Structure of Double Real Radiation at NNLO

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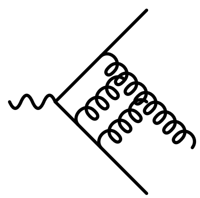
Loop-Fest 2004

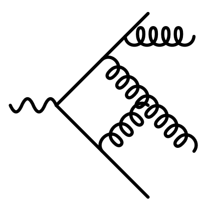
Jet physics at NNLO

Ingredients to NNLO n-jet:

- Two-loop matrix elements

  $|\mathcal{M}|^2_{\text{2-loop},n \text{ partons}}$ 

- One-loop matrix elements

  $|\mathcal{M}|^2_{\text{1-loop},n+1 \text{ partons}}$ 

- One-loop one-particle subtraction terms

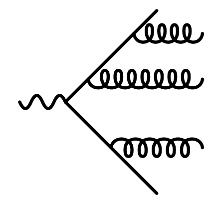
  $\int |\mathcal{M}^{R,1}|^2_{\text{1-loop},n+1 \text{ partons}} \, d\Phi_1$

  D. Kosower, P. Uwer

  Z. Bern et al.

  S. Weinzierl; D. Kosower

- Tree level matrix elements

  $|\mathcal{M}|^2_{\text{tree},n+2 \text{ partons}}$ 

- Tree-level one-particle subtraction terms

  $\int |\mathcal{M}^{R,1}|^2_{\text{tree},n+2 \text{ partons}} \mathrm{d}\Phi_1$

  W. Giele, N. Glover

  S. Catani, M. Seymour

- Tree-level two-particle subtraction terms

  $\int |\mathcal{M}^{R,2}|^2_{\text{tree},n+2 \text{ partons}} \mathrm{d}\Phi_2$

  D. Kosower; S. Weinzierl

  remain to be integrated

Real corrections at NNLO

Double real radiation

$$\mathrm{d}\sigma^{(n+2)} = |\mathcal{M}_{n+2}|^2 \mathrm{d}\Phi_{n+2} \mathcal{F}_n^{(n+2)}(p_1, \ldots, p_{n+2}) \sim \frac{1}{\epsilon^4}$$

with $\mathcal{F}_n^{(n+2)}$ jet definition for combining n+2 partons into n jets

Two approaches:

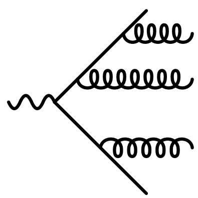
- Direct evaluation
  C. Anastasiou, K. Melnikov, F. Petriello $\longrightarrow$ talk by C. Anastasiou
  - expand $|\mathcal{M}_{n+2}|^2 \mathrm{d}\Phi_{n+2}$ in distributions
  - decompose $\mathrm{d}\Phi_{n+2}$ into sectors corresponding to different singular configurations (Iterated sector decomposition)
    T. Binoth, G. Heinrich $\longrightarrow$ talk by G. Heinrich
  - compute sector integrals numerically
- Evaluation with subtraction term $\longrightarrow$ this talk

both approaches tested on $e^+e^- \to 2j$

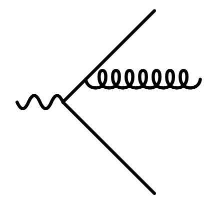
Real corrections at NNLO

Infrared subtraction terms

$n+2$ parton final state forming n jets:



$$\mathrm{d}\Phi_2 \longrightarrow C$$

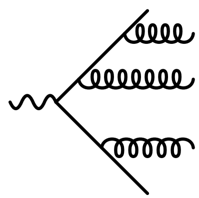


- Singular configurations:
  - triple collinear
  - double single collinear
  - soft/collinear
  - double soft

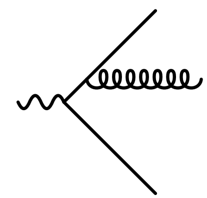
Real corrections at NNLO

Infrared subtraction terms

$n+2$ parton final state forming n jets:



$$\mathrm{d}\Phi_2 \longrightarrow C$$



- Singular configurations:
  - triple collinear
  - double single collinear
  - soft/collinear
  - double soft
- Issue: find subtraction functions which
  - approximate full $n+2$ matrix element in all singular limits
  - are sufficiently simple to be integrated analytically

NLO subtraction

Structure of NLO m-jet cross section

$$\mathrm{d}\sigma_{NLO} = \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^R_{NLO} - \mathrm{d}\sigma^S_{NLO}\right) + \left[\int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^S_{NLO} + \int_{\mathrm{d}\Phi_m} \mathrm{d}\sigma^V_{NLO}\right].$$

Dipole subtraction S. Catani, M. Seymour

$$\mathrm{d}\sigma^R_{NLO} - \mathrm{d}\sigma^S_{NLO} =$$

$$N_{in} \sum_{m+1} \mathrm{d}\Phi_{m+1}(p_1, \ldots, p_{m+1}, Q) \frac{1}{S_{m+1}} \Bigg[|\mathcal{M}_{m+1}(p_1, \ldots p_{m+1})|^2 \, \mathcal{F}_J^{(m+1)}(p_1, \ldots, p_{m+1})$$

$$- \sum_{\text{pairs } i,j} \sum_{k \neq i,j} \mathcal{D}_{ijk} \, |\mathcal{M}_m(p_1, ..\tilde{p}_{ij}, \tilde{p}_k, \ldots, p_{m+1})|^2 \, \mathcal{F}_J^{(m)}(p_1, ..\tilde{p}_{ij}, \tilde{p}_k, \ldots, p_{m+1}) \Bigg]$$

NLO subtraction

Structure of NLO m-jet cross section

$$d\sigma_{NLO} = \int_{d\Phi_{m+1}} \left(d\sigma^R_{NLO} - d\sigma^S_{NLO} \right) + \left[\int_{d\Phi_{m+1}} d\sigma^S_{NLO} + \int_{d\Phi_m} d\sigma^V_{NLO} \right].$$

Dipole subtraction S. Catani, M. Seymour

$$d\sigma^R_{NLO} - d\sigma^S_{NLO} =$$

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$$- \sum_{\text{pairs } i,j} \sum_{k \neq i,j} \mathcal{D}_{ijk} \, |\mathcal{M}_m(p_1, ..\tilde{p}_{ij}, \tilde{p}_k, \ldots, p_{m+1})|^2 \, \mathcal{F}_J^{(m)}(p_1, ..\tilde{p}_{ij}, \tilde{p}_k, \ldots, p_{m+1}) \Bigg]$$

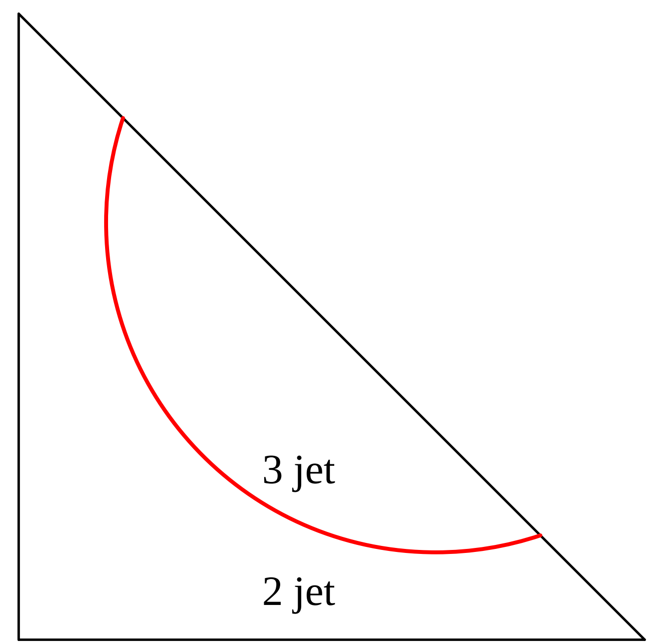
For two jets

$$\begin{aligned} d\sigma^R_{NLO} - d\sigma^S_{NLO} \quad &= \quad N_{in} \, d\Phi_3(p_1, \ldots, p_3, Q) \, |\mathcal{M}_3(p_1, \ldots p_3)|^2 \; \times \\ &\left(\mathcal{F}_2^{(3)}(p_1, p_2, p_3) \; - \frac{1}{2} \mathcal{F}_2^{(2)}(\widetilde{p}_{13}, \tilde{p}_2) - \frac{1}{2} \mathcal{F}_2^{(2)}(\widetilde{p}_{23}, \tilde{p}_1) \right) \end{aligned}$$

NLO subtraction

Two-jet cross section

$$
\begin{aligned}
d\sigma_{\mathrm{NLO}}^{2j} \quad = \quad & d\Phi_3 \, |\mathcal{M}_3(p_1, ...p_3)|^2 \, \left[(\mathcal{F}_2^{(3)} - \mathcal{F}_2^{(2)}) \right] \\
& + d\Phi_2 \, |\mathcal{M}_2|^2 \, \mathcal{F}_2^{(2)} \left(\int_{d\Phi_D} |M_3|^2 + |M_2^{V,1}|^2 \right)
\end{aligned}
$$

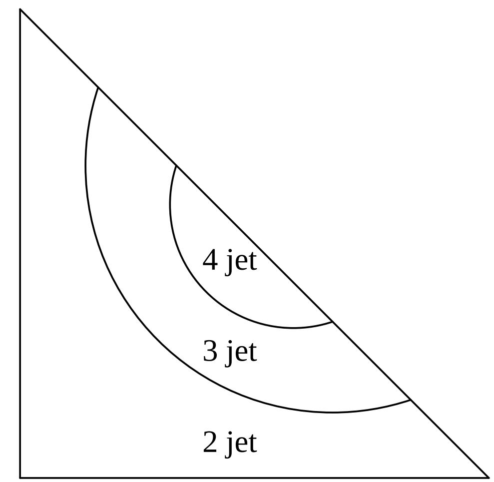


Interpretation: subtraction by subtracting and adding
three parton inclusive contribution $\gamma^* \to q\bar{q}g$

NNLO subtraction

NNLO two-jet cross section

A. Gehrmann-De Ridder, N. Glover, TG



$$
\begin{aligned}
d\sigma_{NNLO} &= \left[d\sigma^{R}_{NNLO} - d\sigma^{S,0}_{NNLO} + d\sigma^{S,1}_{NNLO} \right] \\
&+ \left[d\sigma^{V,1}_{NNLO} - d\sigma^{VS,1}_{NNLO} - d\sigma^{S,1}_{NNLO} \right] \\
&+ \left[d\sigma^{V,2}_{NNLO} + d\sigma^{S,0}_{NNLO} + d\sigma^{VS,1}_{NNLO} \right] \\
&= d\Phi_4 \left[|\mathcal{M}_4|^2 \left(\mathcal{F}_2^{(4)} - \mathcal{F}_2^{(2)} \right) + \sum_{ijk} |\mathcal{M}_3|^2 \, D_{ijk} \mathcal{F}_3^{(3)} \right] \\
&+ d\Phi_3 \left[|\mathcal{M}_3^{V,1}|^2 \left(\mathcal{F}_2^{(3)} - \mathcal{F}_2^{(2)} \right) - \sum_{ijk} |\mathcal{M}_3|^2 \left(\int_{d\Phi_D} D_{ijk} \right) \mathcal{F}_3^{(3)} \right] \\
&+ d\Phi_2 \, |\mathcal{M}_2|^2 \left[|M_2^{V,2}|^2 + \int_{d\Phi_T} |M_4|^2 + \int_{d\Phi_D} |M_3^{V,1}|^2 \right] \mathcal{F}_2^{(2)} \, .
\end{aligned}
$$

where: $P_2 d\Phi_D = d\Phi_3$, $P_2 d\Phi_T = d\Phi_4$, $|\mathcal{M}_2|^2 |M_i|^2 = |\mathcal{M}_i|^2$

Phase space at NNLO

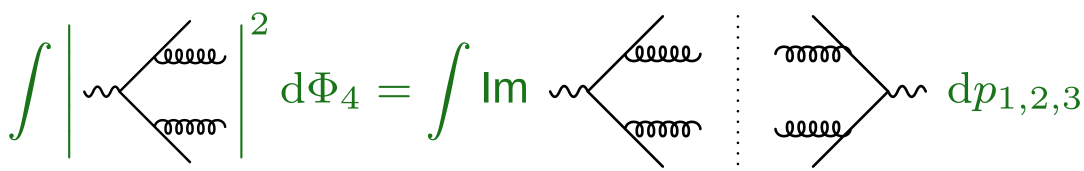
Four-particle phase space integrals $\int \mathrm{d}\Phi_T |\mathcal{M}|^2_4$

A. Gehrmann-De Ridder, G. Heinrich, TG

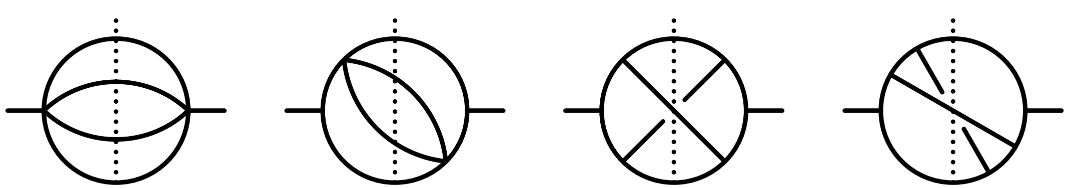
- use (C. Anastasiou, K. Melnikov)

$$\frac{\mathrm{d}^{d-1}p}{2E} = d^d p \delta_+(p^2) = \frac{1}{2\pi i} \mathrm{d}^d p \left(\frac{1}{p^2 + i\epsilon} - \frac{1}{p^2 - i\epsilon} \right)$$

to convert to cuts of three-loop propagator integrals



- use IBP to reduce to master integrals



Phase space at NNLO

Four-particle phase space integrals $\int \mathrm{d}\Phi_T |\mathcal{M}|_4^2$

compute master integrals

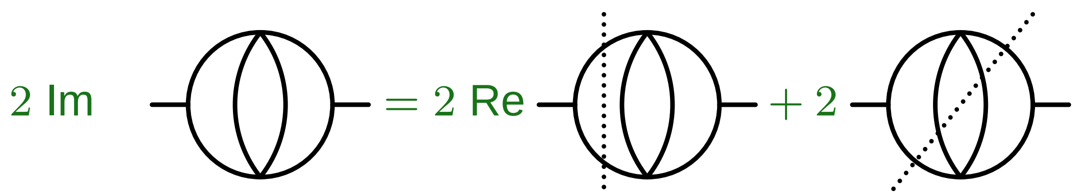
- by direct integration

Phase space at NNLO

Four-particle phase space integrals $\int \mathrm{d}\Phi_T |\mathcal{M}|_4^2$

compute master integrals

- by direct integration
- from unitarity



Phase space at NNLO

Four-particle phase space integrals $\int \mathrm{d}\Phi_T |\mathcal{M}|_4^2$

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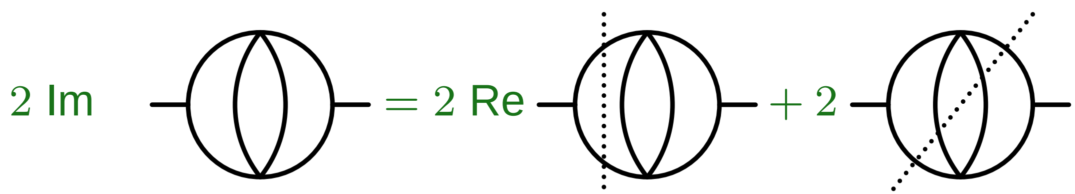

- purely numerically: iterated sector decomposition
  $\longrightarrow$ talk by G. Heinrich

Phase space at NNLO

Four-particle phase space integrals $\int \mathrm{d}\Phi_T |\mathcal{M}|_4^2$

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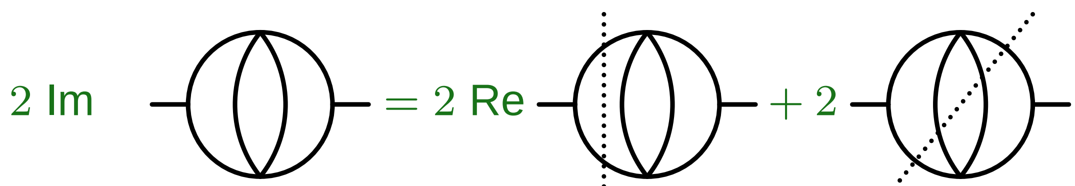

- purely numerically: iterated sector decomposition
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same approach yields $\int \mathrm{d}\Phi_D |\mathcal{M}^{V,1}|_3^2$

Infrared structure at NNLO

Contributions to $\gamma^* \to 2j$ at NNLO

A. Gehrmann-De Ridder, N. Glover, TG

Two partons:

$$\mathcal{T}_{q\bar{q}} = 4\pi\alpha \sum_q e_q^2 \left[\mathcal{T}^{(2)}_{q\bar{q}}(q^2) + \left(\frac{\alpha_s(q^2)}{2\pi}\right) \mathcal{T}^{(4)}_{q\bar{q}}(q^2) + \left(\frac{\alpha_s(q^2)}{2\pi}\right)^2 \mathcal{T}^{(6)}_{q\bar{q}}(q^2) + \mathcal{O}(\alpha_s^3(q^2)) \right]$$

Three partons:

$$\begin{aligned} \langle \mathcal{M}|\mathcal{M}\rangle_{q\bar{q}g} \quad = \quad & 4\pi\alpha \sum_q e_q^2 8\pi^2 \left[\left(\frac{\alpha_s(q^2)}{2\pi}\right) \langle \mathcal{M}^{(0)}|\mathcal{M}^{(0)}\rangle_{q\bar{q}g} \right. \\ & \left. + \left(\frac{\alpha_s(q^2)}{2\pi}\right)^2 \left(\langle \mathcal{M}^{(0)}|\mathcal{M}^{(1)}\rangle_{q\bar{q}g} + \langle \mathcal{M}^{(1)}|\mathcal{M}^{(0)}\rangle_{q\bar{q}g} \right) + \mathcal{O}(\alpha_s^3(q^2)) \right] \end{aligned}$$

Four partons $q\bar{q}q'\bar{q}'$, $q\bar{q}q\bar{q}$, $q\bar{q}gg$:

$$\langle \mathcal{M}|\mathcal{M}\rangle_{q\bar{q}ij} = 4\pi\alpha \sum_q e_q^2 64\pi^4 \left[\left(\frac{\alpha_s(q^2)}{2\pi}\right)^2 \langle \mathcal{M}^{(0)}|\mathcal{M}^{(0)}\rangle_{q\bar{q}ij} + \mathcal{O}(\alpha_s^3(q^2)) \right]$$

Infrared structure at NNLO

Contributions to $\gamma^* \to 2j$ at NNLO

Integrated subtraction terms

$$\mathcal{T}^{(4)}_{q\bar{q}g}(q^2) = 8\pi^2 \int \mathrm{d}\Phi_D \langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle_{q\bar{q}g}$$

$$\mathcal{T}^{(6)}_{q\bar{q}g}(q^2) = 8\pi^2 \int \mathrm{d}\Phi_D \langle \mathcal{M}^{(0)} | \mathcal{M}^{(1)} \rangle_{q\bar{q}g} + \langle \mathcal{M}^{(1)} | \mathcal{M}^{(0)} \rangle_{q\bar{q}g}$$

$$\mathcal{T}^{(6)}_{q\bar{q}ij}(q^2) = 64\pi^4 \int \mathrm{d}\Phi_T \langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle_{q\bar{q}ij}$$

Infrared structure at NNLO

Contributions to $\gamma^* \to 2j$ at NNLO

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\mathcal{T}^{(6)}_{q\bar{q}ij}(q^2) &= 64\pi^4 \int \mathrm{d}\Phi_T \langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle_{q\bar{q}ij}
\end{aligned}
$$

Infrared poles at NLO

$$
\mathcal{P}oles^{(1\times 0)}_{q\bar{q}} = -\mathcal{P}oles^{(0\times 0)}_{q\bar{q}g} = 2\Re \langle \mathcal{M}^{(0)} | \boldsymbol{I}^{(1)}(\epsilon) | \mathcal{M}^{(0)} \rangle
$$

Infrared structure at NNLO

Contributions to $\gamma^* \to 2j$ at NNLO

Integrated subtraction terms

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\end{aligned}$$

Infrared poles at NLO

$$\mathcal{P}oles^{(1\times 0)}_{q\bar{q}} = -\mathcal{P}oles^{(0\times 0)}_{q\bar{q}g} = 2\Re \langle \mathcal{M}^{(0)} | \boldsymbol{I}^{(1)}(\epsilon) | \mathcal{M}^{(0)} \rangle$$

Infrared singularity operator S. Catani

$$\boldsymbol{I}^{(1)}(\epsilon) = -\frac{e^{\epsilon\gamma}}{2\Gamma(1-\epsilon)} \left[\frac{N^2-1}{2N} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + c_1 \right) \left(-\frac{\mu^2}{q^2} \right)^{\epsilon} \right]$$

Infrared structure at NNLO

Infrared poles of virtual two-loop corrections

S. Catani

$$\mathcal{P}oles_{q\bar{q}}^{(2\times 0)} = 2\Re\Bigg[-\frac{1}{2}\langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(\epsilon)\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle - \frac{\beta_0}{\epsilon}\langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle$$
$$+\langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(1)}\rangle$$
$$+e^{-\epsilon\gamma}\frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{\beta_0}{\epsilon}+K\right)\langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(2\epsilon)|\mathcal{M}^{(0)}\rangle$$
$$+\langle\mathcal{M}^{(0)}|\boldsymbol{H}^{(2)}(\epsilon)|\mathcal{M}^{(0)}\rangle\Bigg]$$

$$\mathcal{P}oles_{q\bar{q}}^{(1\times 1)} = \Re\Big[2\langle\mathcal{M}^{(1)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle - \langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)\dagger}(\epsilon)\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle\Big]$$

with $\boldsymbol{H}^{(2)}(\epsilon) \sim 1/\epsilon$

Infrared structure at NNLO

Infrared poles of one-loop subtraction term

$$\mathcal{P}oles^{(1\times0)}_{q\bar{q}g} = 2\mathcal{R}\Bigg[-\langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(1)}\rangle + \frac{\beta_0}{\epsilon}\langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle$$

$$-\langle\mathcal{M}^{(1)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle - \langle\mathcal{M}^{(0)}|\boldsymbol{H}^{(2)}_V(\epsilon)|\mathcal{M}^{(0)}\rangle + \frac{1}{2}\langle\mathcal{M}^{(0)}|\boldsymbol{S}^{(2)}_V(\epsilon)|\mathcal{M}^{(0)}\rangle\Bigg]$$

Infrared structure at NNLO

Infrared poles of one-loop subtraction term

$$\mathcal{P}oles_{q\bar{q}g}^{(1\times 0)} = 2\mathcal{R}\Bigg[-\langle \mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(1)}\rangle + \frac{\beta_0}{\epsilon}\langle \mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle$$

$$-\langle \mathcal{M}^{(1)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle - \langle \mathcal{M}^{(0)}|\boldsymbol{H}_V^{(2)}(\epsilon)|\mathcal{M}^{(0)}\rangle + \frac{1}{2}\langle \mathcal{M}^{(0)}|\boldsymbol{S}_V^{(2)}(\epsilon)|\mathcal{M}^{(0)}\rangle\Bigg]$$

- must fix finite constant $c_1 = 43/4 - \pi^2/3$ in $\boldsymbol{I}^{(1)}(\epsilon)$

Infrared structure at NNLO

Infrared poles of one-loop subtraction term

$$\mathcal{P}oles^{(1\times 0)}_{q\bar{q}g} = 2\mathcal{R}\Bigg[-\langle \mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(1)}\rangle + \frac{\beta_0}{\epsilon}\langle \mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle$$

$$-\langle \mathcal{M}^{(1)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle - \langle \mathcal{M}^{(0)}|\boldsymbol{H}^{(2)}_V(\epsilon)|\mathcal{M}^{(0)}\rangle + \frac{1}{2}\langle \mathcal{M}^{(0)}|\boldsymbol{S}^{(2)}_V(\epsilon)|\mathcal{M}^{(0)}\rangle \Bigg]$$

- must fix finite constant $c_1 = 43/4 - \pi^2/3$ in $\boldsymbol{I}^{(1)}(\epsilon)$
- contribution from one-loop correction to soft gluon current
  S. Catani, M. Grazzini

$$\langle \mathcal{M}^{(0)}|\boldsymbol{S}^{(2)}_V(\epsilon)|\mathcal{M}^{(0)}\rangle =$$

$$-\langle \mathcal{M}^{(0)}|\mathcal{M}^{(0)}\rangle \frac{e^{2\epsilon\gamma}}{1+\epsilon}\left[(N^2-1)\frac{1}{\epsilon^2}\frac{\Gamma^4(1-\epsilon)\Gamma^3(1+\epsilon)}{\Gamma^2(1-2\epsilon)\Gamma(1+2\epsilon)}\right]\int \mathrm{d}\Phi_D \left(\frac{q^2}{s_{13}s_{23}}\right)^{1+\epsilon}$$

Infrared structure at NNLO

Infrared poles of one-loop subtraction term

$$\mathcal{P}oles_{q\bar{q}g}^{(1\times 0)} = 2\mathcal{R}\Bigg[-\langle \mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(1)}\rangle + \frac{\beta_0}{\epsilon}\langle \mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle$$

$$-\langle \mathcal{M}^{(1)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle - \langle \mathcal{M}^{(0)}|\boldsymbol{H}_V^{(2)}(\epsilon)|\mathcal{M}^{(0)}\rangle + \frac{1}{2}\langle \mathcal{M}^{(0)}|\boldsymbol{S}_V^{(2)}(\epsilon)|\mathcal{M}^{(0)}\rangle \Bigg]$$

- must fix finite constant $c_1 = 43/4 - \pi^2/3$ in $\boldsymbol{I}^{(1)}(\epsilon)$
- contribution from one-loop correction to soft gluon current
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$$\langle \mathcal{M}^{(0)}|\boldsymbol{S}_V^{(2)}(\epsilon)|\mathcal{M}^{(0)}\rangle =$$

$$-\langle \mathcal{M}^{(0)}|\mathcal{M}^{(0)}\rangle \frac{e^{2\epsilon\gamma}}{1+\epsilon}\left[(N^2-1)\frac{1}{\epsilon^2}\frac{\Gamma^4(1-\epsilon)\Gamma^3(1+\epsilon)}{\Gamma^2(1-2\epsilon)\Gamma(1+2\epsilon)}\right]\int \mathrm{d}\Phi_D \left(\frac{q^2}{s_{13}s_{23}}\right)^{1+\epsilon}$$

- partial contribution $\boldsymbol{H}_V^{(2)}(\epsilon) \sim 1/\epsilon$ to $\boldsymbol{H}^{(2)}(\epsilon)$

Infrared structure at NNLO

Infrared poles of two-particle subtraction term

$$
\begin{aligned}
\mathcal{P}oles^{(0\times 0)}_{q\bar{q}(ij)} \quad = \quad & \Re\Bigg[\langle \mathcal{M}^{(0)} | \boldsymbol{I}^{(1)}(\epsilon) \boldsymbol{I}^{(1)}(\epsilon) | \mathcal{M}^{(0)} \rangle + \langle \mathcal{M}^{(0)} | \boldsymbol{I}^{(1)\dagger}(\epsilon) \boldsymbol{I}^{(1)}(\epsilon) | \mathcal{M}^{(0)} \rangle \\
& -2 e^{-\epsilon\gamma} \frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)} \left(\frac{\beta_0}{\epsilon} + K \right) \langle \mathcal{M}^{(0)} | \boldsymbol{I}^{(1)}(2\epsilon) | \mathcal{M}^{(0)} \rangle - 2 \langle \mathcal{M}^{(0)} | \boldsymbol{H}^{(2)}_{R}(\epsilon) | \mathcal{M}^{(0)} \rangle \\
& - \langle \mathcal{M}^{(0)} | \boldsymbol{S}^{(2)}_{V}(\epsilon) | \mathcal{M}^{(0)} \rangle \Bigg]
\end{aligned}
$$

Infrared structure at NNLO

Infrared poles of two-particle subtraction term

$$\begin{aligned}\mathcal{P}oles_{q\bar{q}(ij)}^{(0\times 0)} \quad = \quad & \Re\Bigg[\langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(\epsilon)\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle + \langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)\dagger}(\epsilon)\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle \\ & -2e^{-\epsilon\gamma}\frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{\beta_0}{\epsilon}+K\right)\langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(2\epsilon)|\mathcal{M}^{(0)}\rangle - 2\,\langle\mathcal{M}^{(0)}|\boldsymbol{H}_R^{(2)}(\epsilon)|\mathcal{M}^{(0)}\rangle \\ & -\langle\mathcal{M}^{(0)}|\boldsymbol{S}_V^{(2)}(\epsilon)|\mathcal{M}^{(0)}\rangle\Bigg]\end{aligned}$$

- cancel all remaining terms from two-parton and three-parton final states

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- recover two-loop result for R_{had}

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