Three-jet event shapes: old and new

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in collaboration with

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based on arXiv:1606.03453

Kavli Institute for Theoretical Physics

University of California, Santa Barbara

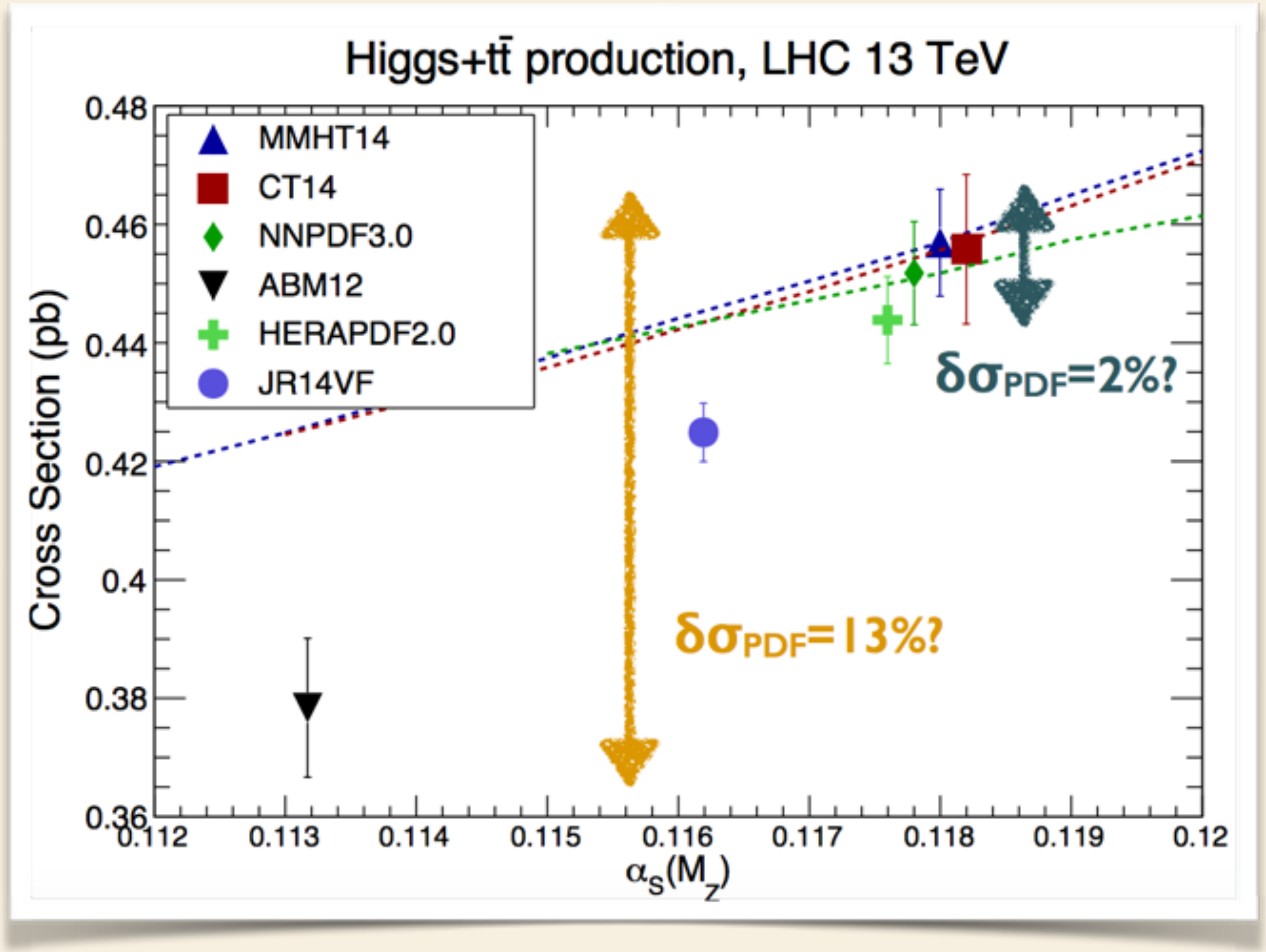
LHC Run II and the Precision Frontier workshop, KITP

June 15, 2016

Outline

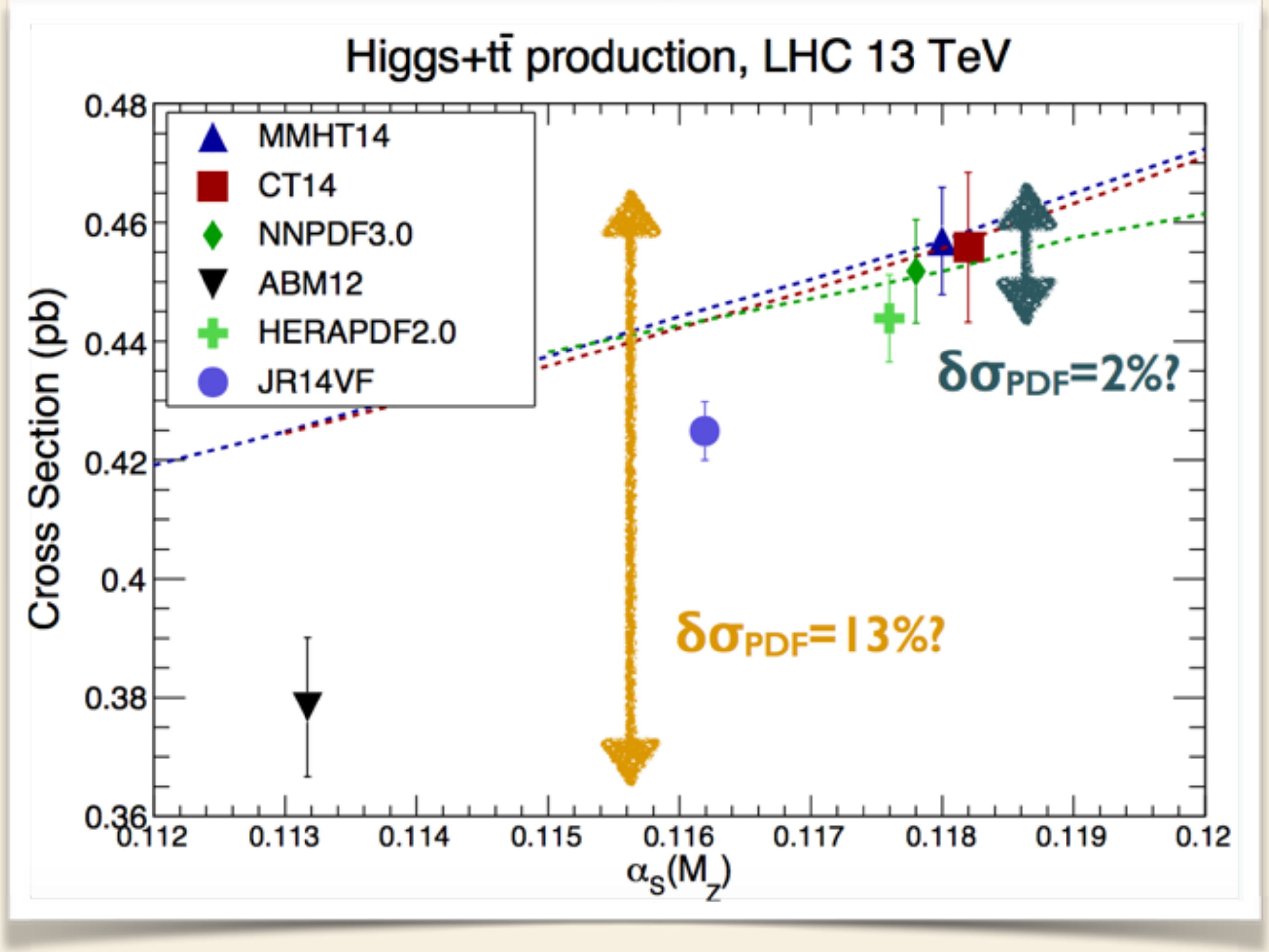
- Why event shapes?
- Why NNLO?
- Our CoLoRFulNNLO method:
  recipe in a nut-shell with historical remarks
- Main difficulty
- Rewards of solution
- Event shapes: old and new
- Conclusions

Why event shapes?



Why event shapes?

- value of the strong coupling matters



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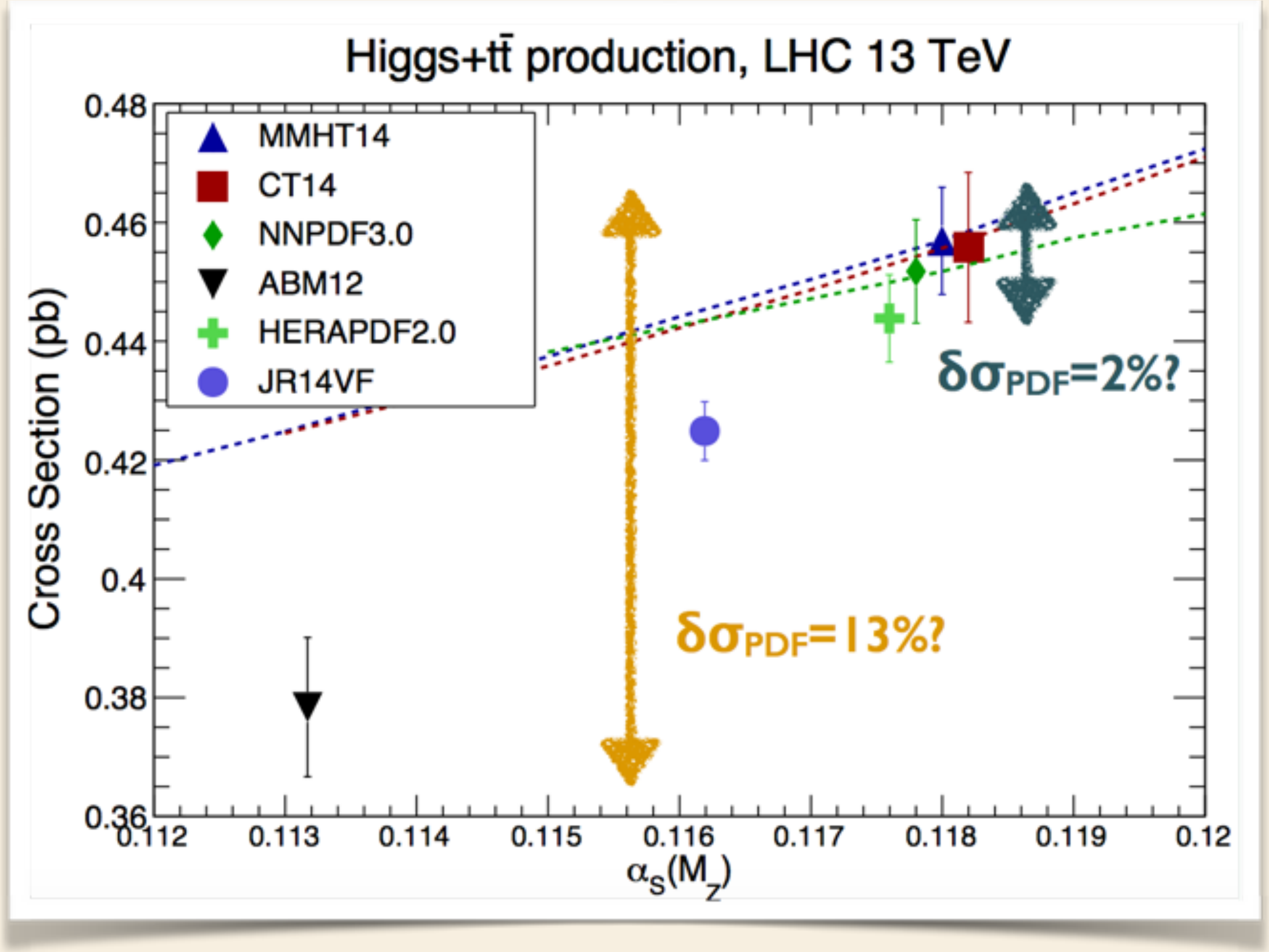
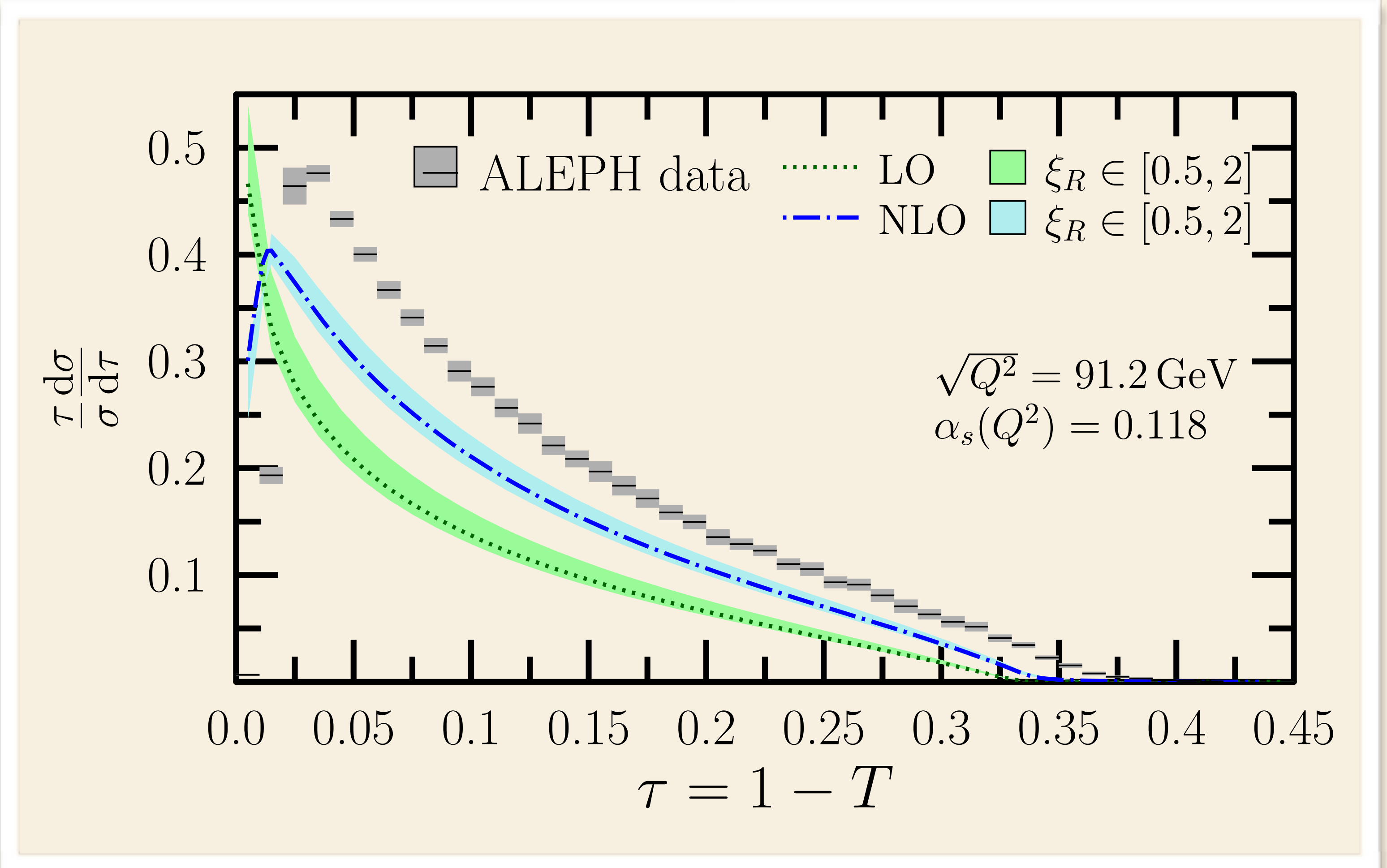


- three-jet event shapes
  - ✓ are sensitive to α_s
  - ✓ are measured extensively
  - ✓ can be computed from first principles (assuming local parton-hadron duality)

Why NNLO?



ALEPH data
LO
NLO
$\xi_R \in [0.5, 2]$
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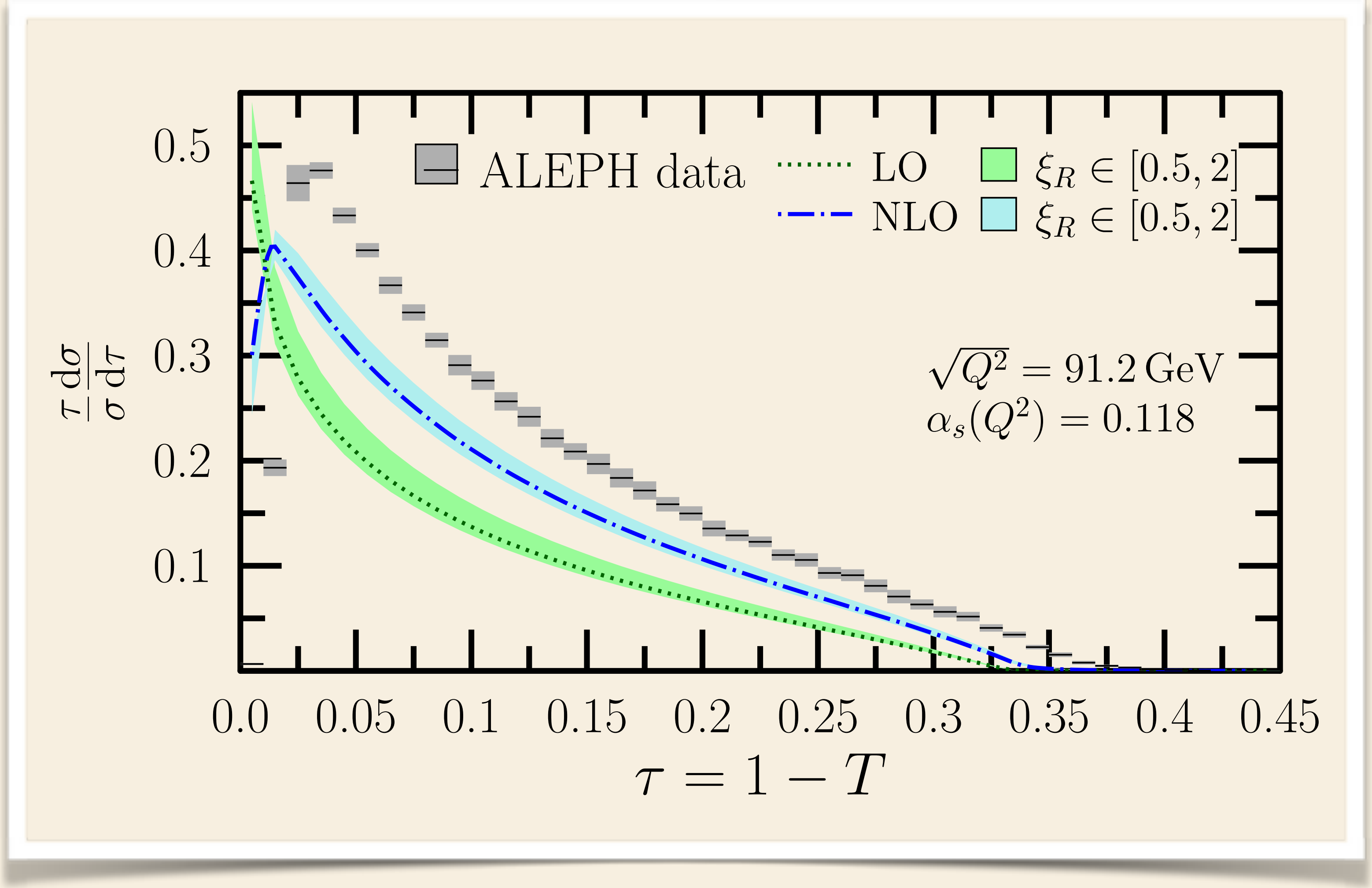
$\sqrt{Q^2} = 91.2\,\mathrm{GeV}$
$\alpha_s(Q^2) = 0.118$
$\frac{\tau}{\sigma}\frac{d\sigma}{d\tau}$
0.5
0.4
0.3
0.2
0.1
0.0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
$\tau = 1 - T$

Why NNLO?

- LO vs. NLO vs. data:



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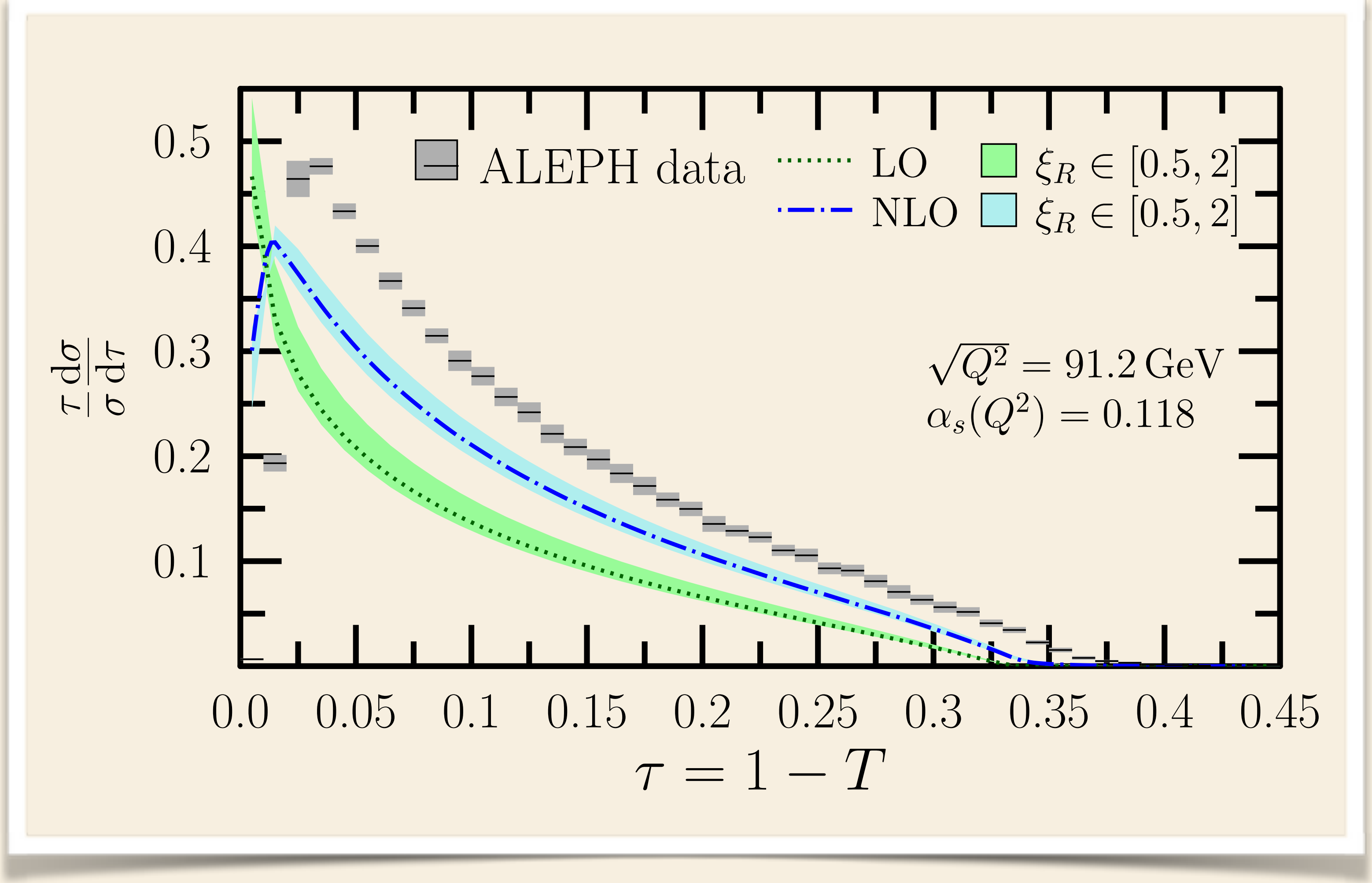
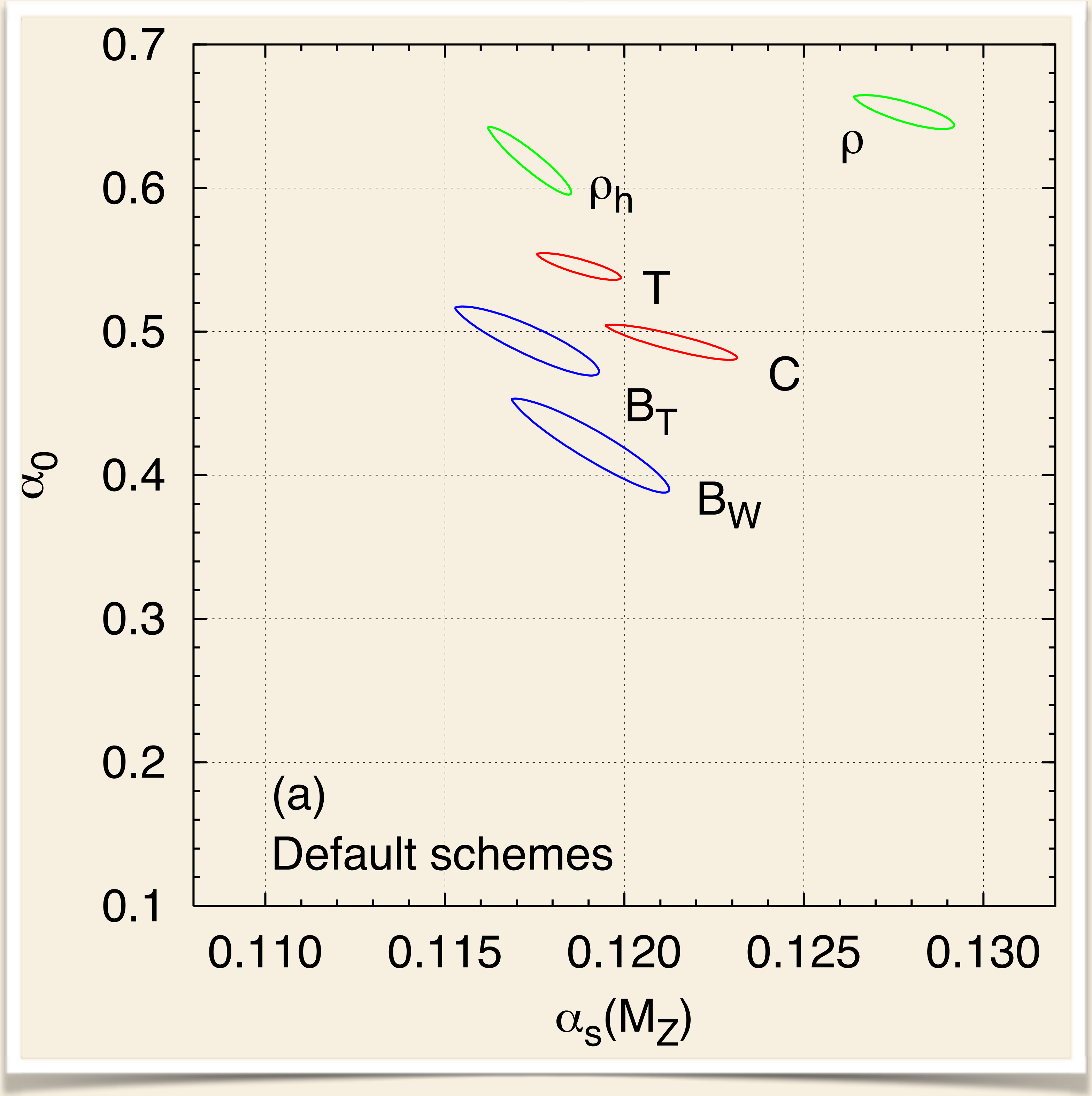


- three-jet event shapes
  - ✓ suffer large NLO corrections
  - ✓ NNLL or NNNLL resummation available
  - ✓ analytic model for hadronization available

Shapes at NLO+NLL+power corr.+had. mass



D. Wicke, G. Salam hep-ph/0102343

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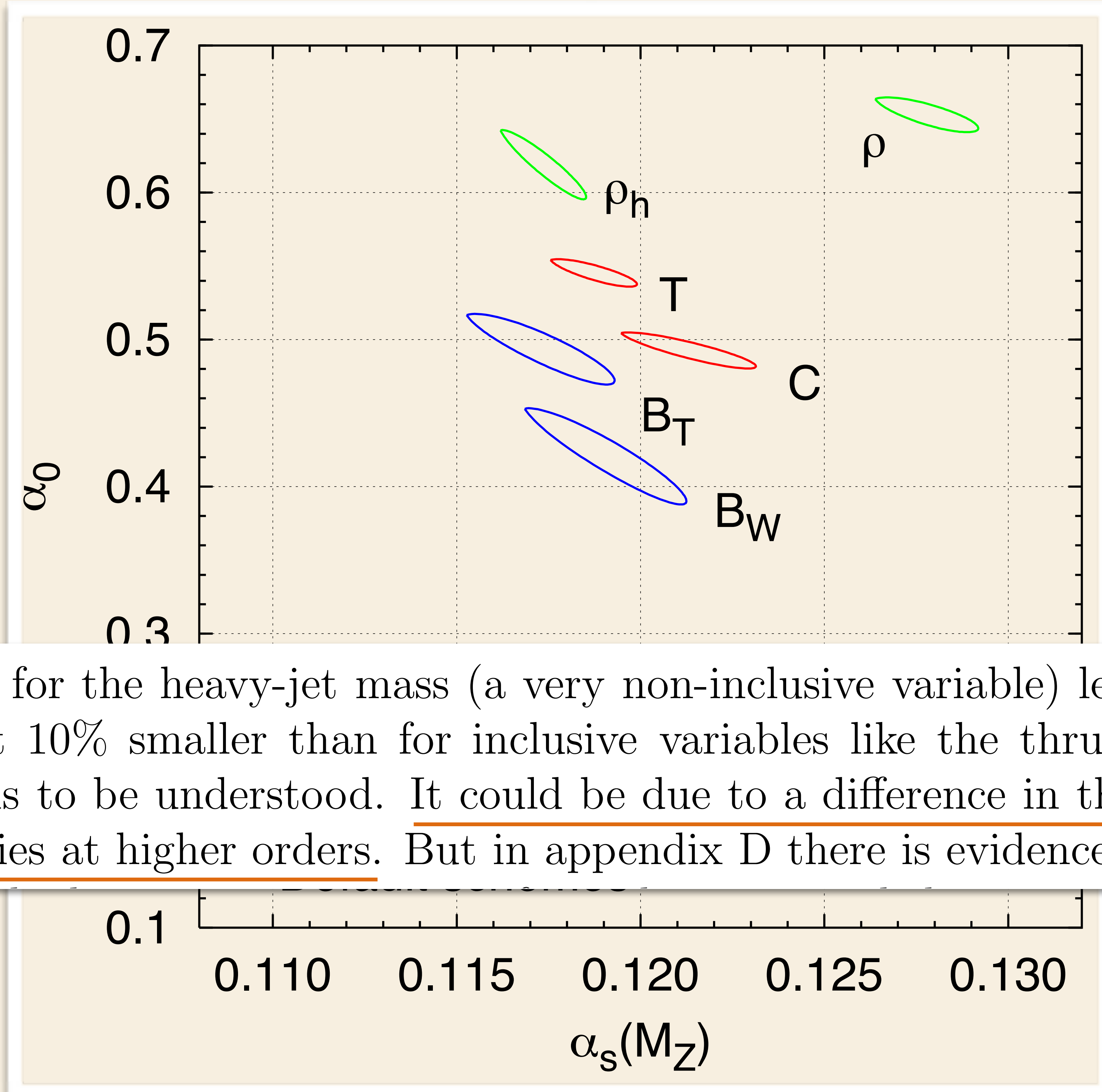

Secondly fits for the heavy-jet mass (a very non-inclusive variable) lead to values for α_s which are about 10% smaller than for inclusive variables like the thrust or the mean jet mass. This needs to be understood. It could be due to a difference in the behaviour of the perturbation series at higher orders. But in appendix D there is evidence from Monte Carlo

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CoLoRFulNNLO

Problem

$$\sigma_m^{\mathrm{NNLO}} = \sigma_{m+2}^{\mathrm{RR}} + \sigma_{m+1}^{\mathrm{RV}} + \sigma_m^{\mathrm{VV}}$$
$$\equiv \int_{m+2} \mathrm{d}\sigma_{m+2}^{\mathrm{RR}} J_{m+2} + \int_{m+1} \mathrm{d}\sigma_{m+1}^{\mathrm{RV}} J_{m+1} + \int_m \mathrm{d}\sigma_m^{\mathrm{VV}} J_m$$

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  - in σ^{RR} kinematical singularities as one or two partons become unresolved yielding ϵ-poles at $O(\epsilon^{-4}, \epsilon^{-3}, \epsilon^{-2}, \epsilon^{-1})$ after integration over phase space, no explicit ϵ-poles
  - in σ^{RV} kinematical singularities as one parton becomes unresolved yielding ϵ-poles at $O(\epsilon^{-2}, \epsilon^{-1})$ after integration over phase space + explicit ϵ-poles at $O(\epsilon^{-2}, \epsilon^{-1})$
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How to combine to obtain finite cross section?

Approaches

- Sector decomposition

  Anastasiou, Melnikov, Petriallo et al 2004-

- Antennae subtraction

  Gehrmann, Gehrmann-De Ridder, Glover et al 2004-

- q_T-slicing

  S. Catani, M. Grazzini et al 2007-

- SecToR-Improved Phase sPacE for Real radiation (STRIPPER)

  Czakon et al 2010-

- τ_N-slicing

  Boughezal et al 2015-
  Gaunt et al 2015-

- Completely Local SubtRactions for Fully Differential Predictions at NNLO (CoLoRFulNNLO)

  ZT, Somogyi et al 2005-

personal opinion: a completely satisfactory solution is not yet available

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such schemes are known at NLO (CS-dipoles, FKS etc)

How to build a local subtraction scheme?

S. Catani, S. Dittmaier,
M.H. Seymour,ZT
hep-ph/0201036

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steps proven to be too difficult at NNLO:

given up

Structure

of subtractions is governed by the jet functions

$$\sigma^{\rm NNLO} = \sigma^{\rm RR}_{m+2} + \sigma^{\rm RV}_{m+1} + \sigma^{\rm VV}_{m} = \sigma^{\rm NNLO}_{m+2} + \sigma^{\rm NNLO}_{m+1} + \sigma^{\rm NNLO}_{m}$$

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$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \left\{ \left({\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{{\rm RR},{\rm A}_1}_{m+2} \right) J_{m+1} - \left[{\rm d}\sigma^{{\rm RV},{\rm A}_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{{\rm RR},{\rm A}_1}_{m+2} \right)^{{\rm A}_1} \right] J_m \right\}$$

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RR,A_2 regularizes doubly-unresolved limits

G. Somogyi, ZT hep-ph/0609041, hep-ph/0609043
G. Somogyi, ZT, V. Del Duca hep-ph/0502226, hep-ph/0609042
Z. Nagy, G. Somogyi, ZT hep-ph/0702273

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RR,A_1 regularizes singly-unresolved limits

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RR,A_{12} removes overlapping subtractions

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Use known ingredients

- Universal IR structure of QCD (squared) matrix elements
  - ϵ-poles of one- and two-loop amplitudes
  - soft and collinear factorization of QCD matrix elements

tree-level 3-parton splitting, double soft current:

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- Extension over whole phase space using momentum mappings (not unique):

$$\{p\}_{n+s} \longrightarrow \{\tilde{p}\}_n$$

Momentum mappings

$$\{p\}_{n+s} \longrightarrow \{\tilde{p}\}_n$$

- implement exact momentum conservation
- recoil distributed democratically

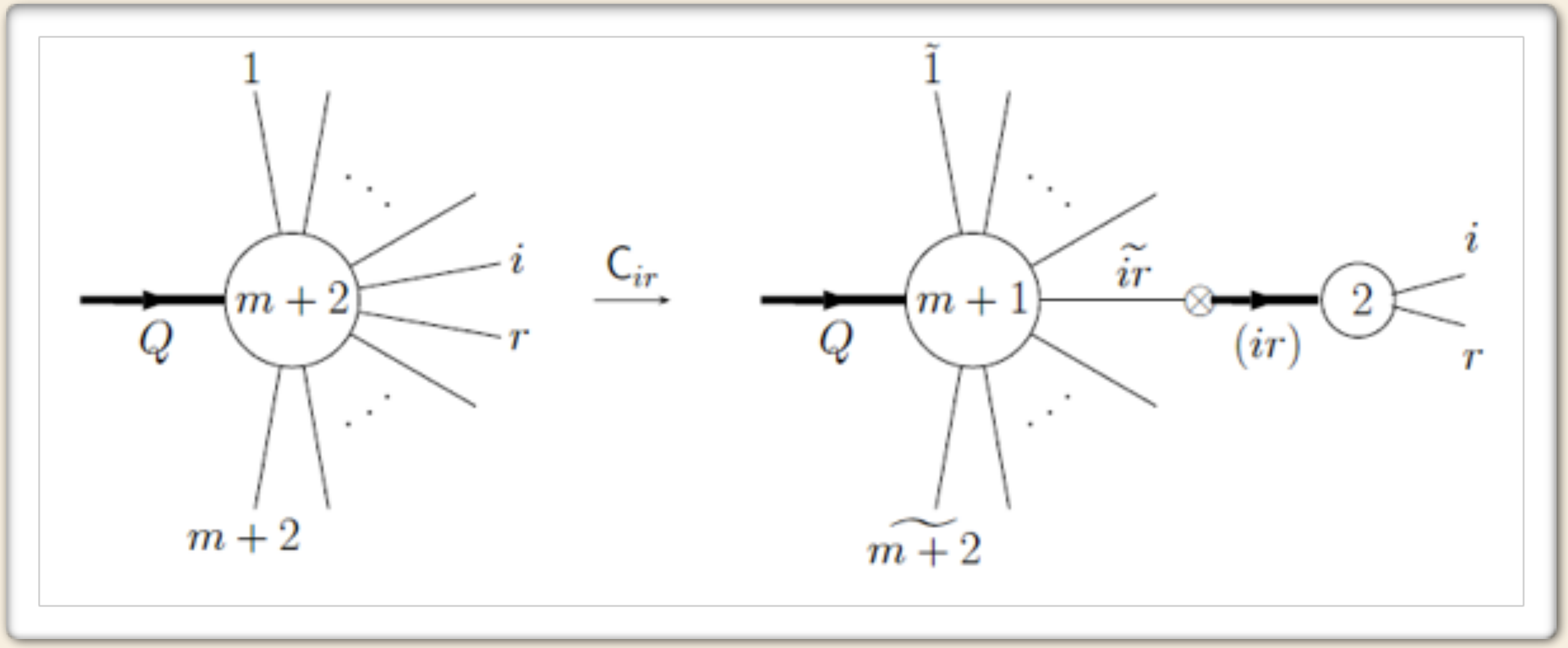
  $\Rightarrow$ can be generalized to any number s of unresolved partons

- different mappings for collinear and soft limits
  - collinear limit $p_i || p_r$: $\{p\}_{n+1} \xrightarrow{\mathsf{C}_{ir}} \{\tilde{p}\}_n^{(ir)}$
  - soft limit $p_s \to 0$: $\{p\}_{n+1} \xrightarrow{\mathsf{S}_s} \{\tilde{p}\}_n^{(s)}$

Momentum mappings

$$\{p\}_{n+s} \to \{\tilde{p}\}_n$$

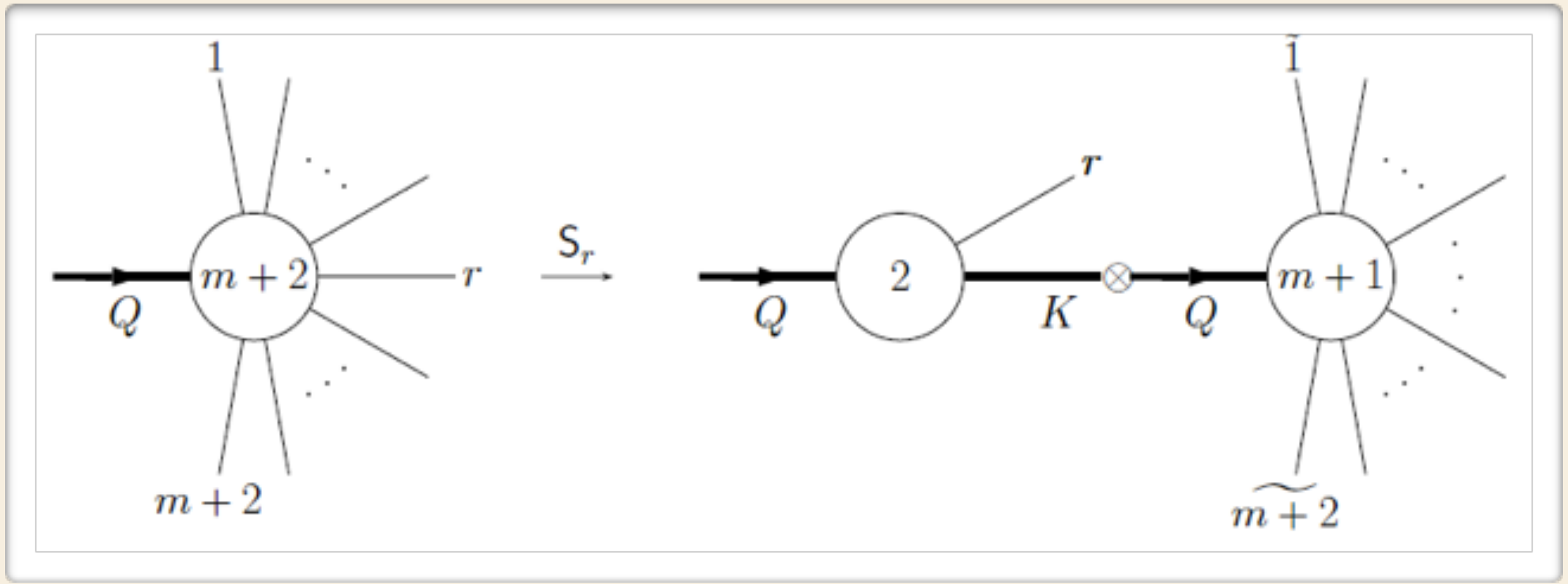
- implement exact momentum conservation
- recoil distributed democratically
- different mappings for collinear and soft limits
- lead to phase-space factorization
- can be generalized to any s trivially



Momentum mappings

$$\{p\}_{n+s} \longrightarrow \{\tilde{p}\}_n$$

- implement exact momentum conservation
- recoil distributed democratically
- different mappings for collinear and soft limits
- lead to phase-space factorization
- can be generalized to any s trivially



Momentum mappings

define subtractions

$$\sigma^{\rm NNLO} = \sigma^{\rm RR}_{m+2} + \sigma^{\rm RV}_{m+1} + \sigma^{\rm VV}_{m} = \sigma^{\rm NNLO}_{m+2} + \sigma^{\rm NNLO}_{m+1} + \sigma^{\rm NNLO}_{m}$$

$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \left\{ {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m - \left({\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \left\{ \left({\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right) J_{m+1} - \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \right\}$$

$$\sigma^{\rm NNLO}_{m} = \int_{m} \left\{ {\rm d}\sigma^{\rm VV}_{m} + \int_2 \left({\rm d}\sigma^{\rm RR,A_2}_{m+2} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} \right) + \int_1 \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] \right\} J_m$$

G. Somogyi, ZT hep-ph/0609041, hep-ph/0609043
G. Somogyi, ZT, V. Del Duca hep-ph/0502226, hep-ph/0609042
Z. Nagy, G. Somogyi, ZT hep-ph/0702273

implementation for general m in MCCSM code

Adam Kardos 2015

MCCSM built in checks

```
iterm:     5 , g (3) -> g (7) || b (3) || b~(6)
UBorn: e+ e- -> g  b~ b
                \->  g  b  b~
iexp=   1  , Cirs/RR=    1.0080327185410246935976025656525l
iexp=   0  , Cirs/RR=    1.00499213240252449541142746655114
iexp=   0  , Cirs/RR=    1.00188210122417253945669893587992
iexp=  -1  , Cirs/RR=    1.00062799209472484026964163812994
iexp=  -2  , Cirs/RR=    1.00020195770962404594799550034461
iexp=  -3  , Cirs/RR=    1.00006420440823578844890246156009
iexp=  -4  , Cirs/RR=    1.00002033728517247971408852818993
iexp=  -5  , Cirs/RR=    1.00000643462401283847535738494549
iexp=  -6  , Cirs/RR=    1.00000203514784404384857605445244
iexp=  -7  , Cirs/RR=    1.00000064360436586785172322902143
iexp=  -8  , Cirs/RR=    1.00000020352898187202854767170299
iexp=  -9  , Cirs/RR=    1.00000006436185636550093621075798
iexp= -10  , Cirs/RR=    1.00000002035304016617958048905596
iexp= -11  , Cirs/RR=    1.00000000643619983435100933779238
iexp= -12  , Cirs/RR=    1.00000000203530543521165651669029
iexp= -13  , Cirs/RR=    1.00000000064362017775031206617771
iexp= -14  , Cirs/RR=    1.00000000020353075587311446332694
iexp= -15  , Cirs/RR=    1.00000000006435764917616243177692
iexp= -16  , Cirs/RR=    1.00000000002031691222958531635813
iexp= -17  , Cirs/RR=    1.00000000000562358122843217515565
```

MCCSM built in checks

Checking subtractions locally in IR limits, e.g.triple-collinear limit in arbitrary phase space point:

```
iterm:     5 , g (3) -> g (7) || b (3) || b~(6)
UBorn: e+ e- -> g  b~ b
                \-> g  b  b~
iexp=   1  , Cirs/RR=    1.0080327185410246935976025656525 1
iexp=   0  , Cirs/RR=    1.0049921324025244954114274665511 4
iexp=   0  , Cirs/RR=    1.0018821012241725394566989358799 2
iexp=  -1  , Cirs/RR=    1.0006279920947248402696416381299 4
iexp=  -2  , Cirs/RR=    1.0002019577096240459479955003446 1
iexp=  -3  , Cirs/RR=    1.0000642044082357884489024615600 9
iexp=  -4  , Cirs/RR=    1.0000203372851724797140885281899 3
iexp=  -5  , Cirs/RR=    1.0000064346240128384753573849454 9
iexp=  -6  , Cirs/RR=    1.0000020351478440438485760544524 4
iexp=  -7  , Cirs/RR=    1.0000006436043658678517232290214 3
iexp=  -8  , Cirs/RR=    1.0000002035289818720285476717029 9
iexp=  -9  , Cirs/RR=    1.0000000643618563655009362107579 8
iexp= -10  , Cirs/RR=    1.0000000203530401661795804890559 6
iexp= -11  , Cirs/RR=    1.0000000064361998343510093377923 8
iexp= -12  , Cirs/RR=    1.0000000020353054352116565166902 9
iexp= -13  , Cirs/RR=    1.0000000006436201777503120661777 1
iexp= -14  , Cirs/RR=    1.0000000002035307558731144633269 4
iexp= -15  , Cirs/RR=    1.0000000000643576491761624317769 2
iexp= -16  , Cirs/RR=    1.0000000000203169122295853163581 3
iexp= -17  , Cirs/RR=    1.0000000000056235812284321751556 5
```

MCCSM built in checks

```
CSirs: g (6) -> g (6) || g (7) , g (5) -> 0  VALID
iter no.  1 scale no.  1 1.0626663494874406131036910247 5825 *-WARN-*
iter no.  2 scale no.  1 .999333391187566641313172350855109
iter no.  3 scale no.  1 .999936056716206679301961328662179
iter no.  4 scale no.  1 .999993217158857353081669676825320
iter no.  5 scale no.  1 .999999289527334562367472371577073
iter no.  6 scale no.  1 .999999927955557480464159147841895
iter no.  7 scale no.  1 .999999992764231332748306260947794
iter no.  8 scale no.  1 .999999999275434672484589563284781
iter no.  9 scale no.  1 .999999999927512229318504406669479
iter no. 10 scale no.  1 .999999999992750235327996735663320
iter no. 11 scale no.  1 .999999999999274992304311327282204
iter no. 12 scale no.  1 .999999999999927498242894752910729
iter no. 13 scale no.  1 .999999999999992749794474709275527
iter no. 14 scale no.  1 .999999999999999275003843983911918
iter no. 15 scale no.  1 .999999999999999927675414662535521
```

double unresolved

```
Cir: b (3) -> b (3) || g (7)  VALID
iter no.  1 scale no.  1 .961486708018718654422606471529938 *-WARN-*
iter no.  2 scale no.  1 1.00602959209786220837235112804777
iter no.  3 scale no.  1 1.00066580047174234782868128197356
iter no.  4 scale no.  1 1.00006749924864464471460885374332
iter no.  5 scale no.  1 1.00000675951123416892158622562722
iter no.  6 scale no.  1 1.00000067604739572862393476710447
iter no.  7 scale no.  1 1.00000006760570270606858225599869
iter no.  8 scale no.  1 1.00000000676057990234915689940388
iter no.  9 scale no.  1 1.00000000067605808655274887283141
iter no. 10 scale no.  1 1.00000000006760580961845340615602
iter no. 11 scale no.  1 1.00000000000676058097147183507127
iter no. 12 scale no.  1 1.00000000000067605809725802473631
iter no. 13 scale no.  1 1.00000000000006760580921822736597
iter no. 14 scale no.  1 1.00000000000000676057794954317165
iter no. 15 scale no.  1 1.00000000000000067615396661119602
```

single unresolved

MCCSM built in checks

Checking finiteness in singular regions, e.g. regularized RR:

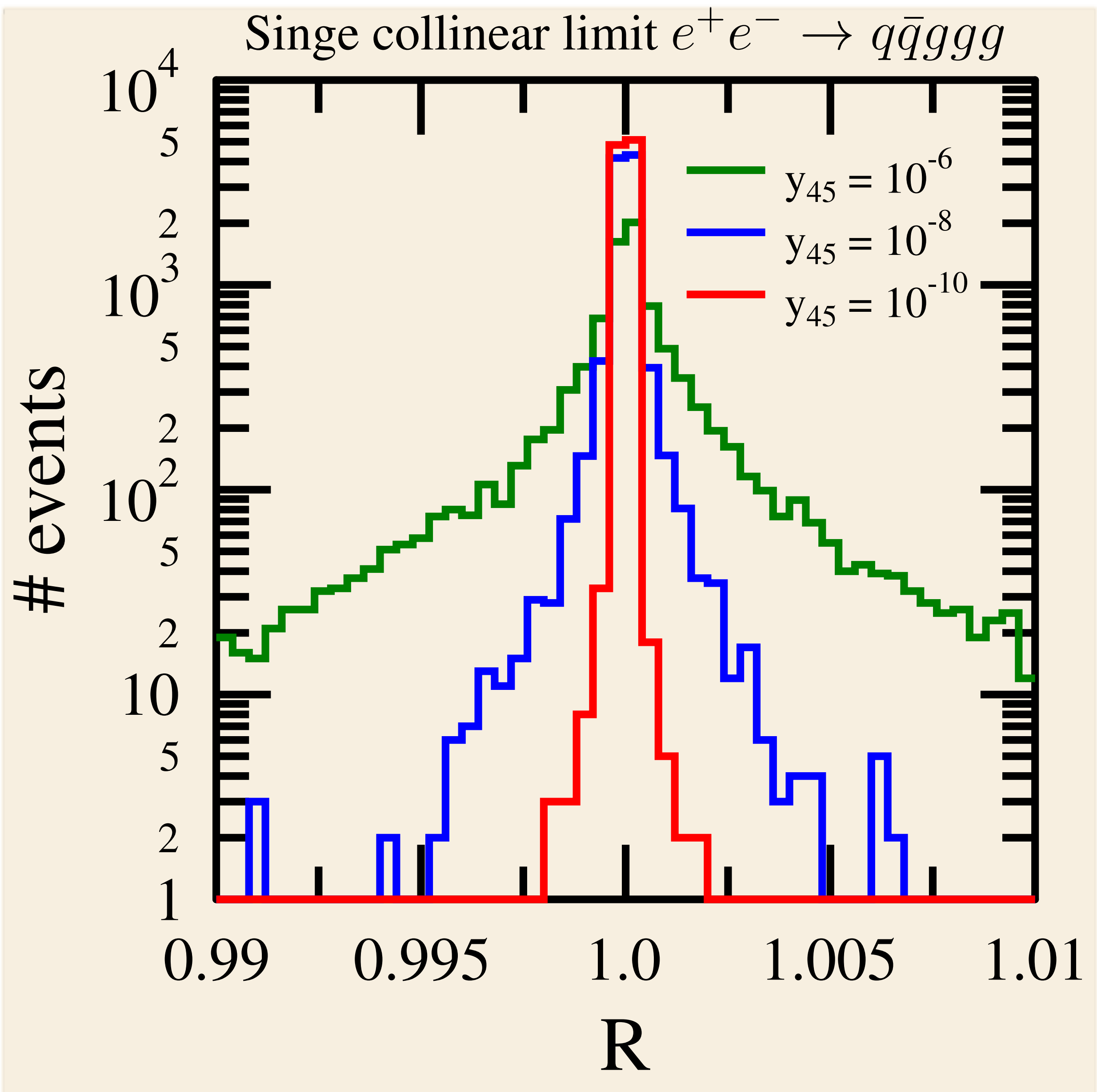
```
CSirs: g (6) -> g (6) || g (7) , g (5) -> 0  VALID
iter no.  1 scale no.  1 1.0626663494874406131036910247582 5 *-WARN-*
iter no.  2 scale no.  1 .9993333911875666413131723508551 09
iter no.  3 scale no.  1 .9999360567162066793019613286621 79
iter no.  4 scale no.  1 .9999932171588573530816696768253 20
iter no.  5 scale no.  1 .9999992895273345623674723715770 73
iter no.  6 scale no.  1 .9999999279555574804641591478418 95
iter no.  7 scale no.  1 .9999999927642313327483062609477 94
iter no.  8 scale no.  1 .9999999992754346724845895632847 81
iter no.  9 scale no.  1 .9999999999275122293185044066694 79
iter no. 10 scale no.  1 .9999999999927502353279967356633 20
iter no. 11 scale no.  1 .9999999999992749923043113272822 04
iter no. 12 scale no.  1 .9999999999999274982428947529107 29
iter no. 13 scale no.  1 .9999999999999927497944747092755 27
iter no. 14 scale no.  1 .9999999999999992750038439839119 18
iter no. 15 scale no.  1 .9999999999999999276754146625355 21
```

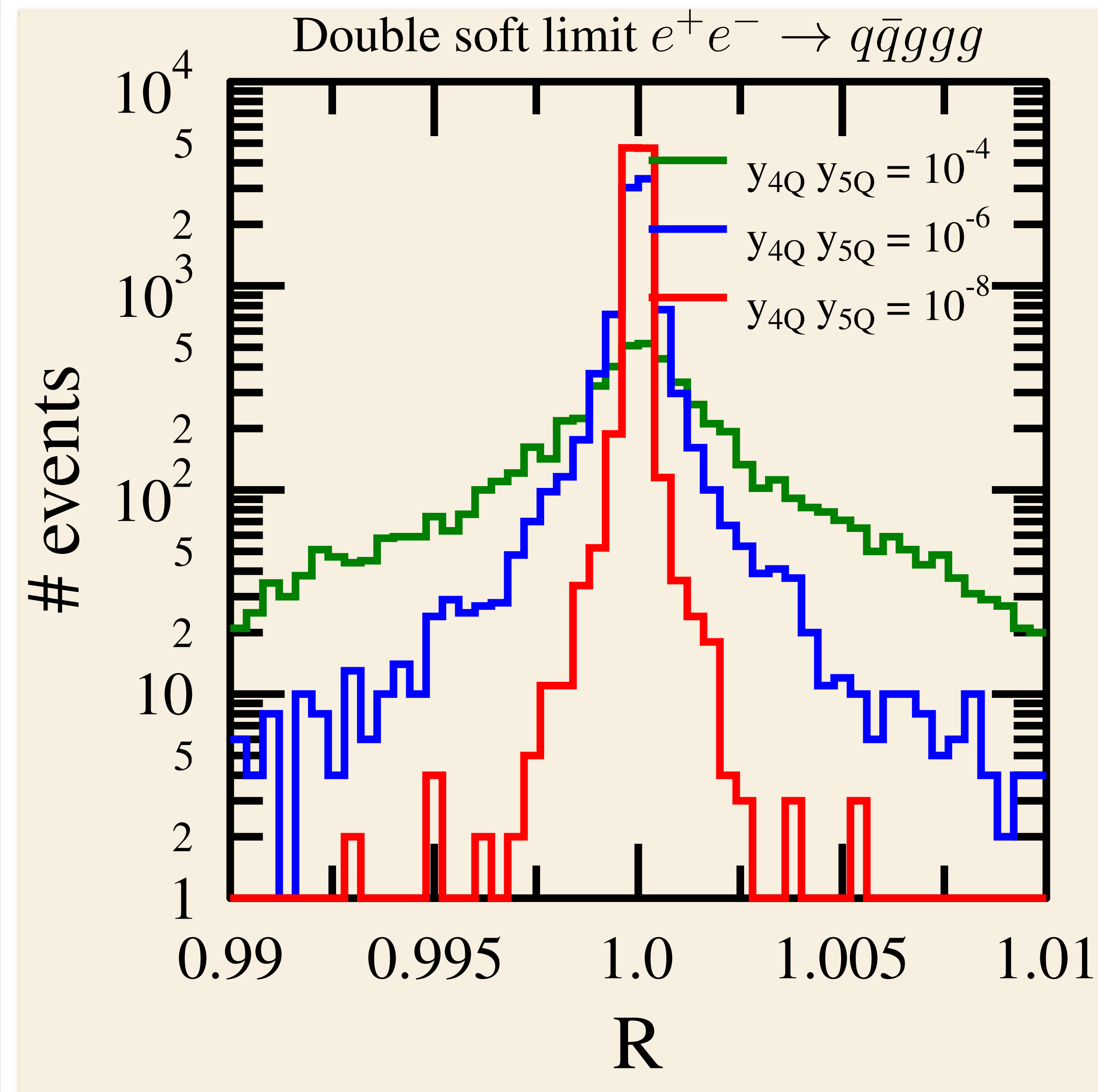
double unresolved

```
Cir: b (3) -> b (3) || g (7)  VALID
iter no.  1 scale no.  1 .96148670801871865442260647152993 8 *-WARN-*
iter no.  2 scale no.  1 1.0060295920978622083723511280477 7
iter no.  3 scale no.  1 1.0006658004717423478286812819735 6
iter no.  4 scale no.  1 1.0000674992486446447146088537433 2
iter no.  5 scale no.  1 1.0000067595112341689215862256272 2
iter no.  6 scale no.  1 1.0000006760473957286239347671044 7
iter no.  7 scale no.  1 1.0000000676057027060685822559986 9
iter no.  8 scale no.  1 1.0000000067605799023491568994038 8
iter no.  9 scale no.  1 1.0000000006760580865527488728314 1
iter no. 10 scale no.  1 1.0000000000676058096184534061560 2
iter no. 11 scale no.  1 1.0000000000067605809714718350712 7
iter no. 12 scale no.  1 1.0000000000006760580972580247363 1
iter no. 13 scale no.  1 1.0000000000000676058092182273659 7
iter no. 14 scale no.  1 1.0000000000000067605779495431716 5
iter no. 15 scale no.  1 1.0000000000000006761539666111960 2
```

single unresolved

Kinematic singularities cancel in RR

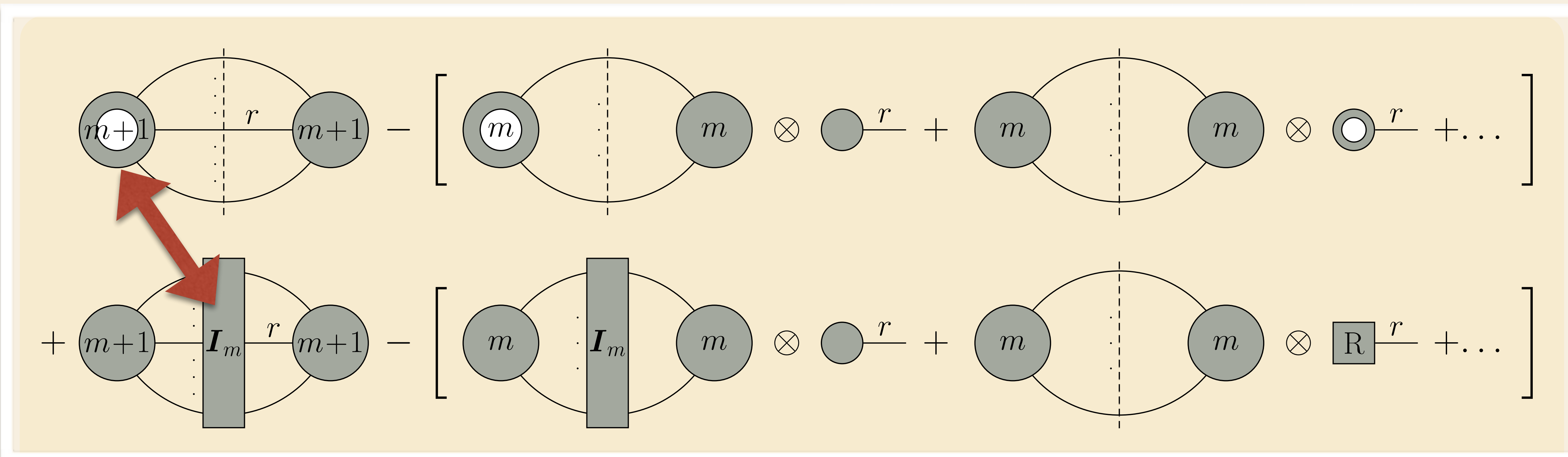




R = subtraction/RR

Cancellation of singularities in RV

Poles cancel vertically pairwise



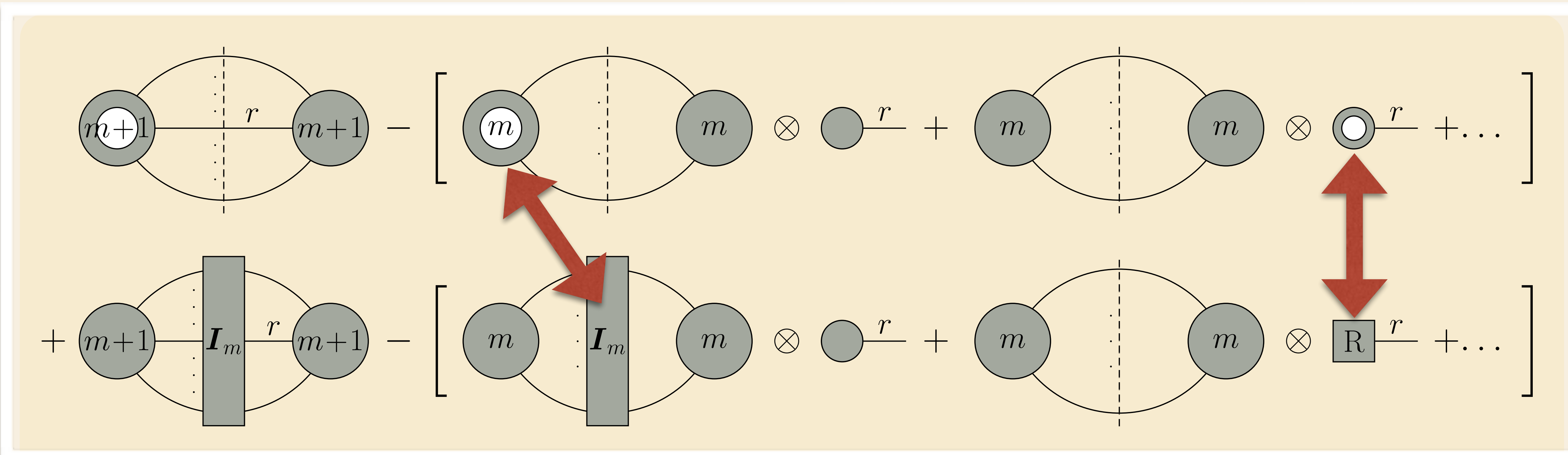
$$\sigma_{m+2}^{\mathrm{NNLO}} = \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} J_m - \left(\mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} J_m \right) \right\}$$

$$\sigma_{m+1}^{\mathrm{NNLO}} = \int_{m+1} \left\{ \left(\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right) J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] J_m \right\}$$

$$\sigma_{m}^{\mathrm{NNLO}} = \int_{m} \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_{m} + \int_2 \left(\mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} \right) + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] \right\} J_m$$

Cancellation of singularities in RV

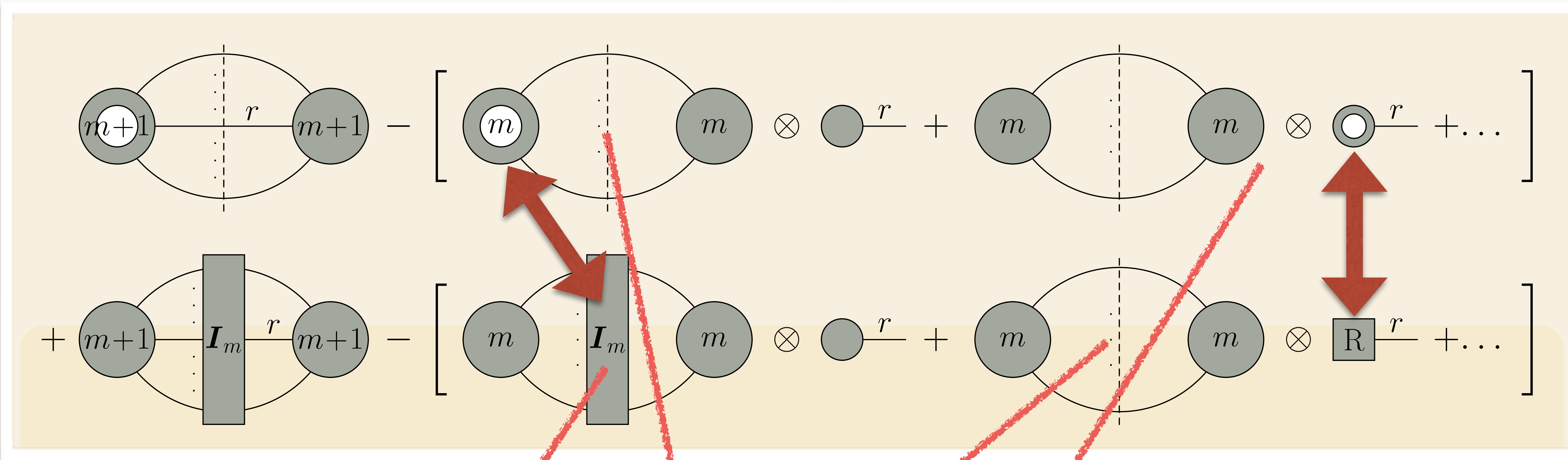
Poles cancel vertically pairwise



$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \left\{ {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m - \left({\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \left\{ \left({\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right) J_{m+1} - \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \right\}$$

$$\sigma^{\rm NNLO}_{m} = \int_{m} \left\{ {\rm d}\sigma^{\rm VV}_{m} + \int_2 \left({\rm d}\sigma^{\rm RR,A_2}_{m+2} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} \right) + \int_1 \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] \right\} J_m$$



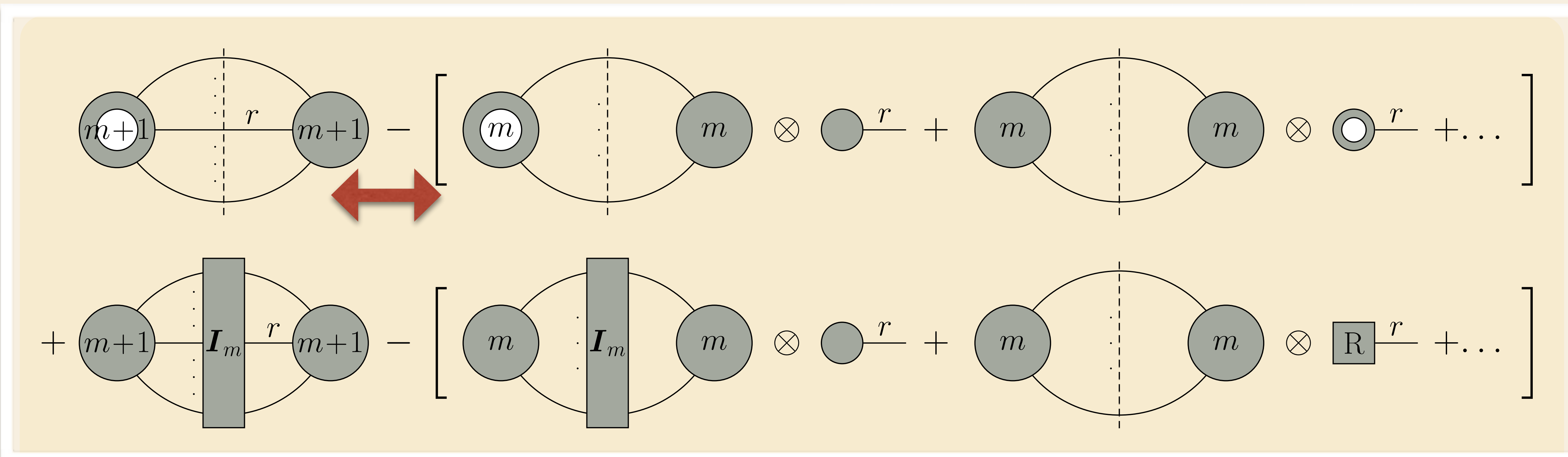

```
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~ Cir ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
e+ e- -> b  b~ b  b~
Checking pole cancellation in point 1
iterm:    1 , g (3) -> b (3) || b~(4)
UBorn: e+ e- -> g  b  b~
                \->  b  b~


Cancellation for Cir00I + Cir01:
             Cir00I                     Cir01                     norm. sum
O(e^-2) :    18.826825462152872        -18.826825462153515       -3.4155581733924357E-014
O(e^-1) :    63.517133810744149        -63.517133810746685       -3.9936272537449989E-014

Cancellation for CirR00 + Cir10:
             CirR00                     Cir10                     norm. sum
O(e^-2) :   -1.1074603213031107         1.1074603213031107       -0.0000000000000000
O(e^-1) :    39.321998994866775        -39.321998994866760        3.6139705707884122E-016
```

Cancellation of singularities in RV

Kinematic singularities cancel horizontally



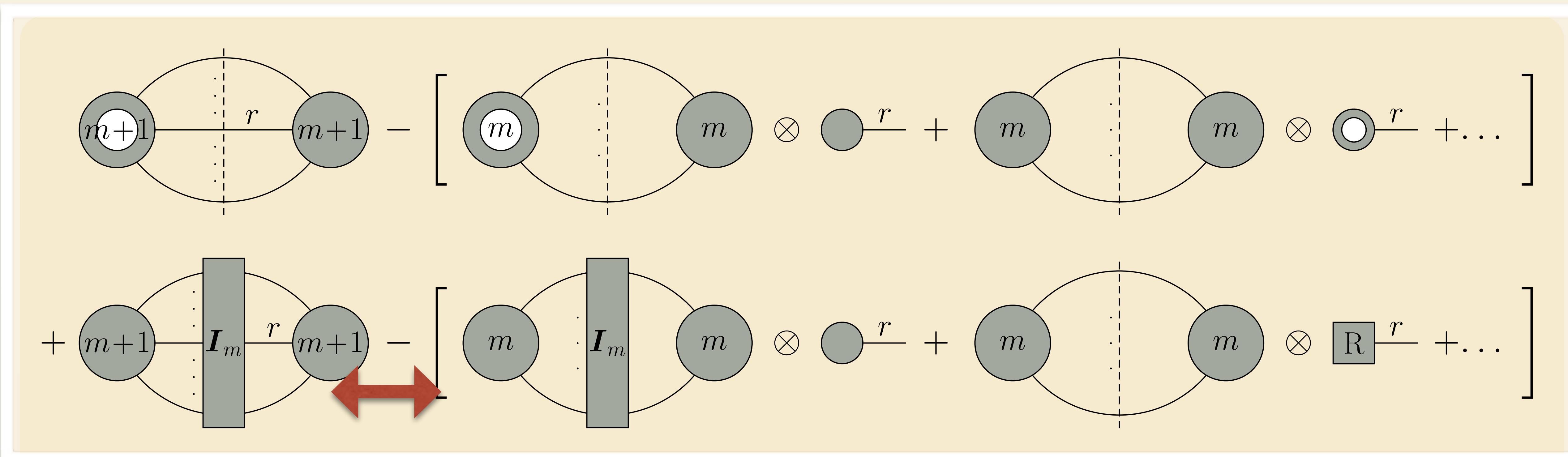
$$\sigma^{\mathrm{NNLO}}_{m+2} = \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} J_m - \left(\mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m+1} = \int_{m+1} \left\{ \left(\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right) J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] J_m \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m} = \int_{m} \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_{m} + \int_2 \left(\mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} \right) + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] \right\} J_m$$

Cancellation of singularities in RV

Kinematic singularities cancel horizontally

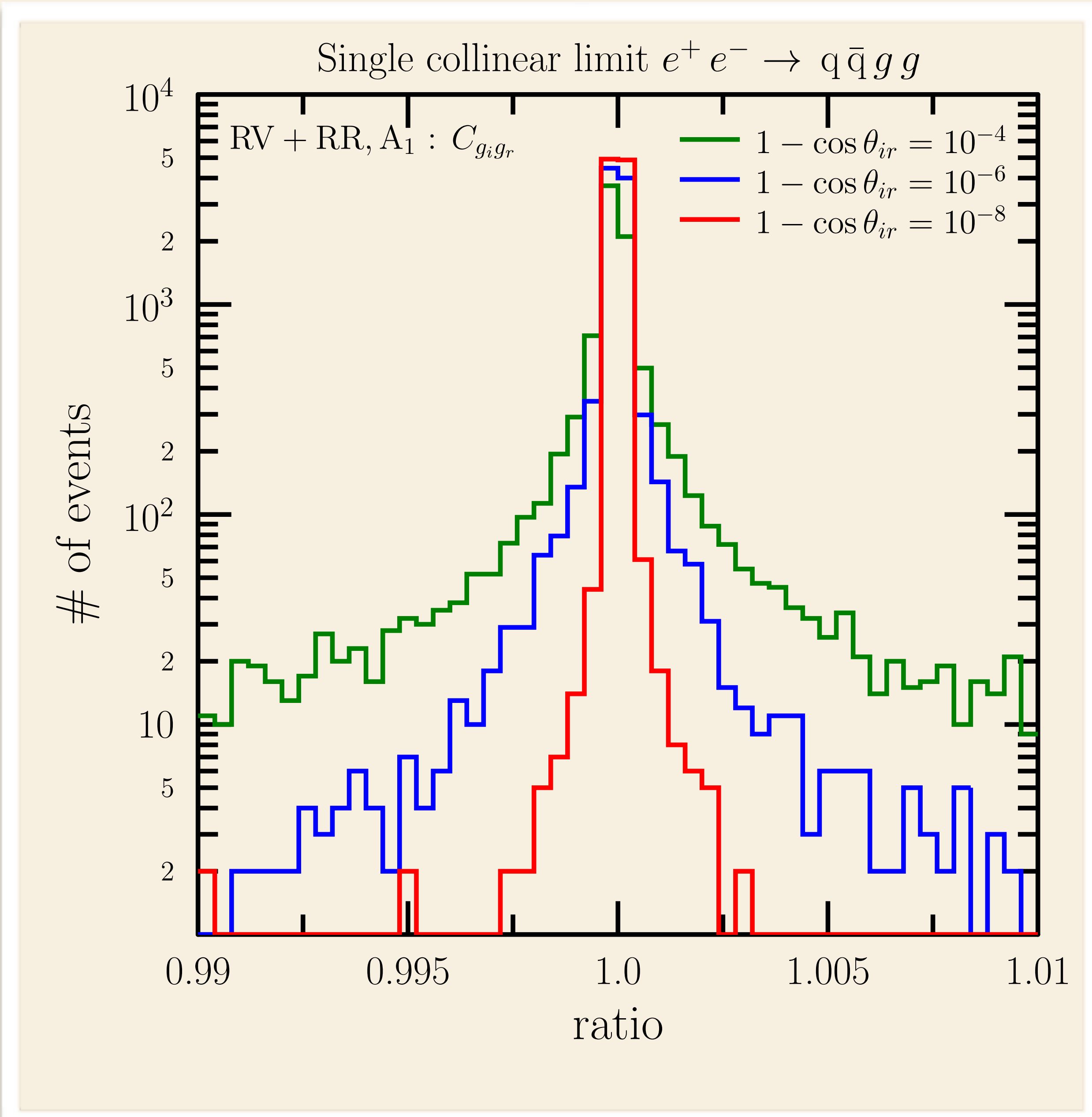


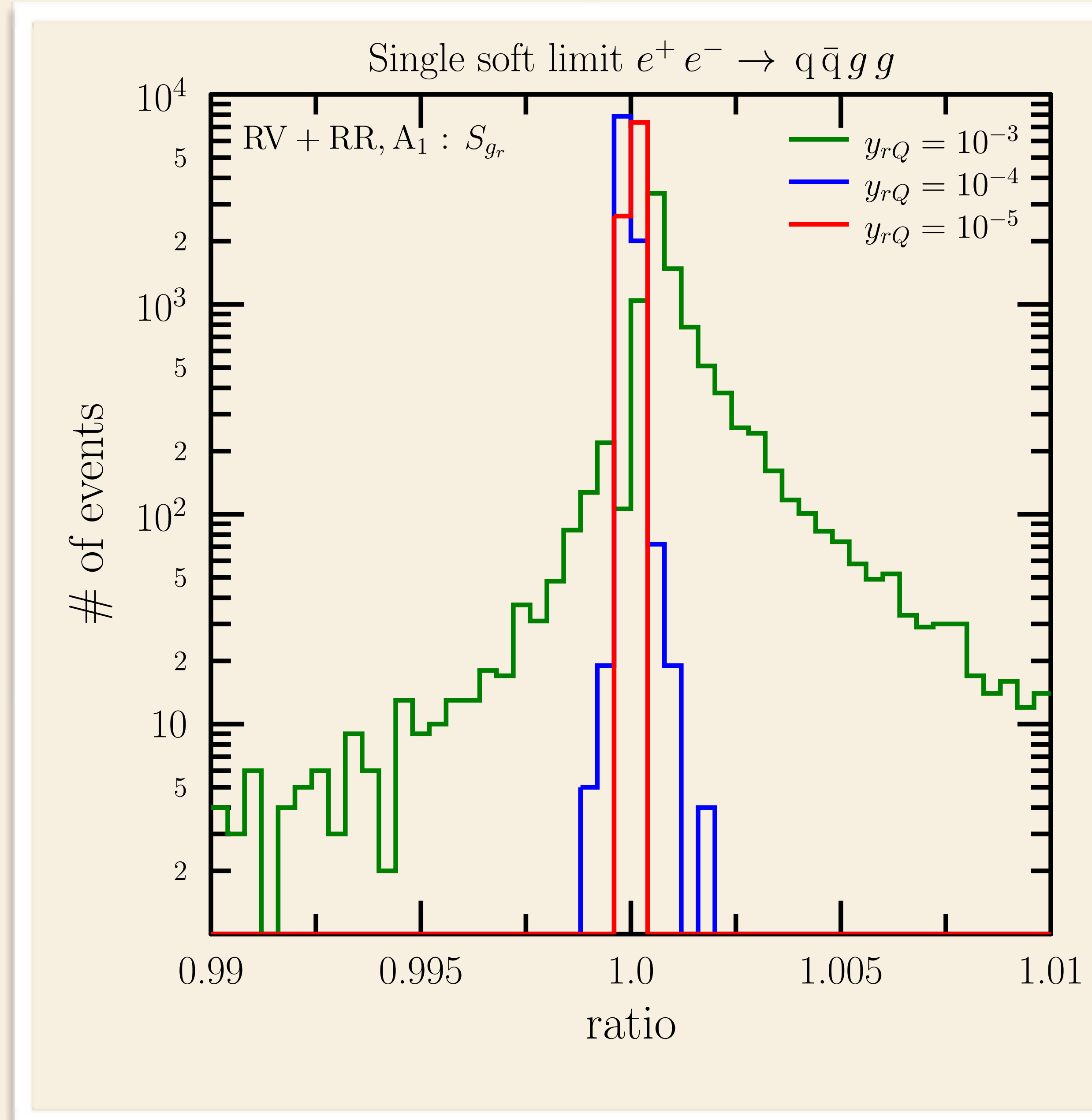
$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \left\{ d\sigma^{\rm RR}_{m+2} J_{m+2} - d\sigma^{\rm RR,A_2}_{m+2} J_m - \left(d\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - d\sigma^{\rm RR,A_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \left\{ \left(d\sigma^{\rm RV}_{m+1} + \int_1 d\sigma^{\rm RR,A_1}_{m+2} \right) J_{m+1} - \left[d\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 d\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \right\}$$

$$\sigma^{\rm NNLO}_{m} = \int_{m} \left\{ d\sigma^{\rm VV}_{m} + \int_2 \left(d\sigma^{\rm RR,A_2}_{m+2} - d\sigma^{\rm RR,A_{12}}_{m+2} \right) + \int_1 \left[d\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 d\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] \right\} J_m$$

Kinematic singularities cancel in RV





R = subtraction/(RV+RR,A$_1$)

Regularized RR and RV contributions

can now be computed by numerical Monte Carlo integrations

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{RR}}_{m+2} + \sigma^{\mathrm{RV}}_{m+1} + \sigma^{\mathrm{VV}}_{m} = \sigma^{\mathrm{NNLO}}_{m+2} + \sigma^{\mathrm{NNLO}}_{m+1} + \sigma^{\mathrm{NNLO}}_{m}$$

$$\sigma^{\mathrm{NNLO}}_{m+2} = \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A}_2}_{m+2} J_m - \left(\mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR,A}_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m+1} = \int_{m+1} \left\{ \left(\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \right) J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV,A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \right)^{\mathrm{A}_1} \right] J_m \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m} = \int_{m} \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_{m} + \int_2 \left(\mathrm{d}\sigma^{\mathrm{RR,A}_2}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A}_{12}}_{m+2} \right) + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV,A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \right)^{\mathrm{A}_1} \right] \right\} J_m$$

G. Somogyi, ZT hep-ph/0609041, hep-ph/0609043
G. Somogyi, ZT, V. Del Duca hep-ph/0502226, hep-ph/0609042
Z. Nagy, G. Somogyi, ZT hep-ph/0702273

implementation for general m in MCCSM code

Adam Kardos 2015

Difficulty

Integrated approximate xsections

$$\sigma^{\rm NNLO} = \sigma^{\rm RR}_{m+2} + \sigma^{\rm RV}_{m+1} + \sigma^{\rm VV}_{m} = \sigma^{\rm NNLO}_{m+2} + \sigma^{\rm NNLO}_{m+1} + \sigma^{\rm NNLO}_{m}$$

$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \Big\{ {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m - \Big({\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \Big) \Big\}$$

$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \Big\{ \Big({\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \Big) J_{m+1} - \Big[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \Big(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \Big)^{\rm A_1} \Big] J_m \Big\}$$

$$\sigma^{\rm NNLO}_{m} = \int_{m} \Big\{ {\rm d}\sigma^{\rm VV}_{m} + \int_2 \Big({\rm d}\sigma^{\rm RR,A_2}_{m+2} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} \Big) + \int_1 \Big[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \Big(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \Big)^{\rm A_1} \Big] \Big\} J_m$$

After integrating over unresolved momenta & summing over unresolved colors and flavors, the subtraction terms can be written as products of insertion operators (in color space) and lower point cross sections:

$$\int_p {\rm d}\sigma^{\rm RR,A_p} = \boldsymbol{I}^{(0)}_p(\{p\}_n;\epsilon) \otimes {\rm d}\sigma^{\rm B}_n$$

Integrated approximate xsections

$$\int_p d\sigma^{\mathrm{RR,A}_p} = \int_p \left[d\phi_{m+2}(\{p\}) \sum_R \mathcal{X}_R(\{p\}) \right]$$

$$= \int_p \left[d\phi_n(\{\tilde{p}\}^{(R)})[dp_p^{(R)}] \sum_R \left(8\pi\alpha_s\mu^{2\epsilon}\right)^p Sing_R(p_p^{(R)}) \otimes |\mathcal{M}_n^{(0)}(\{\tilde{p}\}_n^{(R)})|^2 \right]$$

$$= \left(8\pi\alpha_s\mu^{2\epsilon}\right)^p \sum_R \left[\int_p [dp_p^{(R)}] Sing_R(p_p^{(R)}) \right] \otimes d\phi_n(\{\tilde{p}\}^{(R)}) |\mathcal{M}_n^{(0)}(\{\tilde{p}\}_n^{(R)})|^2$$

$$= \underbrace{\left(8\pi\alpha_s\mu^{2\epsilon}\right)^p \sum_R \left[\int_p [dp_p^{(R)}] Sing_R(p_p^{(R)}) \right]}_{\boldsymbol{I}_p^{(0)}(\{p\}_n;\epsilon)} \otimes d\sigma_n^{\mathrm{B}}$$

the integrated counter-terms $[X]_R \propto \int_p [dp_p^{(R)}] Sing_R(p_p^{(R)})$ are

independent of the process & observable

$\Rightarrow$ need to compute only once

Summation over unresolved flavors

- integrated counter-terms $[X]_{f_i\dots}$ carry flavor indices of unresolved patrons

  ⇒ need to sum over unresolved flavors:

  straightforward, though tedious, result can be summarized in flavor-summed integrated counter-terms

P. Bolzoni, G. Somogyi, ZT arXiv:0905.4390

- symbolically:

$$\left(X^{(0)}\right)_{f_i\dots}^{(j,l)\dots} = \sum [X^{(0)}]_{f_k\dots}^{(j,l)\dots}$$

- and precisely, for instance, two-flavor sum:

$$\sum_{\{m+2\}} \frac{1}{S_{\{m+2\}}} \sum_t \sum_{k\neq t} [X_{kt}^{(0)}]_{f_k f_t}^{(\dots)} \equiv \sum_{\{m\}} \frac{1}{S_{\{m\}}} \left(X_{kt}^{(0)}\right)^{(\dots)}$$

Integrating out unresolved momenta

two types of singly-unresolved

$$\sigma^{\rm NNLO} = \sigma^{\rm RR}_{m+2} + \sigma^{\rm RV}_{m+1} + \sigma^{\rm VV}_{m} = \sigma^{\rm NNLO}_{m+2} + \sigma^{\rm NNLO}_{m+1} + \sigma^{\rm NNLO}_{m}$$

$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \left\{ {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m - \left({\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \left\{ \left({\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right) J_{m+1} - \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \right\}$$

$$\sigma^{\rm NNLO}_{m} = \int_{m} \left\{ {\rm d}\sigma^{\rm VV}_{m} + \int_2 \left({\rm d}\sigma^{\rm RR,A_2}_{m+2} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} \right) + \int_1 \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] \right\} J_m$$

G. Somogyi, ZT arXiv:0807.0509
U. Aglietti, V. Del Duca, C. Duhr, G. Somogyi, ZT arXiv:0807.0514
P. Bolzoni, S. Moch, G. Somogyi, ZT arXiv:0905.4390

Collinear integrals

convolution of the integral of AP-splitting function over ordinary phase space

$$\int_0^{\alpha_0} \mathrm{d}\alpha\,(1-\alpha)^{2d_0-1}\,\frac{s_{\widetilde{ir}Q}}{2\pi}\int \mathrm{d}\phi_2(p_i,p_r;p_{(ir)})\,\frac{1}{s_{ir}^{1+\kappa\epsilon}}P^{(\kappa)}_{f_i f_r}(z_i,z_r;\epsilon)\,, \qquad \kappa = 0,\,1$$

$$\mathrm{d}\phi_2(p_i,p_r;p_{(ir)}) = \frac{s_{ir}^{-\epsilon}}{8\pi}\,\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\,\mathrm{d}s_{ir}\,\mathrm{d}v\,\delta\big(s_{ir}-Q^2\alpha\big(\alpha+(1-\alpha)x\big)\big)$$

$$\times\;[v\,(1-v)]^{-\epsilon}\,\Theta(1-v)\Theta(v)$$

G. Somogyi, ZT arXiv:0807.0509

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$$\frac{z_r^{k+\delta\epsilon}}{s_{ir}^{1+\kappa\epsilon}}\,g_I^{(\pm)}(z_r)\,, \qquad z_r=\frac{\alpha Q^2+(1-\alpha)v s_{\widetilde{ir}Q}}{2\alpha Q^2+(1-\alpha)s_{\widetilde{ir}Q}}$$

δ	Function	$g_I^{(\pm)}(z)$
0	g_A	1
∓ 1	$g_B^{(\pm)}$	$(1-z)^{\pm\epsilon}$
0	$g_C^{(\pm)}$	$(1-z)^{\pm\epsilon}\,{}_2F_1(\pm\epsilon,\pm\epsilon,1\pm\epsilon,z)$
± 1	$g_D^{(\pm)}$	${}_2F_1(\pm\epsilon,\pm\epsilon,1\pm\epsilon,1-z)$

Soft integrals

convolution of the integral of eikonal factors over ordinary phase space

$$\mathcal{J} \propto -\int_0^{y_0} \mathrm{d}y\,(1-y)^{d_0'-1} \frac{Q^2}{2\pi} \int \mathrm{d}\phi_2(p_r, K; Q) \left(\frac{s_{ik}}{s_{ir}s_{kr}}\right)^{1+\kappa\epsilon}$$

$$\mathrm{d}\phi_2(p_r, K; Q) = \frac{(Q^2)^{-\epsilon}}{16\pi^2} \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \mathrm{d}\varepsilon_r\, \varepsilon_r^{1-2\epsilon} \delta(y-\varepsilon_r)$$
$$\times\, \mathrm{d}(\cos\vartheta)\, \mathrm{d}(\cos\varphi) (\sin\vartheta)^{-2\epsilon} (\sin\varphi)^{-1-2\epsilon}$$

G. Somogyi, ZT arXiv:0807.0509
U. Aglietti, V. Del Duca, C. Duhr, G. Somogyi, ZT arXiv:0807.0514
P. Bolzoni, S. Moch, G. Somogyi, ZT arXiv:0905.4390

Computing the integrals

- Use algebraic and symmetry relations to reduce to a basic set $\Rightarrow$ MI's (but no IBP was used), not minimal
- two strategies:

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2. angular integrals in terms of MB representations
3. resolve ϵ-poles by analytic continuation
4. MB integrals -> Euler-type integrals, pole coefficients are finite parametric integrals

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5. evaluate parametric integrals of pole coefficients in terms of multiple polylogs, or numerically e.g. by SecDec

Status of (287) integrals

Int	status
$\mathcal{I}_{1\mathcal{C},0}^{(k)}$	✔
$\mathcal{I}_{1\mathcal{C},1}^{(k)}$	✔
$\mathcal{I}_{1\mathcal{C},2}^{(k)}$	✔
$\mathcal{I}_{1\mathcal{C},3}^{(k)}$	✔
$\mathcal{I}_{1\mathcal{C},4}^{(k)}$	✔
$\mathcal{I}_{1\mathcal{C},5}^{(k,l)}$	✔
$\mathcal{I}_{1\mathcal{C},6}^{(k,l)}$	✔
$\mathcal{I}_{1\mathcal{C},7}^{(k)}$	✔
$\mathcal{I}_{1\mathcal{C},8}$	✔

Int	status
$\mathcal{I}_{1\mathcal{S},0}$	✔
$\mathcal{I}_{1\mathcal{S},1}$	✔
$\mathcal{I}_{1\mathcal{S},2}$	$(m > 3)$ ✔
$\mathcal{I}_{1\mathcal{S},3}^{(k)}$	✔
$\mathcal{I}_{1\mathcal{S},4}$	✔
$\mathcal{I}_{1\mathcal{S},5}$	✔
$\mathcal{I}_{1\mathcal{S},6}$	✔
$\mathcal{I}_{1\mathcal{S},7}$	✔

Int	status
$\mathcal{I}_{1\mathcal{CS},0}$	✔
$\mathcal{I}_{1\mathcal{CS},1}$	✔
$\mathcal{I}_{1\mathcal{CS},2}^{(k)}$	✔
$\mathcal{I}_{1\mathcal{CS},3}$	✔
$\mathcal{I}_{1\mathcal{CS},4}$	✔

Int	status
$\mathcal{I}_{12\mathcal{C},1}^{(k,l)}$	✔
$\mathcal{I}_{12\mathcal{C},2}^{(k,l)}$	✔
$\mathcal{I}_{12\mathcal{C},3}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{C},4}^{(k,l)}$	✔
$\mathcal{I}_{12\mathcal{C},5}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{C},6}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{C},7}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{C},8}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{C},9}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{C},10}^{(k)}$	✔

Int	status
$\mathcal{I}_{2\mathcal{S},1}$	✔
$\mathcal{I}_{2\mathcal{S},2}$	✔
$\mathcal{I}_{2\mathcal{S},3}$	✔
$\mathcal{I}_{2\mathcal{S},4}$	✔
$\mathcal{I}_{2\mathcal{S},5}$	✔
$\mathcal{I}_{2\mathcal{S},6}$	✔
$\mathcal{I}_{2\mathcal{S},7}$	✔
$\mathcal{I}_{2\mathcal{S},8}$	✔
$\mathcal{I}_{2\mathcal{S},9}$	✔
$\mathcal{I}_{2\mathcal{S},10}$	✔
$\mathcal{I}_{2\mathcal{S},11}$	✔
$\mathcal{I}_{2\mathcal{S},12}$	✔
$\mathcal{I}_{2\mathcal{S},13}$	✔
$\mathcal{I}_{2\mathcal{S},14}$	✔
$\mathcal{I}_{2\mathcal{S},15}$	✔
$\mathcal{I}_{2\mathcal{S},16}$	✔
$\mathcal{I}_{2\mathcal{S},17}$	✔
$\mathcal{I}_{2\mathcal{S},18}$	✔
$\mathcal{I}_{2\mathcal{S},19}$	✔
$\mathcal{I}_{2\mathcal{S},20}$	✔
$\mathcal{I}_{2\mathcal{S},21}$	✔
$\mathcal{I}_{2\mathcal{S},22}$	✔
$\mathcal{I}_{2\mathcal{S},23}$	✔

Int	status
$\mathcal{I}_{12\mathcal{S},1}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{S},2}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{S},3}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{S},4}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{S},5}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{S},6}$	✔
$\mathcal{I}_{12\mathcal{S},7}$	✔
$\mathcal{I}_{12\mathcal{S},8}$	✔
$\mathcal{I}_{12\mathcal{S},9}$	✔
$\mathcal{I}_{12\mathcal{S},10}$	✔
$\mathcal{I}_{12\mathcal{S},11}$	✔
$\mathcal{I}_{12\mathcal{S},12}$	✔
$\mathcal{I}_{12\mathcal{S},13}$	✔

Int	status
$\mathcal{I}_{12\mathcal{CS},1}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{CS},2}$	✔
$\mathcal{I}_{12\mathcal{CS},3}$	✔

Int	status	Int	status
$\mathcal{I}_{2\mathcal{C},1}^{(j,k,l,m)}$	✔	$\mathcal{I}_{2\mathcal{CS},1}^{(k)}$	✔
$\mathcal{I}_{2\mathcal{C},2}^{(j,k,l,m)}$	✔	$\mathcal{I}_{2\mathcal{CS},2}^{(k)}$	✔
$\mathcal{I}_{2\mathcal{C},3}^{(j,k,l,m)}$	✔	$\mathcal{I}_{2\mathcal{CS},2}^{(2)}$	✔
$\mathcal{I}_{2\mathcal{C},4}^{(j,k,l,m)}$	✔	$\mathcal{I}_{2\mathcal{CS},3}^{(k)}$	✔
$\mathcal{I}_{2\mathcal{C},5}^{(-1,-1,-1,-1)}$	✔	$\mathcal{I}_{2\mathcal{CS},4}^{(k)}$	✔
$\mathcal{I}_{2\mathcal{C},6}^{(k,l)}$	✔	$\mathcal{I}_{2\mathcal{CS},5}^{(k)}$	✔

✓: pole coefficients and logarithmic terms in finite part are computed analytically, power terms in finite part numerically, in some cases analytically

G. Somogyi, C. Duhr

Structure of insertion operators

recall general form for Born sections

$$\int_p d\sigma^{\mathrm{RR},\mathrm{A}_p} = \boldsymbol{I}_p^{(0)}(\{p\}_n;\epsilon) \otimes d\sigma_n^{\mathrm{B}}$$

Insertion operators involve all possible color connections with given number of unresolved patrons with kinematic coefficients

for 1 unresolved parton on tree SME $|M^{(0)}|^2$:

$$\boldsymbol{I}_1^{(0)}(\{p\}_{m+1};\epsilon) = \frac{\alpha_s}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon \sum_i \left[\mathrm{C}_{1,f_i}^{(0)} \boldsymbol{T}_i^2 + \sum_k \mathrm{S}_1^{(0),(i,k)} \boldsymbol{T}_i \boldsymbol{T}_k \right]$$

kinematic functions contain poles starting from $O(\epsilon^{-2})$ for collinear and from $O(\epsilon^{-1})$ for soft

G. Somogyi, ZT hep-ph/0609041

Structure of insertion operators

recall general form for Born sections

$$\int_p \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_p} = \boldsymbol{I}_p^{(0)}(\{p\}_n;\epsilon) \otimes \mathrm{d}\sigma_n^{\mathrm{B}}$$

for 2 unresolved patrons on tree SME $|M^{(0)}|^2$:

$$\boldsymbol{I}_2^{(0)}(\{p\}_m;\epsilon) = \left[\frac{\alpha_s}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon\right]^2 \Bigg\{ \sum_i \left[\mathrm{C}_{2,f_i}^{(0)} \boldsymbol{T}_i^2 + \sum_k \mathrm{C}_{2,f_i f_k}^{(0)} \boldsymbol{T}_k^2\right] \boldsymbol{T}_i^2$$
$$+ \sum_{j,l} \left[\mathrm{S}_2^{(0),(j,l)} C_\mathrm{A} + \sum_i \mathrm{CS}_{2,f_i}^{(0),(j,l)} \boldsymbol{T}_i^2\right] \boldsymbol{T}_j \boldsymbol{T}_l$$
$$+ \sum_{i,k,j,l} \mathrm{S}_2^{(0),(i,k)(j,l)} \{\boldsymbol{T}_i \boldsymbol{T}_k, \boldsymbol{T}_j \boldsymbol{T}_l\} \Bigg\}$$

the iterated doubly-unresolved has the same color structure, kinematic coefficients differ

G. Somogyi et al arXiv:0905.4390, arXiv:1301.3504, arXiv:1301.3919

Structure of insertion operators

general form at one loop

$$\int_1 \mathrm{d}\sigma_{m+1}^{\mathrm{RV},\mathrm{A}_1} = \boldsymbol{I}_1^{(0)}(\{p\}_m;\epsilon) \otimes \mathrm{d}\sigma_m^{\mathrm{V}} + \boldsymbol{I}_1^{(1)}(\{p\}_m;\epsilon) \otimes \mathrm{d}\sigma_m^{\mathrm{B}}$$

for 1 unresolved parton on loop SME $|M^{(1)}|^2$:

$$\boldsymbol{I}_1^{(1)}(\{p\}_m;\epsilon) = \left[\frac{\alpha_{\mathrm{s}}}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon\right]^2 \sum_i \left[\mathrm{C}_{1,f_i}^{(1)} C_{\mathrm{A}} \boldsymbol{T}_i^2 + \sum_k \mathrm{S}_1^{(1),(i,k)} C_{\mathrm{A}} \boldsymbol{T}_i \boldsymbol{T}_k \right.$$
$$\left. + \sum_{\substack{k,l \\ k\neq l}} \mathrm{S}_1^{(1),(i,k,l)} \sum_{a,b,c} f_{abc} T_i^a T_k^b T_l^c \right]$$

present for m > 3 (four or more hard partons)

G. Somogyi, ZT arXiv:0807.0509

Structure of insertion operators

singly-unresolved integrated singly unresolved:

$$\int_1 \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A}_1} \right)^{\mathrm{A}_1} = \left[\frac{1}{2} \left\{ \boldsymbol{I}_1^{(0)}(\{p\}_m;\epsilon), \boldsymbol{I}_1^{(0)}(\{p\}_m;\epsilon) \right\} + \boldsymbol{I}_{1,1}^{(0,0)}(\{p\}_m;\epsilon) \right] \otimes \mathrm{d}\sigma_m^{\mathrm{B}}$$

for 1 unresolved parton contributions on iterated I:

$$\boldsymbol{I}_{1,1}^{(0,0)}(\{p\}_m;\epsilon) = \left[\frac{\alpha_{\mathrm{s}}}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2} \right)^\epsilon \right]^2 \sum_i \left[\mathrm{C}_{1,1,f_i}^{(0,0)} C_{\mathrm{A}} \boldsymbol{T}_i^2 + \sum_k \mathrm{S}_{1,2}^{(0,0),(i,k)} C_{\mathrm{A}} \boldsymbol{T}_i \boldsymbol{T}_k \right]$$

kinematic functions contain poles starting from $O(\epsilon^{-3})$ only

G. Somogyi, ZT arXiv:0807.0509

Structure of insertion operators

- the color structures are independent of the precise definition of subtractions (momentum mappings), only subleading coefficients of ϵ-expansion in kinematic functions may depend
- we have computed all insertion operators analytically (defined in our subtraction scheme) up to $O(\epsilon^{-2})$ for arbitrary m
- we have computed all insertion operators analytically (defined in our subtraction scheme) up to $O(\epsilon^{-0})$ for m=2 and up to $O(\epsilon^{-1})$ together with the logs of $O(\epsilon^{-0})$ for m=3

G. Somogyi, Z. Szőr, Z. Tulipánt, ZT
with contributions by D. Tommasini and R. Derco

Rewards

Cancellation of poles

- we checked the cancellation of the leading and first subleading poles (defined in our subtraction scheme) for arbitrary m
- for m=2,
  - the insertion operators are independent of the kinematics (momenta are back-to-back, so MI's are needed at the endpoints only)
  - color algebra is trivial: $\boldsymbol{T}_1\boldsymbol{T}_2 = -\boldsymbol{T}_1^2 = -\boldsymbol{T}_2^2 = -C_{\mathrm{F}}$
- so can demonstrate the cancellation of poles
- e.g. for H→bb

V. Del Duca, C. Duhr, G. Somogyi, F. Tramontano, ZT, arXiv:1501.07226

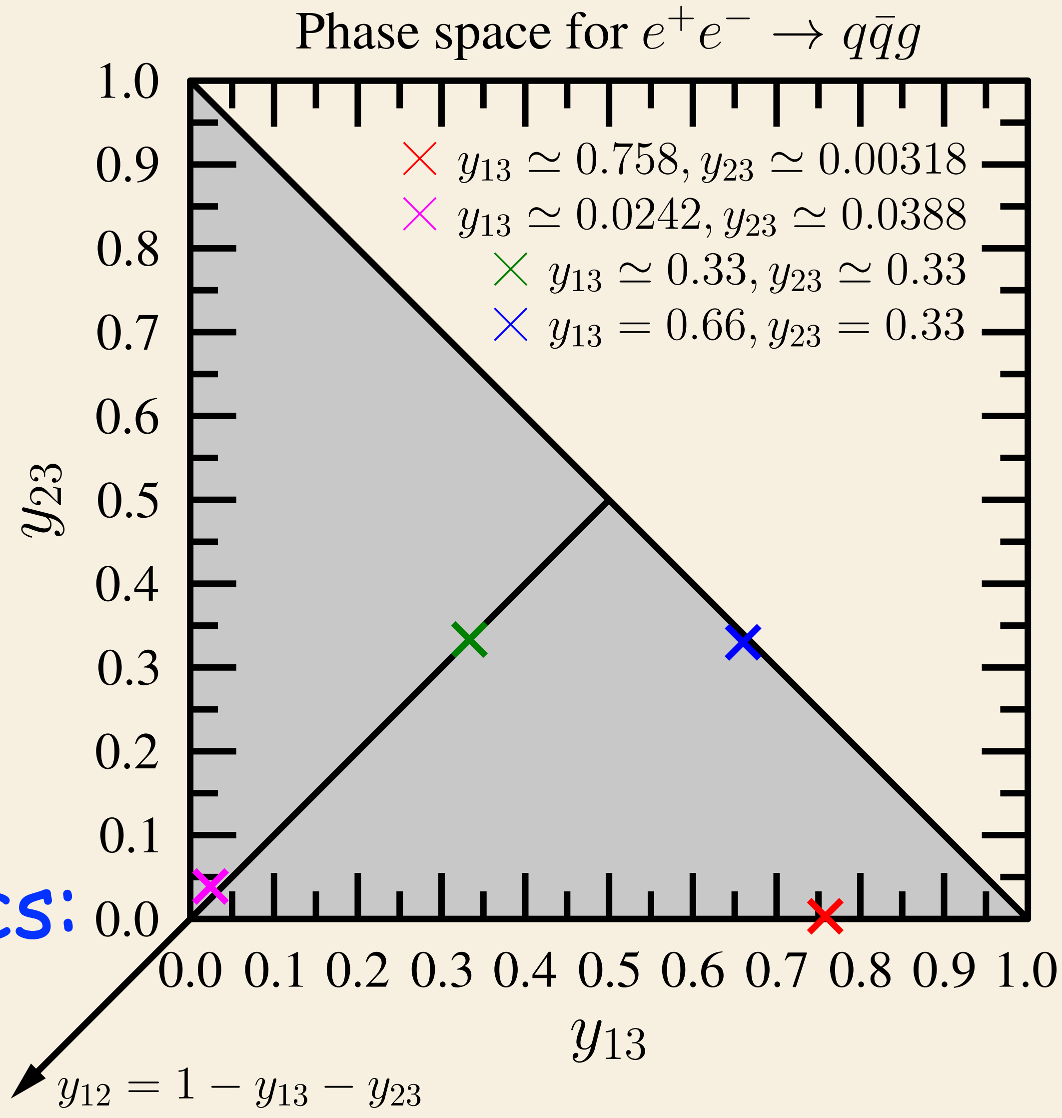
Cancellation of poles

- we checked the cancellation of the leading and first subleading poles (defined in our subtraction scheme) for arbitrary m
- for m=2
- for m=3,
  - color algebra can be performed explicitly:

$$\boldsymbol{T}_1\boldsymbol{T}_2 = \frac{1}{2}C_{\mathrm{A}} - C_{\mathrm{F}}$$

$$\boldsymbol{T}_1\boldsymbol{T}_3 = \boldsymbol{T}_2\boldsymbol{T}_3 = -\frac{1}{2}C_{\mathrm{A}}$$

  - the insertion operators depend on 3-jet kinematics:



Example: e$^+$e$^-$ → m(=3) jets at μ^2 = s

$$\sigma_m^{\rm NNLO} = \int_m \left\{ {\rm d}\sigma_m^{\rm VV} + \int_2 \left[{\rm d}\sigma_{m+2}^{\rm RR,A_2} - {\rm d}\sigma_{m+2}^{\rm RR,A_{12}} \right] + \int_1 \left[{\rm d}\sigma_{m+1}^{\rm RV,A_1} + \left(\int_1 {\rm d}\sigma_{m+2}^{\rm RR,A_1} \right)^{\rm A_1} \right] \right\} J_m$$

$$ {\rm d}\sigma_3^{\rm VV} = \mathcal{P}oles\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right) + \mathcal{F}inite\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right)$$

$$\begin{aligned}
&\mathcal{P}oles\left(A_3^{(2\times0)}(1_q,3_g,2_{\bar q}) + A_3^{(1\times1)}(1_q,3_g,2_{\bar q})\right)\\
&= 2\Bigg[-\left(\boldsymbol{I}^{(1)}_{q\bar q g}(\epsilon)\right)^2 - \frac{\beta_0}{\epsilon}\boldsymbol{I}^{(1)}_{q\bar q g}(\epsilon)\\
&\quad + e^{-\epsilon\gamma}\frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{\beta_0}{\epsilon}+K\right)\boldsymbol{I}^{(1)}_{q\bar q g}(2\epsilon) + \boldsymbol{H}^{(2)}_{q\bar q g}\Bigg] A_3^0(1_q,3_g,2_{\bar q})\\
&\quad + 2\,\boldsymbol{I}^{(1)}_{q\bar q g}(\epsilon)\, A_3^{(1\times 0)}(1_q,3_g,2_{\bar q})\ . \qquad (4.59)
\end{aligned}$$

$$\boldsymbol{I}^{(1)}_{q\bar q g}(\epsilon) = \mathcal{R}e\boldsymbol{I}^{(1)}_0(p_q,p_{\bar q},p_g;\epsilon)$$

$$\begin{aligned}
\boldsymbol{H}^{(2)}_{q\bar q g} = \frac{e^{\epsilon\gamma}}{4\,\epsilon\,\Gamma(1-\epsilon)}\Bigg[&\left(4\zeta_3 + \frac{589}{432} - \frac{11\pi^2}{72}\right)N^2 + \left(-\frac{1}{2}\zeta_3 - \frac{41}{54} - \frac{\pi^2}{48}\right)\\
&+\left(-3\zeta_3 - \frac{3}{16} + \frac{\pi^2}{4}\right)\frac{1}{N^2} + \left(-\frac{19}{18} + \frac{\pi^2}{36}\right)NN_F + \left(-\frac{1}{54} - \frac{\pi^2}{24}\right)\frac{N_F}{N} + \frac{5}{27}N_F^2.\Bigg]\ .
\end{aligned}$$

A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover, G. Heinrich arXiv:0710.0346 (4.61)

Example: $e^+e^- \longrightarrow$ m(=3) jets at $\mu^2 = s$

$$\sigma_m^{\rm NNLO} = \int_m \left\{ d\sigma_m^{\rm VV} + \int_2 \left[d\sigma_{m+2}^{\rm RR,A_2} - d\sigma_{m+2}^{\rm RR,A_{12}} \right] + \int_1 \left[d\sigma_{m+1}^{\rm RV,A_1} + \left(\int_1 d\sigma_{m+2}^{\rm RR,A_1} \right)^{\rm A_1} \right] \right\} J_m$$

$$d\sigma_3^{\rm VV} = \mathcal{P}oles\left(A_3^{(2\times0)} + A_3^{(1\times1)}\right) + \mathcal{F}inite\left(A_3^{(2\times0)} + A_3^{(1\times1)}\right)$$

$$\mathcal{P}oles\left(A_3^{(2\times0)} + A_3^{(1\times1)}\right) + \mathcal{P}oles \sum \int d\sigma^{\rm A} =$$ 200k Mathematica lines

= zero numerically in any phase space point:

```
                  0.             2    0. nf
          0. + --- + 0. Nc   + ----- + 0. Nc nf
                 2                Nc
                Nc
Out[1]= ------------------------------------ +
                                   2
                                  e

                  0.             2   0. nf
          0. + --- + 0. Nc   + ----- + 0. Nc nf
                 2                Nc
                Nc                                        0
---------------------------------------------- + O[e]
                          e
```

Example: $e^+e^- \longrightarrow$ m(=3) jets at μ^2 = s

$$\sigma_m^{\rm NNLO} = \int_m \Big\{ {\rm d}\sigma_m^{\rm VV} + \int_2 \Big[{\rm d}\sigma_{m+2}^{\rm RR,A_2} - {\rm d}\sigma_{m+2}^{\rm RR,A_{12}}\Big] + \int_1 \Big[{\rm d}\sigma_{m+1}^{\rm RV,A_1} + \Big(\int_1 {\rm d}\sigma_{m+2}^{\rm RR,A_1}\Big)^{\rm A_1}\Big]\Big\} J_m$$

$$ {\rm d}\sigma_3^{\rm VV} = \mathcal{P}oles\big(A_3^{(2\times0)} + A_3^{(1\times1)}\big) + \mathcal{F}inite\big(A_3^{(2\times0)} + A_3^{(1\times1)}\big)$$

$$\begin{aligned}{\rm d}\sigma_3^{\rm NNLO} = \Big\{ & {\rm d}\sigma_3^{\rm VV} + {\rm d}\sigma_3^{\rm B} \otimes \Big[\boldsymbol{I}_2^{(0)}(\epsilon) - \boldsymbol{I}_{12}^{(0)}(\epsilon) + \boldsymbol{I}_1^{(1)}(\epsilon) + \boldsymbol{I}_{1,1}^{(0,0)}(\epsilon) + \frac{1}{2}\{\boldsymbol{I}_1^{(0)}(\epsilon), \boldsymbol{I}_1^{(0)}(\epsilon)\}\Big] \\ & + {\rm d}\sigma_3^{\rm V} \otimes \boldsymbol{I}_1^{(0)}(\epsilon)\Big\} J_3\,. \end{aligned}$$

$$\boldsymbol{J}_2 \equiv \boldsymbol{I}_2^{(0)} - \boldsymbol{I}_{12}^{(0)} + \boldsymbol{I}_1^{(1)} + \boldsymbol{I}_{1,1}^{(0,0)} + \frac{1}{4}\Big\{\boldsymbol{I}_1^{(0)}, \boldsymbol{I}_1^{(0)}\Big\}$$

$$\begin{aligned}\boldsymbol{J}_2(\{p\}_3;\epsilon) = \frac{\alpha_{\rm s}}{2\pi}\frac{S_\epsilon}{S_\epsilon^{\overline{\rm MS}}}\left(\frac{\mu^2}{Q^2}\right)^\epsilon \frac{1}{2\epsilon}\Bigg[& \left(\beta_0 + 2\epsilon K - \epsilon^2\beta_0\frac{\pi^2}{4}\right)\boldsymbol{I}_1^{(0)}(\{p\}_3;2\epsilon) \\ & - \beta_0 \boldsymbol{I}_1^{(0)}(\{p\}_3;\epsilon) - \frac{\alpha_{\rm s}}{2\pi}\frac{S_\epsilon}{S_\epsilon^{\overline{\rm MS}}}\left(\frac{\mu^2}{Q^2}\right)^\epsilon \Big(2H_q(n_{\rm f}) + H_g(n_{\rm f})\Big)\Bigg] \\ & + {\rm O}(\epsilon^0)\,. \end{aligned}$$

Example: $e^+e^- \longrightarrow$ m(=3) jets at $\mu^2 = s$

$$\sigma_m^{\rm NNLO} = \int_m \left\{ d\sigma_m^{\rm VV} + \int_2 \left[d\sigma_{m+2}^{\rm RR,A_2} - d\sigma_{m+2}^{\rm RR,A_{12}} \right] + \int_1 \left[d\sigma_{m+1}^{\rm RV,A_1} + \left(\int_1 d\sigma_{m+2}^{\rm RR,A_1} \right)^{\rm A_1} \right] \right\} J_m$$

$$d\sigma_3^{\rm VV} = \mathcal{P}oles\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right) + \mathcal{F}inite\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right)$$

$\mathcal{P}oles\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right) + \mathcal{P}oles \sum \int d\sigma^{\rm A} =$ 200k Mathematica lines

= zero analytically using symbol techniques (C. Duhr)

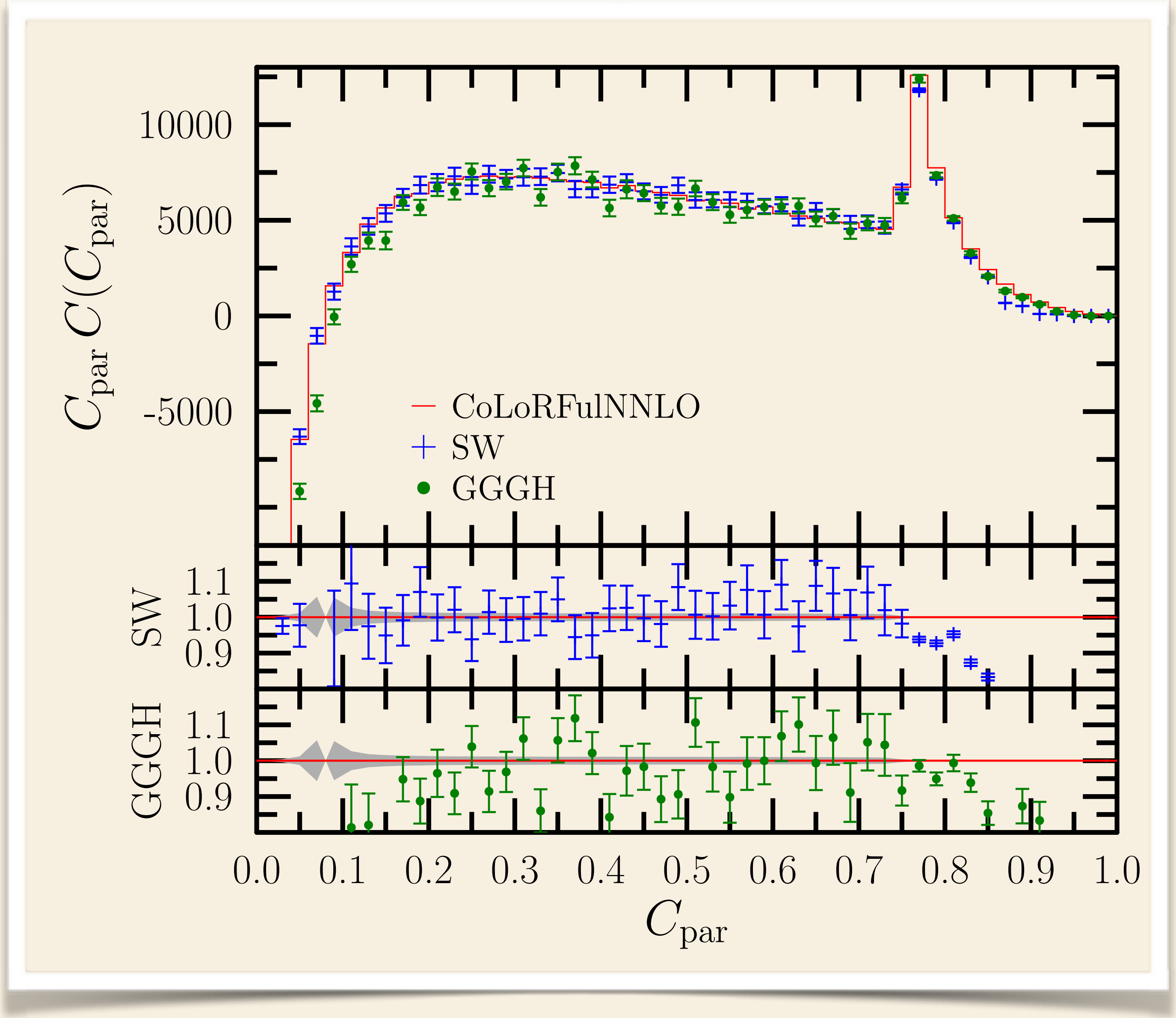
Message:

$$\sigma_3^{\rm NNLO} = \int_3 \left\{ d\sigma_3^{\rm VV} + \sum \int d\sigma^{\rm A} \right\}_{\epsilon=0} J_3$$

indeed finite in d=4 dimensions

Application

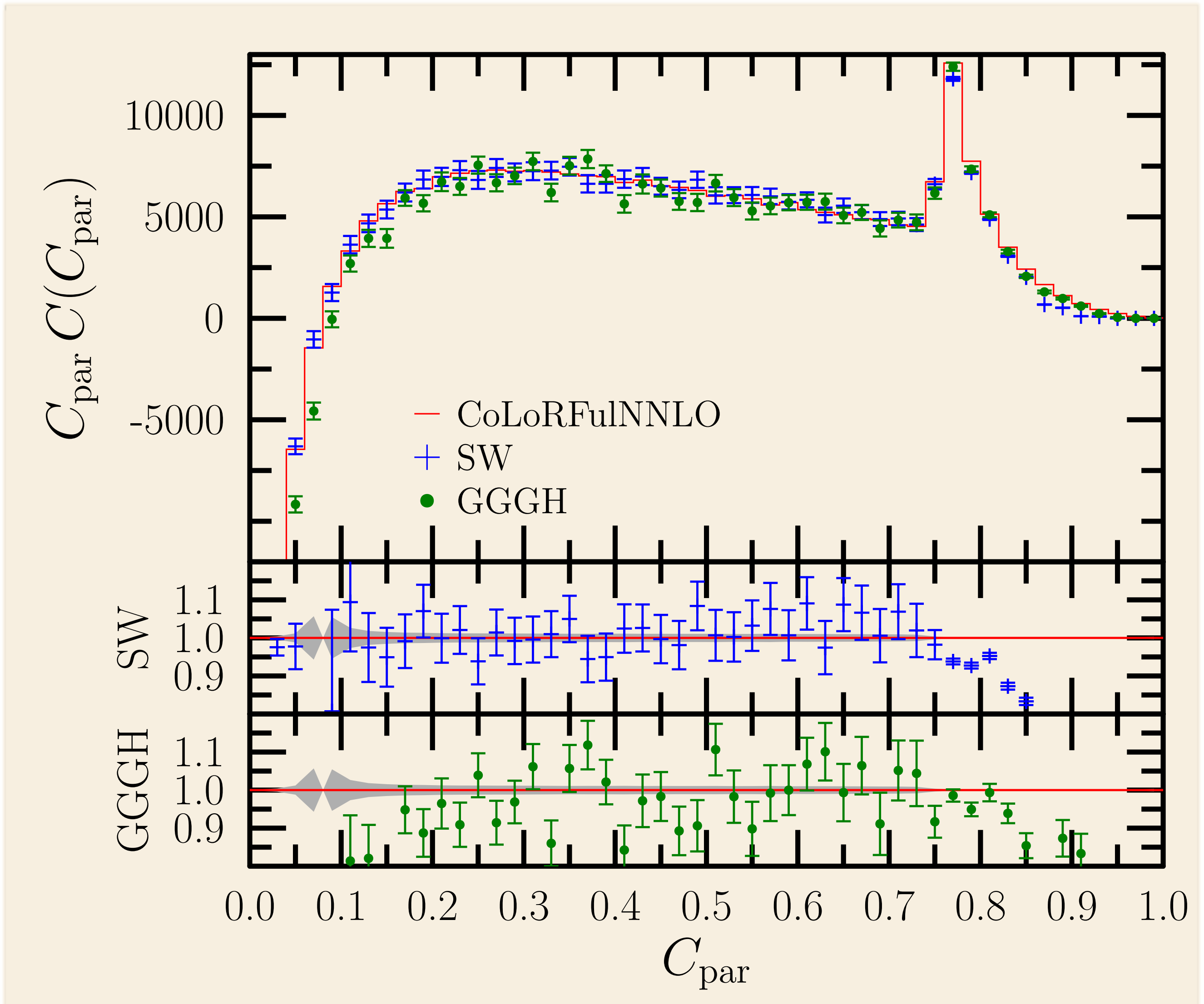
Three-jet event shapes: old

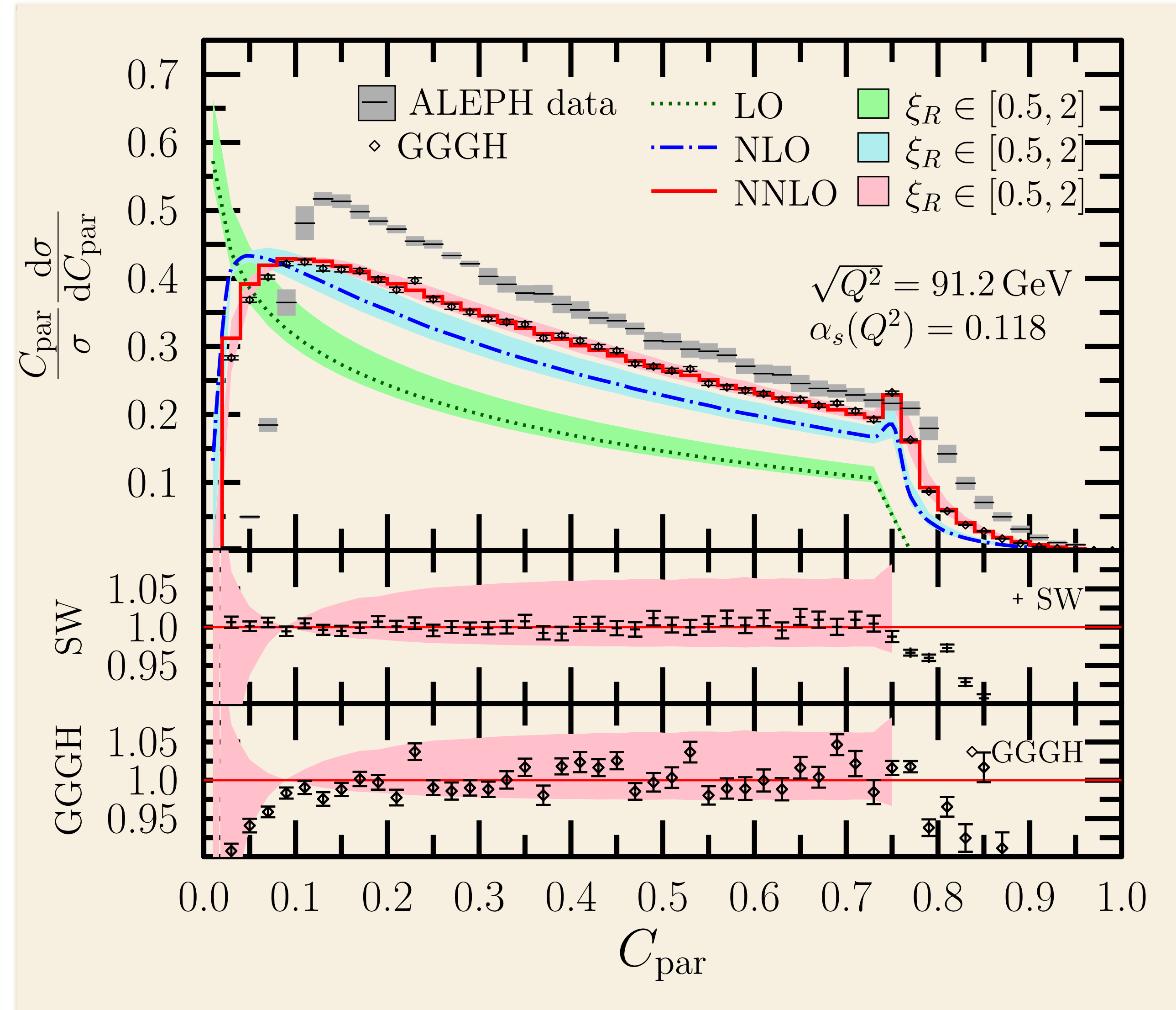


$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}O} = \frac{\alpha_s}{2\pi}A(O) + \left(\frac{\alpha_s}{2\pi}\right)^2 B(O) + \left(\frac{\alpha_s}{2\pi}\right)^3 C(O) + \mathrm{O}(\alpha_s^4)$$

$$C_{\rm par} = \frac{3}{2}\frac{\sum_{i,j}|\vec{p}_i||\vec{p}_j|\sin^2\theta_{ij}}{\left(\sum_i|\vec{p}_i|\right)^2}$$

Three-jet event shapes: old





$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}O} = \frac{\alpha_{\rm s}}{2\pi}A(O) + \left(\frac{\alpha_{\rm s}}{2\pi}\right)^2 B(O) + \left(\frac{\alpha_{\rm s}}{2\pi}\right)^3 C(O) + \mathrm{O}(\alpha_{\rm s}^4)$$

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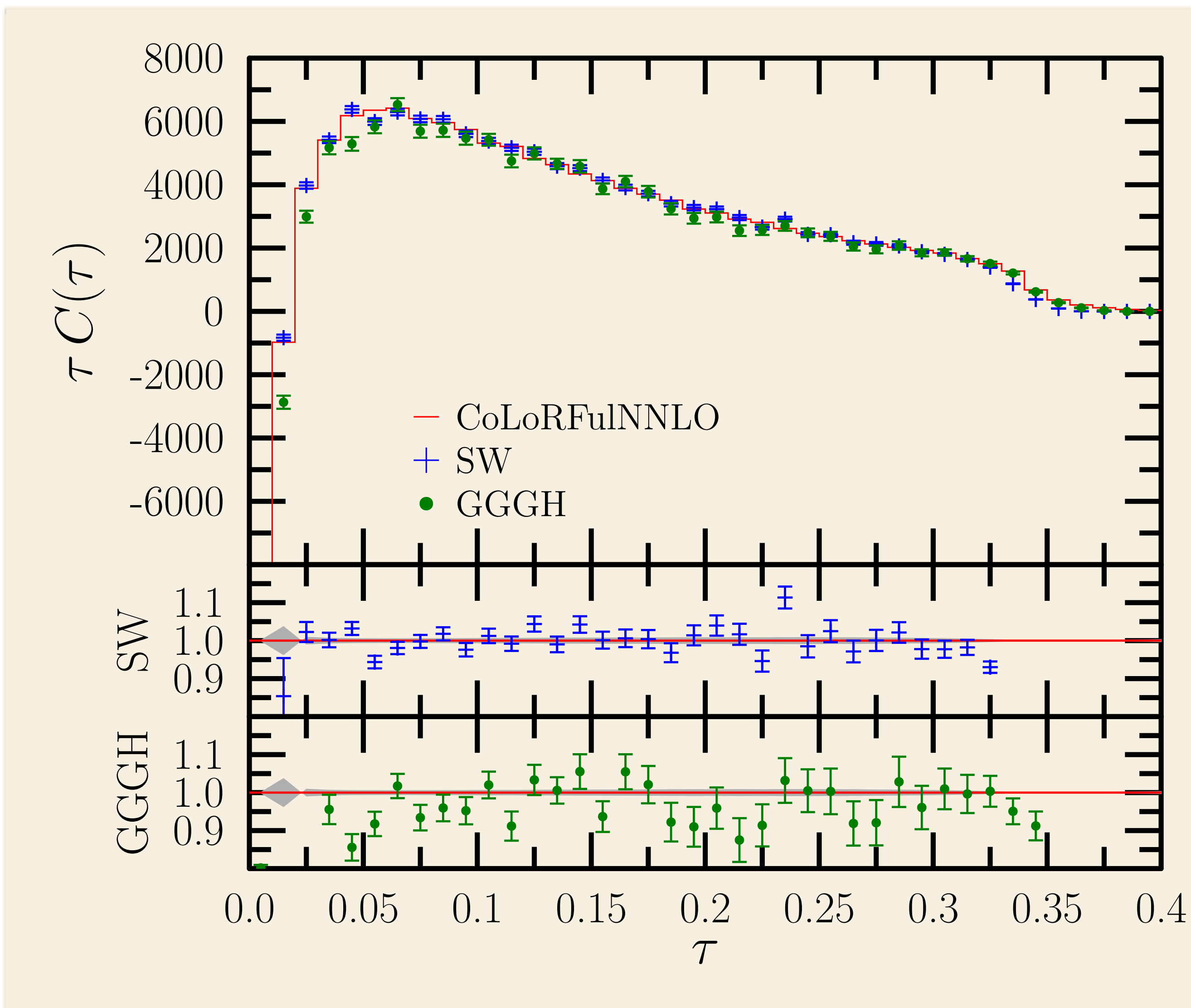
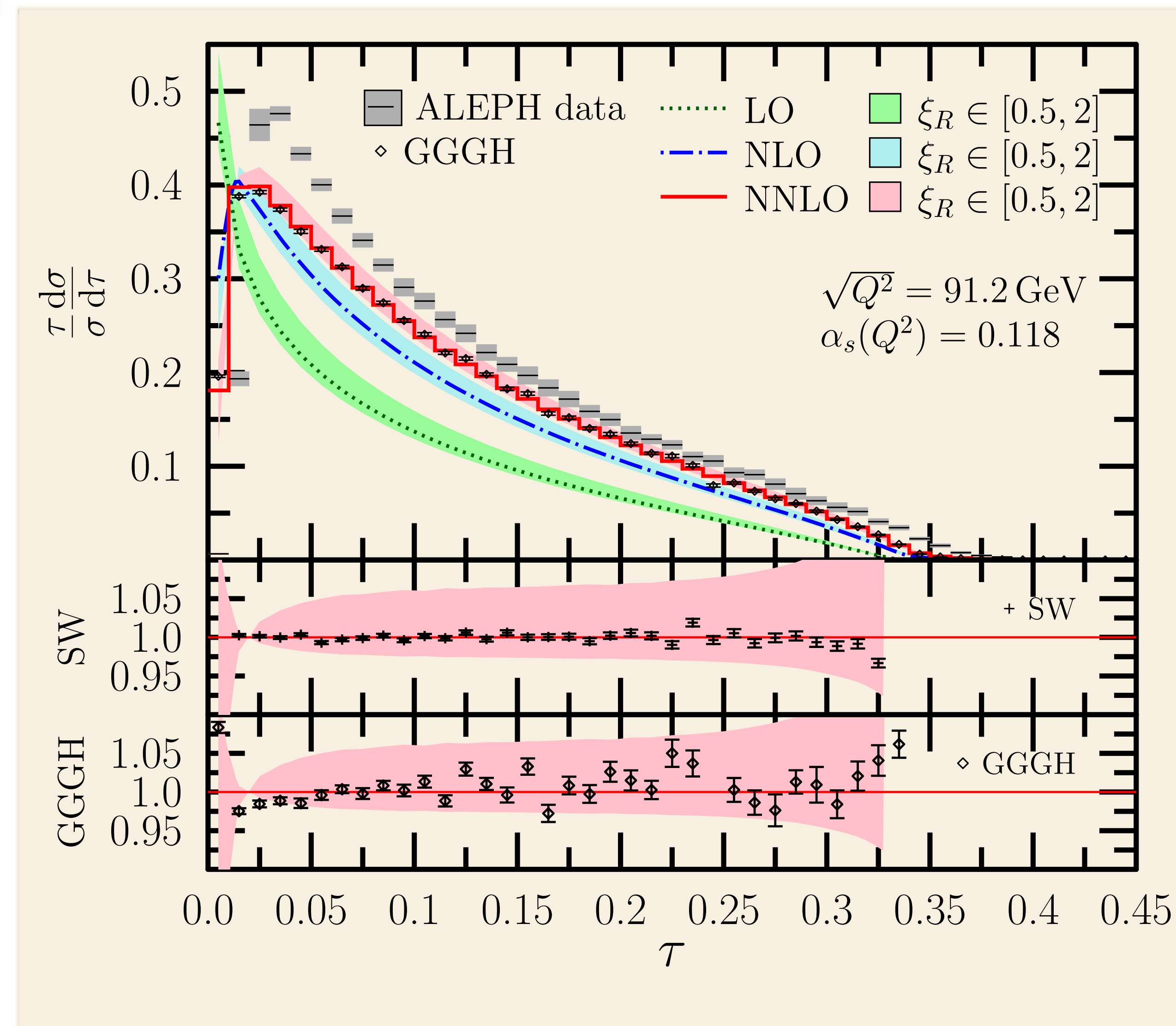
Three-jet event shapes: old

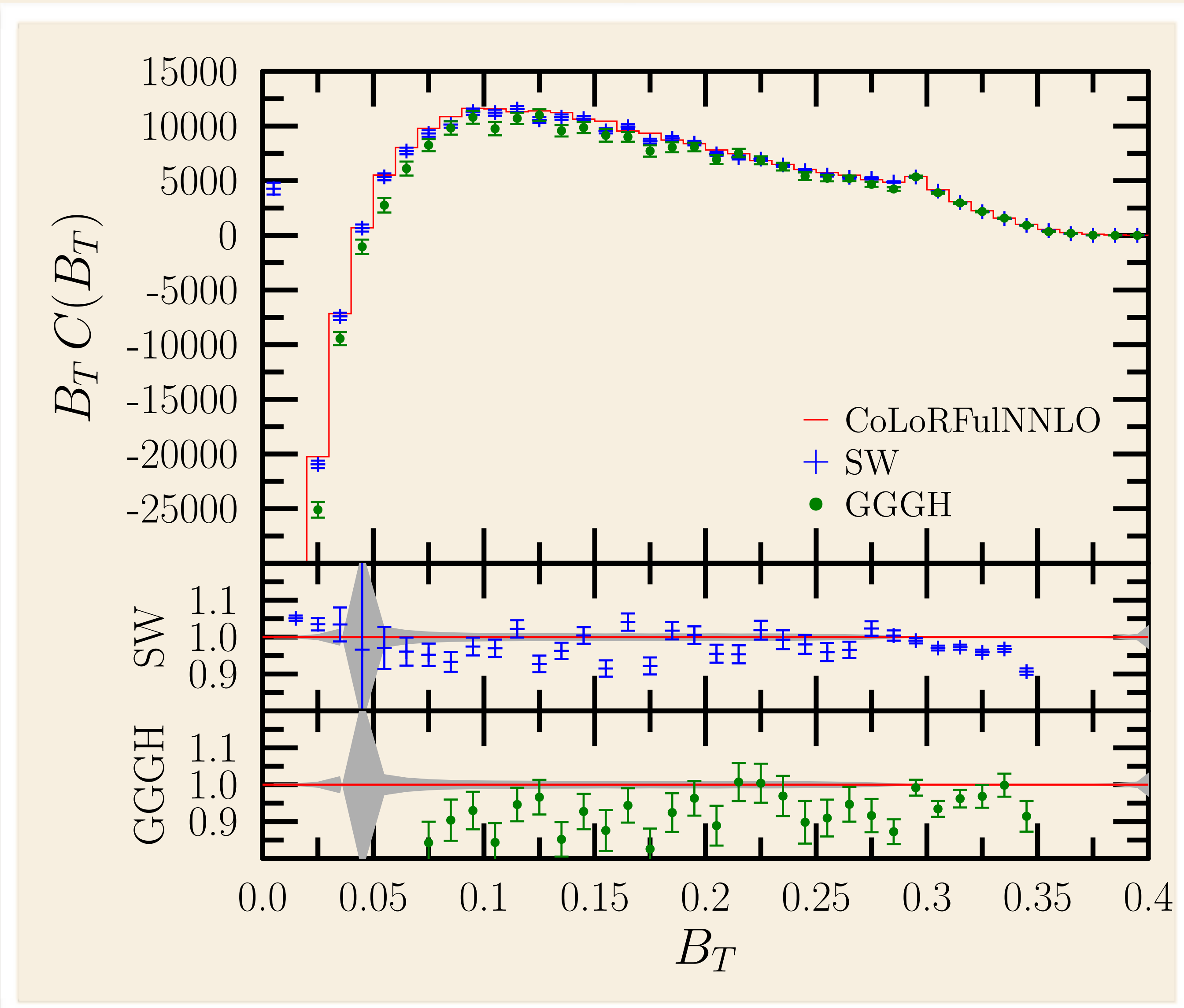


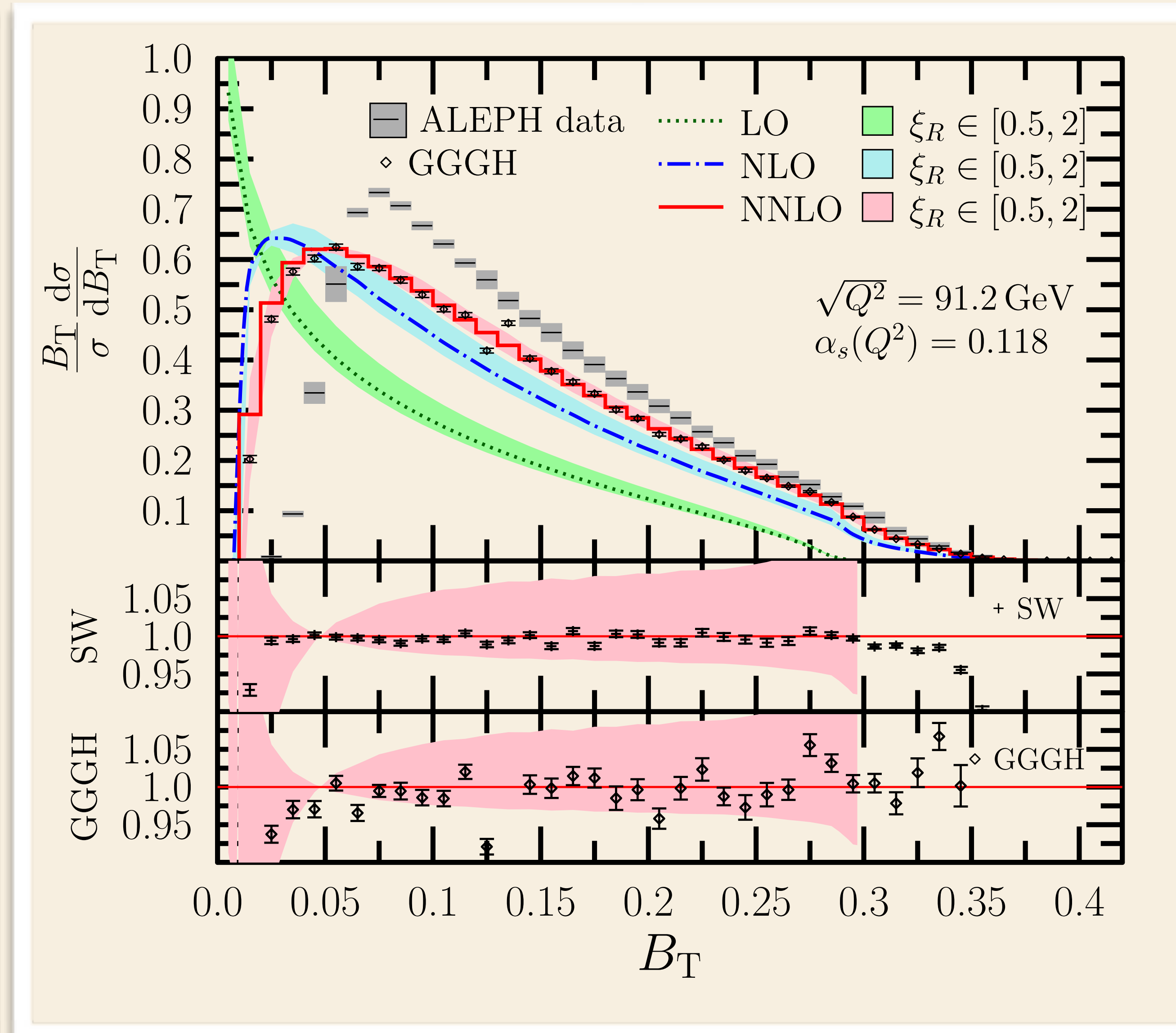


τ = 1-T

$$T = \max_{\vec{n}} \left(\frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|} \right)$$

Three-jet event shapes: old

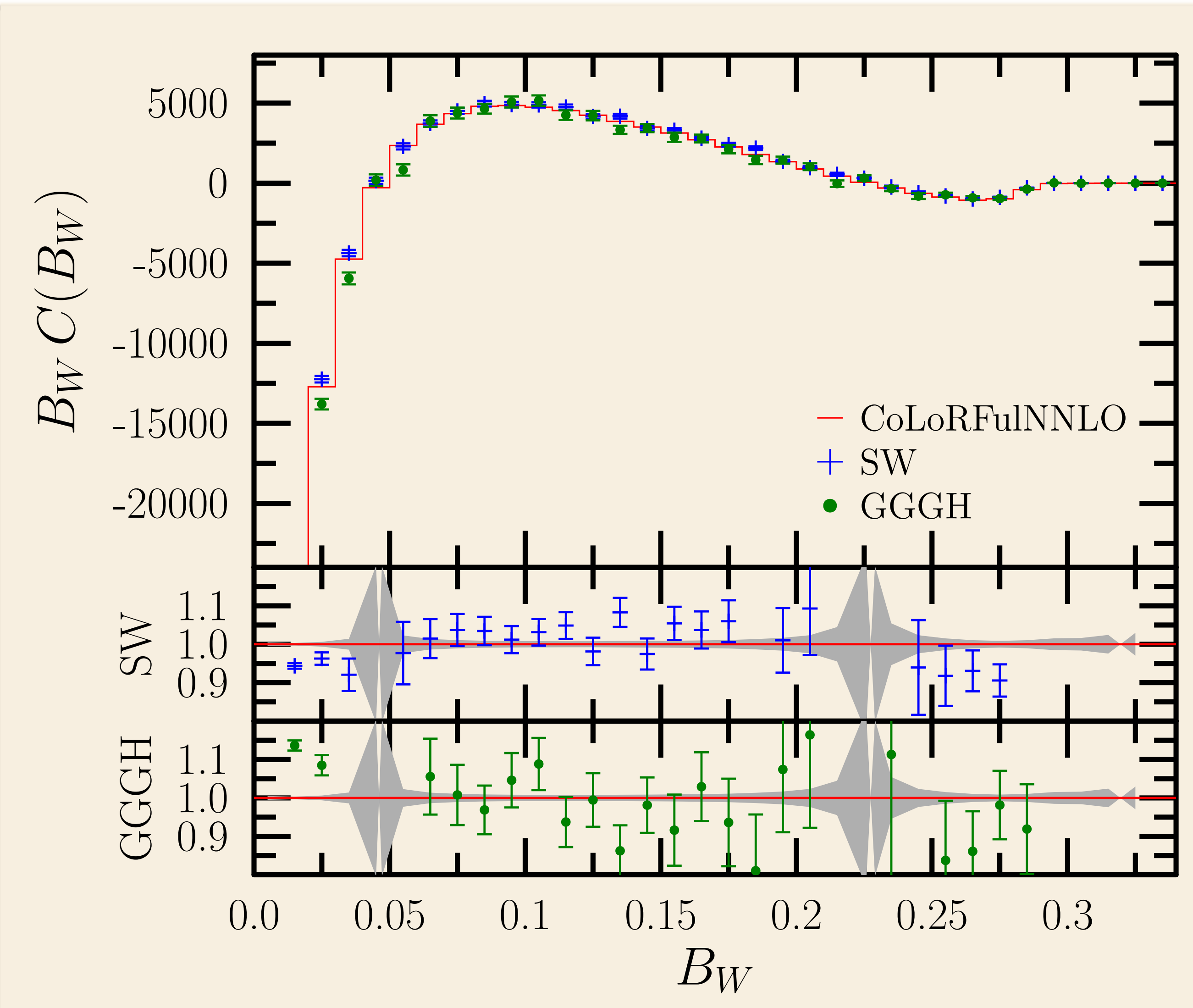


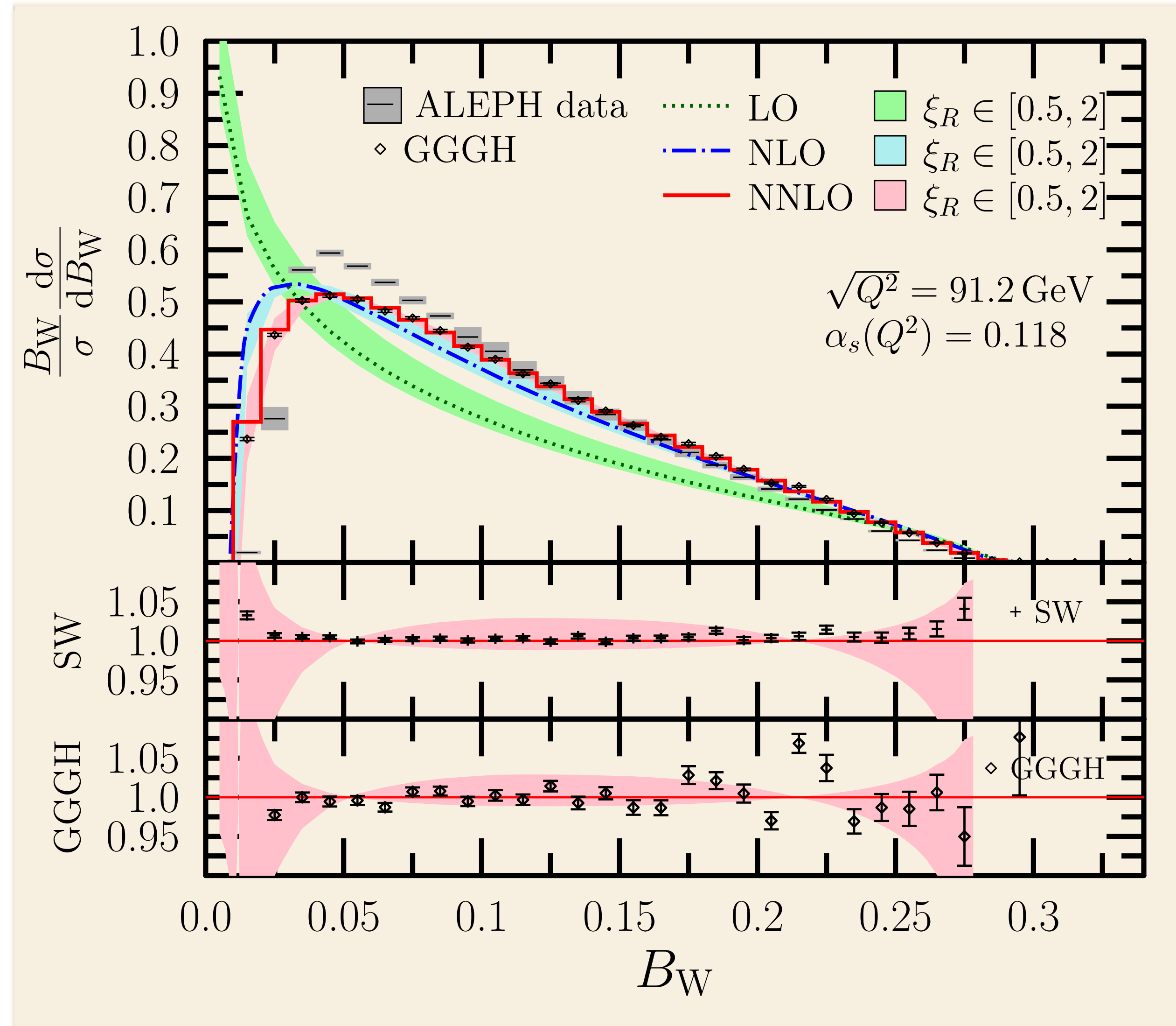


$$B_i = \frac{\sum_{j\in H_i} |\vec{p_j} \times \vec{n}_T|}{2\sum_{j\in H_i} |\vec{p_j}|}, \qquad i = L, R.$$

$$B_T = B_L + B_R$$

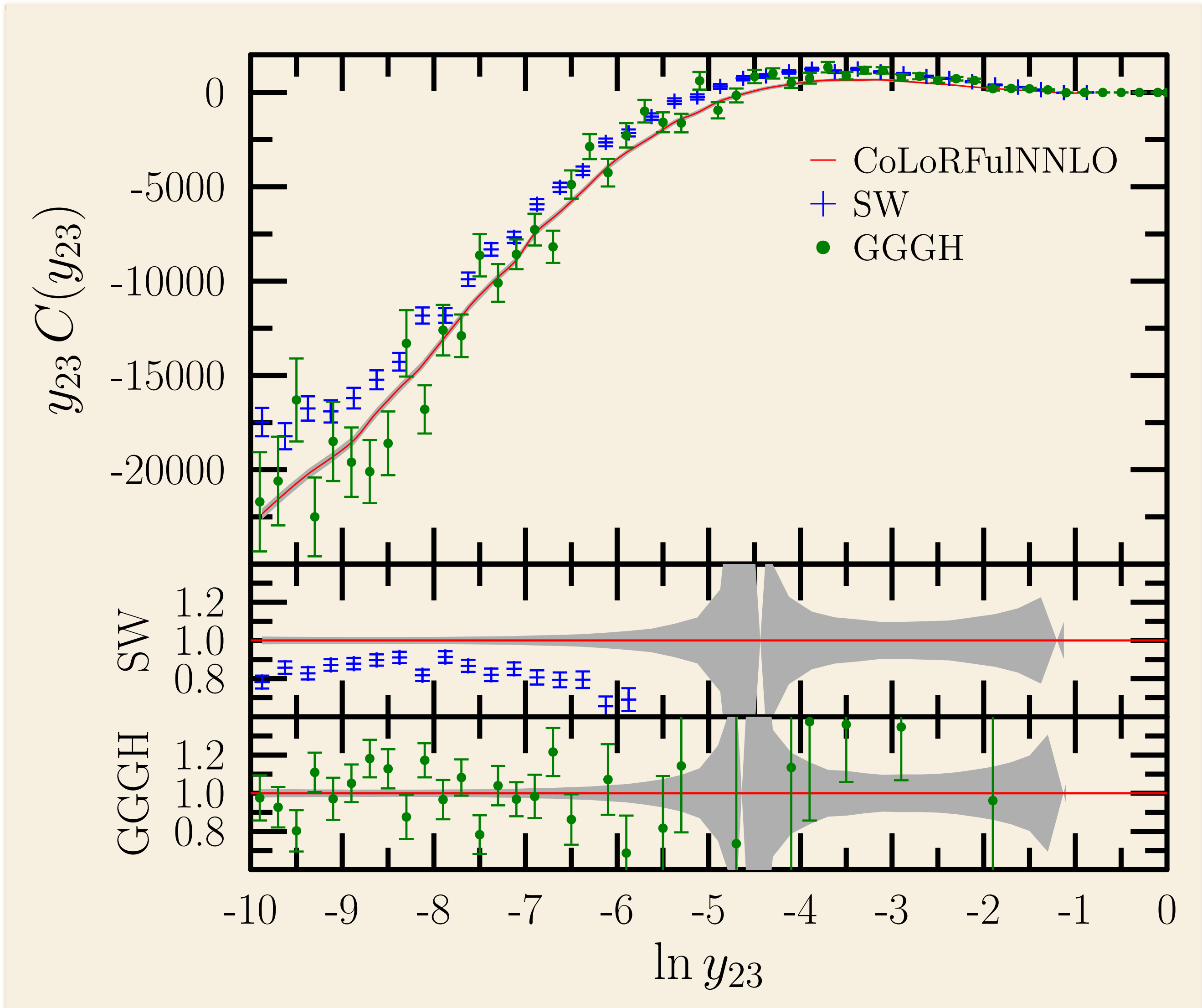
Three-jet event shapes: old





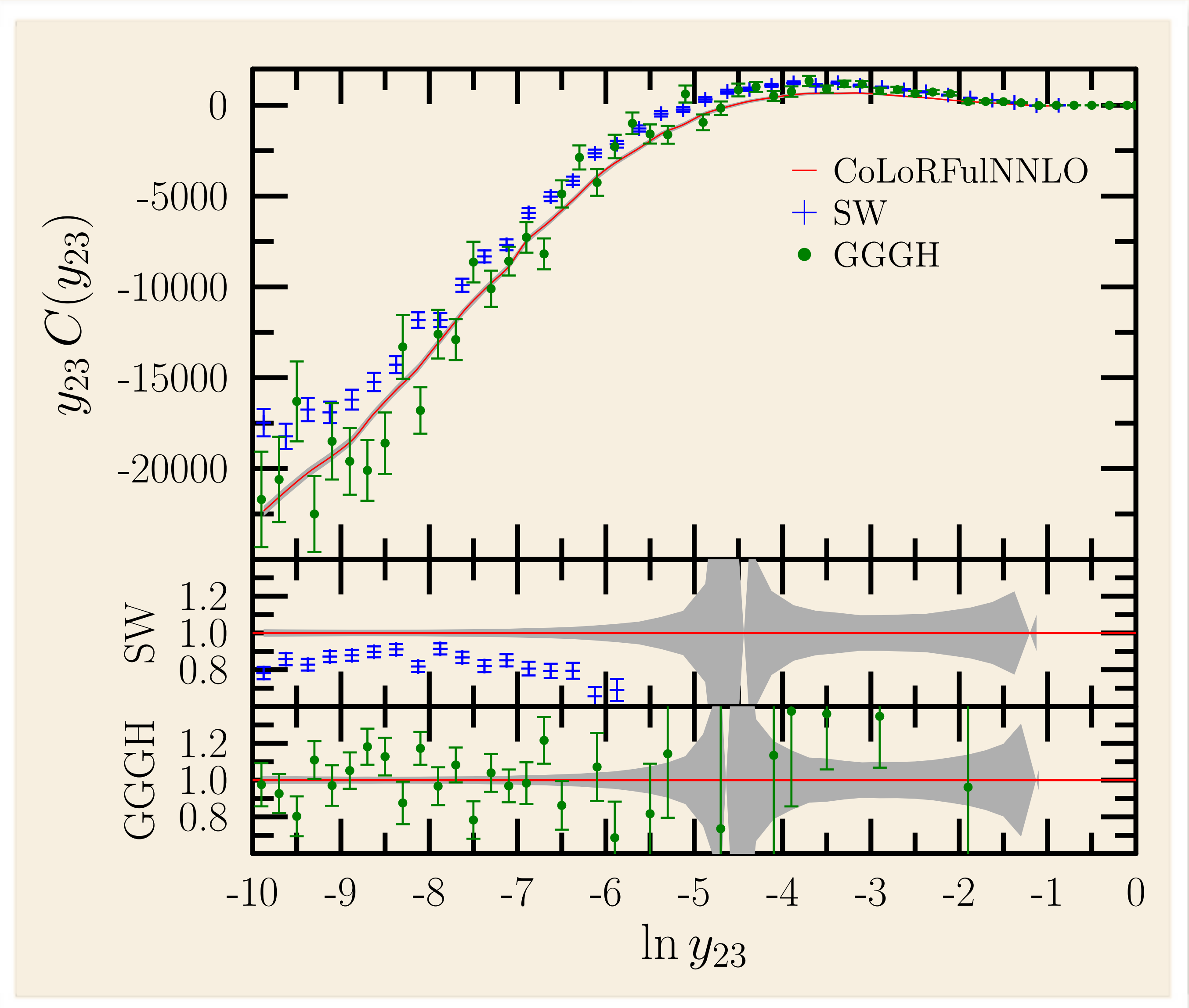
$$B_i = \frac{\sum_{j \in H_i} |\vec{p_j} \times \vec{n}_T|}{2 \sum_{j \in H_i} |\vec{p_j}|}\,, \qquad i = L, R\,. \qquad\qquad B_W = \max(B_L, B_R)$$

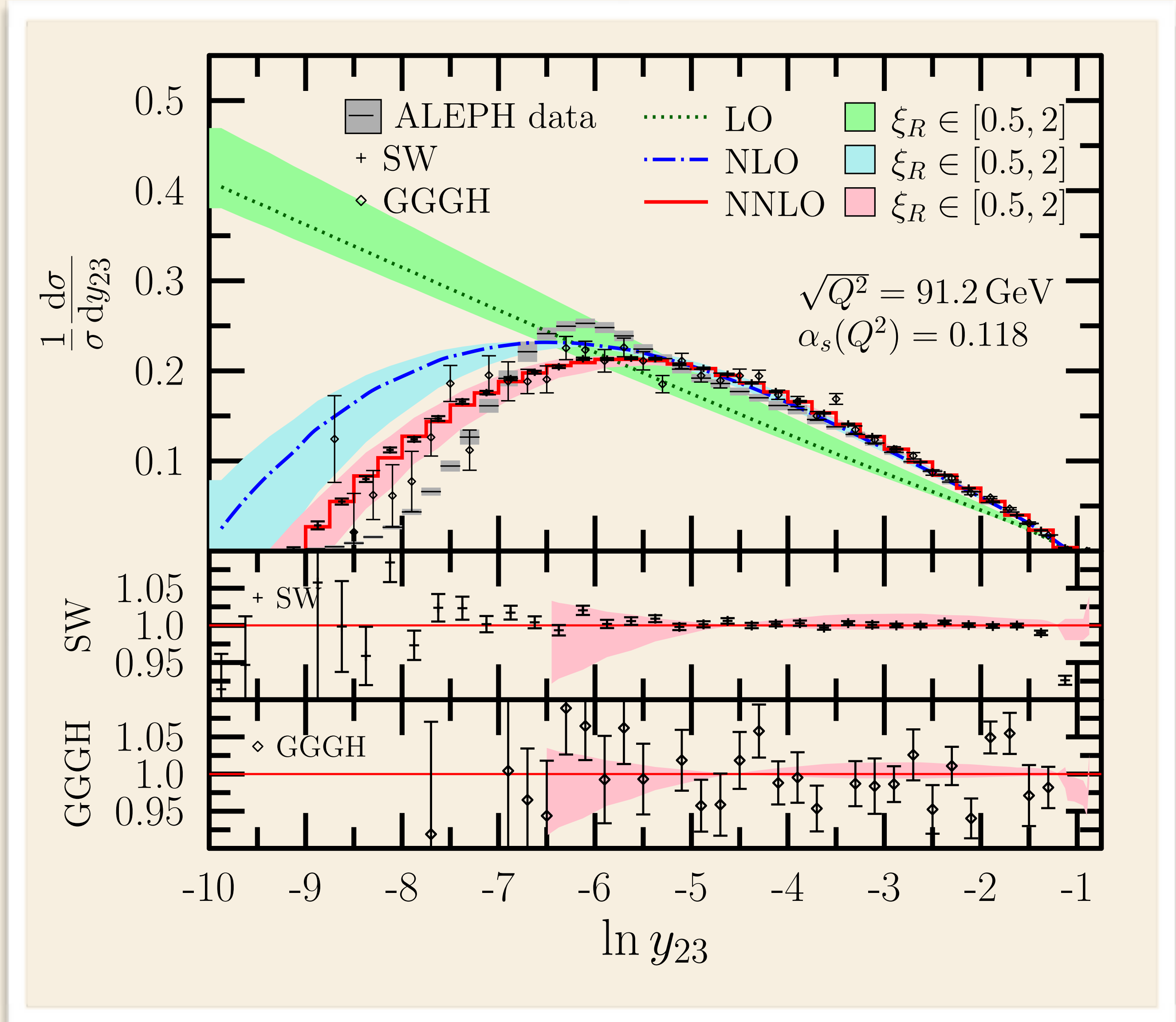
Three-jet event shapes: old



$y_{23} = y_{cut}$ that separates the event from being considered as 2 or 3 jet event using Durham clustering

Three-jet event shapes: old





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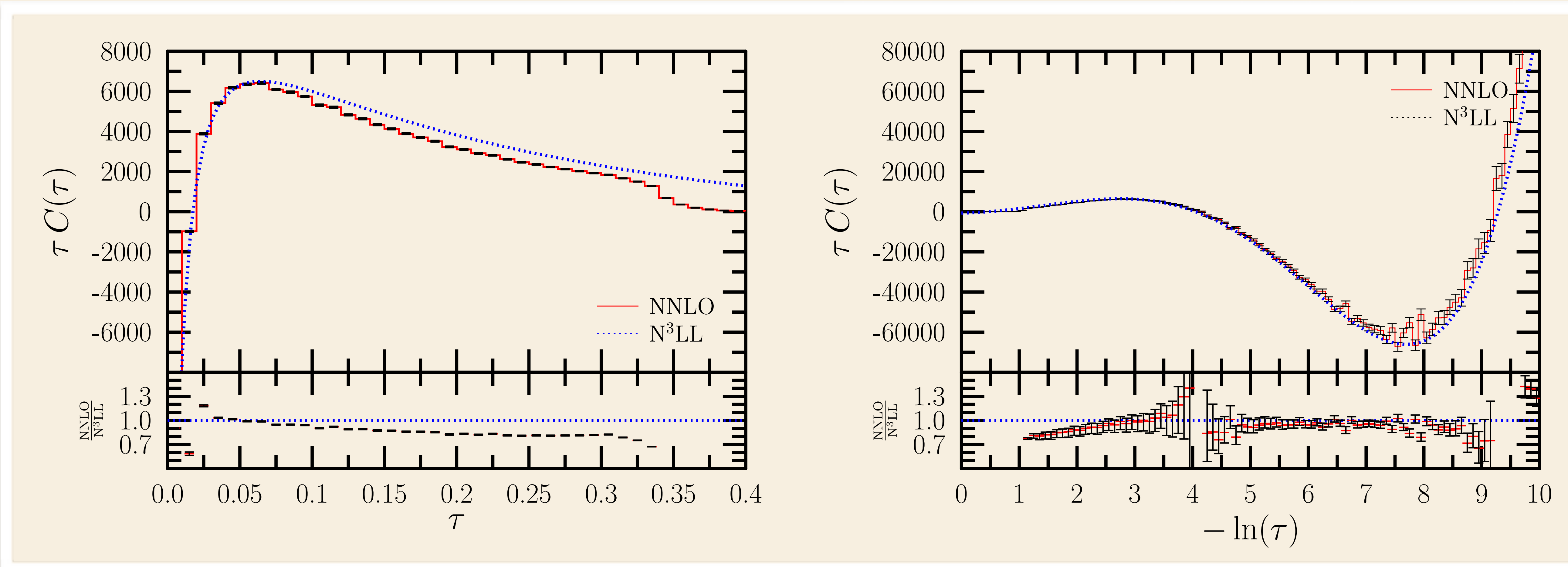

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N^{3}LL resummation from
T. Becher, M.D. Schwartz arXiv:0803.0343

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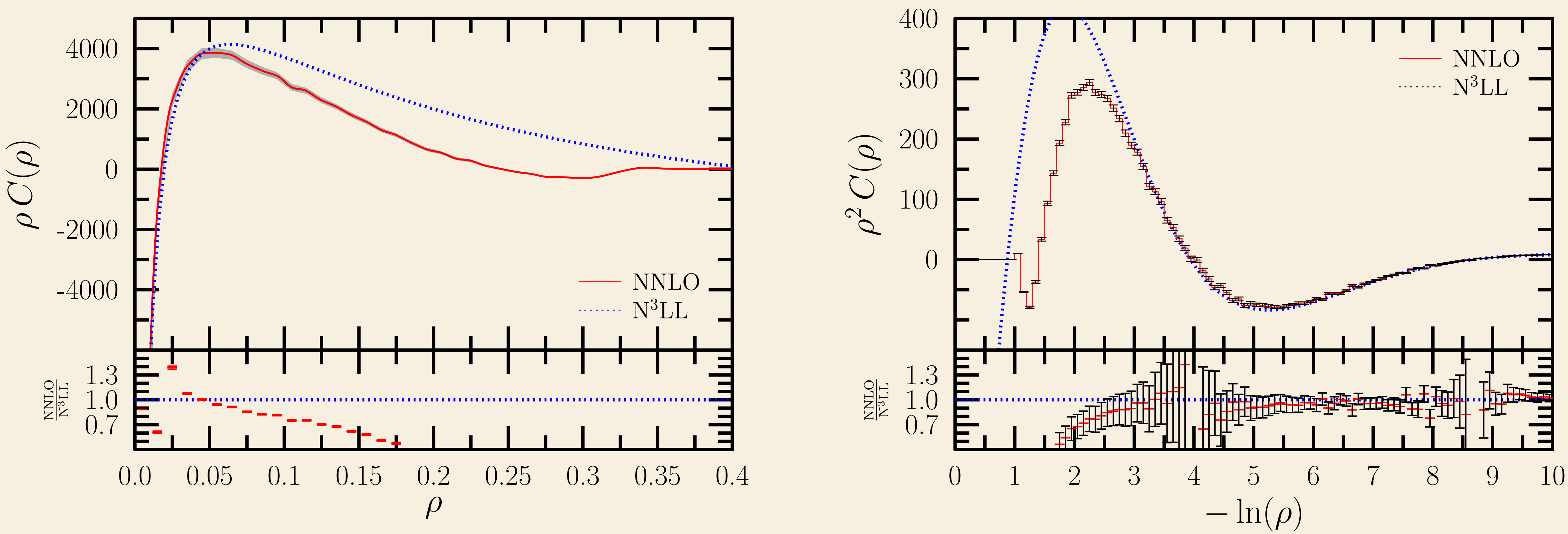
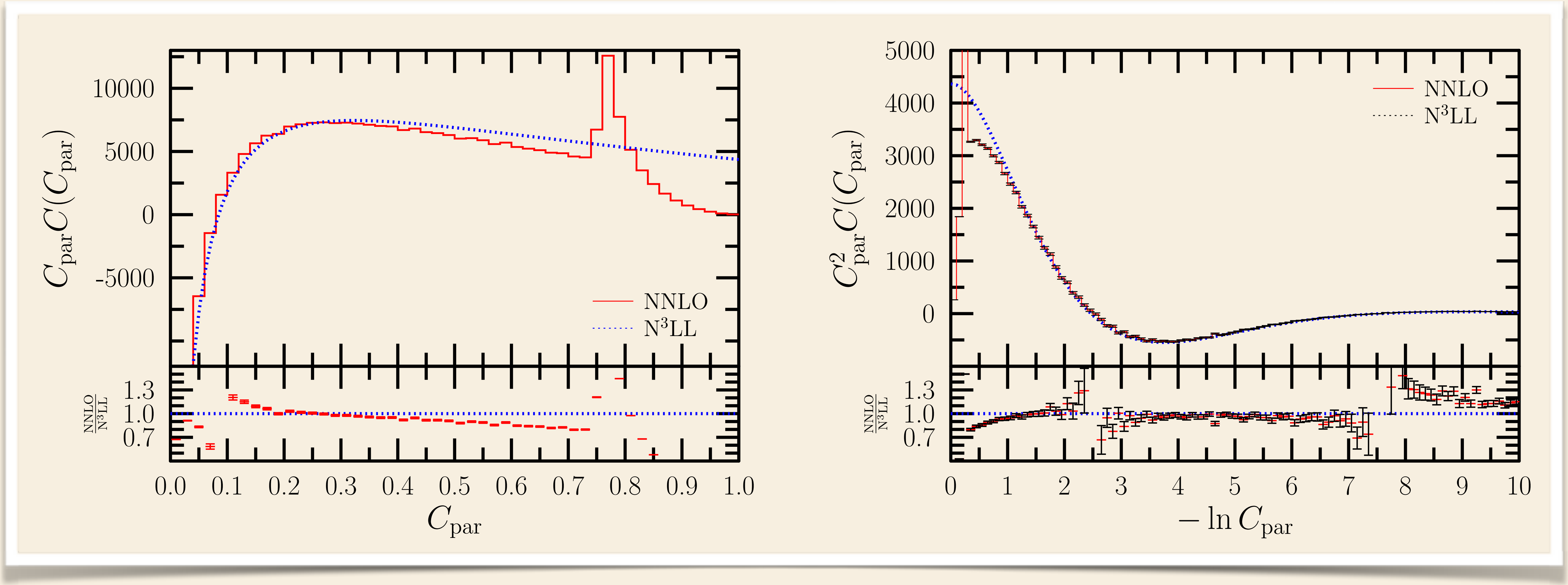


$$\frac{M_i^2}{s} = \frac{1}{E_{\text{vis}}^2}\left(\sum_{j\in H_i} p_j\right)^2, \qquad i = L, R \qquad \rho = \max\left(\frac{M_L^2}{s}, \frac{M_R^2}{s}\right)$$

N^{3}LL resummation from
Y-T. Chien, M.D. Schwartz arXiv:1005.1644

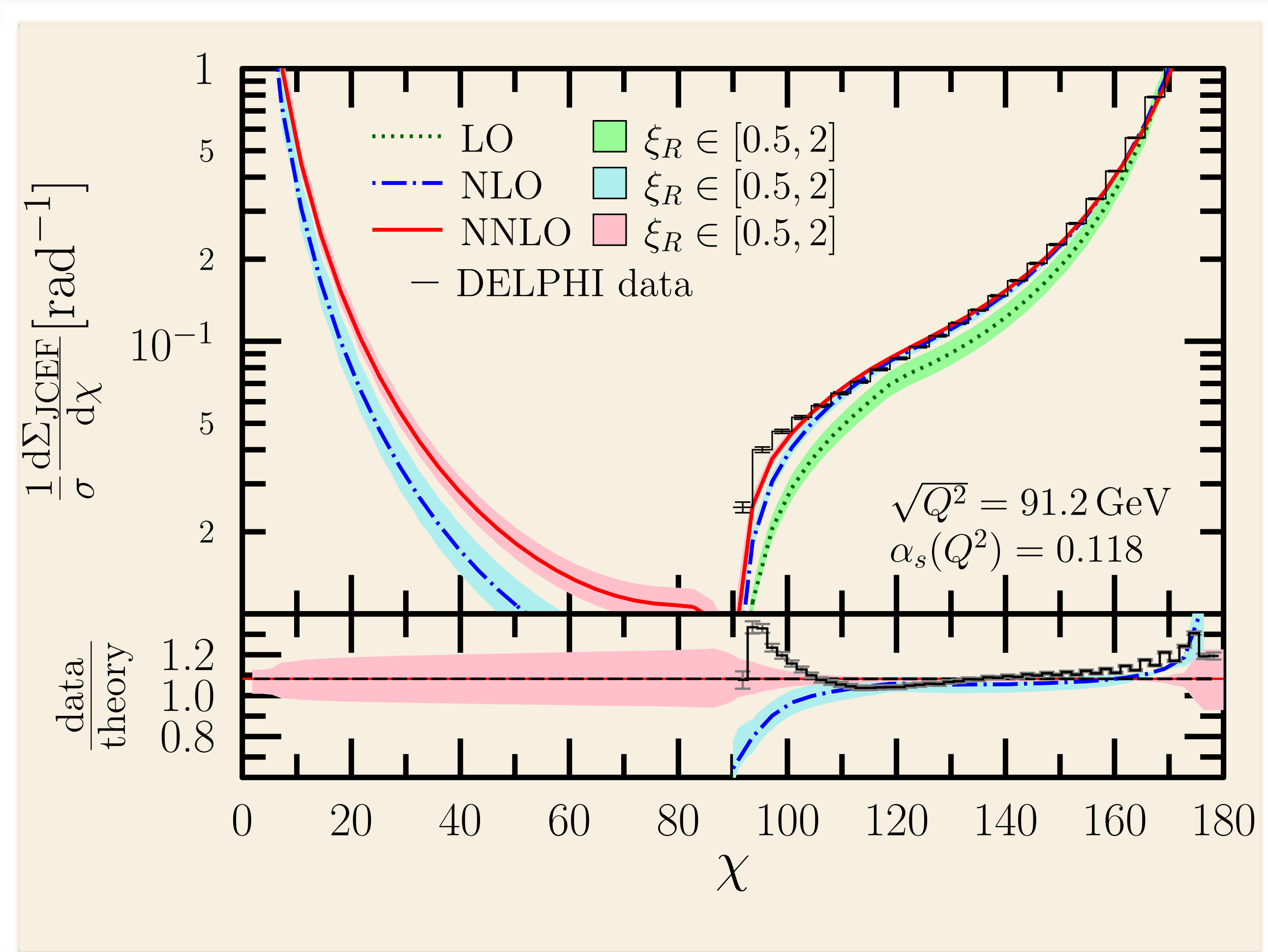
Three-jet event shapes: old

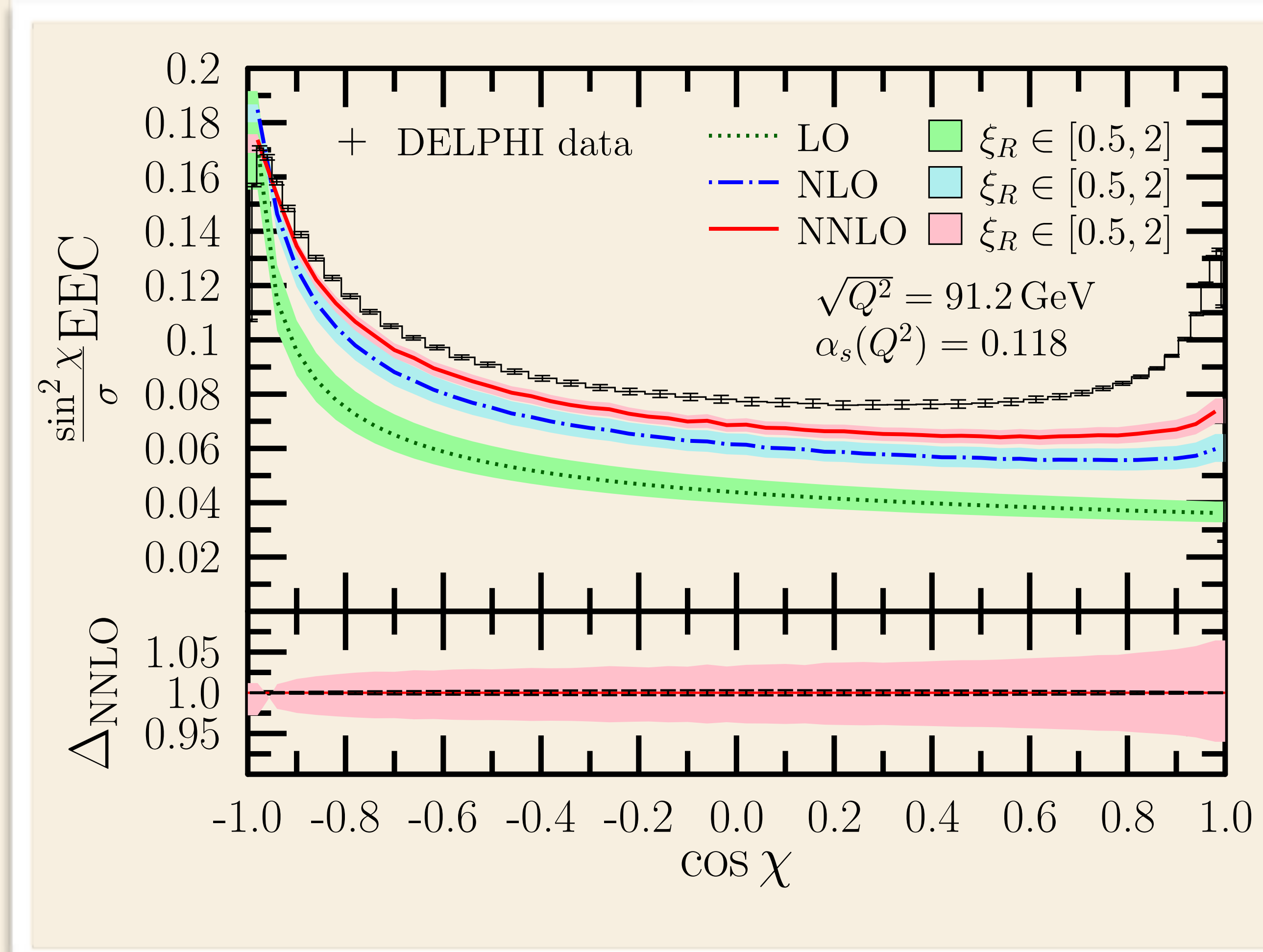


$$C_{\rm par} = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2 \theta_{ij}}{\left(\sum_i |\vec{p}_i|\right)^2}$$

N^{3}LL resummation from
A. Hoang et al arXiv:1411.6633

Three-jet event shapes: new

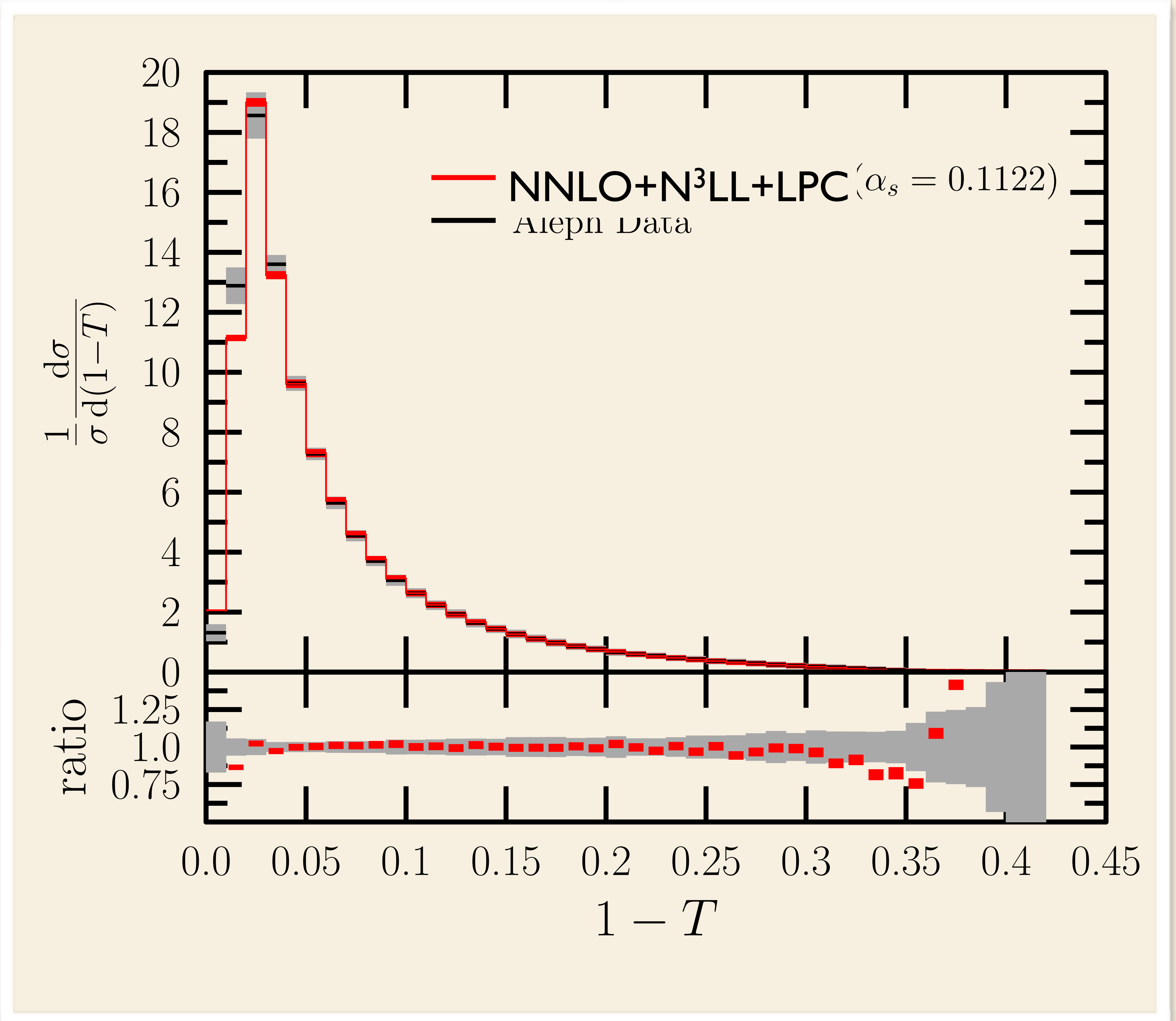




$$\frac{d\Sigma_{\rm JCEF}}{d\cos\chi} = \sum_i \int \frac{E_i}{Q} d\sigma_{e^+e^-\to i+X}\,\delta\!\left(\cos\chi - \frac{\vec{p}_i\cdot\vec{n}_T}{|\vec{p}_i|}\right)$$

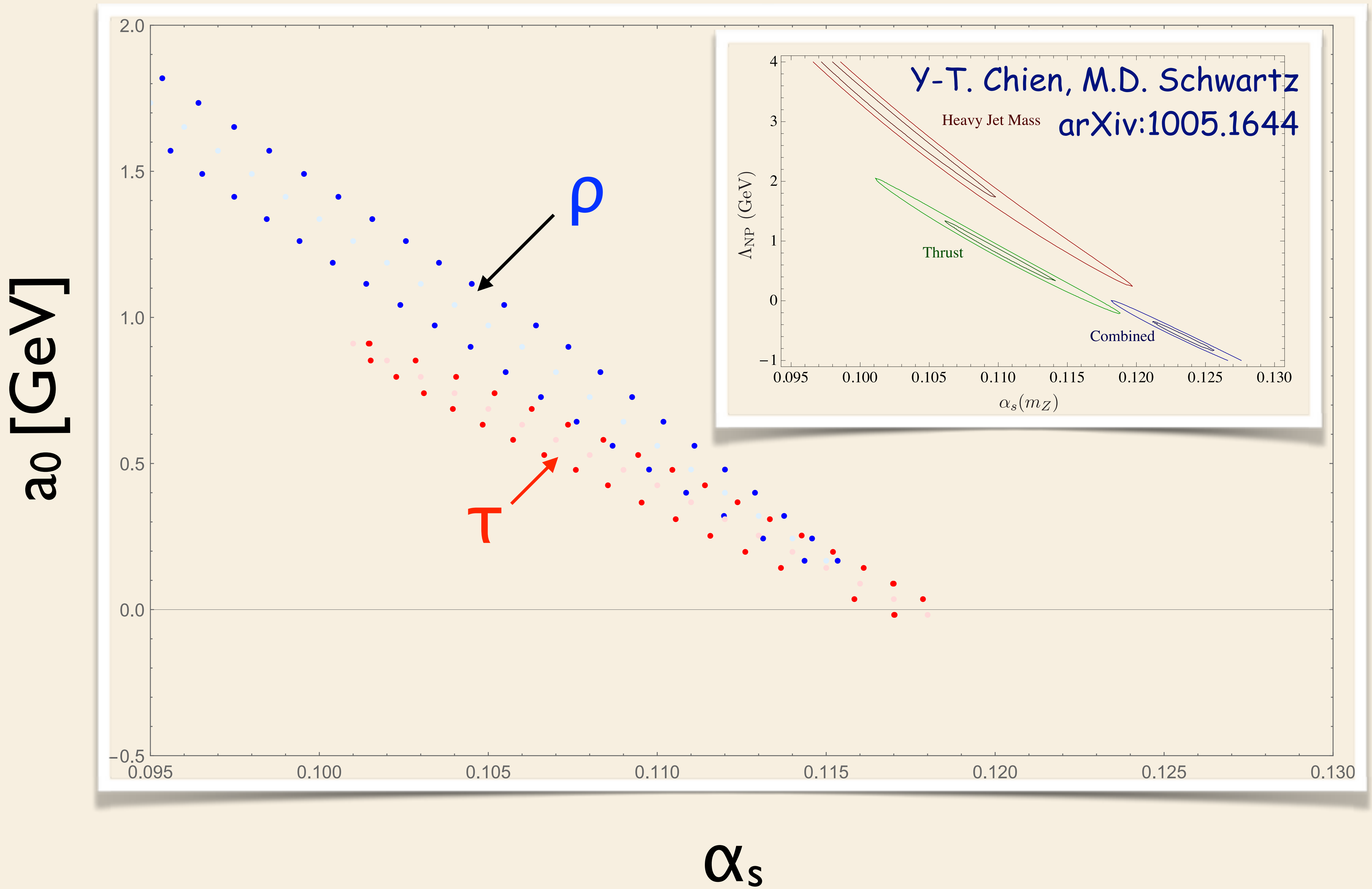
$$\mathrm{EEC}(\chi) = \frac{1}{\sigma_{\rm had}} \sum_{i,j} \int \frac{E_i E_j}{Q^2} \times d\sigma_{e^+e^-\to i\,j+X}\,\delta(\cos\chi + \cos\theta_{ij})$$

In progress



R. Albers, ZT in progress
with help of M.D.Schwartz

In progress



R. Albers, ZT in progress
with help of M.D.Schwartz

MCCSM performance

MCCSM performance

Approximate timing without binning on one core

(Intel(R) Xeon(R) CPU E5-2695 v2 @ 2.40GHz)

	B	V	R	VV	RV	RR
# of PS points	100M	100M	100M	10M	10M	10M
Timing	12min	8.3h	3.5h	7.5h	22h	5.5h

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# of PS points	100M	100M	100M	10M	10M	10M
Timing	12min	8.3h	3.5h	7.5h	22h	5.5h

- ✓ Regularized double-real contribution is smooth using 15B phase space points: in 27.5 hs on 300 cores
- ✓ Regularized real-virtual contribution is smooth using 1.5B phase space points: in 11 hs on 300 cores
- ✓ Regularized double-virtual contribution is smooth using 50M phase space points: in 7.5 min one 300 cores

Conclusions

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- ✓ Numerical implementation in MCCSM: converges well
- ✓ Precise (NNLO+N^3LL+LPC) predictions for three-jet event shapes in progress

Appendix

Pole-cancelation: $H \to b\bar{b}$ at $\mu = m_H$

$$\sigma_m^{\rm NNLO} = \int_m \left\{ d\sigma_m^{\rm VV} + \int_2 \left[d\sigma_{m+2}^{\rm RR,A_2} - d\sigma_{m+2}^{\rm RR,A_{12}} \right] + \int_1 \left[d\sigma_{m+1}^{\rm RV,A_1} + \left(\int_1 d\sigma_{m+2}^{\rm RR,A_1} \right)^{\rm A_1} \right] \right\} J_m$$

$$d\sigma_{H\to b\bar{b}}^{\rm VV} = \left(\frac{\alpha_{\rm s}(\mu^2)}{2\pi} \right)^2 d\sigma_{H\to b\bar{b}}^{\rm B} \Bigg\{ + \frac{2C_{\rm F}^2}{\epsilon^4} + \left(\frac{11 C_{\rm A} C_{\rm F}}{4} + 6 C_{\rm F}^2 - \frac{C_{\rm F} n_{\rm f}}{2} \right) \frac{1}{\epsilon^3}$$
$$+ \left[\left(\frac{8}{9} + \frac{\pi^2}{12} \right) C_{\rm A} C_{\rm F} + \left(\frac{17}{2} - 2\pi^2 \right) C_{\rm F}^2 - \frac{2 C_{\rm F} n_{\rm f}}{9} \right] \frac{1}{\epsilon^2}$$
$$+ \left[\left(-\frac{961}{216} + \frac{13\zeta_3}{2} \right) C_{\rm A} C_{\rm F} + \left(\frac{109}{8} - 2\pi^2 - 14\zeta_3 \right) C_{\rm F}^2 + \frac{65 C_{\rm F} n_{\rm f}}{108} \right] \frac{1}{\epsilon} \Bigg\}$$

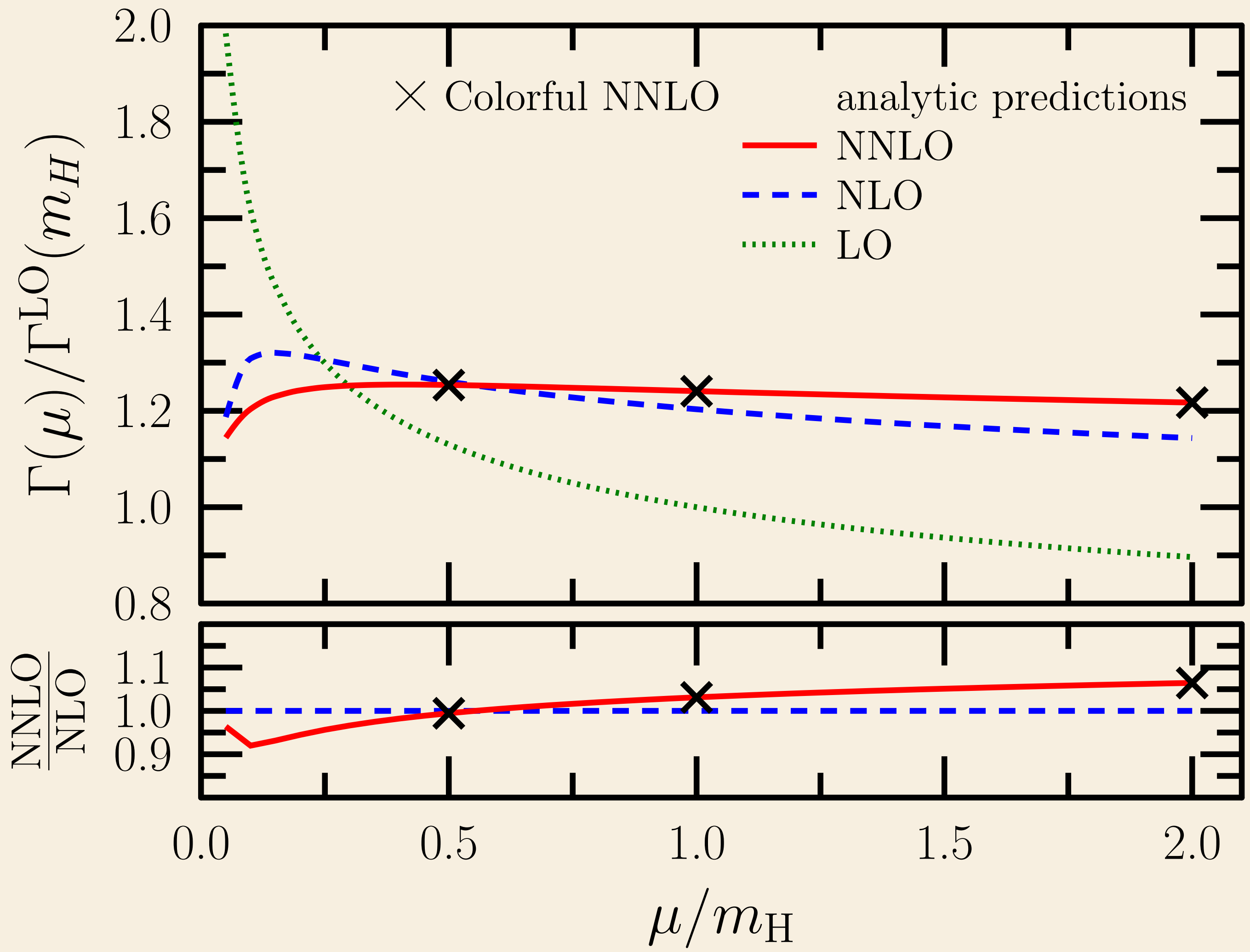
C. Anastasiou, F. Herzog, A. Lazopoulos, arXiv:0111.2368

$$\sum \int d\sigma^{\rm A} = \left(\frac{\alpha_{\rm s}(\mu^2)}{2\pi} \right)^2 d\sigma_{H\to b\bar{b}}^{\rm B} \Bigg\{ - \frac{2C_{\rm F}^2}{\epsilon^4} - \left(\frac{11 C_{\rm A} C_{\rm F}}{4} + 6 C_{\rm F}^2 + \frac{C_{\rm F} n_{\rm f}}{2} \right) \frac{1}{\epsilon^3}$$
$$- \left[\left(\frac{8}{9} + \frac{\pi^2}{12} \right) C_{\rm A} C_{\rm F} + \left(\frac{17}{2} - 2\pi^2 \right) C_{\rm F}^2 - \frac{2 C_{\rm F} n_{\rm f}}{9} \right] \frac{1}{\epsilon^2}$$
$$- \left[\left(-\frac{961}{216} + \frac{13\zeta_3}{2} \right) C_{\rm A} C_{\rm F} + \left(\frac{109}{8} - 2\pi^2 - 14\zeta_3 \right) C_{\rm F}^2 + \frac{65 C_{\rm F} n_{\rm f}}{108} \right] \frac{1}{\epsilon} \Bigg\}$$

V. Del Duca, C. Duhr, G. Somogyi, F. Tramontano, Z. Trócsányi, arXiv:1501.07226

Example: H→b$\bar{b}$

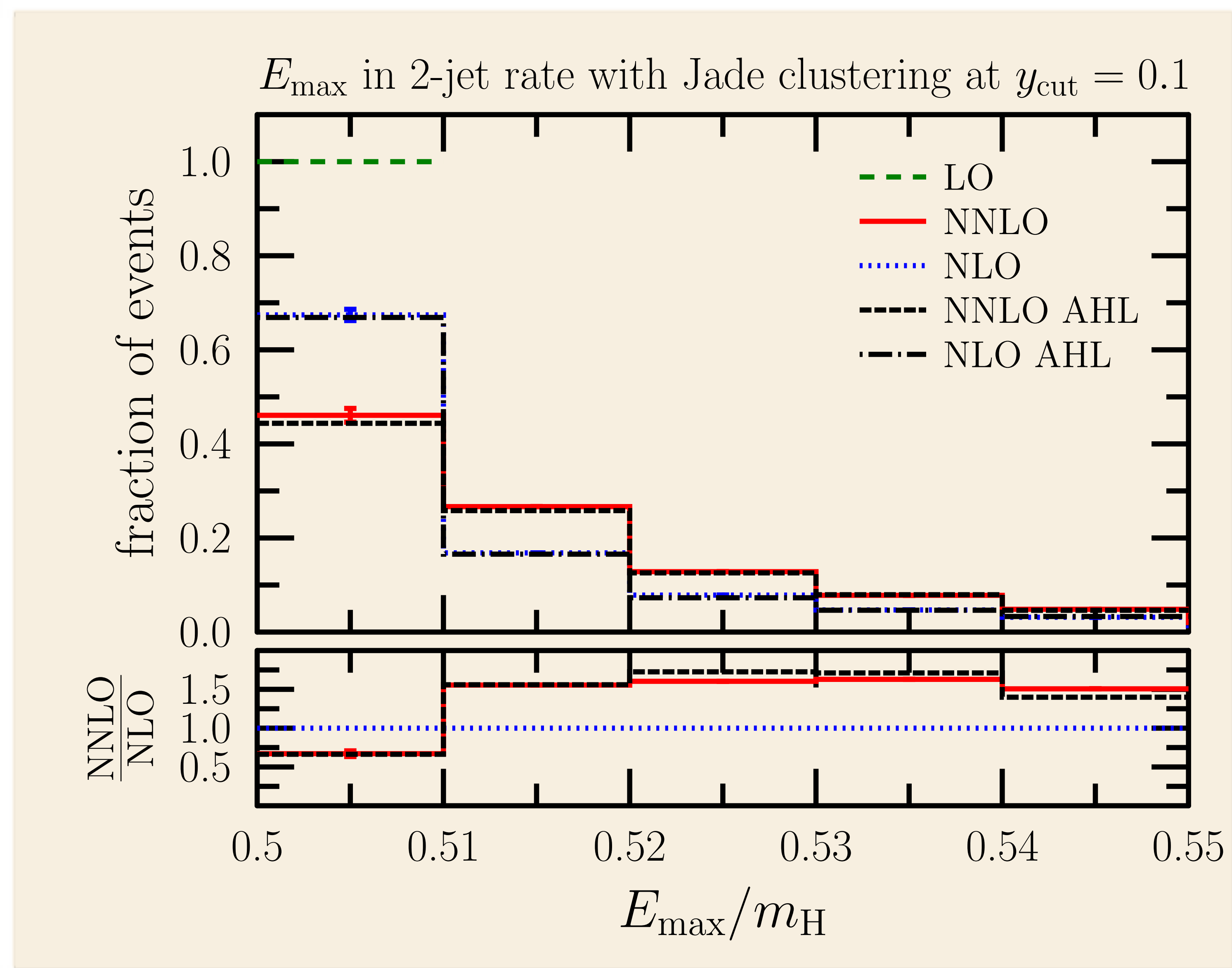
$$\Gamma^{\rm NNLO}_{H\to b\bar{b}}(\mu = m_H) = \Gamma^{\rm LO}_{H\to b\bar{b}}(\mu = m_H)\left[1 - \left(\frac{\alpha_s}{\pi}\right) 5.666667 - \left(\frac{\alpha_s}{\pi}\right)^2 29.149 + \mathcal{O}(\alpha_s^3)\right]$$



Scale dependence of the inclusive decay rate Γ(H -> b$\bar{b}$)

analytic: K.G. Chetyrkin hep-ph/9608318

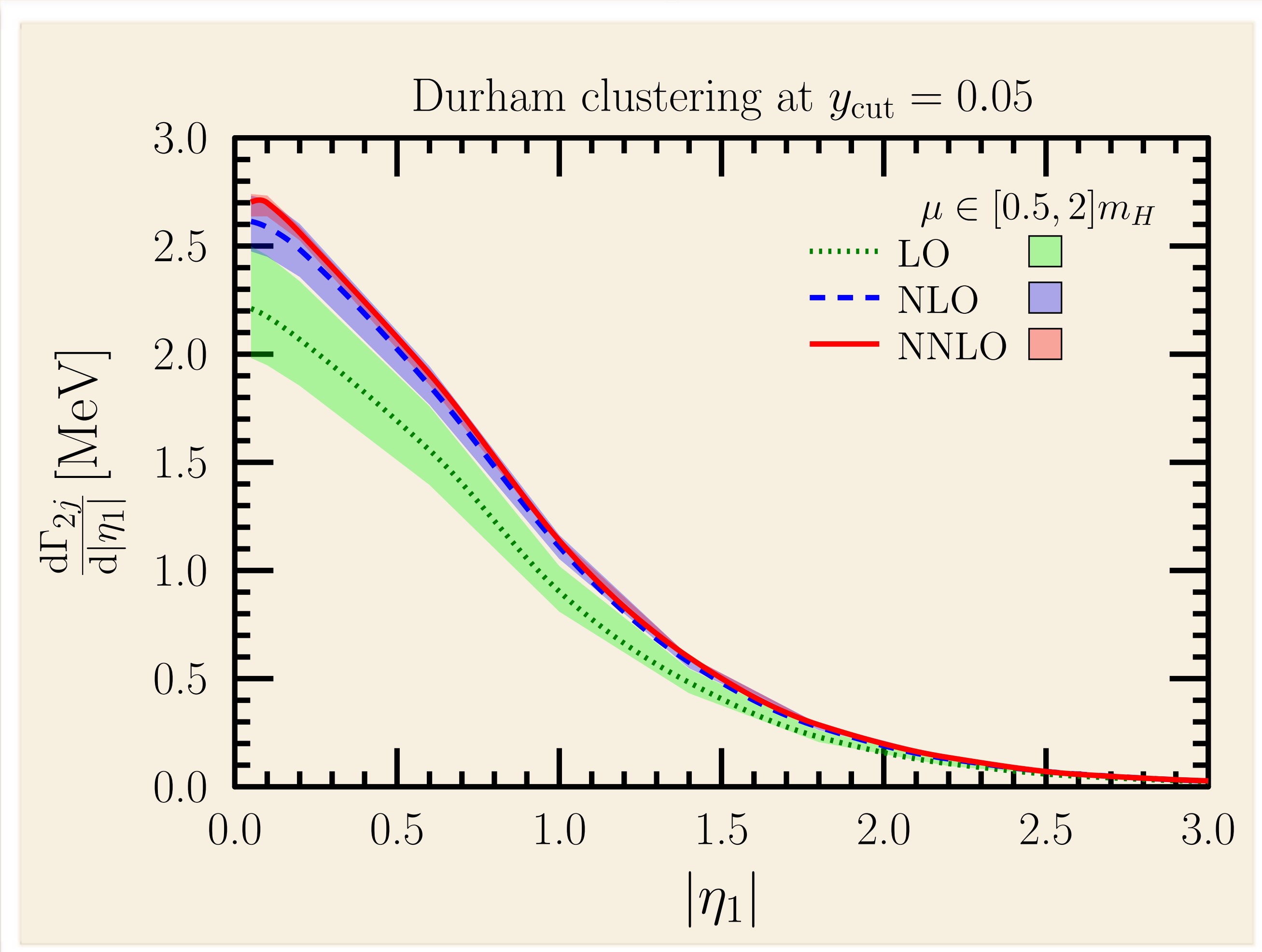
Example: $H \rightarrow b\bar{b}$ at $\mu = m_H$

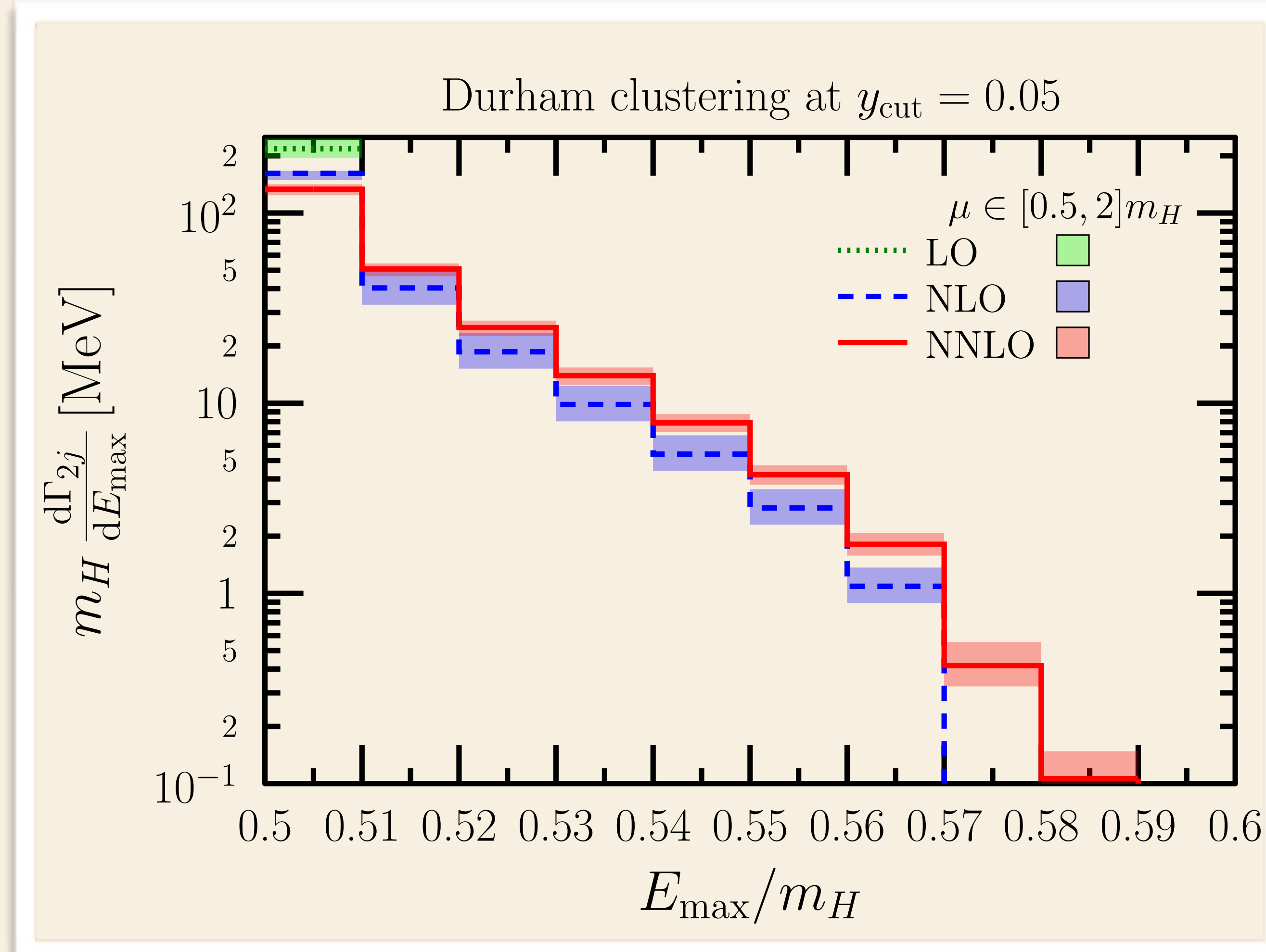


Energy spectrum of the leading jet in the rest frame of the Higgs boson. Jets are clustered using the JADE algorithm with $y_{cut} = 0.1$

AHL = C. Anastasiou, F. Herzog, A. Lazopoulos arXiv:0111.2368

Example: $H \to b\bar{b}$





rapidity distribution energy spectrum

of the leading jet in the rest frame of the Higgs boson.

jets are clustered using the Durham algorithm with $y_{cut} = 0.05$

Can constrain subtractions

We can constrain subtractions near singular regions ($\alpha_0<1$)

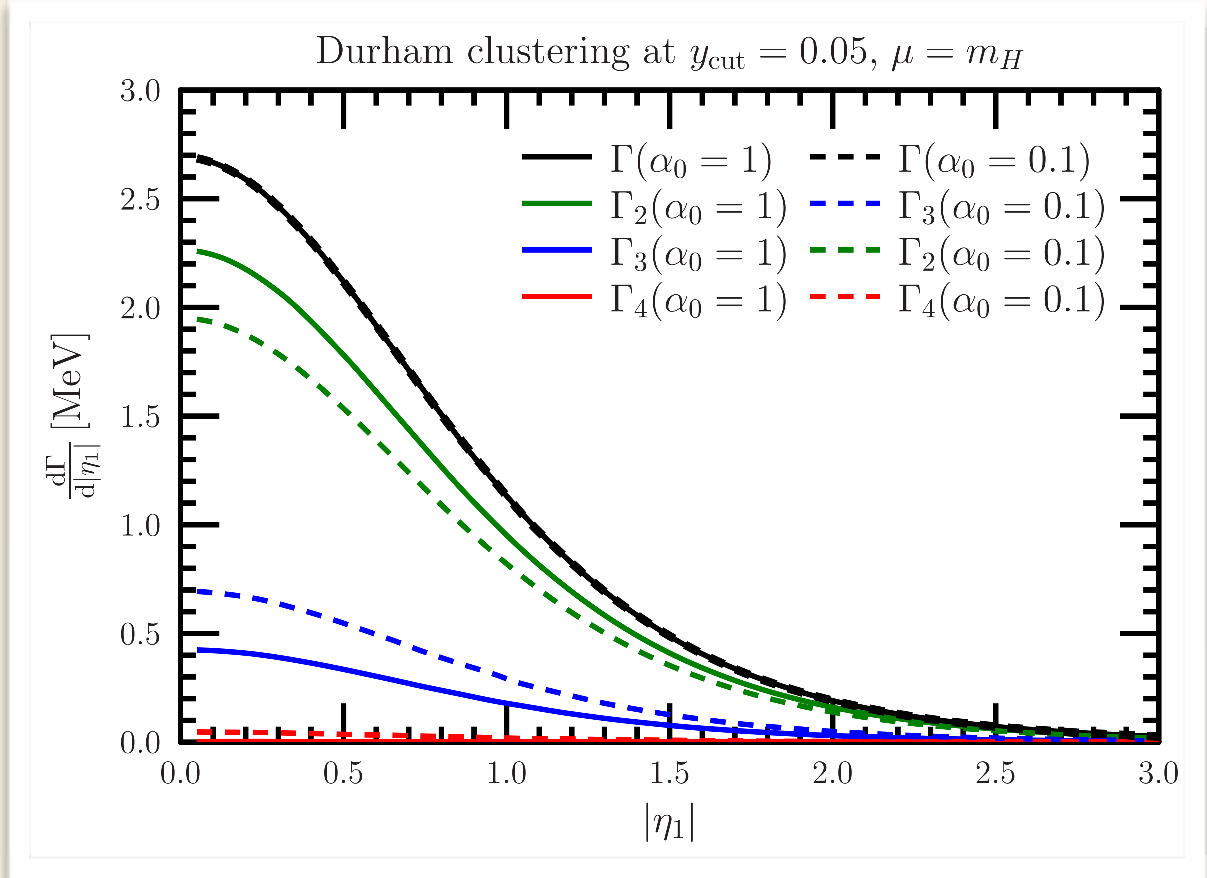
E.g. $H \to b\bar{b}$: poles cancel numerically ($\alpha_0 = 0.1$)

$$\mathrm{d}\sigma^{\mathrm{VV}}_{H\to b\bar{b}} + \sum \int \mathrm{d}\sigma^{\mathrm{A}} = \frac{5.4\times 10^{-8}}{\epsilon^4} + \frac{3.9\times 10^{-5}}{\epsilon^3} + \frac{3.3\times 10^{-3}}{\epsilon^2} + \frac{6.7\times 10^{-3}}{\epsilon} + \mathcal{O}(1)$$

$$Err\left(\sum \int \mathrm{d}\sigma^{\mathrm{A}}\right) = \frac{3.1\times 10^{-5}}{\epsilon^4} + \frac{5.0\times 10^{-4}}{\epsilon^3} + \frac{8.1\times 10^{-3}}{\epsilon^2} + \frac{7.7\times 10^{-2}}{\epsilon} + \mathcal{O}(1)$$

Predictions remain the same:

rapidity distribution of the leading jet in the rest frame of the Higgs boson. jets are clustered using the Durham algorithm (flavour blind) with y_{cut} = 0.05



Subtractions may help efficiency

We can constrain subtractions near singular regions (α_0<1), leading to fewer calls of subtractions:

α_0	1	0.1
timing (rel.)	1	0.40
$\langle N_{\rm sub} \rangle$	52	14.5

$\langle N_{sub} \rangle$ is the average number of subtraction calls