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Bounds on M_W, M_t, $\sin^2\theta_{eff}^{lept}$

(A. Ferroglia, G. Ossola, A.S., hep-ph/0401196)

General consensus: the SM gives a very good description of a multitude of phenomena from atomic energies to ≈ 100 GeV, about 10 orders of magnitude!

On the other hand, a fundamental pillar of the theory, the Higgs boson, has not been found and some observables put sharp constraints on its mass.

For instance, as pointed out about 2 years ago: using the 2002 average value

$(M_W)_{exp} = 80.451 \pm 0.033$ GeV, in conjunction with

$(M_t)_{exp} = 174.3 \pm 5.1$ GeV, $\Delta\alpha_h^{(5)} = 0.02761 \pm 0.00036$,

$\alpha_s(M_Z) = 0.118 \pm 0.002$ led to the prediction

$$M_H = 23^{+49}_{-23}\ \text{GeV} \quad ; \quad M_H^{95} = 122\ \text{GeV}.$$

To be compared with the 95% C.L. lower bound

$(M_H)_{L.B.} = 114.4$ GeV from direct searches!

(Ferroglia, Ossola, Passera, A.S. 2002).

Since then, the situation has changed significantly: with the new ave. $(M_W)_{exp} = 80.426 \pm 0.034$ GeV,

$$\rightarrow \quad M_H = 45^{+69}_{-36}\ \text{GeV}, \quad M_H^{95} = 184\ \text{GeV},$$

less restrictive. Further relaxed if combined with $(s^2_{eff})_{exp} = 0.23150 \pm 0.00016$

$$\rightarrow \quad M_H = 112^{+69}_{-45}\ \text{GeV}, \quad M_H^{95} = 243\ \text{GeV},$$

not far from the values derived from the global fit:

$$M_H = 96^{+60}_{-38}\ \text{GeV}, \quad M_H^{95} = 219\ \text{GeV}.$$

The fact remains, however, that the central value of M_H derived from $(M_W)_{exp}$ is well below $(M_H)_{LB.}$!

In fact, it can be argued that the M_W measurement is particularly significant for three reasons: i) it places sharp restrictions on M_H ii) the LEP2 and collider measurements are in excellent agreement and iii) the two-loop evaluation of the relevant electroweak correction, Δr, has been recently completed

(Awramik and Czakon; Omshchenko and Veretin).

The clash between the predicted M_H derived from $(M_W)_{exp}$ and $(M_t)_{exp}$ with $(M_H)_{L.B.}$ suggests that it is possible to obtain sharp bounds on M_W, M_t by comparing the experimental results at various C.L. with the SM theoretical function $M_W = M_W(M_H, M_t)$, taking into account $(M_H)_{L.B.}$.

Specifically, we ask the question: given the present experimental values of M_W and M_t, and assuming the validity of the SM, what are the allowed values for these parameters at various C.L., irrespective of other observables, when $(M_H)_{L.B.}$ is taken into account?

$(M_H)_{L.B.}$ restricts the available parameter space and this leads to bounds on M_W, M_t that are significantly sharper than those derived from the experimental measurements. They are also sharper than analogous bounds derived in global analyses.

This conforms with the idea that, aside from the global analyses, it is also important to separately confront the theory with the observables that are most sensitive to M_H. The point is that striking discrepancies and inconsistencies may be blurred in the global analysis. (Degrassi et al., Chanowitz, Marciano, A.S. ...)

A recent analysis with A. Ferroglia and G. Ossola confronts separately the theory with the couples $(M_W, M_t)_{exp}$ and $(s^2_{eff}, M_t)_{exp}$.

M_W and M_t

Fig. 1 shows the 68%, 80%, 90%, 95% C.L. contours derived from $(M_W)_{exp} = 80.426 \pm 0.034$ GeV and $(M_t)_{exp} = 174.3 \pm 5.1$ GeV, together with the SM theoretical curves $M_W(M_H, M_t)$ for $M_H = 114.4$ (dashed B.C.), 139, 180, 224 GeV and $\Delta\alpha_h^{(5)} = 0.02761$. To simplify the analysis we take the restriction $M_H \geq 114.4$ GeV to be a sharp cutoff. At a given C.L. the allowed region lies within the corresponding ellipse and below the dashed curve, which we call the boundary curve (B.C.)

The theoretical curves employ the simple formula

$$M_W = M_W^0 - d_1 A_1 - d_5 A_1^2 - d_2 A_2 + d_3 A_3 - d_4 A_4$$

$$A_1 = \ln(M_H/100), \quad A_2 = (\Delta\alpha_h^{(5)}/0.02761) - 1,$$

$$A_3 = (M_t/174.3)^2 - 1, \quad A_4 = (\alpha_s(M_Z)/0.118) - 1$$

By adjusting M_W^0 and d_i, this formula reproduces accurately in the range $20\,GeV \leq M_H \leq 300\,GeV$ the results of calculations carried out in different renormalization schemes: on-shell (OS), $\overline{MS}$, and effective (EFF). (Ferroglia, Ossola, Passera, A.S., PRD 65, 113002, 2002). These calculations include two-loop effects enhanced by $(M_t^2/M_W^2)^n$ $(n=1,2)$.

The EFF scheme is a recently proposed framework in which $s_{eff}^2 \equiv \sin^2\theta_{eff}^{lept.}$ plays the role of the basic electroweak mixing parameter. It shares the good convergence properties of the $\overline{MS}$ scheme, but eliminates the residual scale dependence of the $\overline{MS}$ framework in finite orders of PT. (Ferroglia, Ossola, A.S. (2001); A.S. (2003)).

The B.C. barely misses intersecting the 68% C.L. ellipse. To good approximation: the maximum and minimum M_W and M_t in a given allowed region are given by the intersections of the B.C. with the corresponding ellipse. Table 1 shows the allowed M_W and M_t ranges at 80%, 90%, 95% C.L. determined from the intersections with the B.C.. They are significantly reduced. For example, the allowed intervals in the 80% C.L. domain are $\Delta M_W = 36$ MeV, $\Delta M_t = 5.7$ GeV, to be compared with 87 MeV and 13 GeV from the experimental values, a reduction by factors 2.4 and 2.3, respectively. In Fig. 1, the maximum allowed M_H values in the 80%, 90%, 95% C.L. domains are

$M_H \approx 139, 180, 224$ GeV.

In the above, $\Delta\alpha_h^{(5)}$ was fixed at 0.02761.
If varied according to $\Delta\alpha_h^{(5)} = 0.02761 \pm 0.00036$
(Burkhardt and Pietrzyk (2001)), the analysis
is more involved, but we find that the M_W, M_t
ranges in Table 1 are at most affected by minor shifts.

Table 2 presents the M_W, M_t ranges using
the "theory driven" determination $\Delta\alpha_h^{(5)} = 0.02747 \pm 0.00012$
(Troconiz and Yndurain (2002)). In this case, using
the EFF scheme calculation, we find a very narrow
window of compatibility with the 68% C.L. domain.
The allowed ranges are $\Delta M_W = 12\,MeV$, $\Delta M_t = 2\,GeV$,
reduced by factors 5.7 and 5.1 relative to the
intervals derived from $(M_W)_{exp}$, $(M_t)_{exp}$. For the
80%, 90%, 95% C.L. domains, the allowed M_W, M_t
intervals are very similar to those in Table 1.

Tables 3 and 4 repeat the analysis using the SM theoretical formulae in hep-ph/0311148 (Awramik, Czakon, Freitas and Weiglein). They are based on a complete two-loop calculation of Δr in the OS scheme (Awramik, Czakon; Onishchenko, Veretin). The 68% CL domain is again not compatible with the SM curves subject to the $(M_H)_{L.B.}$ restriction (now for both $\Delta\alpha_h^{(5)}$ determinations). In the 80%, 90%, 95% C.L. domains, the allowed M_W, M_t values are similar but somewhat more restrictive. In particular, the minimum M_W values are nearly the same, but the maximum ones are from 10 to 7 MeV smaller. As an example, the allowed 80% C.L. intervals for $\Delta\alpha_h^{(5)} = 0.02761$ are $\Delta M_W = 24$ MeV, $\Delta M_t = 4$ GeV, reduced by factors 3.6 and 3.3 relative to those derived from $(M_W)_{exp}$, $(M_t)_{exp}$.

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Table 5 shows the mid-points of the allowed M_W and M_t ranges in Table 2 (EFF Scheme with $\Delta\alpha_h^{(5)} = 0.02747$) with variations that cover the full intervals. To very good approximation, the mid-points (80.402, 177.7) GeV are independent of the C.L. and are shifted from $(M_W)^c_{exp}$ and $(M_t)^c_{exp}$ by $-0.71\,\sigma_{M_W}$ and $+0.67\,\sigma_{M_t}$. The mid-points in Tables 1, 3, 4 are also independent of the C.L. and given by (80.401, 177.9) GeV, (80.397, 178.3) GeV, (80.398, 178.1) GeV. The largest shifts occur for Table 3 (Awramik, $\Delta\alpha_h^{(5)} = 0.02761$) and are $-0.85\,\sigma_{M_W}$ and $+0.78\,\sigma_{M_t}$.

The theoretical formulae are subject to errors associated with the truncation of PT and the QCD corrections.

What is the effect of such errors on the determination of the allowed ranges?

Illustration: Using the estimated theoretical errors of Ferroglia et al. (2002), we find that the 90% C.L. ranges in Table 1 ($\Delta\alpha_h^5 = 0.02761$) become $\Delta M_W = 60 \pm 5$ MeV and $\Delta M_t = 9.5^{+0.8}_{-0.9}$ GeV. Using the estimated theoretical errors of Awramic et al (2003), we find that the 90% C.L. intervals in Table 3 (also $\Delta\alpha_h^5 = 0.02761$) become $\Delta M_W = 53^{+3}_{-4}$ MeV and $\Delta M_t = 8.8^{+0.6}_{-0.8}$ GeV. Thus, in this 90% C.L. illustration, the effect of the theoretical errors is to change the size of the allowed intervals by < 10%. Note that these modifications are significantly smaller than the experimental errors $\sigma_{M_W} = 15$ MeV, $\sigma_{M_t} = 2$ GeV expected at TEV-LHC.

$\sin^2\theta_{eff}^{lept.}$ and M_t

On the experimental side we consider two possibilities: the world average $s^2_{eff} \equiv \sin^2\theta^{lept}_{eff} = 0.23150 \pm 0.00016$ and the ave. from leptonic observables: $(s^2_{eff})_\ell = 0.23113 \pm 0.00021$. Difference reflects the well-known dichotomy between the leptonic and hadronic determinations.

On the theoretical side, we employ the relevant electroweak correction Δr_{eff} (A.S. (2000); B.A. Kniehl and A.S. (2000); A. Ferroglia, G. Ossola and A.S. (2001)). Unlike Δr, it has not been fully evaluated at the two-loop level. We use the simple formula

$$s^2_{eff} = (s^2_{eff})^\circ + c_1 A_1 + c_5 A_1^2 + c_2 A_2 - c_3 A_3 + c_4 A_4$$

with the constants corresponding to the EFF scheme. (Ferroglia et al., PRD 65, 113002, (2002)).

Fig. 2 shows the 68%, 80%, 90%, 95% C.L. domains derived from $(s^2_{eff})_{exp} = 0.23150 \pm 0.00016$ and $(M_t)_{exp}$, and the SM theoretical curves $s^2_{eff}(M_H, M_t)$ for $M_H = 114.4$ (B.C.), 193, 218, 253, 289 GeV and $\Delta\alpha_h^{(5)} = 0.02761$. At a given C.L., the allowed region lies within the corresponding ellipse and above the B.C.. Since the center of the ellipses lies in the allowed regions, the reduction in parameter space is much smaller than in the (M_W, M_t) case. In particular, the maximum s^2_{eff} and M_t values are not affected by $(M_H)_{L.B.}$ and the minimum values are increased by relatively small amounts.

Fig. 3 shows the situation in the case of the leptonic ave. $(s^2_{eff})_\ell = 0.23113 \pm 0.00021$.

The 68% C.L. domain is clearly forbidden.

The effect of varying $\Delta\alpha_h^{(5)}$ according to $(\Delta\alpha_h^{(5)})^c \pm \sigma_{\Delta\alpha}$ is more pronounced than in the (M_W, M_t) case and, accordingly, the allowed intervals are derived from a χ^2 analysis. They are shown in Tables 6 and 7 for $\Delta\alpha_h^5 = 0.02761 \pm 0.00036$ and $\Delta\alpha_h^{(5)} = 0.02747 \pm 0.00012$. The allowed 80% C.L. intervals in Table 6 are $(\Delta s_{eff}^2)_\ell = 0.00020$ and $\Delta M_t = 5.2$ GeV which are reduced by factors 2.7 and 2.5 relative to the 80% C.L. ranges from the experimental values. To good approximation, the mid-points of the allowed intervals are again independent of the C.L. and given by (0.23129; 177.3 GeV) and (0.23129; 177.4 GeV) in Tab. 6, 7.

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For a later application we also consider the mid-points of the intervals defined by the inter-sections of the B.C. with the C.L. ellipses. They are $(0.23133; 178.1\,GeV)$ for $\Delta\alpha_h^{(5)} = 0.02761$ and $(0.23130; 177.5\,GeV)$ for $\Delta\alpha_h^{(5)} = 0.02747$. The shifts of these B.C. mid-points from the current experimental central values are $(+0.95\,\sigma_{s^2_{eff}}; +0.75\,\sigma_{M_t})$ and $(+0.81\,\sigma_{s^2_{eff}}; +0.63\,\sigma_{M_t})$.

Using the theoretical errors discussed by Ferroglia et al. (2002), we find that the 90% C.L. intervals in Table 7 ($\Delta\alpha_h^{(5)} = 0.02747$) become $(\Delta s^2_{eff})_\ell = 0.00033 \pm 0.00005$ and $\Delta M_t = (10.5^{+1.0}_{-1.9})\,GeV$. Thus, in this example the effect of the theoretical errors is more pronounced than in the (M_W, M_t) case and amount to $\lesssim 15\%$ in $\Delta(s^2_{eff})_\ell$ and $\lesssim 18\%$ in ΔM_t.

BENCHMARK SCENARIO

The allowed M_W, M_t, s^2_{eff} domains may be regarded as theoretically favored in the SM framework when $(M_H)_{L.B.}$ is taken into account, irrespective of other observables. If the SM is correct, $(M_W)^c_{exp}$, $(M_t)^c_{exp}$, and $(s^2_{eff,l})^c_{exp}$ should approach the allowed regions as errors decrease. The precise end-point of this trajectory is, of course, not known. On the other hand, the mid-points of the allowed regions provide natural representative examples. The fact that they are independent of the C.L. makes them particularly attractive benchmarks. We therefore consider, as an illustration of plausible future developments, a hypothetical scenario in which the experimental central points move to the mid-points of the current allowed regions. This requires a shift of -0.71 to $-0.85\,\sigma_{M_W}$ in $(M_W)^c_{exp}$ and $+0.67$ to $0.78\,\sigma_{M_t}$ in $(M_t)^c_{exp}$. A similar shift in $(M_W)^c_{exp}$ occurred recently: $80.451 \to 80.426$ GeV ($-0.76\,\sigma_{M_W}$). Also, the most precise M_W measurement, the LEP2 value 80.412 ± 0.042 GeV, is closer than the ave. to the mid-points. Finally, a preliminary value $M_t = 180.1 \pm 5.4$ GeV from the DØ collaboration suggests that $(M_t)^c_{exp}$ may increase.

Assuming that H remains undiscovered, what would be the M_H prediction in such hypothetical scenario? Assume $\sigma_{M_W} = 15$ MeV and $\sigma_{M_t} = 2$ GeV, projected for TEV-LHC, $\Delta\alpha_h^{(5)} = 0.02761 \pm 0.00036$. We find:

$M_H = 114^{+46}_{-35}$ GeV ; $M_H^{95} = 195$ GeV (Ferroglia et al.)

$M_H = 114^{+47}_{-37}$ GeV ; $M_H^{95} = 198$ GeV (Awramik et al.)

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Including the effect of $(M_H)_{L.B.}$,

$M_H^{95} \rightarrow 214$ GeV (Ferroglia...); $M_H^{95} \rightarrow 218$ GeV (Awramic...)

Similarly, we consider a hypothetical scenario in which the experimental central values of $s^2_{eff;(\ell)}$ and M_t move to the mid-points (0.23133; 178.1 GeV). We assume again $\sigma_{M_t} = 2$ GeV, $\Delta\alpha_h^{(5)} = 0.02761 \pm 0.00036$, and $\sigma_{s^2_{eff;(\ell)}} = 0.00001$, as projected for Giga Z. These inputs lead to

$$M_H = 115^{+37}_{-29} \text{ GeV}; \quad M_H^{95} = 180 \text{ GeV}.$$

Including the $(M_H)_{L.B.}$ effect, $M_H^{95} \rightarrow 196$ GeV.

The central values $M_H = 114, 115$ GeV in these 3 predictions reflect the fact that the mid-points are very close to the $M_H = 114.4$ GeV B.C.

It is interesting to compare these predictions with those we would obtain if the present central values of M_W, M_t, and $s^2_{eff;(l)}$ remain unaltered, while the errors shrink to $\sigma_{M_W} = 15$ MeV, $\sigma_{M_t} = 2$ GeV, $\sigma_{s^2_{eff;(l)}} = 0.00001$.

These inputs would lead to

$M_H = 45^{+25}_{-18}$ GeV ; $M_H^{95} = 90$ GeV (Ferroglia et al.)

$M_H = 36^{+23}_{-17}$ GeV ; $M_H^{95} = 79$ GeV (Awramik et al.)

$M_H = 59^{+21}_{-16}$ GeV ; $M_H^{95} = 96$ GeV

Clearly, these estimates would be in sharp disagreement with $(M_H)_{L.B.}$!

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CONCLUSIONS

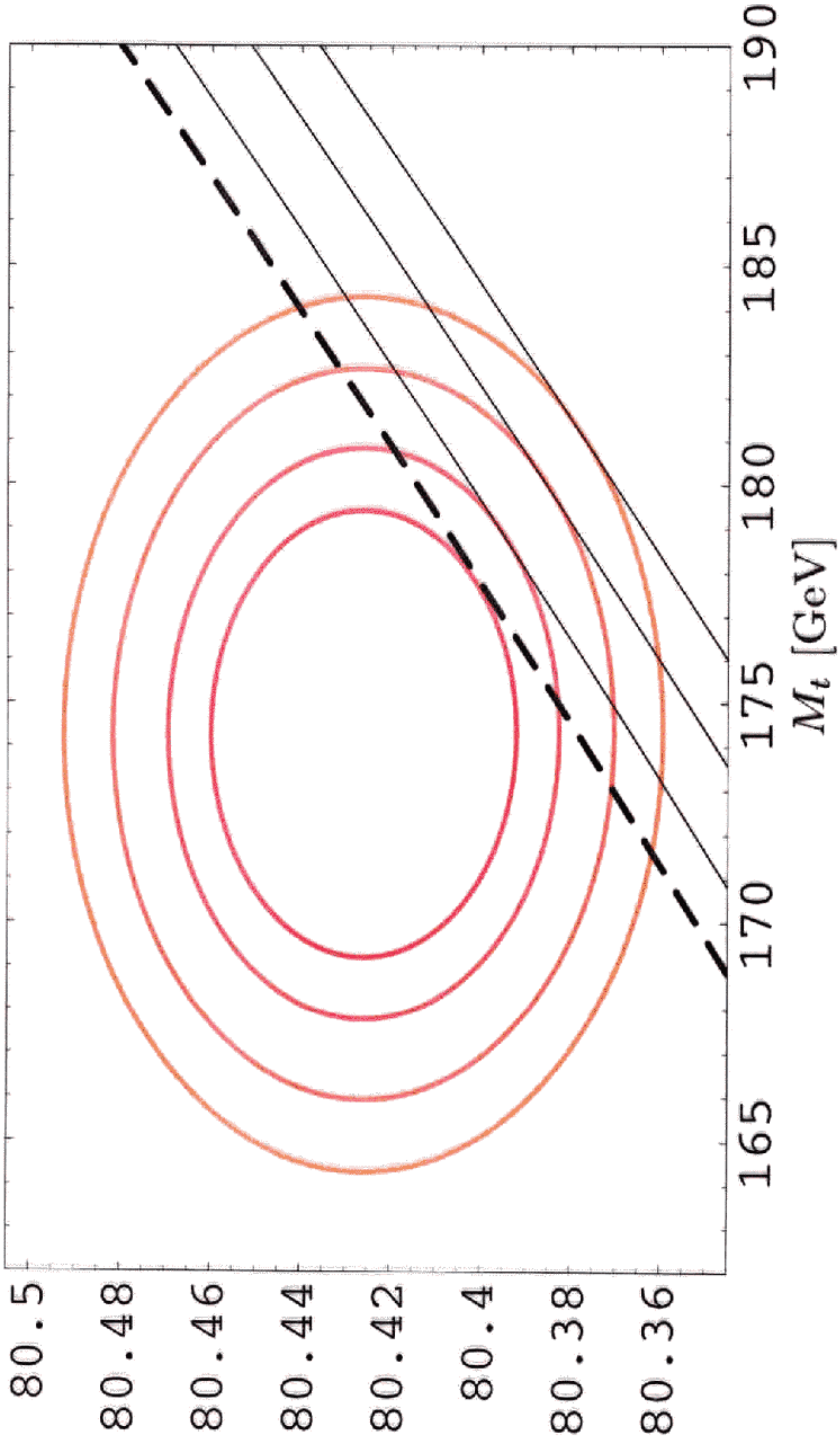
i) Assuming that the SM is correct and taking into account the $(M_H)_{L.B.}$, comparison of the current experimental values of M_W, M_t, and $(s^2_{eff})_{(\ell)}$ with the theory at various C.L. leads to sharp bounds on these parameters, irrespective of other observables.

ii) Compatibility with the 68% C.L. regions only occurs in one of the alternatives we have considered, and is marginal. In the experimental 80% C.L. domain, the current allowed ranges vary from $(M_W = 80.402 \pm 0.020;\ M_t = 177.7 \pm 3.1)$ GeV to $(M_W = 80.397 \pm 0.012;\ M_t = 178.3 \pm 2.0)$ GeV, depending on $\Delta\alpha_h^{(5)}$ and on the theoretical expressions.

In the $((s^2_{eff})_{(\ell)}, M_t)$ analysis, the 80% C.L. allowed intervals are $((s^2_{eff})_{(\ell)} = 0.23129 \pm 0.00010;\ M_t = 177.3 \pm 2.6$ GeV$)$.

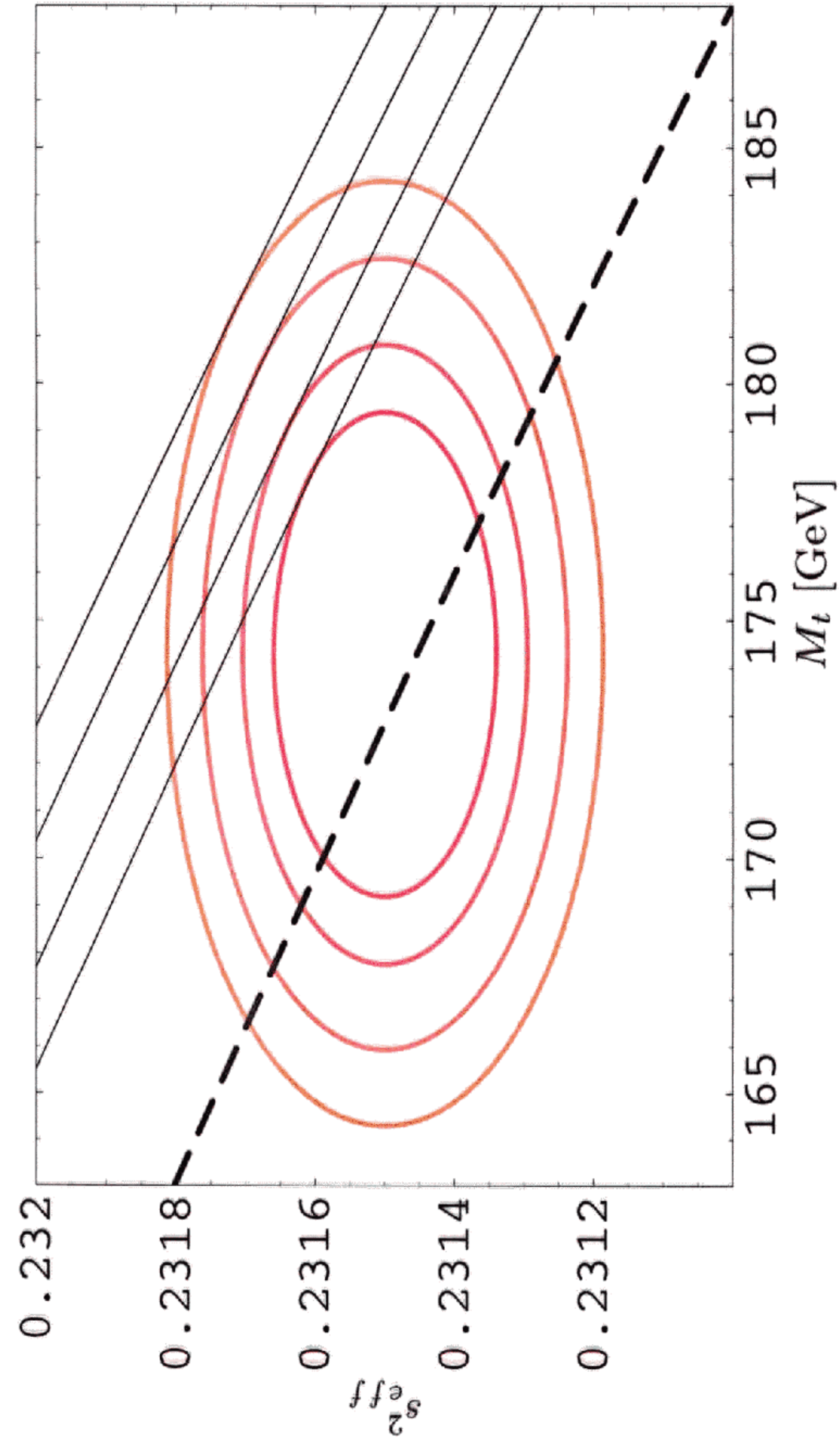
iii) As an illustration of plausible future evolution of the experimental results under the assumption that the SM is correct, we have considered a hypothetical benchmark scenario, involving shifts $\lesssim 1\sigma$ in the central values of M_W, M_t, $(s^2_{eff})_{(l)}$.

iv) As the accuracy increases, it will be very interesting to see whether the central values approach the allowed regions preferred by the SM, remain where they are, or move in the opposite direction. In the last two cases a sharp disagreement with the SM would emerge, thus providing strong evidence for new physics!

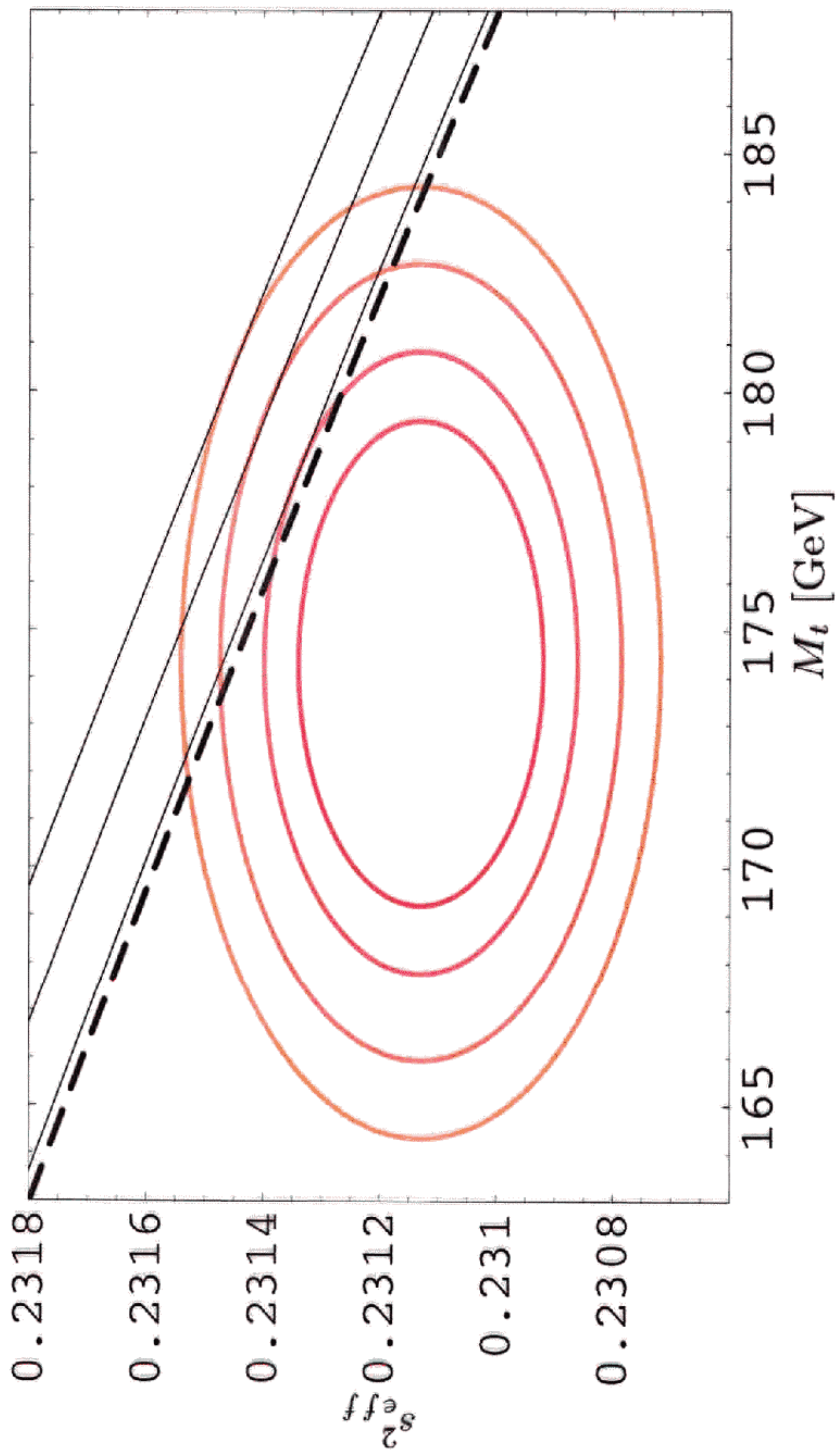


165
170
175
180
185
190
M_t [GeV]
80.36
80.38
80.4
80.42
80.44
80.46
80.48
80.5
M_W [GeV]

1



2


0.2318
0.2316
0.2314
0.2312
0.231
0.2308
s^2_{eff}
165
170
175
180
185
M_t [GeV]

3

EFF scheme $\Delta\alpha_h^{(5)} = 0.02761$	range M_W [GeV]	range M_t [GeV]
80% C.L.	80.383 – 80.419	175.0 – 180.7
90% C.L.	80.371 – 80.431	173.1 – 182.6
95% C.L.	80.362 – 80.441	171.6 – 184.1

Table 1: Comparison of the experimental values $(M_W)_{exp} = 80.426 \pm 0.034$ GeV and $(M_t)_{exp} = 174.3 \pm 5.1$ GeV, at various C.L., with SM theoretical expressions based on Ref. [1] and $\Delta\alpha_h^{(5)} = 0.02761$. The Table shows the ranges for M_W and M_t that, according to the SM, are compatible with the restriction $M_H \geq 114.4$ GeV. In order to belong to the allowed regions, pairs of M_W and M_t values from these intervals should be chosen so that they lie within the corresponding C.L. domains. Within each C.L. domain, the M_W, M_t ranges decrease as M_H increases from 114.4 GeV.

EFF scheme $\Delta\alpha_h^{(5)} = 0.02747$	range M_W [GeV]	range M_t [GeV]
68% C.L.	80.396 – 80.408	176.7 – 178.7
80% C.L.	80.383 – 80.422	174.6 – 180.8
90% C.L.	80.371 – 80.434	172.7 – 182.6
95% C.L.	80.362 – 80.443	171.3 – 184.0

Table 2: Same as in Table 1, for $\Delta\alpha_h^{(5)} = 0.02747$.

Awramik et al. [5] $\Delta\alpha_h^{(5)} = 0.02761$	range M_W [GeV]	range M_t [GeV]
80% C.L.	80.385 – 80.409	176.3 – 180.3
90% C.L.	80.370 – 80.423	173.9 – 182.7
95% C.L.	80.361 – 80.433	172.3 – 184.2

Table 3: Same as in Table 1, with SM theoretical expressions from Ref.[5].

Awramik et al. [5] $\Delta\alpha_h^{(5)} = 0.02747$	range M_W [GeV]	range M_t [GeV]
80% C.L.	80.384 – 80.413	175.7 – 180.5
90% C.L.	80.371 – 80.426	173.6 – 182.7
95% C.L.	80.361 – 80.435	172.0 – 184.2

Table 4: Same as in Table 3, for $\Delta\alpha_h^{(5)} = 0.02747$.

C.L.	$(M_W)_{exp}$ [GeV]	Allowed M_W [GeV]	Allowed M_t [GeV]	$(M_t)_{exp}$ [GeV]
68% C.L.	80.426 ± 0.034	80.402 ± 0.006	177.7 ± 1.0	174.3 ± 5.1
80% C.L.	80.426 ± 0.044	80.402 ± 0.020	177.7 ± 3.1	174.3 ± 6.5
90% C.L.	80.426 ± 0.056	80.402 ± 0.032	177.7 ± 5.0	174.3 ± 8.4
95% C.L.	80.426 ± 0.067	80.402 ± 0.041	177.7 ± 6.4	174.3 ± 10.0

Table 5: Allowed M_W, M_t ranges from Table 2, expressed as mid-points and variations covering the corresponding intervals. They are compared with the ranges extracted from the experimental values.

EFF scheme $\Delta\alpha_h^{(5)} = 0.02761$	range $\sin^2\theta_{eff}^{lept}$	range M_t [GeV]
80% C.L.	0.23119 – 0.23139	174.7 – 179.9
90% C.L.	0.23111 – 0.23147	172.1 – 182.6
95% C.L.	0.23105 – 0.23153	170.4 – 184.3

Table 6: Comparison of the experimental values $(s^2_{eff})_l = 0.23113 \pm 0.00021$ and $M_t = 174.3 \pm 5.1\,\text{GeV}$ at various C.L. with SM theoretical expressions from Ref. [1] and $\Delta\alpha_h^{(5)} = 0.02761 \pm 0.00036$. The Table shows the ranges for s^2_{eff} and M_t that, according to the SM, are compatible with the restriction $M_H \geq 114.4\,\text{GeV}$. In order to belong to the allowed regions, pairs of s^2_{eff} and M_t values from these intervals should be chosen so that they lie within the corresponding C.L. domains.

EFF scheme $\Delta\alpha_h^{(5)} = 0.02747$	range $\sin^2\theta_{eff}^{lept}$	range M_t [GeV]
80% C.L.	0.23119 – 0.23140	174.2 – 180.6
90% C.L.	0.23113 – 0.23146	172.2 – 182.7
95% C.L.	0.23107 – 0.23151	170.6 – 184.2

Table 7: Same as in Table 6, for $\Delta\alpha_h^{(5)} = 0.02747 \pm 0.00012$.