$\nu = \frac{5}{2}$ qubit: what makes us hopeful?

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- adiabatic continuity between Coulomb GS and Pfaffian for systems with $N \leq 18$ electrons
- phase diagram in pseudopotential plane: Gapped phase coincides with Pfaffian phase
- preliminary evidence for braiding in systems with 4 quasiholes
- overview of the history of $\nu = \frac{5}{2}$

Adiabatic continuity between Pfaffian and Coulomb GS?

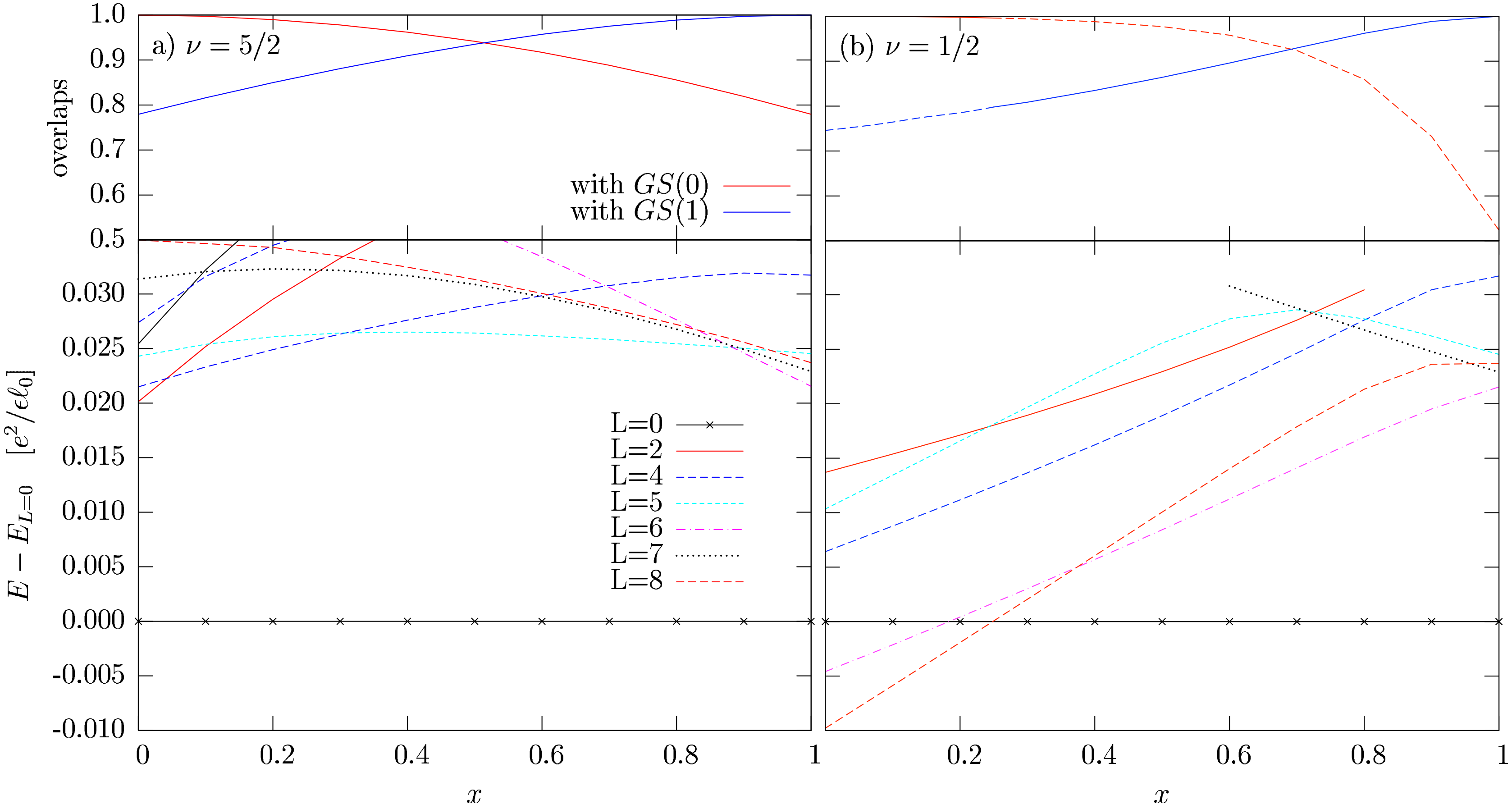
M. Storni, RM, Sankar Das Sarma (arXiv:0812.2691)

Study system in the presence of a hypothetical interaction

$$V_{int} = (1 - x)\, V_{Coulomb} + x\, \lambda V_{3body}$$

which interpolates between Coulomb and the three-body interaction when x is varied from 0 to 1. The parameter λ sets the energy scale of the 3-body interaction such that the gap at $x = 1$ coincides with the Coulomb gap in the second Landau level.

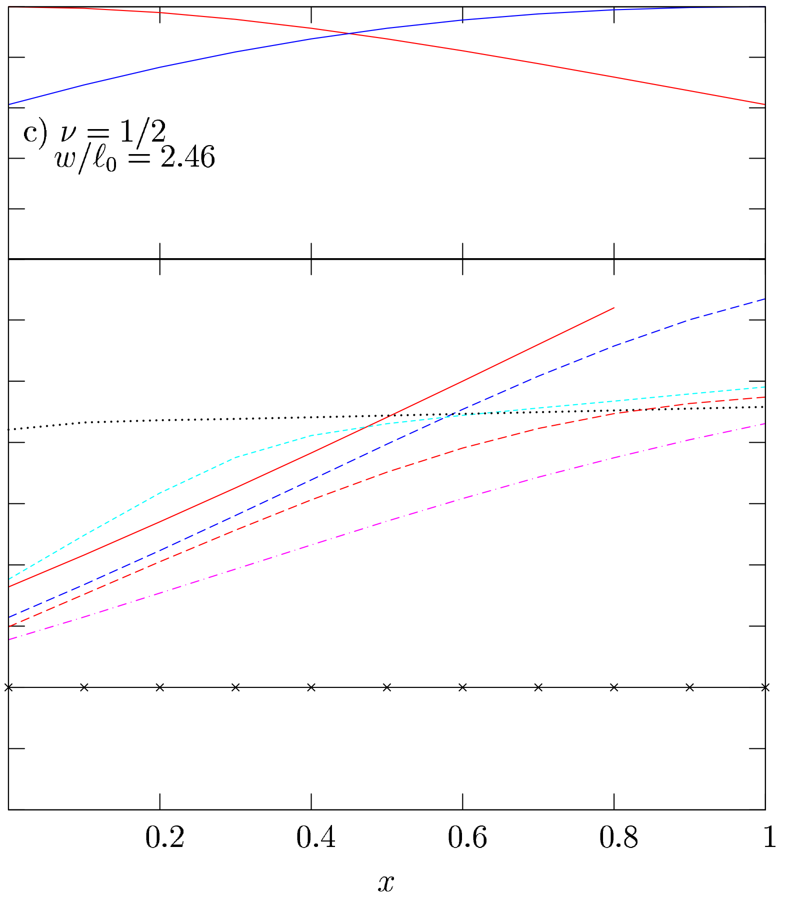
We compare $\nu = 5/2$ with $\nu = 1/2$: (N=16)



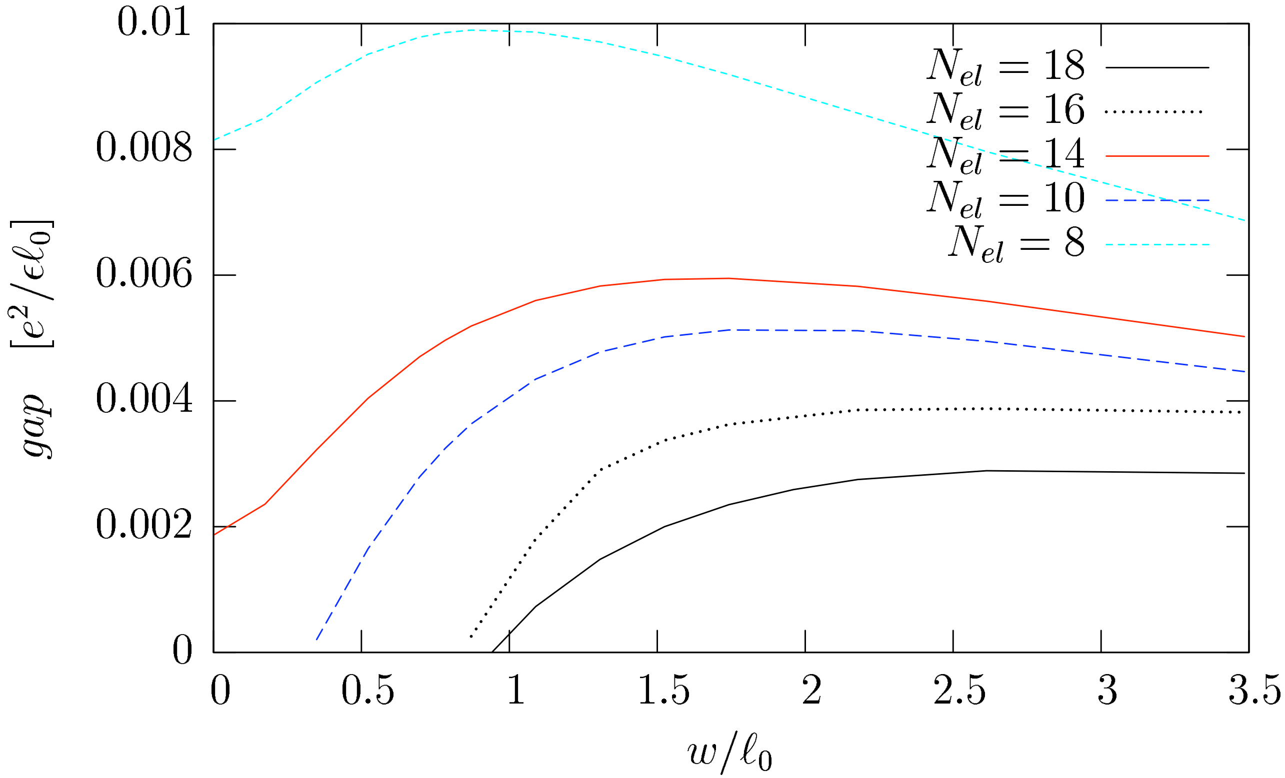
- evidence of adiabatic continuity between Pfaffian and Coulomb GS at $\nu = 2 + \frac{1}{2}$
- no adiabatic continuity between Pfaffian and Coulomb GS at $\nu = \frac{1}{2}$

What about the effects of a finite width (of the wave function perpendicular to the plane of the 2DEG?

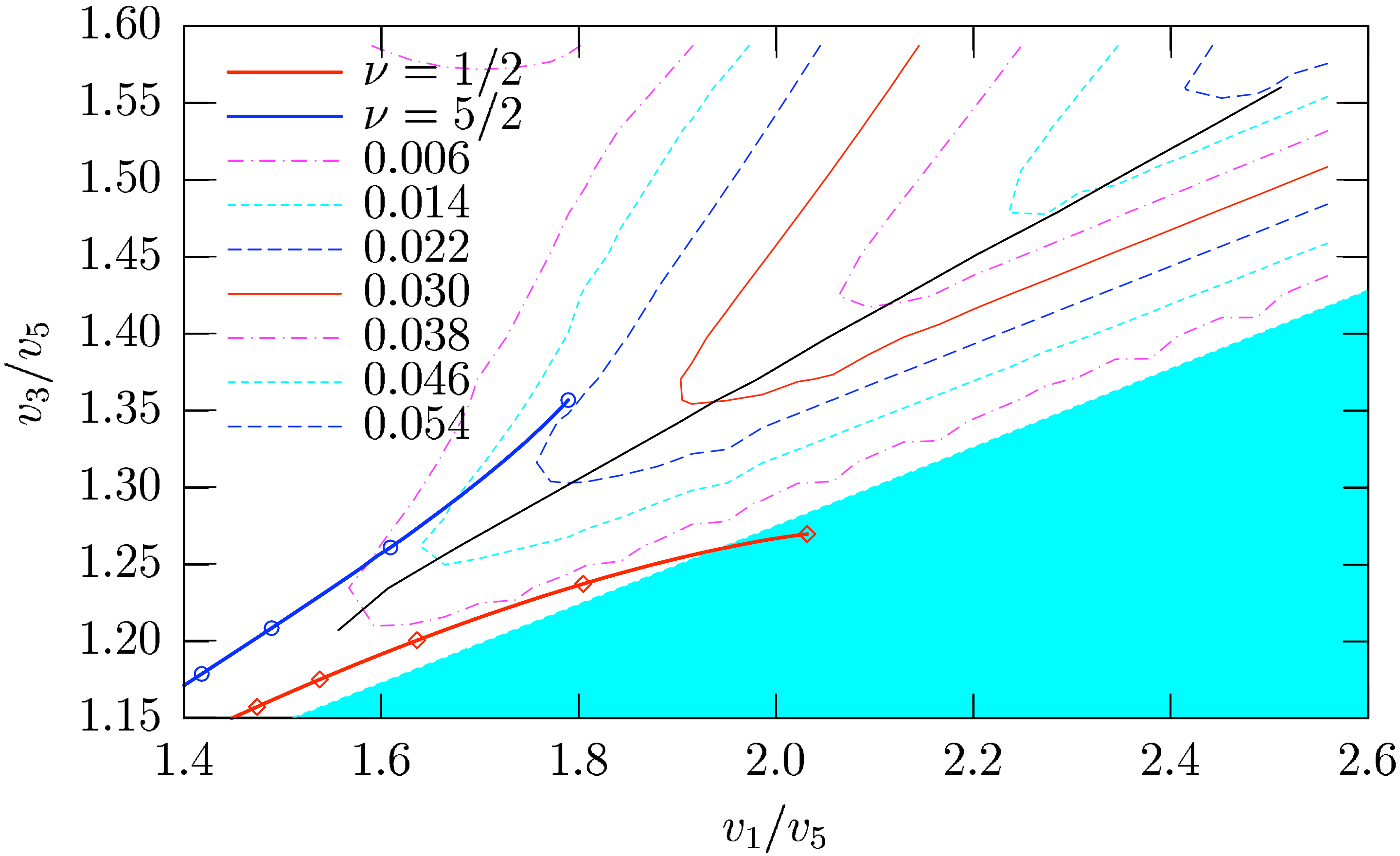
May there be a Pfaffian-like state realized at $\nu = 1/2$ for sufficiently large width?



gap for larger system sizes do not allow a definite prediction if for any value of the width parameter, there may exist a Pfaffian-phase at $\nu = 1/2$

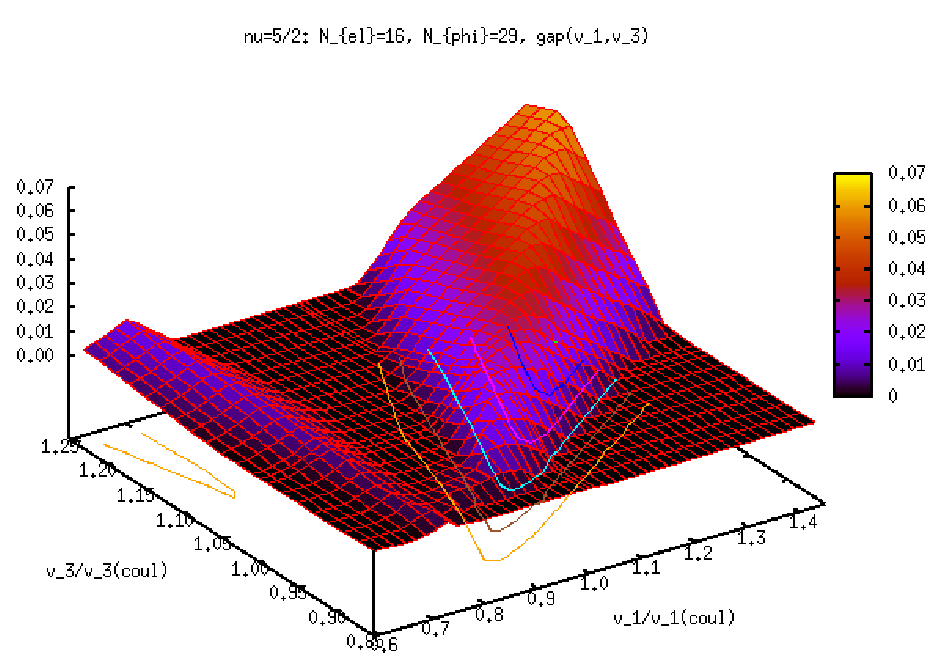


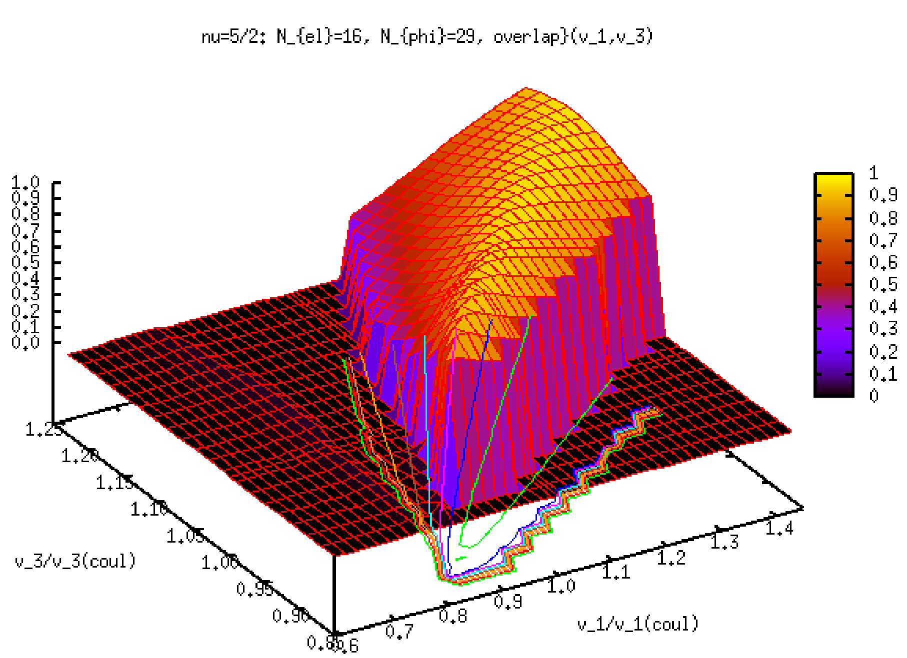
Phase diagram in the v_1, v_3-plane at N=16



Blue (Red) curve denotes the physically accessible (v_1, v_3) points in lowest (second) Landau level when varying the finite width of the wf in the z-direction. The points refer to values $w/\ell_0 = 0, 1, 2, 3, 4$ starting right. The domain coloured in light blue is compressible.

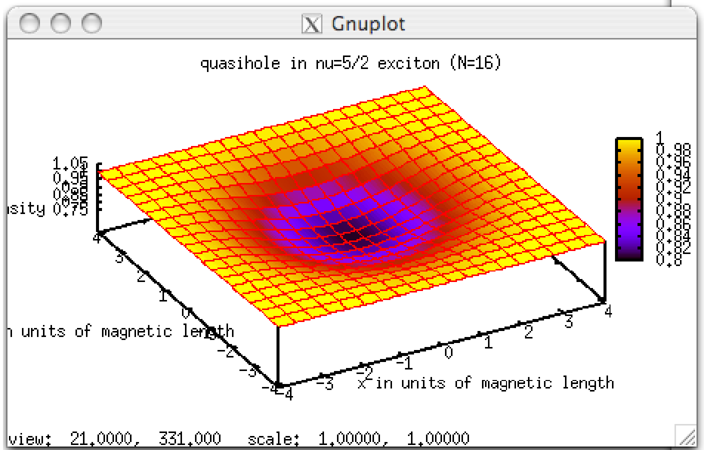
The red line referring to the (v_1, v_3)-values accessible at $\nu = 1/2$ are so close to the compressible domain that no definite conclusion can be reached on the existence of a Pfaffian phase at $\nu = 1/2$.

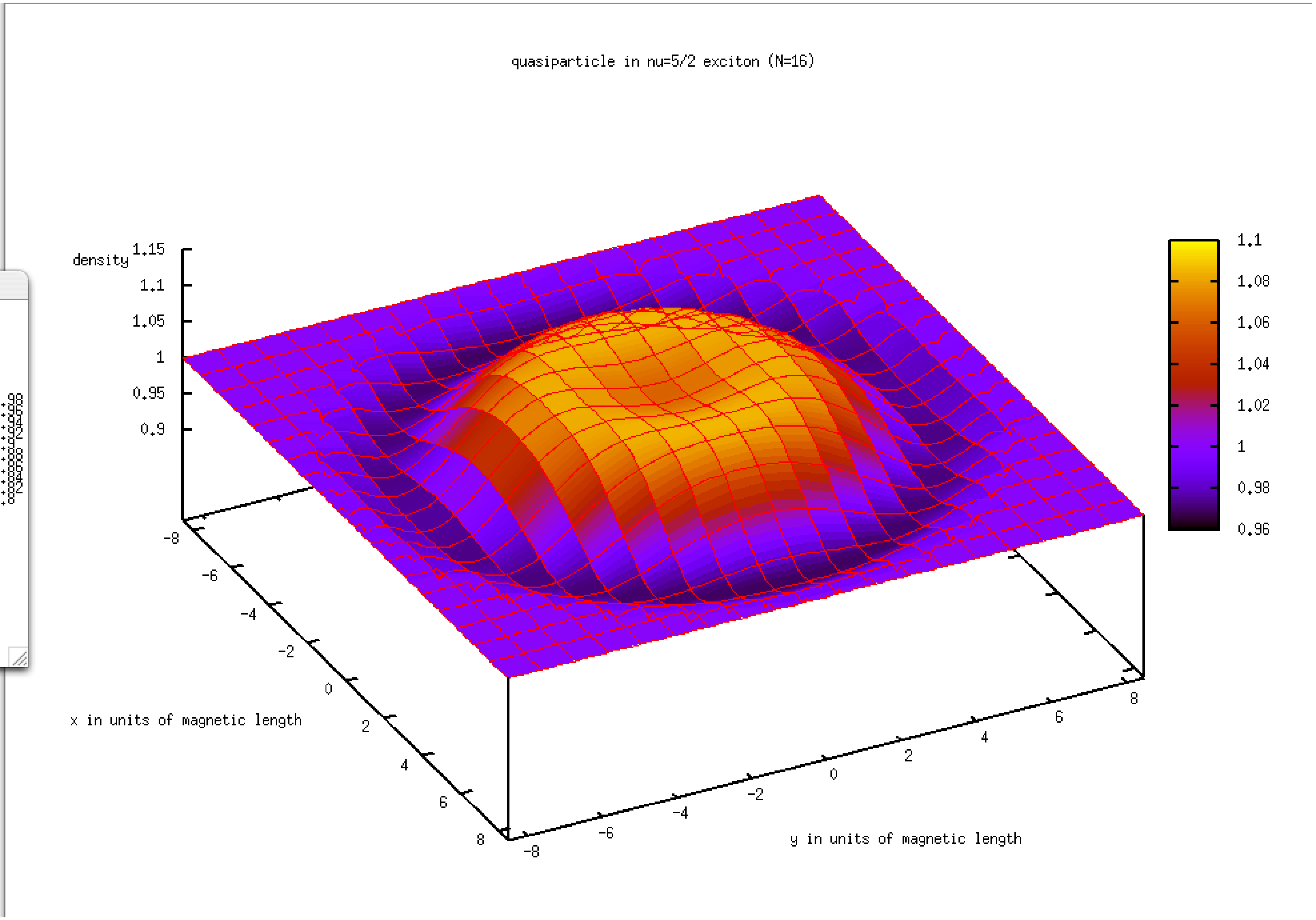




Gapped phase coincides with (v_1, v_3)-domain of finite overlap between the GS(v_1, v_3) and the Pfaffian

What about quasiholes and quasiparticles in the Coulomb GS at 5/2?

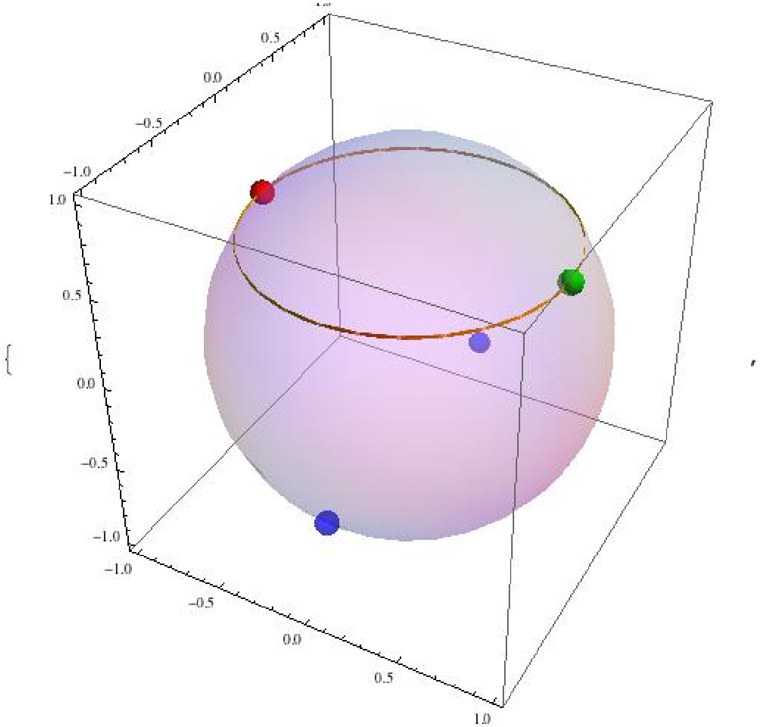




Densities shown are the real ones in the second LL (not their lowest LL image).

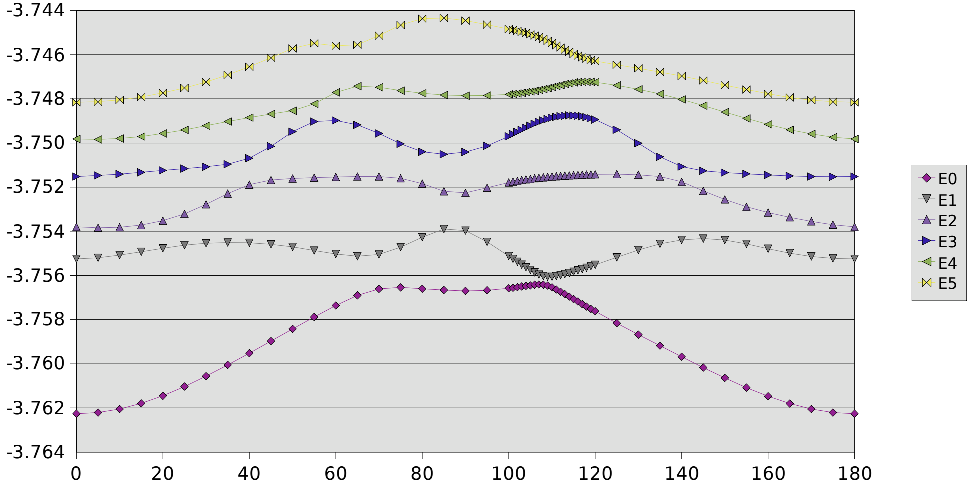
quasihole and quasiparticle are very large with diameter $d_{qh} \approx 8\ell_0$ and $d_{qp} \approx 15\ell_0$

Can we braid quasiholes? Use 4 quasiholes on sphere!

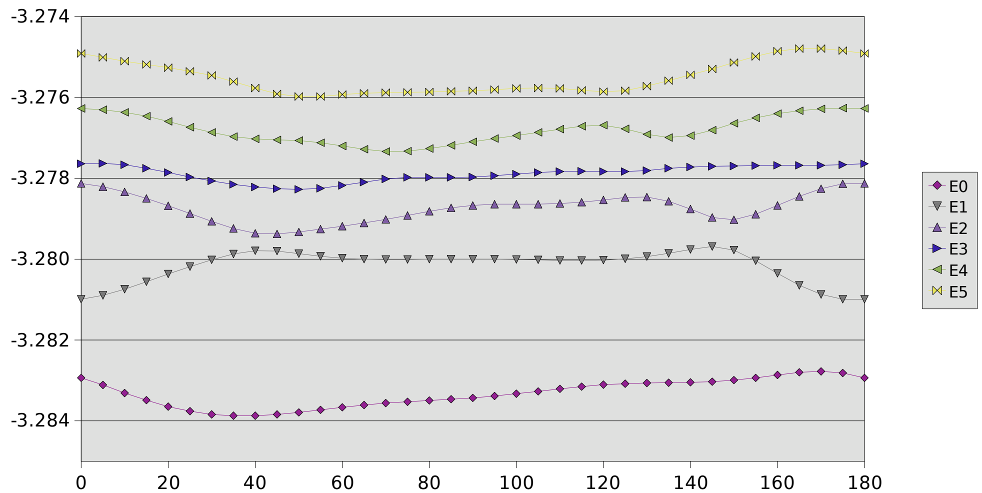


we "braid" by rotating the top two around the north pole by 180 degrees, thereby exchanging their positions.

at $\nu = 5/2$



at $\nu = 1/3$



$\nu = 5/2$

- Lowest two levels are coupled together on braiding
- Tiny level splitting at avoided crossing, but overlaps between states large when crossing
- Maximum spacing between lowest two levels $\approx$ gap / 4

$\nu = 1/3$

- Lowest level entirely uncoupled

This makes us hopeful:

- Adiabatic continuity at $\nu = 5/2$ between Pfaffian and Coulomb GS for all sizes studied ($N \leq$18).
- The gapped phase at $\nu = 5/2$ observed in the plane of pseudopontials v_1, v_3 coincides with the domain of non-zero overlap between the overlap of the GS(v_1, v_3) with the Pfaffian state.
- Maximum overlap between GS and Pfaffian essentially coincides with gap maximum when varying v_1 and keeping v_3 fixed.
- Evidence of braiding seen in evolution of spectrum when positions of quasiparticles are quasi-adiabatically interchanged

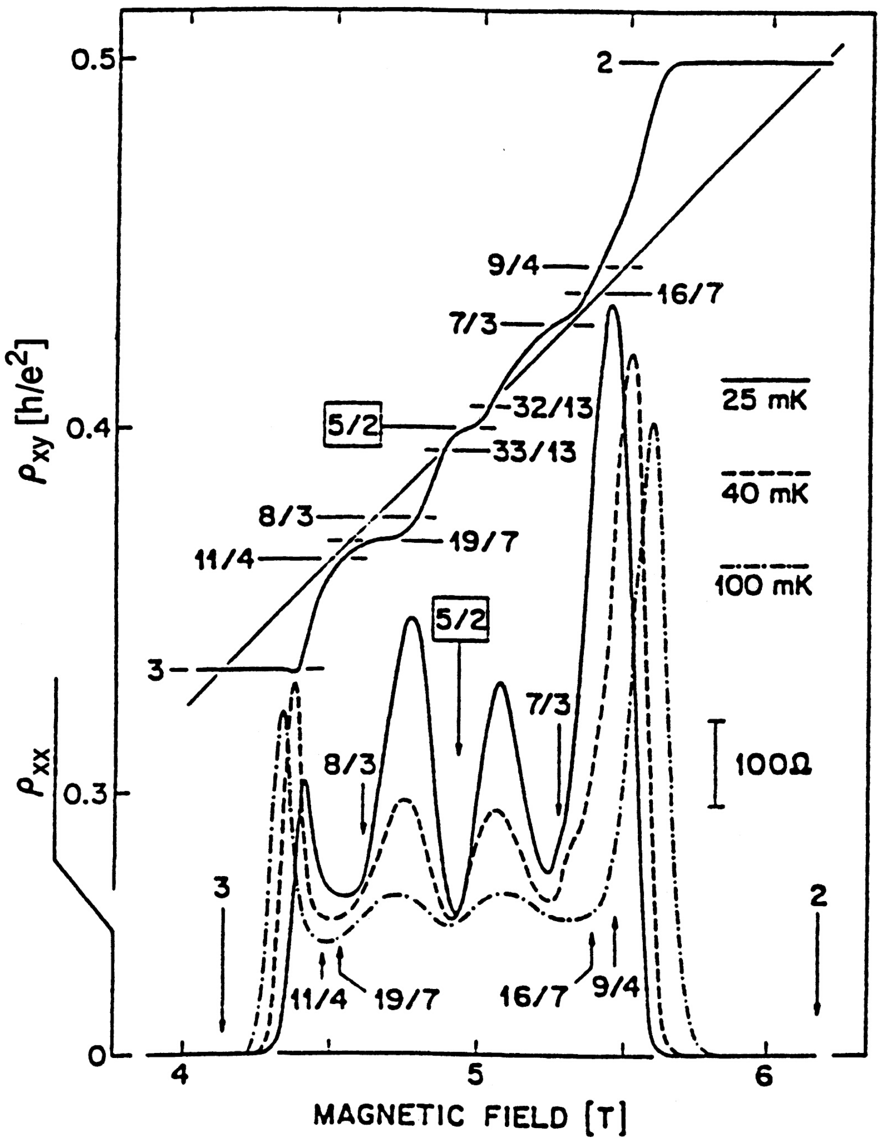
Open Problems:

- No adiabatic continuity at $\nu = 1/2$ between Pfaffian and Coulomb GS. Yet, for some system sizes and for finite width, adiabatic continuity is observed: Existence of Pfaffian state at $\nu = 1/2$ in the thermodynamic limit under special conditions?
- What is the nature of the phas at $\nu = 5/2$ when v_1 is reduced by about 10-15 percent below its Coulomb value in the second LL?
- Theory of disorder effects in FQH states needed!
- Role of spin at $\nu = 5/2$?

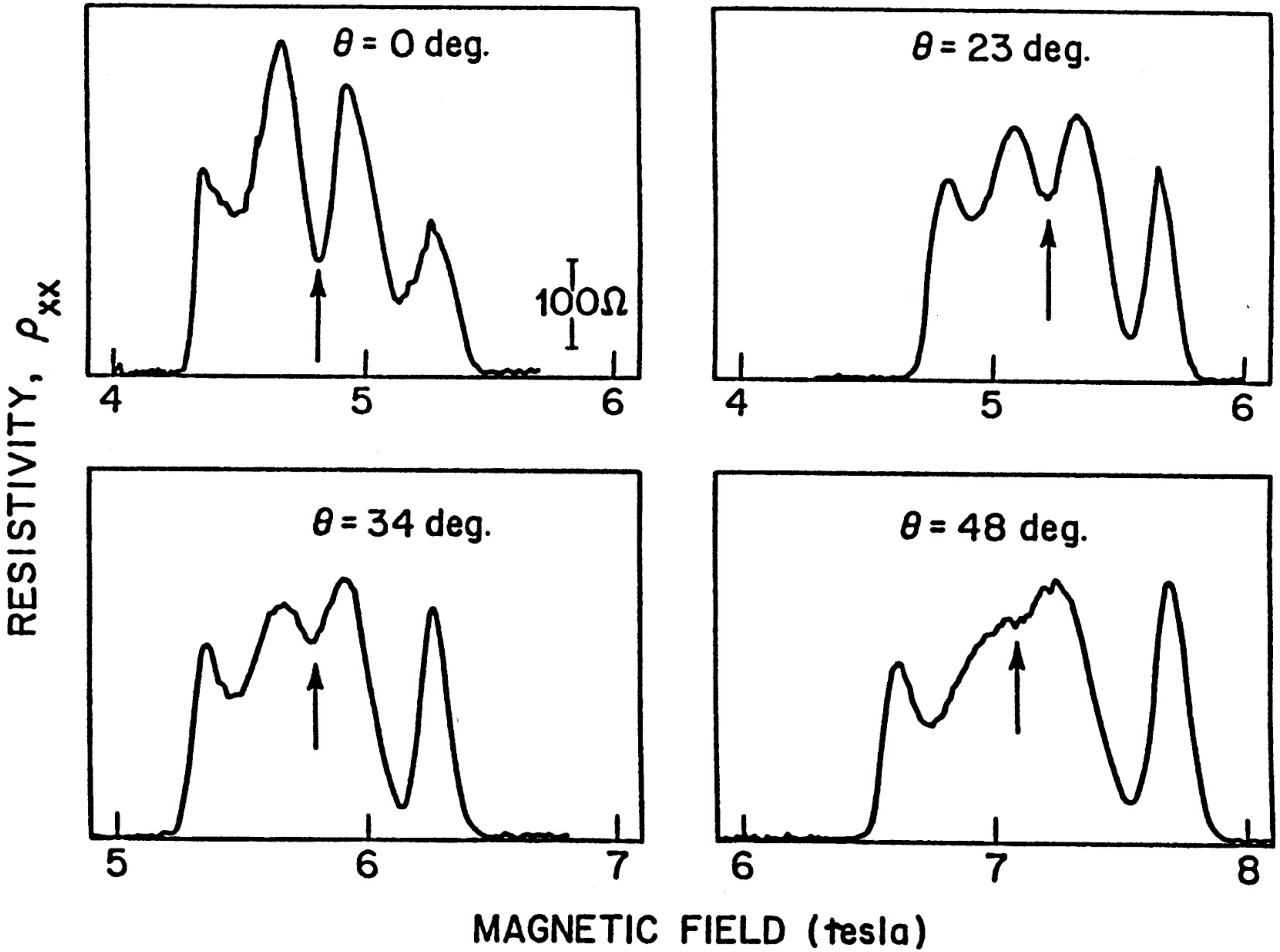
Appendix: a short overview of the early history of $\nu = 5/2$

Cf. also references concerning $\nu = 5/2$ in Nayak, Simon, Stern, Freedman and Das Sarma, RMP 80, 1083 (2008)

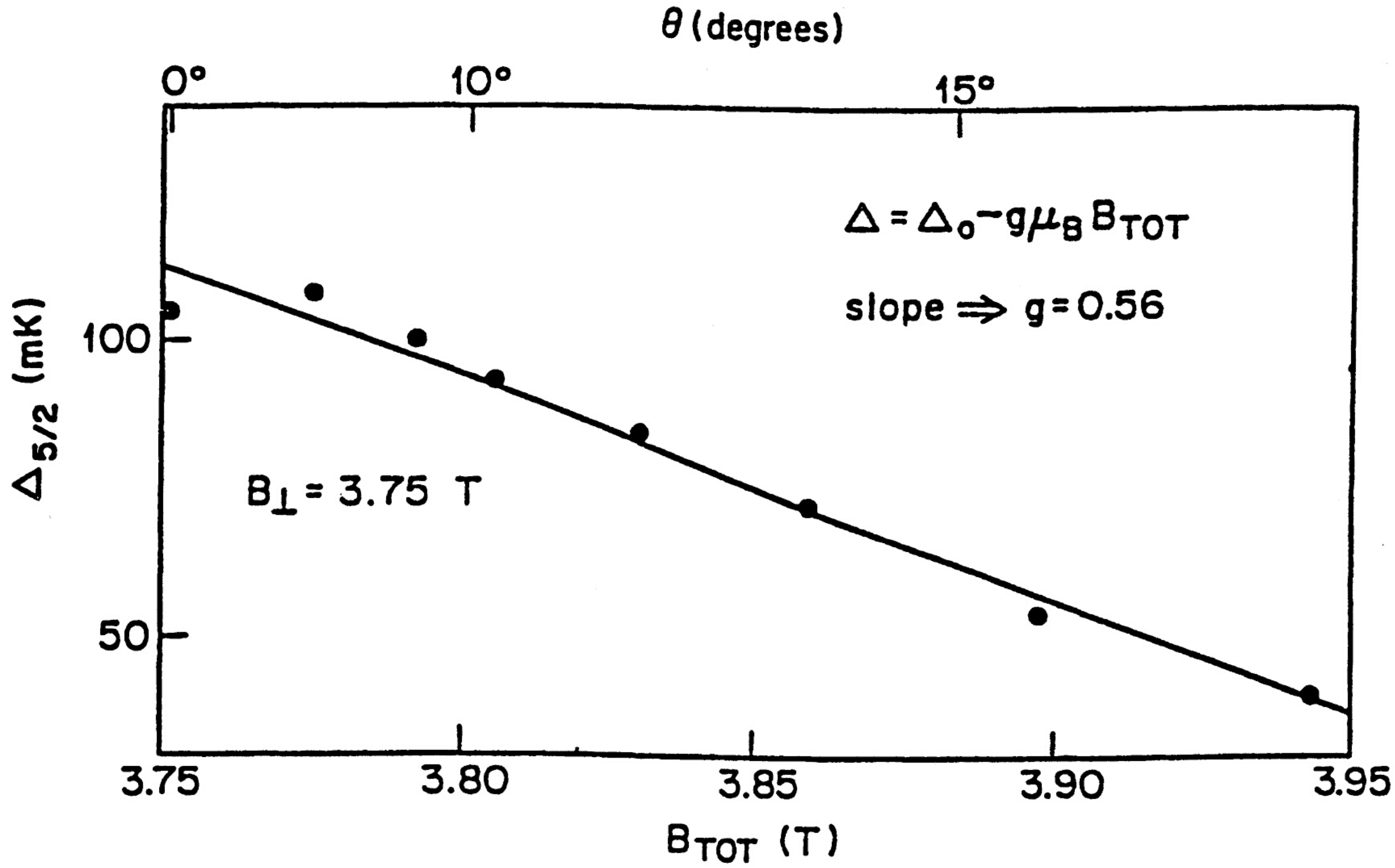
First observation of FQH state at $\nu = 5/2$: Willett et al. PRL 59, 1776 (1987)



Collapse of $\nu = 5/2$ state in tilted field:
Eisenstein et al. PRL 61, 997 (1988)



Activation energy of ρ_{xx} in a tilted field: Eisenstein et al., Surf. Sci. 229, 31 (1990)



Conclusions from Experiment

FQH-plateau at $\nu = 5/2$

Gap decreases in tilted field – gap reduction $\propto B_{tot}$

Transition to compressible state for $B_{tot} \geq B_{tot}^c$

Simplest scenario - generally believed for 10 years until 1998

- FQH state at most partially polarized or fully unpolarized (cf. $\nu = 8/5$)
- lowest energy excitations involve spin flip - gain in Zeeman energy
- Transition to gapless fully polarized state at $B = B_c$ induced by Zeeman energy

FQH states in half-filled Landau levels
Theoretical ideas

Halperin (1983): generalization of Laughlin wf (bilayers or spin)

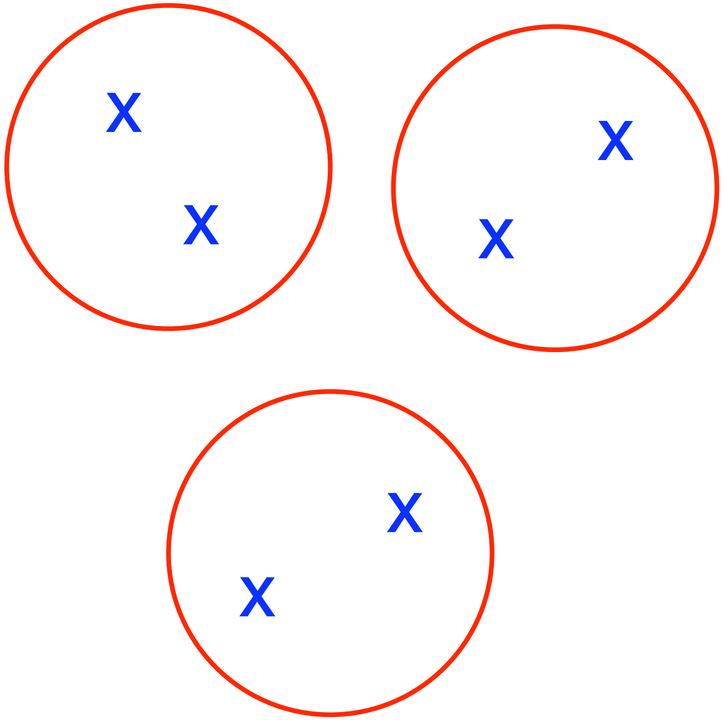
$$\Psi_{mmn} = \prod_{i<k}^{N/2} (z_i - z_k)^m (w_i - w_k)^m (z_i - w_k)^n \times \text{Gaussian}$$

$$\text{Fill Factor } \nu = \frac{2}{m+n}$$

- example $\nu = 2/5$ spin-singlet state for $m = 3, \ \ n = 2$ (observed by Eisenstein et al. 1988)
- $\nu = 1/2$: for bilayer systems: z_i electrons in layer 1, w_i electrons in layer 2

Halperin (1983) Pair Wave Function

grouping electrons into pairs, triplets or k-tuplets



charge of cluster $e^* = k\, e$

magnetic length for electrons $\ell_0 = \sqrt{\hbar c/eB}$

magnetic length for clusters $\ell_* = \sqrt{\hbar c/k\, e\, B} = \ell_0/\sqrt{k}$

Filling fraction for electrons $\nu = n \times 2\pi \ell_0^2$

density of clusters $n^* = n/k$

What is filling ν^* for cluster particles? $\nu^* = n^* \times 2\pi \ell_*^2 = \frac{\nu}{k^2}$

Laughlin wf for k-tuplet of electrons

$$\psi_{\nu^*} = \prod_{i<j}^{N/k} (Z_i - Z_k)^{m^*} \times \exp(-\sum_{i=1}^{N/k} |Z_i|^2/(4\ell_*^2))$$

What are the charges of excitations in this system?

Form quasihole wave function in terms of cluster coordinates Z_i with quasihole at Z_0

$$\psi_{\nu^*}^{(+)} = \prod_{i=1}^{N/k} (Z_i - Z_0) \times \psi_{\nu^*}$$

Quasihole charge $q^* = \frac{e^*}{m^*}$

Paired state $(k = 2)$ at $\nu = \frac{1}{2}$, $\nu^* = \frac{1}{8}$

$$m^* = \frac{1}{\nu^*} = \frac{k^2}{\nu} = 8, \qquad q^* = \frac{e}{4}$$

Adding 1 unit of flux creates 2 quasiholes with charge $\frac{e}{4}$

Haldane and Rezayi (1988) spin-singlet state (s-wave paired state).

Let $z_i = z_i^{\uparrow}$ and $w_i = z_i^{\downarrow}$

$$\Psi_{HR} = \Psi_{331} \times \text{permanent}\frac{1}{z_i - w_k} \quad \equiv \Psi_2 \times det\frac{1}{(z_i^{\uparrow} - z_k^{\downarrow})^2}$$

Moore and Read (1991) cf. also **Greiter, Wen and Wilczek (1991)**: spin polarized p-wave paired wave function

$$\Psi_{MR} = \Psi_2 \times \text{Pf}\frac{1}{z_i - z_k}$$

Pfaffian (antisymmetric function of all variables) defined by

$$\text{Pf}\frac{1}{z_i - z_k} = \sum_{P \in S_N} (-1)^{\sigma_P} \prod_{i=1}^{N/2} \frac{1}{z_{P[i]} - z_{P[i+N/2]}}$$

is exact ground state (zero-energy state) for special 3-body interaction

$$V_{3body} = \prod_{i<k<m}^{N} \mathcal{S}\left(\nabla_k^2 \nabla_m^4 \delta(z_i - z_k)\delta(z_i - z_m) \right)$$

Note $\Psi_{MR} \equiv A\Psi_{331}$ on disk and sphere, A is the antisymmetrizer. More complicated on torus.

2 quasihole excitation:

$$\Psi_{MR+2qh} = \Psi_2 \times \text{Pf}\,\frac{(z_i - w)(z_k - u) + (u \longleftrightarrow w)}{z_i - z_k}$$

4 quasihole excitation:

$$\Psi_{MR+4qh} = \Psi_2 \times \text{Pf}\,\frac{(z_i - w_1)(z_i - u_1)(z_k - w_2)(z_k - u_2) + (u_\ell \longleftrightarrow w_\ell)}{z_i - z_k}$$

Note: There exists a second, linearly independent wf with 4 quasiholes at positions w_1, u_1, w_2, u_2: interchanging $u_1 \leftrightarrow w_2$

$$\Psi'_{MR+4qh} = \Psi_2 \times \text{Pf}\,\frac{(z_i - w_1)(z_i - w_2)(z_k - u_1)(z_k - u_2) + (u_\ell \longleftrightarrow w_\ell)}{z_i - z_k}$$

Nayak and Wilczek (1996), Milovanovic and Read (1996):

$2n$-quasiholes: 2^{n-1} fold degeneracy for Pfaff-interaction $\Longrightarrow$ non-abelian statistics

Halperin (1983): microscopic implementation of pair wf (RM+Halperin 1986,1987 and RM 1998)

$$\Psi_{HM} = \Psi_1 \; \mathcal{S} \; (\prod_{i<k}^{N/2} (z_{2i}z_{2i-1} + z_{2k}z_{2k-1} - 2Z_iZ_k)^2 \,) \quad Z_i = \frac{1}{2}(z_{2i} + z_{2i-1})$$

2 quasihole excitation:

$$\Psi_{HM+2qh} = \Psi_1 \; \mathcal{S} \, \{ \; \prod_{i=1}^{N/2}((z_{2i} - w)(z_{2i-1} - u)) \; \prod_{i<k}^{N/2} (z_{2i}z_{2i-1} + z_{2k}z_{2k-1} - 2Z_iZ_k)^2 \, \}$$

Different pairing mechanism in Ψ_{MR} vs. Ψ_{HM}

Moore Read Pfaffian: Ψ_{MR} characterized by non-abelian statistics $(q = 1/4)$

Halperin pair wf: Ψ_{HM} abelian(?) fractional statistics $(q = 1/4)$

Unbiased numerical study (RM, Phys. Rev. Lett. 80, 1505 (1998))

- Study spin-polarized and -unpolarized systems
- Exact diagonalization
- spherical geometry (Haldane)
- Neglect Landau level mixing - study half-filled $n = 1$ Landau level

FQH states on sphere:

Unique relation between number of electrons N and number of flux units N_Φ

$$N_\Phi = \frac{1}{\nu} N \;+\; k \qquad k:\ \text{shift (Wen and Zee (92))}$$

Examples:

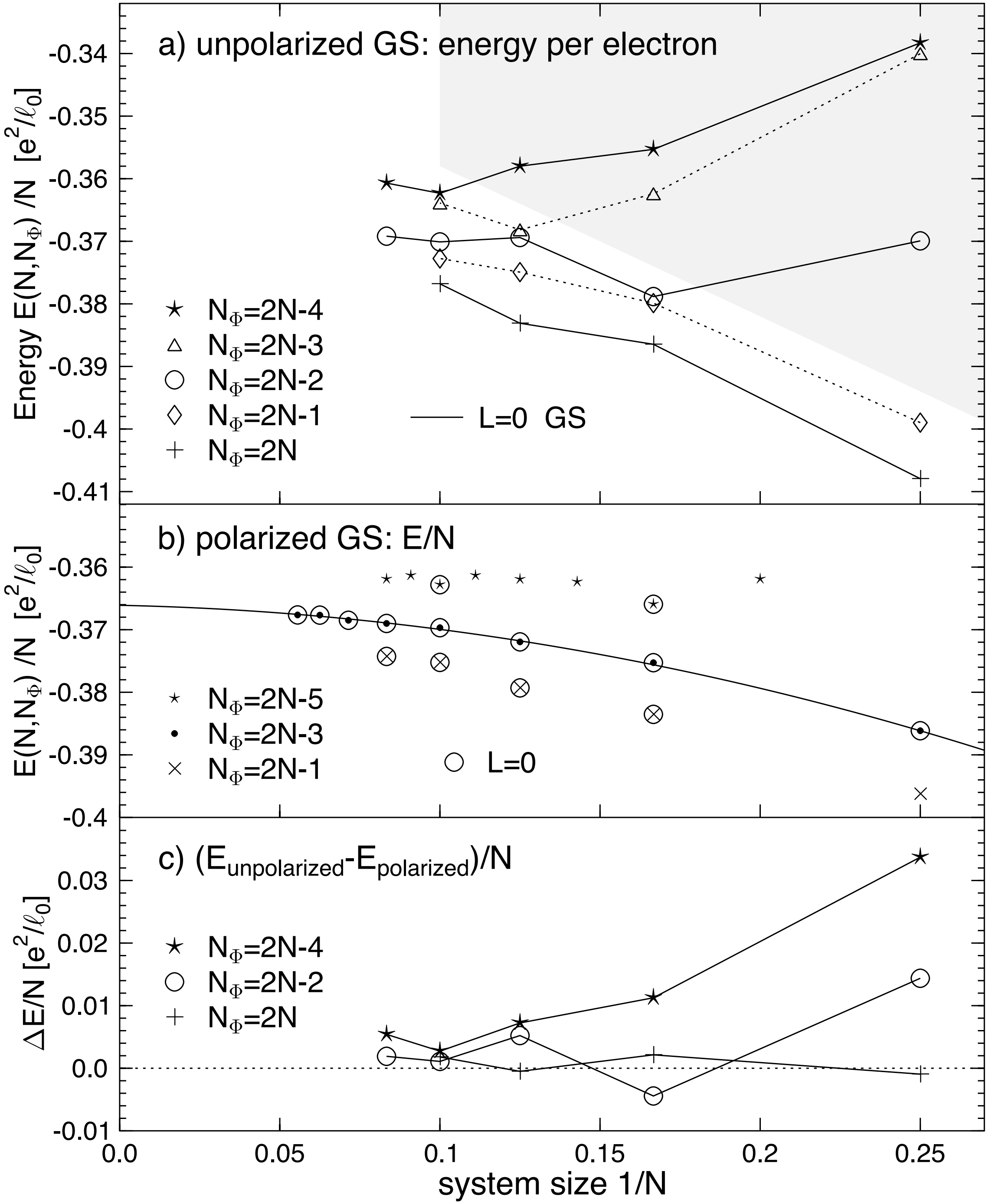
$$\nu = 1/3:\ \ N_\Phi = 3N - 3,\ \ k = -3$$

$$\nu = \frac{2}{5}:\ \text{polarized}\ \ N_\Phi = \frac{5}{2}N - 4, \quad k = -4 \qquad \text{unpolarized}:\ N_\Phi = \frac{5}{2}N - 3, \quad k = -3$$

Shift k for FQH state at $\nu = 5/2$ unknown, predictions:

$$k = -4 \quad \text{Haldane} - \text{Rezayi}$$

$$k = -3 \quad \text{Moore} - \text{Read},\ \text{Halperin pair wf}$$



a) unpolarized GS: energy per electron
Energy E(N,NΦ) /N [e²/ℓ0]
NΦ=2N-4
NΦ=2N-3
NΦ=2N-2
NΦ=2N-1
NΦ=2N
L=0 GS
b) polarized GS: E/N
E(N,NΦ) /N [e²/ℓ0]
NΦ=2N-5
NΦ=2N-3
NΦ=2N-1
L=0
c) (Eunpolarized-Epolarized)/N
ΔE/N [e²/ℓ0]
NΦ=2N-4
NΦ=2N-2
NΦ=2N
system size 1/N

Unpolarized system $S = 0$

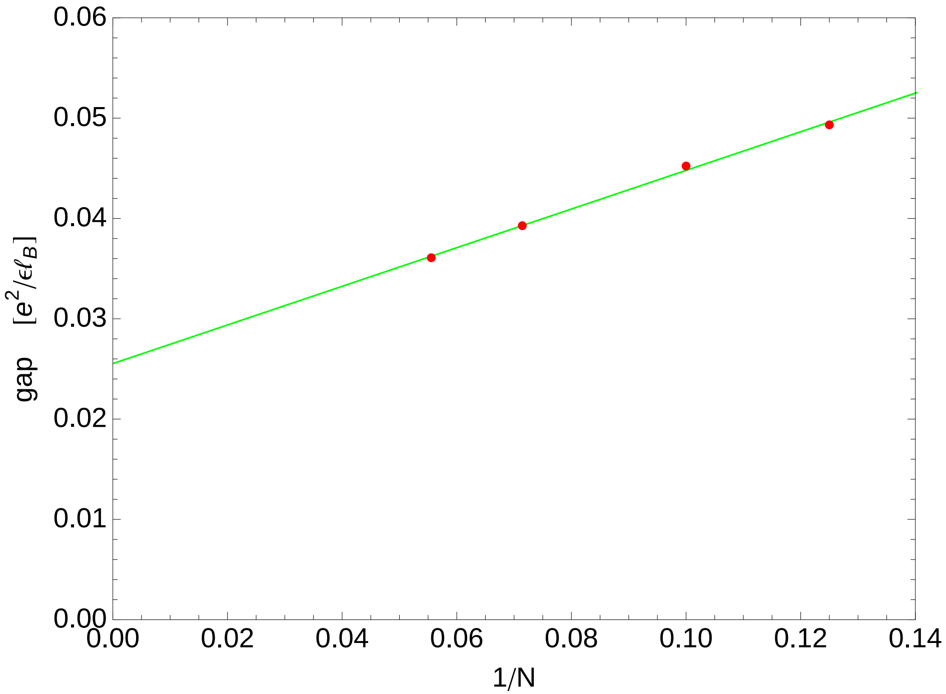
- very large finite size effects
- spin-singlet is GS only at $N = 6$, $N_\Phi = 10$ (even for vanishing Zeeman energy, $g = 0$)
- no consistent energy gap values
- no local singlet: pair correlation function resembles that of polarized state with long-wavelength spinwave excitation to establish $S = 0$. Real GS would be polarized.

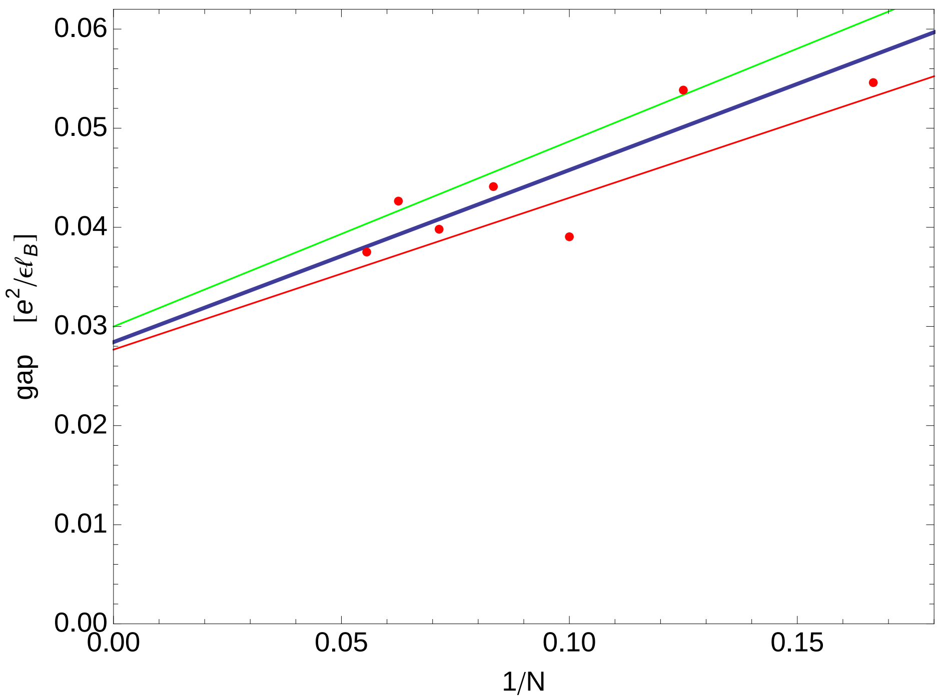
Polarized system

- $L = 0$ GS at $N_\Phi = 2N - 3, \quad k = -3$ for all even tested $(N \leq 18)$
- For all other values of the shift k we obtain GS with $L = O(N^0) = O(1)$, consistent with excitations in an incompressible background.

Is there a gap?

Energy gap at $\nu = 5/2$





Left plot: gap derived from individual quasiparticle and quasihole excitations

Right plot: gap derived from exciton with largest even angular momentum $L \leq N/2$

Note: these energy values have been corrected for finite size effect due to Coulomb attraction between quasihole at north pole and quasiparticle at south pole (with separation equal to $2\,R_{sphere} \sim \sqrt{N}$) giving rise to a $1/\sqrt{N}$ contribution to the exciton energy.

$$\Delta \approx 0.027 \pm 0.003\ e^2/\epsilon\ell_0$$

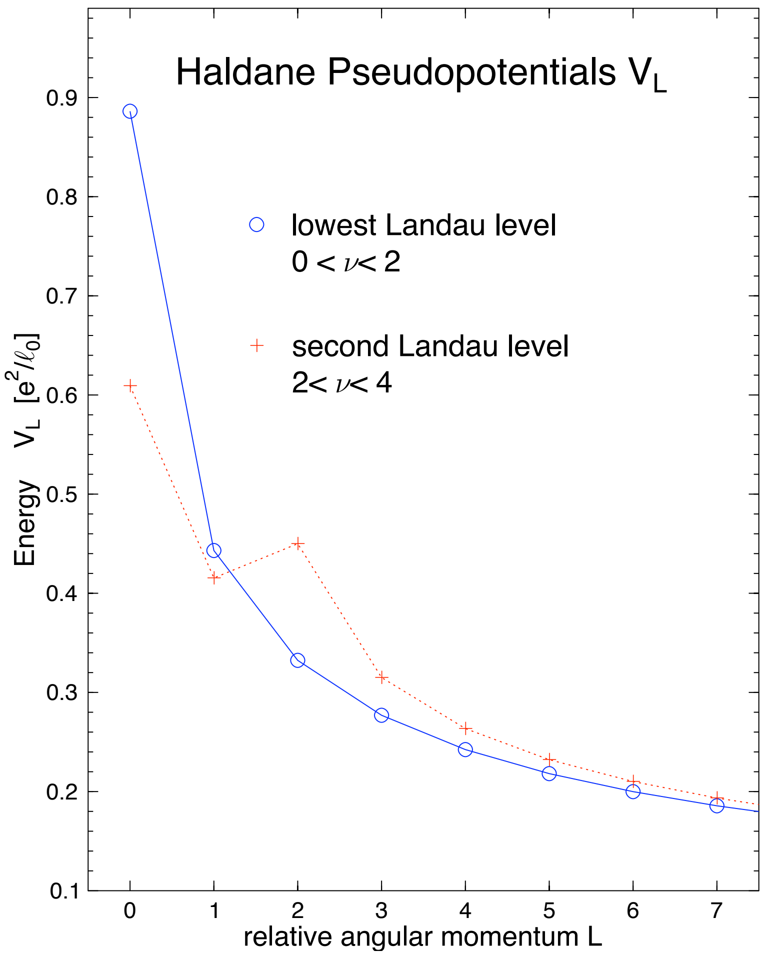
The Haldane pseudopotential

2 electrons in lowest Landau level with relative angular momentum L

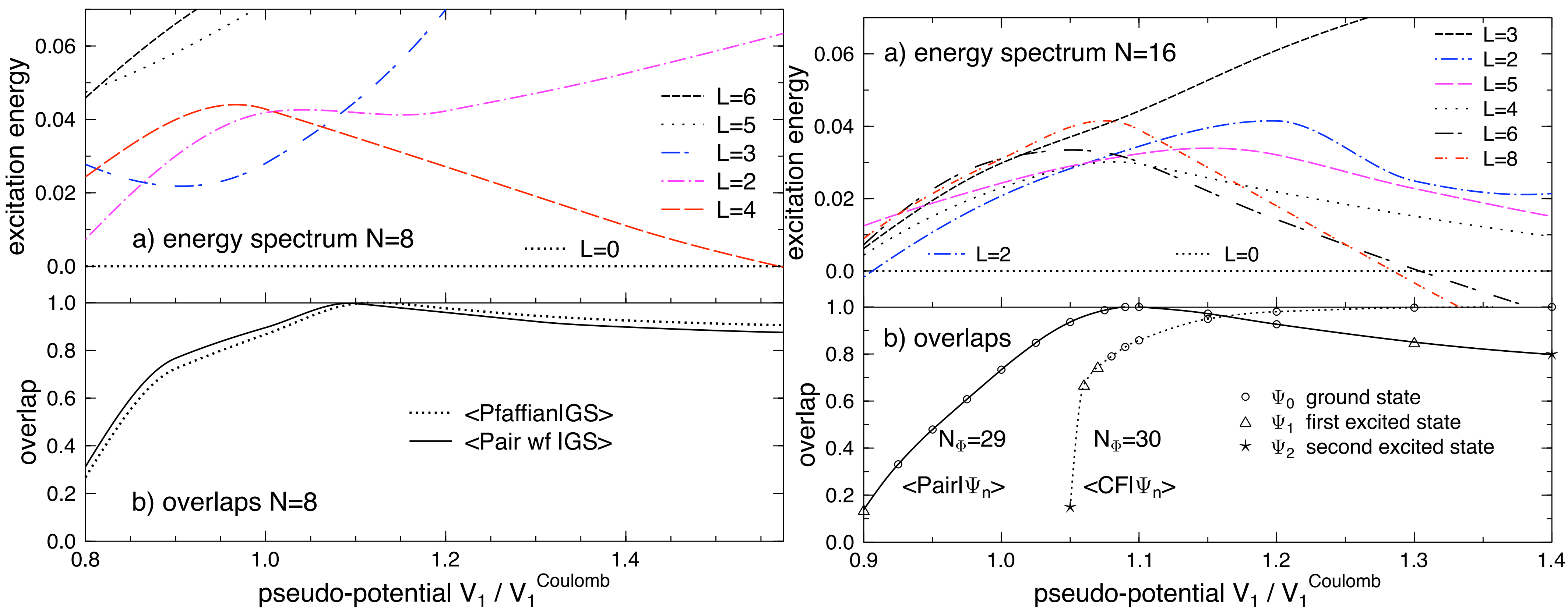
$$\phi_{L,n}(z_1, z_2) = (z_1 - z_2)^L (z_1 + z_2)^n e^{-(|z_1|^2+|z_2|^2)/4}$$

Haldane pseudoptential: energy of two-electron state

$$V_L = \frac{< \phi_{L,n}|V|\phi_{L,n} >}{< \phi_{L,n}|\phi_{L,n} >}$$

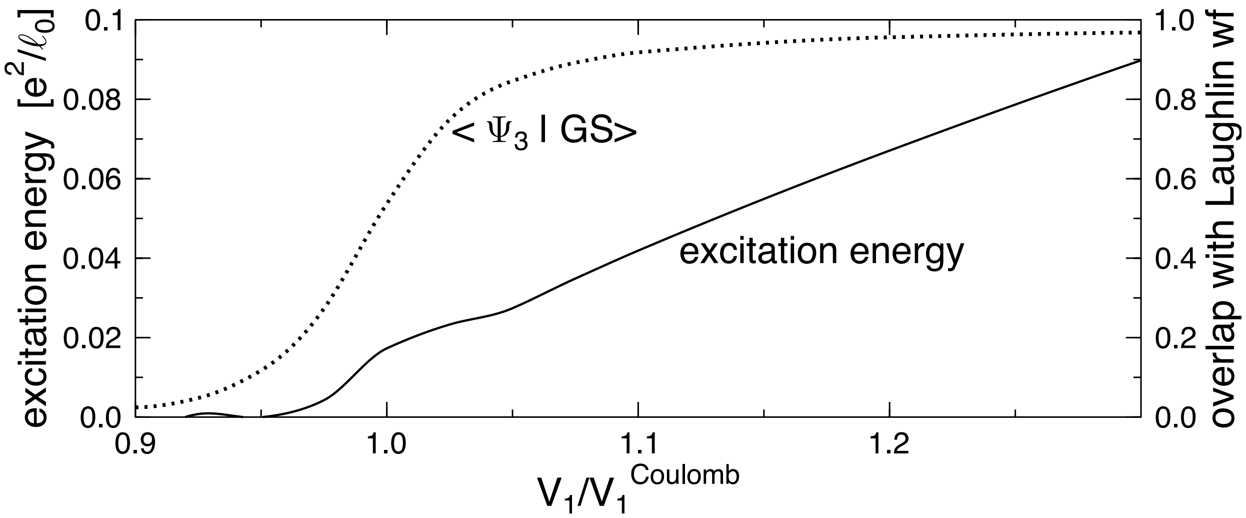


Test system by varying V_1



- Paired state stable in the window $0.95 \lesssim V_1/V_1^{Coulomb} \lesssim 1.2$
- Gap $\Delta_{5/2} \approx 0.025e^2/\epsilon\ell_0$ at $V_1 = V_1^{Coulomb}$
- Gap is maximum for V_1 which maximizes overlap of GS with Ψ_{MR} or pair wave function Ψ_{HM}
- For $V_1 \gtrsim 1.2$ transition to Composite Fermion liquid state (like in the lowest half-filled Landau level)
- For $V_1 < 0.9$ transition to symmetry broken state (at $L = 2$). Charge density wave state à la Fogler and Shklovskii.

Compare to $\nu = 7/3$ state, also observed in Eisenstein's experiment



- Excitation gap $\Delta_{7/3} \approx 0.02e^2/\epsilon\ell_0$ at $V_1 = V_1^{Coulomb}$, similar to $\Delta_{5/2}$
- For $V_1 < 0.96$ transition to symmetry broken state (here at $L = 2$).

J.P. Eisenstein (1998) private communication

- In tilted field, activation energy at $\nu = 7/3$ decreases with increasing tilt angle in a similar way as at $\nu = 5/2$.
- The $\nu = 7/3$ state also disappears for large enough tilt angle

comments on $\nu = 7/3$

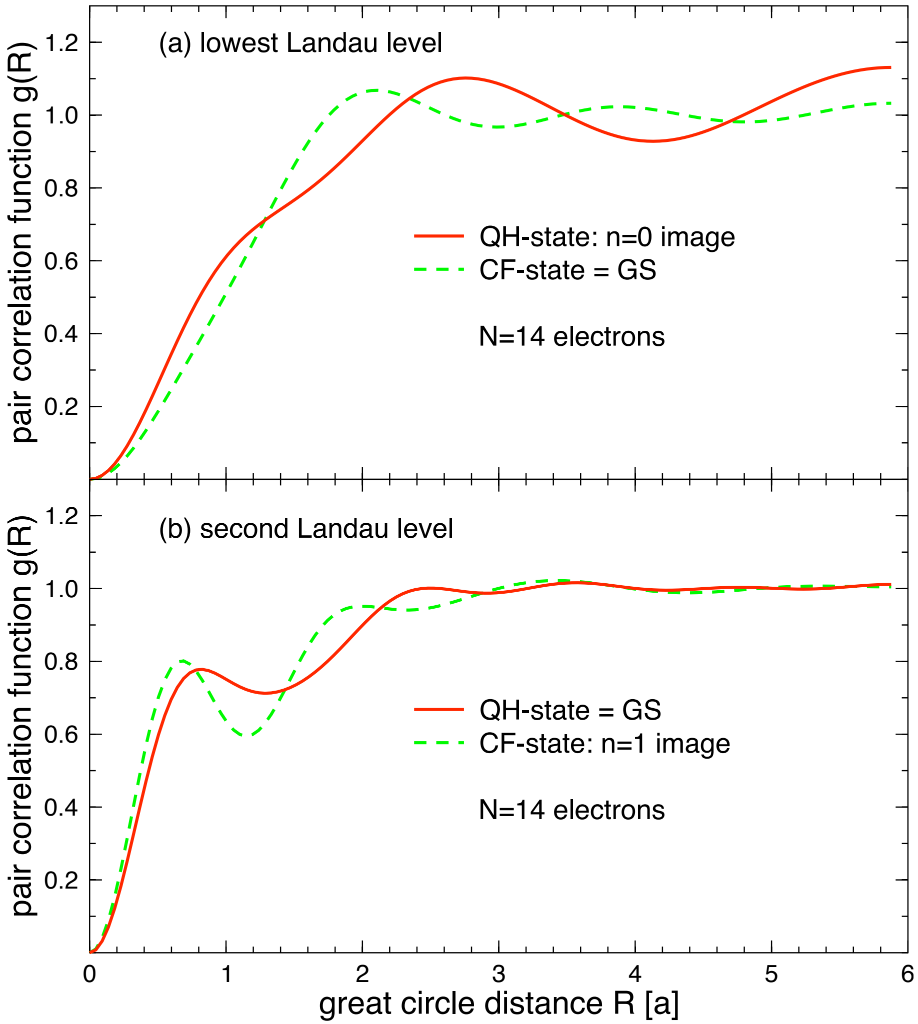
- gap reduction due to Zeeman energy is extremely unlikely
- The GS is most likely polarized
- If excitations involved reversed spin, gap would increase with increasing tilt
- If excitations are polarized, the gap does not depend on the Zeeman energy

scenario for gap reduction in tilted field at $\frac{5}{2}$ and $\frac{7}{3}$

- both $\frac{5}{2}$ and $\frac{7}{3}$ FQH states are spin-polarized
- interaction is modified by tilting the magnetic field
- reduction of gap at $\frac{5}{2}$ and $\frac{7}{3}$
- transition to compressible state at sufficiently large tilt angle, possibly charge density wave à la Fogler and Shklovskii ???

Why does this happen at $\nu = \frac{5}{2}$ but not at $\nu = \frac{1}{2}$?

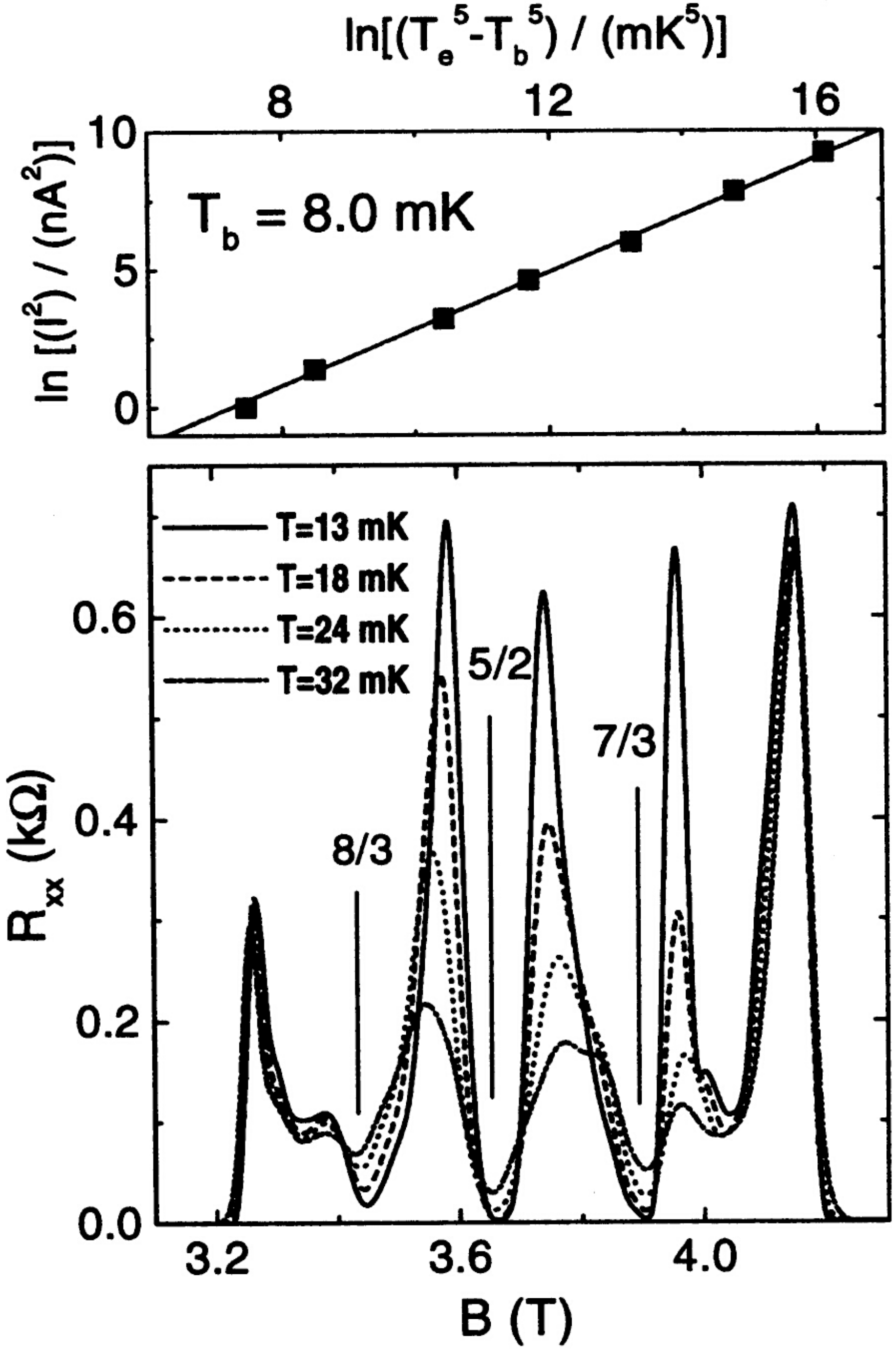
Compare Coulomb GS in half filled lowest LL with that in half-filled second LL



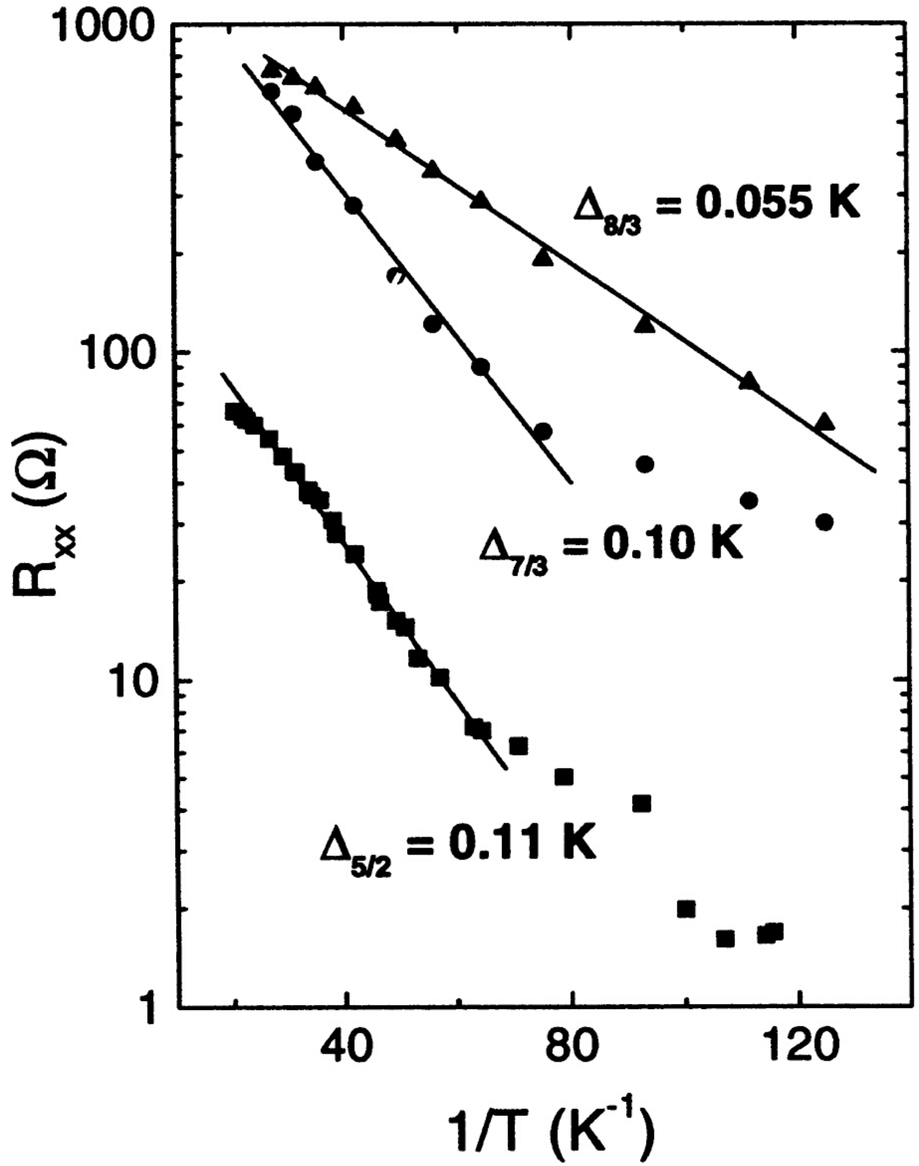
Coulomb energy is low if correlation hole is as close to origin as possible

- composite fermion liquid best at $\nu = 1/2$
- paired state is best at $\nu = 5/2$

Exact quantization of even-denominator FQH state at $\nu = 5/2$ W. Pan et al., Phys. Rev. Lett. 83, 3530 (1999)

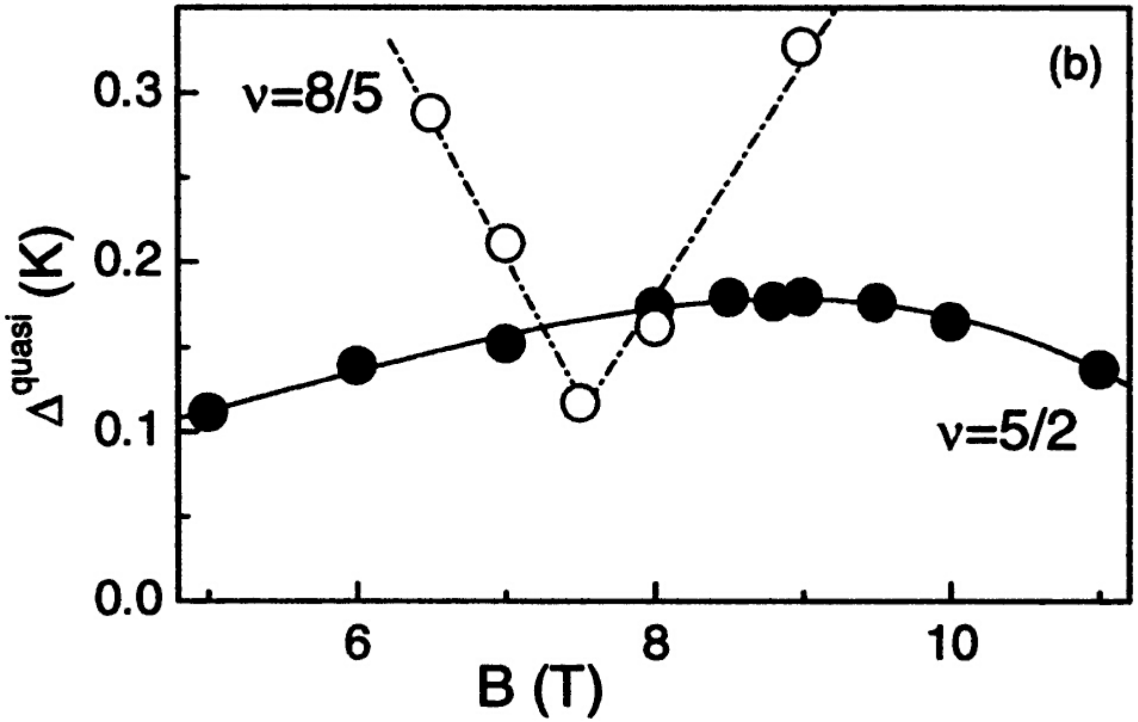


W. Pan et al., Phys. Rev. Lett. 83, 3530 (1999)



W. Pan et al. Sol. St. Comm. 119, 641 (2001): Gap as a function of electron density

$\nu = 5/2$ vs. $\nu = 8/5$ (unpolarized at low density)

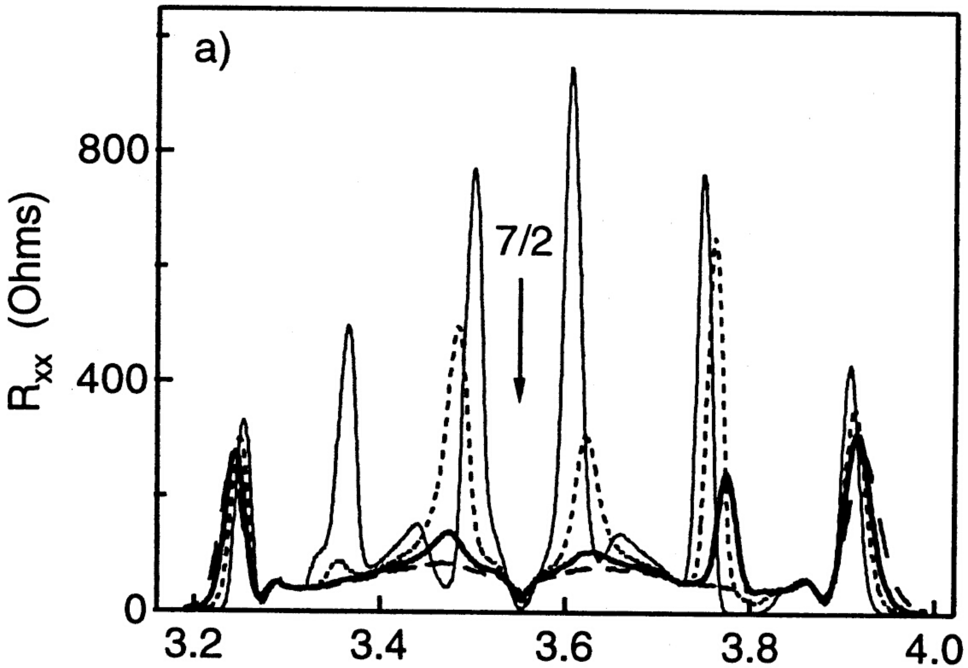


Smooth dependence of gap on magnetic field, no break in slope or sign of discontinuity, indicates that neither ground- nor excited state at $\nu = 5/2$ is changing its character, while Zeeman energy changes by factor ≈ 2K.

Thus, no phase transition in the spin sector appears to occur in this large range of fields. At the largest field the system is likely polarized, implying that it should be SPIN-POLARIZED in the whole range of magnetic field shown.

Eisenstein et al. PRL 88,076801 (2002)

First observation of $\nu = 7/2$ state



The problem of transport vs. ideal gaps

Nicholas d'Ambrumenil and RM, Phys. Rev. B68, 113309 (2003)

Theoretical gaps much larger than experimentally observed

Eisenstein et al. 2002 at $\nu = 5/2$ and $7/2$

$$\Delta_{5/2}^{exp} \approx 0.31\mathrm{K} \qquad \Delta_{7/2}^{exp} \approx 0.07\mathrm{K}$$

M+d'Ambrumenil, 2003

$$\Delta_{5/2}^{th} \approx 1.6\mathrm{K} \qquad \Delta_{7/2}^{th} \approx 1.4\mathrm{K}$$

$$f \approx 5 \qquad\qquad 20$$

- Theoretical calculations determine intrinsic gap Δ^i — no disorder
- samples suffer from disorder via the statistical distribution of donors in dopant layer

discovery of 7/2 plateau is a blessing

- states at 5/2 and 7/2 are related by charge conjugation
- $7/2$ implies 2+**2-1/2**———$\frac{1}{2}$ **filled hole statein second LL**
- $5/2$ implies 2+**0+1/2**———$\frac{1}{2}$ **filled electron state in second LL**
- physics at 5/2 and 7/2 should be essentially the same, if Landau level mixing is weak perturbation.

FQH gaps result from Coulomb interaction of electrons

gaps scale with Coulomb energy $E_c = \frac{e^2}{\epsilon \ell_0}$ $\qquad \ell_0 = \sqrt{\frac{\hbar c}{e B}}$

$$\Delta_\nu = \delta_\nu \, E_c \;\sim\; \delta_\nu \sqrt{B_\nu}$$

As $\delta_{5/2} = \delta_{7/2}$ — intrinsic gaps $\Delta^i_{5/2}$ and $\Delta^i_{7/2}$ are related by

$$\frac{\Delta^i_{5/2}}{\Delta^i_{7/2}} = \sqrt{\frac{B_{5/2}}{B_{7/2}}} = \sqrt{\frac{7}{5}} \qquad \text{limit of no LL mixing}$$

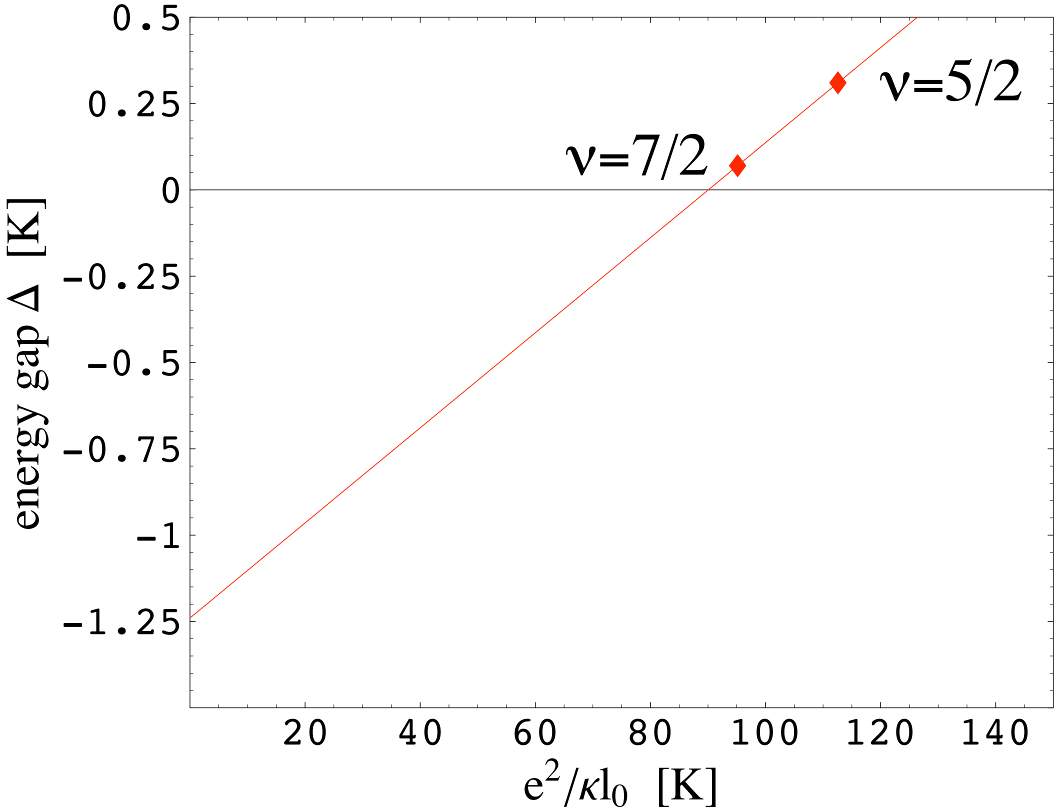
to analyze Δ^{exp}: use symmetry related states at ν and ν' and assume:

- $\delta_\nu = \delta_{\nu'}$
- intrinsic gap is dominated by Coulomb energy E_c
- disorder induced reduction of gap is the same at ν and ν': $\Gamma_\nu = \Gamma_{\nu'}$

$$\Delta^{exp}_\nu = \Delta^i_\nu - \Gamma_\nu = \delta_\nu \, E_c - \Gamma_\nu$$

plot Δ^{exp}_ν as function of E_c (not magnetic field B)

Plot Δ_ν^{exp} vs Coulomb energy $E_c = \frac{e^2}{\epsilon \ell_0} \sim \sqrt{B}$



Slope is measure of intrinsic gap $\Delta^i = \delta \times E_c$: $\delta \approx 0.014$

theoretical values including LL mixing $\delta_{5/2} \approx 0.016$, $\delta_{7/2} \approx 0.015$

$$\Delta^i_{5/2} \approx 1.55\mathrm{K} \qquad \Delta^{th}_{5/2} \approx 1.6\mathrm{K}$$

$$\Delta^i_{7/2} \approx 1.32\mathrm{K} \qquad \Delta^{th}_{7/2} \approx 1.4\mathrm{K}$$

Recent theoretical work on $\nu = 5/2$

- $\nu = 5/2$: non-abelian statistics for topologically protected quantum computation: Das Sarma, Freedman and Nayak 2005
- breaking of particle-hole symmetry at 5/2: Pfaffian vs. Anti-Pfaffian: Lee at al. 2007, Levin et al. 2007
- suggestions for experimental verification of non-abelian statistics by interference studies: Stern and Halperin 2006, Bonderson et al. 2006
- numerical investigation of quasihole systems at 5/2: Töke, Regnault and Jain 2006, 2007
  conclusion: spectrum for Coulomb interaction qualitatively different from Pfaffian phase
- numerical investigation of 5/2 state in disk geometry: Wan, Hu, Rezayi and Yang, 2006,2008
  Pfaff phase stable in window of physical parameters, possible appearance of Anti-Pfaffian phase