CP violation in 5D QED

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Overview

- Introduction

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- Conclusions and future plans

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A simple model

- ***QED on a 5-dim cylinder with Lagrangian***

$$\mathcal{L}_{QED} = -\frac{1}{4} F_{MN}^2 + \sum_{i=1,2} \bar{\psi}_i \left(i\gamma^M D_M - M_i \right) \psi_i + \mathcal{L}_{gf},$$

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Scherk& Schwarz, 1979
Hosotani, 1983 (in this context)

$$\psi_i(x^\mu, y + L) = e^{i\alpha_i} \psi_i(x^\mu, y) \qquad A^M(x^\mu, y + L) = A^M(x^\mu, y)$$

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$$x^M \to \epsilon^M x^M,\ A^M \to -\epsilon^M A^M,\ \psi_i \to \eta_i \gamma^0\gamma^2\psi_i^\star, \quad |\eta_i| = 1,\ \epsilon^{0,4} = -\epsilon^{1,2,3} = +1$$

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- Look for non-trivial CP violating vacua

Fourier expansions and all that

- The KK modes: Hosotani, 1983

$$\psi_i(x,y) = \frac{1}{\sqrt{L}} \sum_{n=-\infty}^{\infty} \psi_{i,n}(x) e^{i\bar{\omega}_{i,n} y}; \qquad \bar{\omega}_{in} = (2\pi n + \alpha_i)/L$$

$$A^M(x,y) = \frac{1}{\sqrt{L}} \left[\sum_{n=-\infty}^{\infty} A_n^M(x) e^{i\omega_n y} + a\, \delta_4^M \right]; \qquad \omega_n = 2\pi n/L$$

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- The fermion Lagrangian

$$\mathcal{L}_\psi = \sum_{in} \bar{\psi}_{i,n} \left[i\gamma^\mu \partial_\mu - M_i + i\gamma_5 \mu_{i,n} \right] \psi_{i,n} - e \sum_{i,l,n} q_i \bar{\psi}_{i,l} \left(\not{A}_{l-n} + i A^4_{l-n}\, \gamma_5 \right) \psi_{i,n}$$

with $\mu_{i,n} = \bar{\omega}_{i,n} + e q_i\, a$, $\quad e \equiv e_5/\sqrt{L} =$ 4-dim gauge coupling.

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- Diagonalize the fermion mass matrix

$$\psi_{i,n} \to \exp(i\gamma_5 \theta_{i,n}) \psi_{i,n}; \qquad \tan(2\theta_{i,n}) = \frac{\mu_{i,n}}{M_i}; \quad |\theta_{i,n}| \le \pi/4$$

Fourier expansions and all that (cont.)

- Physical fermion masses:

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- Interactions:

$$\mathcal{L}_{A\psi} = -e \sum_i q_i \left\{ A_\mu \sum_k \bar{\psi}_{i,k} \gamma^\mu \psi_{i,k} + \sum_{k \neq l} A_{\mu\ k-l} \bar{\psi}_{i,k} \Gamma^{(v)}_{i,kl} \gamma^\mu \psi_{i,l} \right\}$$

$$\mathcal{L}_{\varphi\psi} = -e \sum_i q_i \left\{ \varphi \sum_k \bar{\psi}_{i,k}\, \Gamma^{(\varphi)}_{i,k}\, \psi_{i,k} + \sum_{k \neq l} A_{4\ k-l} \bar{\psi}_{i,k} \Gamma^{(s)}_{i,kl} \psi_{i,l} \right\}$$

$\varphi \equiv A_{4,n=0} =$ physical scalar with (naively) CPV couplings

$A_\mu \equiv A_{\mu\ 0} =$ 4-dim. photon

and

$$\Gamma^{(\varphi)}_{i,k} \equiv -i\gamma_5 e^{2i\gamma_5 \theta_{i,k}}, \qquad \Gamma^{(s)}_{i,kl} \equiv -i\gamma_5 e^{i\gamma_5(\theta_{i,k}+\theta_{i,l})}, \qquad \Gamma^{(v)}_{i,kl} \equiv e^{i\gamma_5(\theta_{i,k}-\theta_{i,l})}$$

Fourier expansions and all that (cont.)

- Gauge fixing:

$$\mathcal{L}_{gf} = -\frac{\xi}{2}\left(\partial^\mu A_\mu - \xi^{-1}\partial_y A_4\right)^2$$

$$\mathcal{L}_A + \mathcal{L}_{gf} = \frac{1}{2}\sum_n \left\{A^\mu_n \left[(\Box + \omega_n^2)g_{\mu\nu} - (1-\xi)\partial_\mu\partial_\nu\right] A^\nu_{-n} - A^4_n(\Box + \omega_n^2/\xi)A^4_{-n}\right\}$$

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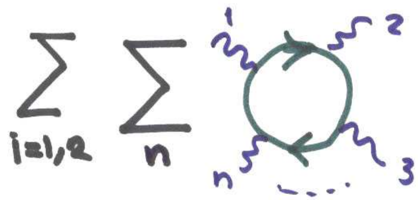
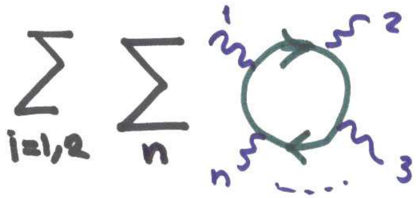
- φ is a gauge singlet: cannot choose the $A_4 = 0$ gauge (similar to finite temp. case).
- $U(1)$ gauge invariance remains unbroken.
- There is no symmetry that forbids $\langle\varphi\rangle \neq 0$

The effective potential

- Standard evaluation 

The effective potential

- Standard evaluation
- Result for one fermion:

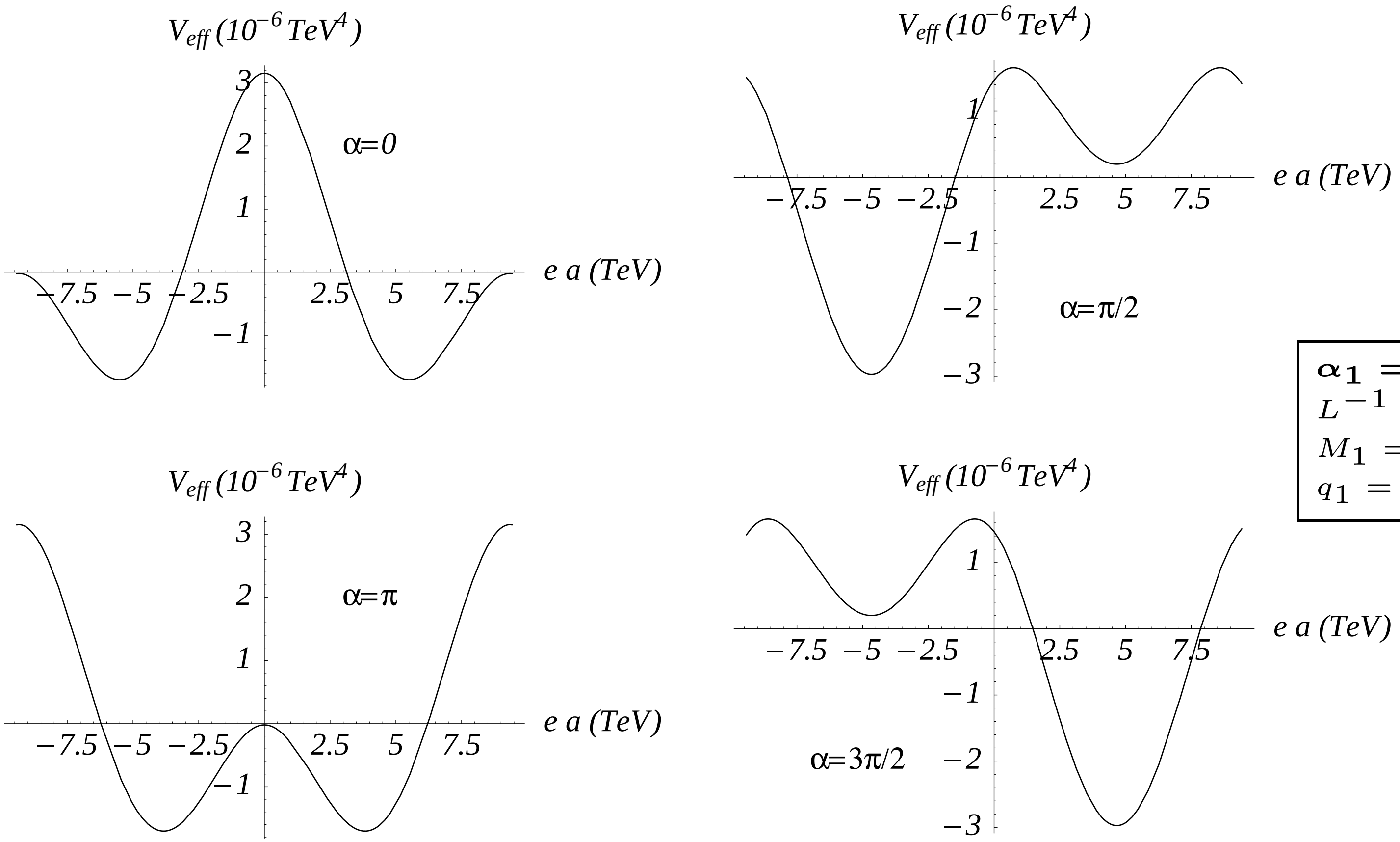


$$V(M;L;\omega) \quad = \quad \frac{1}{32\pi^6 L^4}\left[(LM)^2 Li_3(u) + 3(LM)Li_4(u) + 3Li_5(u) + \mathrm{H.c.}\right]$$

where $\quad u = \exp[L(i\omega - M)],\ \omega = \alpha + eq_\psi La$

Effective potential (cont.)

Full potential: $V_{eff} = \sum_{i=1,2} V(M_i; L; \omega_i)$



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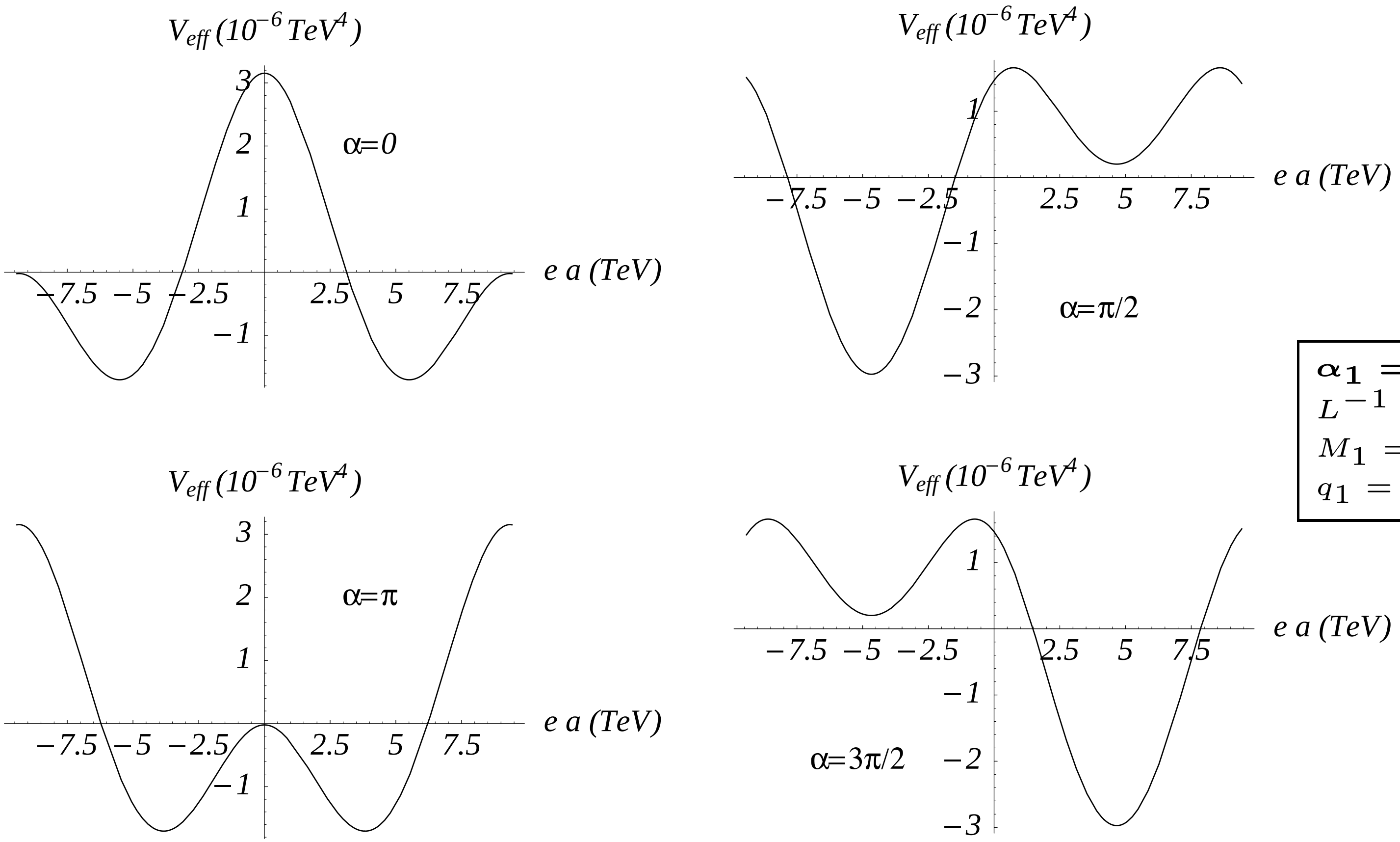

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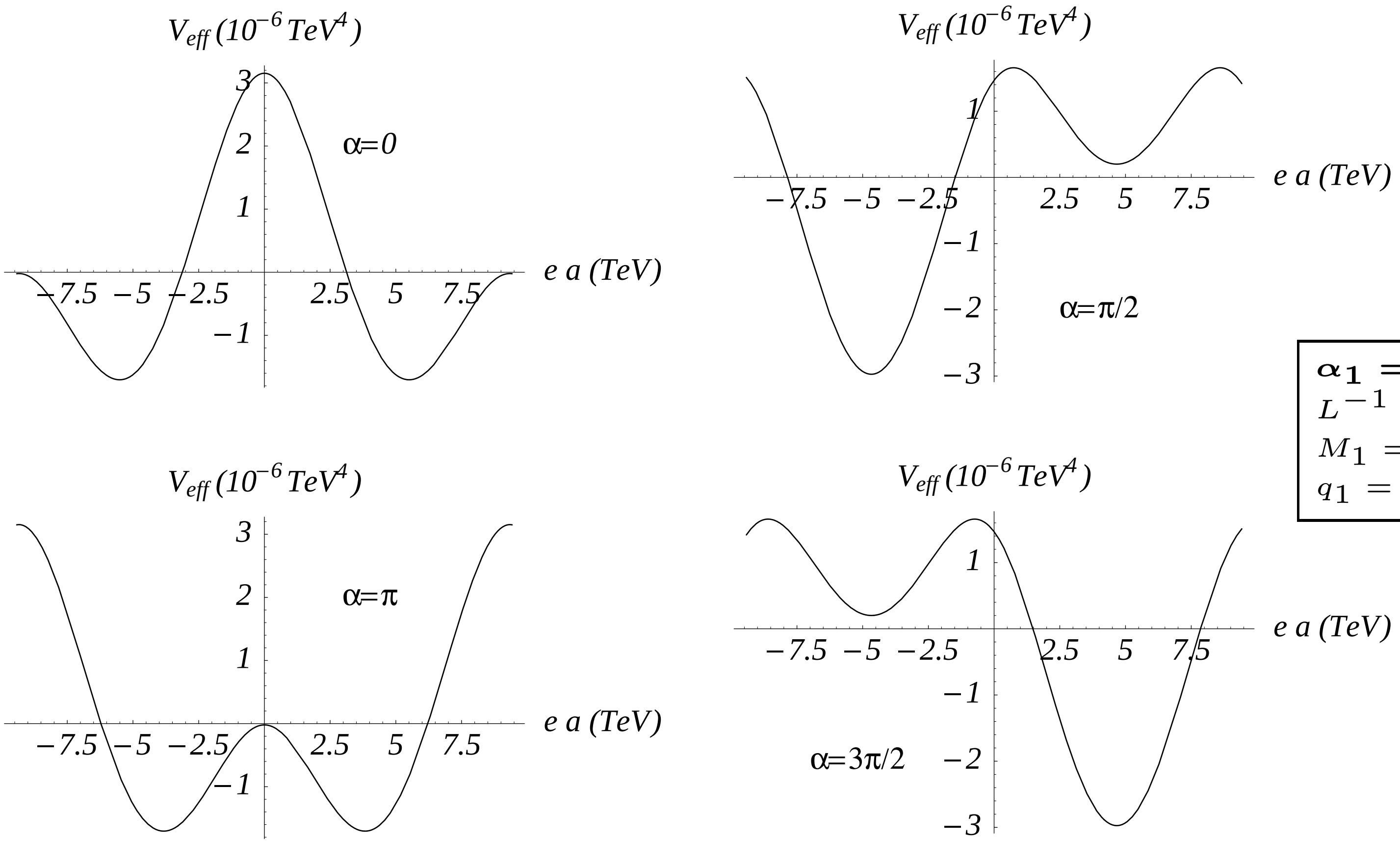

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For $\alpha \neq 0, \pi$, bound conds. contain *explicit* CPV

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  - $\psi_{\text{new}}(x, y+L) = -\psi_{\text{new}}(x,y) \Rightarrow$ no CPV from the boundary conditions.
- Equivalently at the minimum, $m_n = m_{-n-1}$, this allows a generalized definition

$$\psi_n \xrightarrow{\text{CP}} C\overline{(\gamma_0\psi_{-n-1})}^T$$

under which the couplings are invariant.

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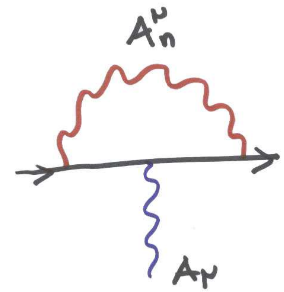

$$A_n^{\mu}: \qquad d^{(v)}_{i,n} = \frac{(eq_i)^3 c^{(-)}_{i,n}}{4\pi^2}\frac{m_{i,n}}{m^2_{i,0}}J^{(v)}(x_{i,n},y_{i,n})$$

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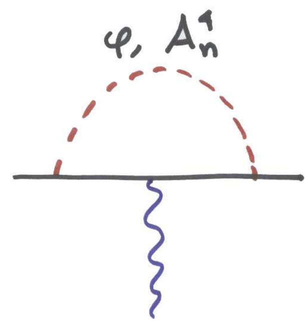

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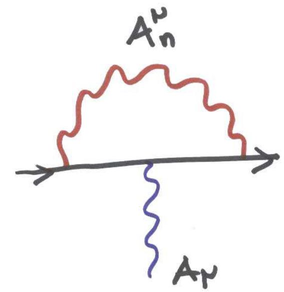

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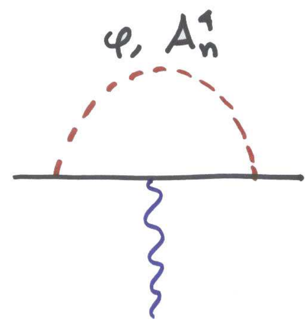
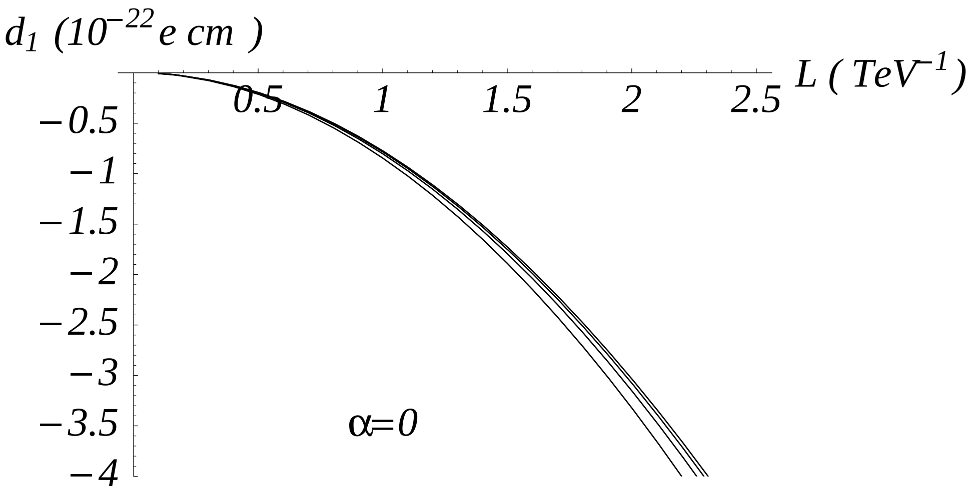
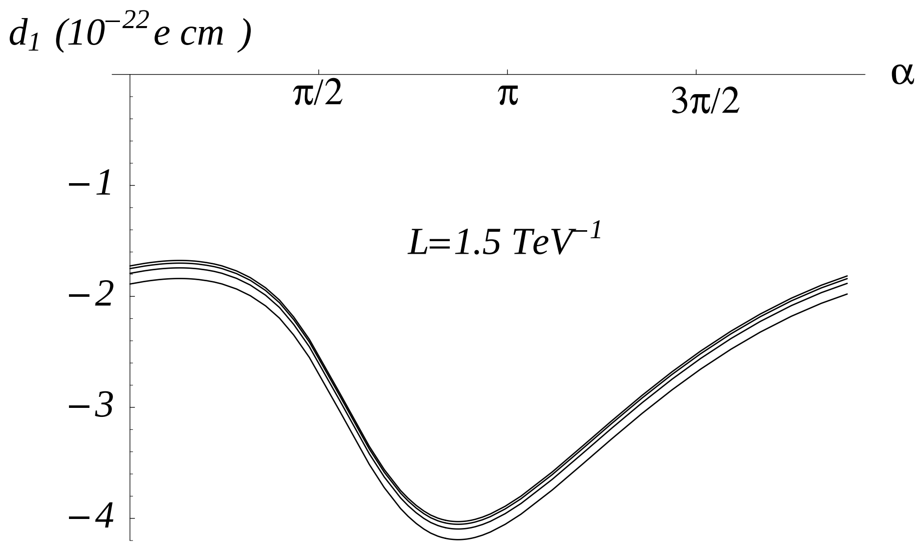


- Where
  - ▷ $c^{(\pm)}_{i,n} = \pm M_i(\mu_{i,n} \pm \mu_{i,0})/(m_{i,n} m_{i,0})$
  - ▷ $x_{i,n} = (\omega_n / m_{i,0})^2, \; y_{i,n} = (m_{i,n}/m_{i,0})^2$
  - ▷ $J^{(s)}(x,y) = 1 + (x - y + 1)\ln\sqrt{y/x} + (2x/\rho - \rho)\,\Theta$
  - ▷ $J^{(v)}(x,y) = -1 + (y - x)\ln\sqrt{y/x} + (\rho - \cot\Theta)\Theta$
  - ▷ $\rho^2 \equiv 4xy - (x + y - 1)^2 \qquad \tan\Theta \equiv \rho/(x + y - 1).$

Phenomenology (cont)

Results





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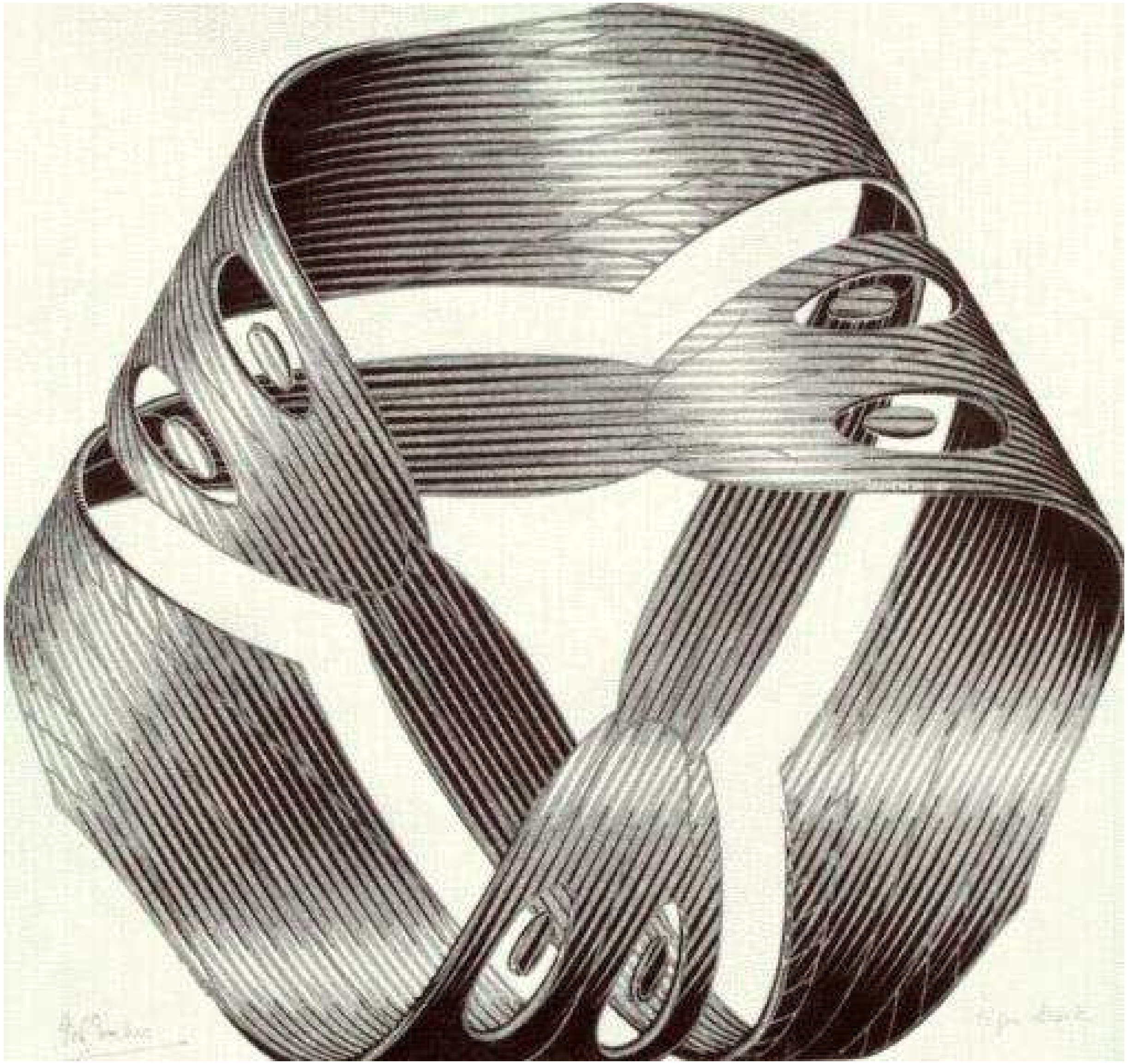
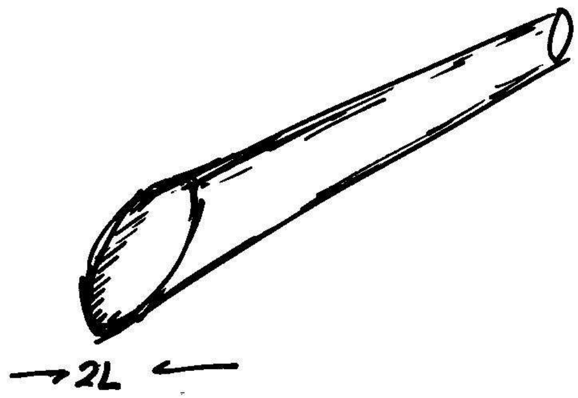
- Non-Abelian extension...in progress



Kaluza Klein

In a 5D cylinder $\sim \mathbb{R}^4 \times S^1$ with coordinates $x^M = (x^\mu, y)$ the metric will be periodic in y...



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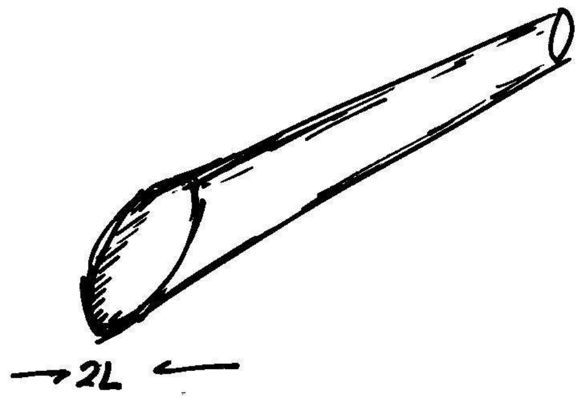

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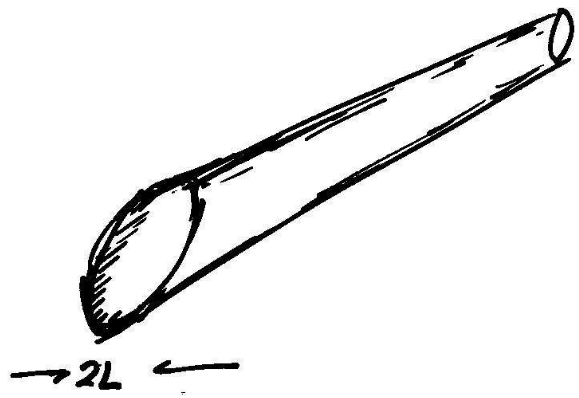

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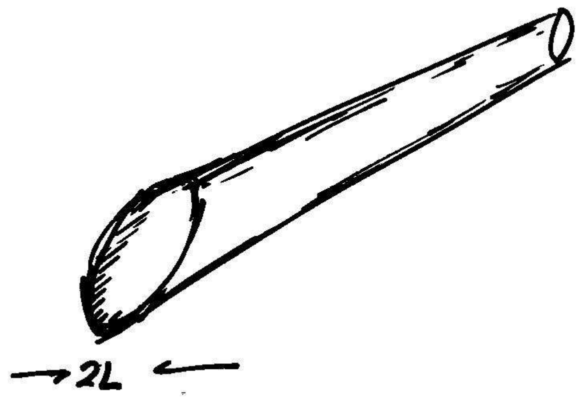

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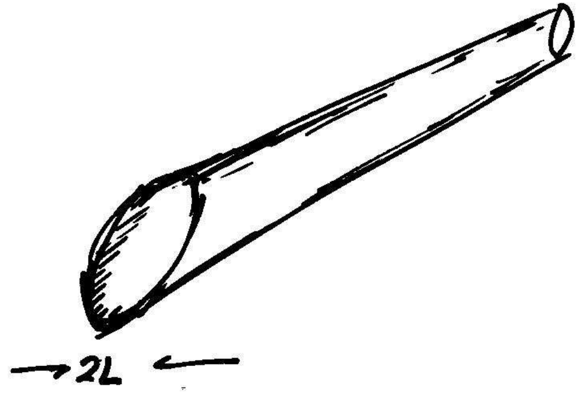

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Gauge couplings suppressed by $1/M_{\rm Pl}$.

(return)

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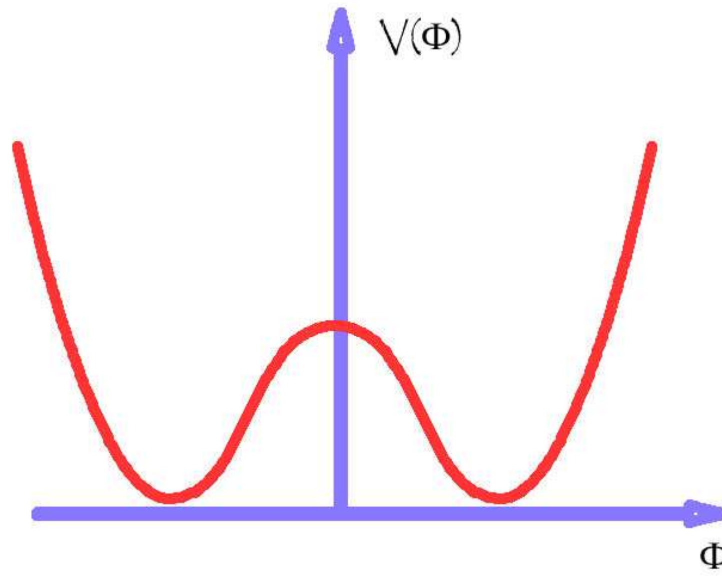
- Then A_μ^a is the gauge field for a gauge theory with group G
  For $G = SU(3) \times SU(2) \times U(1)$, dim$(B) \geq 7$

(return)

Localizing matter to a brane

Imagine a 5D scalar field with potential



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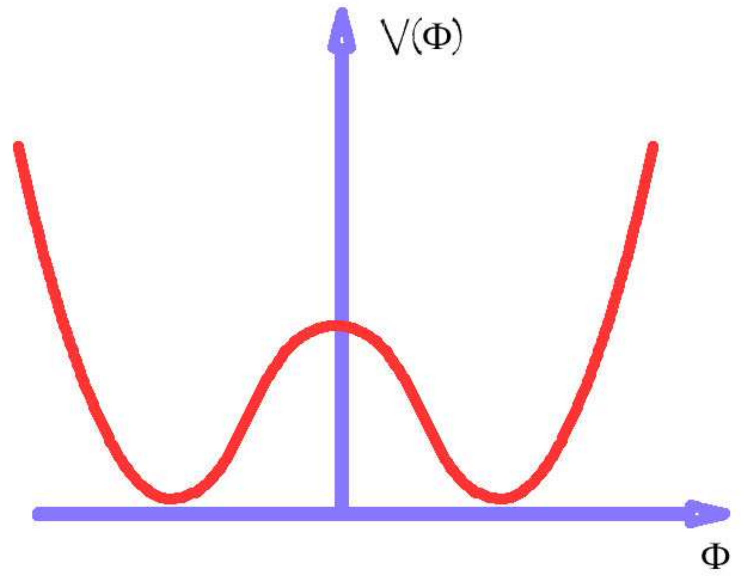
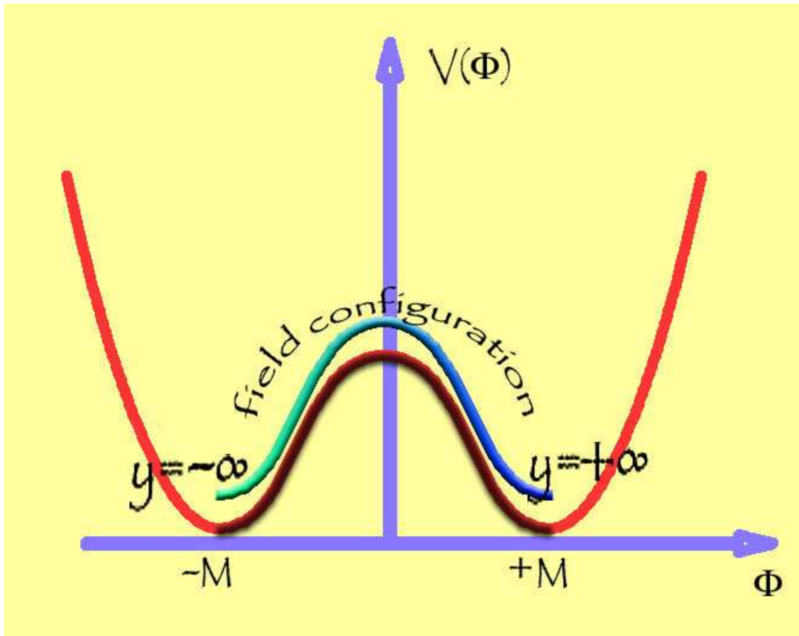


"Bounce" solution: $\Phi(y = \pm\infty) = \pm v$



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- ***Boson couplings:*** $|D\chi|^2 - V(\chi, \Phi)$
  - Can use the above field to induce SSB only off the brane
    $\Rightarrow$ Gauge fields become massive outside the brane

(return)