

Precision Observables in the MSSM: Status and Perspectives

Sven Heinemeyer, CERN

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1. Introduction
2. Precision Observables in the MSSM
3. Status and Perspectives
4. Conclusions

1. Introduction

Q: Which Lagrangian describes the world?

Q': How can one distinguish SM and MSSM?

A: Two possible ways:

- Search for new SUSY particles

new SUSY particles found
↔
SM ruled out

Problem:

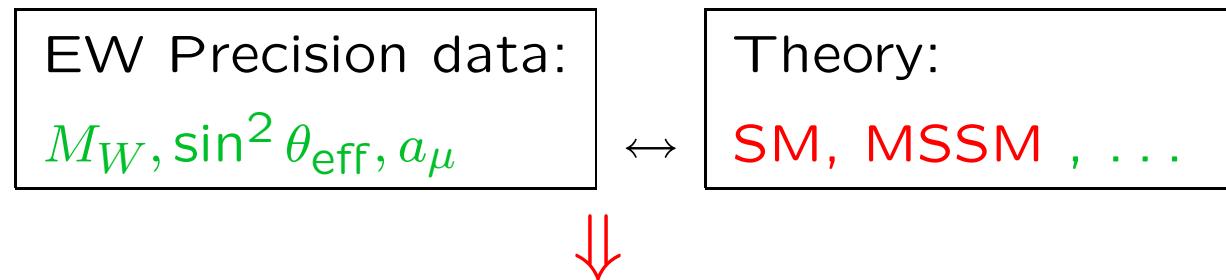
SUSY particles are too heavy for todays colliders, only upper limits of $\mathcal{O}(100 \text{ GeV})$.

- waiting for Tevatron (2005...?)
- waiting for LHC (2007?)

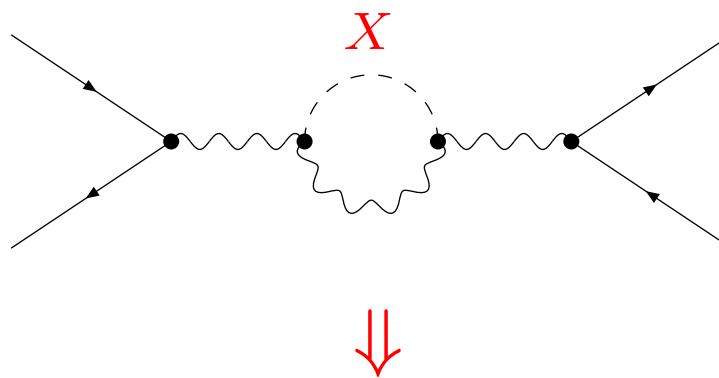
- Search for indirect effects of SUSY via Precision Observables

Precision Observables (POs):

Comparison of electro-weak precision observables with theory:



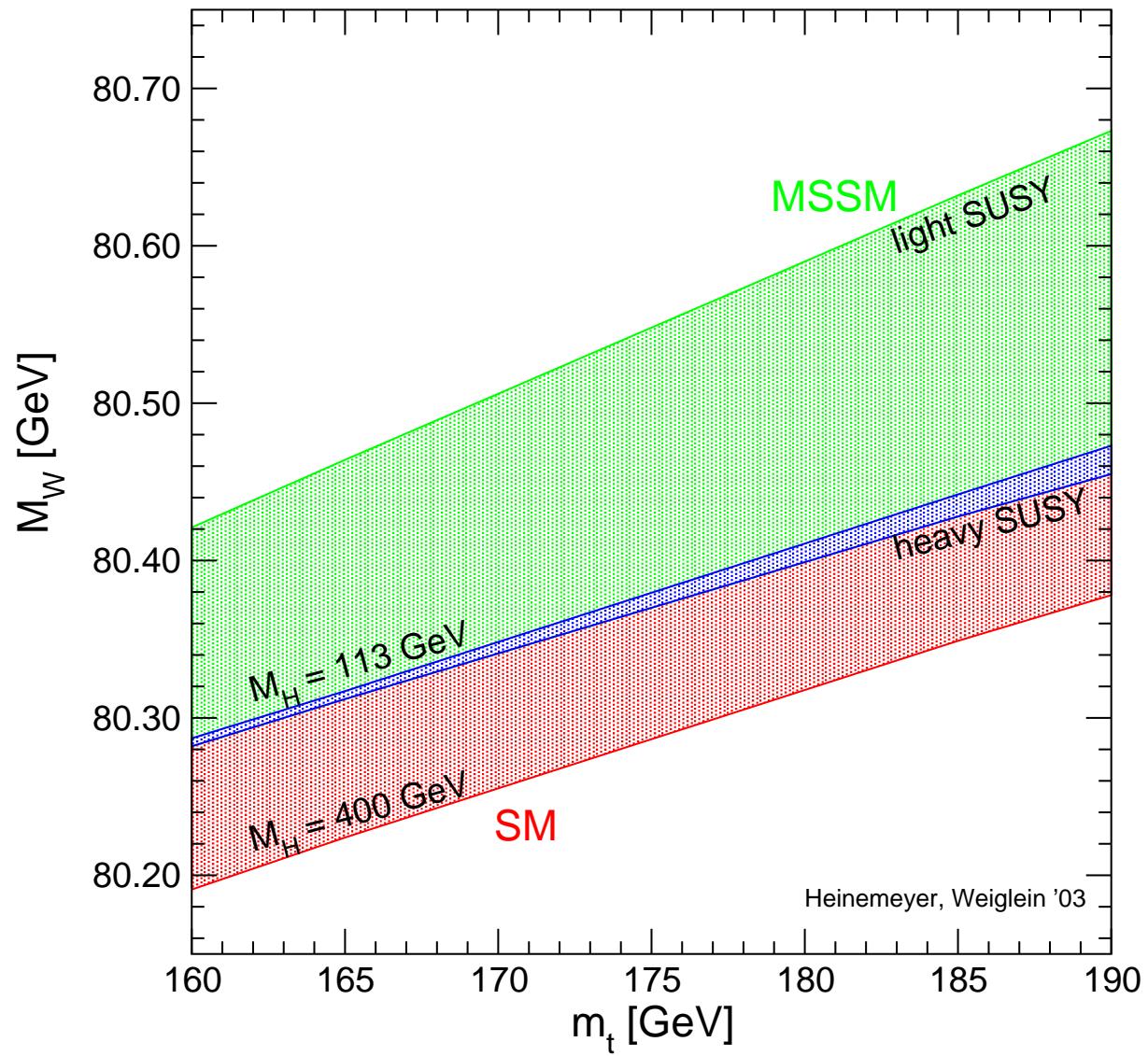
Test of theory at quantum level: **Sensitivity to loop corrections**



Very high accuracy of measurements and theoretical predictions needed

- Which model fits better?
- Does the prediction of a model contradict the experimental data?

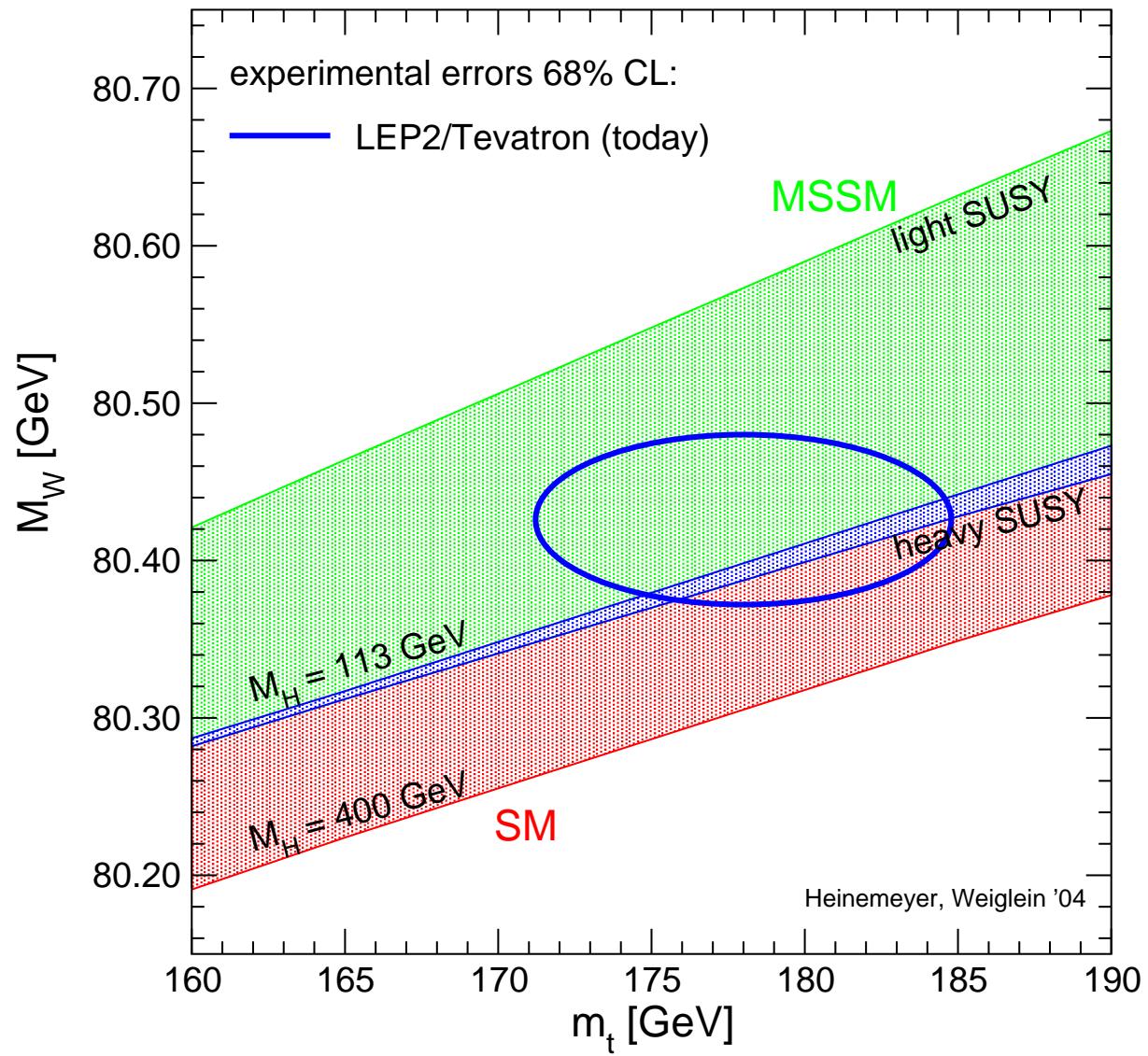
Example: Prediction for M_W in the SM and the MSSM :



MSSM uncertainty:
unknown masses
of SUSY particles

SM uncertainty:
unknown Higgs mass

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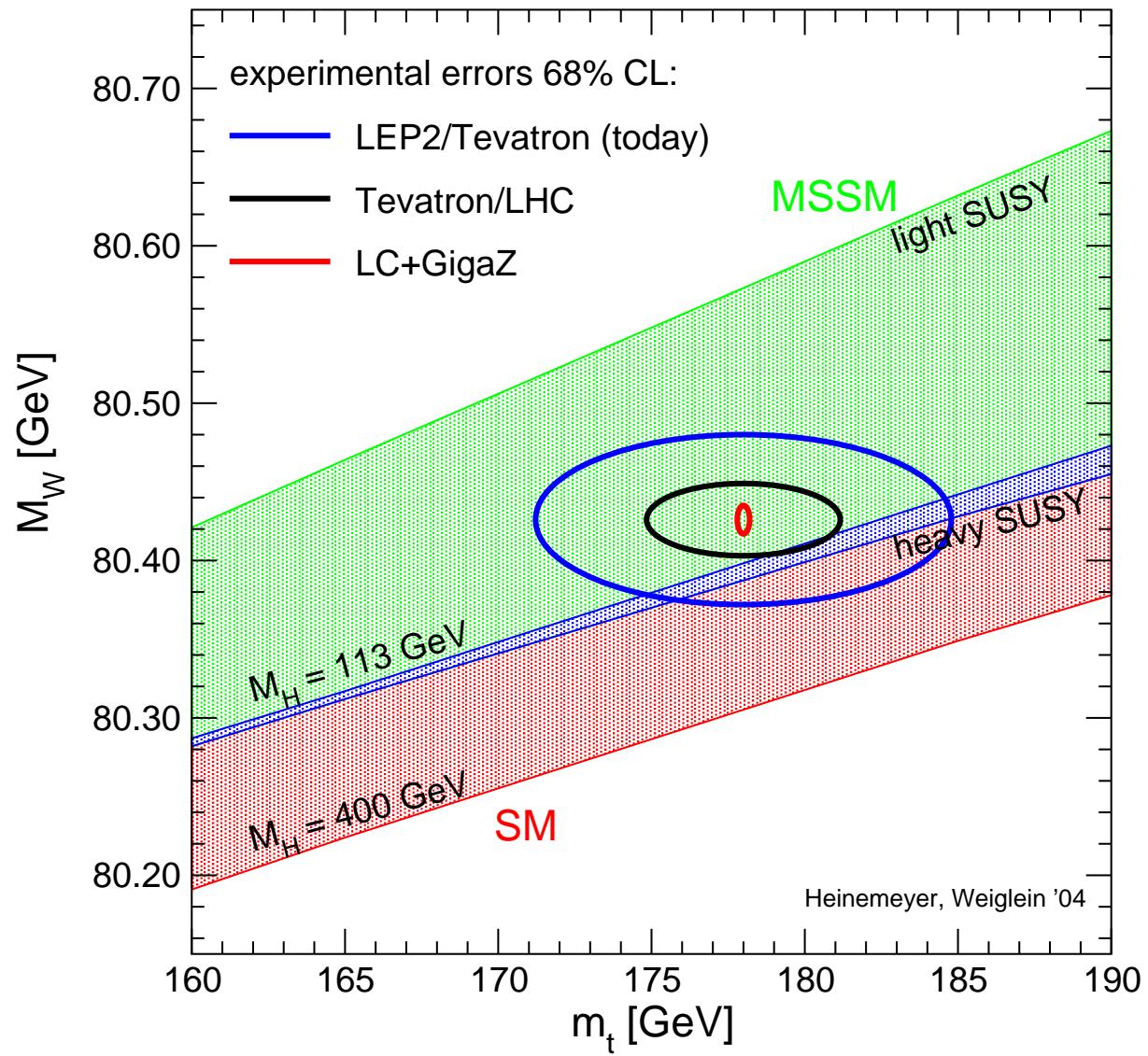
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The Minimal Supersymmetric Standard Model (MSSM)

Superpartners for Standard Model particles

$[u, d, c, s, t, b]_{L,R}$	$[e, \mu, \tau]_{L,R}$	$[\nu_{e,\mu,\tau}]_L$	Spin $\frac{1}{2}$
$[\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}]_{L,R}$	$[\tilde{e}, \tilde{\mu}, \tilde{\tau}]_{L,R}$	$[\tilde{\nu}_{e,\mu,\tau}]_L$	Spin 0
g	$\underbrace{W^\pm, H^\pm}_{\text{}} \quad \underbrace{\gamma, Z, H_1^0, H_2^0}_{\text{}}$		Spin 1 / Spin 0
\tilde{g}	$\tilde{\chi}_{1,2}^\pm$	$\tilde{\chi}_{1,2,3,4}^0$	Spin $\frac{1}{2}$

Enlarged Higgs sector: Two Higgs doublets

physical states: h^0, H^0, A^0, H^\pm

Goldstone bosons: G^0, G^\pm

Input parameters: $\tan \beta = \frac{v_2}{v_1}$, M_A^2 or $M_{H^\pm}^2$

Problem in the MSSM: many scales

Possible extensions:

CMSSM (complex MSSM):

Possibly complex parameters:

- μ : Higgsino mass parameter
 - $A_{t,b,\tau}$: trilinear couplings $\Rightarrow X_{t,b,\tau} = A_{t,b} - \mu^* \{\cot \beta, \tan \beta\}$ complex
 - $M_{1,2}$: gaugino mass parameter (one phase can be eliminated)
 - $m_{\tilde{g}}$: gluino mass
- ⇒ can induce \mathcal{CP} -violating effects

NMFV MSSM (non-minimal flavor violation):

→ Mixing of scalar quark families (beyond CKM)

e.g. sbottom/sstrange mixing :

$$(\tilde{b}_L, \tilde{b}_R, \tilde{s}_L, \tilde{s}_R) \begin{pmatrix} \tilde{B} & 0 \\ 0 & \tilde{S} \end{pmatrix} \begin{pmatrix} \tilde{b}_L \\ \tilde{b}_R \\ \tilde{s}_L \\ \tilde{s}_R \end{pmatrix} \xrightarrow{\quad} (\tilde{b}_L, \tilde{b}_R, \tilde{s}_L, \tilde{s}_R) \begin{pmatrix} \tilde{B} & \neq 0 \\ \neq 0 & \tilde{S} \end{pmatrix} \begin{pmatrix} \tilde{b}_L \\ \tilde{b}_R \\ \tilde{s}_L \\ \tilde{s}_R \end{pmatrix}$$

NMSSM, . . .

2. Precision Observables in the MSSM

Precision observables: M_W , $\sin^2 \theta_{\text{eff}}$, m_h , $(g - 2)_\mu$, b physics, . . .

- 1.) Theoretical prediction for M_W in terms of M_Z , α , G_μ , Δr :

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi \alpha}{\sqrt{2} G_\mu} \left(\frac{1}{1 - \Delta r} \right)$$

\Updownarrow

loop corrections

- 2.) Effective mixing angle:

$$\sin^2 \theta_{\text{eff}} = \frac{1}{4 |Q_f|} \left(1 - \frac{\text{Re } g_V^f}{\text{Re } g_A^f} \right)$$

Higher order contributions:

$$g_V^f \rightarrow g_V^f + \Delta g_V^f, \quad g_A^f \rightarrow g_A^f + \Delta g_A^f$$

Corrections to M_W , $\sin^2 \theta_{\text{eff}}$

→ can be approximated with the **ρ -parameter**:

ρ measures the relative strength between
neutral current interaction and charged current interaction

$$\rho = \frac{1}{1 - \Delta\rho} \quad \Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2}$$

(leading, process independent terms)

$\Delta\rho$ gives the main contribution to EW observables:

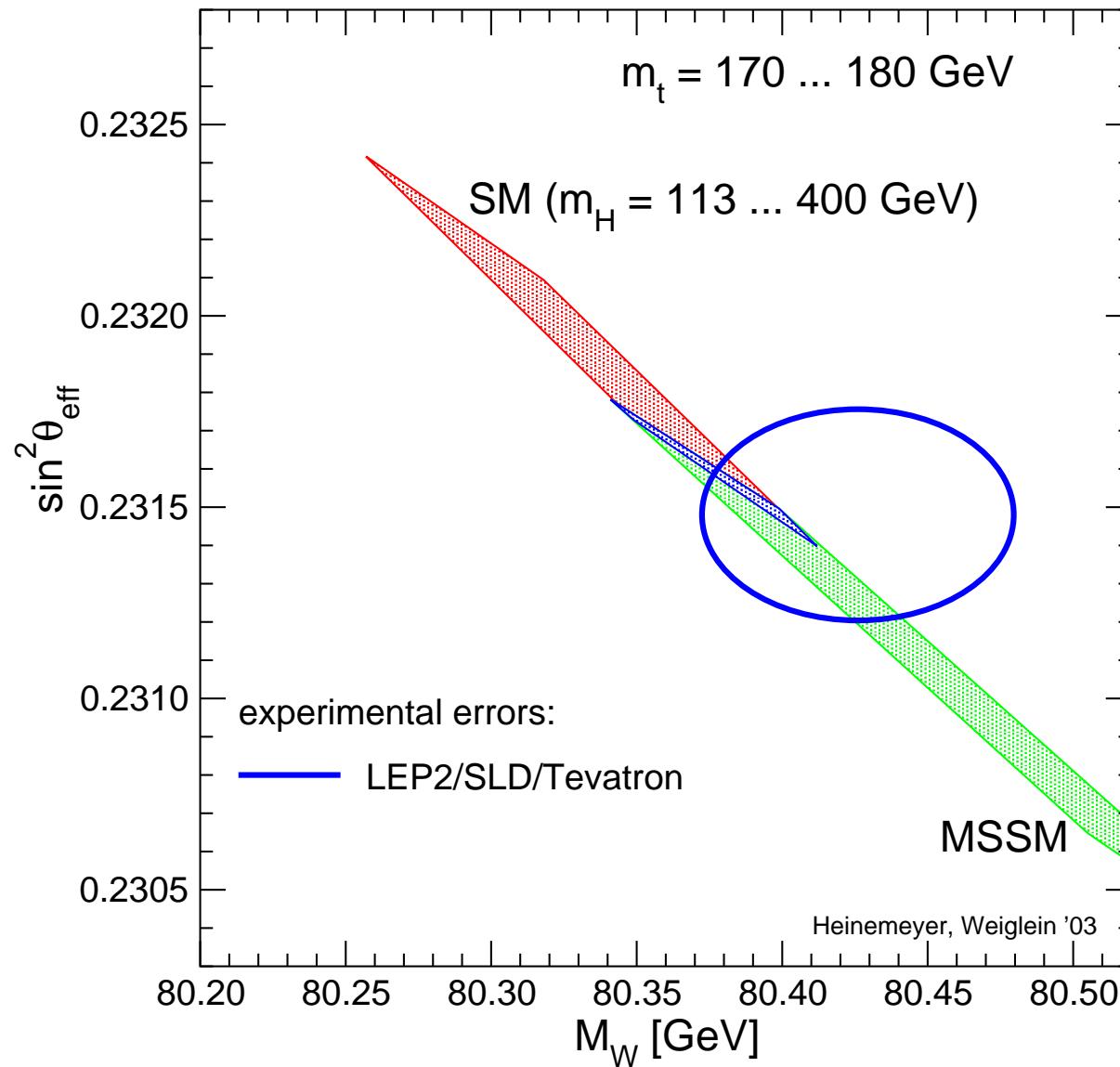
$$\Delta M_W \approx \frac{M_W}{2} \frac{c_W^2}{c_W^2 - s_W^2} \Delta\rho,$$

$$\Delta \sin^2 \theta_W^{\text{eff}} \approx - \frac{c_W^2 s_W^2}{c_W^2 - s_W^2} \Delta\rho$$

⇒ Experimental bound: $\Delta\rho \lesssim 2 \times 10^{-3}$

Example of application:

Prediction for M_W and $\sin^2 \theta_{\text{eff}}$ in the SM and the MSSM :



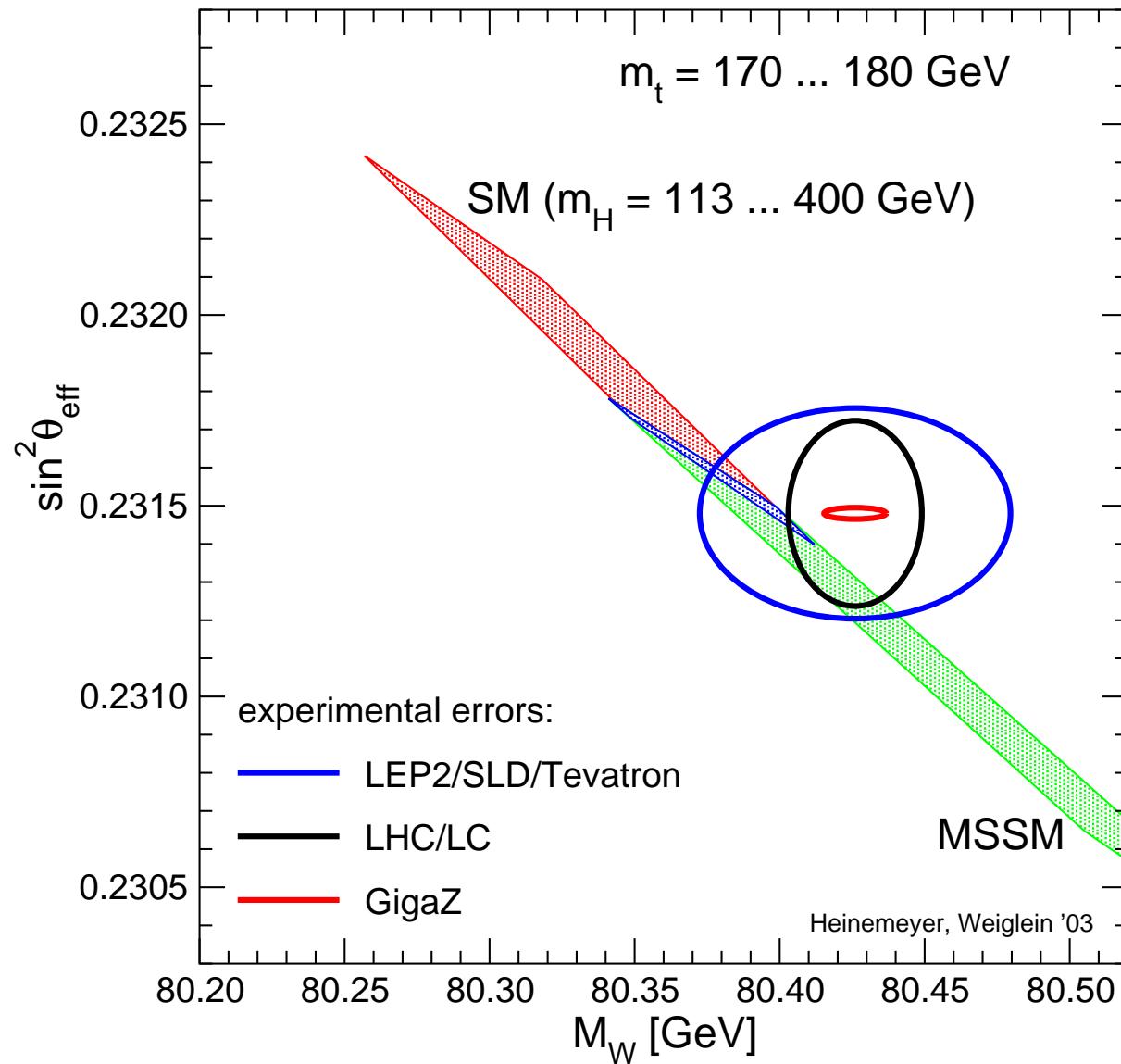
MSSM uncertainty:

unknown masses
of SUSY particles

SM uncertainty:
unknown Higgs mass

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3.) Theoretical prediction of the lightest MSSM Higgs boson mass: m_h

Contrary to the SM: m_h is not a free parameter

MSSM tree-level bound: $m_h < M_Z$, excluded by LEP Higgs searches

Large radiative corrections:

Dominant one-loop corrections: $\sim G_\mu m_t^4 \ln\left(\frac{m_{\tilde{t}1} m_{\tilde{t}2}}{m_t^2}\right)$

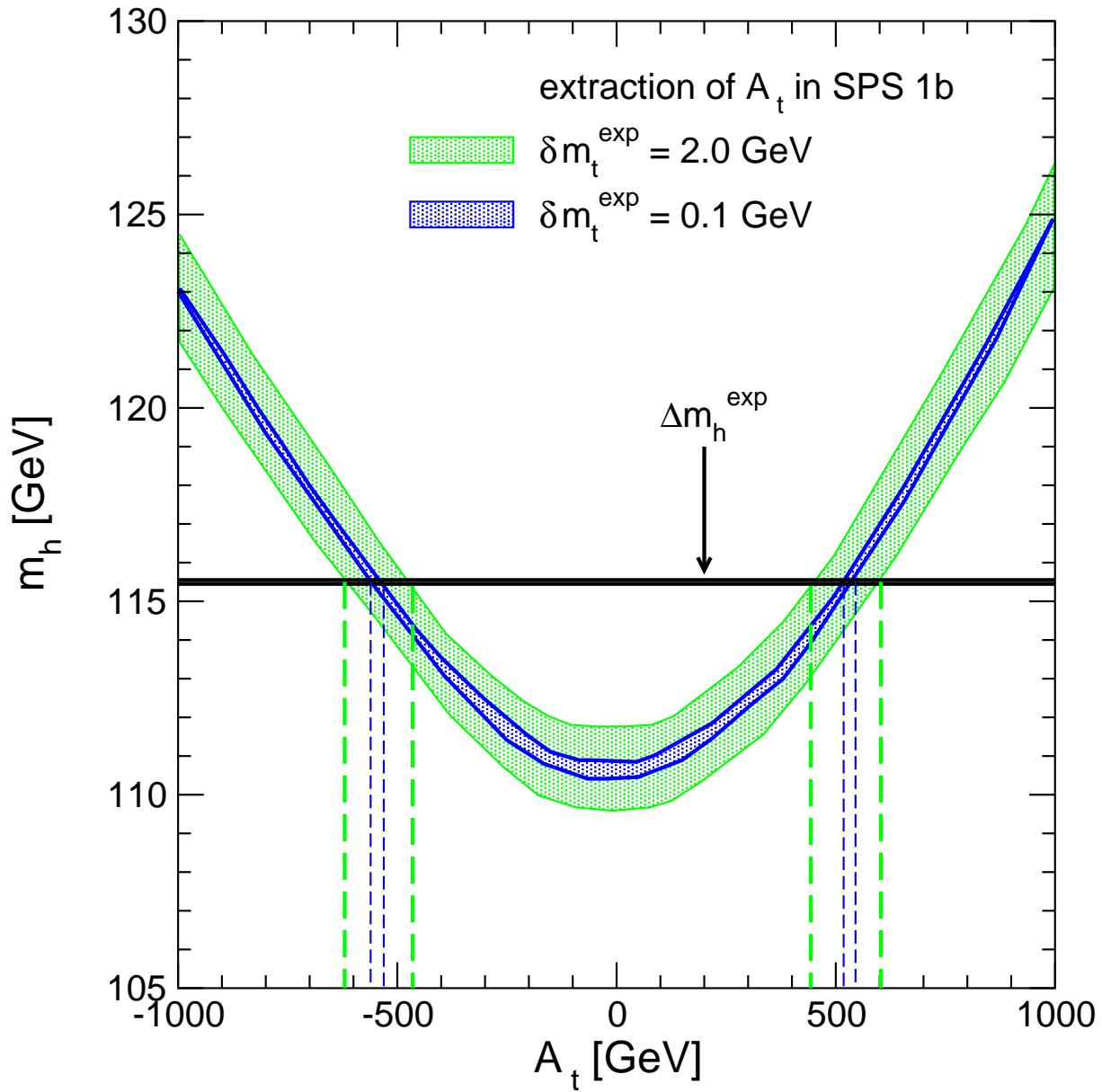
The MSSM Higgs sector is connected to all other sector via loop corrections (especially to the scalar top sector)

Measurement of m_h , Higgs couplings \Rightarrow test of the theory

LHC: $\Delta m_h \approx 0.2$ GeV, LC: $\Delta m_h \approx 0.05$ GeV

$\Rightarrow m_h$ will be (the best?) electroweak precision observable

Example of application: m_h prediction as a function of A_t



SPS1b:

$m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}, m_{\tilde{b}_2}$ known,

A_t unknown

$\tan \beta, M_A$ known,

realistic errors assumed

⇒ extraction of A_t possible

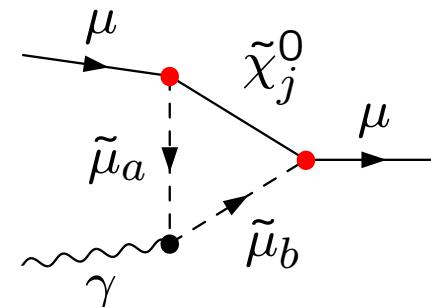
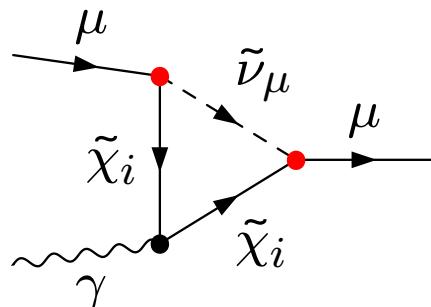
4.) Prediction of the anomalous magnetic moment of the muon: $(g - 2)_\mu$

(→ see talks by Fred Jegerlehner and Kiril Melnikov)

Coupling of muon to magnetic field : $\mu - \mu - \gamma$ coupling

$$\bar{u}(p') \left[\gamma^\mu F_1(q^2) + \frac{i}{2m_\mu} \sigma^{\mu\nu} q_\nu F_2(q^2) \right] u(p) A_\mu \quad F_2(0) = (g - 2)_\mu$$

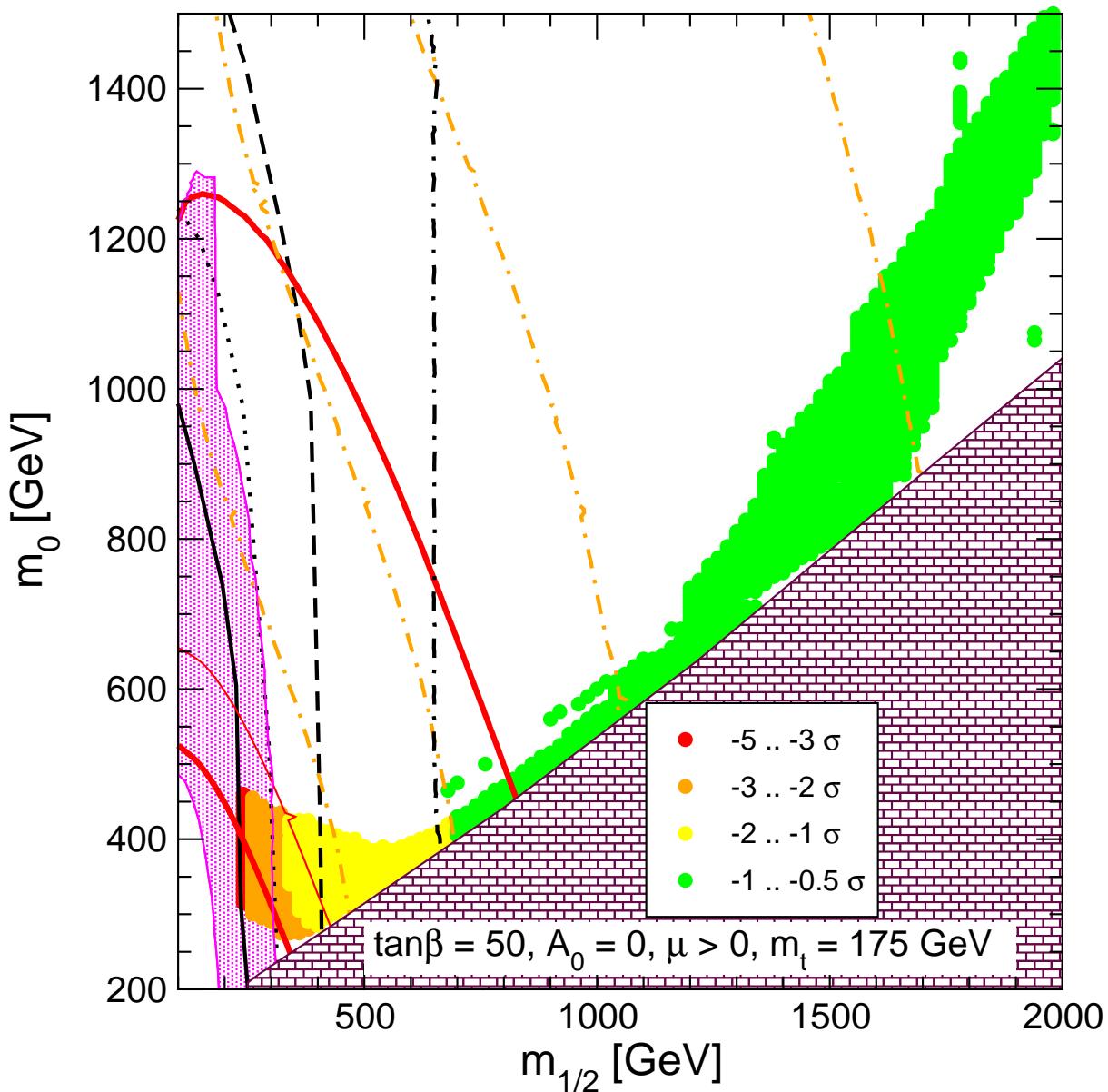
Feynman diagrams for MSSM 1L corrections:



Enhancement factor as compared to SM:

$$\begin{aligned} \mu - \tilde{\chi}_i^\pm - \tilde{\nu}_\mu &: \sim m_\mu \tan \beta \\ \mu - \tilde{\chi}_j^0 - \tilde{\mu}_a &: \sim m_\mu \tan \beta \end{aligned}$$

Example of application: bounds on mSUGRA parameters:



mSUGRA:

$\tan\beta = 50, A_0 = 0, \mu > 0$

$\text{BR}(h \rightarrow WW^*)$, MSSM/SM

[Ellis, S.H., Olive, Weiglein '02]

exp. values from 2002!

e^+e^- : $\delta a_\mu = (33.9 \pm 11.2)$

τ : $\delta a_\mu = (16.7 \pm 10.7)$

3. Status and Perspectives (A)

(focus solely on SUSY, no SM results)

Prediction of M_W and $\sin^2 \theta_{\text{eff}}$:

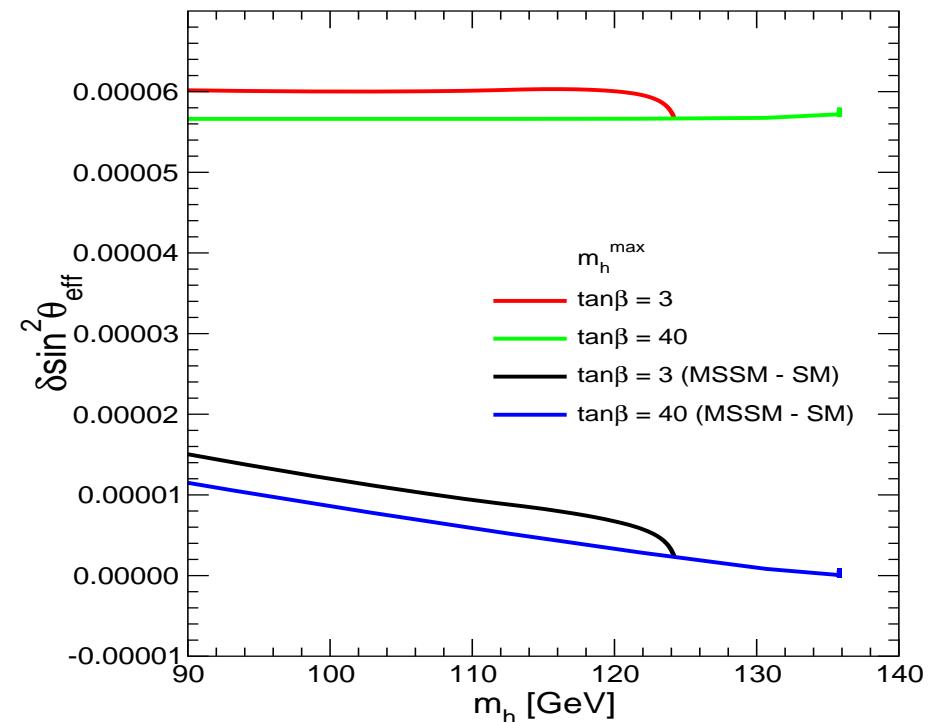
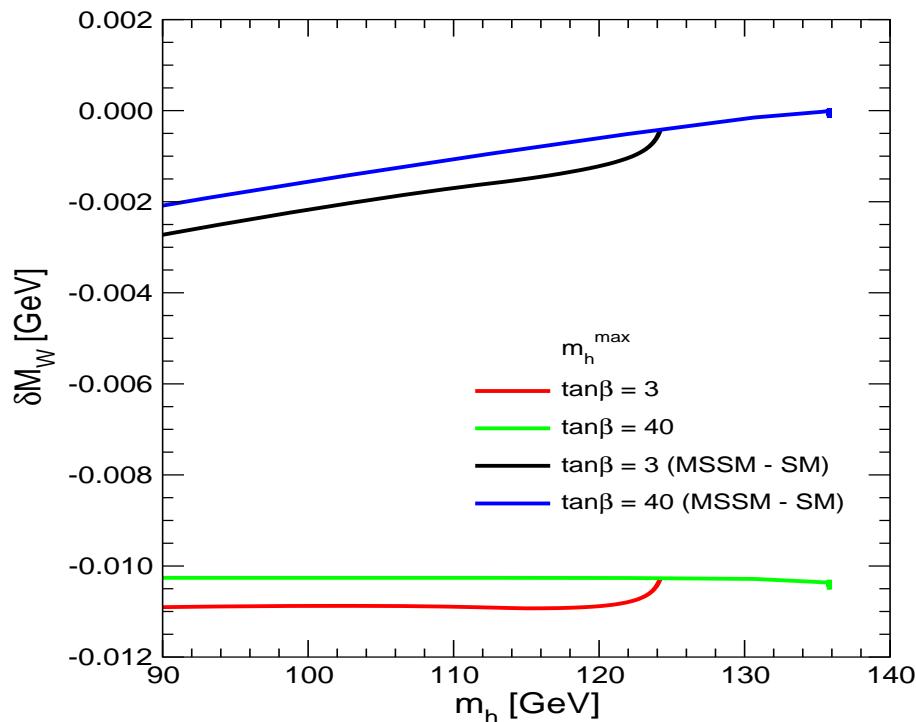
Done:

- MSSM, Δr : full one-loop corrections
[P. Chankowski, A. Dabelstein, W. Hollik, W. Mösle, S. Pokorski, J. Rosiek '94]
[D. Garcia, J. Solà '94]
- MSSM: Z-boson observables, one-loop
[D. Garcia, R. Jiménez, J. Solà '95] [D. Garcia, J. Solà '95]
[A. Dabelstein, W. Hollik, W. Mösle '95] [P. Chankowski, S. Pokorski '96]
- MSSM, $\Delta \rho$: leading $\mathcal{O}(\alpha \alpha_s)$ corrections
[A. Djouadi, P. Gambino, S.H., W. Hollik, C. Jünger, G. Weiglein '97]
[S.H., W. Hollik, G. Weiglein '98]
- MSSM, Δr : leading gluonic $\mathcal{O}(\alpha \alpha_s)$ corr.
[S.H. '98]
- MSSM, $\Delta \rho$: leading $\mathcal{O}(\alpha_t^2, \alpha_t \alpha_b, \alpha_b^2)$ corrections ($M_{\text{SUSY}} \rightarrow \infty$)
[S.H., G. Weiglein '02, '03]

Missing:

- **MSSM, $\Delta\rho$:** leading $\mathcal{O}(\alpha_t^2, \alpha_t\alpha_b, \alpha_b^2)$ corrections ($M_{\text{SUSY}} \neq \infty$)
- **MSSM, Δr , $\sin^2\theta_{\text{eff}}$:** subleading $\mathcal{O}(G_\mu^2 m_t^2)$

Effect of latest corrections:



Corrections to M_W up to -12 MeV, $|\delta M_W^{\text{MSSM-SM}}| \lesssim 3$ MeV

Corrections to $\sin^2\theta_{\text{eff}}$ up to 6×10^{-5} , $\delta \sin^2\theta_{\text{eff}}^{\text{MSSM-SM}} \lesssim 3 \times 10^{-5}$

Current and future errors:

[S.H., G. Weiglein '03]

Current:

$$\delta M_W^{\text{theory}} \approx \pm 10 \text{ MeV},$$

$$\delta m_t : \quad \delta M_W^{\text{para}} \approx \pm 31 \text{ MeV},$$

$$\delta(\Delta\alpha_{\text{had}}) : \quad \delta M_W^{\text{para}} \approx \pm 6.5 \text{ MeV},$$

$$\delta M_W^{\text{exp}} \approx \pm 34 \text{ MeV},$$

$$\delta \sin^2 \theta_{\text{eff}}^{\text{theory}} \approx \pm 12 \times 10^{-5}$$

$$\delta \sin^2 \theta_{\text{eff}}^{\text{para}} \approx \pm 16 \times 10^{-5}$$

$$\delta \sin^2 \theta_{\text{eff}}^{\text{para}} \approx \pm 13 \times 10^{-5}$$

$$\delta \sin^2 \theta_{\text{eff}}^{\text{exp}} \approx \pm 17 \times 10^{-5}$$

Future:

$$\delta M_W^{\text{theory}} \gtrsim \pm 2 \text{ MeV},$$

$$\delta m_t : \quad \delta M_W^{\text{para}} \approx \pm 1 \text{ MeV},$$

$$\delta(\Delta\alpha_{\text{had}}) : \quad \delta M_W^{\text{para}} \approx \pm 1 \text{ MeV},$$

$$[\text{GigaZ}] : \quad \delta M_W^{\text{exp}} \approx \pm 7 \text{ MeV},$$

$$\delta \sin^2 \theta_{\text{eff}}^{\text{theory}} \gtrsim \pm 2 \times 10^{-5}$$

$$\delta \sin^2 \theta_{\text{eff}}^{\text{para}} \approx \pm 0.4 \times 10^{-5}$$

$$\delta \sin^2 \theta_{\text{eff}}^{\text{para}} \approx \pm 1.8 \times 10^{-5}$$

$$\delta \sin^2 \theta_{\text{eff}}^{\text{exp}} \approx \pm 1.3 \times 10^{-5}$$

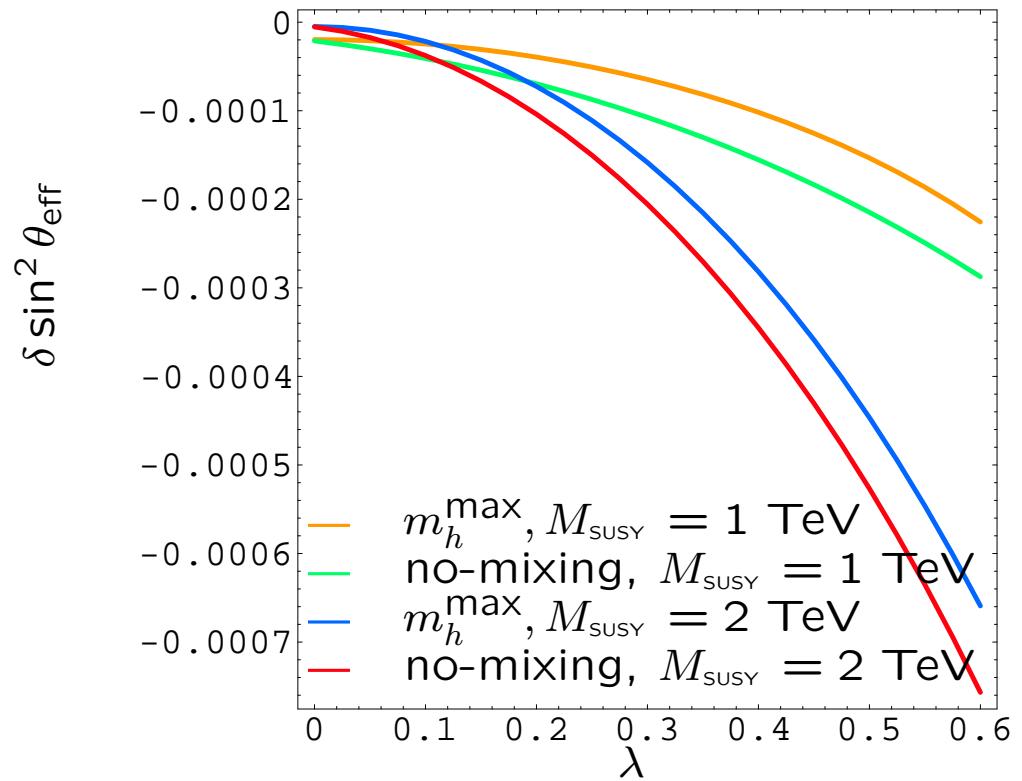
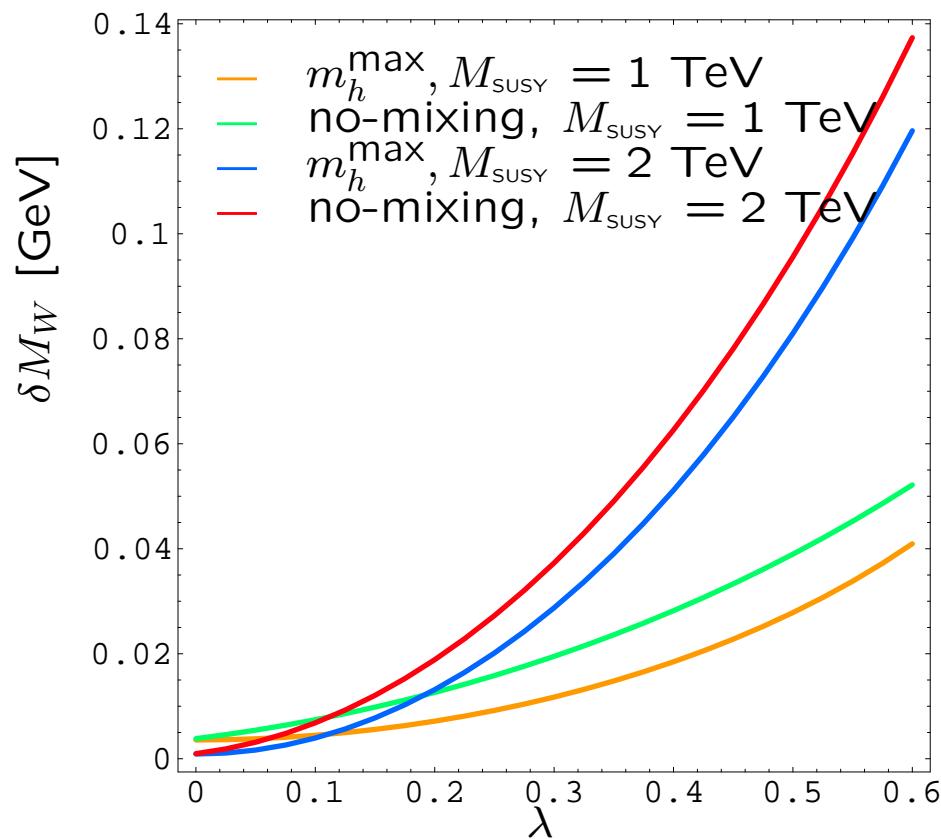
(SUSY parametric errors depend strongly on the scenario)

⇒ M_W under control, $\sin^2 \theta_{\text{eff}}$ barely precise enough

Most recently: Leading corrections in the NMfv MSSM

[S.H., W. Hollik, F. Merz, S. Peñaranda '04]

→ mixing of \tilde{t}/\tilde{c} and of \tilde{b}/\tilde{s} : LL mixing $\sim \lambda \times M_{\text{SUSY}}^2$:



⇒ large NMfv mixing λ can be strongly constrained

Status and Perspectives (B) Prediction of m_h in the r/cMSSM:

More recently done (this millennium):

- subleading (non)-log $\mathcal{O}(\alpha_t^2)$ terms
 $\Rightarrow \delta m_h \lesssim 1 - 4 \text{ GeV}$
[A. Brignole, G. Degrassi, P. Slavich, F. Zwirner '01] [J. Espinosa, R. Zhang '01]
- “ Δm_b ” effects \Rightarrow leading $\mathcal{O}(\alpha_b \alpha_s)$ terms
 $\Rightarrow \delta m_h$ for large $\tan \beta$
[M. Carena, D. Garcia, U. Nierste, C. Wagner '00]
- Remaining $\mathcal{O}(\alpha_b \alpha_s)$ corrections
 $\Rightarrow \delta m_h \lesssim 0 - 3 \text{ GeV}$ for very large $\mu, \tan \beta$
[A. Brignole, G. Degrassi, P. Slavich, F. Zwirner '01]
[S.H., W. Hollik, H. Rzehak, G. Weiglein '04]
- Leading $\mathcal{O}(\alpha_t \alpha_b, \alpha_b^2)$ corrections
 $\Rightarrow \delta m_h \lesssim 0 - 3 \text{ GeV}$ for “extreme” parameters
[A. Dedes, G. Degrassi, P. Slavich '03]
- “Full” 2-loop EP (not for OS calculation)
[S. Martin '02, '03]

More recently done (cont.'):

- evaluation of the Higgs sector of the (c)MSSM:
full 1-loop, $q^2 \neq 0$ (+ leading/subleading 2-loop) $\Rightarrow \delta m_h \lesssim 1 - 7 \text{ GeV}$
[M. Frank, S.H., W. Hollik, G. Weiglein '02]
- full 1-loop corrections for charged Higgs sector
[M. Frank, S.H., W. Hollik, G. Weiglein '02]
- New renormalization ($\overline{\text{MS}}/\text{OS}$)
for 1-loop result $\Rightarrow \delta m_h \approx 1 - 2 \text{ GeV}$
[M. Frank, S.H., W. Hollik, G. Weiglein '02]
- Renormalization at $\mathcal{O}(\alpha_b \alpha_s)$
 $\Rightarrow \delta m_h \lesssim 1 - 2 \text{ GeV}$ for very large $\mu, \tan \beta$
[S.H., W. Hollik, H. Rzehak, G. Weiglein '04]
- evaluation of the Higgs sector of the NMHV MSSM
[S.H., W. Hollik, F. Merz, S. Peñaranda '04]

Missing:

- full 2-loop (incl. full renormalization)
- leading 3-loop (\rightarrow possible, but technical difficulties . . .)

Remaining higher-order uncertainties:

[*G. Degrassi, S.H., W. Hollik, P. Slavich, G. Weiglein '02*]

2-loop momentum independent:

remaining 2-loop, $q^2 = 0$: $\Delta m_h \lesssim 1.5$ GeV

- subleading $\mathcal{O}(\alpha_t \alpha_s)$
- subleading $\mathcal{O}(\alpha_t^2)$
- $\mathcal{O}(\alpha_\tau^2)$
- 2-loop gaugino contributions

$\rightarrow T$

2-loop momentum dependent:

Formally of $\mathcal{O}(\alpha_s \alpha_t m_t^2 m_h^2 / M_W^2)$

(i.e. like the “remaining 2-loop” corrections)

1-loop: $\Delta m_h \lesssim 2$ GeV

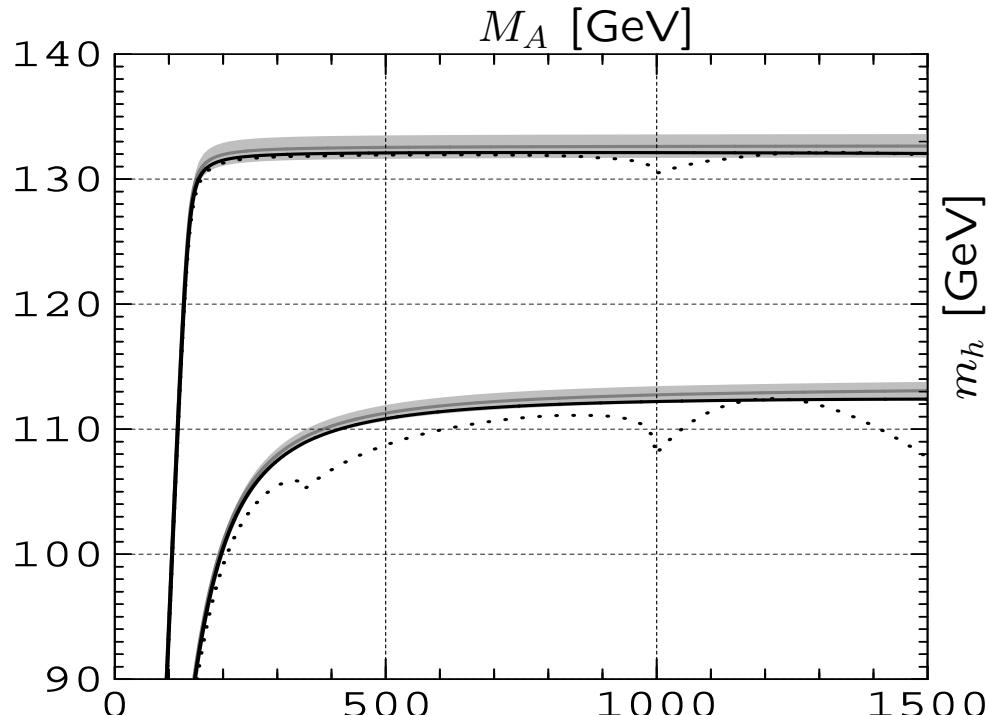
2-loop: $\Delta m_h \lesssim 1$ GeV

Variation of $\overline{\text{MS}}$ renormalization constant:

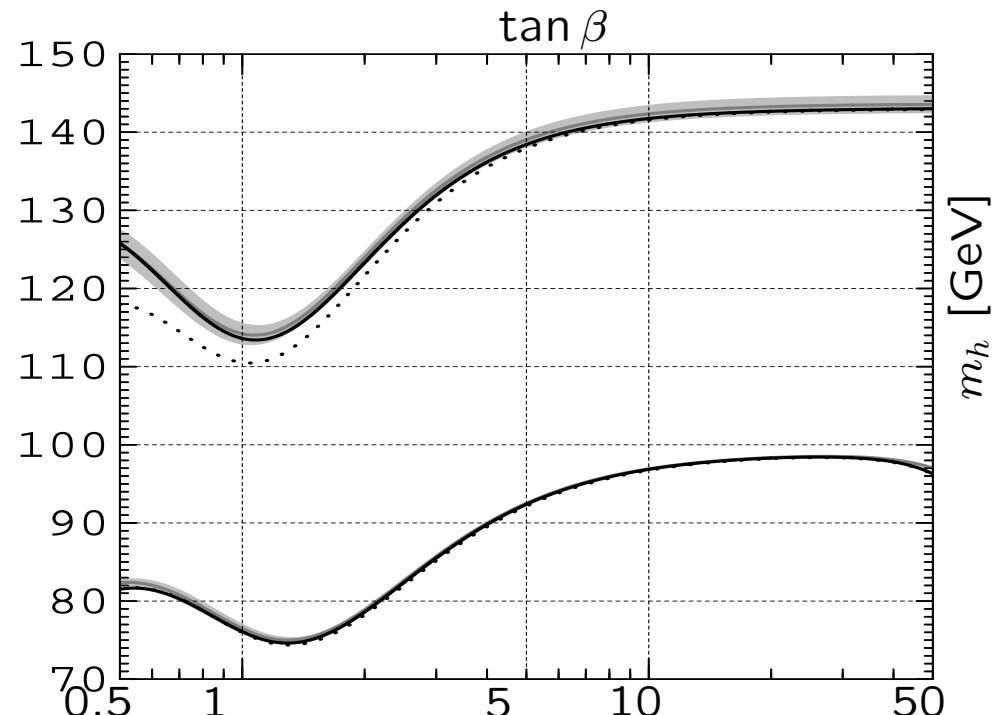
m_h^{\max} scenario, $\mu_{\text{dim}} = 0.5 m_t \dots 2 m_t$

[M. Frank, S.H., W. Hollik, G. Weiglein '02]

$\tan \beta = 2, 20$



$M_A = 100, 500$ GeV



t/\tilde{t} : 3-loop, 4-loop, ...:

1) changing renormalization of m_t at 2-loop:

$\Rightarrow \Delta m_h \lesssim 1.5 \text{ GeV}$ from leading 3-loop corrections

2) explicit formula for simplified case:

$$\Delta m_h^2 = \frac{3\alpha_s}{\pi^3} \log^3 \left(\frac{M_{\text{SUSY}}^2}{m_t^2} \right) \left[\frac{23}{6}\alpha_s^2 - \frac{5}{4}\alpha_s\alpha_t - \frac{33}{64}\alpha_t^2 \right]$$

$(M_A = m_{\tilde{g}} = m_{\tilde{t}_1} = m_{\tilde{t}_2} \equiv M_{\text{SUSY}}, \tan \beta \rightarrow \infty) \Rightarrow \Delta m_h \lesssim 1.5 \text{ GeV}$

3) iterative numerical solution of RGEs: [A. Hoang '97] $\Rightarrow \Delta m_h \lesssim 1.5 \text{ GeV}$

b/\tilde{b} : 3-loop, 4-loop, ...: [S.H., W. Hollik, H. Rzehak, G. Weiglein '04]

changing the renormalization scheme $\Rightarrow \Delta m_h \lesssim 0 - 3 \text{ GeV}$

full intrinsic error: (from unknown higher-order corr.)

today: $\Delta m_h^{\text{intr}} \approx 3 \text{ GeV}$

needed for future: $\Delta m_h^{\text{intr}} \lesssim 0.5 - 0.1 \text{ GeV}$

Parametric uncertainties:

m_t :

today: $\delta m_t^{\text{Tevatron}} \approx 4 \text{ GeV} \Rightarrow \Delta m_h^{m_t} \approx 4 \text{ GeV}$

future: $\delta m_t^{\text{LC}} \approx 100 \text{ MeV} \Rightarrow \Delta m_h^{m_t} \approx 100 \text{ MeV}$

m_b : $\delta m_b \lesssim 100 \text{ MeV} \Rightarrow \text{negligible}$

M_W :

today: $\delta M_W = 34 \text{ MeV} \Rightarrow \Delta m_h^{M_W} \approx 100 \text{ MeV}$

future: $\delta M_W^{\text{GigaZ}} \approx 7 \text{ MeV}$ negligible

α_s :

today: $\delta \alpha_s(M_Z) \approx 0.002 \Rightarrow \Delta m_h^{\alpha_s} \approx 0.3 \text{ GeV}$

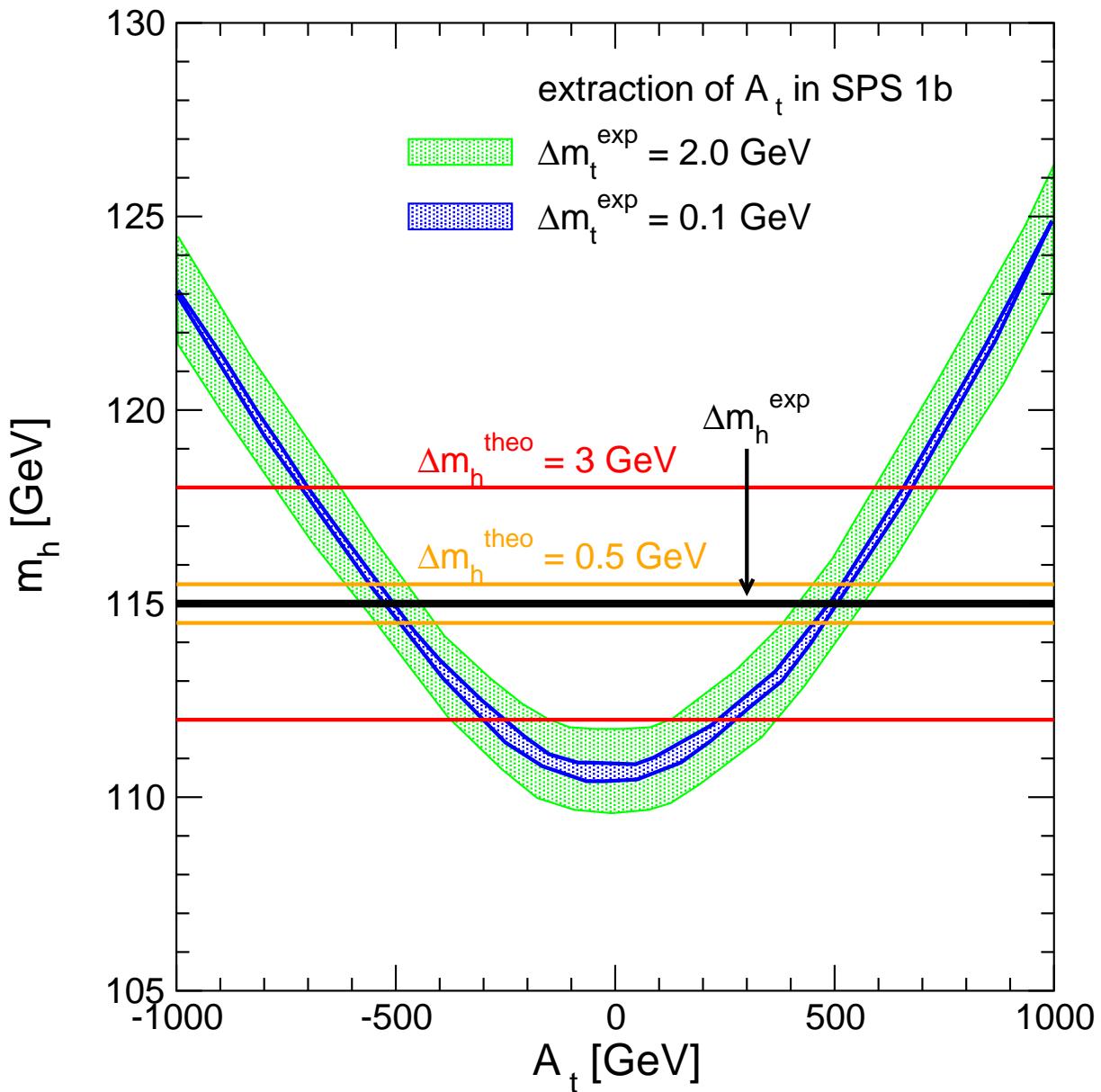
future: $\delta \alpha_s(M_Z) \lesssim 0.001 \Rightarrow \Delta m_h^{\alpha_s} \approx 0.1 - 0.2 \text{ GeV}$

Experimental uncertainties:

$\Delta m_h^{\text{exp,LHC}} \approx 200 \text{ MeV}$

$\Delta m_h^{\text{exp,LC}} \approx 50 \text{ MeV} \Rightarrow \text{can hardly be matched (we do our best!)} \rightarrow T$

Example of effects: m_h prediction as a function of A_t



SPS1b:

$m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}, m_{\tilde{b}_2}$ known,

A_t unknown

$\tan \beta, M_A$ known,

realistic errors assumed

⇒ extraction of A_t possible

⇒ Δm_h^{theo} has to be
under control

Status and Perspectives (C) Prediction of $(g - 2)_\mu$ in the MSSM:

Done:

- full 1-loop corrections
[*T. Moroi '95*]
- leading QED log at $\mathcal{O}(\alpha^2)$
[*G. Degrassi, G. Giudice '98*]
- some leading parts of Barr-Zee diagrams at 2-loop
[*C. Chen, C. Geng '01*] [*A. Arhrib, S. Baek '01*]
→ disagreement by a factor of 4
- all 2-loop diagrams with closed SM fermion/sfermion loop
[*S.H., D. Stöckinger, G. Weiglein '03*]

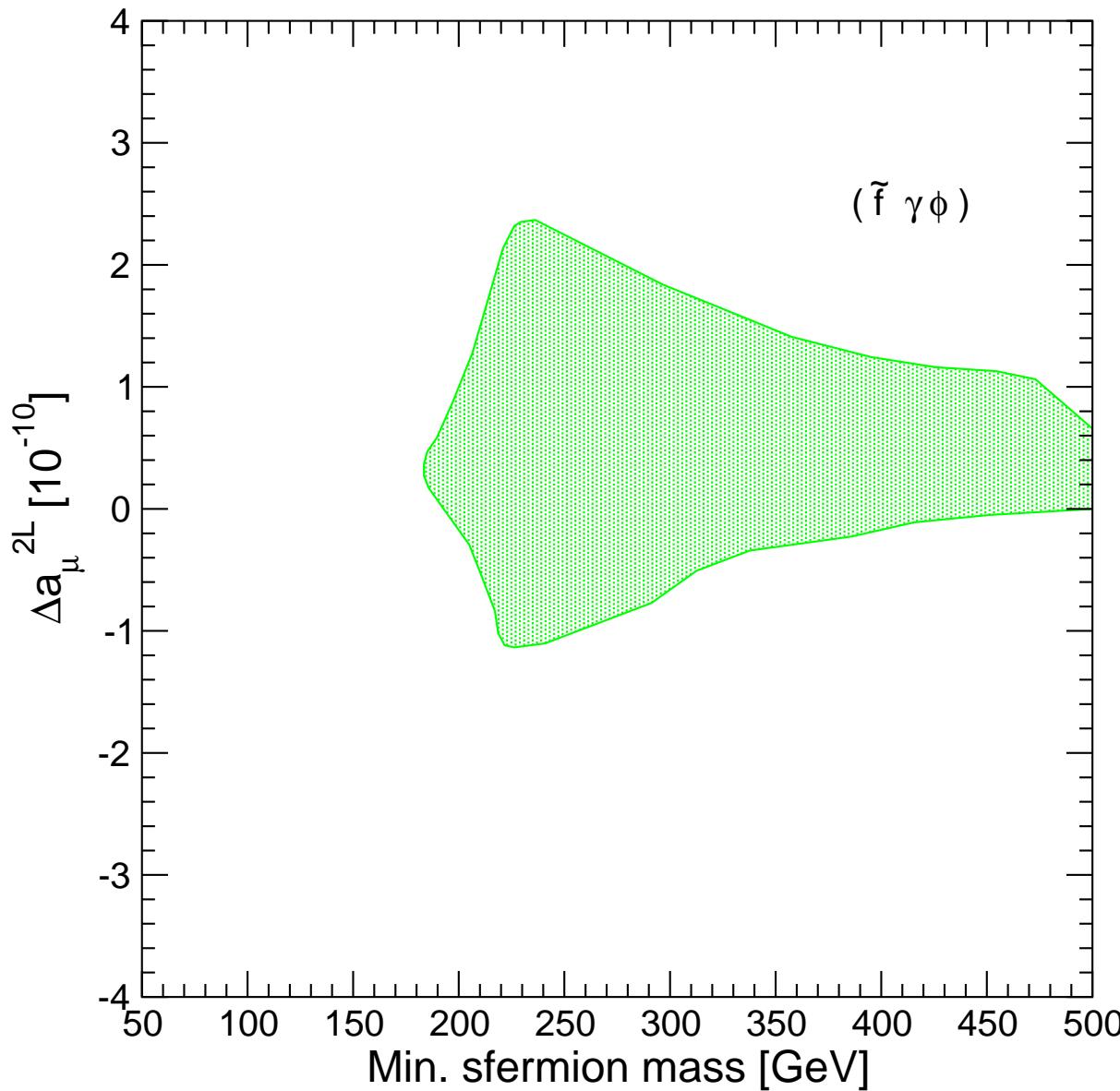
Missing:

- full 2-loop calculation (→ under way)

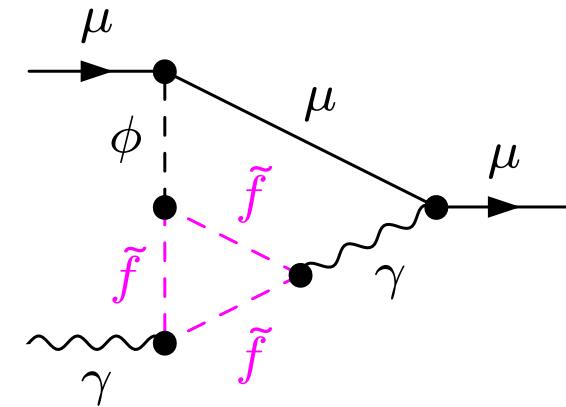
Most problematic: SM evaluation, $\Delta\alpha_{\text{had}}, \dots$

(→ see talks by Fred Jegerlehner and Kiril Melnikov)

Effects of latest corrections (I):

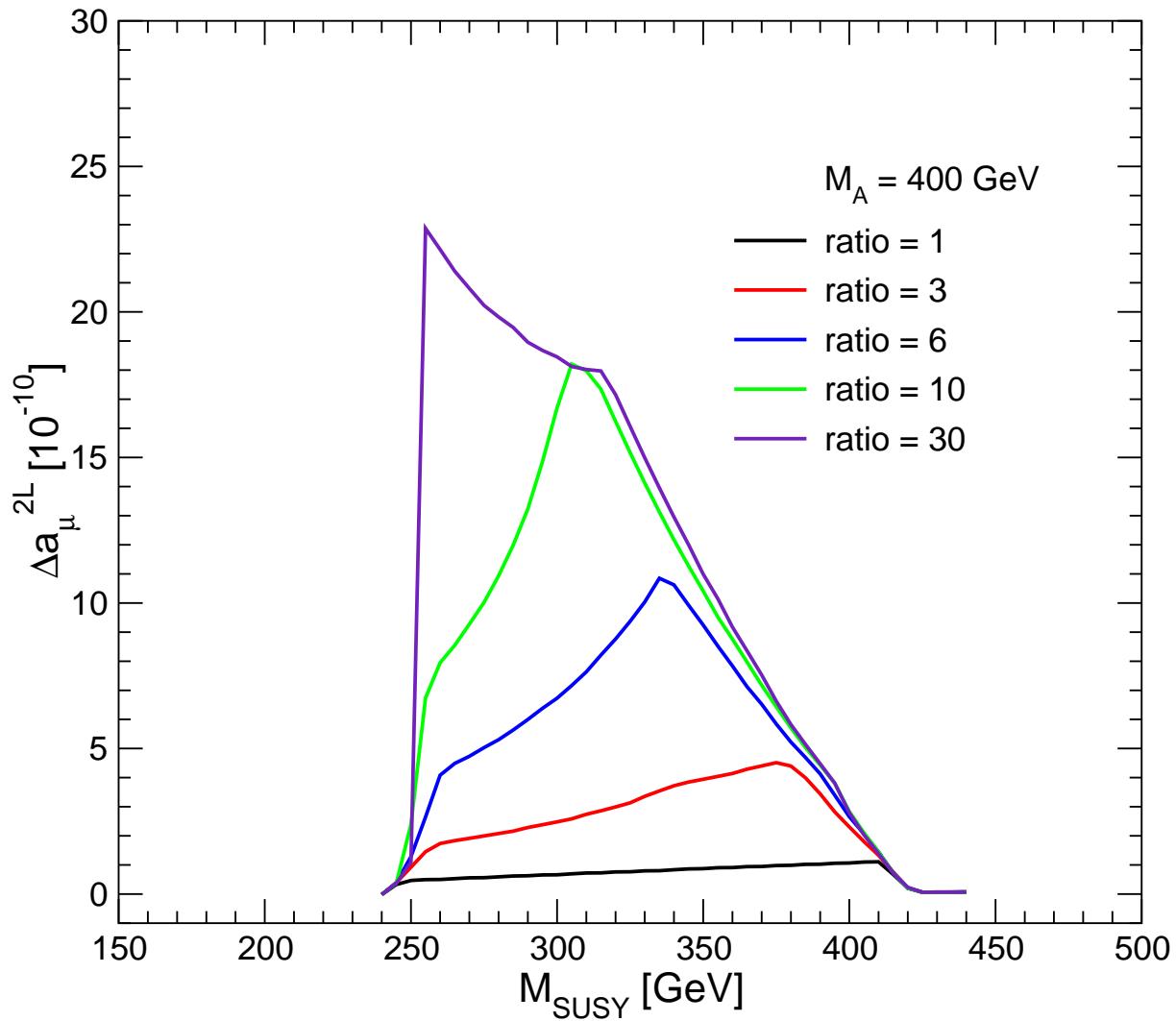


most important subclass:



depending on
 $m_{\tilde{f}}, \mu, A_f, \tan \beta$
significant fraction of
current experimental error
 $\delta a_\mu^{\text{exp}} = \pm 6 \times 10^{-10}$
(Min. sferm. mass =
 $\min\{m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}, m_{\tilde{b}_2}\}$)

Effects of latest corrections (II):



relaxed universality:

$$M_{\text{SUSY}} = M_Q = M_U \neq M_D$$

$$\text{ratio} := M_D/M_U$$

current experimental error:

$$\delta a_\mu^{\text{exp}} = \pm 6 \times 10^{-10}$$

⇒ huge corrections
possible

4. Conclusions

- Precision observables
 - can give valuable information about the “true” Lagrangian
 - can provide bounds on SUSY parameter space
- $M_W, \sin^2 \theta_{\text{eff}}$:
 - today : $\delta M_W^{\text{theory}} \approx \pm 10 \text{ MeV}$, $\delta \sin^2 \theta_{\text{eff}}^{\text{theory}} \approx \pm 12 \times 10^{-5}$
 - future : $\delta M_W^{\text{theory}} \gtrsim \pm 2 \text{ MeV}$, $\delta \sin^2 \theta_{\text{eff}}^{\text{theory}} \gtrsim \pm 2 \times 10^{-5}$
 - [GigaZ] : $\delta M_W^{\text{exp}} \approx \pm 7 \text{ MeV}$, $\delta \sin^2 \theta_{\text{eff}}^{\text{exp}} \approx \pm 1.3 \times 10^{-5}$

$\Rightarrow M_W$ under control, $\sin^2 \theta_{\text{eff}}$ barely precise enough
- m_h :
 - today : $\delta m_h^{\text{theo}} \approx \pm 3 \text{ GeV}$, $\delta m_h^{\text{para}} \gtrsim \pm 5 \text{ GeV}$, $\delta m_h^{\text{exp}} = ???$
 - future : $\delta m_h^{\text{theo}} \lesssim \pm 0.5 \text{ GeV}$, $\delta m_h^{\text{para}} \gtrsim \pm 0.1 \text{ GeV}$, $\delta m_h^{\text{exp}} = 0.05 \text{ GeV}$

\Rightarrow huge effort necessary to exploit physics potential
- $(g-2)_\mu$: will be under control with full 2-loop result

\Rightarrow Most problematic: SM evaluation, $\Delta \alpha_{\text{had}}, \dots$

Experimental situation:

Current/future Experiments

→ provide high accuracy measurements !

Theory situation:

measured observables have to be compared with theoretical predictions
(of your favourite model)

Measured data is only meaningful if it is matched with
theoretical calculations at the same level of accuracy

We have to start **NOW** to achieve necessary accuracy in time

Theoretical calculations should be viewed as an essential part of all
future High Energy Physics programs