Interpretation of Interference Experiments in Real Quantum Hall Systems

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Theoretical work done with Bernd Rosenow, Ady Stern, and Izhar Neder. Motivated by experiments at Harvard by Yiming Zhang, Doug McClure, Angela Kou, and C. M. Marcus, who also contributed to the theoretical picture.

Also, by experiments in Heiblum lab at Weizmann.

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Some reasons why quantum Hall systems are still very interesting

Systems are very rich. Many different phases and phase transitions.

Theory is complex. Uses a wide variety of mathematical techniques, theoretical methods.

Systems exhibit a variety of striking phenomena.

Quantum Hall effects observed in new materials, and with new or improved techniques.

Continuing experimental surprises.

Many unanswered questions.

Many open questions concern the nature of states in bulk 2D systems

Especially:

What is the nature of QHE states observed in the second Landau level: eg. 5/2, 7/3, 8/3, 12/5 ? Are some or all of them non-Abelian? Too difficult for theory or experiment alone to decide.

What is the role of sample parameters, including electron density, well thickness, and disorder, in determining the nature of the ground state?

Other open questions concerning bulk 2D systems

Finite temperature effects, particularly in the presences of disorder:

When should there be "phases" at intermediate temperatures that are not observed at T =0?

What determines the activation gaps actually observed in QHE systems?

Phase transitions in multi-valley systems (e.g., graphene, AlAs)

Phenomena in bi-layer systems.

Phenomena in higher Landau levels, including stripe and bubble phases, microwave-induced resistance oscillations,etc.

Other open questions concern edges, or mesoscopic geometries

Transport in systems with a narrow constriction (quantum point contact), including shot noise and current correlation experiments.

Systems with two or more constrictions, including interferometer geometries.

Experiments on edges may be a way of learning about the bulk states. Or, may be determined by non-universal properties of edges. Interesting in their own right.

Interference experiments have been suggested as a way to determine whether the $\nu=5/2$ state has non-Abelian statistics.

But: Real interference experiments have **complications**, even in simpler cases of integer QHE or odd-denominator fractions, which we need to understand, and have not been well understood in the past.

We shall **focus here** on the **Integer** QHE.

Key Experimental Developments

Improved methods of gating and separately controlling densities in bulk and in regions of constrictions.

Samples of very different sizes.

Continuous measurements of resistance oscillations as a function of both magnetic field B and side-gate voltage V_G.

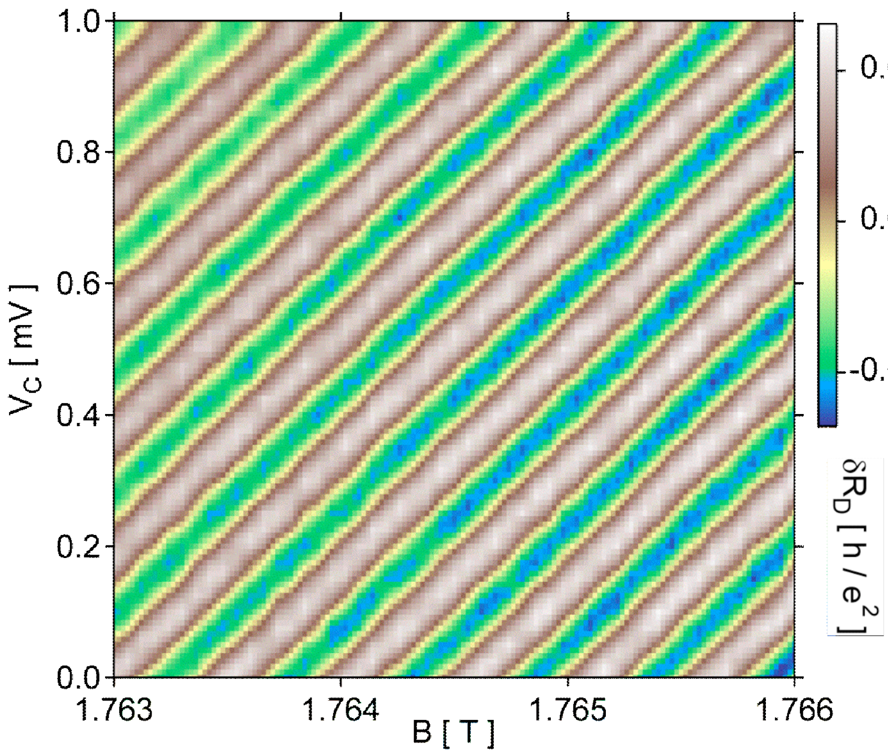
Find regions of qualitatively different behavior distinguished by sign of slope of lines of constant phase.

Yiming Zhang et al:

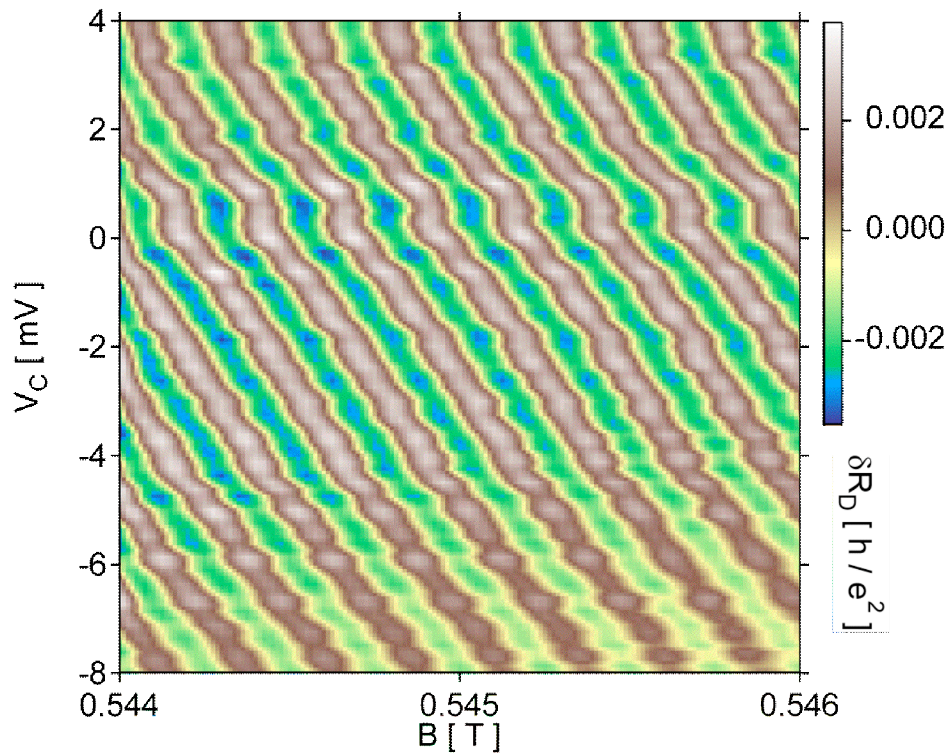
Two types of behavior

Measure in a 2D plane of B and V_G

"Coulomb Dominated"



"Aharonov-Bohm"



Fabry Perot Interferometer--Integer Regime
Assumptions

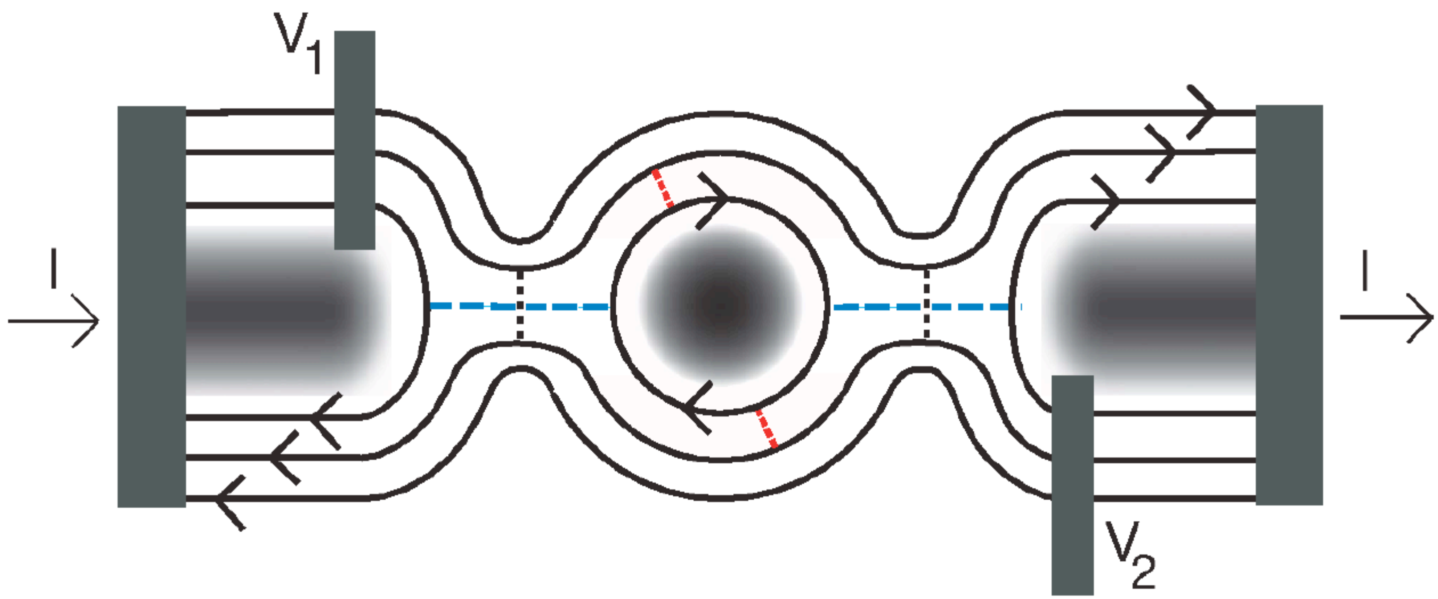
In constrictions: f_T channels are perfectly transmitted, corresponding to levels $j = 1, 2, ..f_T$. (Number may be zero).

One channel, $j_0 = f_T + 1$, is partially backscattered.

Additional channels with $j > j_0$ may exist in bulk, but do not enter constrictions. Can happen if density in bulk is larger than density in constrictions.

Filling factor in constriction is ν_c , with $f_T < \nu_c < f_T + 1$.

Fabry Perot Interferometer--Integer Regime: Illustration



Black dotted lines: weak back scattering: $f_T=1$, $\nu_c = 2-\varepsilon$.

Blue dashed lines: weak forward scattering: $f_T=2$, $\nu_c = 2+\varepsilon$.

Bulk filling factor = $3 \pm \varepsilon$.

Weak Backscattering Limit

Partially transmitted channel, $j_0 = f_T + 1$, is weakly backscattered.

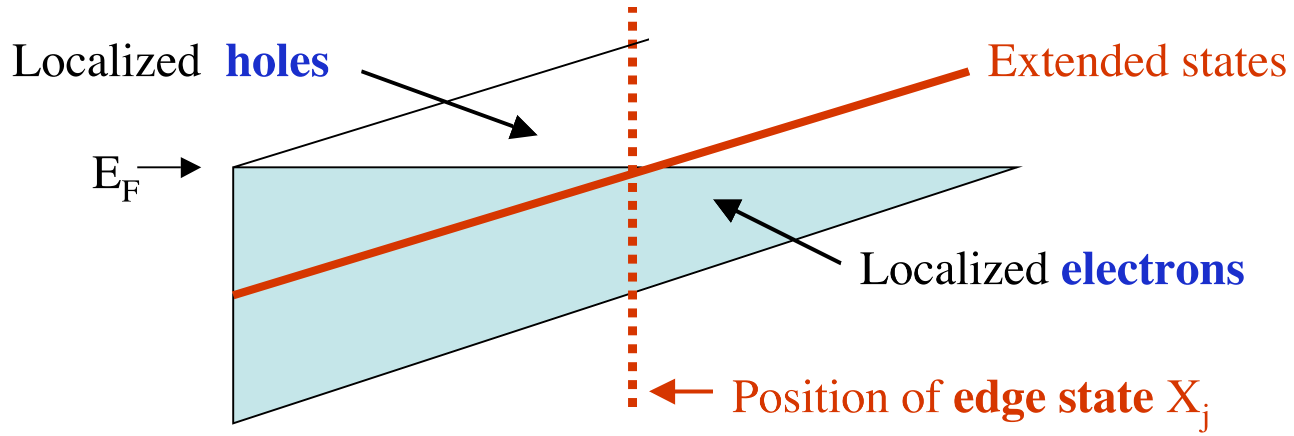
Filling factor in constriction is $\nu_c = j_0 - \varepsilon$.

For small source-drain voltage, R_D will have oscillatory part

$\delta R_D \propto \mathrm{Re}\,[r_1 r_2^* \, e^{i\phi}]$; ϕ is the phase accumulation ϕ_j of the partially transmitted edge state, $j = j_0$.

We must calculate this phase.

Structure of an edge



General picture: The electron density varies gradually at an edge, usually on a length scale larger than the magnetic length. Each (spin-split) Landau level j is a band of localized states, with an extended state in the middle, whose energy depends on position x.

The **edge state** for Landau level j occurs at a (guiding center) position X_j where the extended state crosses the Fermi energy. For $x>X_j$, we find **localized electrons**, in states with $E<E_j$. For $x<X_j$, we have a filled Landau level + **holes** in localized states above $E_{F.}$

Charge in a Landau Level

Phase change around edge, for Landau level j is determined by the number of flux quanta N_j in the area A_j enclosed by the guiding center:

$$\frac{\phi_j}{2\pi} \equiv \frac{BA_j}{\Phi_0} \equiv N_j$$

N_j is an integer for closed edges.

N_j can vary continuously for open edge states.

Total charge in Landau Level is given by $Q_j = N_j + P_j - H_j$

P_j = number of electrons in localized states outside of A_j

H_j = number of holes in localized states inside of A_j

P_j and H_j are integers

Energy Considerations

$$E = \frac{K}{2}\,(Q - n_0\,A_0 - n_0\,\delta A_G - A_0\,\delta n_G)^2 + \frac{B^2}{2}\sum_{ij} U_{ij}\,(\Delta A_i)(\Delta A_j)$$

$$\Delta A_j \equiv A_j - (A_0 + \delta A_G - c_j)$$

$$\delta A_G = \alpha V_G \equiv \text{"area change"}, \quad \delta n_G = \beta V_G \equiv \text{"density change"}$$

$$Q = \sum Q_j = \text{total electron number in the island}$$

$$K^{-1} = \text{capacitance of island} \propto A_0, \quad U_{ij} = ?$$

Justification for Capacitance Charging Model

Electrons and holes in localized states have a small but finite conductivity at finite temperatures. Interior of island behaves like a good screening metal on laboratory time scales; charges equilibrate to give a constant potential in equilibrium.

"Aharonov-Bohm" Regime

Limit $K \to 0$. (Very large area island, with top gate)

$E = (B^2/2) \sum U_{ij} (\Delta A_i)(\Delta A_j)$

Neglect coupling to any edge states trapped in the island ($j>j_0$).

Minimize energy by setting $\Delta A_j = 0$, for $j \leq j_0$.

$\phi /2\pi = B_0 A_0 + A_0 \delta B + B \delta A_G$.

Lines of constant ϕ have: $A_0 \delta B + B \delta A_G$ = constant

Negative slope on plot of (B,V_G)

Field Period $\Delta B = 1/A_0$: one flux quantum in area A_o.

Area Period $\Delta A_G = 1/B$. (Units: $\Phi_0 = 1$)

"Coulomb-Dominated" Regime

Assume $K >> U_{ij}$. (Charging energy dominates)

Also, assume partially transmitted edge j_0 is less stiff than fully transmitted edges. Then minimizing E with respect to A_j gives

$$\phi/2\pi \approx n_0 A_0 + n_0 \delta A_G + A_0 \delta n_G - f_T B (A_0 + \delta A_G) + \text{integer}$$

Integer depends on occupations of localized states; can change if an electron hops from localized state to open edge. This changes phase by 2π, has no effect on interference.

Ignore integer, vary B, V_G, find

$$\delta\phi/2\pi = A_0 \delta n_G + (n_0 - f_T B) \delta A_G - f_T A_0 \delta B \quad \text{[linear in } \delta B \text{ and } V_G]$$

Coulomb-Dominated Regime: Implications

$\delta\phi/2\pi = A_0\, \delta n_G + (n_0 - f_T B)\, \delta A_G - f_T\, A_0\, \delta B$

If $f_T > 0$, lines of constant ϕ have positive slope, in (B, V_G) plane . (Since $\delta A_G = \alpha\, V_G$, $\delta n_G = \beta\, V_G$ with $\alpha, \beta > 0$.)

If $f_T = 0$, ϕ is independent of δB. (Lines are horizontal). This is the case when constrictions are at $\nu_c = 1 - \varepsilon$.

Find: Flux period: $\Delta B = (-1/f_T)\, A_0^{-1}$. (Flux quantum / f_T)

Area Period: $\Delta A_G = 1 / (n_0 - f_T B)$.

Back-gate period: $\Delta n_G = 1/ A_0$.

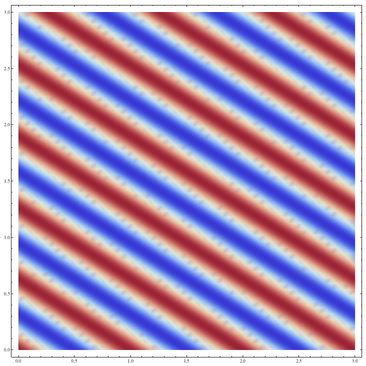
What about cases intermediate between Aharonov-Bohm and Coulomb-dominated regimes?

Intermediate Coupling Cases

Re <e$^{i\phi}$>

$T \neq 0$

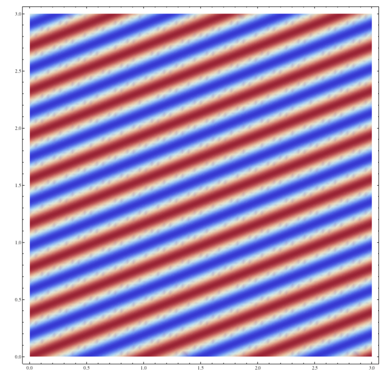
AB

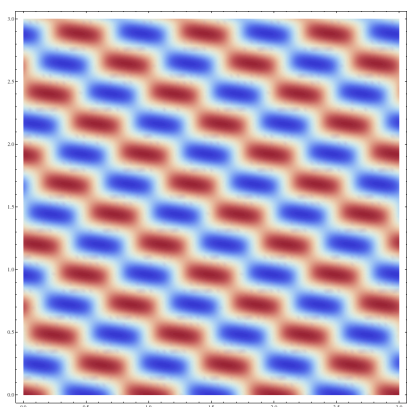


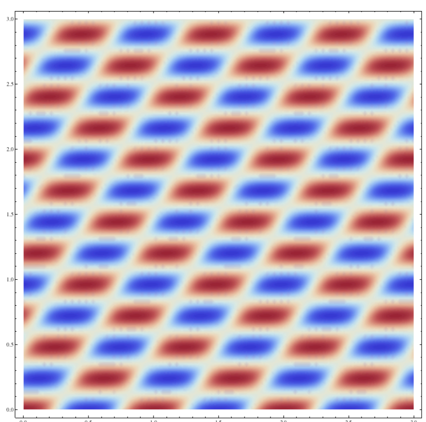
V_G ↑

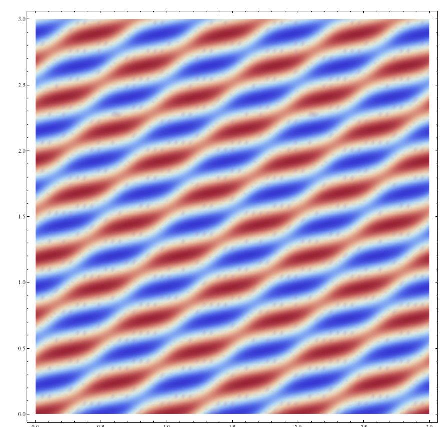
B →

CD









($\nu_c = 2 - \varepsilon$; $f_T = 1$) Calculations by B. Rosenow

Intermediate and strong backscattering

So far we have discussed weak back scattering:

$$\nu_c = f_T+1-\varepsilon \ .$$

Field period is expected to be constant in entire range of constriction fillings $f_T < \nu_c < f_T+1$.

$$A_0 \ \Delta B = (-1/f_T) \ .$$

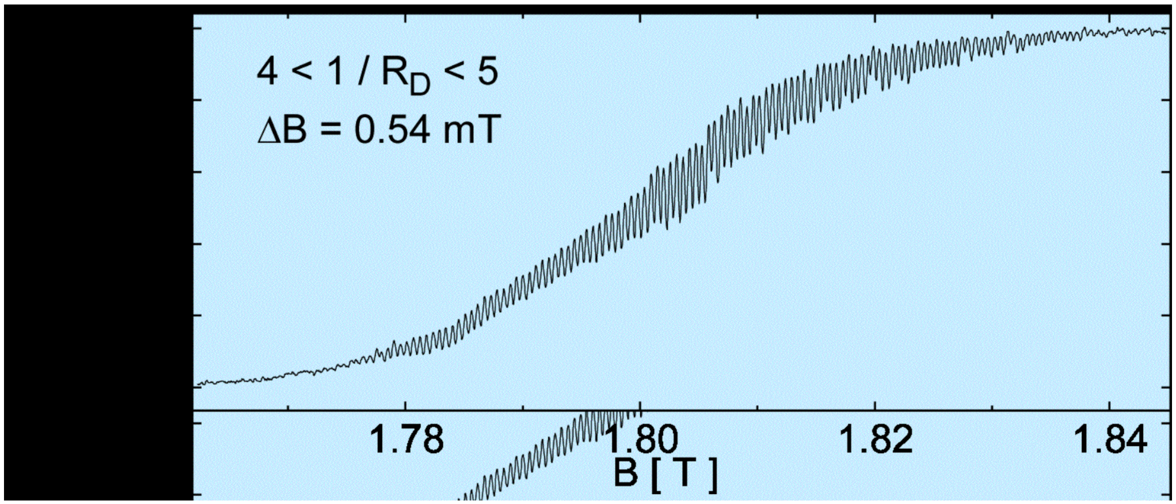
Period changes when you cross a plateau.

Consistent with experiments.

(Gate periods are more complicated.)

Zhang et al, Coulomb-dominated regime

Data - 2 μm² device



Comparison to earlier work:

Present results in Coulomb dominated regime are equivalent to previous results of Rosenow and Halperin (PRL 2007) in the limit of strong backscattering, and also in the limit of weak back scattering, if one takes ratio of coupling constants $\Delta_X / \Delta = 1$ in that paper. Aharonov Bohm regime is $\Delta_X / \Delta = 0$. Previous paper did not explain why one should have $\Delta_X / \Delta = 1$ for small islands.

Also, did not consider sign of the slopes of constant phase lines in the (B, V_G) plane.

Finite Source-Drain Voltage V

Apply finite voltages, V_1 and V_2 to opposite edges. Predict:

$\delta I \propto F(V)\ \cos[\phi_0 + \Phi]$, $V \equiv V_1 - V_2$, where

$F(V)$ is an envelope function that is non-monotonic in V,
ϕ_0 is interference phase at V=0,
$\Phi = (aV_1 + bV_2)$ is an extra phase shift linear in V.

In most cases, a and b are small, so Φ can be neglected.

For the case of a single edge state (ν=1), in the weak backscattering regime, it was found by Chamon et al. (1997) that

$F(V) \propto \sin(V\tau)$

where τ is the time for an electron to move half-way around the island.

Finite V, Multiple Edge States

Total number of edge states in bulk $= f_b > 1$. Bare velocities u_j, with $1 \leq j \leq f_b$. Due to strong Coulomb coupling between modes on a single edge, new velocity eigenmodes v_k. Charge mode with large v_c, remaining modes slow, with $u_k > v_k > u_{k+1}$.

Note: Here we are concerned with fast transport around edge; no time for screening by localized states.

Assume one mode ($j=j_0$) partially transmitted through constrictions, weak backscattering. Ignore extra phase shift Φ.

Result: $\delta I \propto F(V) \cos \phi_0$, but F is not a simple sine function.

However, if velocities of adjacent modes are not too different:

$F(V) \approx (V\tau)^{-x} \sin (V + \eta)$, for $V\tau > 1$,

with $\tau \approx L/2v_j$, for $j=j_0$, and $x = 1/f_b$.

Finite V: Experiments

Zhang et al. apply voltage V to one side, measure dI/dV.

If we use approximate formula: $\delta I \propto \sin(V\,\tau)\ \cos\phi_0$, we find

$$dI/dV \propto \cos(V\tau)\cos\phi_0 ,$$

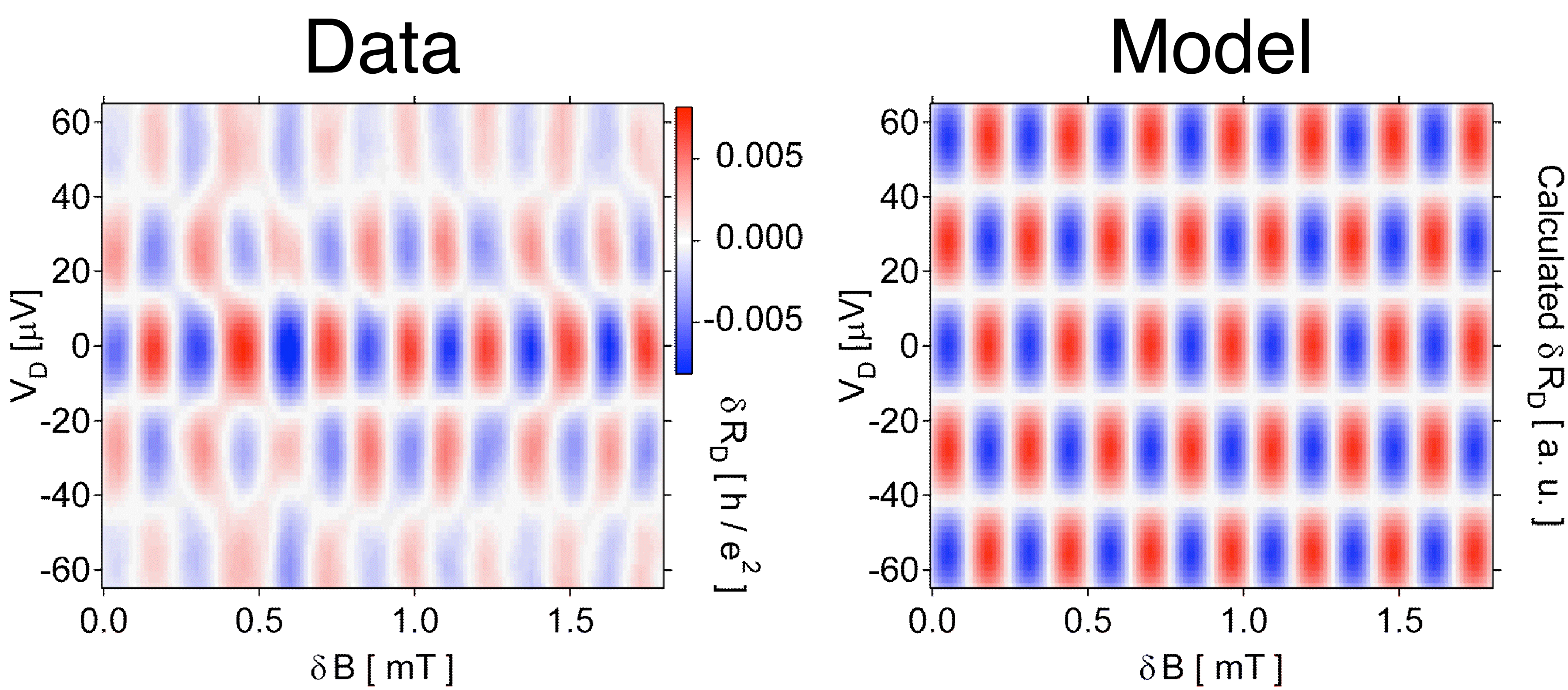
(product of two independent cosines).

Experiments of Zhang et al, in Aharonov Bohm regime, agree with this form.

Y. Zhang et al.:

Non-linear Regime

$$dI_{\mathrm{t}}/dV_{\mathrm{D}} \propto \cos(2\pi\delta BA/\Phi_0)\cos(2\pi V_{\mathrm{D}}/\Delta V_{\mathrm{D}})$$
$$v = ae\Delta V_{\mathrm{D}}/h$$



Fractional Quantized Hall States

Methods can also be applied to fractional quantized Hall edge states. Must take account of fractional charges and fractional statistics (i.e, effective magnetic flux associated with addition of quasiparticles). Do bookkeeping carefully.

Example: For constrictions with $1/3 < \nu_c < 2/5$, find:

Field period $\Delta B = -1/A_0$, (addition of one flux quantum), same as integer regime with $1 < \nu_c < 2$.

Finite Source-Drain Voltage V

Apply finite voltages V_1 and V_2 to opposite edges.

Prediction of Chamon et al (1997) (single edge state)

$$\delta I \propto \sin(V\tau)\ \cos\phi_0\ , \qquad V \equiv V_1 - V_2$$

where ϕ_0 is interference phase at V=0, and τ is the time for a particle to move across one edge of the island. We find extra phase shift linear in voltages, so

$$\delta I \propto \sin(V\tau)\ \cos[\phi_0 + \Phi]\ ,$$

$$\Phi = (aV_1 + bV_2)\,\tau.$$

In many cases, a and b are <<1, so reduces to Chamon et al. In other cases, a, b may be important.

If system is symmetric, (a=b) and if voltage is applied to both edges ($V_1 = -V_2 \equiv V/2$), then $\Phi = 0$.