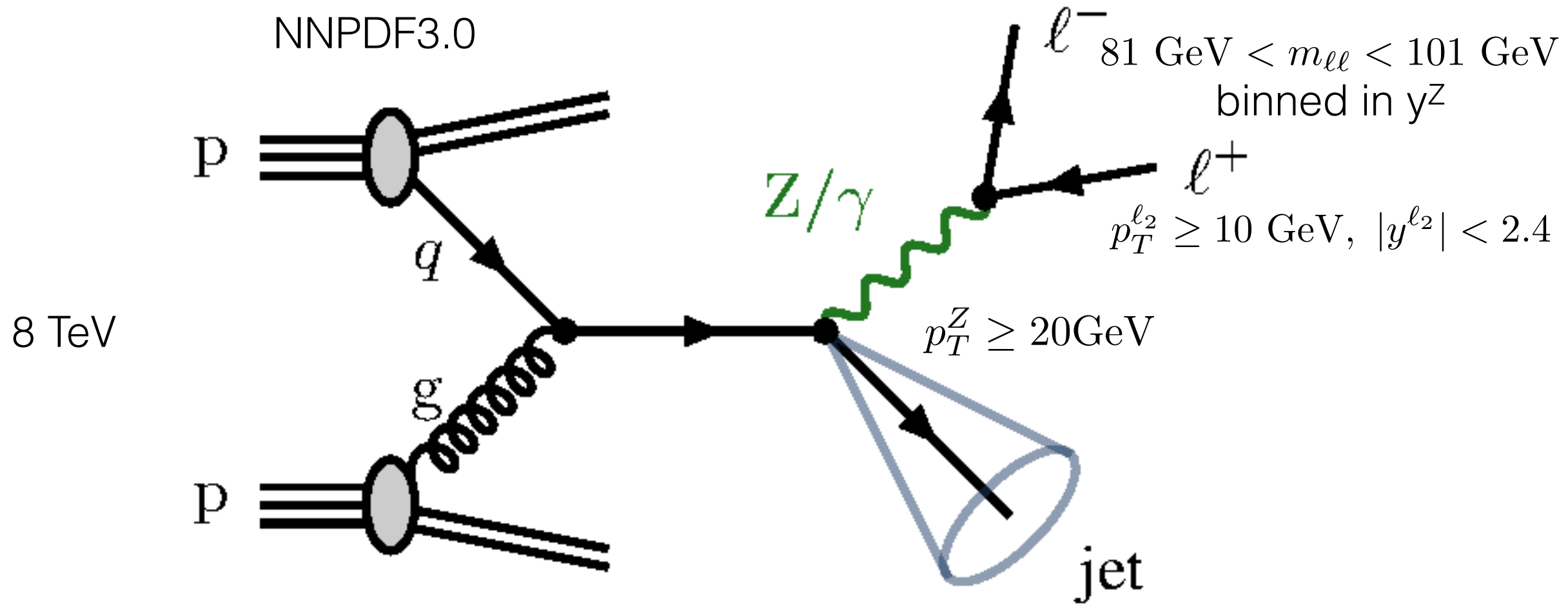


$d\sigma/dp_T^Z$ binned in m_{ll}



Setup-inclusive Z-CMS cuts

$$p_T^{\ell_1} \geq 25 \text{ GeV}, |y^{\ell_1}| < 2.1$$



$$81 \text{ GeV} < m_{\ell\ell} < 101 \text{ GeV}$$

binned in y^Z

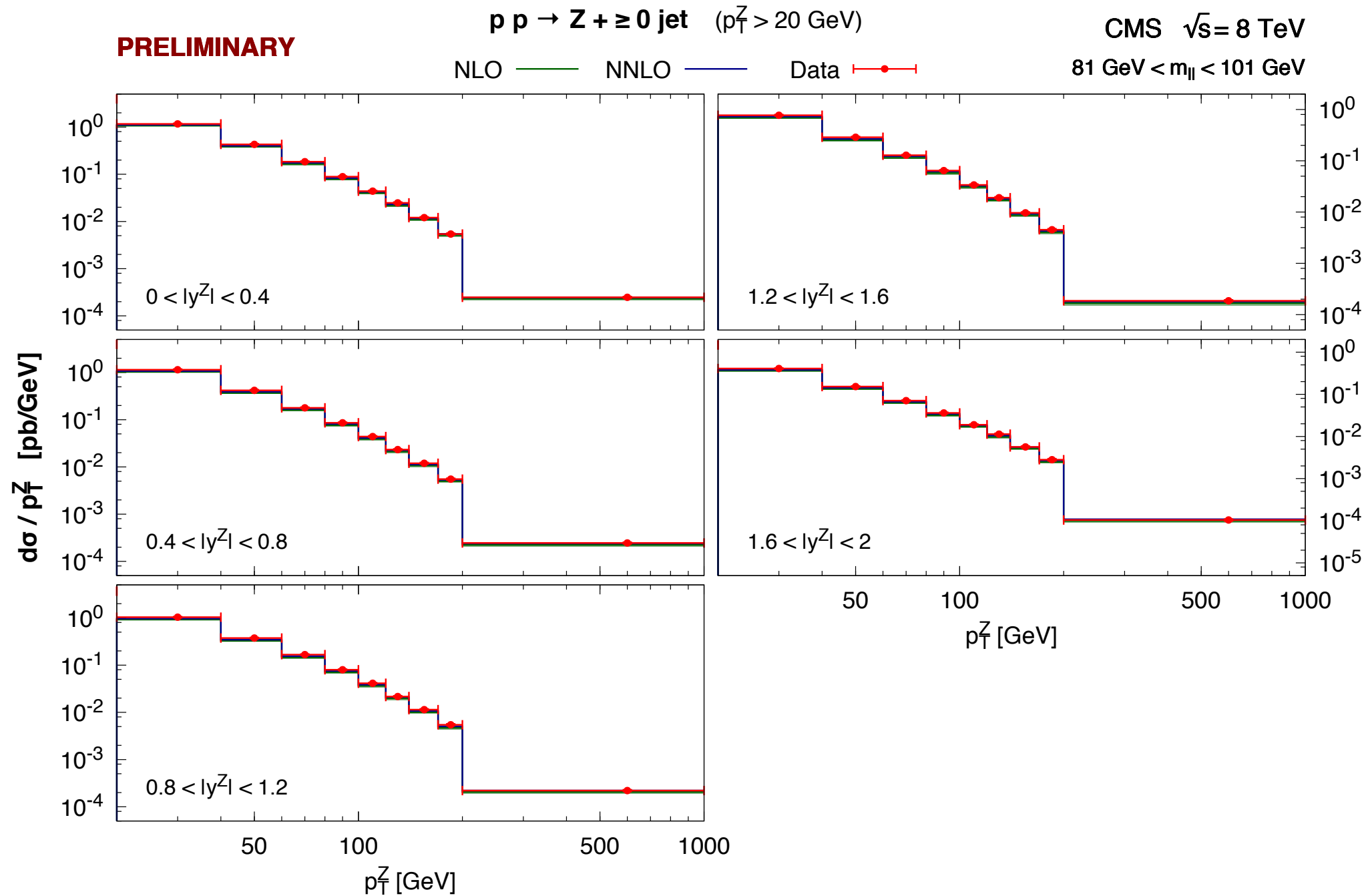
$$p_T^{\ell_2} \geq 10 \text{ GeV}, |y^{\ell_2}| < 2.4$$

$$p_T^Z \geq 20 \text{ GeV}$$

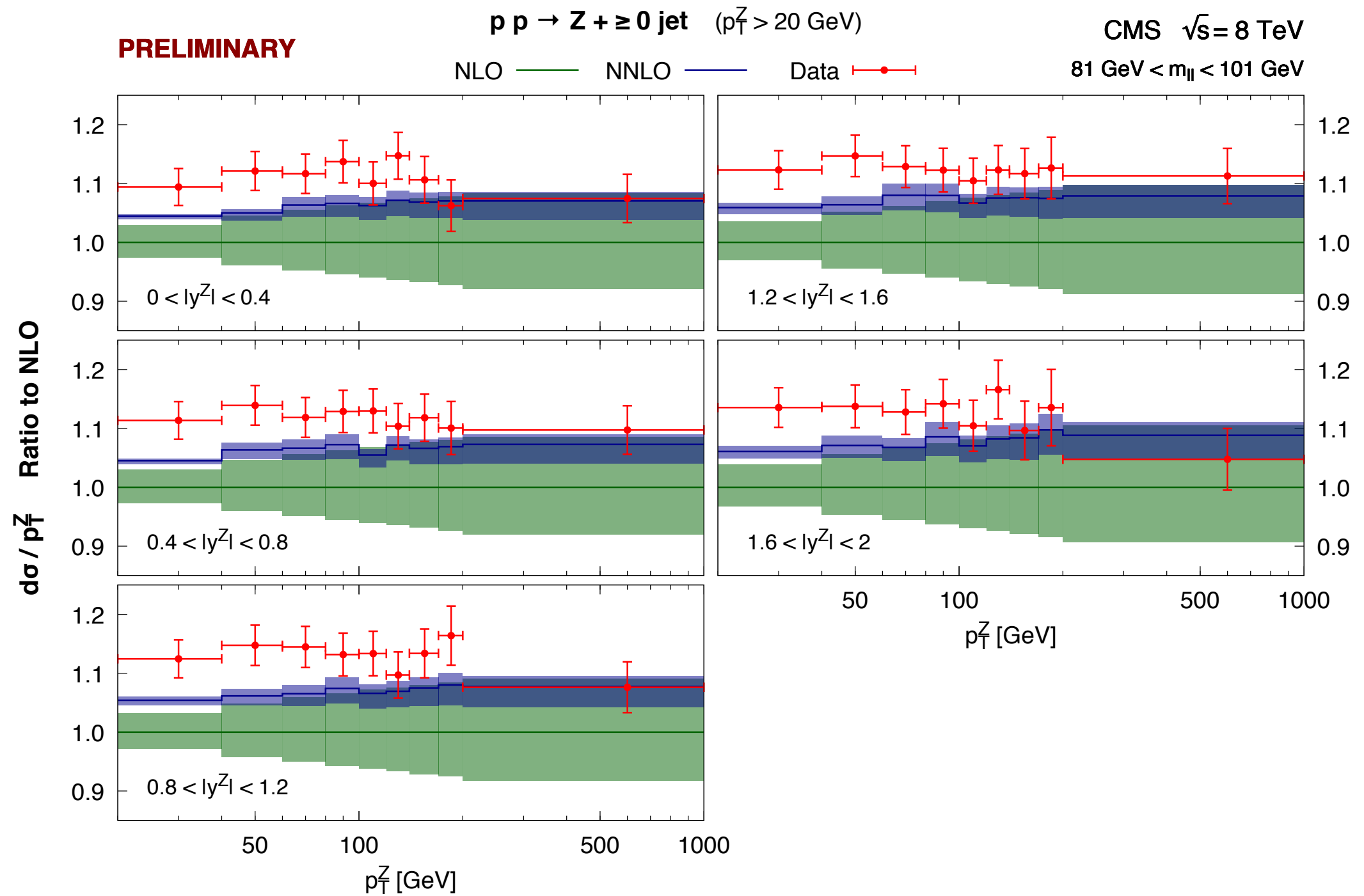
$$\mu_R = \mu_F = \sqrt{m_{\ell\ell}^2 + p_{T,Z}^2} \times \left[\frac{1}{2}, 1, 2 \right]$$

Fully inclusive in QCD radiation

$d\sigma/dp_T^Z$ binned in y^Z



Ratio to NLO

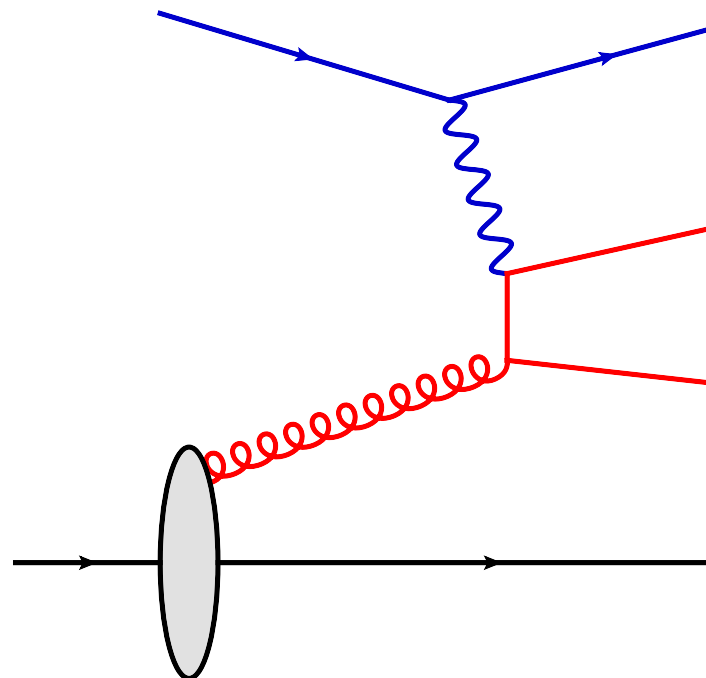


- moderate NNLO corrections (in the right direction!)
- reduces tension with data
- scale variation significantly reduced
- clarifies the theoretical challenge for resolving with data

DIS $(2+1)J$

DIS is not just a way of approximating VBF Higgs!

- wealth of data used extensively for quark PDFs
- $(2+1)J$ directly sensitive to gluon
- would like to include NNLO consistently into PDF fits



Setup

Cuts and parameters as set out in [1406.4709]:

- HERA lab frame $-1 < \eta_j < 2.5$
- Breit frame $5 \text{ GeV} < p_T < 50 \text{ GeV}$

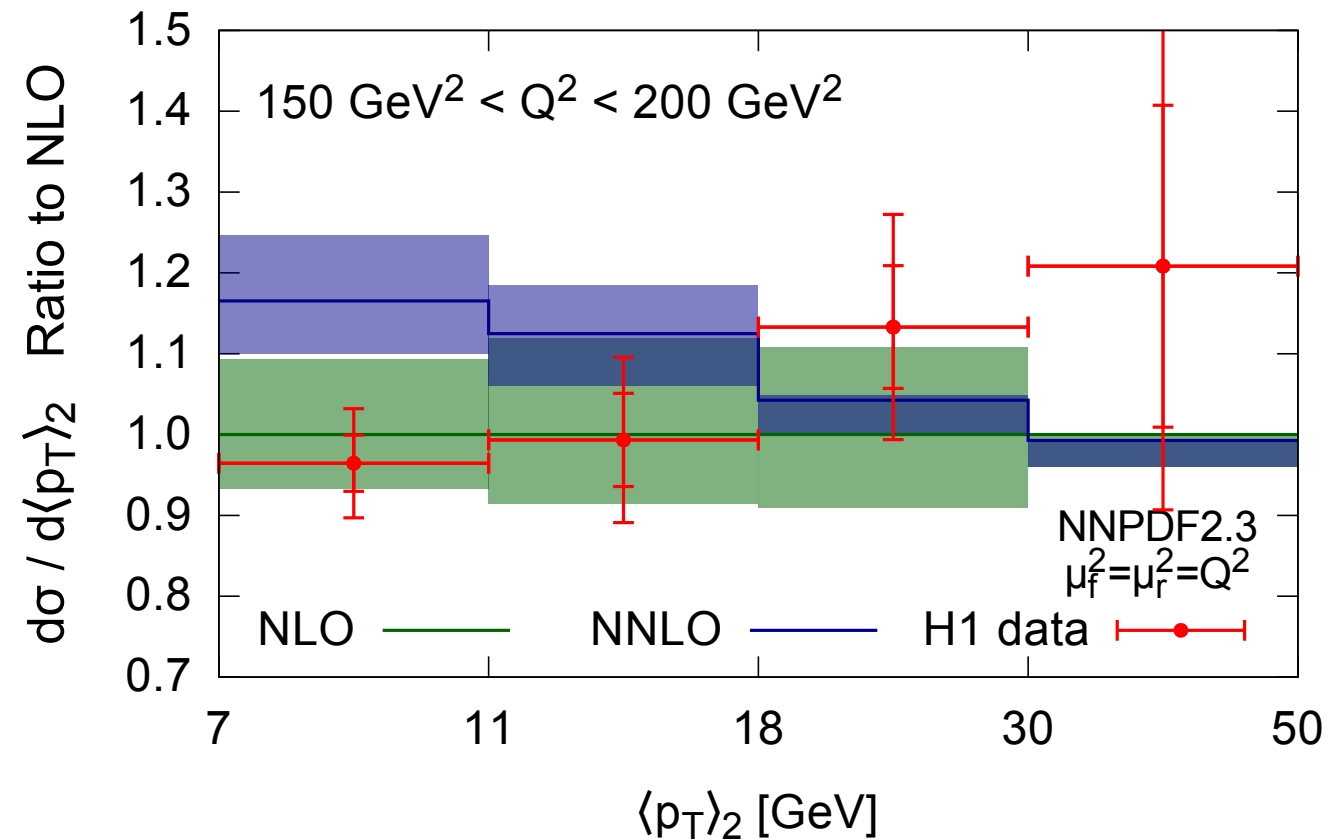
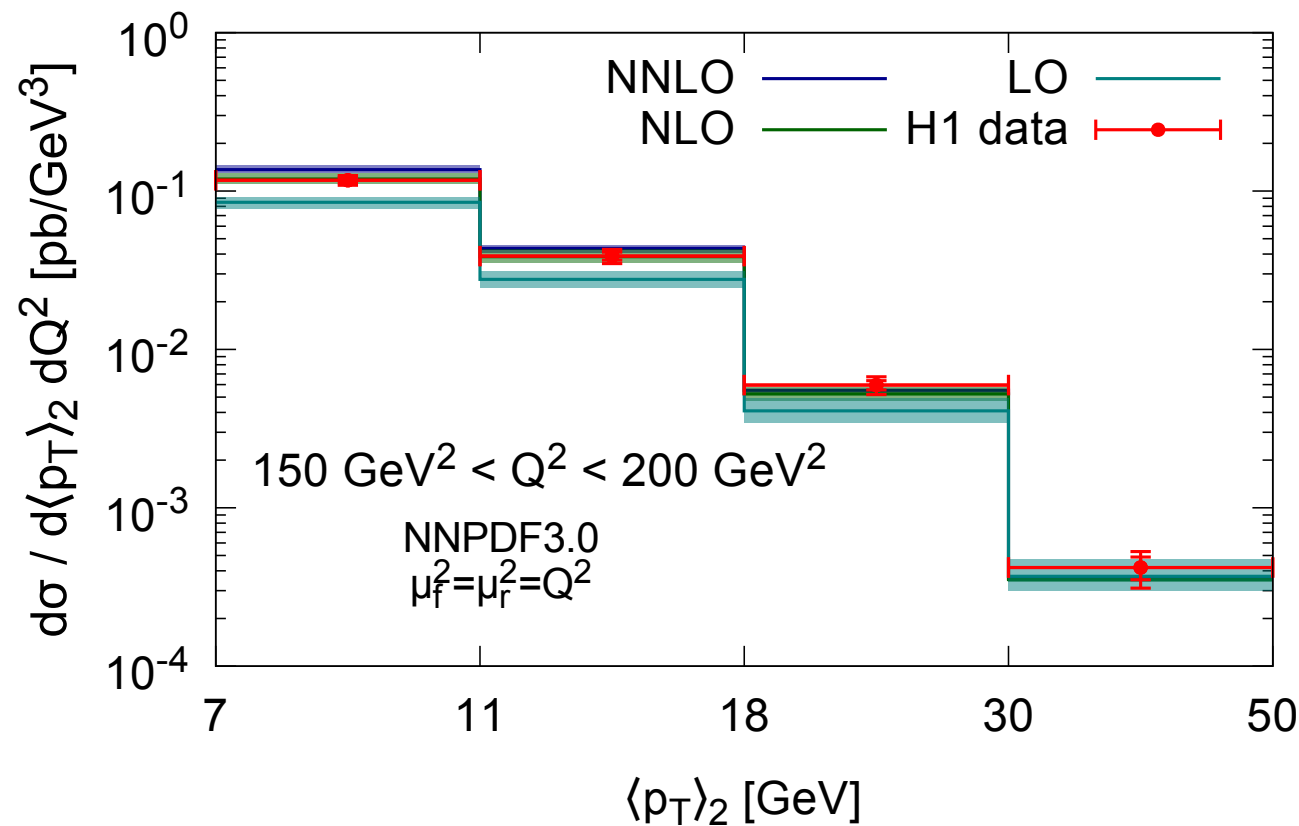
$$m_{12} > 16 \text{ GeV}$$

Dijet and Trijet observables:

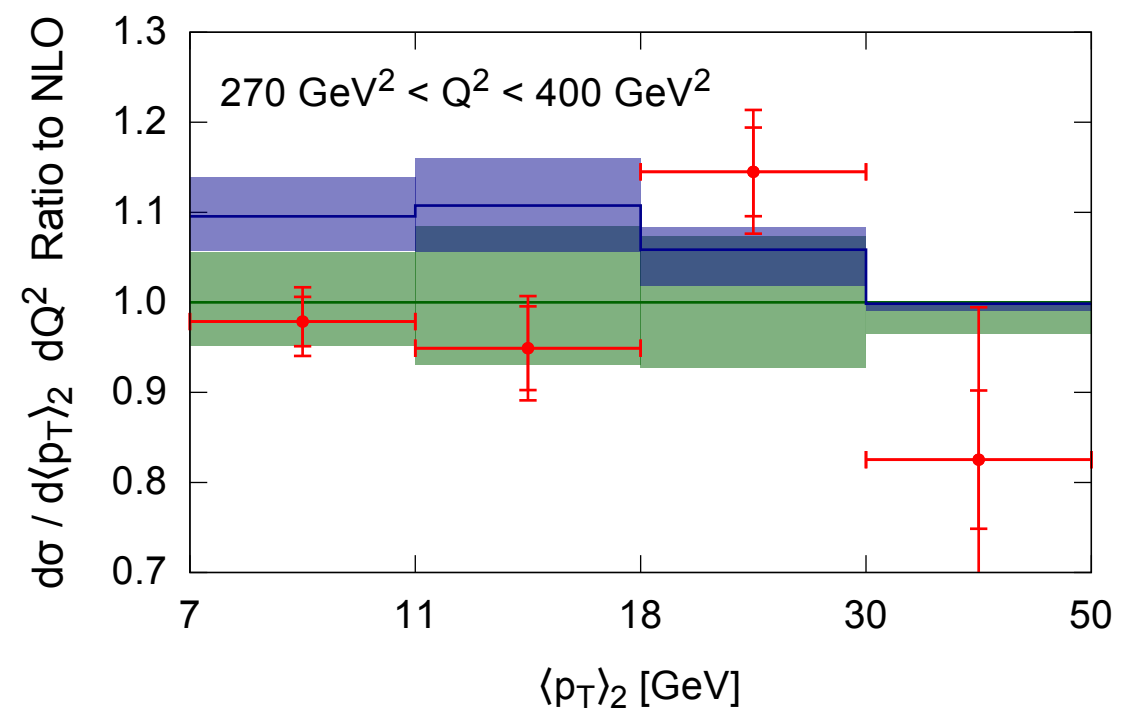
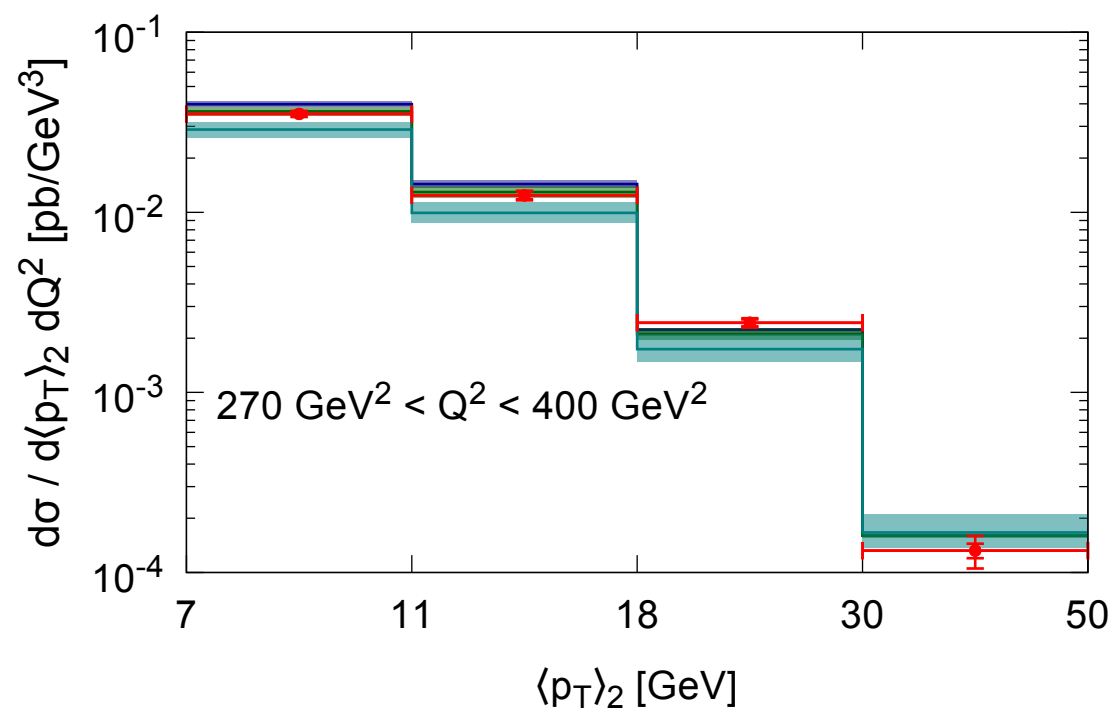
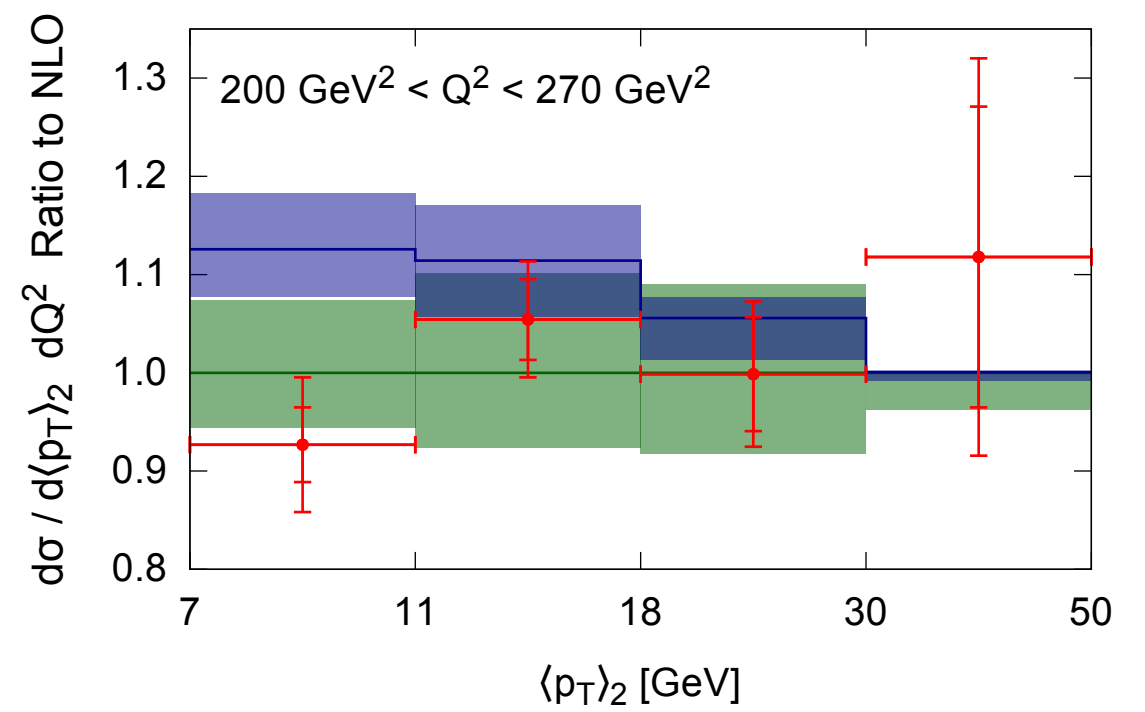
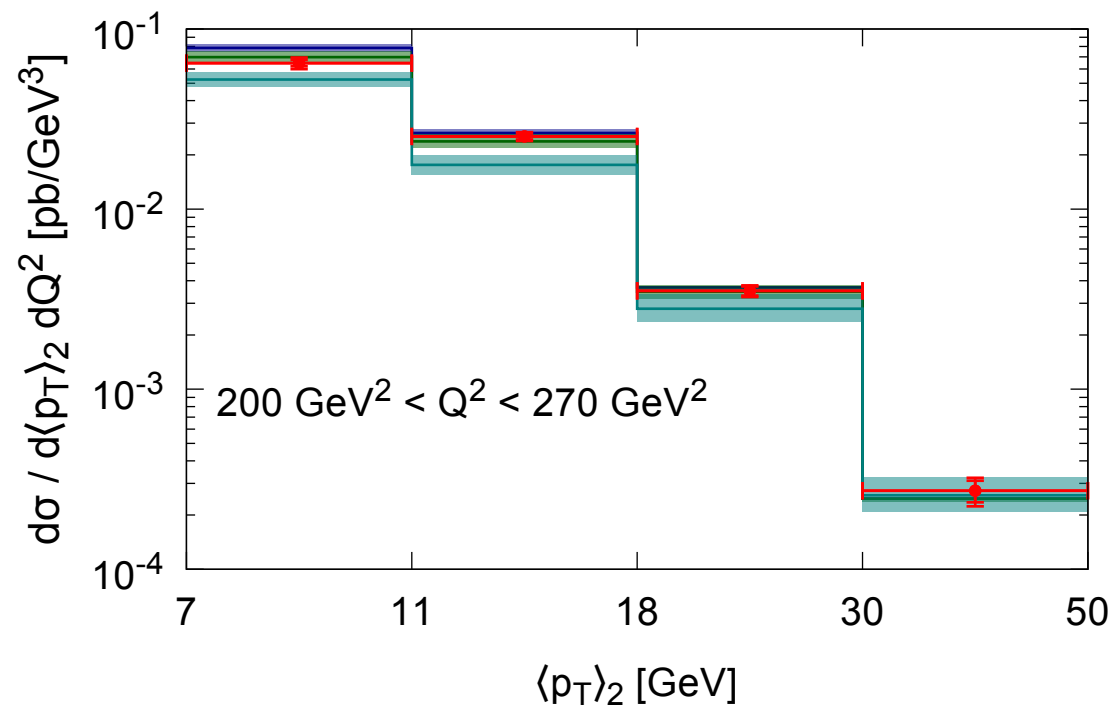
$$\langle p_T \rangle_2 = \frac{1}{2} (p_T^{j_1} + p_T^{j_2}) \quad \xi_2 = x \left(1 + \frac{m_{12}^2}{Q^2} \right)$$

$$\langle p_T \rangle_3 = \frac{1}{2} (p_T^{j_1} + p_T^{j_2} + p_T^{j_3}) \quad \xi_3 = x \left(1 + \frac{m_{123}^2}{Q^2} \right)$$

$$\langle p_T \rangle_2$$



- significant reduction in scale uncertainty $\approx 5\%$, below experimental errors
- suffers from Sudakov shoulder in lowest bin due to selection cuts



Future applications

Would like to improve experimental analysis:

- symmetric p_T and m_{12} cut causing problems
- would like asymmetric cuts on leading and subleading jets

Program ready to be applied to other setups:

- intermediate and low Q^2 ranges

View to phenomenology:

- determination of α_s
- refit PDFs using DIS dijet data

Dijet

- So far published gluons-only full colour [Gehrmann, Gehrmann-De Ridder, Glover, Pires '12; Currie, Gehrmann-De Ridder, Glover, Pires '13]
- recently migrated to *NNLOJET* framework
- completed for all channels (gg , qg , $\bar{q}g$, $q\bar{q}$, qq , qQ , $q\bar{Q}$) at leading colour-leading N_F
 - i.e N^2 , NN_F & N_F^2 NNLO corrections to LO
- currently running and checking results
- hope to have public results imminently for phenomenology

NNLOJET

All calculations now housed within single basic framework:

- completely standalone code at LO, NLO, NNLO
- fully differential parton level event generator
- many processes now included:
 - $H(\gamma\gamma) + 0, 1, 2$ jets
 - $Z(l^+l^-) + 0, 1$ jet
 - DIS dijet
 - LHC+Tevatron dijet
- development and validation of the code ongoing, adding new process

Future Projects

- further refinement of the method to improve efficiency
- H+J new decay channels
- Dijet phenomenology, PDF fits
- H+2j in HEFT @ NNLO
 - same subtraction terms as dijet, need amps
- pp->3J
 - no new ingredients needed for subtraction, need amps
 - $R_{3/2}$ at NNLO hadron collider a real possibility

Summary

Antenna subtraction has moved on from a proof of principle to full blown phenomenology:

- implemented in standalone parton-level event generator *NNLOJET*
- powerful, general and analytic method for producing NNLO accurate distributions
- many processes of interest now complete or nearing completion
- method can cope with exciting new processes without extension
- comparison of NNLO fixed order with data now a reality at LHC