Extreme Fluid Dynamics in White Dwarfs & Neutron Stars

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- Cooling White Dwarfs

  Grav. Energy released from sedimenting ^{22}Ne can slow cooling, but current estimates limited by our knowledge of D and/or macro.

  Re $\ll 1$

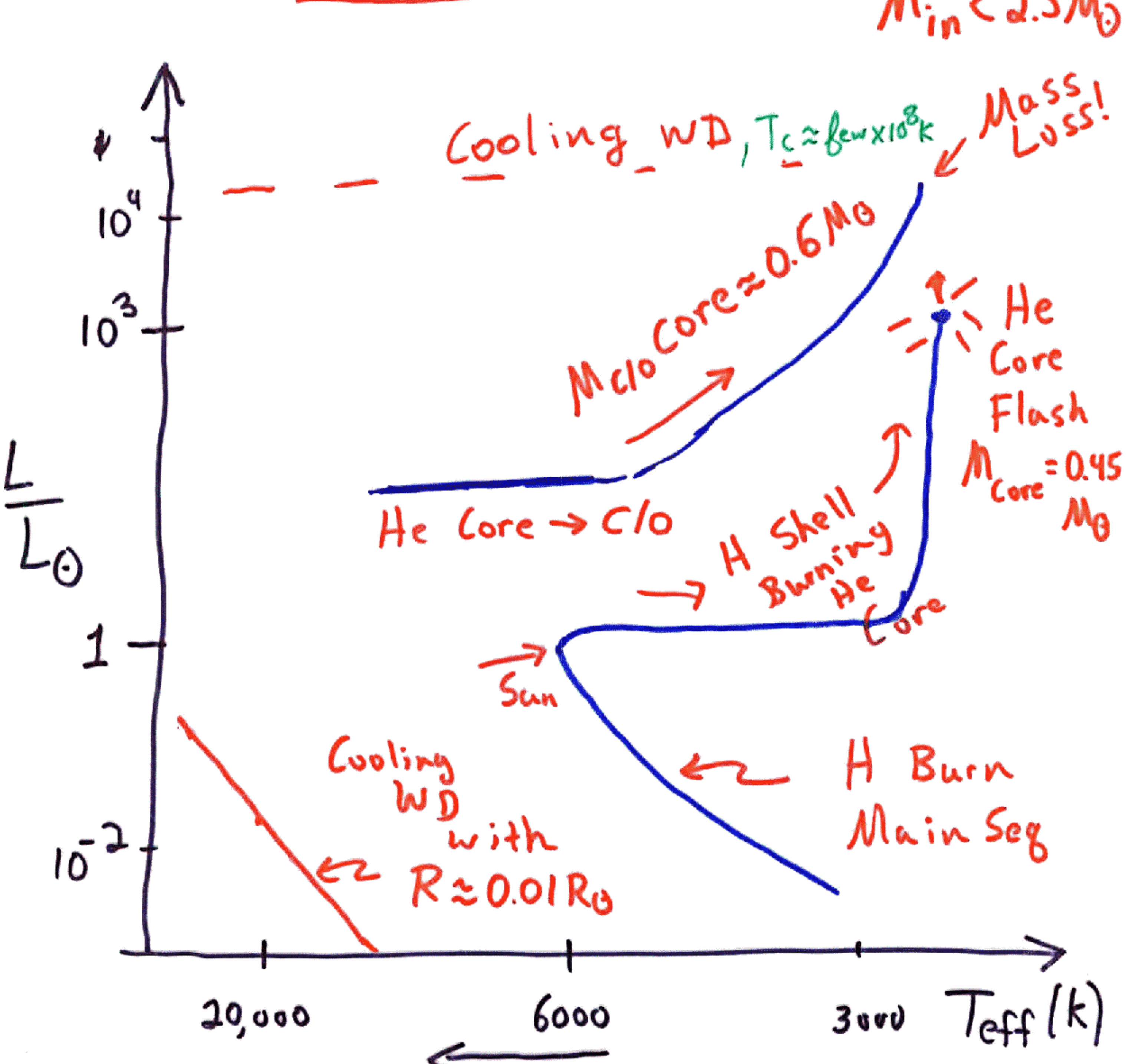
- Explosions on Accreting Neutron Stars

  Observations of nearly coherent oscillations during thermonuclear explosions point to spin modulation of low frequency patterns, but origin unknown - -

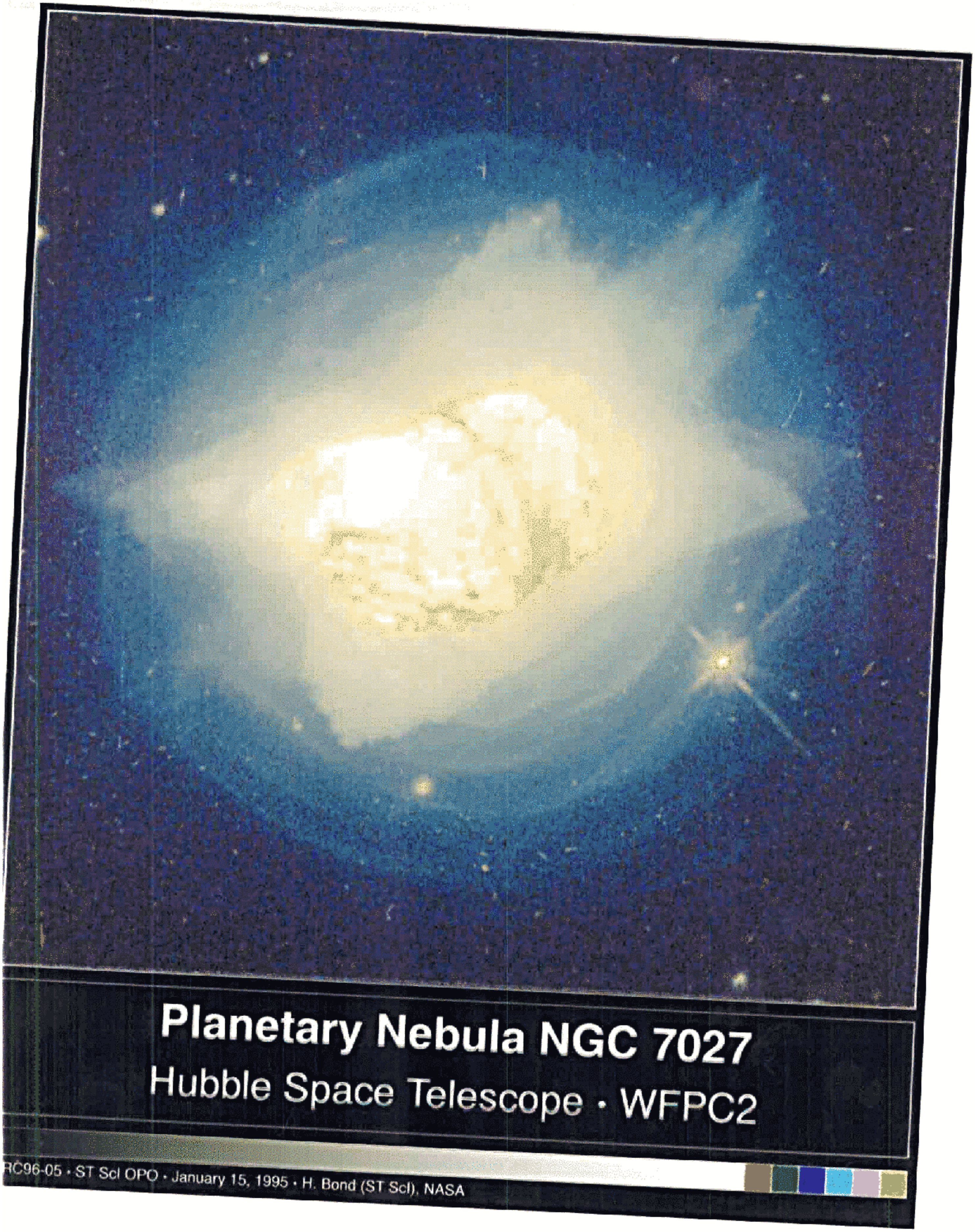
  Re $\gg 1$

$\Rightarrow$ LOTS OF GREAT FLUID CHALLENGES!

Stellar Evolution



- All stars with $M_{in} \lesssim (6-8) M_\odot \Rightarrow$ WD
- Roughly 1 new WD every 10 yrs
- Near sun (out to ~13pc, Holberg et al '02) local WD density is ≈ 50 WD's in r = 13pc.

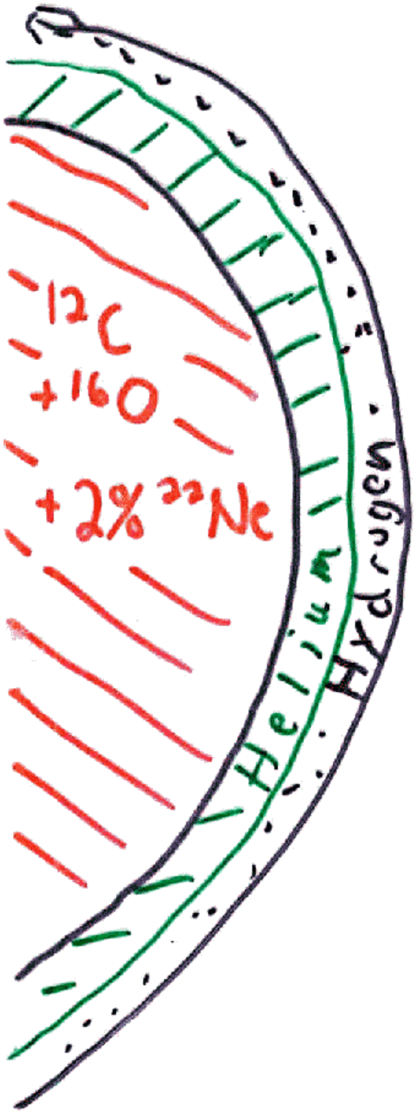


Planetary Nebula NGC 7027

Hubble Space Telescope · WFPC2

RC96-05 · ST ScI OPO · January 15, 1995 · H. Bond (ST ScI), NASA

White Dwarf Cooling



Core:

- Electrons supply nearly all pressure, ions form a
- Coulomb Liquid / Crystal

$$\Gamma = \frac{(Ze)^2}{akT} > 1$$

$$n_i = \frac{1}{\frac{4\pi}{3}a^3} \quad a = \text{ion spacing}$$

$\Gamma \gtrsim 173$ crystallize (Farouki & Hamaguchi)

(Lindemann $\Gamma > 70$..)

- Thermal Content in Ions

$$C_v = \overset{\uparrow}{3} N k_B = 3 k_B \frac{M}{\mu_i m_p}$$

highly correlated liquid & solid $T > \Theta_D$

$\mu_i = 12$ (pure ^{12}C), 16 (pure ^{16}O)

- Degenerate e^-'s in core are excellent conductors

$\Rightarrow$ core nearly isothermal

Mestel Cooling Law 1952!

- Simple treatment of integrating

$$F = -K \frac{dT}{dx}$$

thru H layer, connecting to core

$$L \approx 2.5 L_\odot \left(\frac{M}{0.6 M_\odot}\right)\left(\frac{T_c}{10^8 K}\right)^{7/2}$$

- Heat Equation is Then:

$$\frac{d}{dt} E_{th} = 3 \frac{k_B M}{\mu_i m_p} \frac{dT_c}{dt} = -L$$

mass cancels and $\frac{dT_c}{T_c^{7/2}} \propto dt$

so once $T_c \ll T_{c,i}$

$$T_c \approx 10^8 K \left(\frac{10^6 yr}{t}\right)^{2/5}$$

$$\frac{L}{L_\odot} \approx 2.5 \left(\frac{M}{0.6 M_\odot}\right)\left(\frac{1.6 \times 10^6 yr}{t}\right)^{7/5}$$

L & t Relate!!

Gravitational Settling of ^{22}Ne

Bildsten & Hall '01; Deloye & Bildsten '02

- During H Burning via CNO cycle on main sequence all catalysts pile up at ^{14}N, as $^{14}N(p,\gamma)$ is slow.

  ⇒ Helium Core has ^{14}N at ≈ 2% by mass.

- Once ^{4}He ignites, ^{22}Ne is made.

  $$^{14}N(\alpha,\gamma)^{18}F(\beta^+)^{18}O(\alpha,\gamma)^{22}Ne$$

  ⇒ ^{22}Ne is third most abundant after Carbon & Oxygen.

- ^{22}Ne tends to sink!

  Electric Field : Ionized plasma in grav. field has $\vec{E}$ to keep e^- from rising

In a WD $$\frac{1}{n_e}\frac{dP_e}{dr} = -m_e g - eE$$

but $$\frac{dP_e}{dr} = -\rho g = -Am_p\frac{n_e}{Z}g \Rightarrow \boxed{eE = \frac{A}{Z}m_p g}$$

So ^{12}C + ^{16}O dominate $eE = 2m_p g$; $F=0$

$$\left| F_{22\,Ne} = -22m_p g + 10(2m_p g) = -2m_p g \right|$$

Sinking & Viscosity

^{22}Ne feels a downward force, what is the terminal velocity?

The Diffusion Coefficient in a strongly coupled ($\Gamma \gg 1$) liquid has not been calculated, but the viscosity, η has!

Stokes Law works reasonably well at low Reynolds # on the microscale

$$\Rightarrow \quad F = 2 m_p g = 4\pi a_p \eta \, v_{dr}$$

fluid slips at boundary

Charge neutral sphere

$\uparrow$ Term. Vel.

Using $\eta \simeq \frac{1}{10} \rho \, \omega_p a^2$ (Donko & Nyiri '00) gives a characteristic time to sink!

$$t_s \approx \frac{Z}{18} \left(\frac{e^2}{G m_p^2} \right)^{1/2} \left(\frac{1}{4\pi G \rho} \right)^{1/2} \simeq 13 \, \text{Gyr} \, \frac{Z}{6 (\rho / 10^6)^{1/2}}$$

(Bildsten & Hall '01)

$\Uparrow$ $\simeq$ Hubble!

Crystallization Intervenes!

We only allow for settling in the liquid phase so the energy release is halted once a region is solid!

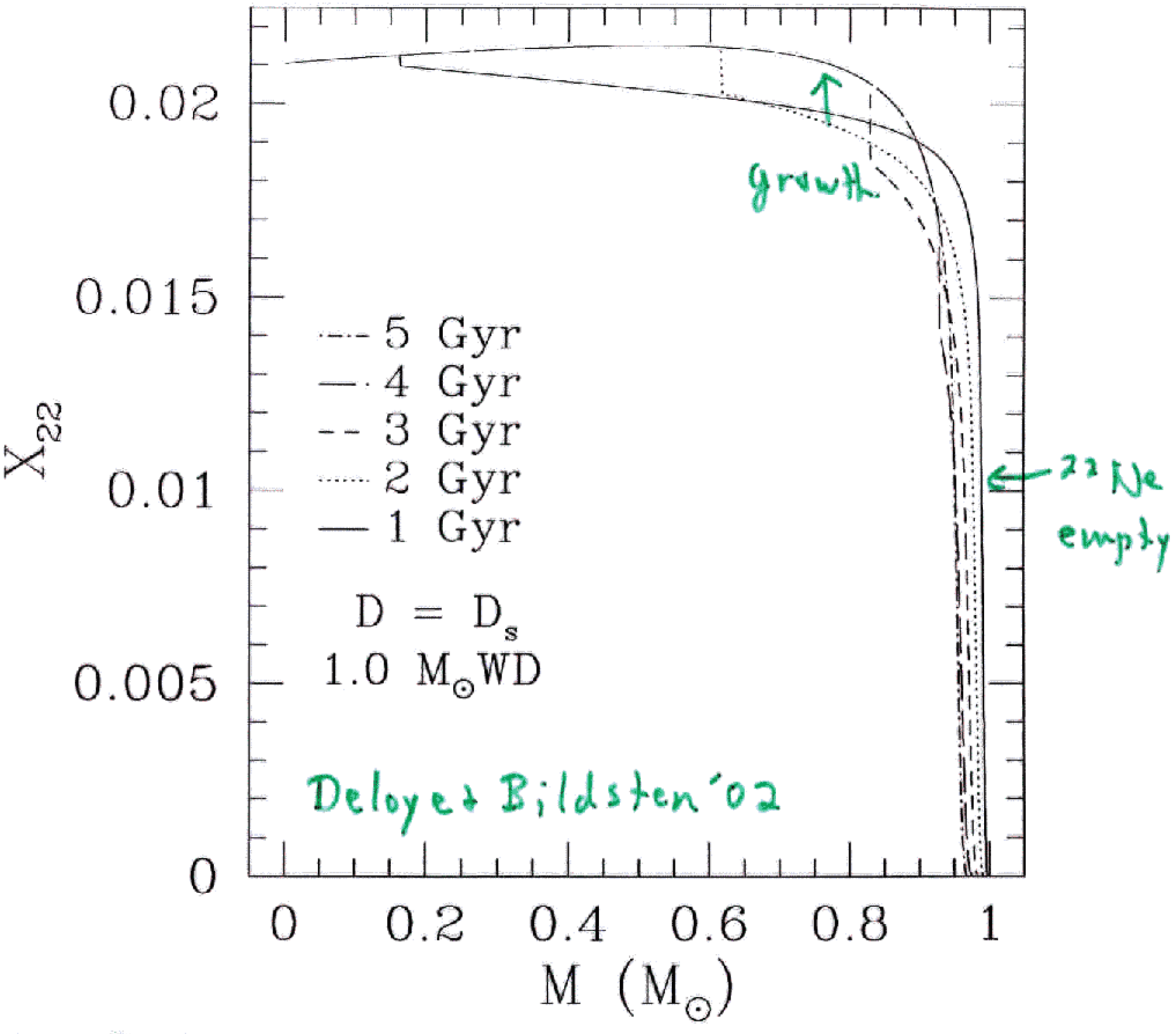


FIG. A6.— ^{22}Ne profile at several epochs in a 1.0 $M_\odot$ C/O WD. Both the effects of rapid settling and rapid crystallization are seen in comparison to a 0.6 $M_\odot$ WD.

Falling ^{22}Ne releases energy

$$L_g \approx \int \vec{F} \cdot \vec{V}\, n_{Ne} 4\pi r^2 dr$$

in liquid

So at late times, total energy released is comparable to the thermal content ⇒ WD Cooling is SLOWED.

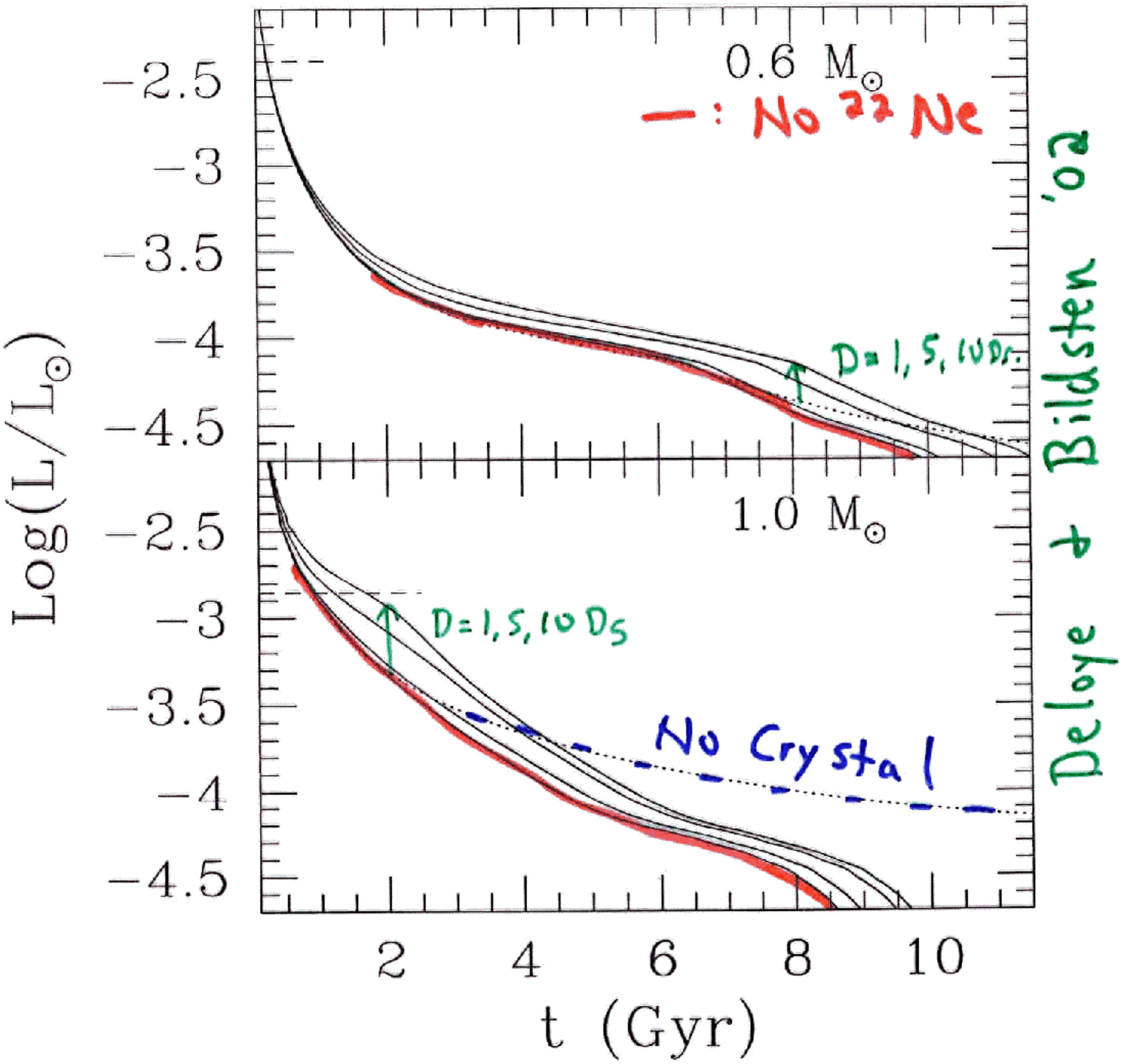


FIG. A9.— WD luminosity as a function of time. Both plots are for C/O WDs. The lower curve in each gives the cooling curve neglecting ^{22}Ne sedimentation in the thermal evolution of the WD. The remaining three solid lines are (from top to bottom) the results for $D = D_s, D = 5D_s$, and $D = 10D_s$ while the dotted line is for a model that enters a glassy state. The glassy transition models stay more luminous at late times due to the continued sedimentation of ^{22}Ne . The horizontal dashed line approximates the ZZ Ceti instability strip.

⇒ Delays of Order 0.5 → 1.5 Gyr for Solar Metallicity.

X-Ray Binaries

Matter is supplied at $\dot{M} \approx 10^{-10} \rightarrow 10^{-8}\ M_\odot/\mathrm{yr}$

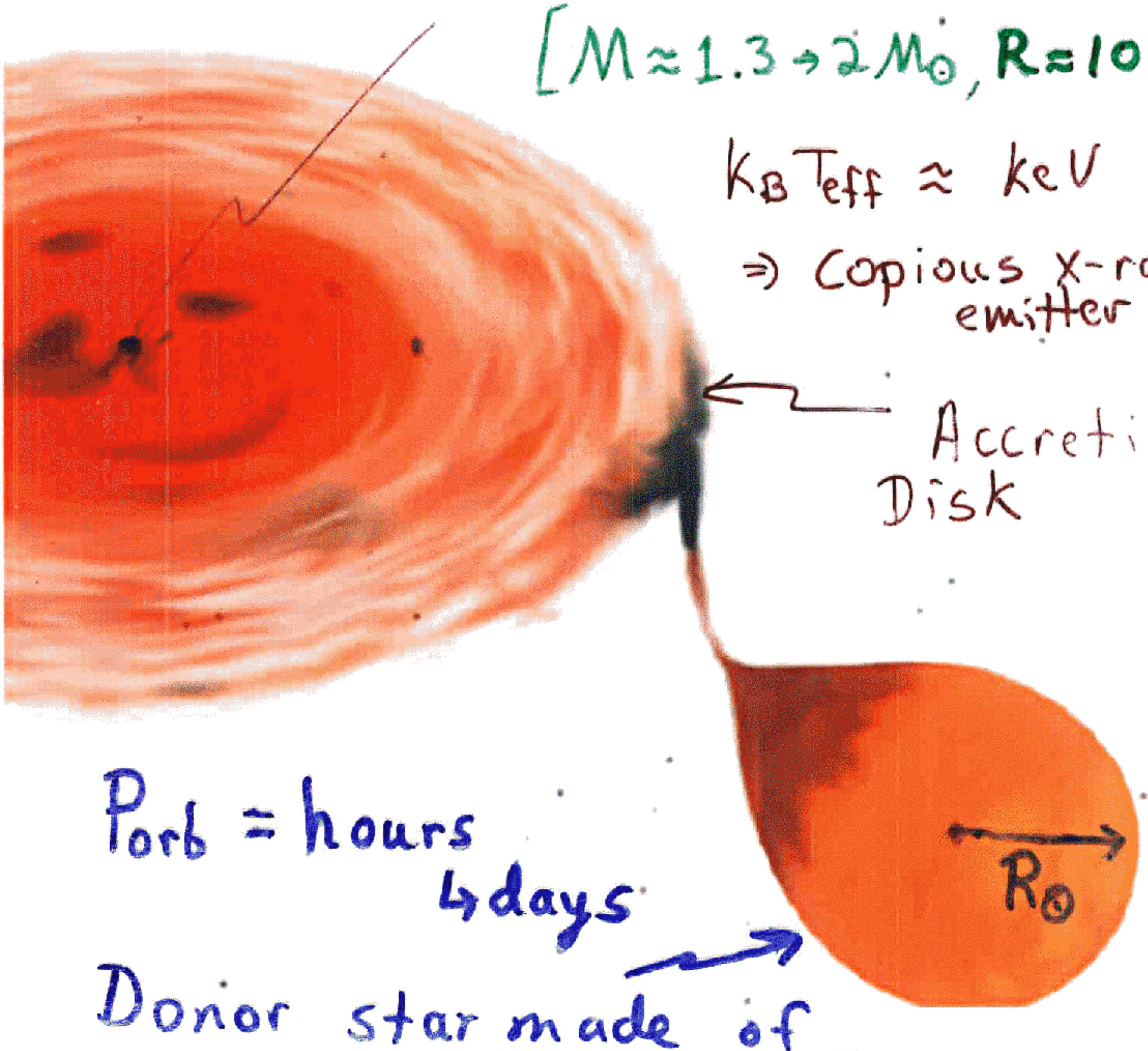
$$\Rightarrow L_{accr} = \frac{GM_{NS}}{R_{NS}}\dot{M} \approx 10^{36} \rightarrow 10^{38}\ \frac{\mathrm{ergs}}{\mathrm{sec}}$$

Neutron Star

$[M \approx 1.3 \rightarrow 2 M_\odot,\ R \approx 10\,\mathrm{km}]$

$k_B T_{eff} \approx \mathrm{keV}$

⇒ Copious X-ray emitter



$P_{orb} \approx$ hours → days

Donor star made of H & He.

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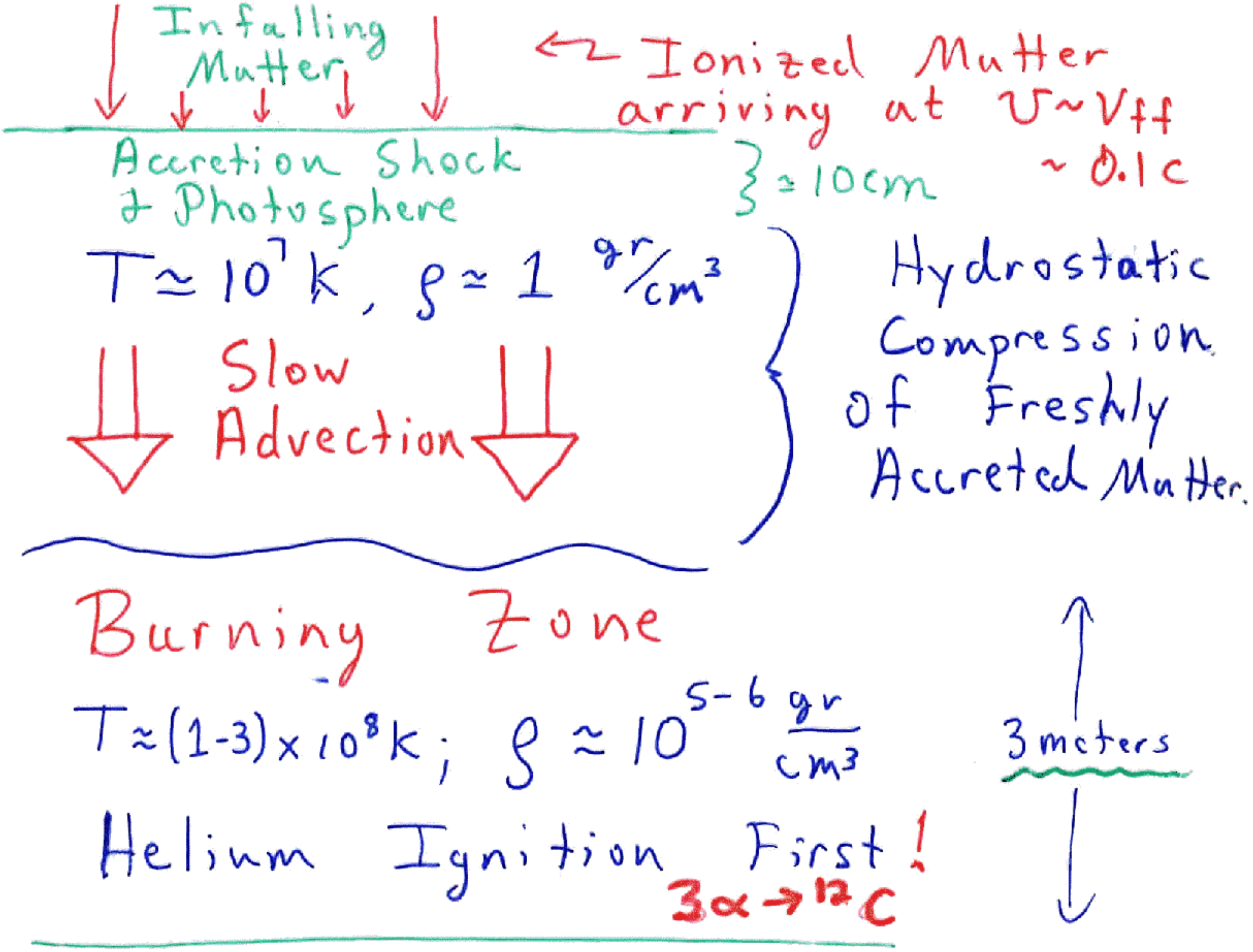
Accretion and Burning of Matter on the Neutron Star

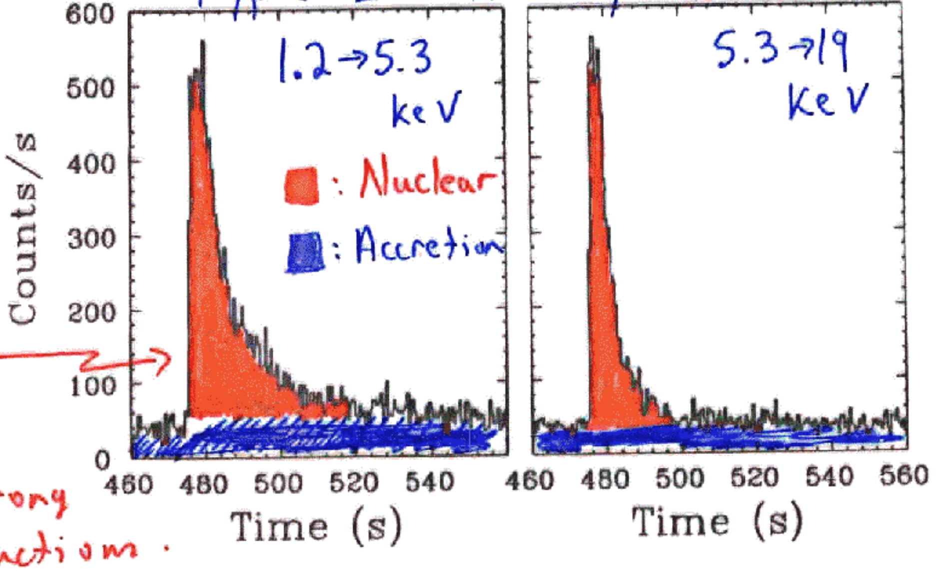


- It takes an hour – day for fresh fuel to reach the burning location
- Burning is confined to a thin shell when a steady-state model is constructed. (Hansen & Van Horn '75)
  ↳ UNSTABLE

Type I X-Ray Bursts

Grindlay et al '76

Belian et al '76



Rapid Rise

⇒ Strong Reactions.

Rapid Nuc. Consumption

Figure 8. Type I X-ray burst from 1702–42 as observed with Exosat in the 1.2 - 5.3 keV band (left) and the 5.3 - 19.0 keV band (right); the softening of the X-ray burst spectrum is apparent as a longer tail in the low-energy burst profile (courtesy T. Oosterbroek).

Many (Woosley + Taam '76; Maraschi + Cavaliere '77 Joss '77; Lamb + Lamb '78) successfully associated the thermal instabilities with Bursts, where:

Observable	Interpretation
$t_{rec} \simeq$ hours - days	Time to accumulate critical fuel pile
$E_{Burst} \simeq 10^{39}$ ergs	$= \left[\dot{M} t_{rec}\right]\left(\frac{7\,\mathrm{MeV}}{\mathrm{nucleon}}\right)$
$t_{decay} \simeq 10$ sec.	Time for cooling the envelope after burn.

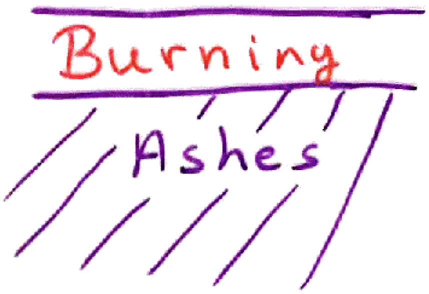
⇒ Limit cycle of accumulation for hours followed by Ignition

Is the Helium Burning Stable to a Thermal Perturb.?

[Hansen & Van Horn 1975]

The overlying material keeps the burning region at constant pressure.

$$P = gy = \text{Constant}$$



So increased nuclear energy generation ↑ T and ε_{nuc} ↑

So the gas responds temporally with C_p

$$\Rightarrow \frac{T\,dS}{dt} = C_p \frac{dT}{dt} = \varepsilon_{nuc} - \frac{1}{\rho}\underset{\sim}{\nabla}\cdot\underset{\sim}{F}$$

In steady-state these terms are equal. What happens when we perturb T?

$\underset{\sim}{F}$ = Heat Flux (Mostly Radiative)

$\varepsilon_{nuc} = \frac{\text{ergs}}{\text{gr}\cdot\text{sec}}$ = Energy generation from thermonuclear reactions.

7 10

For radiative transport mediated by Thomson scattering

$$+\frac{1}{\rho}\underset{\sim}{\nabla}\cdot\underset{\sim}{F} \propto T^4$$

whereas the nuclear burning scales as

$$\varepsilon_{nuc} \propto \rho^2 T^{\nu}$$

where for $3\alpha \rightarrow {}^{12}C$ $\quad \nu \approx \frac{44}{(T/10^8)} - 3$

Clearly it is the temperature dependences which are most crucial, so that roughly

$\nu > 4 + 2$ Unstable, Strong T dependence beats cooling

This tells us that the burning is unstable when

$$T_b \lesssim 5\times 10^8 K \Rightarrow \dot{M} \lesssim \text{few times } \dot{M}_{Edd}$$

⇒ Different Cause Than in Classical Novae!

Is Ignition & Burning Spherically Symmetric?

The thermonuclear runaway grows on ~seconds, but it takes hours to accumulate the unstable pile.....
.... is it truly 1D?

Two Approaches to Relaxing Symmetry

① - Spot ignition, followed by spreading (Shara '82) via a deflagration (too slow, ~100 cm/s : Bildsten '93-95) or detonation (unlikely since growth time $<< h/c_s$ = time for sound to traverse h) has been discussed (Fryxell & Woosley '82)

- Rotation likely very critical as shallow wave speed

$$v_{\rm shallow} \sim \sqrt{gH} \sim 2000\ {\rm km/s} \Rightarrow t \sim \frac{R}{v} \sim 5\ {\rm ms}$$

gives time longer than rotation ($P_{\rm rot} \lesssim 5$ ms)

⇒ Combustion Contained in Rossby Radius

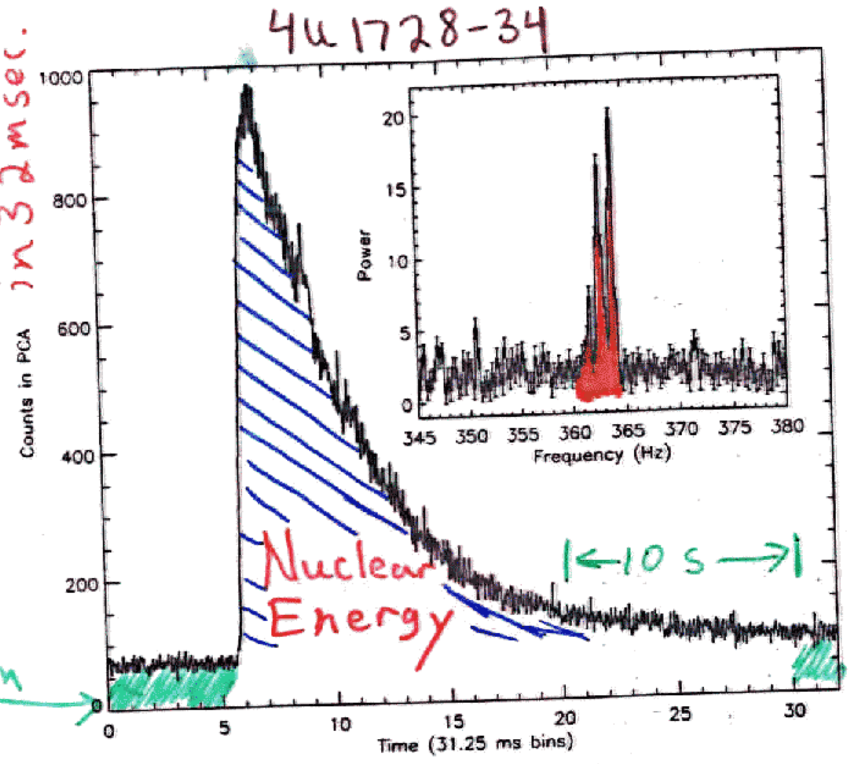
[Spitkovsky, Levin & Ushomirsky '02]

② Symmetry Broken By Instability

$$L_{acc} = \frac{GM}{R}\dot{M} \simeq \left(\frac{200\,\mathrm{MeV}}{m_p}\right)\dot{M}$$

$$\langle L_{nuc}\rangle = \left(\frac{7\,\mathrm{MeV}}{m_p}\right)\dot{M}$$

m_p = proton mass



Accretion Energy

FIG. 1.—Light curve of the burst that occurred at 10:00:45 UTC on 1996 February 16. The main panel shows the total PCA counts in 31.25 ms bins. The inset panel shows a portion of the power spectrum computed from 32 s of 122 μs data. Each bin is 0.25 Hz wide and represents the average of eight original power spectral bins. The error bars include only the uncertainties due to Poisson counting statistics.

Strohmayer et al. (1996)

Rossi X-Ray Timing Explorer,
launched 12/30/95

Proportional Counter Array

Area ≈ 6500 cm^2

$$\frac{\Delta E}{E} \simeq 10\%$$

1° Field of View.

Strohmayer, Zhang & Swank (97)

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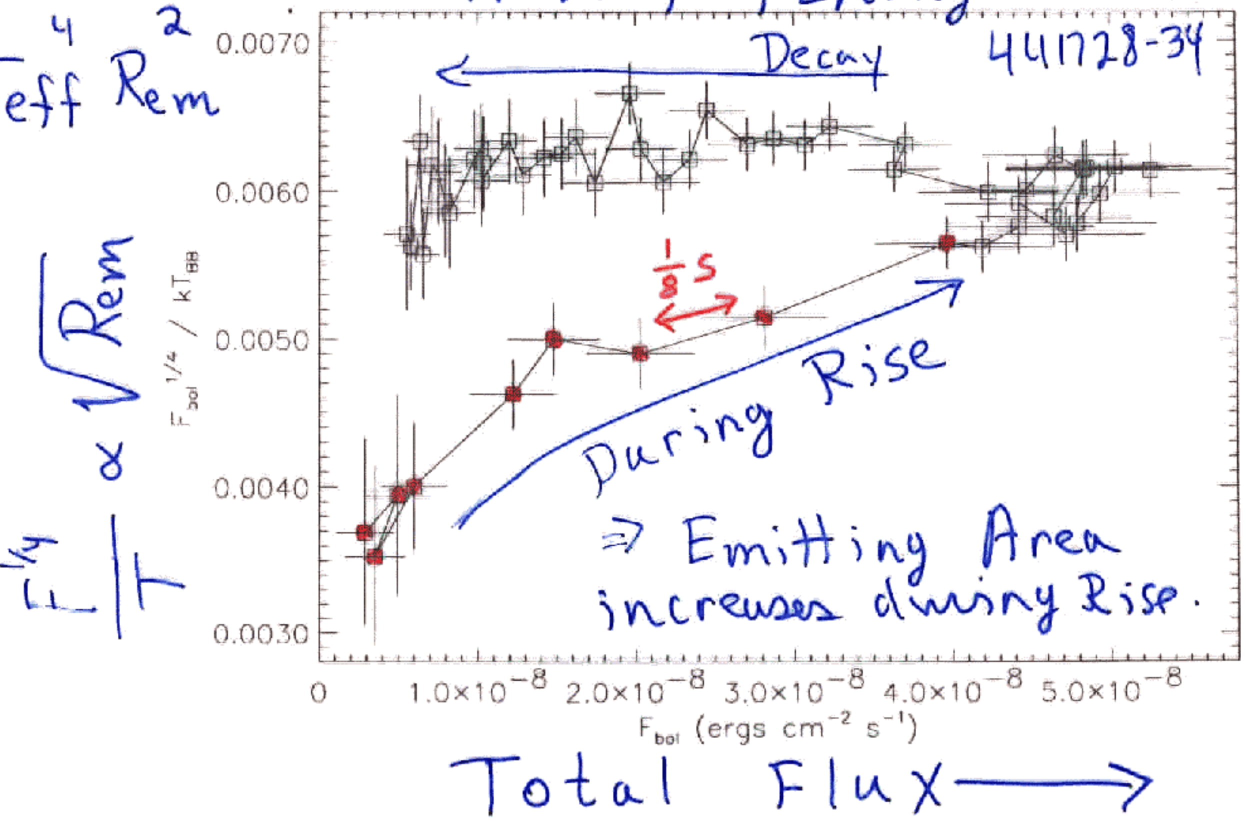
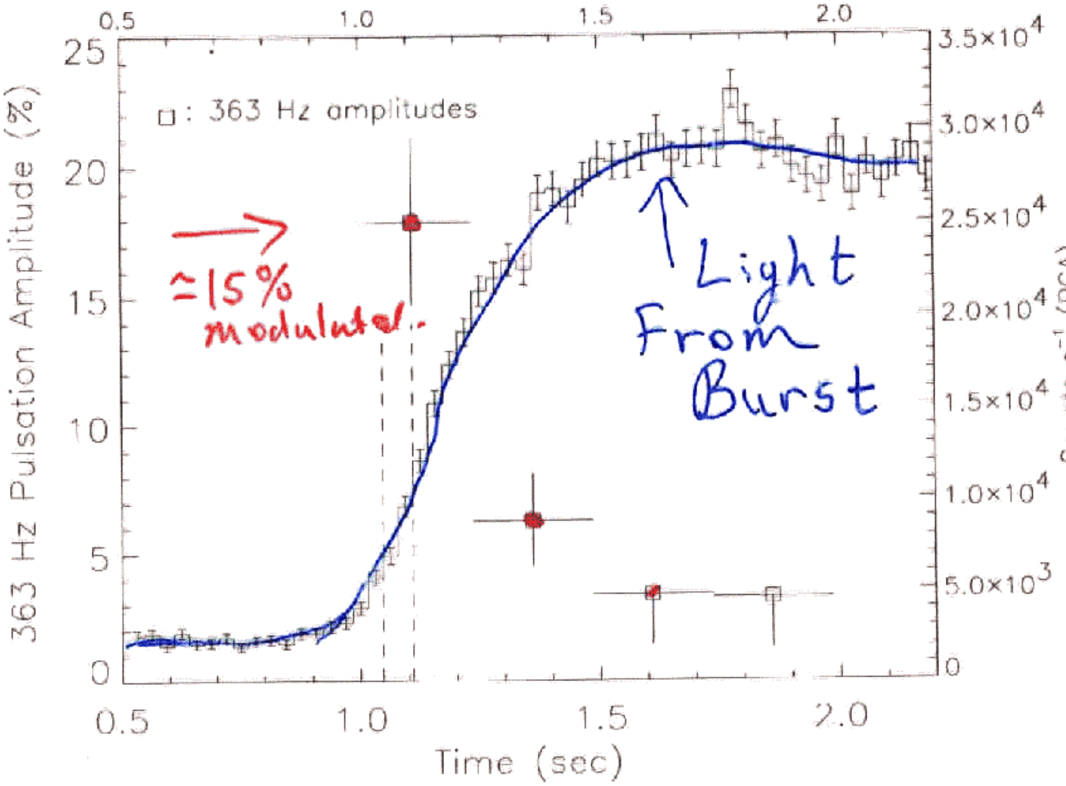
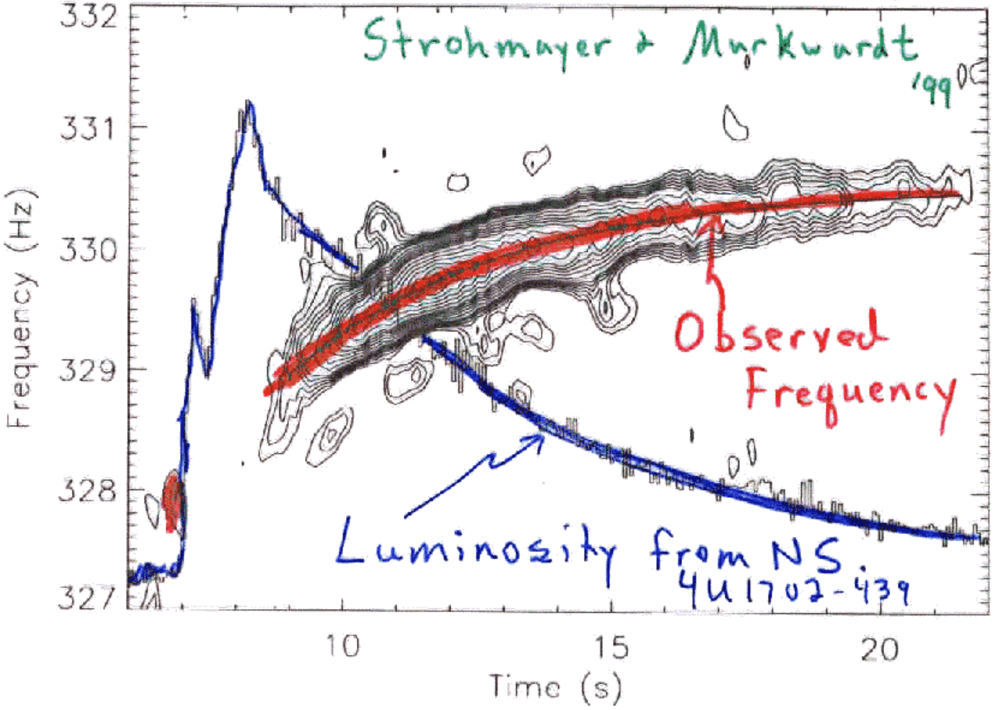
$F \propto T_{eff}^4 R_{em}^2$





$\Rightarrow \quad f_s = 363$ Hz!

PUZZLE



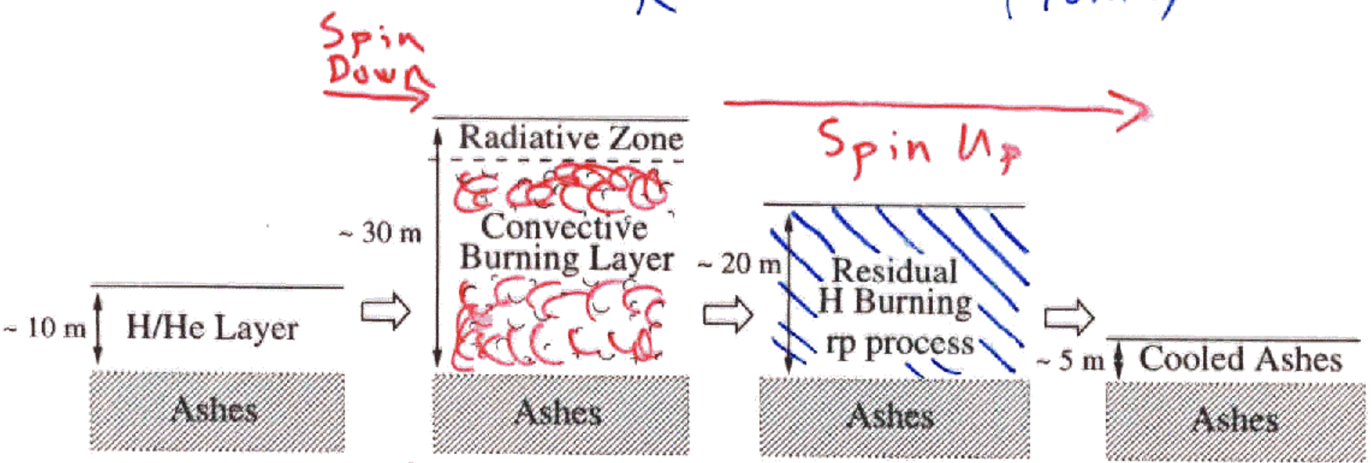
Observed frequency always increases at late times

⇒ NO torque large enough to change ν_s this fast.

Strohmayer et al ('97) suggested that the burning layer is decoupled from the star.

⇒ $\vec{J}$ is conserved.

$$\Delta \nu = \nu_s \frac{2\Delta r}{R} \approx (300\,\mathrm{Hz})\left(\frac{40\,\mathrm{m}}{10\,\mathrm{km}}\right) = \mathrm{Hz}$$



Cumming + Bildsten

Summary of Burst Oscillation

Similar behavior now seen in ~10 separate neutron stars, none with overt $\vec{B}$ fields.

- Oscillations during rise consistent with spreading/rotat. modulated.
- Freq. often drifts, but asymptotic freq very stable on 5 year time scale
- Origin of asymmetry ~10 seconds after burst has started is a puzzle

⇒ Most likely measuring these NS spins (expected to become millisecond radio pulsars), but nature of $m \approx 1$ asymm. unknown.

Conclusions

- Many astrophysical settings where fluid dynamics plays a critical role.
- Other sites have a strong enough $\vec{B}$ that conventional lessons from fluids does not apply.

No matter which problem, I am confident that the DFD community can make significant contributions!