CoLorFulNNLO
a NNLO subtraction method

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Precision QCD

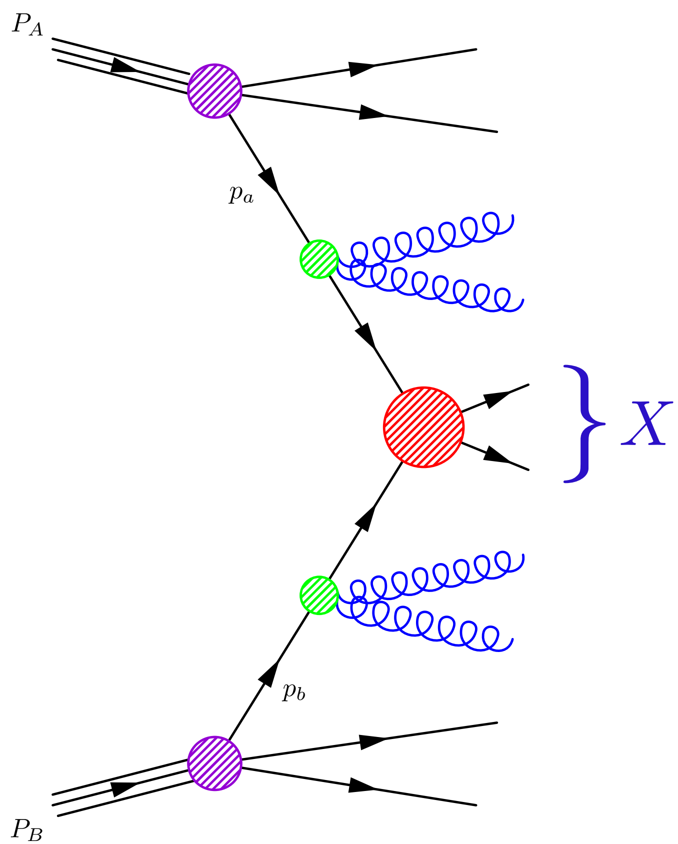
Precise determination of

- strong coupling constant α_s
- parton distributions

Precise prediction for

- Standard Model processes (Higgs, top, etc.)
- new physics processes
- their backgrounds

Cross sections at high Q^2



separate the short- and the long-range interactions through factorisation

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_{a/A}(x_1, \mu_F^2)\, f_{b/B}(x_2, \mu_F^2) \times \hat{\sigma}_{ab\to X}\left(x_1, x_2, \{p_i^\mu\}; \alpha_S(\mu_R^2), \alpha(\mu_F^2), \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2}\right)$$

$$X = W, Z, H, Q\bar{Q}, \text{high-}E_T\text{jets}, \ldots$$

$\hat{\sigma}$ is known as a fixed-order expansion in α_S

$$\hat{\sigma} = C\alpha_S^n(1 + c_1\alpha_S + c_2\alpha_S^2 + \ldots)$$

$c_1 =$ NLO $\quad c_2 =$ NNLO

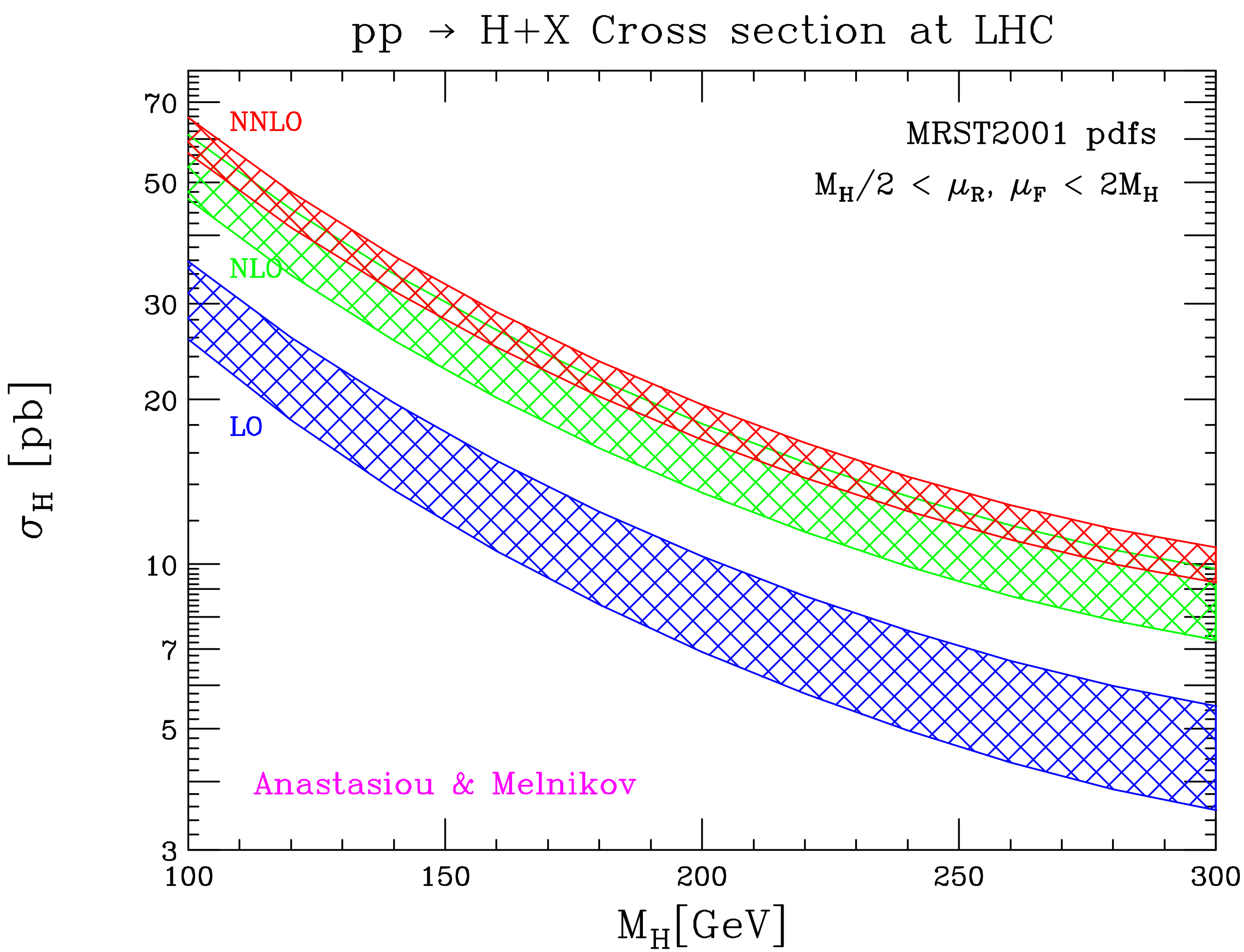
or as an all-order resummation

$$\hat{\sigma} = C\alpha_S^n[1 + (c_{11}L + c_{10})\alpha_S + (c_{22}L^2 + c_{21}L + c_{20})\alpha_S^2 + \ldots]$$

where $L = \ln(M/q_T), \ln(1-x), \ln(1/x), \ln(1-T), \ldots$

$c_{11}, c_{22} =$ LL $\quad c_{10}, c_{21} =$ NLL $\quad c_{20} =$ NNLL

Total cross section for inclusive Higgs production at LHC



C. Anastasiou K. Melnikov 2002

contour bands are

lower

$\mu_R = 2M_H \quad \mu_F = M_H/2$

upper

$\mu_R = M_H/2 \quad \mu_F = 2M_H$

we need at least NLO computations

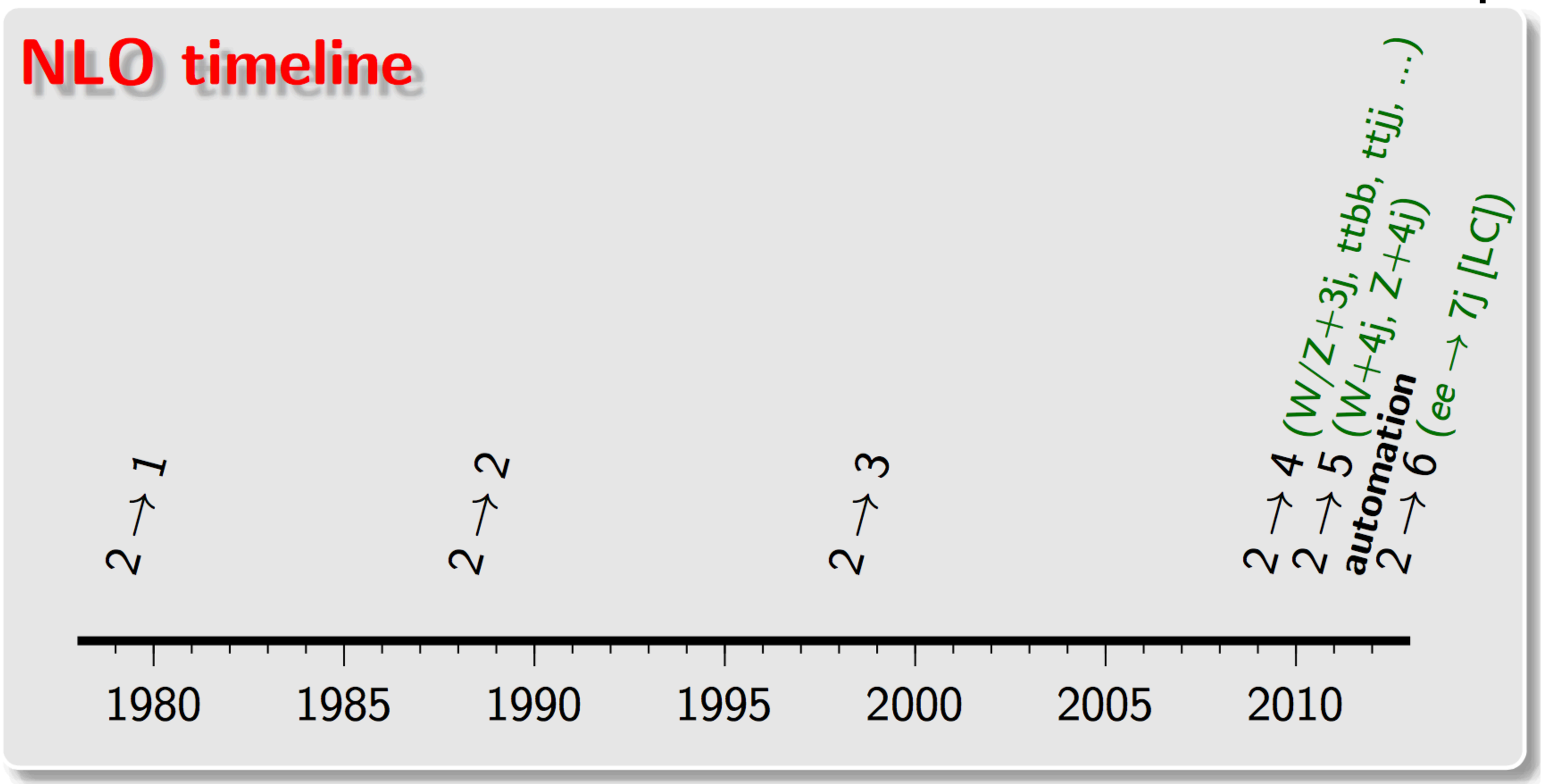
NLO features

- Jet structure: final-state collinear radiation
- PDF evolution: initial-state collinear radiation
- Opening of new channels
- Reduced sensitivity to fictitious input scales: μ_R, μ_F
  - predictive normalisation of observables
    - first step toward precision measurements
    - first estimate of signal and background for Higgs and (possibly) new physics
- Matching with parton-shower MC's:
  MC@NLO   POWHEG

the NLO revolution

At ICHEP 2010, Gavin Salam called ``NLO revolution" the rapid progress in NLO computations

from Gavin Salam's talk at Montpellier 2012



2010: NLO $W+4j$ [BlackHat+Sherpa: Berger et al]	[unitarity]
2011: NLO $WWjj$ [Rocket: Melia et al]	[unitarity]
2011: NLO $Z+4j$ [BlackHat+Sherpa: Ita et al]	[unitarity]
2011: NLO $4j$ [BlackHat+Sherpa: Bern et al]	[unitarity]
2011: first automation [MadNLO: Hirschi et al]	[unitarity + feyn.diags]
2011: first automation [Helac NLO: Bevilacqua et al]	[unitarity]
2011: first automation [GoSam: Cullen et al]	[feyn.diags(+unitarity)]
2011: $e^+e^- \to 7j$ [Becker et al, leading colour]	[numerical loops]

Some reasons for NNLO corrections

- NLO corrections are large:
  Higgs or *ttbar* production in hadron collisions

- NLO uncertainty bands are too large to test theory vs. data:
  ttbar or *bbbar* production in hadron collisions

- NLO is effectively leading order:
  energy distributions in jet cones

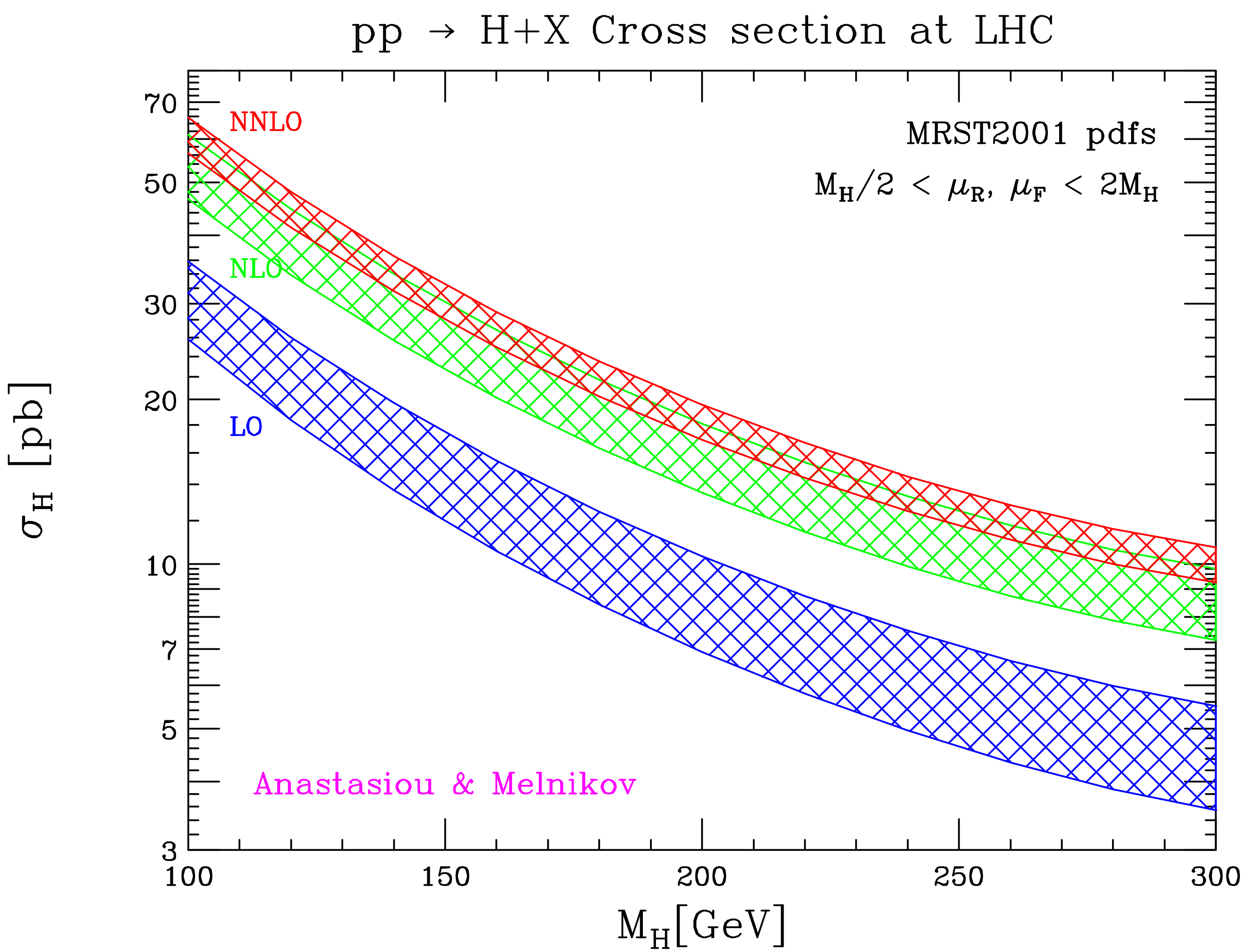
- in the world average of α_s, data are compared to theory, but

  not included in 2015/16 determination of world average :

  - all results not being based on complete NNLO (e+e-, DIS, heavy quarkonia, hadron collider jets, soft and hard fragmentation functions,…)

S. Bethke's talk at ZPW16

Total cross section for inclusive Higgs production at LHC



C. Anastasiou K. Melnikov 2002

contour bands are

lower

$\mu_R = 2M_H \quad \mu_F = M_H/2$

upper

$\mu_R = M_H/2 \quad \mu_F = 2M_H$

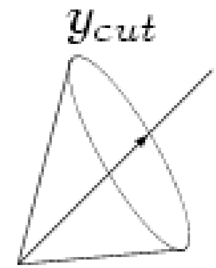
scale uncertainty is about 10%

NNLO prediction stabilises the perturbative series

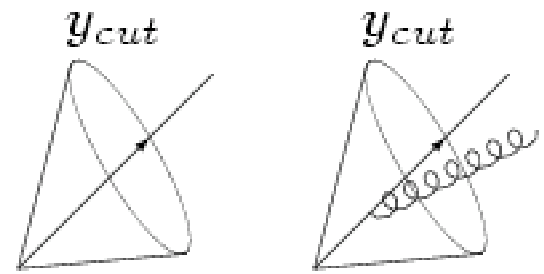
Jet structure

the jet non-trivial structure shows up first at NLO

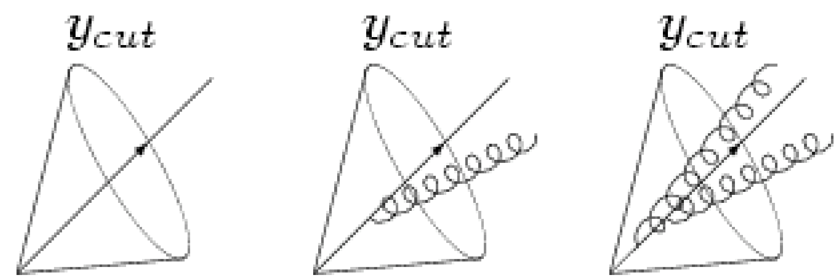
leading order



NLO



NNLO



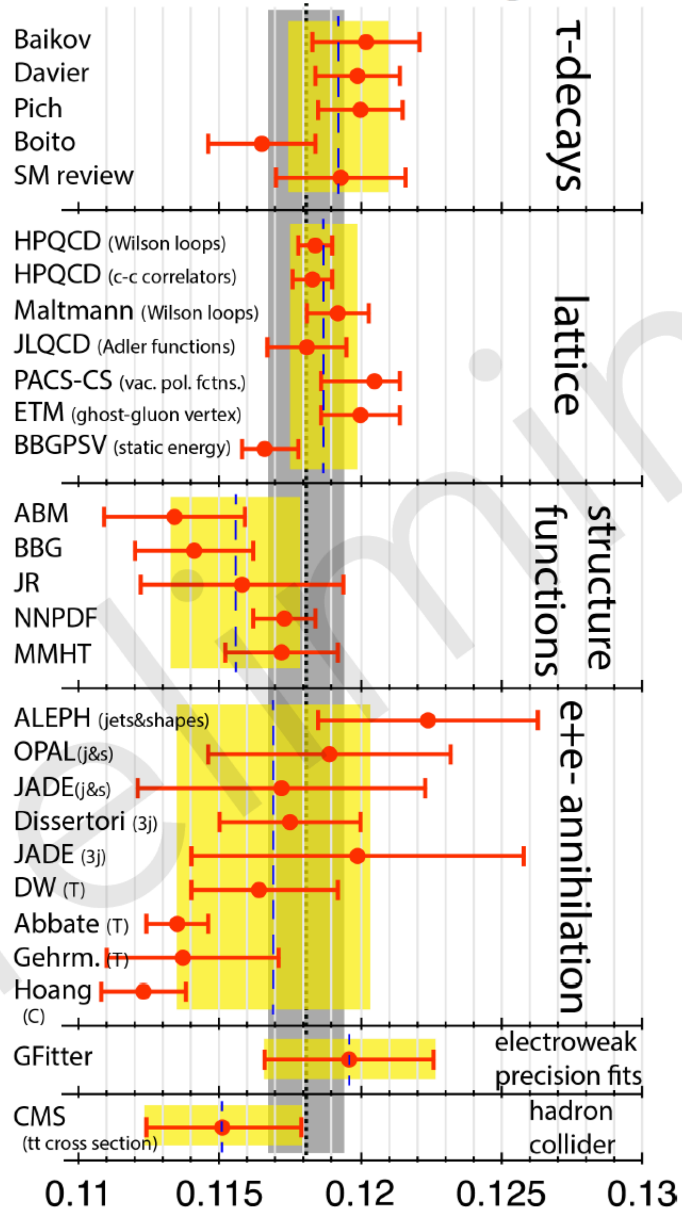
S. Bethke's talk at ZPW16

World Summary of α_s 2015/2016:

- 6 classes of measurements, each pre-averaged
- at least using NNLO QCD…
- including reliable estimates of experimental, theoretical and nonpert. uncertainties, using commonly accepted procedures

S. Bethke's talk at ZPW16

2016 summary of α_s



world average without lattice: $\alpha_s(M_Z)=0.1175(17)$

$\alpha_s(Mz)= 0.1192 \pm 0.0018$

$\alpha_s(Mz)= 0.1187 \pm 0.0012$

$\alpha_s(Mz)= 0.1156 \pm 0.0023$

$\alpha_s(Mz)= 0.1169 \pm 0.0034$

$\alpha_s(Mz)= 0.1196 \pm 0.0030$

$\alpha_s(Mz)= 0.1151 \pm 0.0033$

χ^2 average of unweighted class-averages: $\alpha_s(Mz)= 0.1181 \pm 0.0013$

NNLO state of the art

- $2 \to 1$ processes
- Drell-Yan W, Z production

  total cross section — Hamberg, van Neerven, Matsuura 1990; Harlander, Kilgore 2002

  differential cross section — Melnikov, Petriello 2006

- Higgs production in HEFT

  total cross section — Harlander, Kilgore; Anastasiou, Melnikov 2002; Ravindran, Smith, van Neerven 2003

  differential cross section — Anastasiou, Melnikov, Petriello 2004

NNLO state of the art

- $2 \to 2$ processes
  - $\gamma\gamma$ production — Catani, Cieri, De Florian, Ferrera, Grazzini 2011
  - $Z\gamma$ production — Grazzini Kallweit Rathlev Torre 2013
  - $W\gamma$ production — Grazzini Kallweit Rathlev 2015
  - ZZ production — Cascioli *et al* 2014
  - WW production — Gehrmann *et al* 2014
  - WH production — Ferrera Grazzini Tramontano 2011
  - ZH production — Ferrera Grazzini Tramontano 2014
  - *ttbar* production

    total cross section — Baernreuther, Czakon, Mitov 2012; Czakon, Mitov 2012; Czakon, Fiedler, Mitov 2013

    differential cross section — Czakon, Fiedler, Mitov 2014

NNLO state of the art

- $2 \to 2$ processes

- W + 1 jet production

  Boughezal, Focke, Liu, Petriello 2015

- Z + 1 jet production

  Gehrmann-De Ridder, Gehrmann, Glover, Huss, Morgan 2015
  Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello 2015

- Higgs + 1 jet production in HEFT

  Boughezal, Caola, Melnikov, Petriello, Schulze 2013-15
  Chen, Gehrmann, Glover, Jaquier 2014
  Boughezal, Focke, Giele, Liu, Petriello 2015

- 2 jet production (only gg)

  Curry, Gehrmann-De Ridder, Gehrmann, Glover, Pires 2013

NNLO state of the art

- 1 → 3 processes

  $e^+e^- \to 3\ \mathrm{jets}$

  Gehrmann-De Ridder, Gehrmann, Glover, Heinrich 2007-08
  Becher, Schwartz 2008
  Weinzierl 2009

- 2 → 3 processes

  Higgs + 2 jet production in VBF (in DIS approx)

  Cacciari Dreyer Karlberg Salam Zanderighi 2015

NNLO cross section methods

A variety of (subtraction) methods exist, to get a fiducial cross section at NNLO

- Sector decomposition — Denner Roth 1996; Binoth Heinrich 2000; Anastasiou, Melnikov, Petriello 2004
- Antenna subtraction — Gehrmann-De Ridder, Gehrmann, Glover 2005
- *Colorful*NNLO subtraction — Somogyi, Trocsanyi, VDD 2005-06
- qT subtraction — Catani, Grazzini 2007
- Residue subtraction — Czakon 2010
- N-jettiness subtraction — Boughezal, Focke, Liu, Petriello 2015; Gaunt Stahlhofen Tackmann Walsh 2015

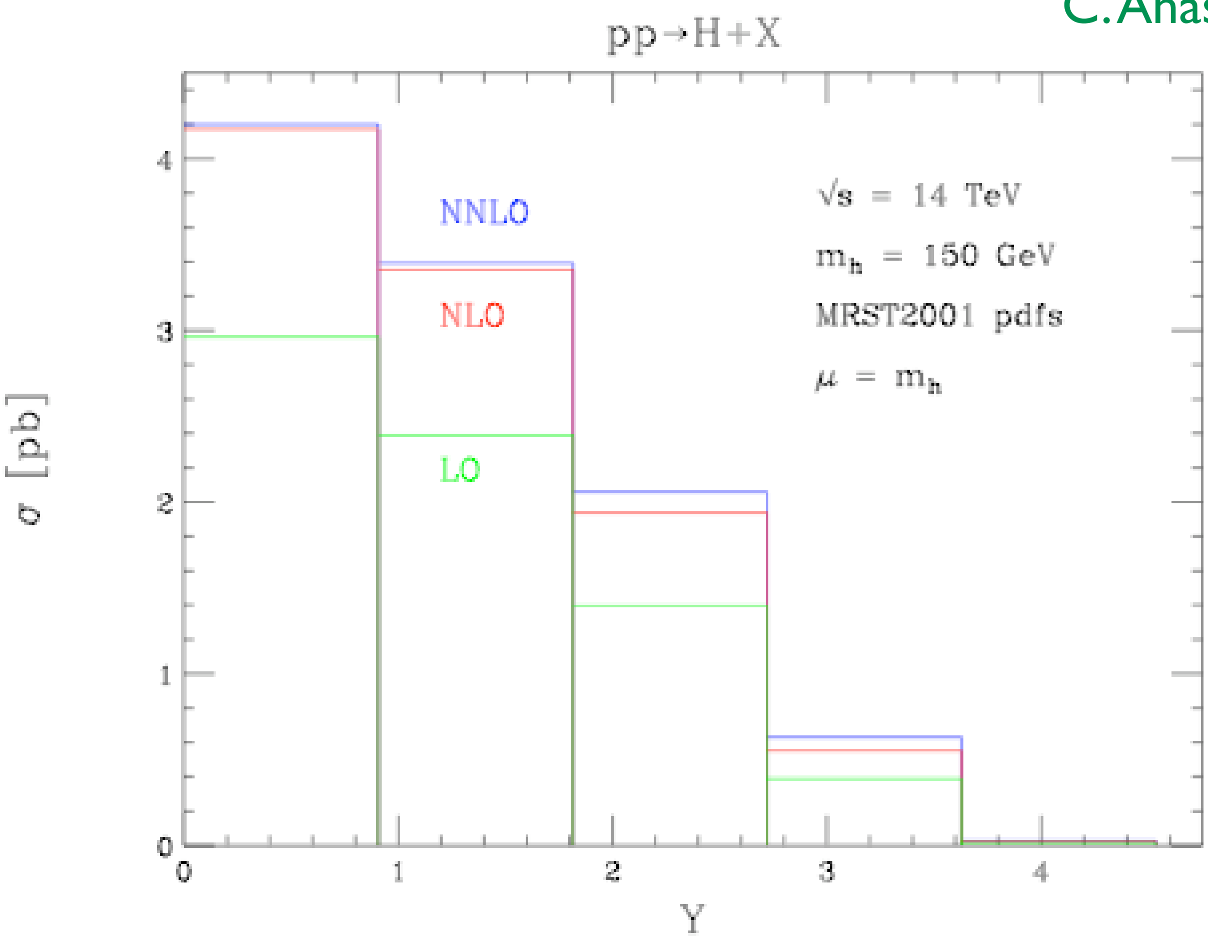
The goal of all those methods is

to compute cross sections at NNLO
with any possible acceptance cuts

Higgs production at LHC

a fully differential cross section:
bin-integrated rapidity distribution, with a jet veto

C. Anastasiou K. Melnikov F. Petriello 2004



jet veto: require

$R = 0.4$

$|\mathbf{p}_T^j| < p_T^{veto} = 40 \text{ GeV}$

for 2 partons

$R_{12}^2 = (\eta_1 - \eta_2)^2 + (\phi_1 - \phi_2)^2$

if $R_{12} > R$

$|\mathbf{p}_T^1|, |\mathbf{p}_T^2| < p_T^{veto}$

if $R_{12} < R$

$|\mathbf{p}_T^1 + \mathbf{p}_T^2| < p_T^{veto}$

- $M_H = 150 \text{ GeV}$ (jet veto relevant in the $H \to W^+W^-$ decay channel)
- K factor is much smaller for the vetoed x-sect than for the inclusive one: average $|\mathbf{p}_T^j|$ increases from NLO to NNLO: less x-sect passes the veto

What is the problem in computing (fiducial or differential) cross sections beyond leading order?

Given a production process and a physical observable to be computed, in order to get a consistent result we must take into account amplitudes with real radiation (bremsstrahlung) and amplitudes with virtual loops.

The virtual amplitudes exhibit explicit ε poles when dimensionally regularised.

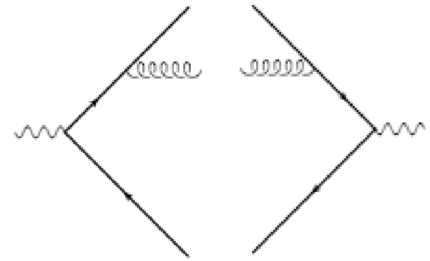
The bremsstrahlung amplitudes sport kinematic singularities as one or more partons become unresolved, which turn into ε poles after phase space integration.

Those ε poles must cancel no matter how the chosen acceptance cuts specify the phase space to be used.

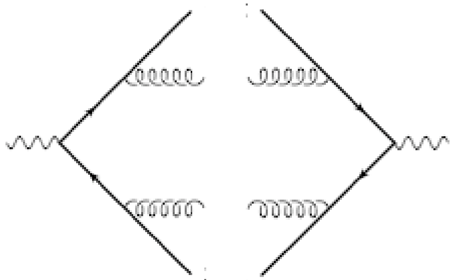
NLO assembly kit

$e^+e^- \to 3$ jets

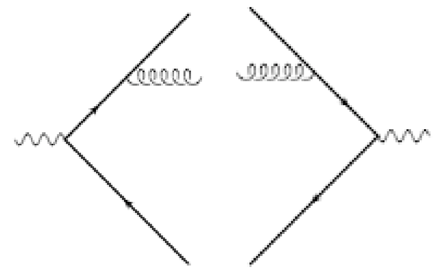
leading order $|\mathcal{M}_n^{tree}|^2$



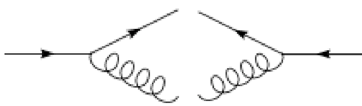
NLO real



IR



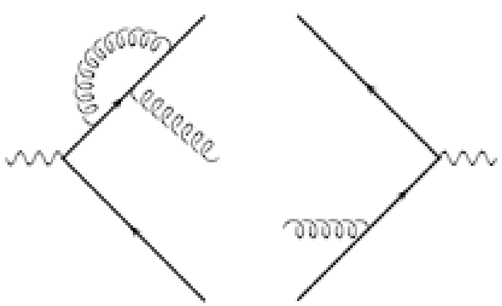
$\otimes$



$$|\mathcal{M}_{n+1}^{\text{tree}}|^2 \quad \to \quad |\mathcal{M}_n^{\text{tree}}|^2 \quad \times \int dPS |P_{\text{split}}|^2$$

$$= \quad -\left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon}\right)$$

NLO virtual



$d = 4 - 2\epsilon$

$$\int d^d l \; 2(\mathcal{M}_n^{loop})^* \mathcal{M}_n^{tree} \quad = \quad \left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon}\right) |\mathcal{M}_n^{tree}|^2 + fin.$$

NLO production rates

Process-independent procedures devised in the 90's

- slicing — Giele, Glover, Kosower 1992-93
- subtraction — Frixione, Kunszt, Signer; Nagy, Trocsanyi 1995
  - dipole — Catani, Seymour 1996
  - antenna — Kosower; Campbell Cullen & Glover 1999

$$\sigma = \sigma^{\rm LO} + \sigma^{\rm NLO} = \int_m d\sigma_m^B \; J_m + \sigma^{\rm NLO}$$

$$\sigma^{\rm NLO} = \int_{m+1} d\sigma_{m+1}^{\rm R} J_{m+1} + \int_m d\sigma_m^{\rm V} J_m$$

these 2 terms are divergent in $d=4$

use universal IR structure to subtract divergences

$$\sigma^{\rm NLO} = \int_{m+1} \left[d\sigma_{m+1}^{\rm R} J_{m+1} - d\sigma_{m+1}^{\rm R,A} J_m \right] + \int_m \left[d\sigma_m^{\rm V} + \int_1 d\sigma_{m+1}^{\rm R,A} \right] J_m$$

the 2 terms are finite in $d=4$

Observable (jet) functions

J_m vanishes when one parton becomes soft or collinear to another one

$$J_m(p_1, ..., p_m) \to 0\,, \quad \text{if} \quad p_i \cdot p_j \to 0$$

⇨ $d\sigma_m^{\rm B}$ is integrable over 1-parton IR phase space

J_{m+1} vanishes when two partons become simultaneously soft and/or collinear

$$J_{m+1}(p_1, ..., p_{m+1}) \to 0\,, \quad \text{if} \quad p_i \cdot p_j \text{ and } p_k \cdot p_l \to 0 \quad (i \neq k)$$

R and V are integrable over 2-parton IR phase space

observables are IR safe

$$J_{n+1}(p_1, .., p_j = \lambda q, .., p_{n+1}) \to J_n(p_1, ..., p_{n+1}) \quad \text{if} \quad \lambda \to 0$$

$$J_{n+1}(p_1, .., p_i, .., p_j, .., p_{n+1}) \to J_n(p_1, .., p, .., p_{n+1}) \quad \text{if} \quad p_i \to zp\,,\ p_j \to (1-z)p$$

for all $n \geq m$

NLO IR limits

collinear operator

$$\mathrm{C}_{ir}|\mathcal{M}^{(0)}_{m+2}(p_i,p_r,\ldots)|^2 \propto \frac{1}{s_{ir}}\langle\mathcal{M}_{m+1}(0)(p_{ir},\ldots)|\hat{P}^{(0)}_{f_i f_r}|\mathcal{M}_{m+1}(0)(p_{ir},\ldots)\rangle$$

soft operator

$$\mathrm{S}_r|\mathcal{M}^{(0)}_{m+2}(p_r,\ldots)|^2 \propto \frac{s_{ik}}{s_{ir}s_{rk}}\langle\mathcal{M}_{m+1}(0)(\ldots)|T_i\cdot T_k|\mathcal{M}_{m+1}(0)(\ldots)\rangle$$

counterterm

$$\sum_r\left(\sum_{i\neq r}\frac{1}{2}\mathrm{C}_{ir}+\mathrm{S}_r\right)|\mathcal{M}^{(0)}_{m+2}(p_i,p_r,\ldots)|^2$$

performs double subtraction in overlapping regions

NLO overlapping divergences

$\mathrm{C}_{ir}\mathrm{S}_r$ can be used to cancel double subtraction

$$\mathrm{C}_{ir}\left(\mathrm{S}_r - \mathrm{C}_{ir}\mathrm{S}_r\right)|\mathcal{M}_{m+2}^{(0)}|^2 = 0$$

$$\mathrm{S}_r\left(\mathrm{C}_{ir} - \mathrm{C}_{ir}\mathrm{S}_r\right)|\mathcal{M}_{m+2}^{(0)}|^2 = 0$$

the NLO counterterm

$$\mathrm{A}_1|\mathcal{M}_{m+2}^{(0)}|^2 = \sum_r \left[\sum_{i\neq r}\frac{1}{2}\mathrm{C}_{ir} + \left(\mathrm{S}_r - \sum_{i\neq r}\mathrm{C}_{ir}\mathrm{S}_r\right)\right]|\mathcal{M}_{m+2}^{(0)}(p_i, p_r, \ldots)|^2$$

- has the same singular behaviour as SME, and is free of double subtractions

$$\mathrm{C}_{ir}\left(1 - \mathrm{A}_1\right)|\mathcal{M}_{m+1}^{(0)}|^2 = 0 \qquad \mathrm{S}_r\left(1 - \mathrm{A}_1\right)|\mathcal{M}_{m+1}^{(0)}|^2 = 0$$

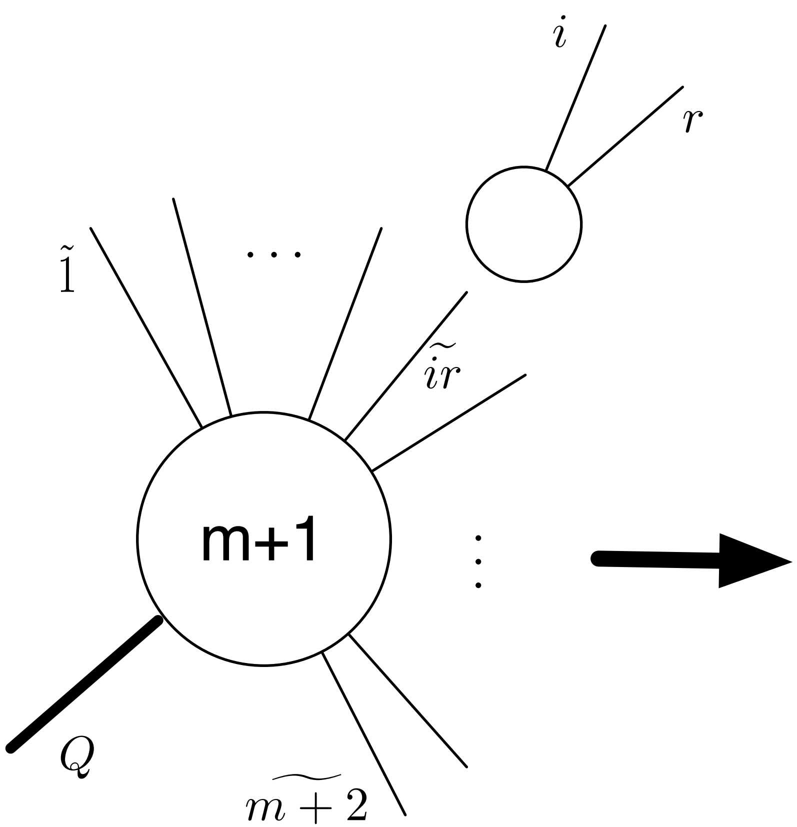
- contains spurious singularities when parton $s \neq r$ becomes unresolved, but they are screened by J_m

Collinear mapping

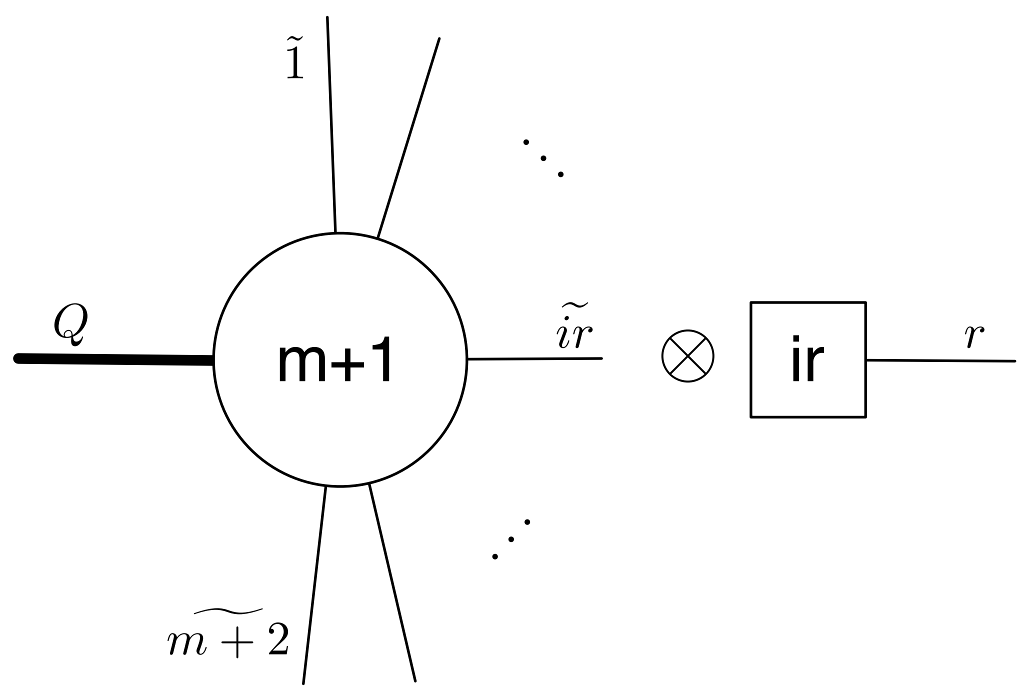
$$\tilde{p}^{\mu}_{ir} = \frac{1}{1-\alpha_{ir}}\left(p^{\mu}_i + p^{\mu}_r - \alpha_{ir} Q^{\mu}\right), \qquad \tilde{p}^{\mu}_n = \frac{1}{1-\alpha_{ir}} p^{\mu}_n, \qquad n \neq i, r$$

$$\alpha_{ir} = \frac{1}{2}\left[y_{(ir)Q} - \sqrt{y^2_{(ir)Q} - 4y_{ir}}\,\right] \qquad y_{ir} = \frac{2p_i \cdot p_r}{Q^2}$$

momentum is conserved $\quad \tilde{p}^{\mu}_{ir} + \sum_n \tilde{p}^{\mu}_n = p^{\mu}_i + p^{\mu}_r + \sum_n p^{\mu}_n$



phase space factorises



NLO counterterm

$$\mathrm{A}_1|\mathcal{M}^{(0)}_{m+2}|^2 = \sum_r \left[\sum_{i\neq r}\frac{1}{2}\mathrm{C}_{ir} + \left(\mathrm{S}_r - \sum_{i\neq r}\mathrm{C}_{ir}\mathrm{S}_r\right)\right]|\mathcal{M}^{(0)}_{m+2}(p_i, p_r, \ldots)|^2$$

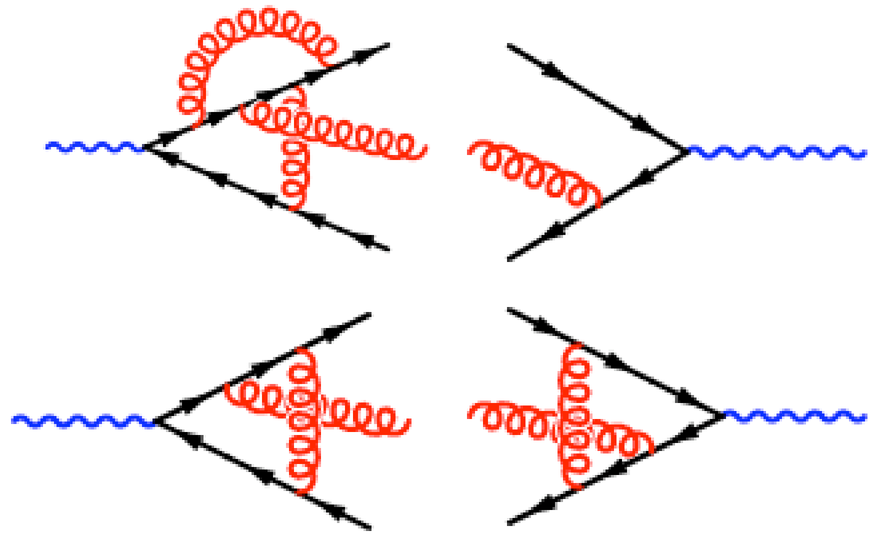
$$\mathrm{d}\sigma^{\mathrm{R},\mathrm{A}_1}_{m+2} = \mathrm{d}\phi_{m+1}\,[\mathrm{d}p_1]\,\boldsymbol{\mathcal{A}}_1|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\int_1 \mathrm{d}\sigma^{\mathrm{R},\mathrm{A}_1}_{m+2} = \mathrm{d}\phi_{m+1}|\mathcal{M}^{(0)}_{m+1}|^2 \otimes \boldsymbol{I}(m+1,\varepsilon)$$

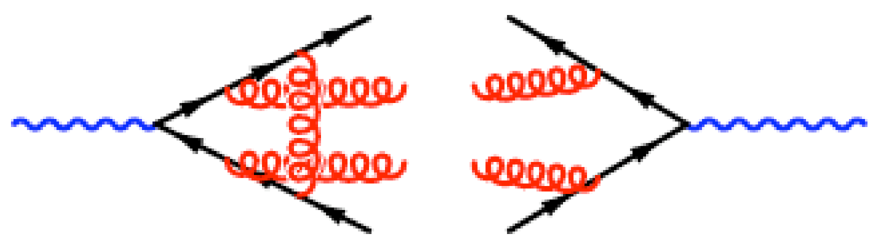
NNLO assembly kit

$e^+e^- \to 3$ jets

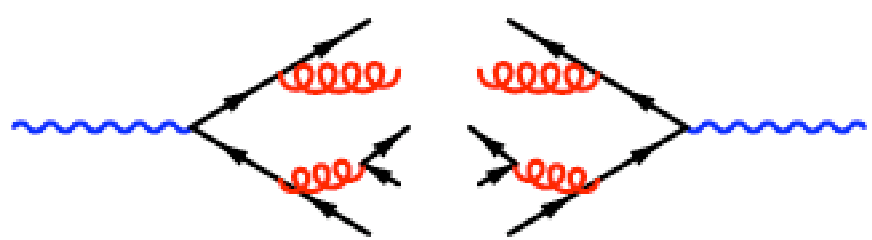
double virtual



real-virtual



double real



NNLO subtraction

$$\sigma^{\mathrm{NNLO}} = \int_{m+2} d\sigma^{\mathrm{RR}}_{m+2} J_{m+2} + \int_{m+1} d\sigma^{\mathrm{RV}}_{m+1} J_{m+1} + \int_{m} d\sigma^{\mathrm{VV}}_{m} J_{m}$$

the 3 terms are separately IR divergent in $d=4$ dimensions

- RR: has kinematic singularities as one or two partons become unresolved.
  After phase space integration, we get up to $1/\epsilon^4$ poles

- RV: has explicit ϵ poles up to $1/\epsilon^2$
  In addition, it has kinematic singularities as one parton becomes unresolved.
  After phase space integration, we get up to $1/\epsilon^2$ poles

- VV: has explicit ϵ poles up to $1/\epsilon^4$
  No kinematic singularities (they are killed by the jet functions)

- KLN theorem ensures that all poles must cancel for an IR safe physical observable

- The RR, RV, VV has must be organised in such a way as to be computable in 4 dimensions, like at NLO

- The ε poles must cancel no matter how the chosen acceptance cuts specify the phase space to be used.

- We must make this cancellation explicit, so the various contributions can be computed numerically

- There are already a few (subtraction) methods which do that. Why another one?

*CoLorFul*NNLO subtraction

*Co*mpletely *Lo*cal subt*r*actions for *Ful*ly differential predictions at NNLO

- general and explicit expressions, including colour and flavour
- fully local counterterms, featuring all colour and spin correlations
- analytic cancellation of ε poles
- option to constrain subtractions to near singular regions
- algorithmic contruction, in principle valid at any order in α_s

NNLO subtraction

$$\sigma^{\rm NNLO} = \int_{m+2} d\sigma^{\rm RR}_{m+2} J_{m+2} + \int_{m+1} d\sigma^{\rm RV}_{m+1} J_{m+1} + \int_m d\sigma^{\rm VV}_m J_m$$

use universal IR structure to build counterterms
which subtract the kinematic singularities

$$\sigma^{\rm NNLO} = \int_{m+2} \left[d\sigma^{\rm RR}_{m+2} J_{m+2} - d\sigma^{\rm RR,A_2}_{m+2} J_m \right]$$

takes care of the doubly-unresolved limits of RR,
but still divergent in the singly-unresolved ones

$$+ \int_{m+1} \left[d\sigma^{\rm RV}_{m+1} J_{m+1} - d\sigma^{\rm RV,A_1}_{m+1} J_m \right]$$

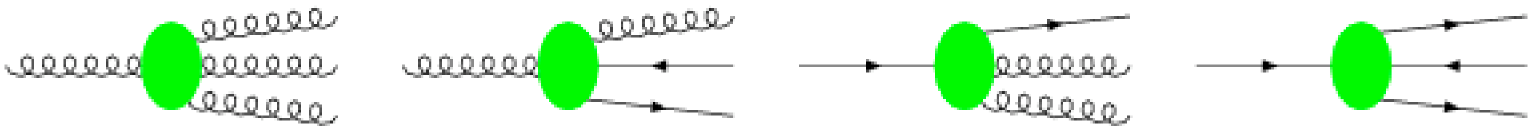
takes care of the singly-unresolved limits of RV,
but still contains 1/ε poles in regions away from the 1-parton IR regions

$$+ \int_m \left[d\sigma^{\rm VV}_m + \int_2 d\sigma^{\rm RR,A_2}_{m+2} + \int_1 d\sigma^{\rm RV,A_1}_{m+1} \right] J_m$$

Collinear and soft currents

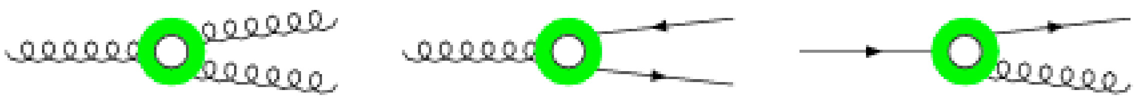
- universal IR structure ⟶ process-independent procedure

- universal collinear and soft currents
  - 3-parton tree splitting functions



J. Campbell N. Glover 1997; S. Catani M. Grazzini 1998; A. Frizzo F. Maltoni VDD 1999; D. Kosower 2002

  - 2-parton one-loop splitting functions



Z. Bern L. Dixon D. Dunbar D. Kosower 1994; Z. Bern W. Kilgore C. Schmidt VDD 1998-99; D. Kosower P. Uwer 1999; S. Catani M. Grazzini 1999; D. Kosower 2003

however,

- there are regions in phase space with overlapping limits
- universal collinear and soft currents are well defined only in the strict limit

CoLorFulNNLO subtraction

- construction based on universal collinear and soft currents
  - general procedure for matching of limits:
    construct subtraction terms that regularise the singularities of the amplitudes in all unresolved parts of the phase space, avoiding multiple subtractions
    G. Somogyi Z. Trocsanyi VDD 2005
  - perform momentum mappings, such that the phase space factorises exactly over the unresolved momenta and such that it respects the structure of the cancellations among subtraction terms
    G. Somogyi Z. Trocsanyi VDD 2006
- fully local in color ⊗ spin space
  - azimuthal correlations fully taken into account in gluon splitting
  - ratio of the sum of counterterms to the real emission cross section tends to unity in any IR limit
- straightforward to constrain subtractions to near singular regions
  - independence of physical results on phase space cutoff

A_2 counterterm

- construct the 2-unresolved-parton counterterm using the IR currents

$$\mathbf{A}_2|\mathcal{M}^{(0)}_{m+2}|^2 = \sum_r \sum_{s\neq r} \Bigg\{ \sum_{i\neq r,s} \Bigg[\frac{1}{6}\mathbf{C}_{irs} + \sum_{j\neq i,r,s} \frac{1}{8}\mathbf{C}_{ir;js} + \frac{1}{2}\mathbf{S}_{rs}$$
$$+\frac{1}{2}\Bigg(\mathbf{CS}_{ir;s} - \mathbf{C}_{irs}\mathbf{CS}_{ir;s} - \sum_{j\neq i,r,s} \mathbf{C}_{ir;js}\mathbf{CS}_{ir;s} \Bigg) \Bigg]$$
$$-\sum_{i\neq r,s} \Bigg[\mathbf{CS}_{ir;s}\mathbf{S}_{rs} + \mathbf{C}_{irs}\left(\frac{1}{2}\mathbf{S}_{rs} - \mathbf{CS}_{ir;s}\mathbf{S}_{rs}\right)$$
$$+\sum_{j\neq i,r,s} \mathbf{C}_{ir;js}\left(\frac{1}{2}\mathbf{S}_{rs} - \mathbf{CS}_{ir;s}\mathbf{S}_{rs}\right) \Bigg] \Bigg\} |\mathcal{M}^{(0)}_{m+2}|^2$$

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performing double and triple subtractions in overlapping regions

$$\mathrm{C}_{irs}\,(1-\mathrm{A}_2)\,|\mathcal{M}^{(0)}_{m+2}|^2 = 0 \qquad \mathrm{S}_{rs}\,(1-\mathrm{A}_2)\,|\mathcal{M}^{(0)}_{m+2}|^2 = 0$$
$$\mathrm{C}_{ir;js}\,(1-\mathrm{A}_2)\,|\mathcal{M}^{(0)}_{m+2}|^2 = 0 \qquad \mathrm{CS}_{ir;s}\,(1-\mathrm{A}_2)\,|\mathcal{M}^{(0)}_{m+2}|^2 = 0$$

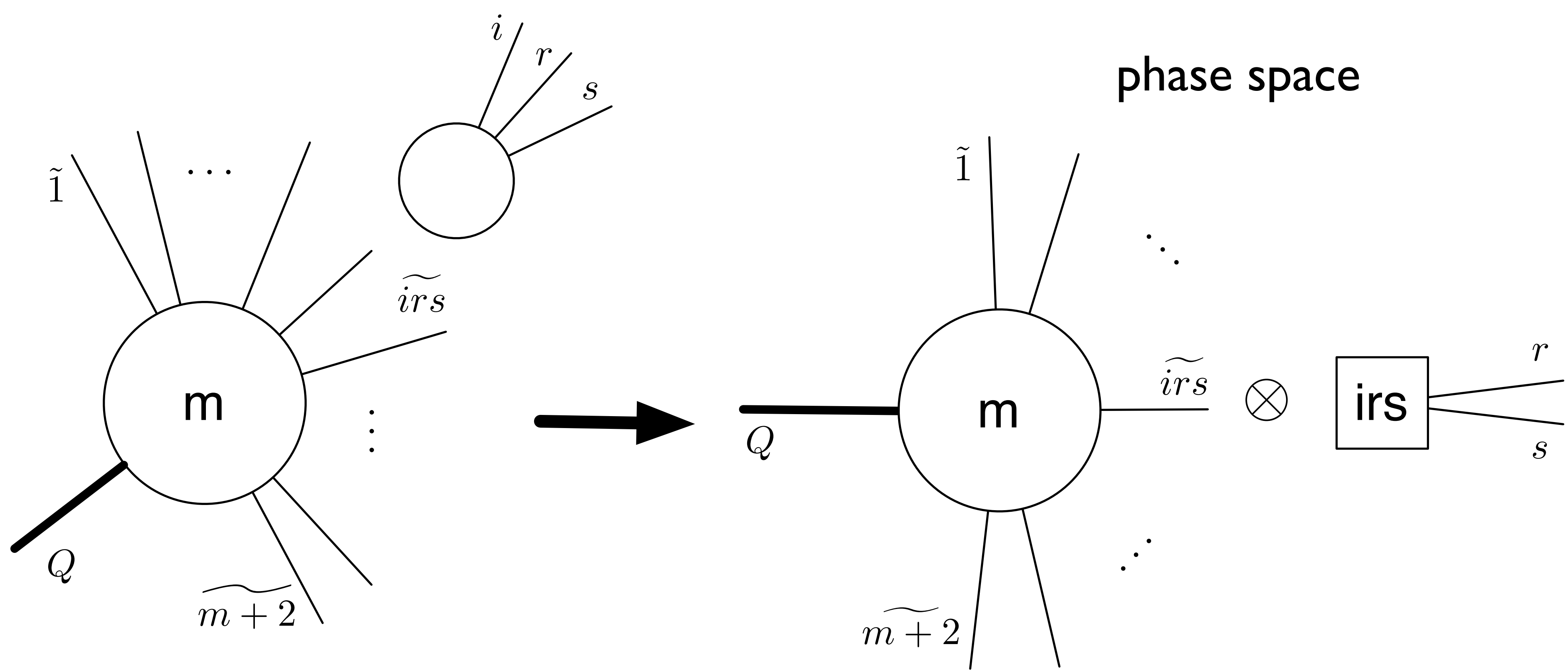
Triple-collinear mapping

$$\tilde{p}^{\mu}_{irs} = \frac{1}{1-\alpha_{irs}}(p^{\mu}_i + p^{\mu}_r + p^{\mu}_s - \alpha_{irs} Q^{\mu})\,, \qquad \tilde{p}^{\mu}_n = \frac{1}{1-\alpha_{irs}} p^{\mu}_n\,, \qquad n \neq i, r, s$$

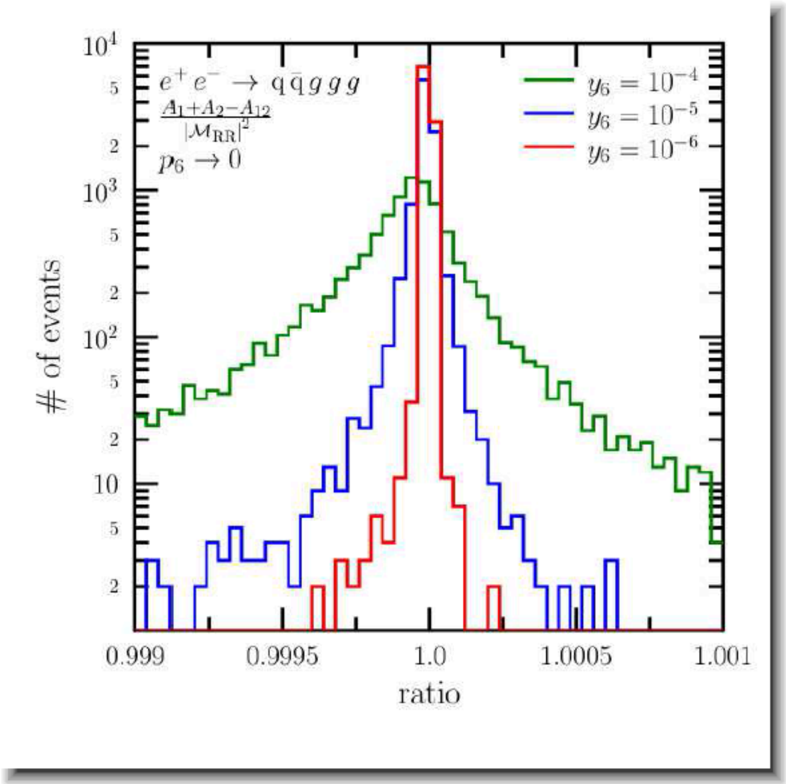
$$\alpha_{irs} = \frac{1}{2}\left[y_{(irs)Q} - \sqrt{y^2_{(irs)Q} - 4 y_{irs}}\,\right]$$

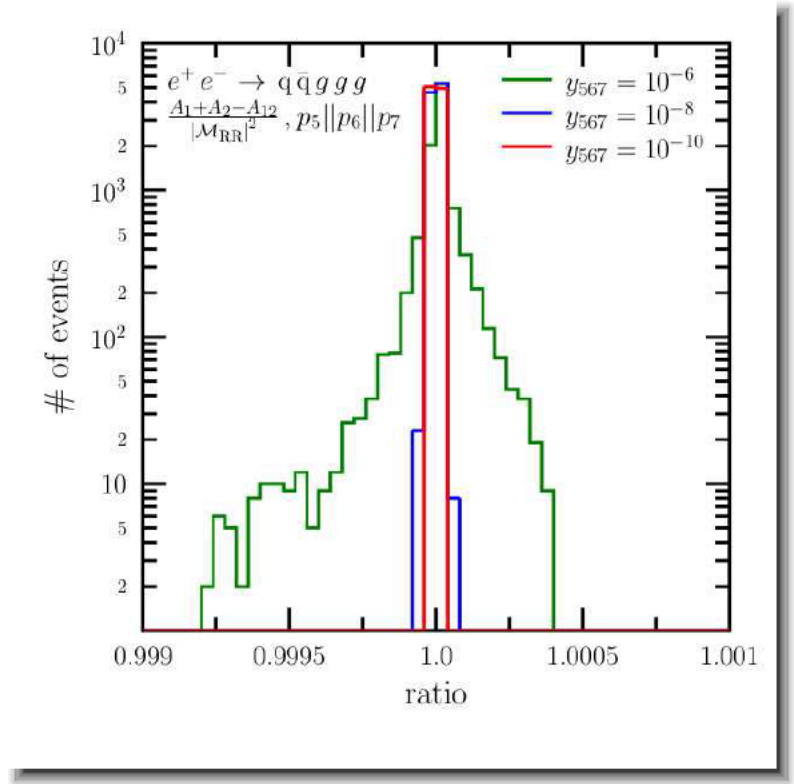
momentum conservation $\tilde{p}^{\mu}_{irs} + \sum_n \tilde{p}^{\mu}_n = p^{\mu}_i + p^{\mu}_r + p^{\mu}_s + \sum_n p^{\mu}_n$

phase space



kinematic singularities cancel in RR



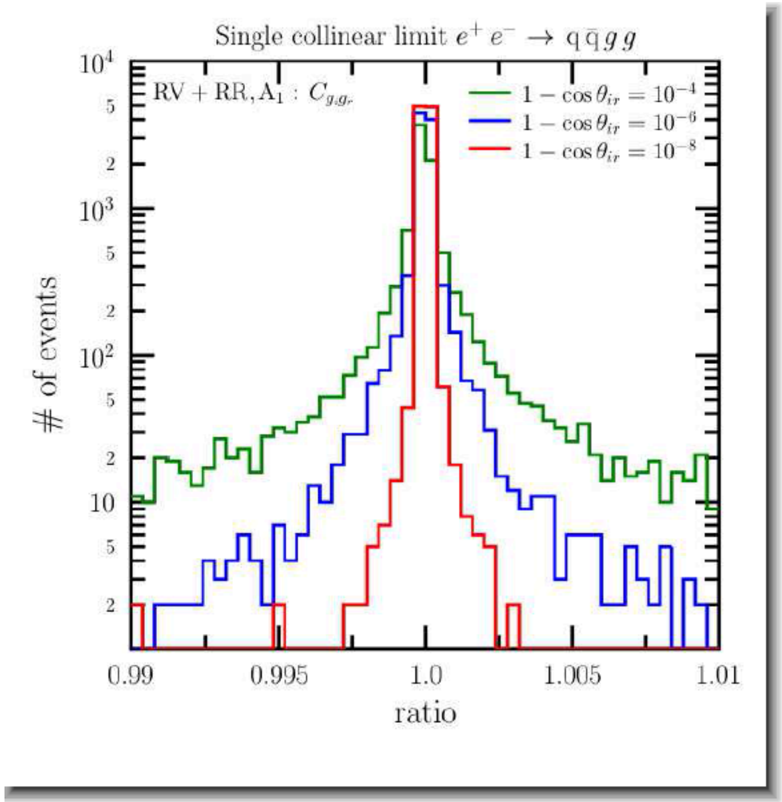


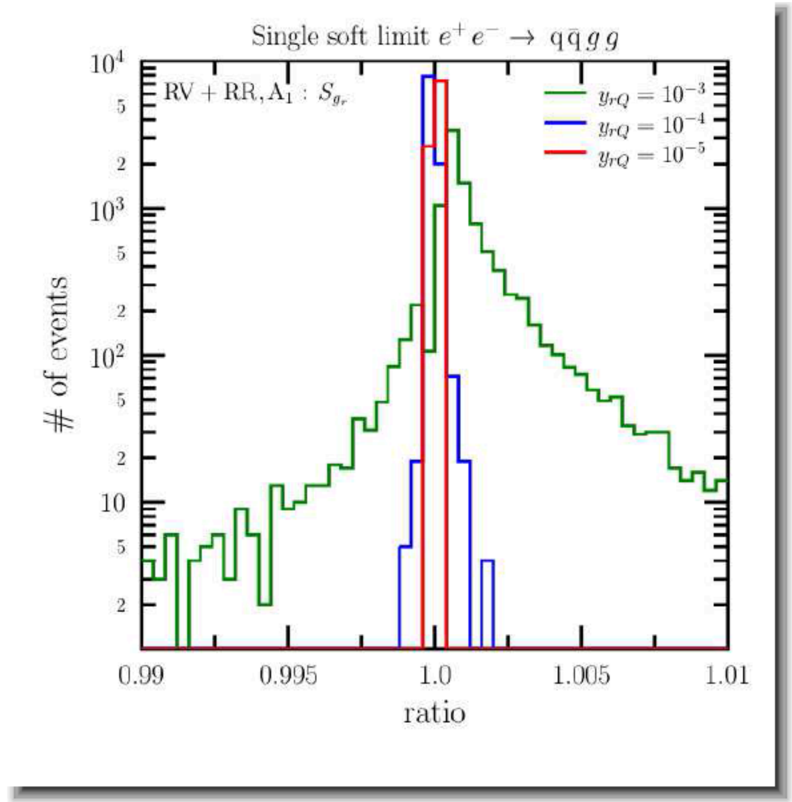
soft

triple collinear

ratio = subtraction terms/RR

- kinematic singularities cancel in RV





collinear

soft

ratio = subtraction terms/(RV + RR,A_1)

RR counterterm

needs a NLO subtraction between the m+2 and the m+1 parton contributions

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{NNLO}}_{\{m+2\}} + \sigma^{\mathrm{NNLO}}_{\{m+1\}} + \sigma^{\mathrm{NNLO}}_{\{m\}}$$

$$\sigma^{\mathrm{NNLO}}_{\{m+2\}} = \int_{m+2} \Big[\mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} J_m$$

must be finite in the doubly-unresolved regions

$$- \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} J_{m+1} + \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} J_m \Big]_{d=4}$$

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A_1 takes care of the singly-unresolved regions and A_{12} of the over-subtracting

RR counterterm = $A_2 + A_1 - A_{12}$

$$\mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} = \mathrm{d}\phi_m \, [\mathrm{d}p_2] \, \boldsymbol{\mathcal{A}}_2 |\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} = \mathrm{d}\phi_{m+1} \, [\mathrm{d}p_1] \, \boldsymbol{\mathcal{A}}_1 |\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} = \mathrm{d}\phi_m \, [\mathrm{d}p_1] \, [\mathrm{d}p_1] \, \boldsymbol{\mathcal{A}}_{12} |\mathcal{M}^{(0)}_{m+2}|^2$$

need to construct A_{12} such that all overlapping regions in
1-parton and 2-parton IR phase space regions are counted only once

$$\mathbf{C}_{ir}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{C}_{ir}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{S}_{r}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{S}_{r}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{C}_{irs}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{C}_{irs}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{C}_{ir;js}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{C}_{ir;js}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{CS}_{ir;s}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{CS}_{ir;s}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{S}_{rs}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{S}_{rs}|\mathcal{M}^{(0)}_{m+2}|^2$$

the definition of A_{12} is rather simple

$$\mathbf{A}_{12}|\mathcal{M}^{(0)}_{m+2}|^2 \equiv \mathbf{A}_1\mathbf{A}_2|\mathcal{M}^{(0)}_{m+2}|^2$$

but showing that it has the right properties is non trivial, and requires considering iterated singly-unresolved limits and strongly-ordered doubly-unresolved limits

Iterated counterterms

$$\boldsymbol{\mathcal{A}}_{12}|\mathcal{M}^{(0)}_{m+2}(\{p\})|^2 = \sum_t \left[\sum_{k\neq t} \frac{1}{2} \mathcal{C}_{kt}\boldsymbol{\mathcal{A}}_2|\mathcal{M}^{(0)}_{m+2}(\{p\})|^2 + \left(\mathcal{S}_t\boldsymbol{\mathcal{A}}_2|\mathcal{M}^{(0)}_{m+2}(\{p\})|^2 - \sum_{k\neq t} \mathcal{C}_{kt}\mathcal{S}_t\boldsymbol{\mathcal{A}}_2|\mathcal{M}^{(0)}_{m+2}(\{p\})|^2 \right) \right]$$

where

$$\mathcal{C}_{kt}\boldsymbol{\mathcal{A}}_2 = \sum_{r\neq k,t} \left[\mathcal{C}_{kt}\mathcal{C}_{ktr} + \mathcal{C}_{kt}\mathcal{CS}_{kt;r} - \mathcal{C}_{kt}\mathcal{C}_{ktr}\mathcal{CS}_{kt;r} - \mathcal{C}_{kt}\mathcal{C}_{rkt}\mathcal{S}_{kt} + \sum_{i\neq r,k,t} \left(\frac{1}{2}\mathcal{C}_{kt}\mathcal{C}_{ir;kt} - \mathcal{C}_{kt}\mathcal{C}_{ir;kt}\mathcal{CS}_{kt;r} \right) \right] + \mathcal{C}_{kt}\mathcal{S}_{kt}$$

and likewise for $\mathcal{S}_t\boldsymbol{\mathcal{A}}_2\,,\ \mathcal{C}_{kt}\mathcal{S}_t\boldsymbol{\mathcal{A}}_2$

Iterated counterterms

- the momentum mapping for each of the iterated counterterms is built out of a composition of either the NLO collinear or the NLO soft mappings, or of both

- the treatment of colour in iterated singly-unresolved limits differs for spin-correlated SME from that of colour-correlated SME

  → no soft factorization formulae for simultaneously colour-correlated and spin-correlated SME.
  This was a no-go in the direction of generalised dipole-type counterterms

RV counterterm

We note that the integrated A_1 counterterm of RR has the same explicit ε poles as RV.
Furthermore, as we said, the RV,A_1 counterterm takes care of the singly-unresolved limits of RV,
but we also need a term which takes care of the singly-unresolved limits of the integrated A_1 counterterm of RR

$$\sigma^{\mathrm{NNLO}}_{\{m+1\}} = \int_{m+1} \left\{ \left[\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right] J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] J_m \right\}_{\varepsilon=0}$$

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NNLO counterterms

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{NNLO}}_{\{m+2\}} + \sigma^{\mathrm{NNLO}}_{\{m+1\}} + \sigma^{\mathrm{NNLO}}_{\{m\}}$$

$$\sigma^{\mathrm{NNLO}}_{\{m+2\}} = \int_{m+2} \Big[\mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} J_m - \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} J_{m+1} + \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} J_m \Big]_{d=4}$$

$$\sigma^{\mathrm{NNLO}}_{\{m+1\}} = \int_{m+1} \left\{ \left[\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right] J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] J_m \right\}_{\varepsilon=0}$$

remainder must be finite by KLN theorem

$$\sigma^{\mathrm{NNLO}}_{\{m\}} = \int_m \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_m + \int_2 \left[\mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} \right] + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] \right\}_{\varepsilon=0} J_m$$

Integrating the counterterms

- momentum mappings used to define the counterterms

$$\{p\}_{n+u} \overset{\mathsf{R}}{\rightarrow} \{\tilde{p}\}_n \quad \Longrightarrow \quad \mathrm{d}\phi_{n+u}(\{p\};Q) = \mathrm{d}\phi_n(\{\tilde{p}\}_n^{(R)};Q)\,[\mathrm{d}p_{u,n}^{(R)}]$$

  - implement exact momentum conservation
  - different collinear and soft mappings, specified by R
  - exact factorisation of phase space

- in colour and spin space, counterterms are products of
  - factorised amplitudes independent of variables in $[\mathrm{d}p_{u,n}^{(R)}]$
  - singular factors, i.e. the universal collinear and soft currents, to be integrated over $[\mathrm{d}p_{u,n}^{(R)}]$

$$\mathcal{C}_R(\{p\}_{n+u}) = (8\pi\alpha_s\mu^{2\epsilon})^u\, Sing_R(p_u^{(R)}) \otimes |M_n^{(0)}(\{\tilde{p}\}_n^{(R)})|^2$$

- compute the integral of the counterterm over unresolved partons

$$\int_u \mathcal{C}_R(\{p\}_{n+u}) = (8\pi\alpha_s\mu^{2\epsilon})^u \left[\int_u Sing_R(p_u^{(R)})\right] \otimes |M_n^{(0)}(\{\tilde{p}\}_n^{(R)})|^2$$

List of integrated counterterms

Int	status
$\mathcal{I}^{(k)}_{1\mathcal{C},0}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},1}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},2}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},3}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},4}$	✔
$\mathcal{I}^{(k,l)}_{1\mathcal{C},5}$	✔
$\mathcal{I}^{(k,l)}_{1\mathcal{C},6}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},7}$	✔
$\mathcal{I}_{1\mathcal{C},8}$	✔

Int	status
$\mathcal{I}_{1\mathcal{S},0}$	✔
$\mathcal{I}_{1\mathcal{S},1}$	✔
$\mathcal{I}_{1\mathcal{S},2}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{S},3}$	✔
$\mathcal{I}_{1\mathcal{S},4}$	✔
$\mathcal{I}_{1\mathcal{S},5}$	✔
$\mathcal{I}_{1\mathcal{S},6}$	✔
$\mathcal{I}_{1\mathcal{S},7}$	✔

Int	status
$\mathcal{I}_{1\mathcal{CS},0}$	✔
$\mathcal{I}_{1\mathcal{CS},1}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{CS},2}$	✔
$\mathcal{I}_{1\mathcal{CS},3}$	✔
$\mathcal{I}_{1\mathcal{CS},4}$	✔

Int	status
$\mathcal{I}^{(k,l)}_{12\mathcal{C},1}$	✔
$\mathcal{I}^{(k,l)}_{12\mathcal{C},2}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},3}$	✔
$\mathcal{I}^{(k,l)}_{12\mathcal{C},4}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},5}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},6}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},7}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},8}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},9}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},10}$	✔

Int	status
$\mathcal{I}_{2\mathcal{S},1}$	✔
$\mathcal{I}_{2\mathcal{S},2}$	✔
$\mathcal{I}_{2\mathcal{S},3}$	✔
$\mathcal{I}_{2\mathcal{S},4}$	✔
$\mathcal{I}_{2\mathcal{S},5}$	✔
$\mathcal{I}_{2\mathcal{S},6}$	✔
$\mathcal{I}_{2\mathcal{S},7}$	✔
$\mathcal{I}_{2\mathcal{S},8}$	✔
$\mathcal{I}_{2\mathcal{S},9}$	✔
$\mathcal{I}_{2\mathcal{S},10}$	✔
$\mathcal{I}_{2\mathcal{S},11}$	✔
$\mathcal{I}_{2\mathcal{S},12}$	✔
$\mathcal{I}_{2\mathcal{S},13}$	✔
$\mathcal{I}_{2\mathcal{S},14}$	✔
$\mathcal{I}_{2\mathcal{S},15}$	✔
$\mathcal{I}_{2\mathcal{S},16}$	✔
$\mathcal{I}_{2\mathcal{S},17}$	✔
$\mathcal{I}_{2\mathcal{S},18}$	✔
$\mathcal{I}_{2\mathcal{S},19}$	✔
$\mathcal{I}_{2\mathcal{S},20}$	✔
$\mathcal{I}_{2\mathcal{S},21}$	✔
$\mathcal{I}_{2\mathcal{S},22}$	✔
$\mathcal{I}_{2\mathcal{S},23}$	✔

Int	status
$\mathcal{I}^{(k)}_{12\mathcal{S},1}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{S},2}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{S},3}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{S},4}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{S},5}$	✔
$\mathcal{I}_{12\mathcal{S},6}$	✔
$\mathcal{I}_{12\mathcal{S},7}$	✔
$\mathcal{I}_{12\mathcal{S},8}$	✔
$\mathcal{I}_{12\mathcal{S},9}$	✔
$\mathcal{I}_{12\mathcal{S},10}$	✔
$\mathcal{I}_{12\mathcal{S},11}$	✔
$\mathcal{I}_{12\mathcal{S},12}$	✔
$\mathcal{I}_{12\mathcal{S},13}$	✔

Int	status
$\mathcal{I}^{(k)}_{12\mathcal{CS},1}$	✔
$\mathcal{I}_{12\mathcal{CS},2}$	✔
$\mathcal{I}_{12\mathcal{CS},3}$	✔

Int	status
$\mathcal{I}^{(j,k,l,m)}_{2\mathcal{C},1}$	✔
$\mathcal{I}^{(j,k,l,m)}_{2\mathcal{C},2}$	✔
$\mathcal{I}^{(j,k,l,m)}_{2\mathcal{C},3}$	✔
$\mathcal{I}^{(j,k,l,m)}_{2\mathcal{C},4}$	✔
$\mathcal{I}^{(-1,-1,-1,-1)}_{2\mathcal{C},5}$	✔
$\mathcal{I}^{(k,l)}_{2\mathcal{C},6}$	✔

Int	status
$\mathcal{I}^{(k)}_{2\mathcal{CS},1}$	✔
$\mathcal{I}^{(k)}_{2\mathcal{CS},2}$	✔
$\mathcal{I}^{(2)}_{2\mathcal{CS},2}$	✔
$\mathcal{I}^{(k)}_{2\mathcal{CS},3}$	✔
$\mathcal{I}^{(k)}_{2\mathcal{CS},4}$	✔
$\mathcal{I}^{(k)}_{2\mathcal{CS},5}$	✔

- coefficients of ε poles computed analytically through Mellin-Barnes representation; finite parts computed numerically
- whole computation checked numerically through sector decomposition

Poles cancel

thanks to KLN theorem, all ε poles must cancel out of

$$\sigma^{\mathrm{NNLO}}_{\{m\}} = \int_m \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_m + \int_2 \left[\mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} \right] + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] \right\}_{\varepsilon=0} J_m$$

- checked the cancellation of $1/\varepsilon^4$ and $1/\varepsilon^3$ poles for any number of jets, *i.e.* for any *m*
- checked the cancellation of all ε poles for *m*=2 $\quad e^+e^- \to q\bar{q}, \quad H \to b\bar{b}$
- checked the cancellation of all ε poles for *m*=3 $\quad e^+e^- \to q\bar{q}g$

$H \to bb$

- double virtual contribution at $\mu^2 = m_H^2$

$$\mathrm{d}\sigma^{\mathrm{VV}}_{H\to b\bar{b}} = \left(\frac{\alpha_{\mathrm{s}}(\mu^2)}{2\pi}\right)^2 \mathrm{d}\sigma^{\mathrm{B}}_{H\to b\bar{b}} \left\{ \frac{2C_{\mathrm{F}}^2}{\epsilon^4} + \left(\frac{11 C_{\mathrm{A}} C_{\mathrm{F}}}{4} + 6C_{\mathrm{F}}^2 - \frac{C_{\mathrm{F}} n_{\mathrm{f}}}{2}\right)\frac{1}{\epsilon^3} + \left[\left(\frac{8}{9} + \frac{\pi^2}{12}\right) C_{\mathrm{A}} C_{\mathrm{F}} + \left(\frac{17}{2} - 2\pi^2\right) C_{\mathrm{F}}^2 - \frac{2 C_{\mathrm{F}} n_{\mathrm{f}}}{9}\right]\frac{1}{\epsilon^2} + \left[\left(-\frac{961}{216} + \frac{13\zeta_3}{2}\right) C_{\mathrm{A}} C_{\mathrm{F}} + \left(\frac{109}{8} - 2\pi^2 - 14\zeta_3\right) C_{\mathrm{F}}^2 + \frac{65 C_{\mathrm{F}} n_{\mathrm{f}}}{108}\right]\frac{1}{\epsilon} \right\}$$

Anastasiou Herzog Lazopoulos 2011

- sum of integrated counterterms at $\mu^2 = m_H^2$

$$\sum \int \mathrm{d}\sigma^{\mathrm{A}} = \left(\frac{\alpha_{\mathrm{s}}(\mu^2)}{2\pi}\right)^2 \mathrm{d}\sigma^{\mathrm{B}}_{H\to b\bar{b}} \left\{ \frac{-2C_{\mathrm{F}}^2}{\epsilon^4} + \left(-\frac{11 C_{\mathrm{A}} C_{\mathrm{F}}}{4} - 6C_{\mathrm{F}}^2 + \frac{C_{\mathrm{F}} n_{\mathrm{f}}}{2}\right)\frac{1}{\epsilon^3} + \left[\left(-\frac{8}{9} - \frac{\pi^2}{12}\right) C_{\mathrm{A}} C_{\mathrm{F}} + \left(-\frac{17}{2} + 2\pi^2\right) C_{\mathrm{F}}^2 + \frac{2 C_{\mathrm{F}} n_{\mathrm{f}}}{9}\right]\frac{1}{\epsilon^2} + \left[\left(\frac{961}{216} - \frac{13\zeta_3}{2}\right) C_{\mathrm{A}} C_{\mathrm{F}} + \left(-\frac{109}{8} + 2\pi^2 + 14\zeta_3\right) C_{\mathrm{F}}^2 - \frac{65 C_{\mathrm{F}} n_{\mathrm{f}}}{108}\right]\frac{1}{\epsilon} \right\}$$

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H →bb

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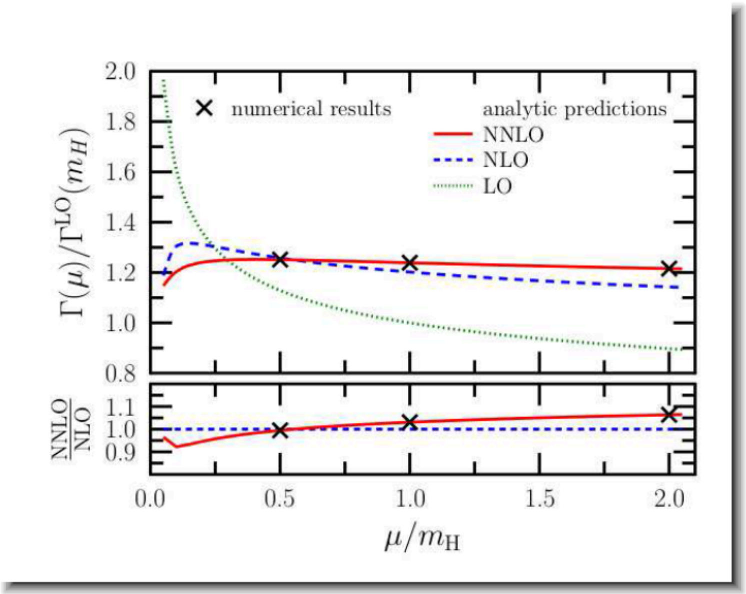
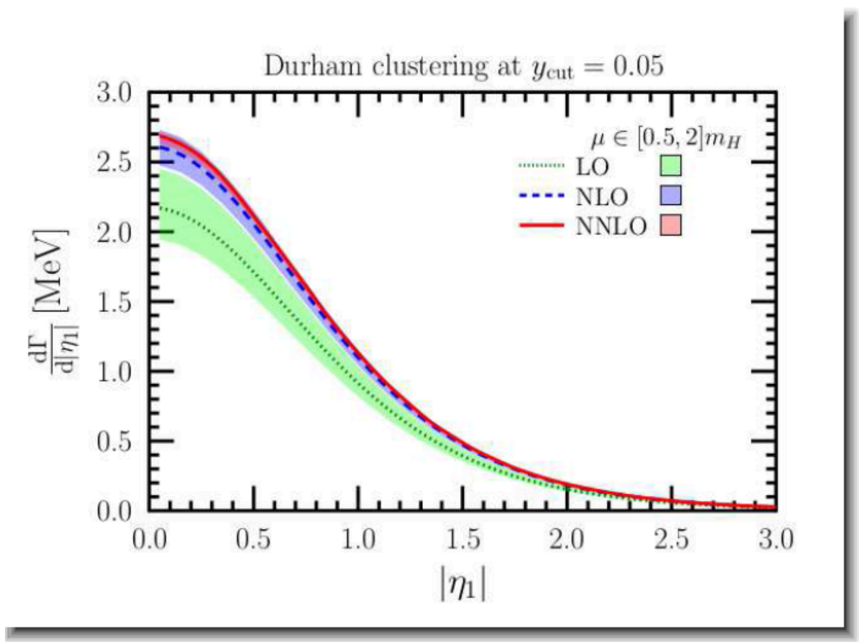
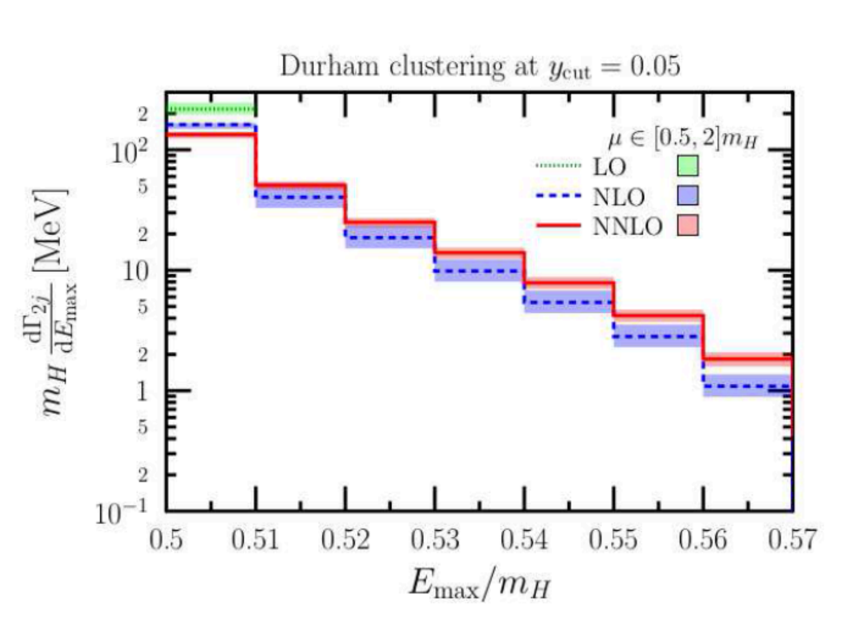


inclusive decay rate





highest energy jet pseudorapidity

leading jet energy

$e^+ e^- \to 3$ jets

- double virtual contribution at $\mu^2 = s$

$$d\sigma_3^{VV} = Poles\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right) + Finite\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right)$$

where

$$\mathcal{P}oles\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right) = 2\Big[-\left(I^{(1)}_{q\bar{q}g}(\epsilon)\right)^2 - \frac{\beta_0}{\epsilon} I^{(1)}_{q\bar{q}g}(\epsilon)$$
$$+ e^{-\epsilon\gamma}\frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{\beta_0}{\epsilon} + K\right) I^{(1)}_{q\bar{q}g}(2\epsilon) + H^{(2)}_{q\bar{q}g}\Big] A_3^0(1_q, 3_g, 2_{\bar{q}})$$
$$+ 2 I^{(1)}_{q\bar{q}g}(\epsilon) A_3^{1\times 0}(1_q, 3_g, 2_{\bar{q}})$$

with

$$H^{(2)}_{q\bar{q}g} = \frac{e^{\epsilon\gamma}}{4\epsilon\Gamma(1-\epsilon)}\Big[\left(4\zeta_3 + \frac{589}{432} - \frac{11\pi^2}{72}\right) N_c + \left(-\frac{1}{2}\zeta_3 - \frac{41}{54} - \frac{\pi^2}{48}\right)$$
$$+ \left(-3\zeta_3 - \frac{3}{16} + \frac{\pi^2}{4}\right)\frac{1}{N_c} + \left(-\frac{19}{18} + \frac{\pi^2}{36}\right) N_c n_f + \left(-\frac{1}{54} - \frac{\pi^2}{24}\right)\frac{n_f}{N_c} + \frac{5}{27} n_f^2\Big]$$

Gehrmann-De Ridder, Gehrmann, Glover, Heinrich 2007

$e^+ e^- \to 3$ jets

- $$Poles\left(A_3^{(2\times0)} + A_3^{(1\times1)}\right) + Poles\sum\int \mathrm{d}\sigma^A = 117k$$ terms

- zero numerically in any phase space point using sector decomposition

- zero analytically using symbol technology C. Duhr 2015

- $$Finite\sum\int \mathrm{d}\sigma^A$$

- we compute finite part of integrated counterterms numerically and fit numbers with a formula which contains polynomials of $\log(y_{ij})$ and $\log(1-y_{ij})$

3-jet event shape variables

$$\frac{O}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}O} = \frac{\alpha_s(Q)}{2\pi}A_O + \left(\frac{\alpha_s(Q)}{2\pi}\right)^2 B_O + \left(\frac{\alpha_s(Q)}{2\pi}\right)^3 C_O$$

$C_O = C_{O;5} + C_{O;4} + C_{O;3}$ is NNLO contribution

RR RV VV

- $C_{O;5}$ and $C_{O;4}$ have been computed and shown to be finite for $e^+e^- \to q\bar{q}ggg$ and $e^+e^- \to q\bar{q}gg$ G. Somogyi 2006

  O = *C* and *O* = *I* - *T*

- Perfect agreement with NLO results for B_O

Thrust

$$T = \text{Max} \frac{\sum_i |\mathbf{p_i} \cdot \mathbf{n}|}{\sum_i |\mathbf{p_i}|}$$

sum over all final-state particles *i*

n defines direction of thrust axis n_T by maximising *T*

T = 1 for back-to-back jets

T = 1/2 for isotropic distribution of particles (spherical events)

2/3 ≤ *T* ≤ 1 for 3-jet events

C parameter

$$C = 3(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1)$$

where λ_α are eigenvalues of $\Theta^{\alpha\beta} = \frac{\sum_i p_i^\alpha p_i^\beta / |\mathbf{p_i}|}{\sum_j |\mathbf{p_j}|}$ $\quad \alpha, \beta = 1, 2, 3$

For massless particles $C = 3 - \frac{3}{2} \sum_{i,j} \frac{(p_i \cdot p_j)^2}{(p_i \cdot Q)(p_j \cdot Q)}$ $\quad Q = \sum_i p_i^\mu$

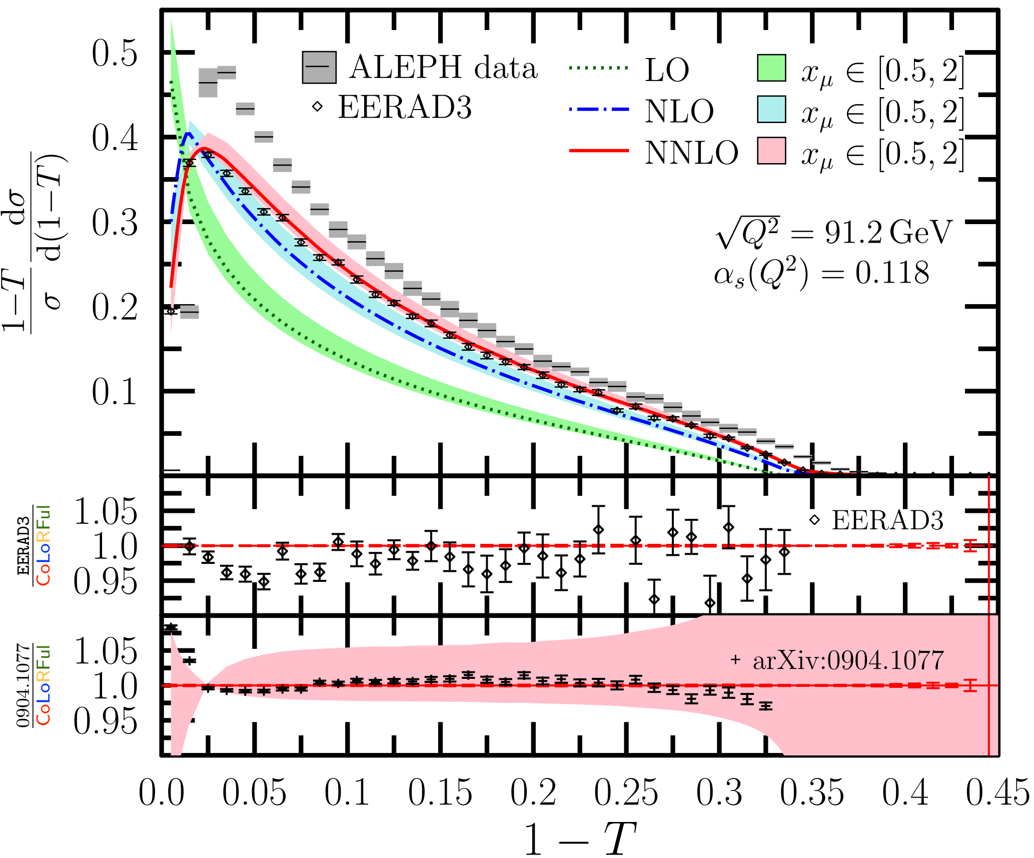
C = 0 for back-to-back jets

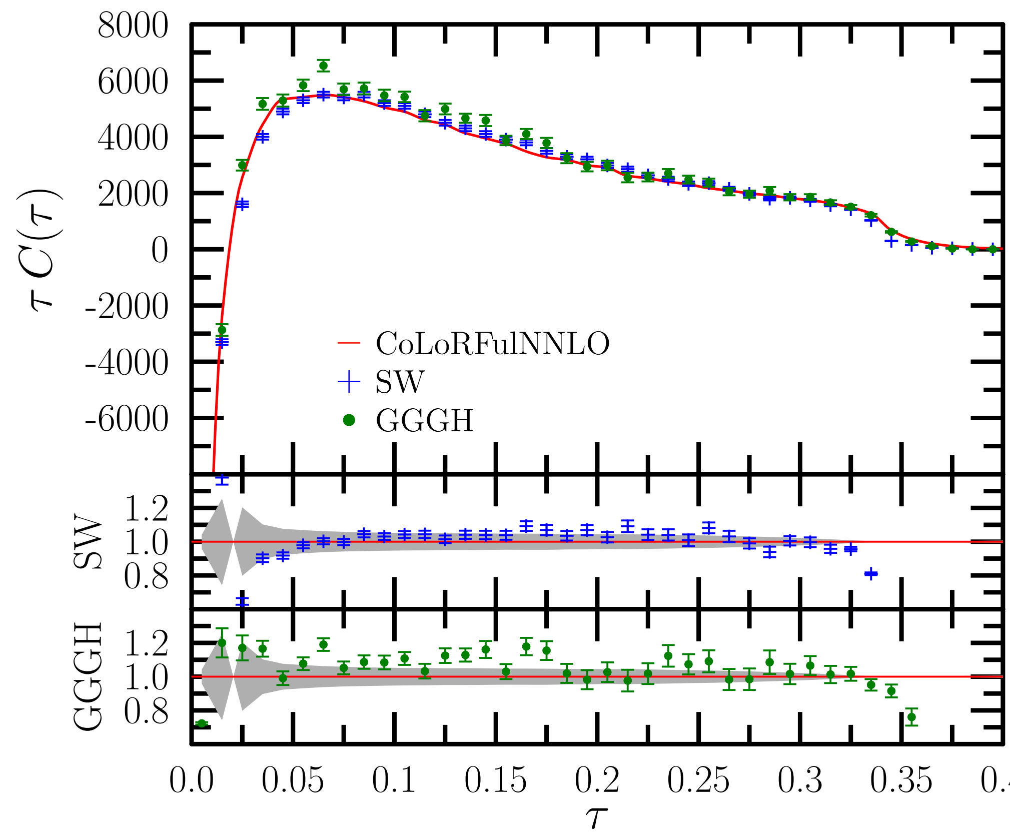
C = 1 for isotropic distribution of (at least 4) jets

0 ≤ *C* ≤ 3/4 for 3-jet events

Thrust

Duhr Kardos Somogyi Trocsanyi VDD, in preparation



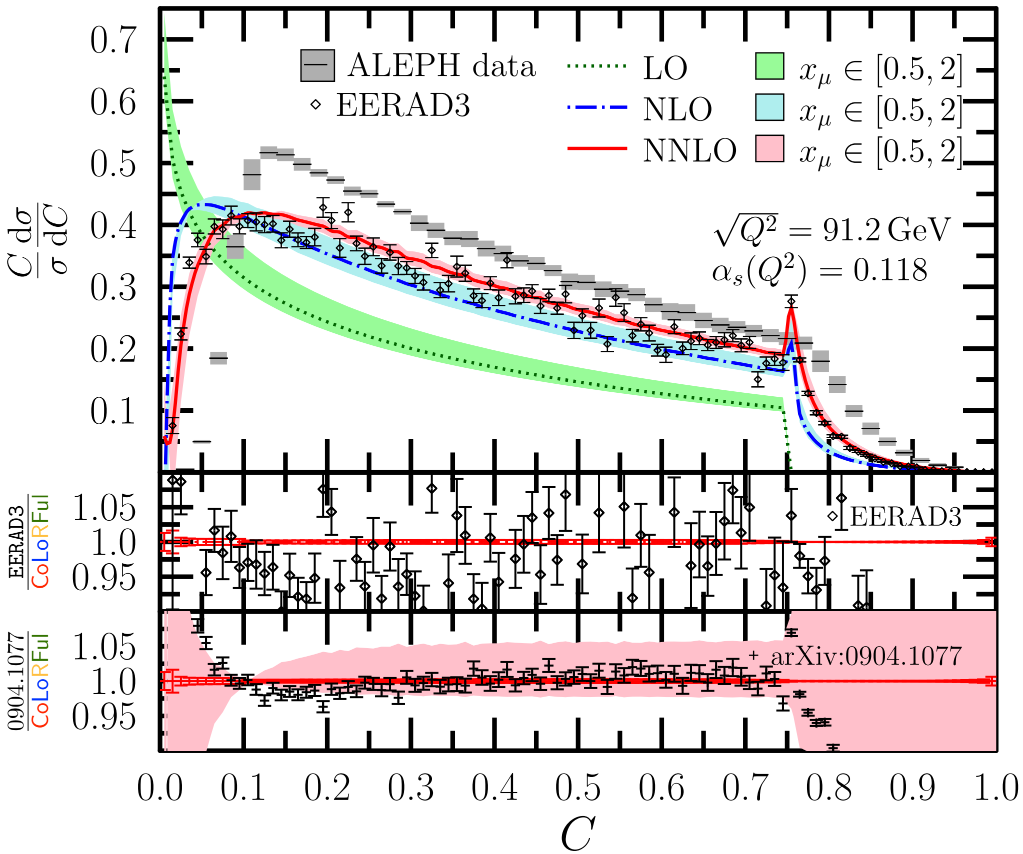


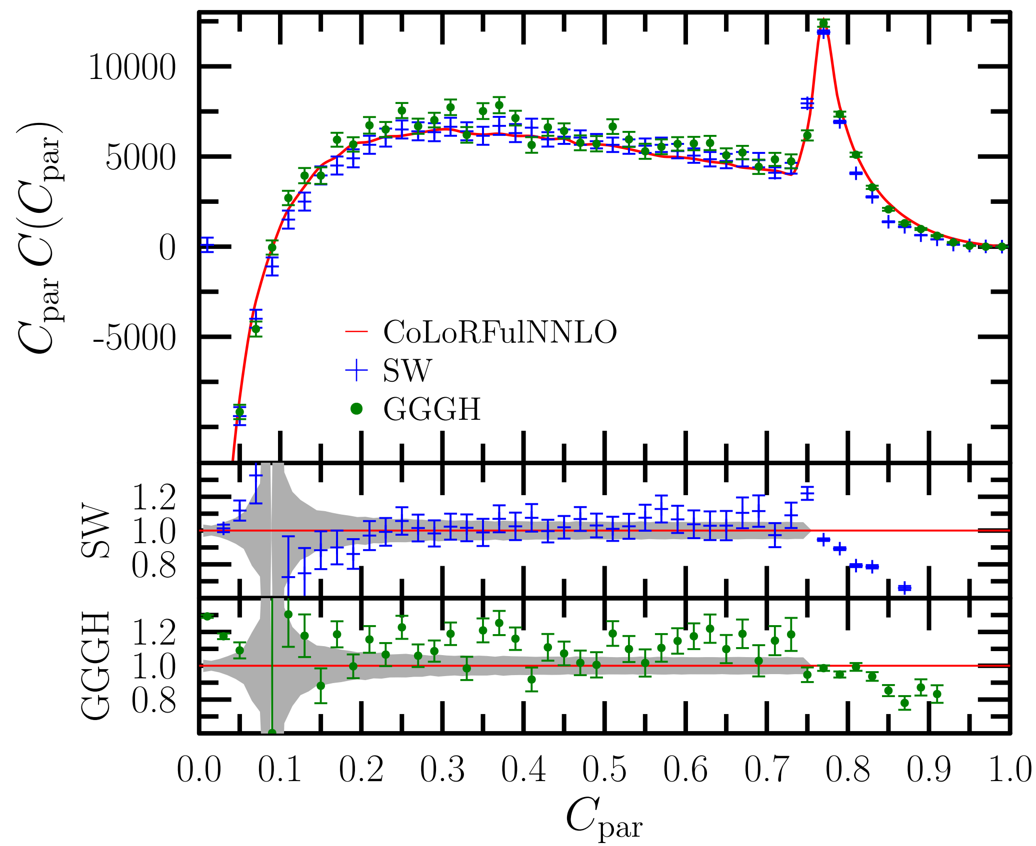
NNLO coefficient

- generally good agreement with EERAD3 & Weinzierl
- differences in the 4-jet region
  (our computation checked vs. aMC@NLO to 1% accuracy)

C parameter

Duhr Kardos Somogyi Trocsanyi VDD, in preparation





NNLO coefficient

Oblateness

- thrust major $$T_M = \max_{\vec{n}\cdot\vec{n}_T=0}\left(\frac{\sum_i |\vec{n}\cdot\vec{p}_i|}{\sum_i |\vec{p}_i|}\right)$$

  n defines direction of thrust-major axis n_{TM} by maximising T_M over all directions orthogonal to thrust axis n_T

- thrust minor $$T_m = \frac{\sum_i |\vec{n}_{T_m}\cdot\vec{p}_i|}{\sum_i |\vec{p}_i|}$$

  thrust-minor axis $\vec{n}_{T_m} = \vec{n}_T \times \vec{n}_{T_M}$
  defined as orthogonal to both the thrust and thrust-major axes

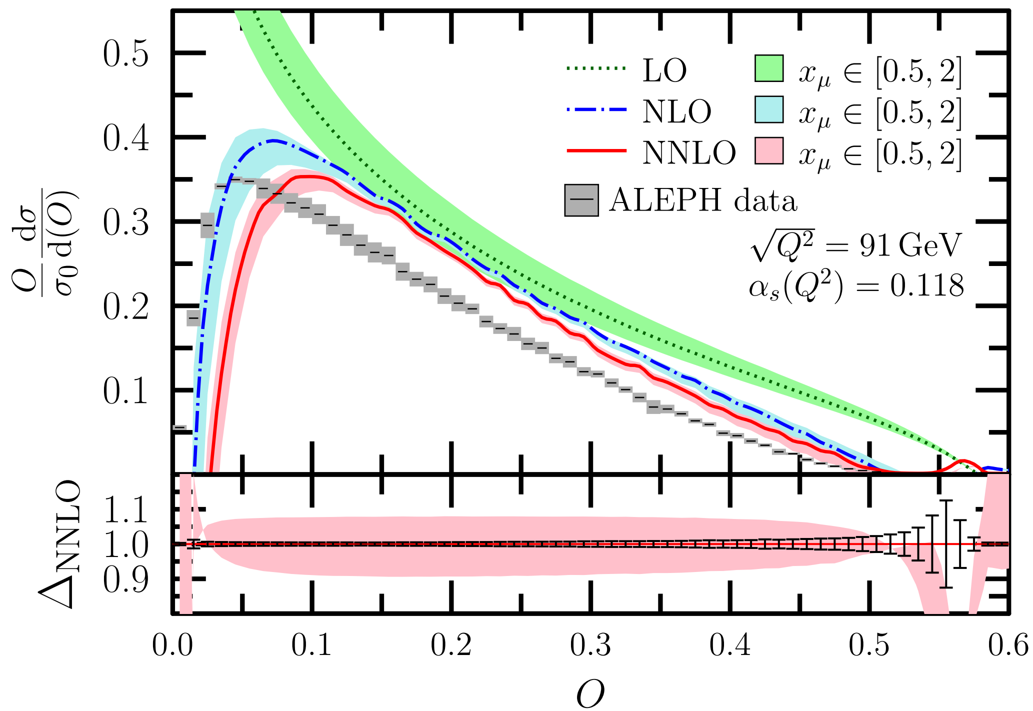
- oblateness is the difference between thrust major and thrust minor

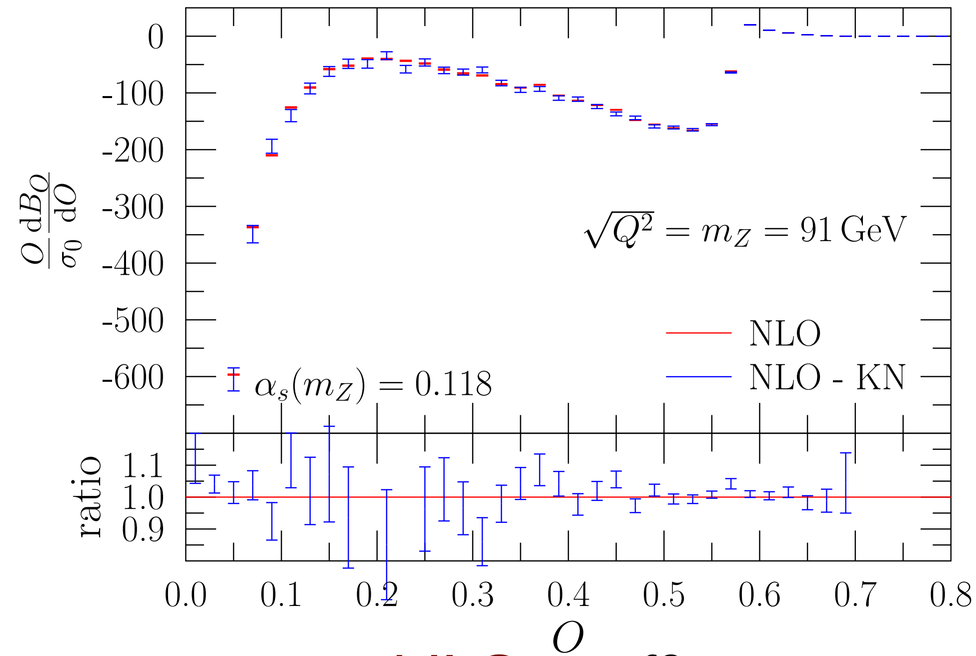
$$O = T_M - T_m$$

$O = 0$ for back-to-back jets
$0 \leq O \leq 1/\sqrt{3}$ for 3-jet events

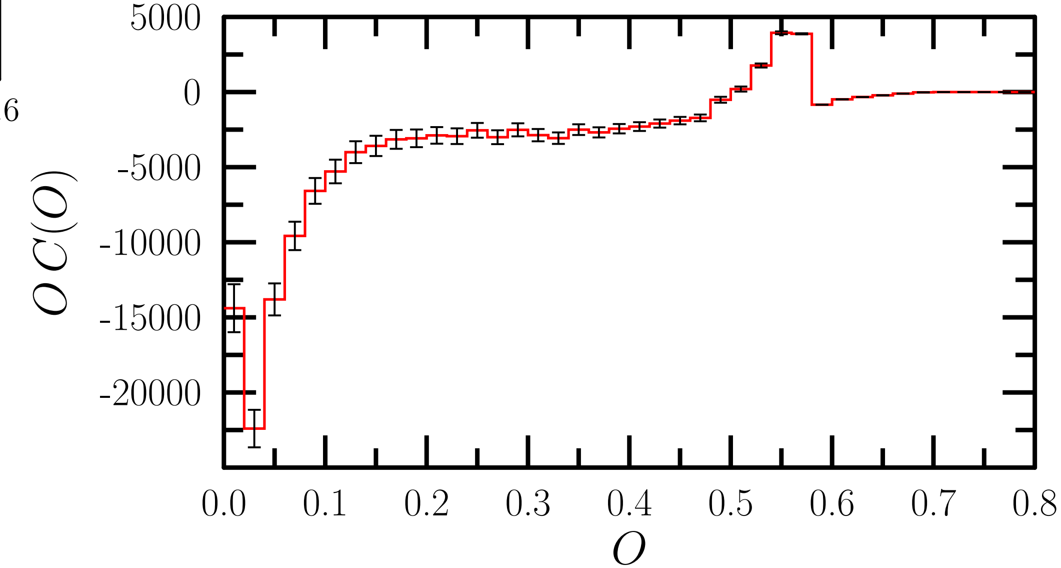
Oblateness

Duhr Kardos Somogyi Trocsanyi VDD, in preparation





NLO coefficient



NNLO coefficient

Energy-energy correlation

energy-energy correlation is the normalised energy-weighted cross section defined in terms of the angle between two particles *i* and *j* in an event

$$EEC(\chi) = \frac{1}{\sigma_{\rm had}} \sum_{i,j} \int \frac{E_i E_j}{Q^2} \mathrm{d}\sigma_{e^+e^- \to i\, j + X} \delta(\cos\chi + \cos\theta_{ij})$$

$\theta_{ij} = \pi - \chi$ angle between the 2 particles

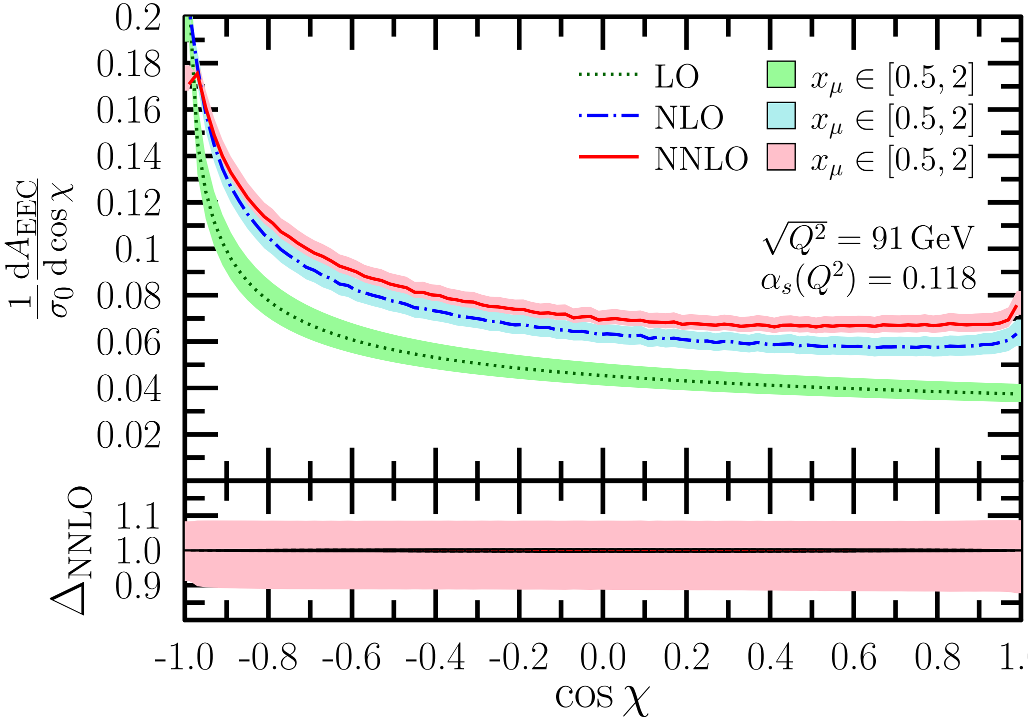
Q = centre-of-mass energy

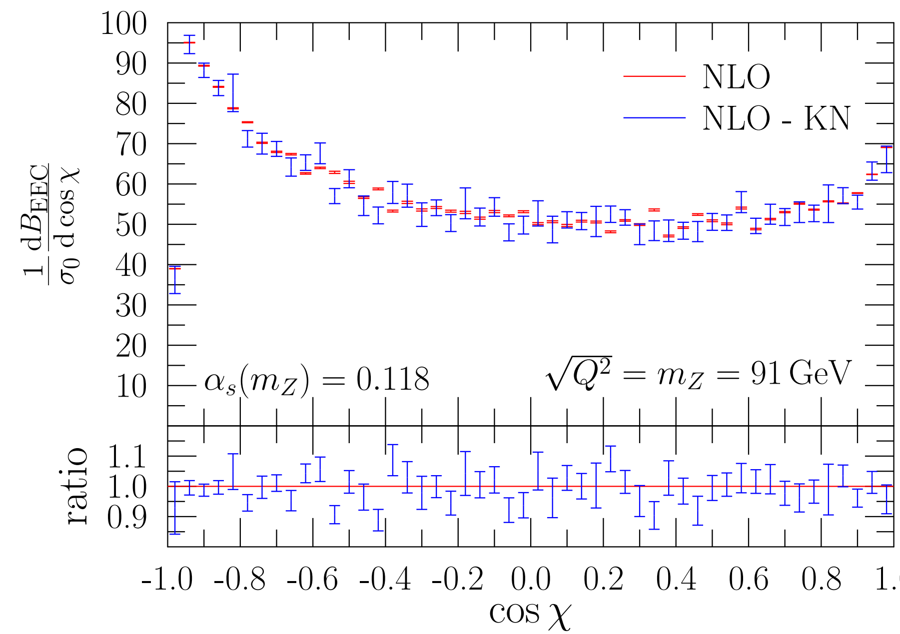
E_i E_j = particle energies

σ_{had} = total cross section

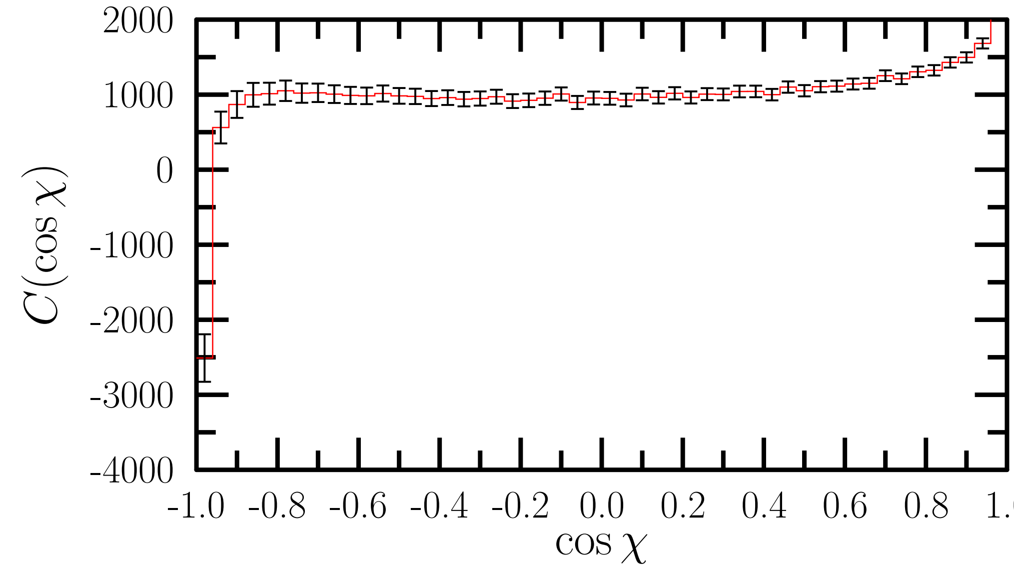
Energy-energy correlation

Duhr Kardos Somogyi Trocsanyi VDD, in preparation





NLO coefficient



NNLO coefficient

code performance

- RR on one core: 10M PS points in 9hrs
  RV on one core: 10M PS points in 31hrs

- code runs on 300 cores

- RR: to match Weinzierl's binning we need ~ 15B PS points → ~ 45hrs
  RV smooth with ~ 1.5B PS points → ~ 15hrs
  VV runs in ~ 2hrs

Conclusions

- we devised a NNLO subtraction scheme for $e^+e^- \longrightarrow n$ jets
- the calculation is organised into 3 contributions, RR, RV, VV, each of which finite in *d*=4 dimensions
- the code can compute any 3-jet event shape at NNLO
- extension to jets in hadron collisions in progress
- possible improvements on the method?
  - finite part of integrated counterterms analytically — Dulat Mistlberger Somogyi, in progress
  - more efficient organisation of counterterms — Bevilacqua, in progress
  - inclusion of external massive fermions