The NuTeV Result for $\sin^2\theta_W$ - a Reanalysis of the Electroweak Radiative Corrections

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outline

- introduction
- issues/subtleties in $O(\alpha)$ corrections
- numerical results
- conclusions

introduction

in neutrino-nucleon (ν-N) scattering one can determine weak mixing angle $\sin^2\theta_W$ from:

$$R^\nu = \frac{\sigma^\nu_{NC}}{\sigma^\nu_{CC}} \approx \frac{1}{2} - \sin^2\theta_W + \frac{20}{27}\sin^4\theta_W$$

$$R^{\bar\nu} = \frac{\sigma^{\bar\nu}_{NC}}{\sigma^{\bar\nu}_{CC}} \approx \frac{1}{6} - \frac{1}{3}\sin^2\theta_W + \frac{20}{27}\sin^4\theta_W$$

(Llewellyn-Smith)

in ratios $\sigma_{NC}(\nu q \to \nu q)/\sigma_{CC}(\nu q \to l q')$ and $\sigma_{NC}(\bar\nu q \to \bar\nu q)/\sigma_{CC}(\bar\nu q \to \bar l q')$ systematic uncertainties partially cancel

approximations in $R^\nu, R^{\bar\nu}$:

- leading order
- valence approximation
- isosymmetry of nuclear target
- contact interaction
- massless quarks and leptons
- integration over full phase space

R^{ν} and $R^{\bar{\nu}}$ easy to measure but less suited for precise determination of $\sin^2\theta_W$ due to
- sea quark PDF uncertainties
- quark mass effects in charm production
- higher orders in perturbation theory

instead consider:

$$R^{-} = \frac{\sigma^{\nu}_{NC} - \sigma^{\bar{\nu}}_{NC}}{\sigma^{\nu}_{CC} - \sigma^{\bar{\nu}}_{CC}} \approx 1 - \sin^2\theta_W$$

(Paschos, Wolfenstein)

approximations in R^{-}:
- leading order
- isosymmetry of nuclear target
- contact interaction
- massless quarks and leptons
- integration over full phase space

but:
- no uncertainties from sea quark PDF
- charm mass effects Cabbibo-suppressed
  $\rightarrow$ negligible

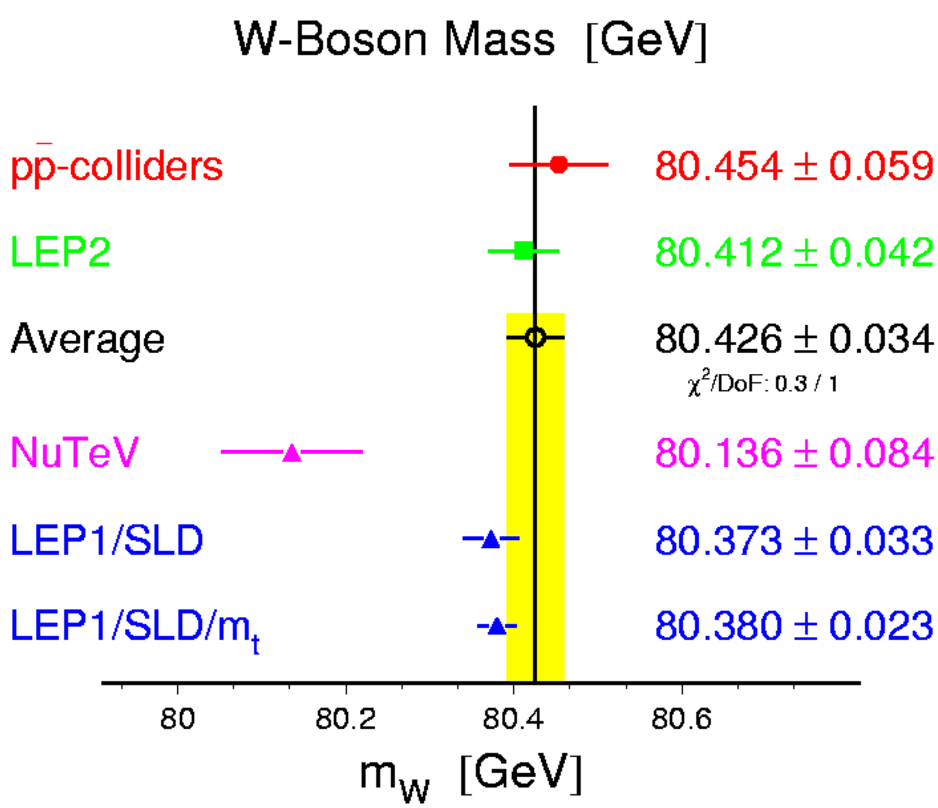
$\rightarrow$ R^{-} systematically cleaner than $R^{\nu/\bar{\nu}}$
$\rightarrow$ accuracies of $\approx 0.1\%$ can be achieved

however:

$\rightarrow$ increased statistics needed

$\rightarrow$ $\nu/\bar{\nu}$ must be distinguished

NuTeV experiment at Fermilab measures R^- to determine $\sin^2\theta_W$ or m_W (equivalent for on-shell definition of $\sin^2\theta_W$):



$\rightarrow$ NuTeV result deviates by 2.9σ!

suggested explanations/related literature
(controversial and incomplete!)

- strange PDF asymmetry $s - \bar{s} \neq 0$
  Kulagin; McFarland, Moch; Davidson et al.; Kretzer et al.

- isospin violation $u_p \neq d_n$
  Kulagin; McFarland, Moch; Davidson et al.; Kretzer et al.

- nuclear effects: Fermi motion, shadowing, binding effects
  Kulagin; McFarland, Moch

- target non-isoscalarity (neutron excess)
  Kulagin; McFarland, Moch; Davidson et al.

- reduced $\bar{\nu}_\mu Z \nu_\mu$ coupling
  Loinaz et al.
  - extra dimensions
    de Gouvea et al.
  - fermion admixture to ν_μ
    Babu, Pati; Davidson et al.
  - Z' boson admixture to SM Z
    Davidson et al.
- non-mixing Z'
  Davidson et al.
- $\nu_e - \nu_s$ oscillations
  Giunti, Laveder
- effective dimension 6 operators
  Davidson et al.
- leptoquarks
  Davidson et al.
- . . . ?

sources of uncertainty:

source	$\delta \sin^2 \theta_W$	δR^ν	$\delta R^{\bar{\nu}}$
total stat.	0.00135	0.00069	0.00159
total exp.	0.00063	0.00044	0.00057
c prod./s sea	0.00047	0.00089	0.00184
charm sea	0.00010	0.00005	0.00004
isovec. contr.	0.00005	0.00004	0.00004
rad. corr.	0.00011	0.00005	0.00006
⋮	⋮	⋮	⋮
total model	0.00064	0.00101	0.00212

(G.P. Zeller et al., **PRL 88** (2002) 091802)

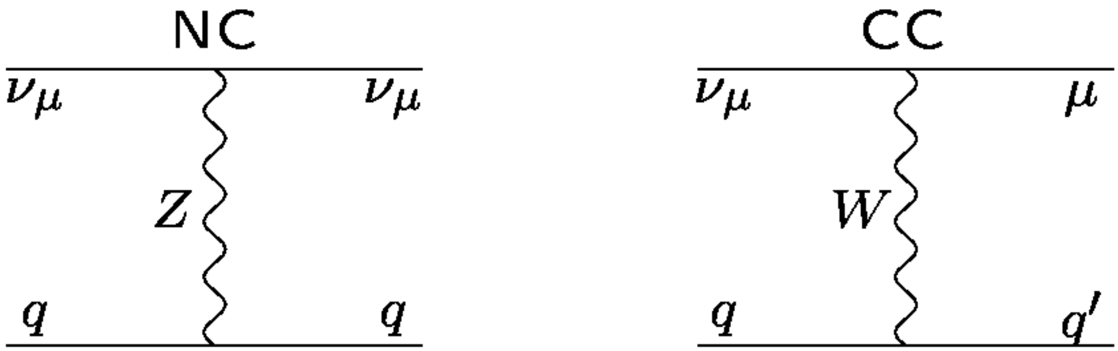
uncertainties in radiative corrections:
- no second independent calculation of EW radiative corrections
- errors determined from variation of input in existing calculation

$\rightarrow$ independent check of EW radiative corrections desirable (necessary)!

existing calculation (basis of exp. analysis):
D. Bardin, V. Dokuchaeva,
JINR-E2-86-260, (1986) (BD)

issues/subtleties in $O(\alpha)$ corrections

calculate $\mathcal{O}(\alpha)$ corrections to:



total cross section:

$$\sigma^{\text{tot}} = \sigma^{\text{born}} + \sigma^{\text{virt}} + \sigma^{\text{real}}$$

with:

$$\begin{aligned} \sigma^{\text{virt}} &= \sigma^{\text{loop}} + \sigma^{\text{ct}} \\ \sigma^{\text{real}} &= \sigma^{\text{hard}} + \sigma^{\text{soft}} + \sigma^{\text{coll}} \end{aligned}$$

amplitudes calculated/evaluated using *FeynArts*, *FeynCalc*, *FormCalc*, *LoopTools*, own routines, crossed results of Dittmaier, Krämer, (Phys. Rev. D **65** (2002) 073007)

all results verified in independent calculations structure of divergences and singularities :

UV divergences:
dimensionally regularized, cancel in $\sigma^{\text{loop}} + \sigma^{\text{ct}}$

IR divergences:
regularized by m_γ, cancel in $\sigma^{\text{virt}} + \sigma^{\text{soft}}$
regularized by E_γ^{max}, cancel in $\sigma^{\text{soft}} + \sigma^{\text{hard}}$

collinear initial-state singularities:
regularized by m_q
cancel in $\sigma^{\text{virt}} + \sigma^{\text{soft}} + \sigma^{\text{hard}} + \sigma^{\text{coll}}$
σ^{coll} is cross section contribution from mass factorization counterterm

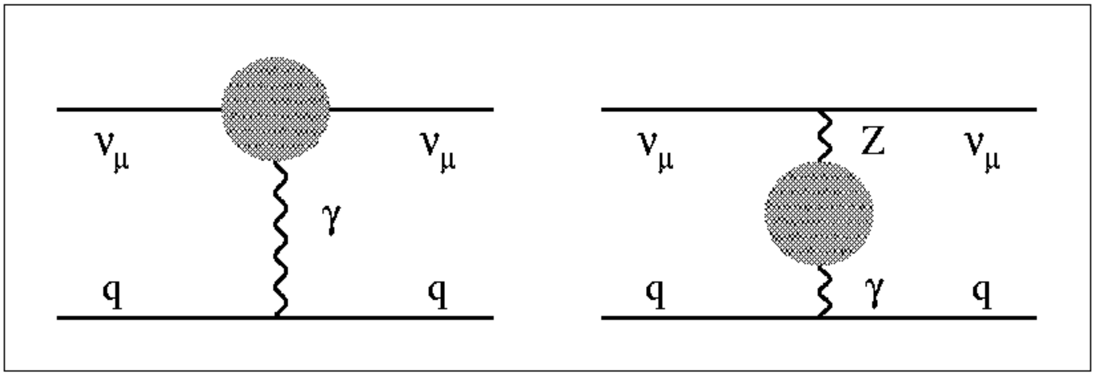
collinear final-state singularities:
regularized by m_q and m_μ
cancel in $\sigma^{\text{virt}} + \sigma^{\text{soft}} + \sigma^{\text{hard}}$
if observable is inclusive enough (KLN)
$\rightarrow$ photon recombination

virtual Corrections

calculation of virtual corrections in
$\alpha(0)$ -scheme: input $\alpha(0), m_Z, m_W$
$\alpha(m_Z)$ -scheme: input $\alpha(m_Z), m_Z, m_W$
G_F -scheme: input G_F, m_Z, m_W
G'_F -scheme: input $G_F, m_Z, \alpha(0)$

calculation of partonic $d\sigma/d\Omega$ in principle straightforward with computer-algebraic tools

caveat I: numerically unstable diagrams with photon in t-channel contribute σ^{crit} to cross section



for $t \to 0$: formfactors F^V and F^Σ for ren. ν-ν-γ-vertex and γ-Z-self-energy must vanish 'fast enough'

expand formfactors in powers of t around 0 !
for $t > t_{\text{cut}} \approx -5\text{MeV}^2$ use first non-vanishing order:

$$\sigma^{\text{crit}} = \frac{\pi\alpha}{s_W c_W} \frac{s^2}{m_Z^2} (I_3 - 2Q_q s_W^2) \left(F^V + F^\Sigma\right)$$

with

$$F^V = \frac{Q_q \alpha^2}{m_W^2 s_W^2} \frac{1}{3} \left(1 + \ln \frac{m_W^2}{m_\mu^2}\right)$$

$$F^\Sigma = \frac{Q_q 2\pi\alpha}{s_W c_W m_Z^2} \left(\frac{\Sigma_T^{\gamma Z}(0) - \Sigma_T^{\gamma Z}(m_Z^2)}{m_Z^2} + \Pi_T^{\gamma Z}(0) \right)$$

Σ_T is transverse part of self-energy, $\Pi_T = \partial\Sigma_T / \partial q^2$
for small t_{cut}: σ^{crit} independent of t_{cut}

caveat **II**: $\Pi_T^{\gamma Z}(0)$ depends on light quark masses

light m_q should be obtained from fit to $\gamma - Z$ mixing via dispersion relations
$\rightarrow$ Jegerlehner, Z. Phys. C **32** (1986) 195
$\rightarrow$ here: small effect

initial-state mass factorisation

inclusion of higher-order QED corrections requires redefinition of parton densities

$$f^0(x) = f(x,Q^2) + \delta f(x,Q^2)$$

with factorisation scale Q^2

explicitly in $\overline{\text{MS}}$-scheme:

$$\delta f_p^q(x,Q^2) = -\int_x^1 \frac{\mathrm{d}z}{z} f_p^q\left(\frac{x}{z},Q^2\right) \frac{\alpha}{2\pi} Q_q^2 \left\{ \ln\left(\frac{Q^2}{m_q^2}\right) \times \left[P_{ff}(z)\right]_+ - \left[P_{ff}(z)(2\ln(1-z)+1)\right]_+ \right\}$$

cross section contribution from mass factorisaton:

$$\sigma^{coll} = \sigma^{born} \otimes \delta f_p^q$$

where $\otimes$ denotes convolution and $P_{ff}(z) = \frac{1+z^2}{1-z}$

however:

physically not complete as NLO QED parton densities not available

but: best we can do

→ code ready for implementation of NLO QED parton densities

→ allows numerical test of cancellation of initial-state mass singularities

effects estimated to be small

(H. Spiesberger, Phys. Rev. **D52**(1995) 4936)
(Kripfganz et al., Z. Phys. **C49** (1991) 501)
(Roth, Weinzierl, hep-ph/0403200)

in existing analysis (BD) :

trivial initial-state mass factorization i.e. no subtraction but regularization with initial state mass $m_q = xm_N$

photon recombination

→ implementation of cut on E_{cal} changes result of numerical analysis

3 ways to implement cut on E_{cal}:

- E_{cal} identified with $E_{q'}$
  → sensitivity to $\alpha \log(m_{q'}/\mu_f)$
- E_{cal} identified with $E_{q'} + E_\gamma$
  → sensitivity to $\alpha \log(m_\mu/\mu_f)$
- E_{cal} identified with $E_{q'} + E_\gamma$ if $\theta_{q'\gamma} < 5^\circ$
  → photon recombination

the quantities δR^ν and $\delta \sin^2\theta_W$

recall Llewellyn-Smith relation:

$$R^\nu = \frac{\sigma^\nu_{NC}}{\sigma^\nu_{CC}} \approx \frac{1}{2} - \sin^2\theta_W + \frac{20}{27}\sin^4\theta_W$$

small variation (write s_W^2 for $\sin^2\theta_W$):

$$\delta s_W^2 \approx \frac{\frac{1}{2} - s_W^2 + \frac{20}{27}s_W^4}{-1 + \frac{40}{27}s_W^2}\left(\frac{\delta\sigma^\nu_{NC}}{\sigma^\nu_{NC}} - \frac{\delta\sigma^\nu_{CC}}{\sigma^\nu_{CC}}\right)$$

definition in the literature (BD) :

$$\Delta s_W^2 = \frac{\frac{1}{2} - s_W^2 + \frac{20}{27}s_W^4}{1 - \frac{40}{27}s_W^2}(\delta R^\nu_{NC} + \delta R^\nu_{CC})$$

with

$$\delta R^\nu_{NC} = \frac{\delta\sigma^\nu_{NC}}{\sigma^\nu_{NC}} \quad \delta R^\nu_{CC} = -\frac{\delta\sigma^\nu_{CC}}{\sigma^\nu_{CC}}$$

observe sign difference: gives correction to s_W^2 determined from Llewellyn-Smith relation with experimental cross sections as input

numerical results

massless quarks outside closed fermion loops
parton densities: CTEQ4L, iso-averaged
factorization scale: $\mu_F = \sqrt{-(p_{l'} - p_\nu)^2}$
ν-beam energy: $E_\nu^{\text{lab}} = 80$ GeV
final hadronic energy: $E_{p'}^{\text{lab}} > 10$ GeV

input parameter schemes:

name	coupling	charge renormalization
$\alpha(0)$	$\alpha(0)$	δZ_e (Thomson limit)
$\alpha(m_Z)$	$\alpha(m_Z)$	$\delta Z_e \to \delta Z_e - 2\Delta\alpha(m_Z)$
G_F	α_{eff} from G_F	$\delta Z_e \to \delta Z_e - 2\Delta r$

relative numerical error in corrections $< 10^{-3}$

Hadronic energy cut: $E_{q'} > 10$ GeV

IPS	R_0^ν	δR_{NC}^ν	δR_{CC}^ν	$\Delta \sin^2 \theta_W$
$\alpha(0)$	0.31766	0.0582	−0.0758	−0.0082
$\alpha(m_Z)$	0.31766	−0.0639	0.0452	−0.0088
G_F	0.31766	0.0003	−0.0185	−0.0085

Hadronic+photonic energy cut: $E_{q'} + E_\gamma > 10$GeV

IPS	R_0^ν	δR_{NC}^ν	δR_{CC}^ν	$\Delta \sin^2 \theta_W$
$\alpha(0)$	0.31766	0.0589	−0.0842	−0.0118
$\alpha(m_Z)$	0.31766	−0.0632	0.0363	−0.0126
G_F	0.31766	0.0011	−0.0272	−0.0122

Cut after $q' - \gamma$ recombination: $(E_{q'}, E_\gamma) > 10$GeV

IPS	R_0^ν	δR_{NC}^ν	δR_{CC}^ν	$\Delta \sin^2 \theta_W$
$\alpha(0)$	0.3177	0.0586	−0.0770	−0.0086
$\alpha(m_Z)$	0.3177	−0.0635	0.0439	−0.0092
G_F	0.3177	0.0008	−0.0198	−0.0090

absolute theor. uncertainties in $\Delta \sin^2 \theta_W$:

$\rightarrow$ scheme dependence $\approx 5 \cdot 10^{-4}$

$\rightarrow$ final state energy cut: $\approx 4 \cdot 10^{-3}$

NuTeV: total theor. uncertainty $\approx 5 \cdot 10^{-5}$

for comparison with BD :

numerical input adopted (light m_f not expl. given, taken from Ref. within BD)

$\alpha = 1/137$	$G_F = 11.7\ \text{TeV}^{-2}$	$m_Z = 93.8$ GeV
$m_t = 180$ GeV	$\sin^2\theta_W = 0.227$	$m_H = 100$ GeV

scheme ambiguity in BD : G_F or G'_F ?

$\rightarrow$ G_F vs. G'_F input parameter scheme:

G_F : m_W from $m_Z, \sin^2\theta_W$
$\rightarrow m_W = 82.5$ GeV, $\alpha_{\text{eff}} = 1/123.4$

G'_F : m_W from numerical solution of

$$m_W^2\left(1 - \frac{m_W^2}{m_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_F}\frac{1}{1-\Delta r(m_W, m_Z, m_H, m_t \ldots)}$$

$\rightarrow m_W = 83.8$ GeV, $\alpha_{\text{eff}} = 1/134.4$

main conceptual difference to BD :
$\rightarrow$ $\overline{\text{MS}}$ vs. BD

results of tuned comparison with BD
(energy cut on hadronic final state)

Hadronic energy cut: $E_{q'} > 10\,\text{GeV}$

IPS	FS	R_0^ν	δR^ν_{NC}	δR^ν_{CC}	$\Delta \sin^2\theta_W$
result of BD	BD	–	−0.0021	−0.0223	−0.0114
G_F	$\overline{\text{MS}}$	0.31455	0.0010	−0.0202	−0.0090
G'_F	$\overline{\text{MS}}$	0.33113	−0.0018	−0.0186	−0.0095
G_F	BD	0.31455	−0.0026	−0.0184	−0.0098
G'_F	BD	0.33113	−0.0050	−0.0169	−0.0103

→ absolute dependence on FS $< 8 \cdot 10^{-4}$

→ input parameter scheme dep. $< 5 \cdot 10^{-4}$

→ energy cut dep. $< 4 \cdot 10^{-3}$ (not shown)

→ BD result not reproduced

systematic deviations in $\delta R^\nu_{NC/CC}$?

open questions:

- is our comparison with BD meaningful? (authors contacted)
- effects of m_c in final-state (implemented, not yet checked)
- what changes for $V_{CKM} \neq 1$
- $\delta R^{\nu}_{NC/CC}$ suffer from strange and charm PDF uncertainties
- QED corrections to parton densities; assumed small
- what is total theory error in $\mathcal{O}(\alpha)$ corrections?

conclusions

- issues/subtleties in $\mathcal{O}(\alpha)$ corrections:
  in δR^{ν}_{NC}: $\Pi_T^{\gamma Z}(0)$
  initial state mass factorization
  treatment of final state collinear photons

- absolute uncertainties in $\sin^2\theta_W$:
  input parameter scheme: $\approx 5 \cdot 10^{-4}$
  cut on final state energy: $< 4 \cdot 10^{-3}$
  ($5 \cdot 10^{-3}$ to resolve discrepancy)

- NuTeV for total theory error: $5 \cdot 10^{-5}$
  off by more than an order of magnitude!

- BD results not reproduced — systematic problem?

- still open questions

- NuTeV error should be reanalyzed