# Precise Predictions for Higgs production at hadron colliders

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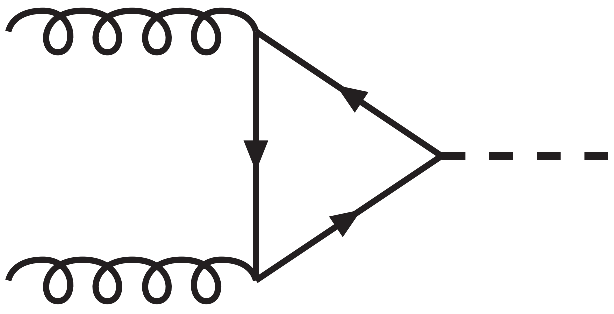
KITP Santa Barbara
LHC Run II and the precision frontier

# Higgs production at the LHC

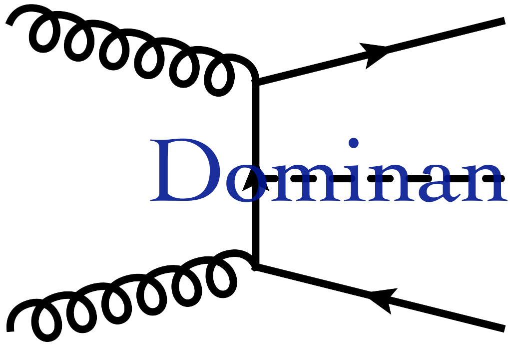
- Establishing whether the BEH mechanism and its boson is SM-like will be of utmost importance for the run of the LHC.

- Higgs-boson production modes at the LHC:

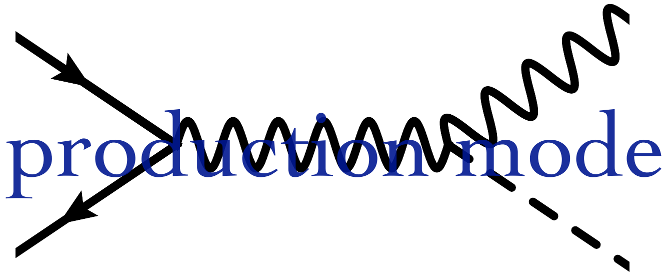


Gluon fusion

Dominant production mode at the LHC



TTH

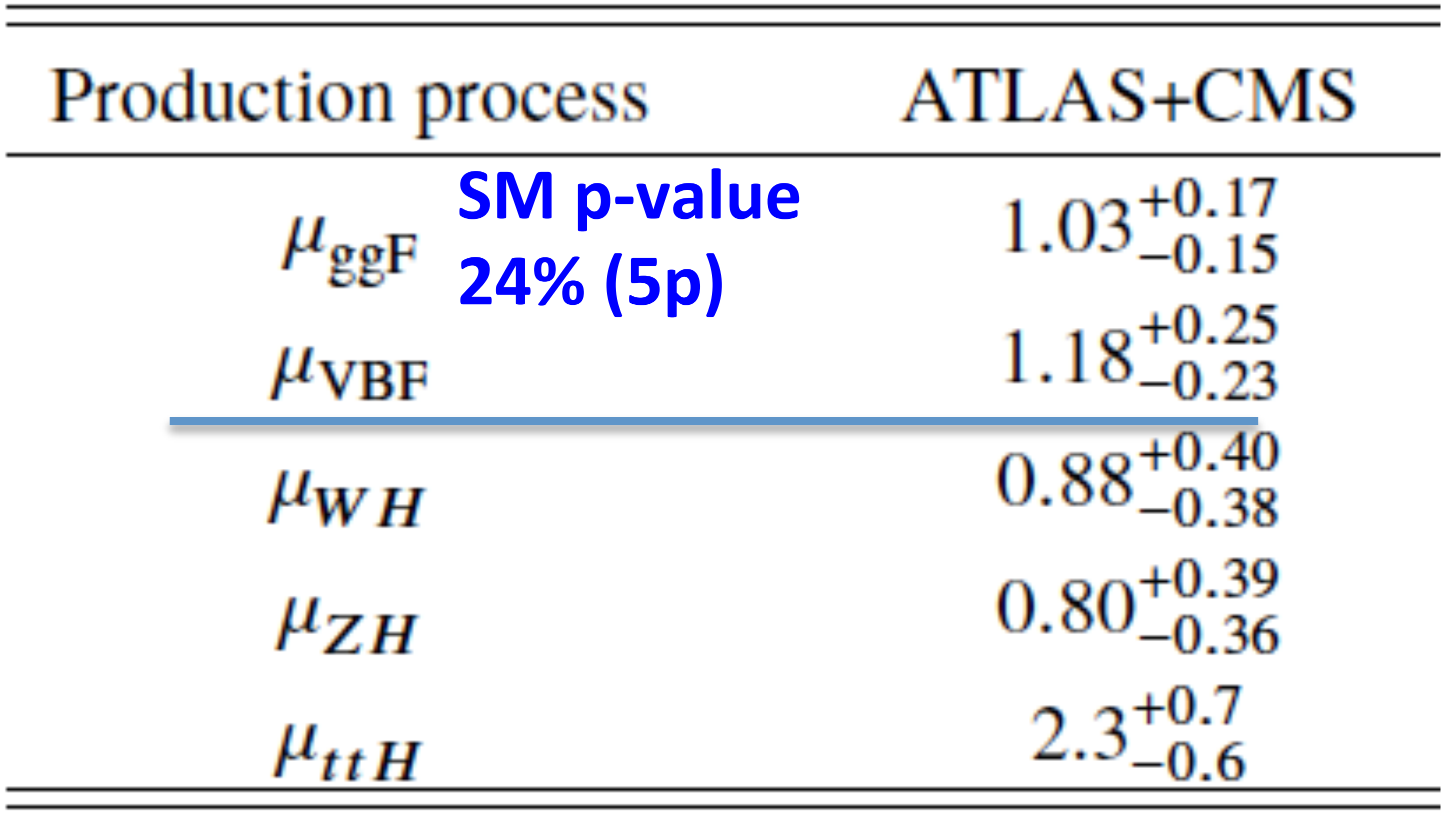


Higgs strahlung

VBF

- We want to know the gluon-fusion cross section precisely!

# Higgs production at the LHC



| Production process | ATLAS+CMS |
|---|---|
| $\mu_{\mathrm{ggF}}$ | $1.03^{+0.17}_{-0.15}$ |
| $\mu_{\mathrm{VBF}}$ | $1.18^{+0.25}_{-0.23}$ |
| $\mu_{WH}$ | $0.88^{+0.40}_{-0.38}$ |
| $\mu_{ZH}$ | $0.80^{+0.39}_{-0.36}$ |
| $\mu_{ttH}$ | $2.3^{+0.7}_{-0.6}$ |

**SM p-value 24% (5p)**

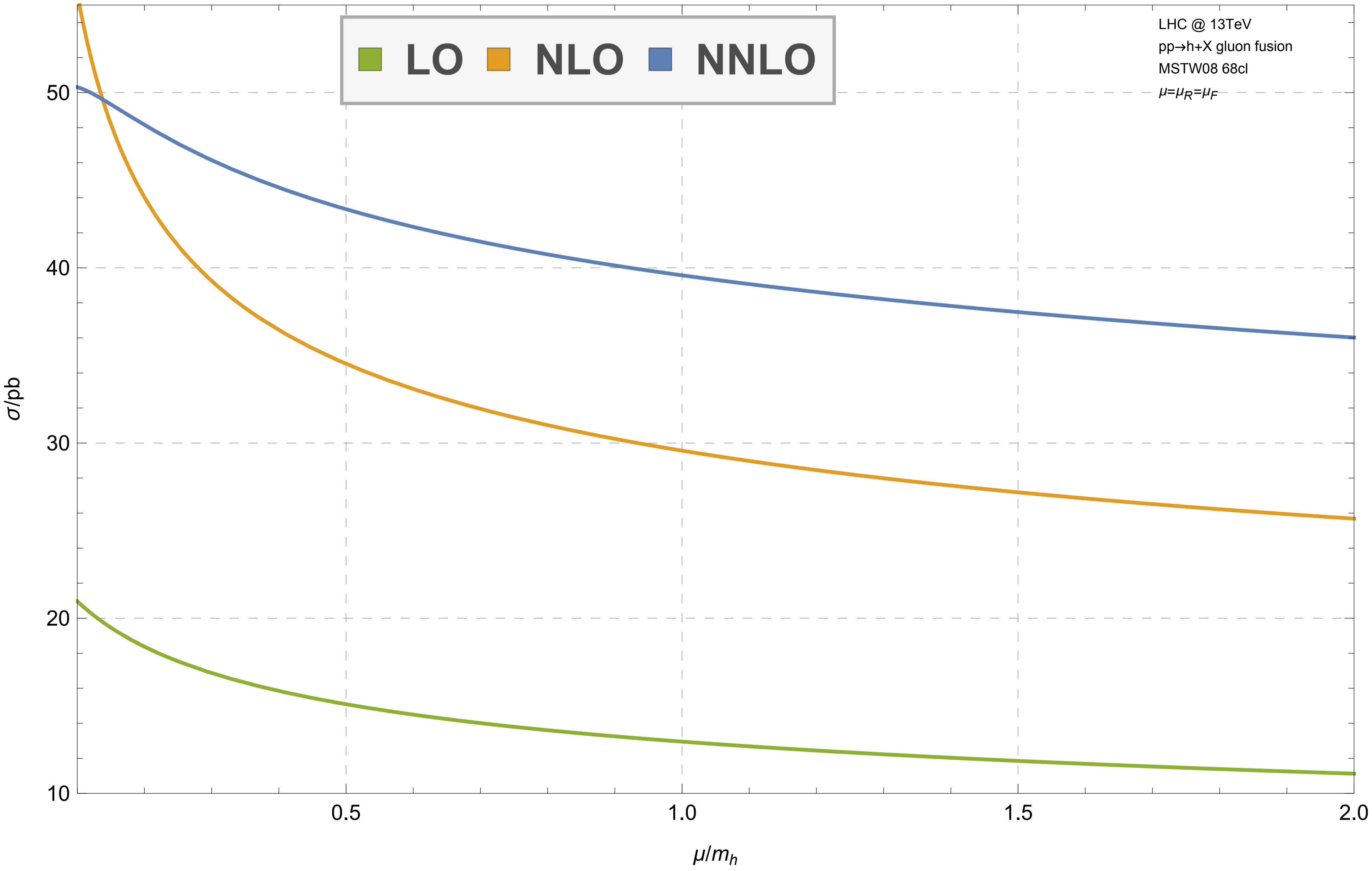
$$\mu = 1.09^{+0.11}_{-0.10} = 1.09^{+0.07}_{-0.07}\,(\mathrm{stat})\,^{+0.04}_{-0.04}\,(\mathrm{expt})\,^{+0.03}_{-0.03}\,(\mathrm{thbgd})^{+0.07}_{-0.06}\,(\mathrm{thsig})$$

[E. Gross @ KITP 2016]

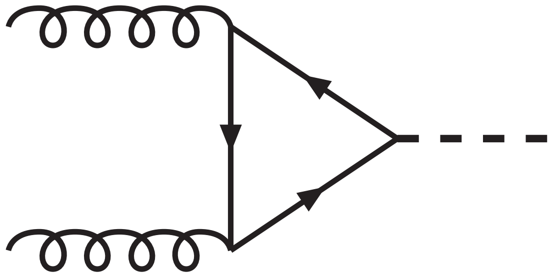
# Higgs production at the LHC

- Known at NLO and NNLO, but plagued by large perturbative uncertainties. [Dawson; Djouadi, Spira, Zerwas; Harlander, Kilgore; Anastasiou, Melnikov; Ravindran, Smith, van Neerven]

$$\sigma_{p\,p\to H} = \sigma_0(\mu) + \alpha_s(\mu)\,\sigma_1(\mu) + \alpha_s(\mu)^2\,\sigma_2(\mu) + \ldots \qquad \alpha_s(m_Z) = 0.118$$

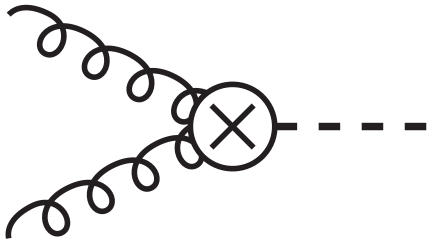


# The gluon fusion cross section



- The dominant Higgs production mechanism at the LHC is gluon fusion.
  - ➡ Loop-induced process.
- For a light Higgs boson, the dimension five operator describing a tree-level coupling of the gluons to the Higgs boson is

$$\mathcal{L} = \mathcal{L}_{QCD,5} - \frac{1}{4v} C_1 \, H \, G^a_{\mu\nu} \, G^{\mu\nu}_a$$



- Top-mass corrections known at NNLO.

[Harlander, Ozeren; Pak, Rogal, Steinhauser; Ball, Del Duca, Marzani, Forte, Vicini; Harlander, Mantler, Marzani, Ozeren]

- For now, I will concentrate on the effective theory, and comment on quark mass effects at the end.

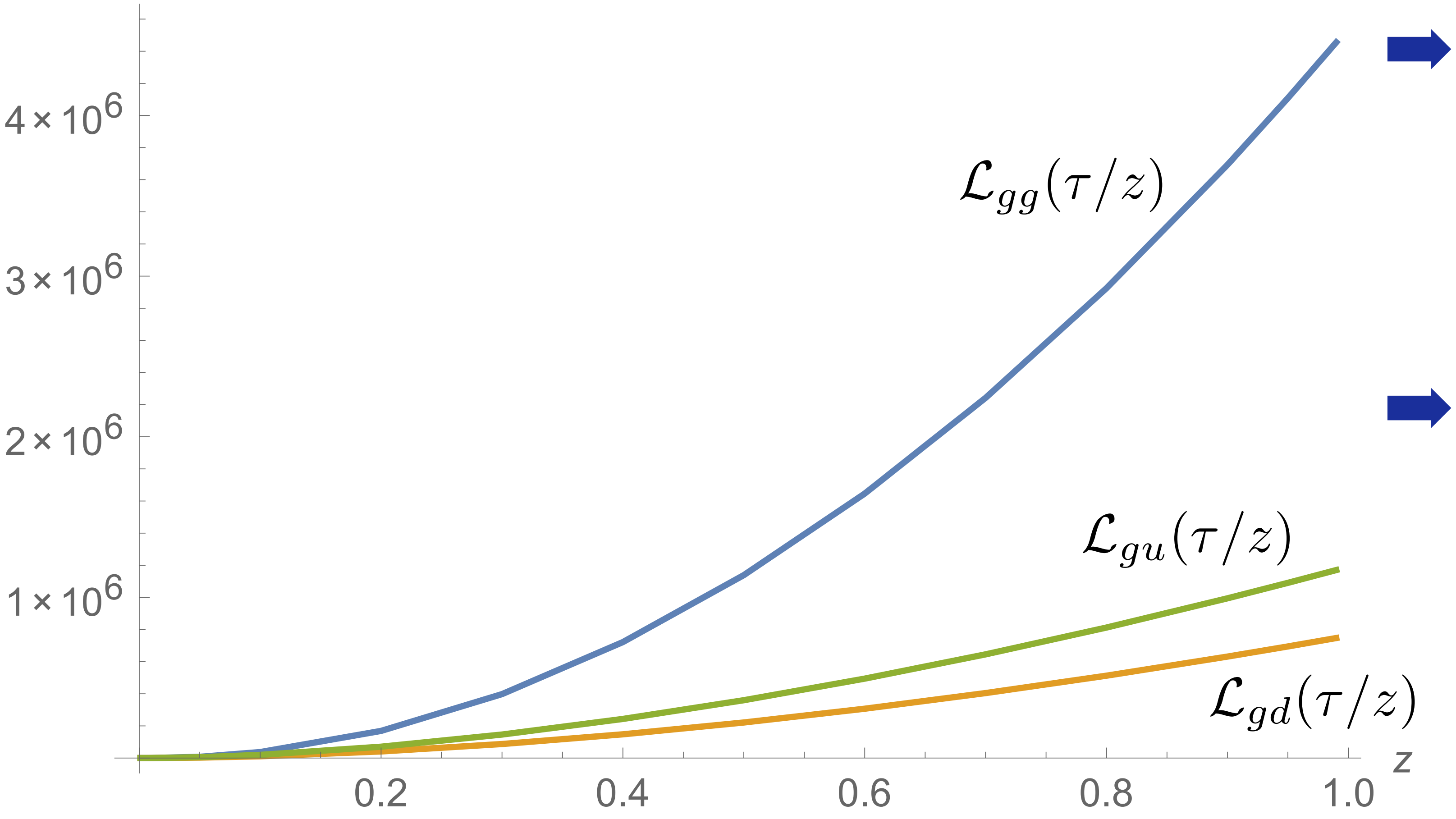
# The gluon fusion cross section

- The gluon fusion cross section is given in perturbation theory by

$$\sigma = \tau \sum_{ij} \int_\tau^1 \frac{dz}{z} \mathcal{L}_{ij}(\tau/z) \frac{\hat{\sigma}_{ij}(z)}{z}$$

$$z = \frac{m_H^2}{\hat{s}}$$

$$\tau = \frac{m_H^2}{S} \simeq 10^{-4}$$



➡ Main contribution from region where $z \simeq 1$.

➡ Physically: production at threshold + emission of soft partons.

# The threshold expansion

- Steep fall of the gluon luminosity!
  - ➡ Approximate partonic cross sections by threshold expansion:

$$\hat{\sigma}(z) = \sigma_{-1} + \sigma_0 + (1-z)\,\sigma_1 + \mathcal{O}(1-z)^2$$

  - ➡ NNLO result was obtained in this way.
- Goal: Compute cross section as a series around threshold!
- Challenge: Never has an N3LO computation been performed so far...
  - ➡ Uncharted territory!
  - ➡ New conceptual challenges.

# Outline

- Computing at N3LO.

- Phenomenology at N3LO:
  - ➡ Scale variation & higher-order QCD effects.
- Other corrections:
  - ➡ PDFs, quark masses and electroweak effects.

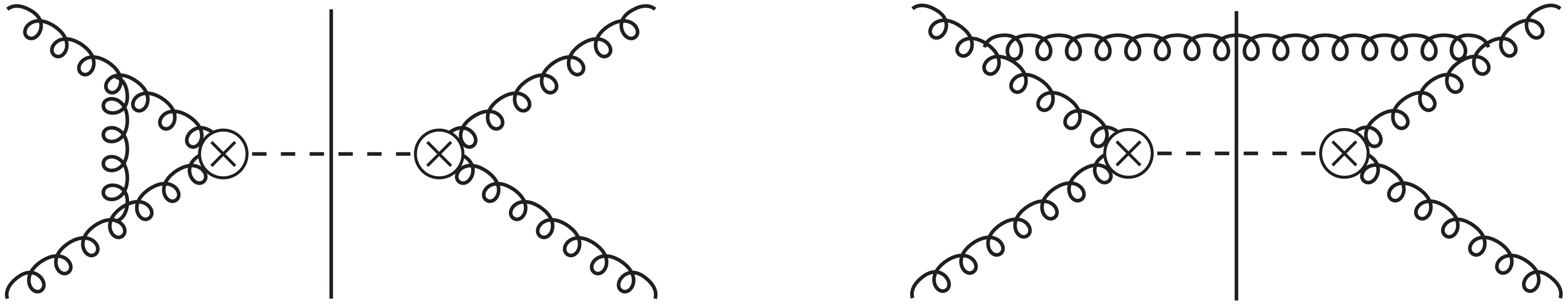
# Computing at N3LO

## Pushing the boundary

# The gluon fusion cross section

- At NLO, there are two contributions (~1991):

[Dawson; Djouadi, Spira, Zerwas]



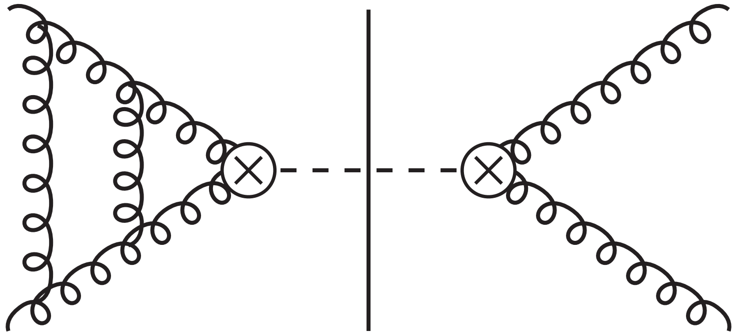
Virtual corrections (‘loops’) Real emission

- Both contributions are individually divergent:
  - ➡ UV divergences are handled by renormalization.
  - ➡ IR divergences cancelled by PDF counterterms.

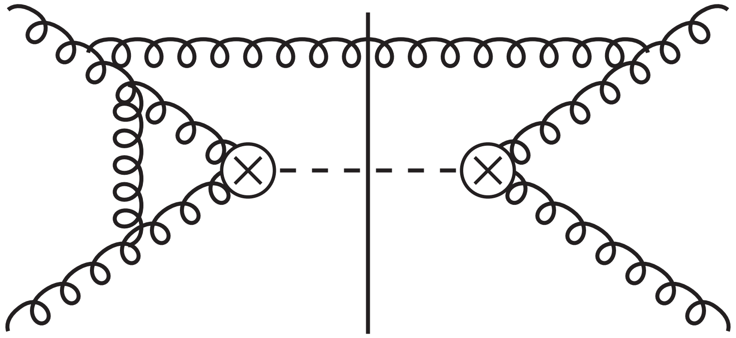
# The gluon fusion cross section

- At NNLO, there are three contributions (2002):

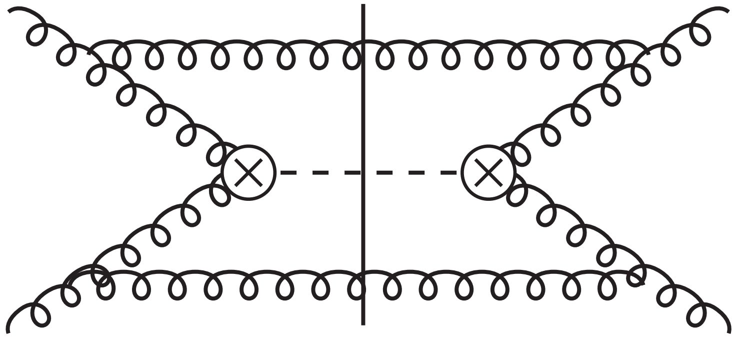
[Harlander, Kilgore; Anastasiou, Melnikov; Ravindran, Smith, van Neerven]



Double virtual



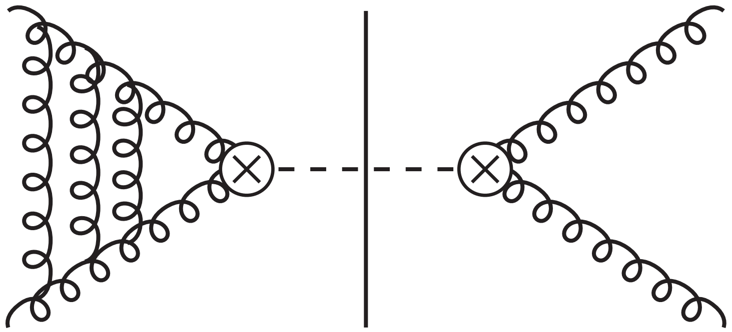
Real-virtual



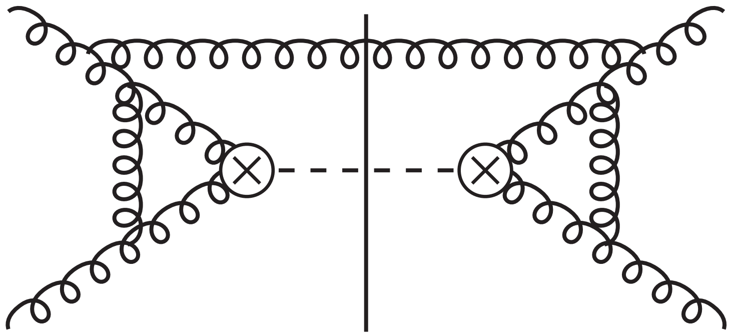
Double real

# The gluon fusion cross section

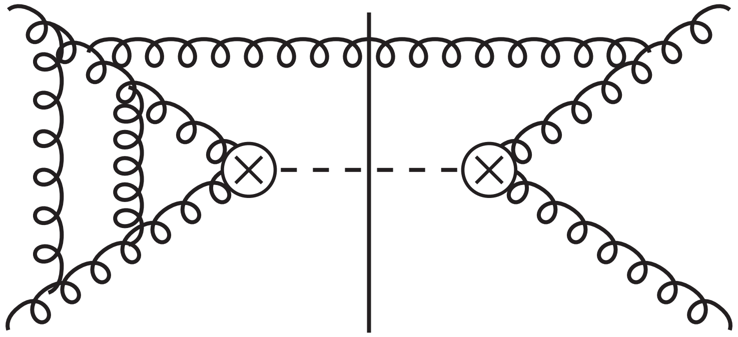
- At N3LO, there are five contributions:



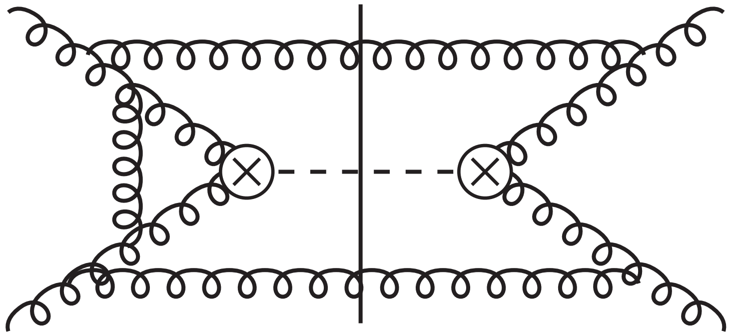
Triple virtual



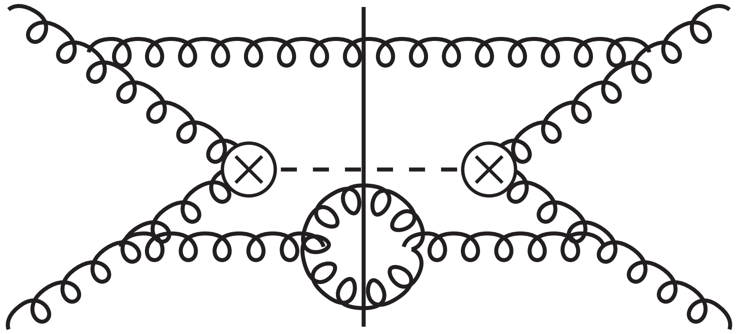
Real-virtual squared



Double virtual real



Double real virtual

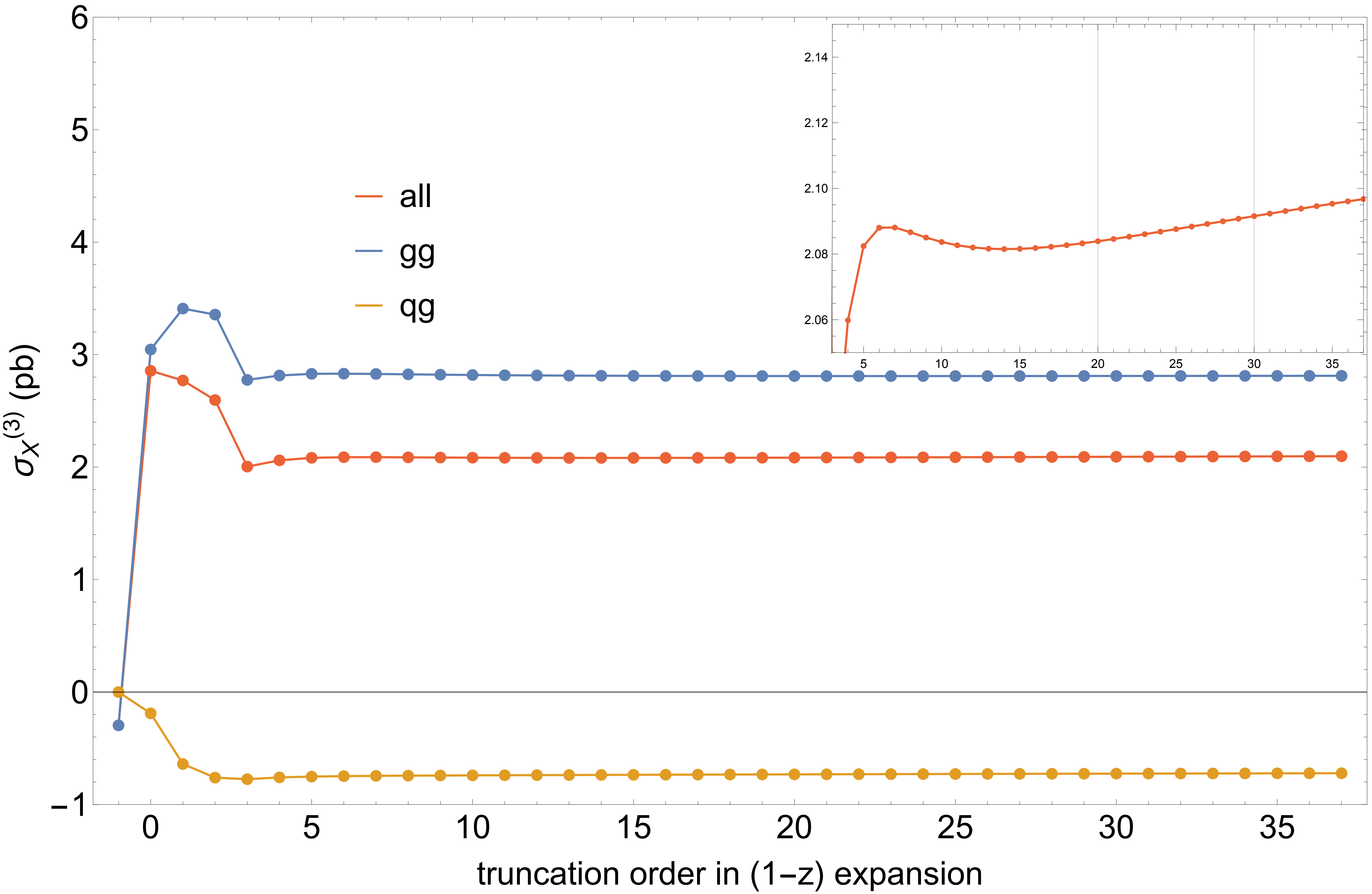


Triple real

# The General Strategy

- Generate all Feynman diagrams (with QGraf), and translate them into analytic expressions. [~100.000 @ N3LO (~1.000 @ NNLO)]
- Reduce everything to a minimal set of phase-space integrals that we need to compute (master integrals). [1.028 (27)]
- Master integrals satisfy a set of coupled ordinary differential equations in the single variable $z$.
- Find a truncated power series solution to the differential equations.
- Fix boundary conditions by requiring by matching to the limit $z \to 1$, where all the QCD radiation is soft.
- Compute boundary conditions explicitly. [72 (5)]

# Convergence of the expansion

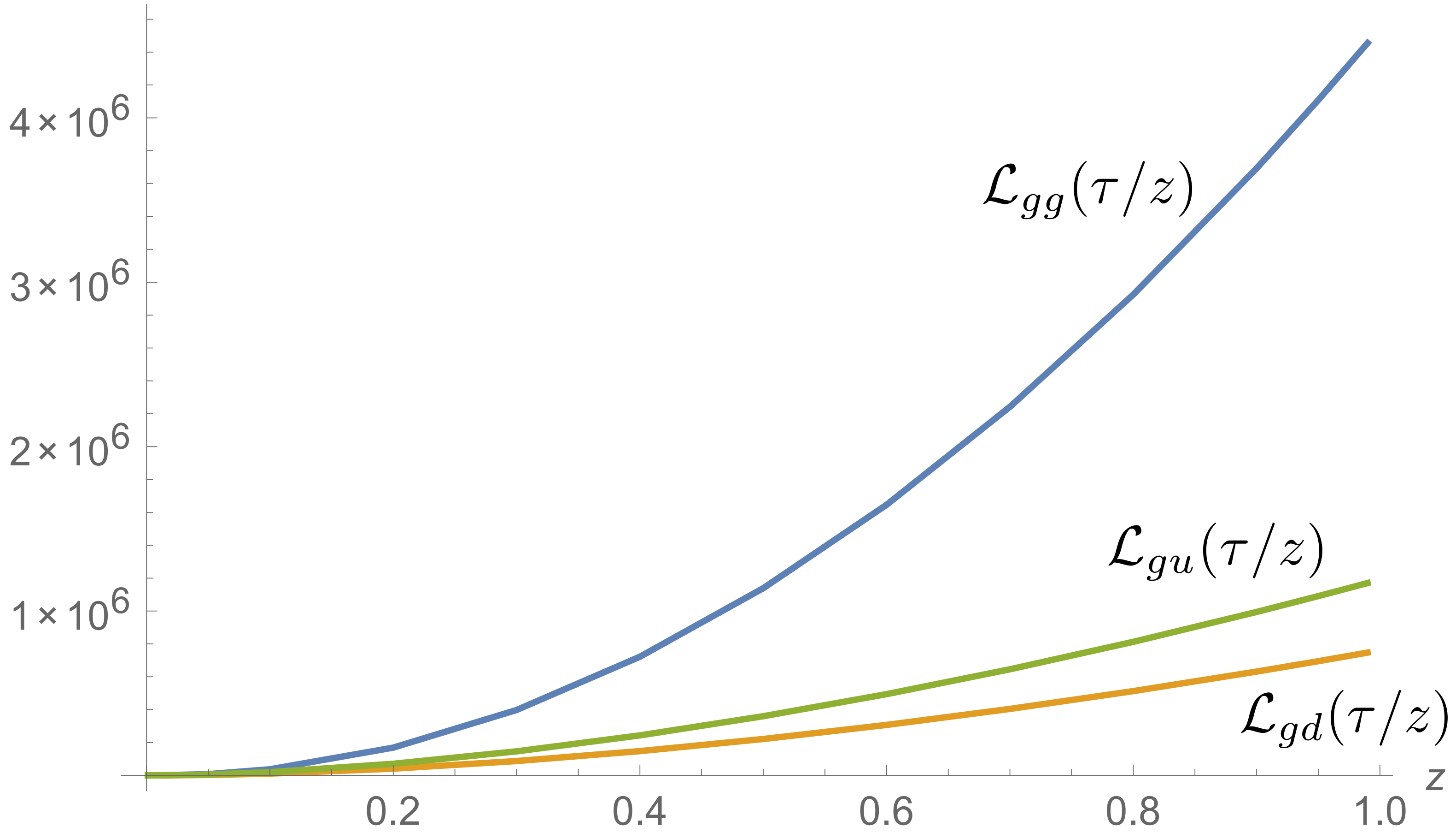


# Convergence of the expansion

- Reason for the slow convergence:

$$\sigma = \tau \sum_{ij} \int_\tau^1 \frac{dz}{z} \mathcal{L}_{ij}(\tau/z) \boxed{\frac{\hat\sigma_{ij}(z)}{z}} \qquad \tau = \frac{m_H^2}{S} \simeq 10^{-4}$$

$$\sim \frac{\log^5 z}{z} \quad @ \text{ N3LO}$$



# Convergence of the expansion

- Estimated truncation uncertainty:

$$\delta(\text{trunc}) = 10 \times \frac{\sigma^{(3)}_{EFT}(37) - \sigma^{(3)}_{EFT}(27)}{\sigma^{\text{N}^3\text{LO}}_{EFT}} = 0.37\%$$

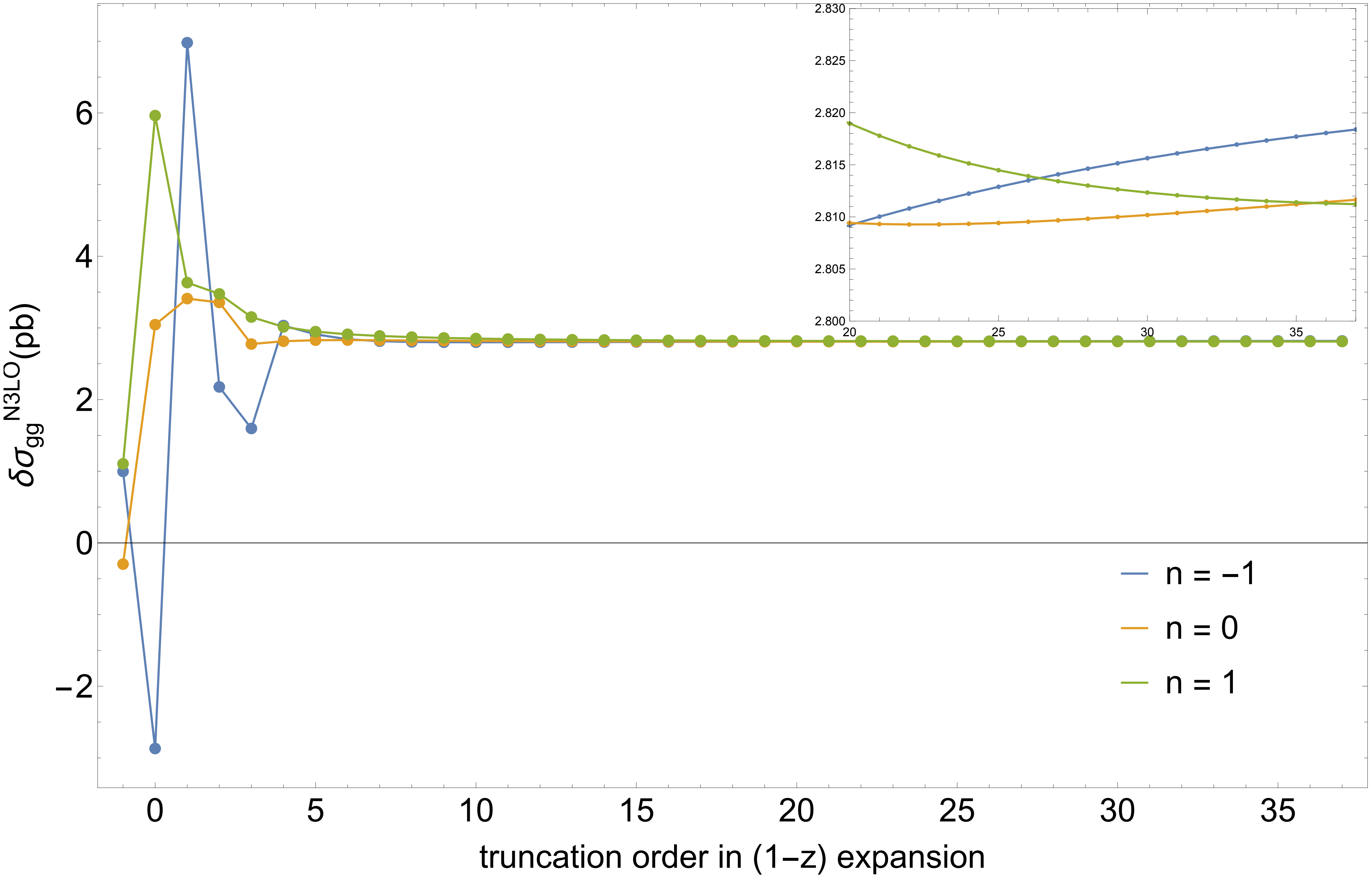
- A quantifier of the convergence: a modified QCD factorisation formula.

$$\sigma_{EFT} = \tau^{1+n} \sum_{ij} \left( f_i^{(n)} \otimes f_j^{(n)} \otimes \frac{\hat{\sigma}_{ij,EFT}}{z^{1+n}} \right)(\tau) \qquad f_i^{(n)}(z) \equiv \frac{f_i(z)}{z^n}$$

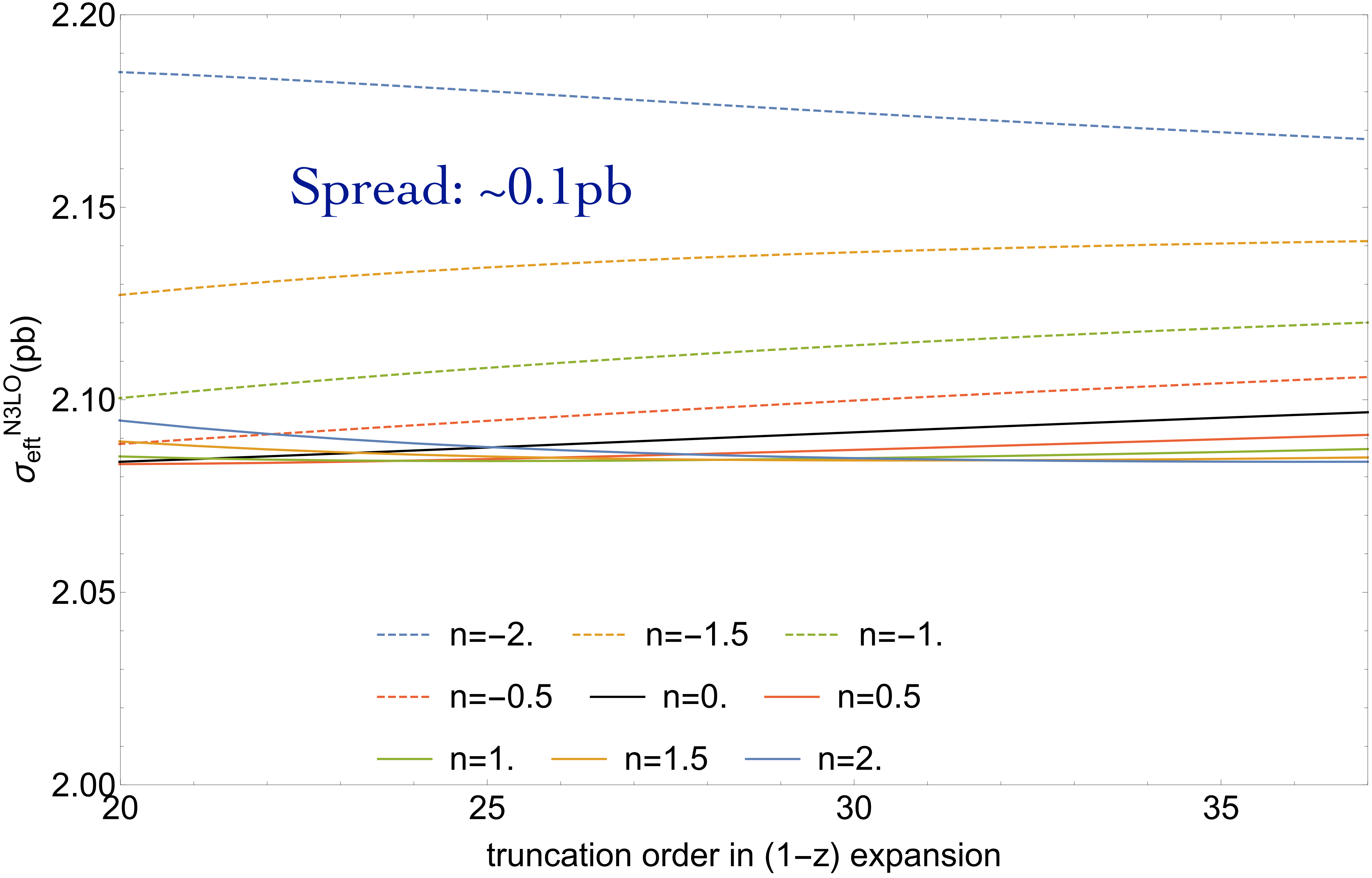
- Total cross section is independent of n, but threshold expansion depends on it.

- We can use the spread of the cross section with n as a quantifier for the convergence.

# Convergence of the expansion



# Convergence of the expansion



Spread: ~0.1pb

# Convergence of the expansion

- This is consistent with known exact results for logarithms:

$$\sigma^{(3)}_{EFT} = f_0(z) + f_1(z)\,\log(1-z) + f_2(z)\,\log^2(1-z) + f_3(z)\,\log^3(1-z) + f_4(z)\,\log^4(1-z) + f_5(z)\,\log^5(1-z)$$

Known exactly

$$\left.\sigma^{(3)}_{EFT}\right|_{\text{expansion}} - \left.\sigma^{(3)}_{EFT}\right|_{\text{full logs}} = 0.004\,\text{pb}$$

- Conclusion: The threshold expansion gives a reliable result for the N3LO cross section!

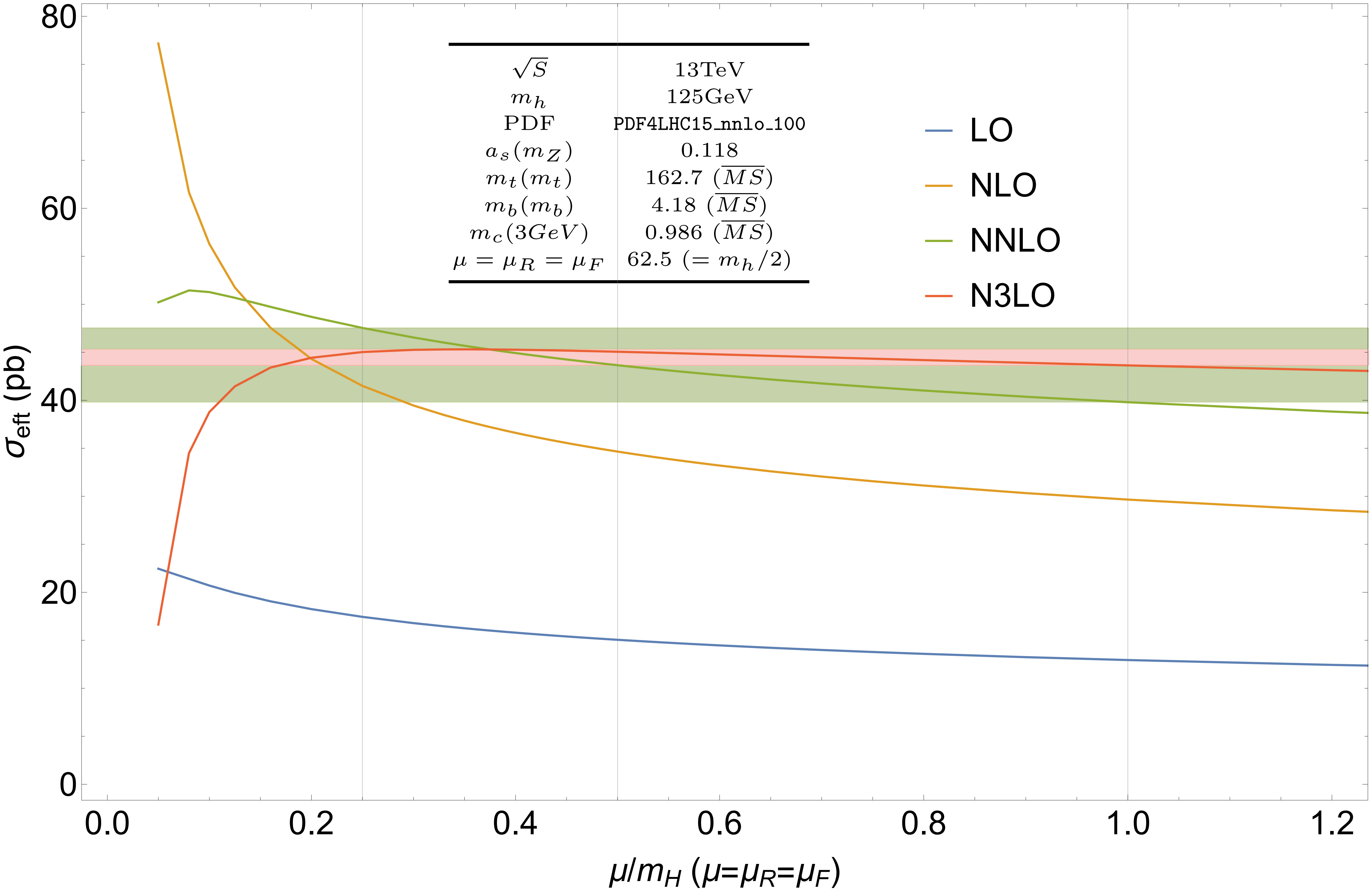
# Phenomenology at N3LO

## Scale variation
## &
## higher orders in QCD

# Higgs @ N3LO

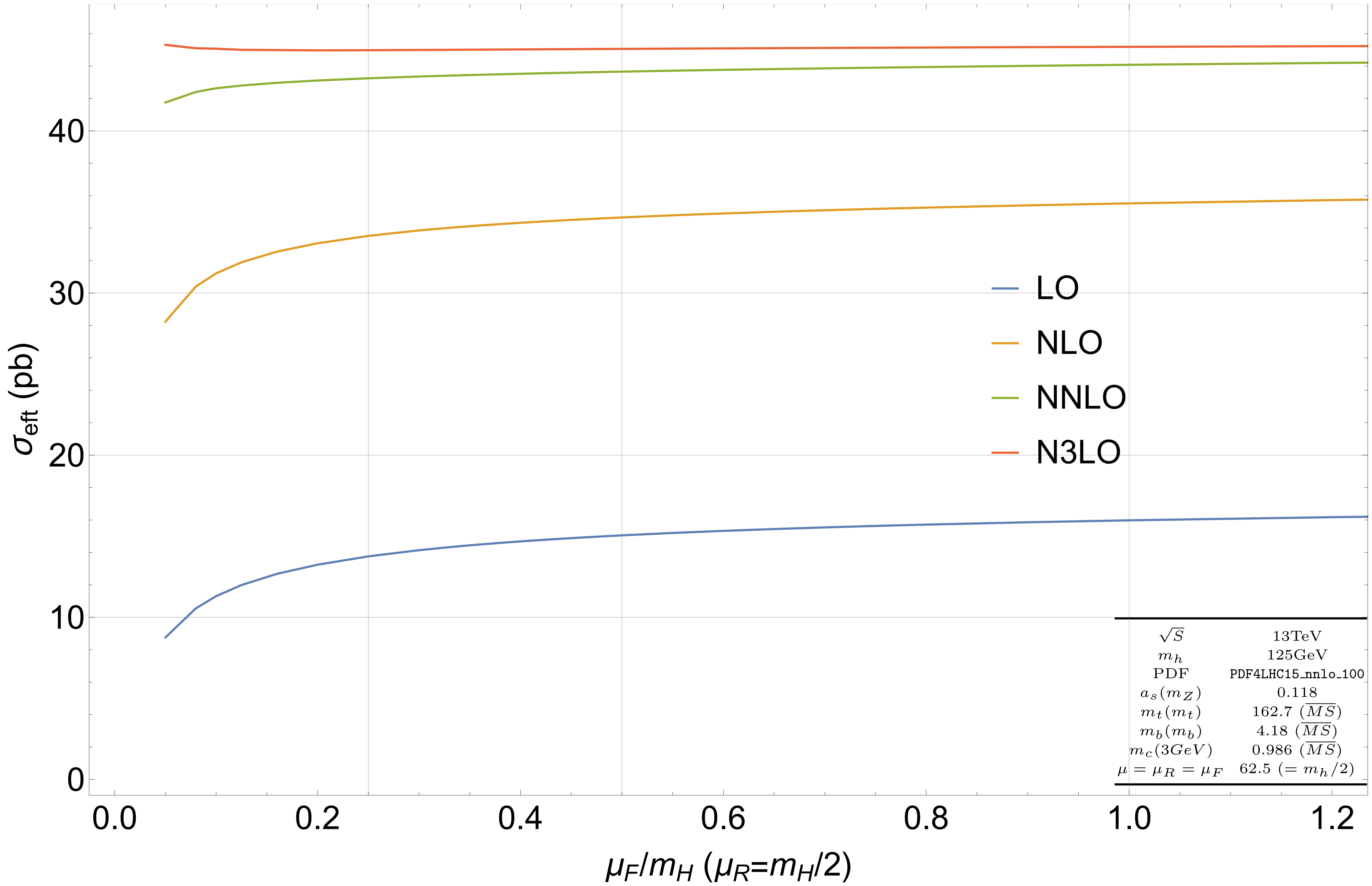
- We can now for the first time study the N3LO phenomenology of a QCD cross section at a hadron collider!
- Interesting questions to ask:
  - ➡ How much does the cross section still depend on the arbitrary renormalisation and factorisation scales?
  - ➡ Is there a preferred scale choice?
  - ➡ How well does the perturbative QCD series converge?
  - ➡ What is the uncertainty on the value of the Higgs cross section at N3LO, and what are the dominant sources of uncertainty?

# Scale variation



| | |
|---|---|
| $\sqrt{S}$ | 13TeV |
| $m_h$ | 125GeV |
| PDF | `PDF4LHC15_nnlo_100` |
| $a_s(m_Z)$ | 0.118 |
| $m_t(m_t)$ | 162.7 $(\overline{MS})$ |
| $m_b(m_b)$ | 4.18 $(\overline{MS})$ |
| $m_c(3GeV)$ | 0.986 $(\overline{MS})$ |
| $\mu = \mu_R = \mu_F$ | 62.5 $(= m_h/2)$ |

# Scale variation



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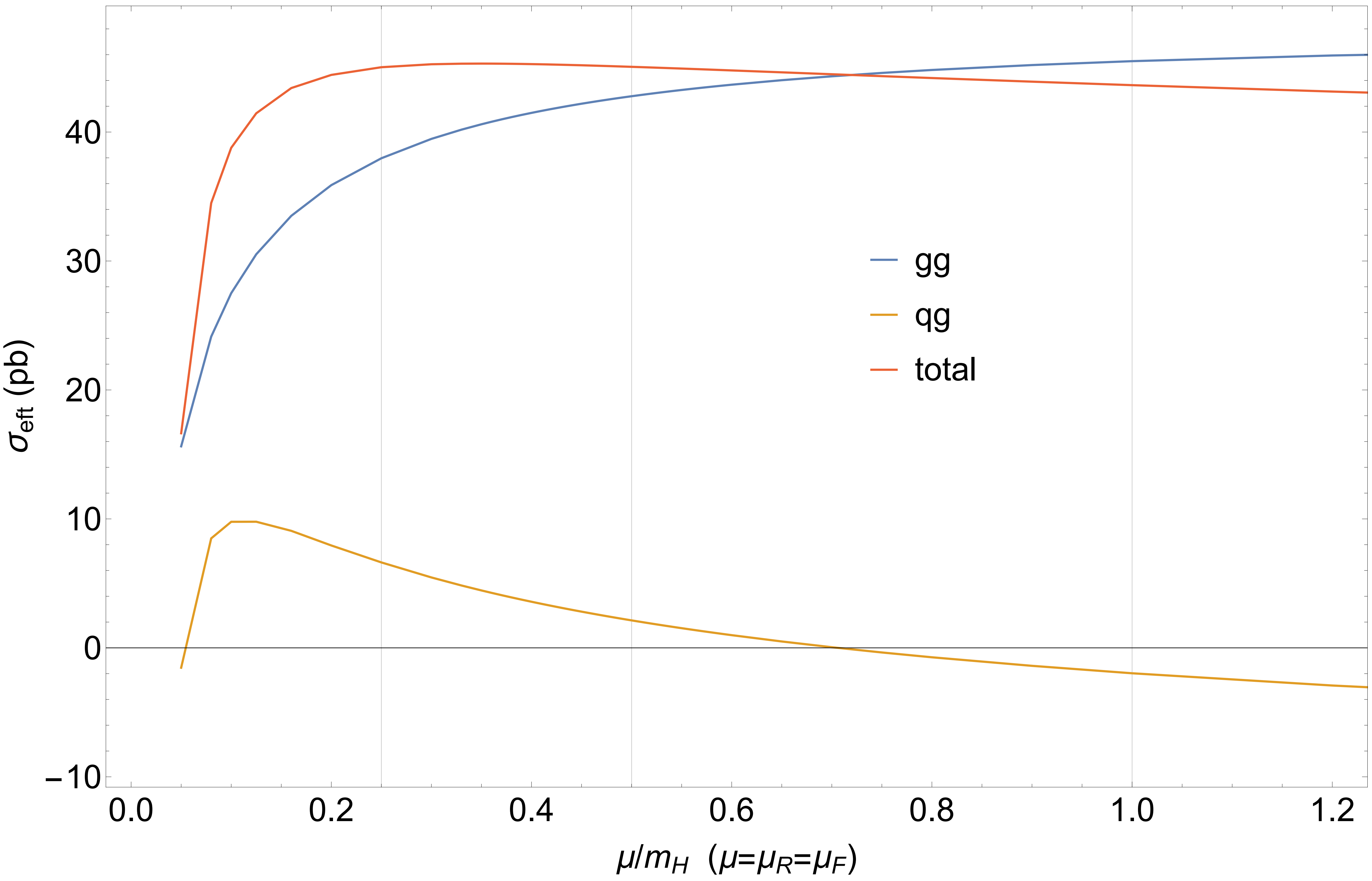


# Scale variation

- For $\mu \in [m_H/4, m_H]$ the N3LO band is nicely contained inside the NNLO band.
- Scale variation at N3LO almost entirely due to renormalisation scale.
- Scale uncertainty $\mu \in [m_H/4, m_H]$ per order:

$$\Delta^{\text{scale}}_{EFT,k} = \pm \frac{\sigma^{\max}_{EFT,k} - \sigma^{\min}_{EFT,k}}{\sigma^{\max}_{EFT,k} + \sigma^{\min}_{EFT,k}} 100\%$$

| | | $\Delta^{\text{scale}}_{EFT,k}$ |
|---|---|---|
| LO | $(k=0)$ | $\pm 14.8\%$ |
| NLO | $(k=1)$ | $\pm 16.6\%$ |
| NNLO | $(k=2)$ | $\pm 8.8\%$ |
| $\text{N}^3\text{LO}$ | $(k=3)$ | $\pm 1.9\%$ |

- Important question: Is scale variation a reliable estimator of missing higher-order corrections?

➡ We know that it is not at low orders!

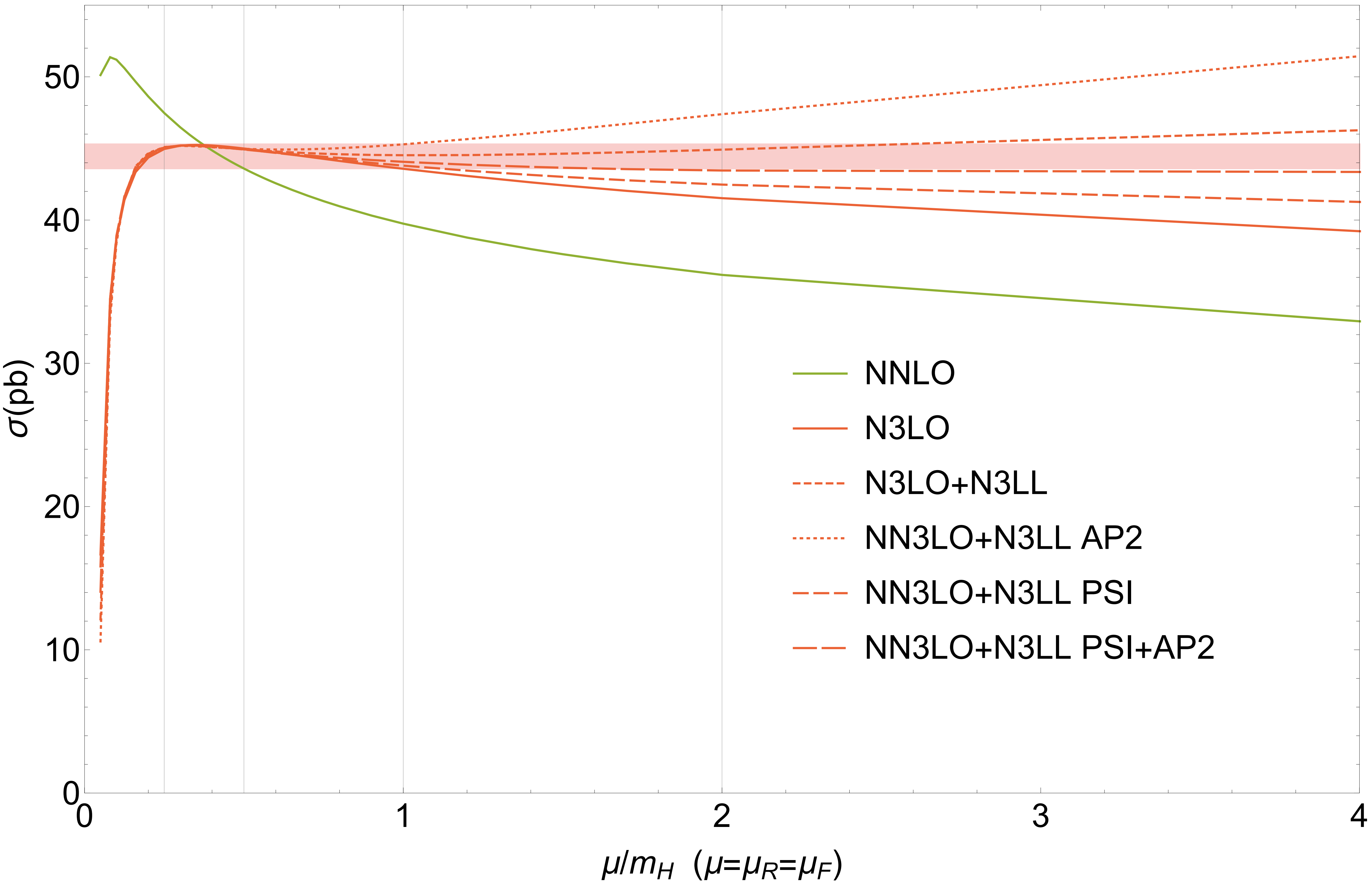
# Missing higher orders

- We estimate the effect of missing higher orders in different ways.
- In particular, in the limit where the final-state QCD radiation is soft, the dominant effect of the radiation can be predicted to all orders.
  - ➡ Leading soft-gluon effects can be resummed into an exponential.
  - ➡ Threshold resummation.

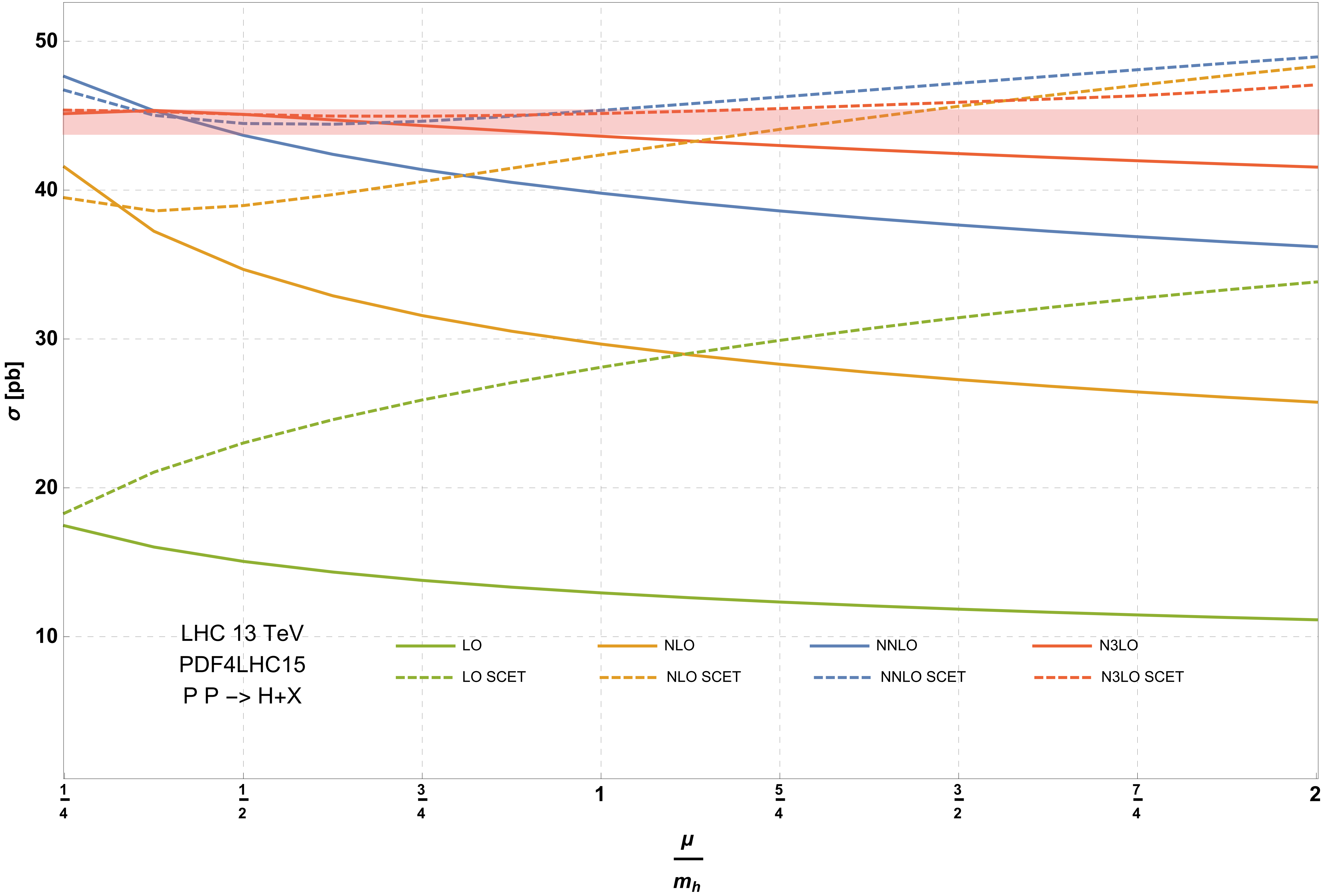
[Collins, Soper, Stermann; Catani, Mangano, Nason, Trentadue]

- There are different formalisms for doing this:
  - ➡ They all exponentiate the same leading soft effects.
  - ➡ They differ by the inclusion of subleading effects.

# Threshold resummation



# Soft-collinear effective theory



# Higgs @ N3LO

- We seem to have good perturbative control on the Higgs cross section:
  - ➡ For $\mu \in [m_H/4, m_H]$ there is good apparent convergence of the perturbative series.
  - ➡ Residual scale dependence ~1.9% (~10% @ NNLO).
  - ➡ Estimation of higher-order effects through resummation methods indicates that scale variation is a reliable estimator for missing perturbative orders.

- We are in a good shape!
  - ➡ But need to be careful not to neglect any other source of uncertainty that may challenge the 1.9%!

# Other corrections

PDFs,
Quark masses &
Electroweak corrections

# Other uncertainties

$$\sigma = \tau \sum_{ij} \int_{\tau}^{1} \frac{dz}{z} \mathcal{L}_{ij}(\tau/z) \frac{\hat{\sigma}_{ij}(z)}{z} \qquad \alpha_s(m_Z) = 0.118$$

- What are the residual uncertainties on the partonic cross sections $\hat{\sigma}_{ij}$ and the parton luminosities $\mathcal{L}_{ij}$ ?

- Other effects entering the perturbative partonic cross sections:
  - ➡ Finite quark mass effects: so far we have considered the top-quark infinitely heavy and all other quarks massless.
  - ➡ Electroweak effects: so far we have only discussed QCD corrections.

# Quark-mass effects

- At LO and NLO, we know the exact result including all quark mass effects.

[Djouadi, Graudenz, Spira, Zerwas;
Anastasiou, Beerli, Bucherer, Daleo, Kunszt]

➡ EFT known to work well if rescaled by the LO ratio.

$$R_{LO} \equiv \frac{\sigma^{LO}_{ex;t}}{\sigma^{LO}_{EFT}}$$

| | | | |
|---|---|---|---|
| $\sigma^{LO}_{EFT}$ | 15.05 pb | $\sigma^{NLO}_{EFT}$ | 34.66 pb |
| $R_{LO}\,\sigma^{LO}_{EFT}$ | 16.00 pb | $R_{LO}\,\sigma^{NLO}_{EFT}$ | 36.84 pb |
| $\sigma^{LO}_{ex;t}$ | 16.00 pb | $\sigma^{NLO}_{ex;t}$ | 36.60 pb |
| $\sigma^{LO}_{ex;t+b}$ | 14.94 pb | $\sigma^{NLO}_{ex;t+b}$ | 34.96 pb |
| $\sigma^{LO}_{ex;t+b+c}$ | 14.83 pb | $\sigma^{NLO}_{ex;t+b+c}$ | 34.77 pb |

- Scale uncertainty in rescaled EFT is 1.6% (vs. 1.9% in EFT) (because $R_{LO}$ runs in the $\overline{\text{MS}}$ scheme).

- Parametric dependence on the quark masses is negligible.

- There is a strong dependence on the renormalisation scheme at NLO for light quarks (~30% at NLO for the b quark).

# Quark-mass effects

- At NNLO, we do not know any quark mass effects exactly.
  - ➡ Beyond our technical capabilities at this point.

- NNLO top-mass effects have been computed as an expansion in the inverse top-mass. [Harlander, Mantler, Marzani, Ozeren]
  - ➡ Give a contribution of ~+1%, with an uncertainty of ~1%.

- At NLO, the bottom quark contributed a substantial negative contribution to the cross section.
  - ➡ We do not know t-b interference at NNLO.

$$\delta(tbc)^{\overline{\text{MS}}} = \pm \left| \frac{\delta\sigma^{NLO}_{ex;t} - \delta\sigma^{NLO}_{ex;t+b+c}}{\delta\sigma^{NLO}_{ex;t}} \right| \left(R_{LO}\delta\sigma^{NNLO}_{EFT} + \delta_t\hat{\sigma}^{NNLO}_{gg+qg,EFT}\right) \simeq \pm 0.31\,\text{pb}$$

# Electroweak corrections

- Exact NLO EW corrections are known. [Actis, Passarino, Sturm, Uccirati]

- There is an ambiguity of how to combine the QCD and electroweak interactions:

$$1+\underbrace{\alpha_s\,\delta_{QCD}}_{\text{large} \sim 100\%}+\alpha_{EW}\,\delta_{EW}+\mathcal{O}(\alpha_s\alpha_{EW})=(1+\underbrace{\alpha_s\,\delta_{QCD}}_{\text{large} \sim 100\%})(1+\alpha_{EW}\,\delta_{EW})+\mathcal{O}(\alpha_s\alpha_{EW})$$

  ➡ Formally, these two expressions are equivalent in perturbation theory.

- Complete factorisation approach:

  ➡ Gives rise to an increase of ~5%.

# Electroweak corrections

- The factorisation issue could be settled by an exact computation of mixed QCD-EW corrections.
- Mixed EW-QCD corrections are only known as an EFT where the weak bosons are integrated out. [Anastasiou, Boughezal, Petriello]

$$C_{QCD} \to C_{QCD} + \underbrace{\lambda_{EW}}_{\text{NLO EW}}(1 + \underbrace{C_{1w}a_s}_{\text{EW-QCD in EFT approach}} + C_{2w}a_s^2 + \ldots)$$

➡ Modified Wilson coefficient.

➡ EFT approach misses threshold effects at $\mathcal{O}(\alpha_s\alpha_{EW})$, but the leading EW threshold effects should already be captured by NLO-EW.

➡ Numerical impact is similar to 'complete factorisation' for EW corrections, ~5.1%.

# Other uncertainties

$$\sigma = \tau \sum_{ij} \int_\tau^1 \frac{dz}{z} \mathcal{L}_{ij}(\tau/z) \frac{\hat{\sigma}_{ij}(z)}{z} \qquad \alpha_s(m_Z) = 0.118$$

- What are the residual uncertainties on the partonic cross sections $\hat{\sigma}_{ij}$ and the parton luminosities $\mathcal{L}_{ij}$ ?
- Uncertainties affecting the parton luminosities and the strong coupling constant:
  - ➡ Parton densities functions (PDFs) are not calculable from first principles but need to be extracted from data.
  - ➡ Extraction and parametrisation of the PDFs introduce uncertainties
  - ➡ So far, PDFs have only been extracted from NNLO data.

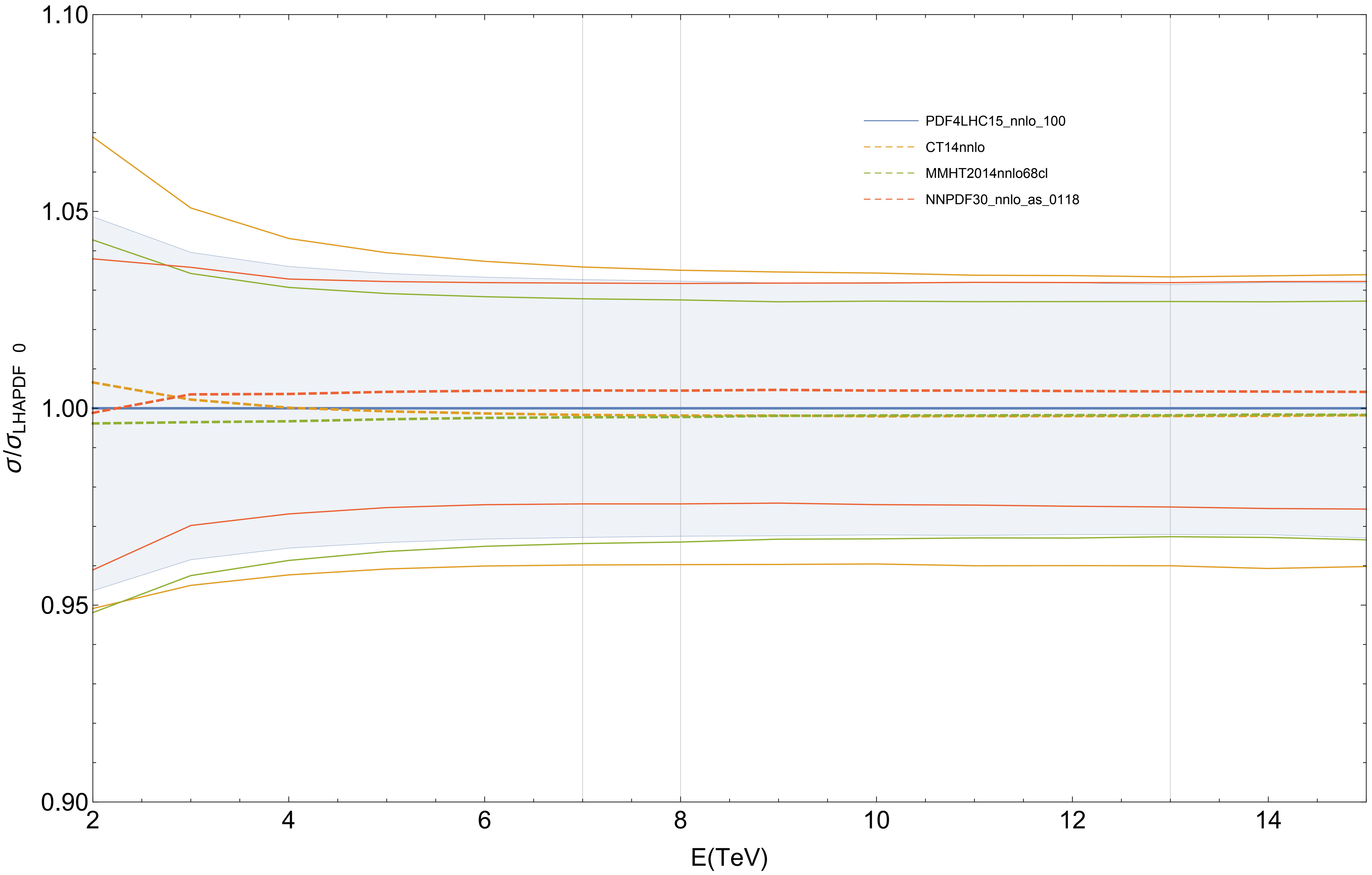
# PDF + aS uncertainty

- We follow the PDF4LHC recommendation:
  - ➡ PDF and $\alpha_s$ error are added in quadrature

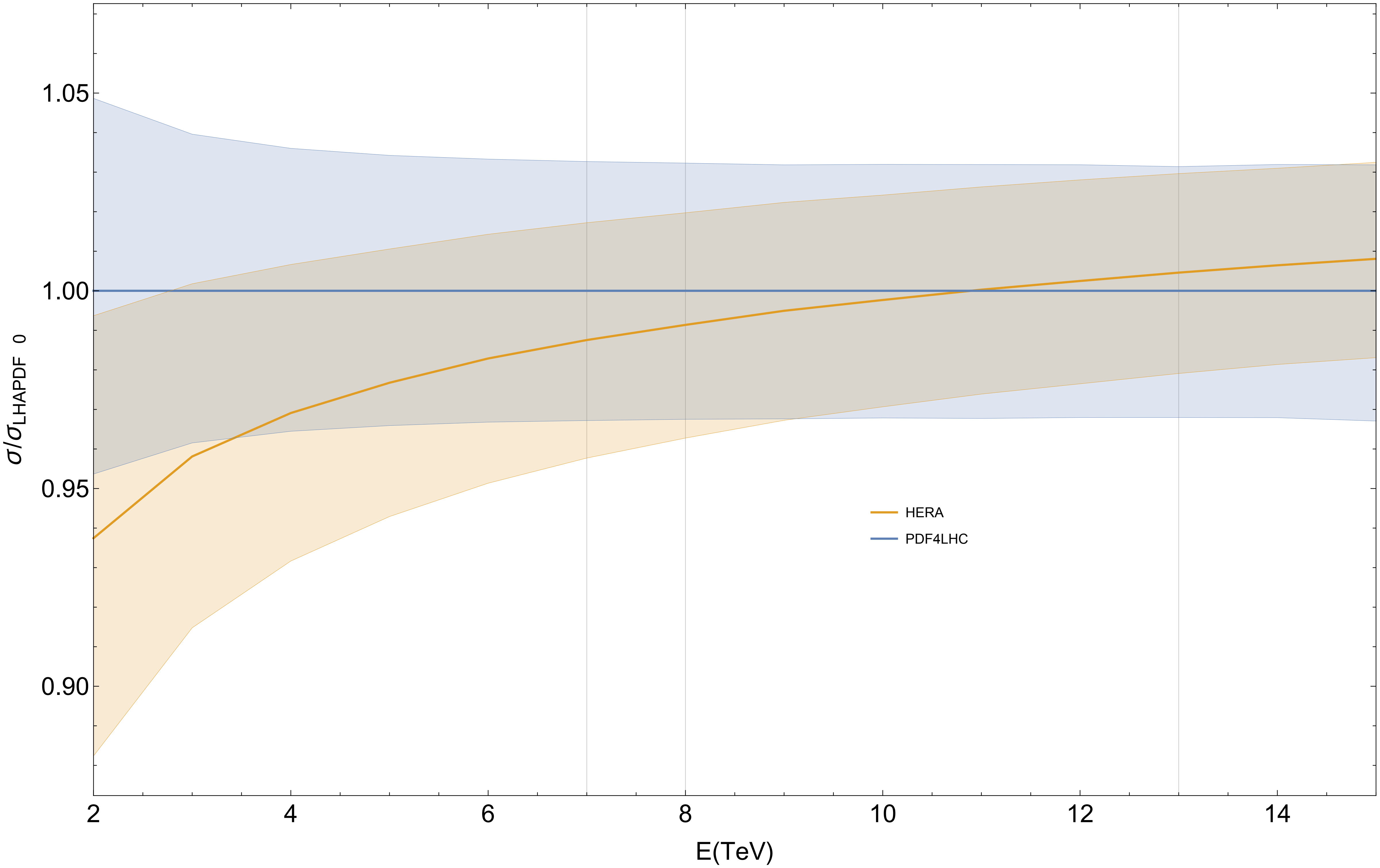
$$\delta_{\pm}(PDF + \alpha_s) = \sqrt{\delta_{\pm}(PDF)^2 + \delta_{\pm}(\alpha_s)^2}$$

  - ➡ $\delta_{\mathrm{PDF}}$ obtained by using Hessian or Monte Carlo methods.
  - ➡ $\alpha_s$ uncertainty obtained by varying $\alpha_s$ up and down by 0.00115 around PDG world average (0.118).
- There are various different PDF sets publicly available:
  - ➡ MMHT, CTQ, NNPDF, ABM, HeraPDF,…
  - ➡ MMHT, CTQ and NNPDF have been combined into a the PDF4LHC set.

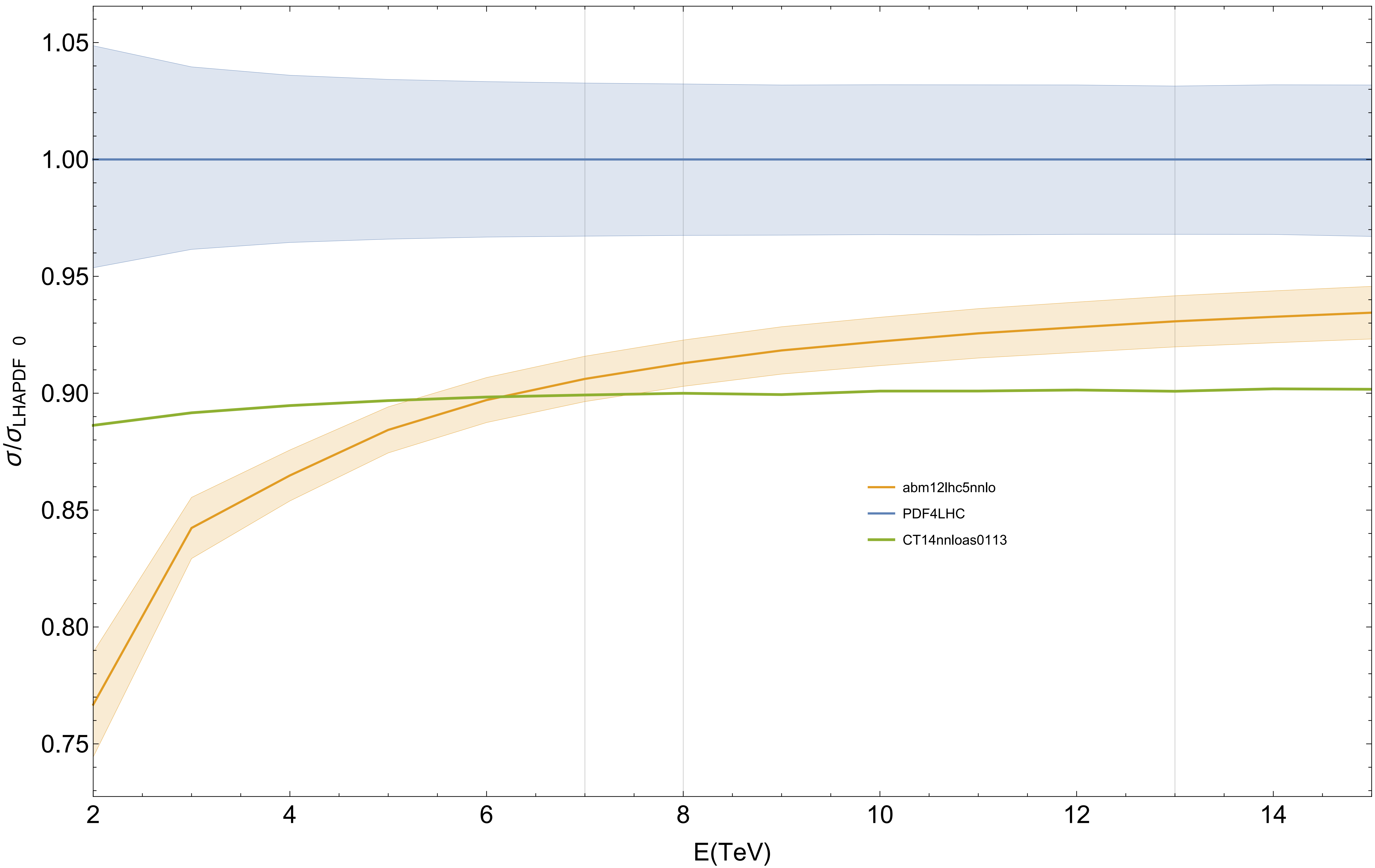
# PDF + aS uncertainty



# PDF + aS uncertainty



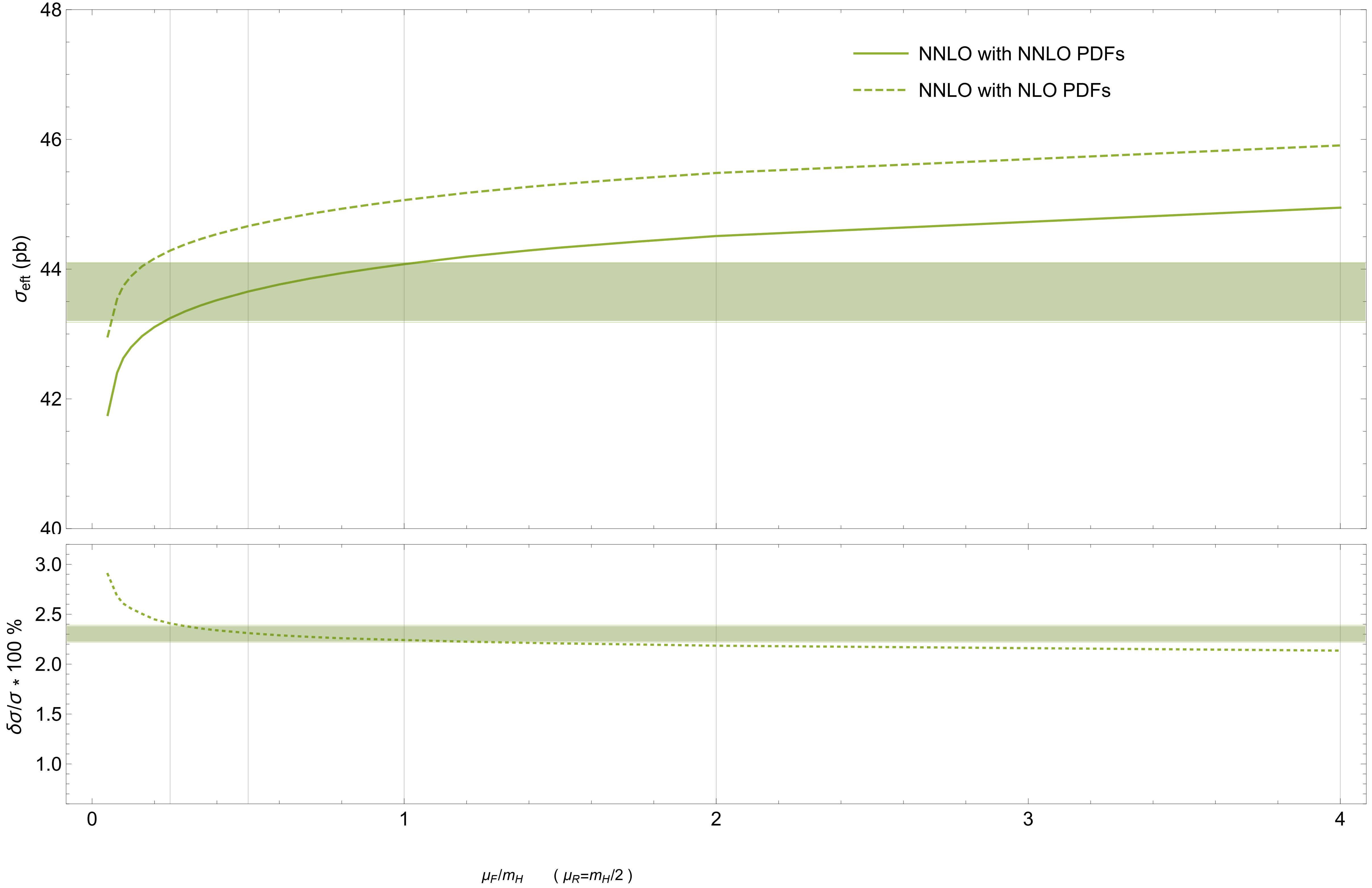
# PDF + aS uncertainty



# PDF + aS uncertainty

- We observe that MMHT, CTQ, NNPDf and HeraPDF give very similar predictions at LHC energies.
- ABM gives a rather different prediction.
  - ➡ ABM uses a different value of $\alpha_s$ resulting from their fit and different theoretical assumptions (e.g., different treatment of the charm-quark mass).
- So far there are no PDF sets that have been extracted using N3LO input.
  - ➡ All our predictions at N3LO were made using NNLO PDFs.
  - ➡ Inconsistent, because partonic cross section at N3LO is combined with NNLO PDFs.
  - ➡ Need to estimate the uncertainty this induces.

# Missing N3LO PDFs



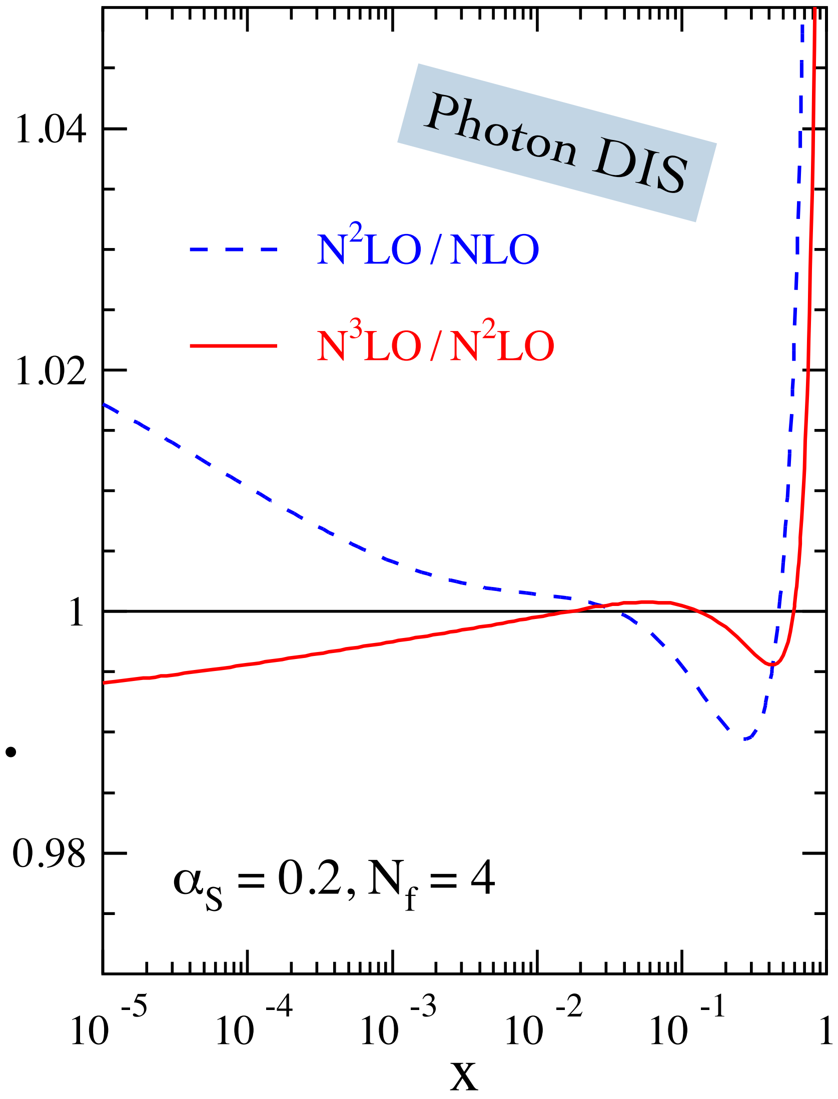
# Missing N3LO PDFs

- Using NLO PDFs at NNLO results in a 2-2.5% error at NNLO.
- From this, we estimate the uncertainty of using NNLO PDFs at N3LO

$$\delta(\text{PDF} - \text{TH}) = \frac{1}{2} \left| \frac{\sigma_{EFT}^{(2),NNLO} - \sigma_{EFT}^{(2),NLO}}{\sigma_{EFT}^{(2),NNLO}} \right|$$

$$= \frac{1}{2} 2.31\% = 1.16\%$$

- The factor 1/2 takes into account that this estimate is most likely overly conservative.
  - ➡ cf. convergence pattern of DIS.



[Moch, Vermaseren, Vogt]

# Summary

- We have obtained the most precise theoretical prediction of the gluon-fusion cross section available to date!

$$\boxed{\sigma = 48.58\,\text{pb}^{+2.22\,\text{pb}\,(+4.56\%)}_{-3.27\,\text{pb}\,(-6.72\%)}\ (\text{theory}) \pm 1.56\,\text{pb}\,(3.20\%)\ \ (\text{PDF}+\alpha_s)}$$

➡ To be compared with

$$\sigma^{NNLO} = 47.02\ \text{pb}\ {}^{+5.13\ \text{pb}\ (10.9\%)}_{-5.17\ \text{pb}\ (11.0\%)}\ (\text{theory})^{+1.48\ \text{pb}\ (3.14\%)}_{-1.46\ \text{pb}\ (3.11\%)}\ (\text{PDF}+\alpha_s)$$

➡ Theoretical uncertainty reduced by roughly a factor of 2!

- Breakdown of the uncertainties:

| $\delta$(scale) | $\delta$(trunc) | $\delta$(PDF-TH) | $\delta$(EW) | $\delta(t,b,c)$ | $\delta(1/m_t)$ |
|---|---|---|---|---|---|
| $^{+0.10\ \text{pb}}_{-1.15\ \text{pb}}$ | ±0.18 pb | ±0.56 pb | ±0.49 pb | ±0.40 pb | ±0.49 pb |
| $^{+0.21\%}_{-2.37\%}$ | ±0.37% | ±1.16% | ±1% | ±0.83% | ±1% |

# Summary

$$\boxed{\sigma = 48.58\,\mathrm{pb}^{+2.22\,\mathrm{pb}\,(+4.56\%)}_{-3.27\,\mathrm{pb}\,(-6.72\%)}\ (\mathrm{theory}) \pm 1.56\,\mathrm{pb}\,(3.20\%)\ (\mathrm{PDF}+\alpha_s)}$$

| $\delta(\mathrm{scale})$ | $\delta(\mathrm{trunc})$ | $\delta(\mathrm{PDF\text{-}TH})$ | $\delta(\mathrm{EW})$ | $\delta(t,b,c)$ | $\delta(1/m_t)$ |
|---|---|---|---|---|---|
| $^{+0.10\ \mathrm{pb}}_{-1.15\ \mathrm{pb}}$ | $\pm 0.18$ pb | $\pm 0.56$ pb | $\pm 0.49$ pb | $\pm 0.40$ pb | $\pm 0.49$ pb |
| $^{+0.21\%}_{-2.37\%}$ | $\pm 0.37\%$ | $\pm 1.16\%$ | $\pm 1\%$ | $\pm 0.83\%$ | $\pm 1\%$ |

- Places where we can improve:
  - ➡ top-bottom interference at NNLO in QCD.
  - ➡ N3LO PDFs.
  - ➡ Exact mixed QCD-EW corrections.
  - ➡ NNLO corrections including exact top-mass dependence.