New developments in the AHE: *phenomenological regime, unified linear theories, and a new member of the spintronic Hall family*

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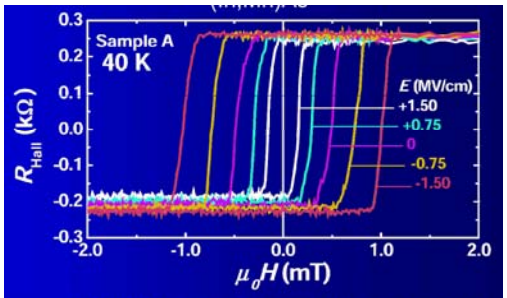
Low Dimensional Systems Workshop
KITP, Santa Barbara, CA
May 14th , 2009

Research fueled by:

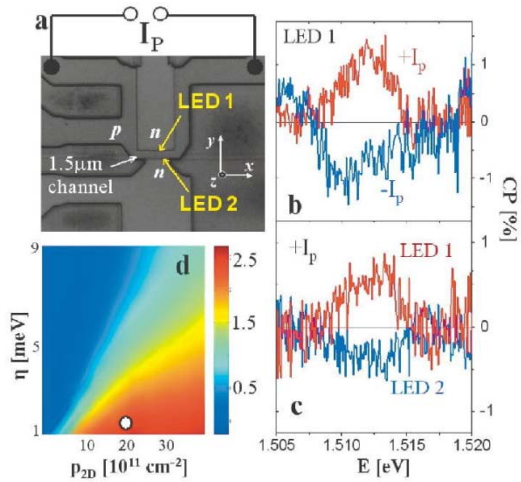
Anomalous Hall transport: lots to think about

AHE

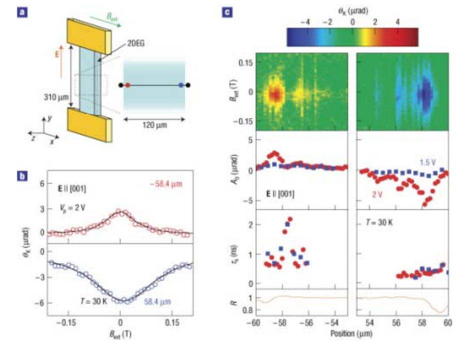


D. Chiba *et al.*, Science **301**, 943 (2003)

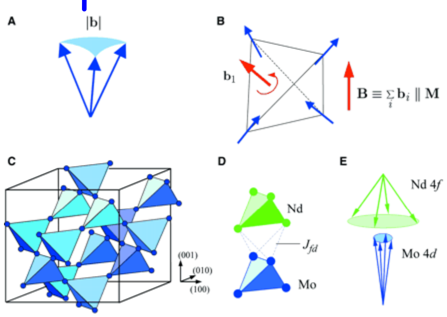
SHE



Wunderlich et al

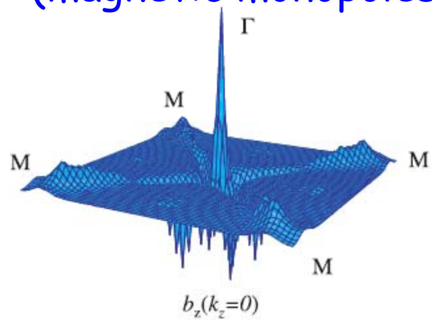

Kato et al

AHE in complex spin textures



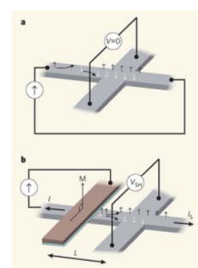
Taguchi et al

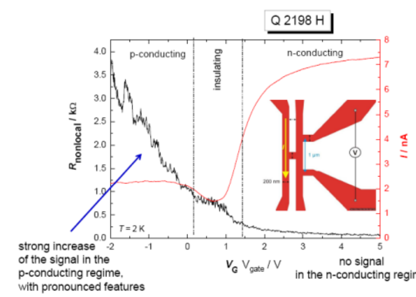
Intrinsic AHE (magnetic monopoles?)



Fang et al

Inverse SHE


Valenzuela et al

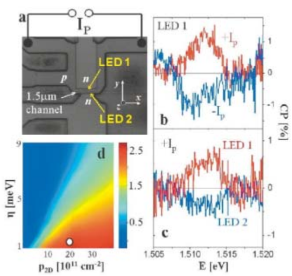


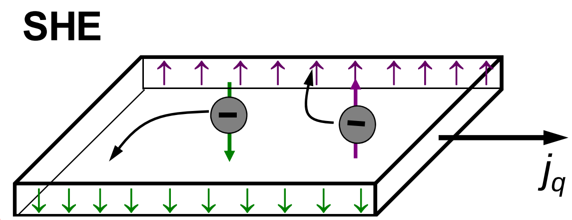
Brune et al

OUTLINE

- Introduction
- SIHE experiment
  - Making the device
  - Basic observation
  - Analogy to AHE
  - Photovoltaic and high T operation
  - The effective Hamiltonian
  - Spin-charge Dyanmcis
- AHE in spin injection Hall effect:
  - AHE basics
  - Strong and weak spin-orbit couple contributions of AHE
  - SIHE observations
  - AHE in SIHE
- Spin-charge dynamics of SIHE with magnetic field:
  - Static magnetic field steady state
  - Time varying injection
- AHE general prospective
  - Phenomenological regimes
  - New challenges

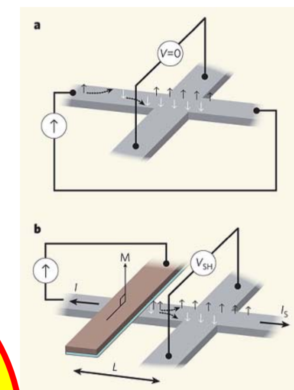
The family of spintronic Hall effects

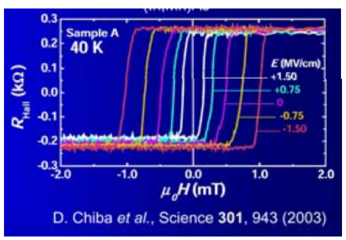




SHE
B=0
charge current gives spin current

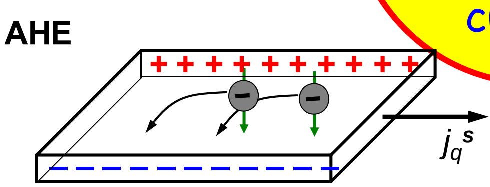
Optical detection





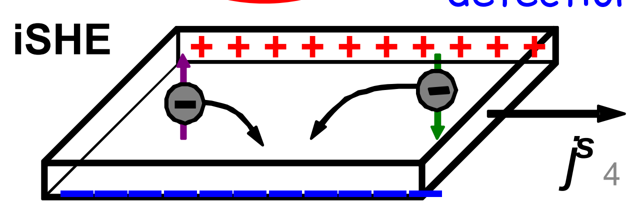
AHE
B=0
polarized charge current gives charge-spin current

Electrical detection



SHE^{-1}
B=0
spin current gives charge current

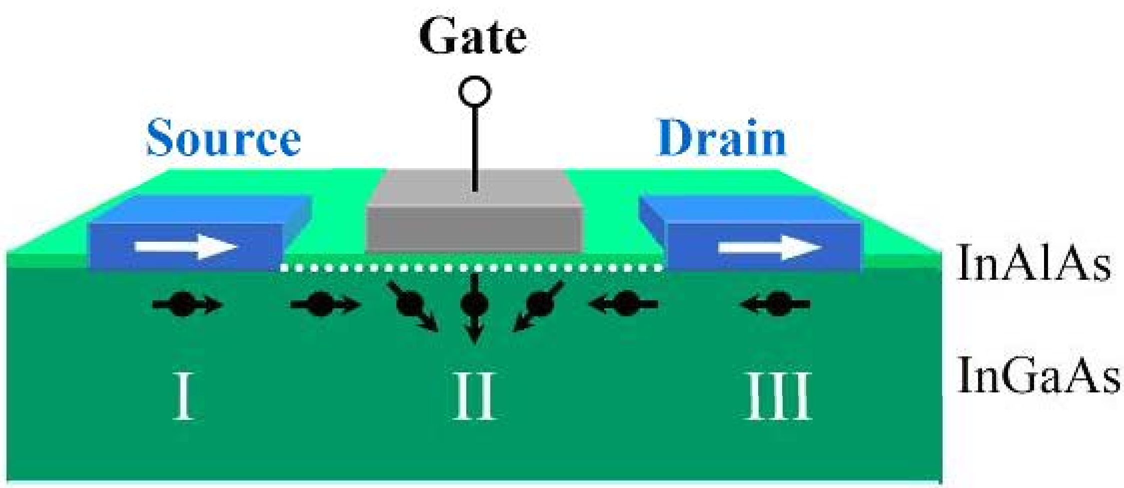
Electrical detection



Towards a spin-based non-magnetic FET device: can we electrically measure the spin-polarization?

Can we achieve direct spin polarization detection through an electrical measurement in an all paramagnetic semiconductor system?

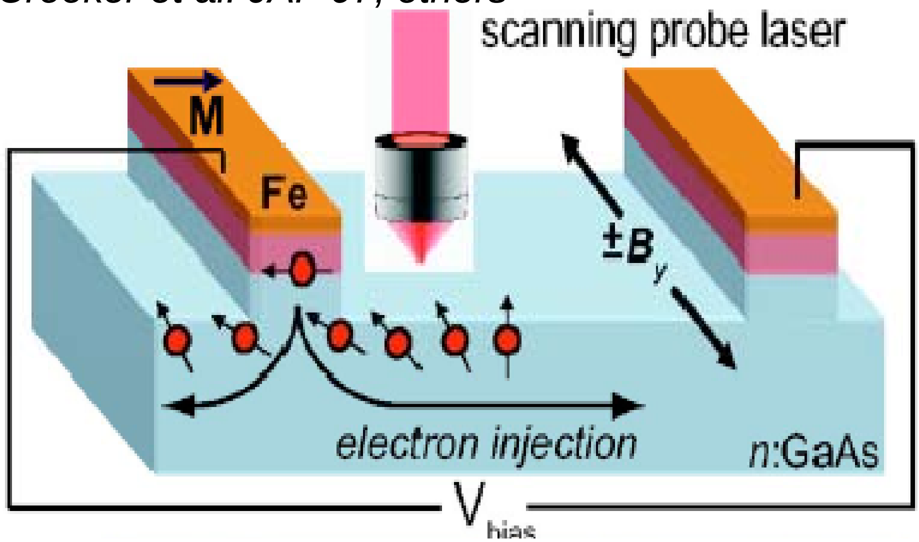
Long standing paradigm: Datta-Das FET



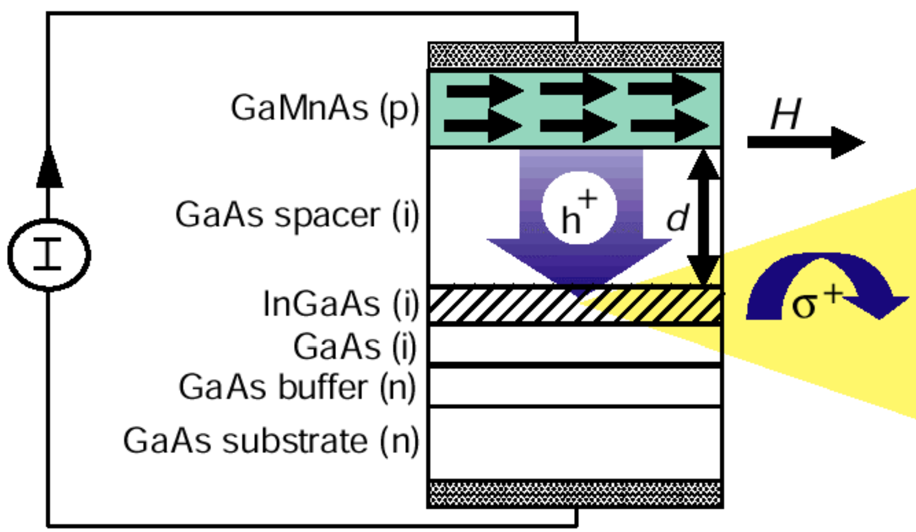
Unfortunately it has not worked:
- no reliable detection of spin-polarization in a diagonal transport configuration
- No long spin-coherence in a Rashba SO coupled system

Spin-detection in semiconductors

Crooker et al. JAP'07, others



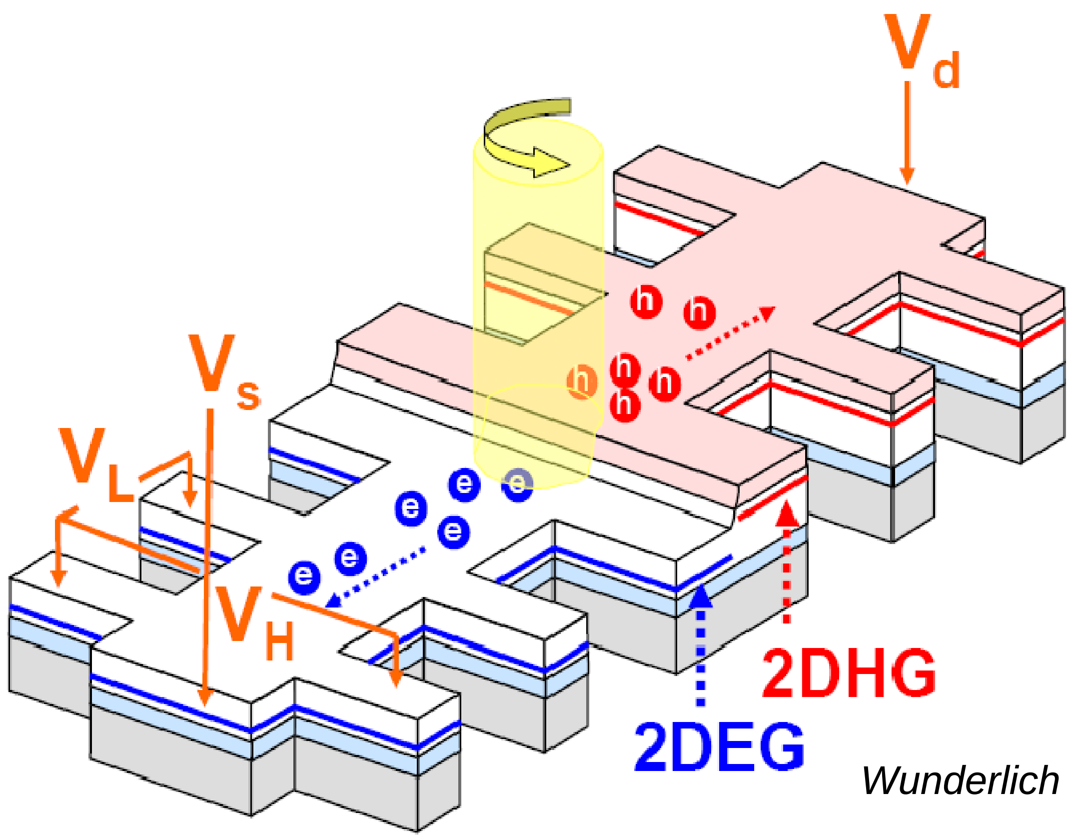
Ohno et al. Nature'99, others



- **Magneto-optical imaging**
  - ✓ non-destructive
  - ✗ lacks nano-scale resolution and only an optical lab tool
- **MR Ferromagnet**
  - ✓ electrical
  - ✗ destructive and requires semiconductor/magnet hybrid design & B-field to orient the FM
- **spin-LED**
  - ✓ all-semiconductor
  - ✗ destructive and requires further conversion of emitted light to electrical signal

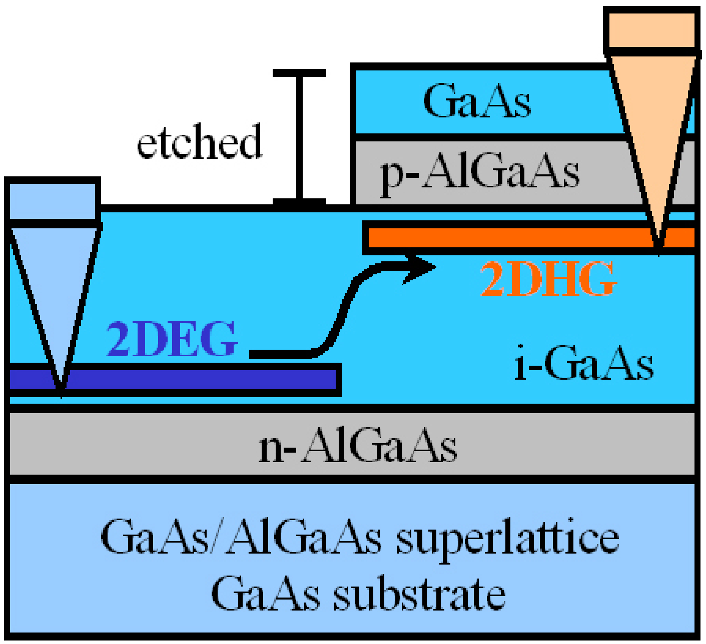
Spin-injection Hall effect

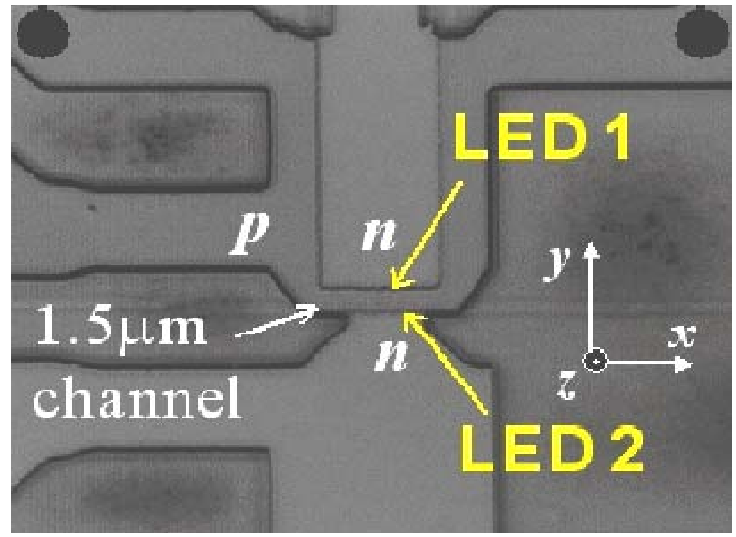
- ✓ non-destructive
- ✓ electrical
- ✓ 100-10nm resolution with current lithography
- ✓ *in situ* directly along the SmC channel
  (all-SmC requiring no magnetic elements in the structure or B-field)



Wunderlich et al. arXives:0811.3486

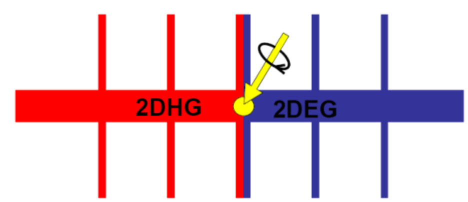
Utilize technology developed to detect SHE in 2DHG and measure polarization via Hall probes





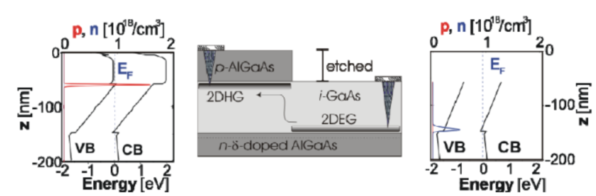
J. Wunderlich, B. Kaestner, J. Sinova and T. Jungwirth, Phys. Rev. Lett. 94 047204 (2005)

B. Kaestner, et al, JPL 02; B. Kaestner, et al Microelec. J. 03; Xiulai Xu, et al APL 04, Wunderlich et al PRL 05

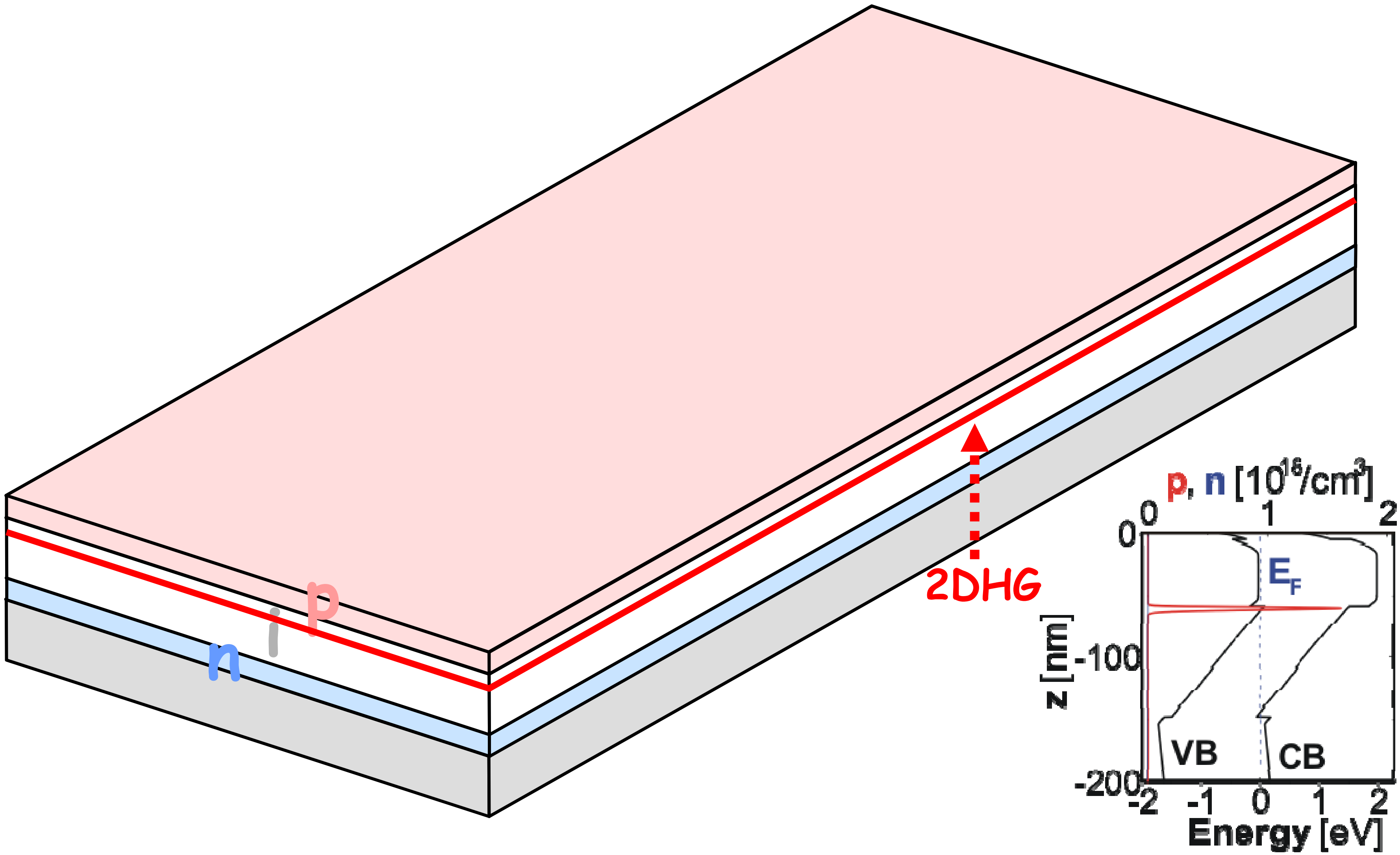


Proposed experiment/device: Coplanar photocell in reverse bias with Hall probes along the 2DEG channel

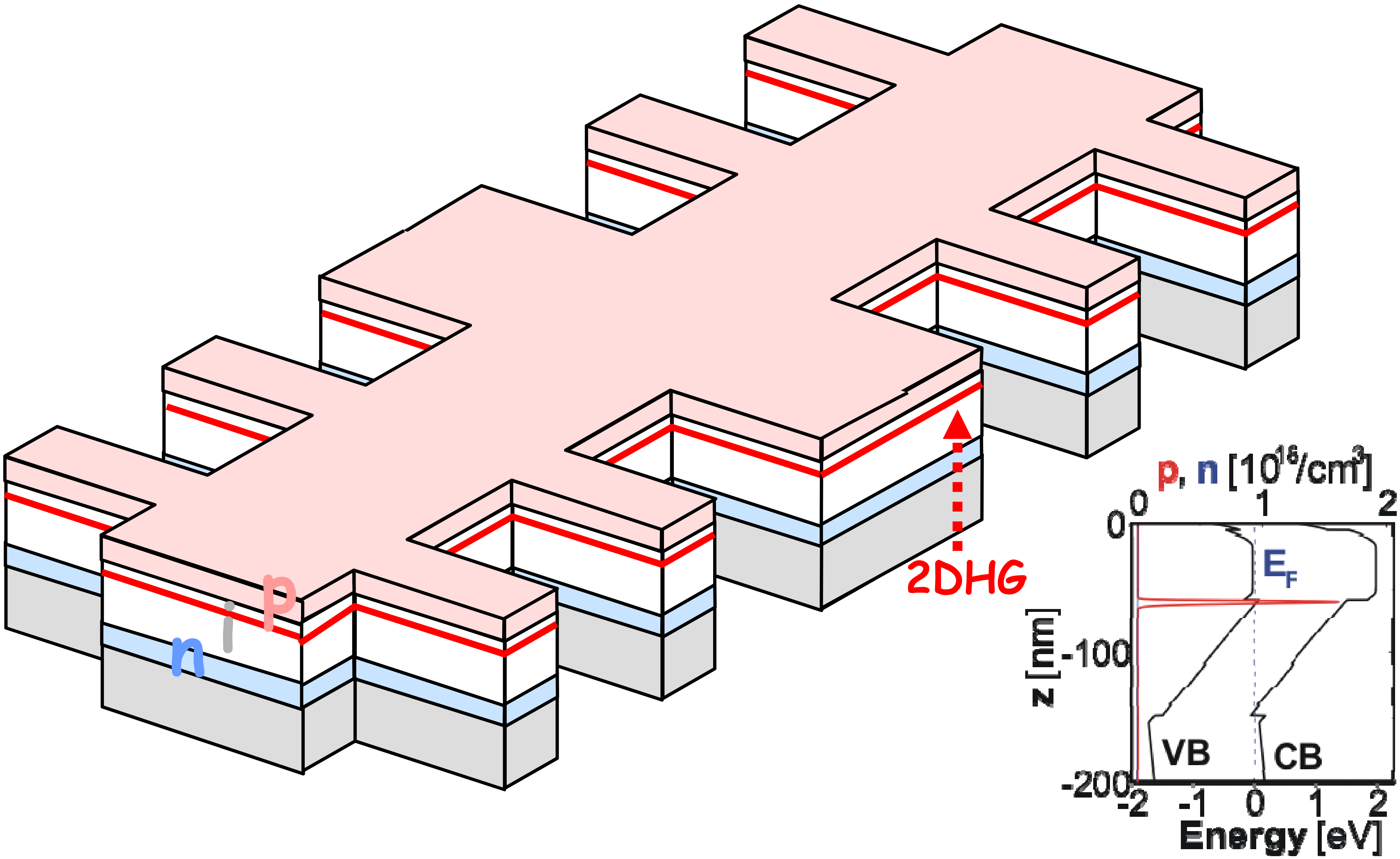
Borunda, Wunderlich, Jungwirth, Sinova et al PRL 07



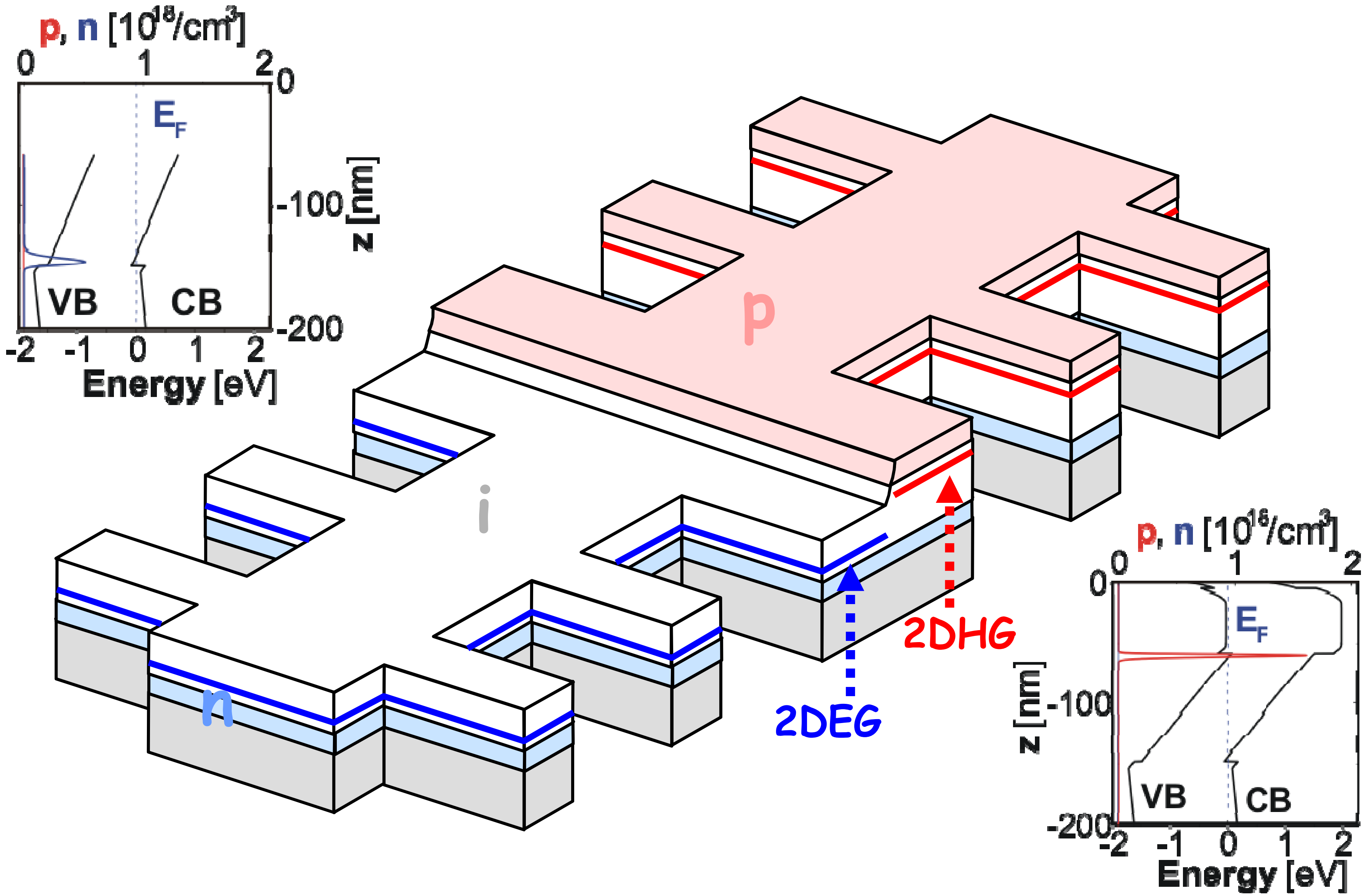
Device schematic - material



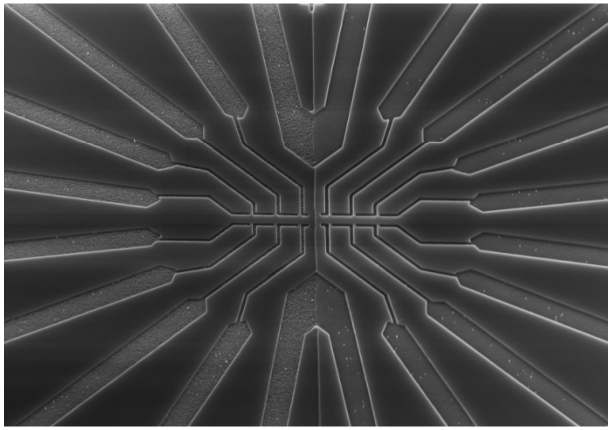
Device schematic - trench

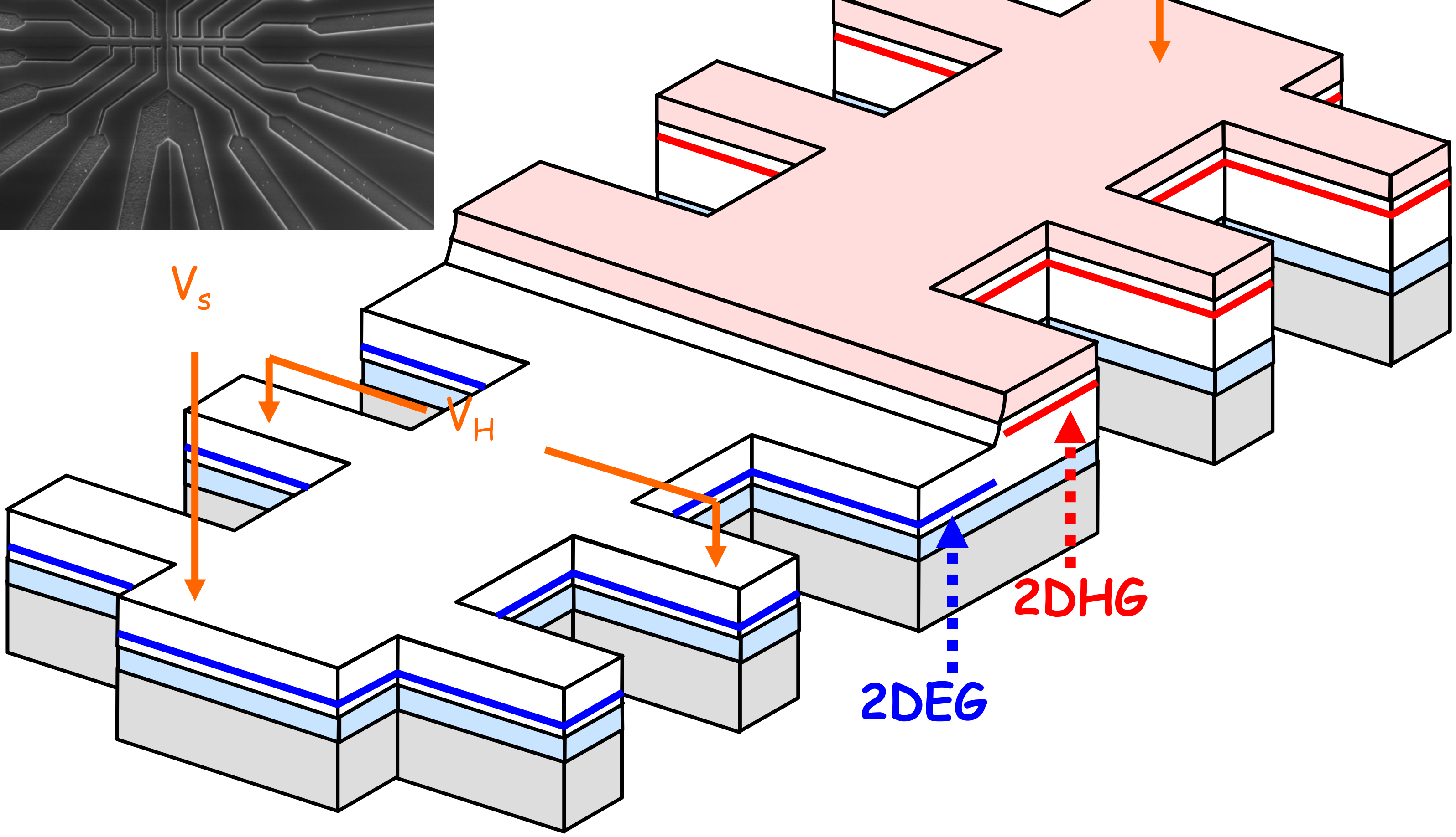


Device schematic – n-etch

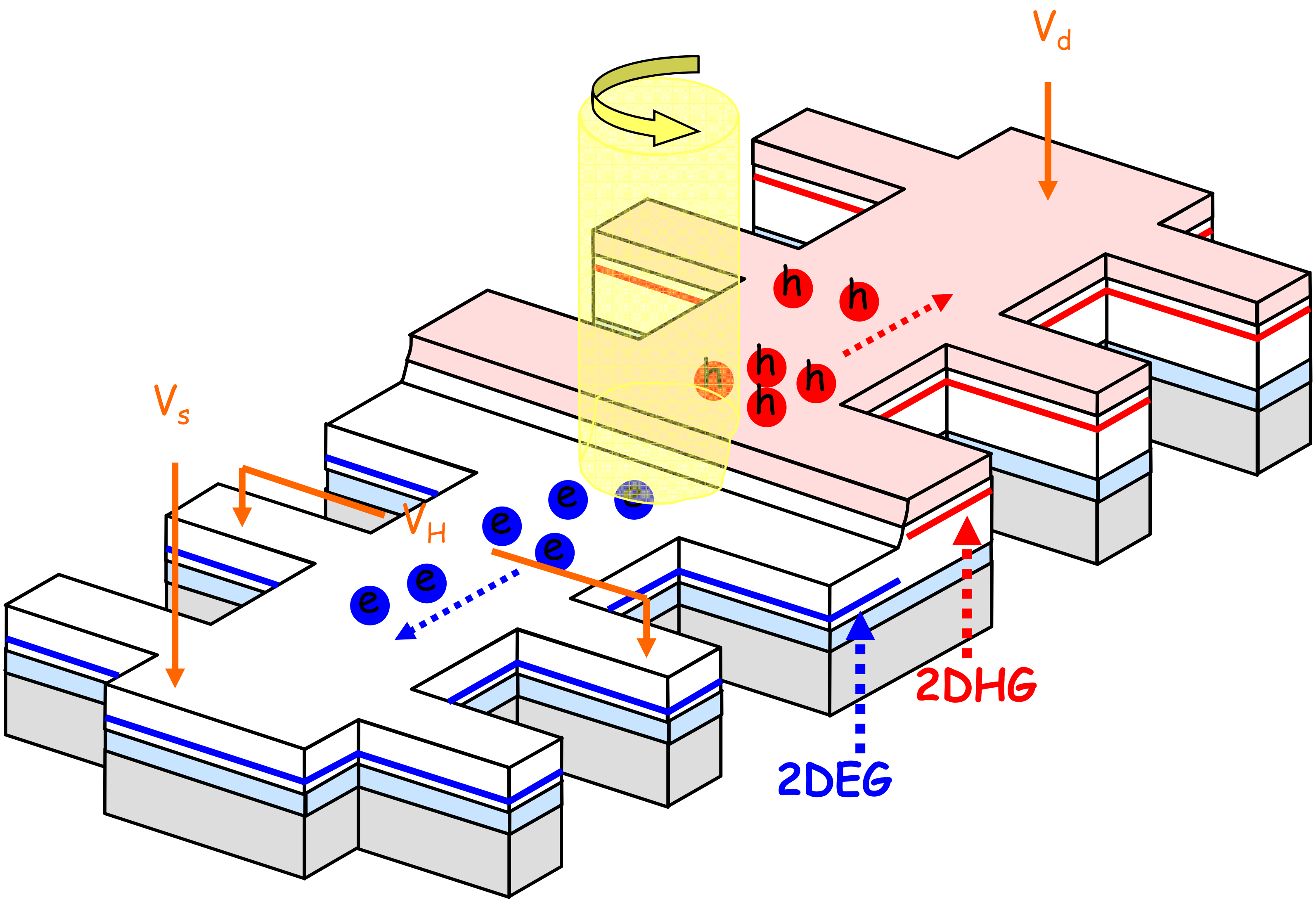


Device schematic - Hall measurement



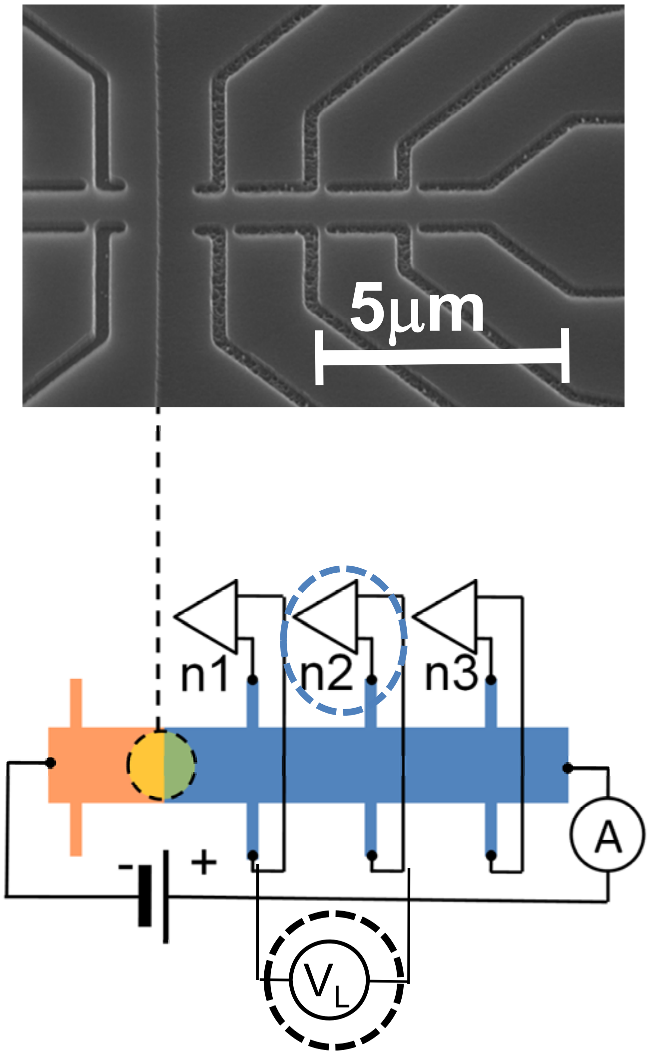


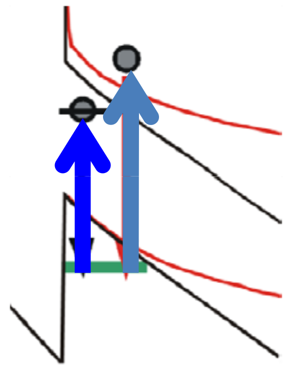
Device schematic - SIHE measurement

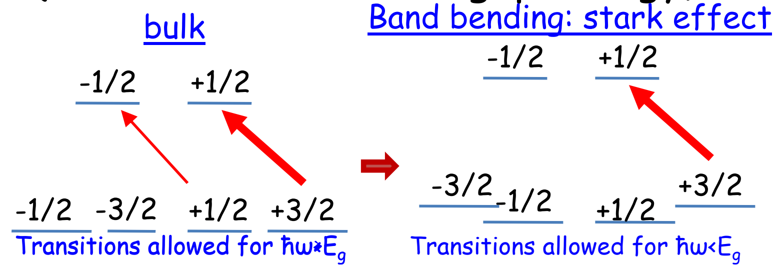


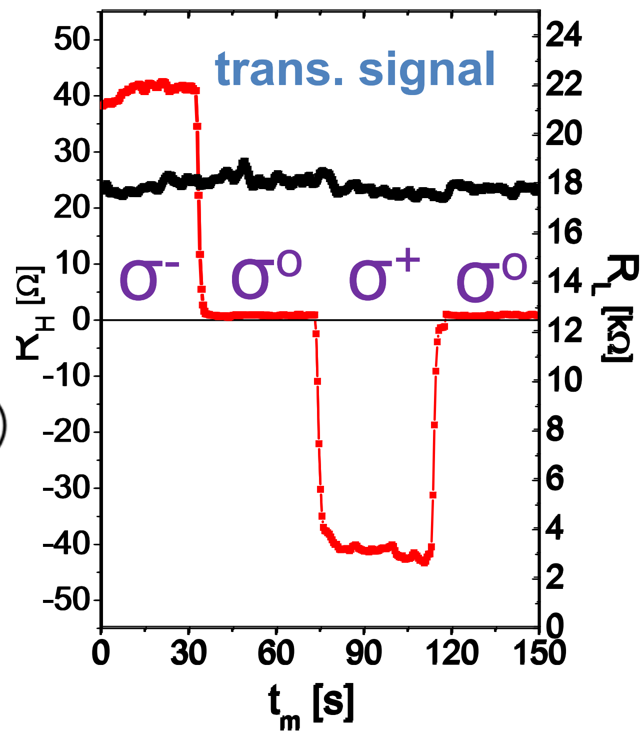
Reverse- or zero-biased: Photovoltaic Cell

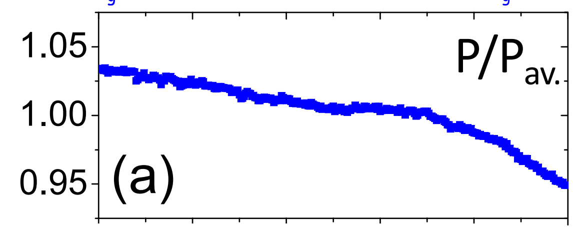
Red-shift of confined 2D hole → free electron trans.
due to built in field and reverse bias
light excitation with λ = **850nm**
(well below bulk band-gap energy)



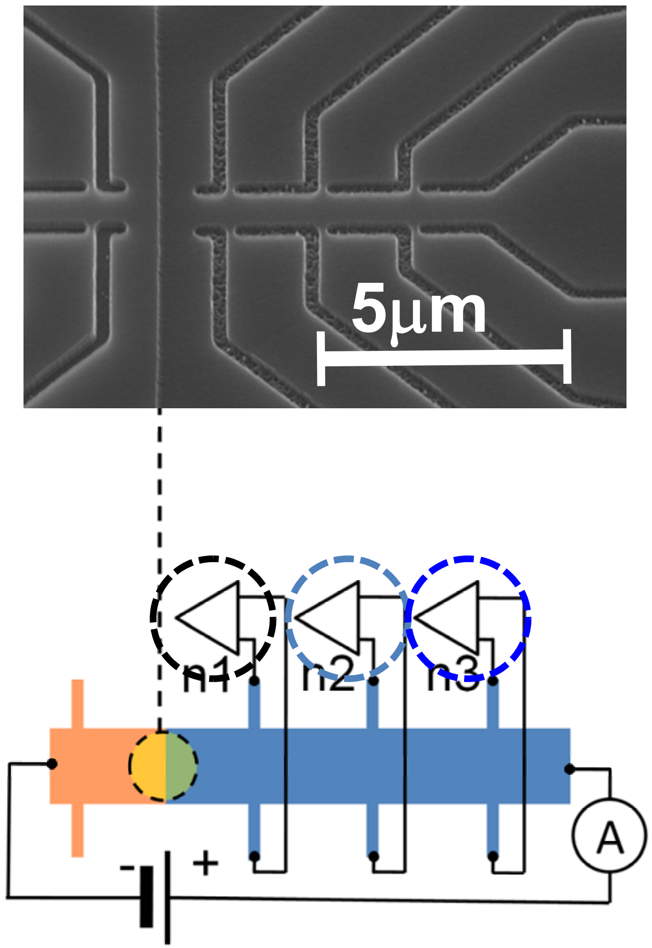


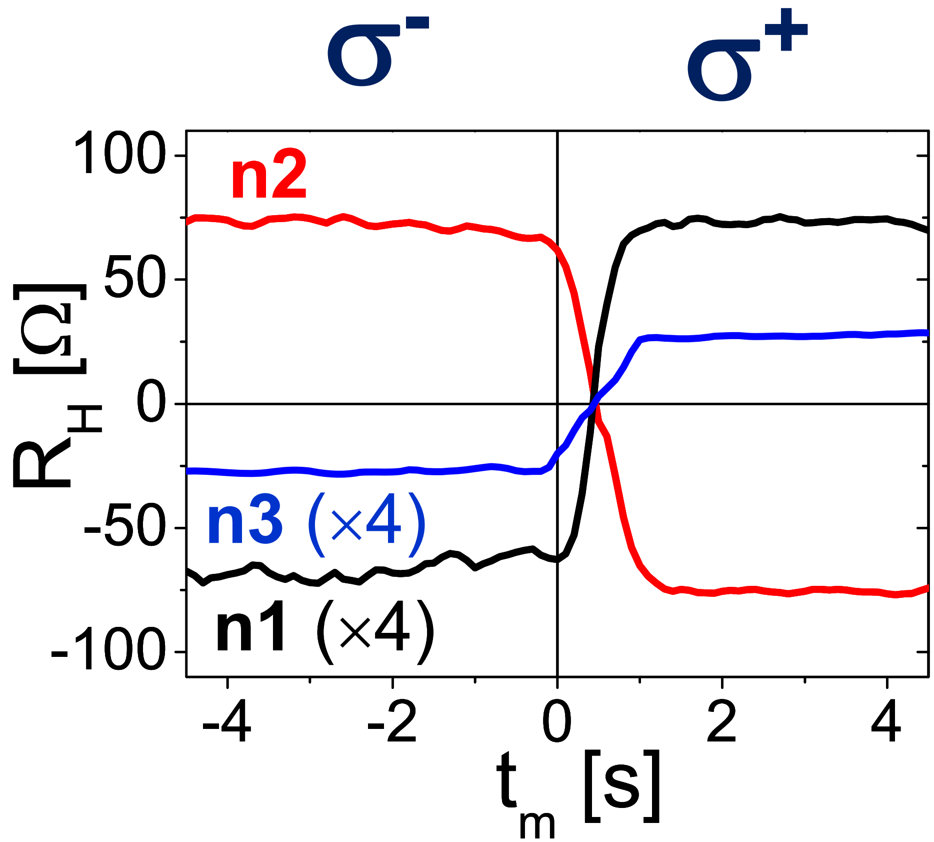






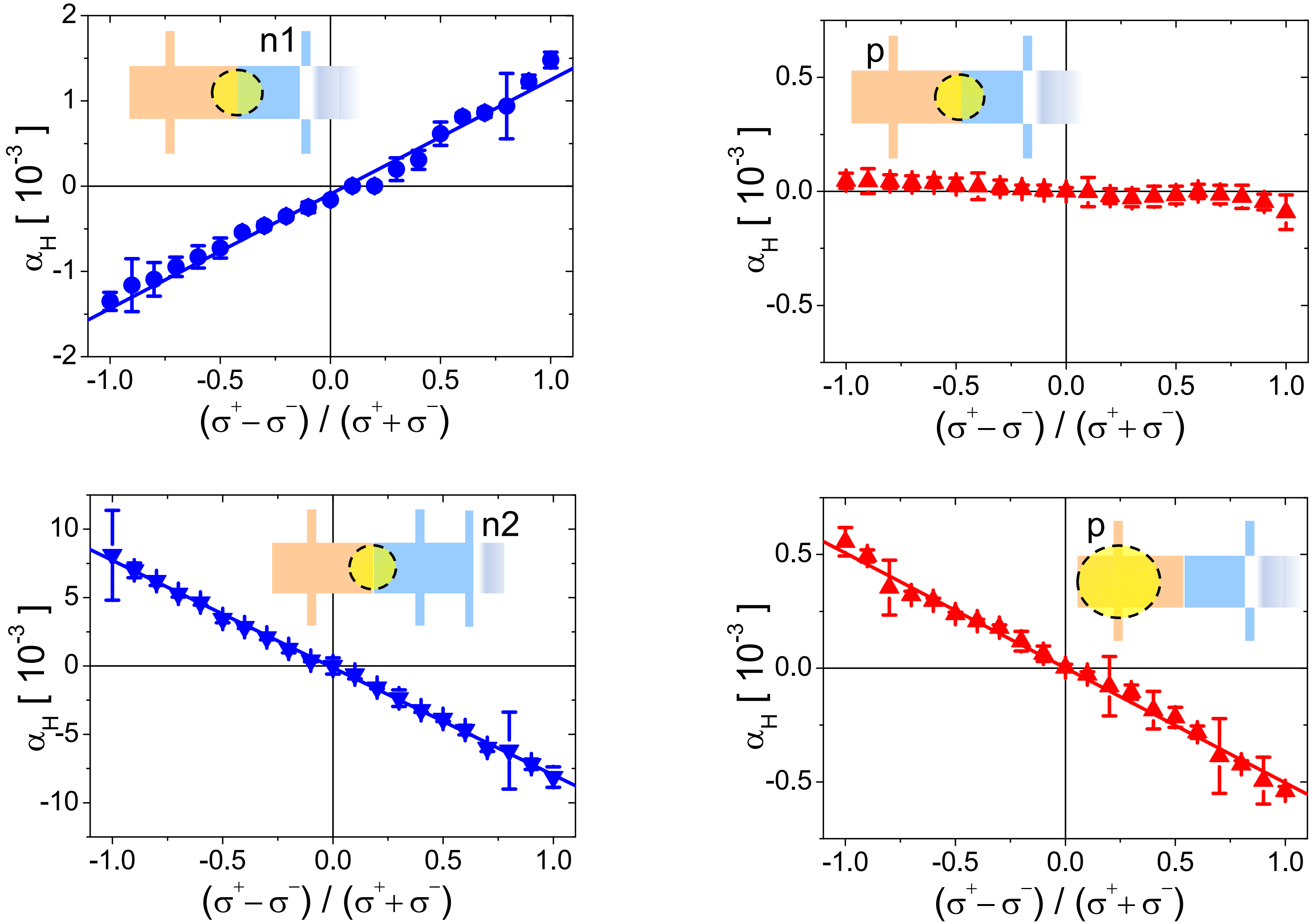
Spin injection Hall effect: experimental observation





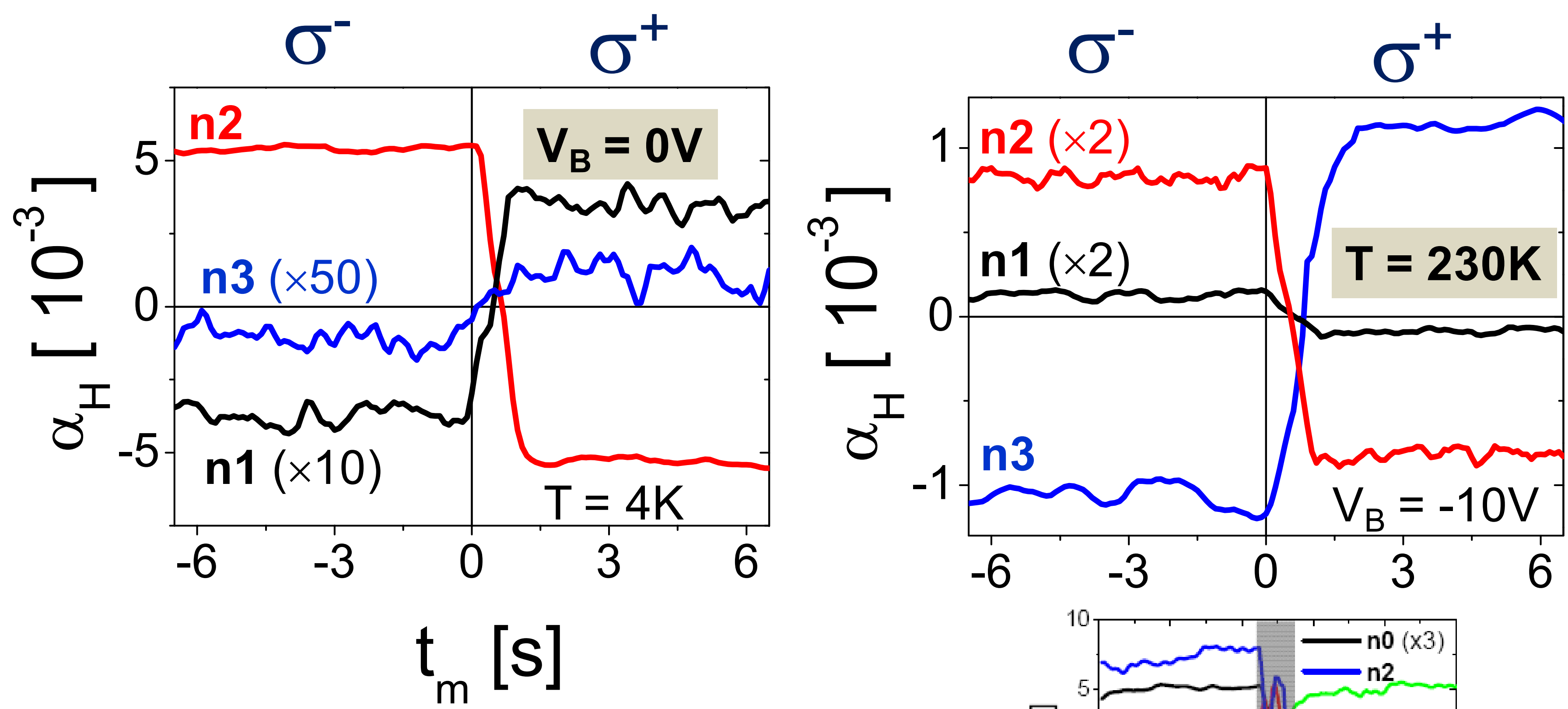
Local Hall voltage changes sign and magnitude along the stripe

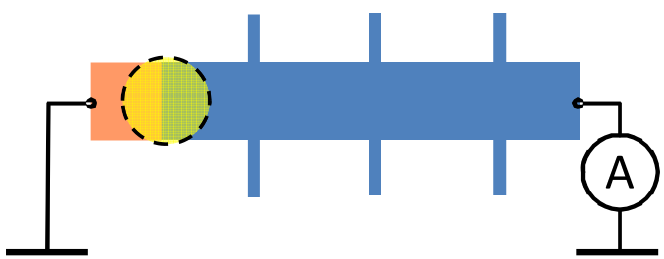
Spin injection Hall effect ←→ Anomalous Hall effect

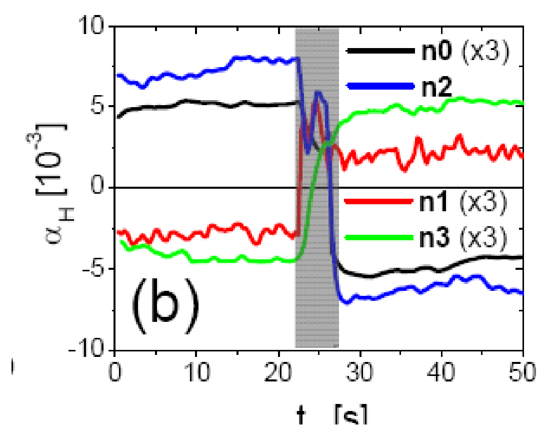


Persistent Spin injection Hall effect

Zero bias-and high temperature operation







THEORY CONSIDERATIONS
Spin transport in a 2DEG with Rashba+Dresselhaus SO

The 2DEG is well described by the effective Hamiltonian:

$$H_{\text{2DEG}} = \frac{\hbar^2 k^2}{2m} + \alpha\left(k_y\sigma_x - k_x\sigma_y\right) + \beta\left(k_x\sigma_x - k_y\sigma_y\right) + \lambda^*\vec{\sigma}\cdot(\vec{k}\times\nabla \mathrm{V}_{\text{dis}}(\vec{r}))$$

$\lambda^* = \frac{P^2}{3}\left(\frac{1}{E_g^2} - \frac{1}{(E_g+\Delta_{so})^2}\right) \approx 5.3\ \mathring{\mathrm{A}}^2$ for GaAs, $\beta = -B\langle k_z^2\rangle$ with $B = 10\ \mathrm{eV}\,\mathring{\mathrm{A}}^3$ for GaAs, $\alpha = \lambda^* E_z$

For our 2DEG system: $\beta \approx -0.02\ \mathrm{eV}\,\mathring{\mathrm{A}}$, $m = 0.067 m_e$

$\alpha \approx 0.01 - 0.03\ \mathrm{eV}\,\mathring{\mathrm{A}}$ (for $E_Z \approx 0.01 - 0.03\,\mathrm{eV}/\mathring{\mathrm{A}}$)

Hence $\alpha \sim -\beta$

What is special about $\alpha \sim -\beta$?

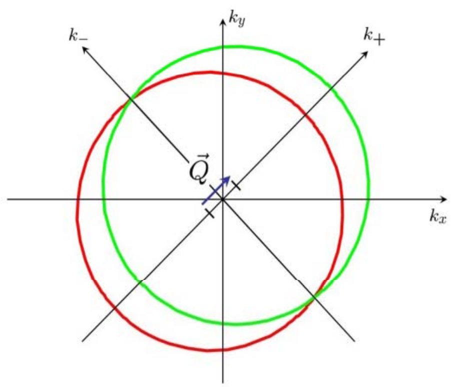
$$H_{\text{2DEG}} \approx \frac{\hbar^2 k^2}{2m} + \alpha(k_y - k_x)(\sigma_x + \sigma_y)$$

Ignoring the term
$\lambda^* \vec{\sigma} \cdot (\vec{k} \times \nabla V_{\text{dis}}(\vec{r}))$
for now

- spin along the [110] direction is conserved
- long lived precessing spin wave for spin perpendicular to [110]
- The nesting property of the Fermi surface:

$$\varepsilon_{\downarrow}\left(\vec{k}\right) = \varepsilon_{\uparrow}\left(\vec{k} + \vec{Q}\right)$$

$$Q = \frac{4m\alpha}{\hbar^2}$$



The long lived spin-excitation: "spin-helix"

- Finite wave-vector spin components

$$S_Q^- = \sum_{\vec{k}} c^+_{\vec{k}\downarrow} c_{\vec{k}+\vec{Q}\uparrow}, \quad S_Q^+ = \sum_{\vec{k}} c^+_{\vec{k}+\vec{Q}\uparrow} c_{\vec{k}\downarrow}, \quad S_0^z = \sum_{\vec{k}} c^+_{\vec{k}\uparrow} c_{\vec{k}\uparrow} - c^+_{\vec{k}\downarrow} c_{\vec{k}\downarrow}$$

$$\left[S_0^z, S_Q^\pm\right] = \pm 2 S_Q^\pm, \quad \left[S_Q^+, S_Q^-\right] = S_0^z$$

- Shifting property essential

$$\left[H_{\mathrm{ReD}}, c^+_{\vec{k}+\vec{Q}\uparrow} c_{\vec{k}\downarrow}\right] = \left(\varepsilon_\uparrow\left(\vec{k}+\vec{Q}\right) - \varepsilon_\downarrow\left(\vec{k}\right)\right) c^+_{\vec{k}+\vec{Q}\uparrow} c_{\vec{k}\downarrow} = 0$$

⇒ An exact SU(2) symmetry

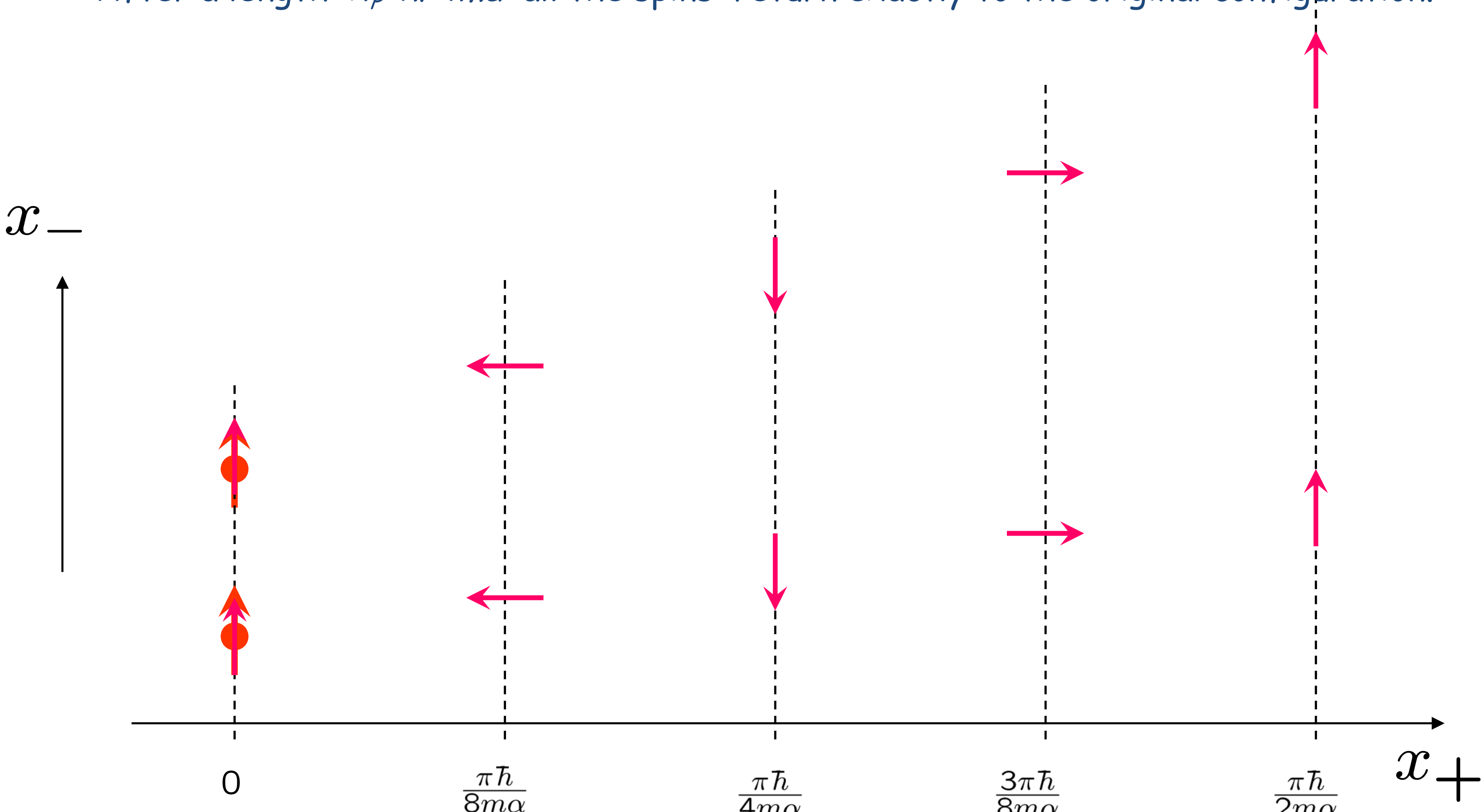
Only Sz, zero wavevector U(1) symmetry previously known:

J. Schliemann, J. C. Egues, and D. Loss, Phys. Rev. Lett. **90**, 146801 (2003).

K. C. Hall *et. al.*, Appl. Phys. Lett **83**, 2937 (2003).

Physical Picture: Persistent Spin Helix $\alpha = -\beta$

- Spin configurations do not depend on the particle initial momenta.
- For the same x_+ distance traveled, the spin precesses by exactly the same angle.
- After a length $x_P = h/4m\alpha$ *all* the spins return exactly to the original configuration.



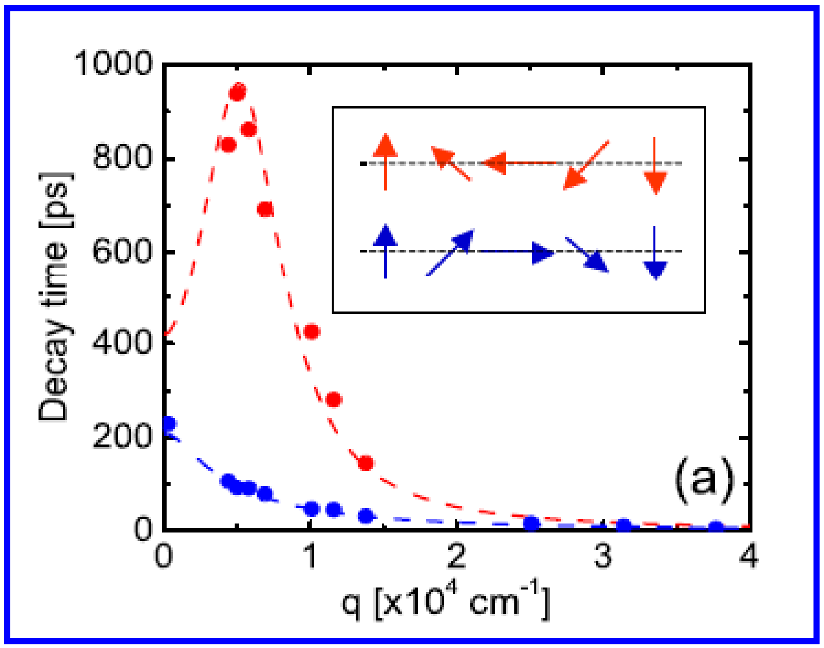
Thanks to SC Zhang, Stanford University

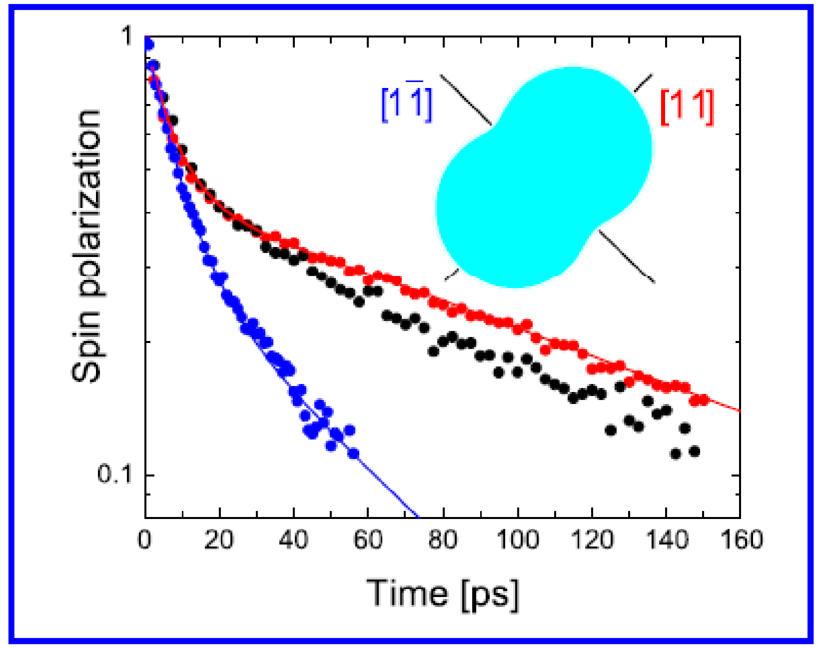
Persistent state spin helix verified by pump-probe experiments

Nondiffusive Spin Dynamics in a Two-Dimensional Electron Gas

C. P. Weber,[1] J. Orenstein,[1] B. Andrei Bernevig,[2] Shou-Cheng Zhang,[2] Jason Stephens,[3] and D. D. Awschalom[3]

PRL 98, 076604 (2007)   PHYSICAL REVIEW LETTERS   week ending 16 FEBRUARY 2007





Similar wafer parameters to ours

The Spin-Charge Drift-Diffusion Transport Equations

For arbitrary α,β spin-charge transport equation is obtained for diffusive regime

$$\partial_t n = D\nabla^2 n + B_1 \partial_{x+} S_{x-} - B_2 \partial_{x-} S_{x+}$$

$$\partial_t S_{x+} = D\nabla^2 S_{x+} - B_2 \partial_{x-} n - C_1 \partial_{x+} S_z - T_1 S_{x+}$$

$$\partial_t S_{x-} = D\nabla^2 S_{x-} - B_1 \partial_{x+} n - C_2 \partial_{x-} S_z - T_2 S_{x-}$$

$$\partial_t S_z = D\nabla^2 S_z + C_2 \partial_{x-} S_{x-} + C_2 \partial_{x+} S_{x+} - (T_1 + T_2) S_z$$

$$B_{1/2} = 2(\alpha \mp \beta)^2 (\alpha \pm \beta) k_F^2 \tau^2, \quad T_{1/2} = \frac{2}{m} (\alpha \pm \beta)^2 \frac{k_F^2 \tau}{\hbar^2}$$

$$D = v_F^2 \tau / 2, \quad \text{and } C_{1/2}^2 = 4DT_{1/2}$$

For propagation on [1-10], the equations decouple in two blocks.
Focus on the one coupling S_{x+} and S_z:

$$\partial_t S_{x-} = D\nabla^2 S_{x-} - C_2 \partial_{x-} S_z - T_2 S_{x-}$$

$$\partial_t S_z = D\nabla^2 S_z + C_2 \partial_{x-} S_{x-} - (T_1 + T_2) S_z$$

For Dresselhauss = 0, the equations reduce to **Burkov, Nunez and MacDonald,** PRB 70, 155308 (2004);
Mishchenko, Shytov, Halperin, PRL 93, 226602 (2004)

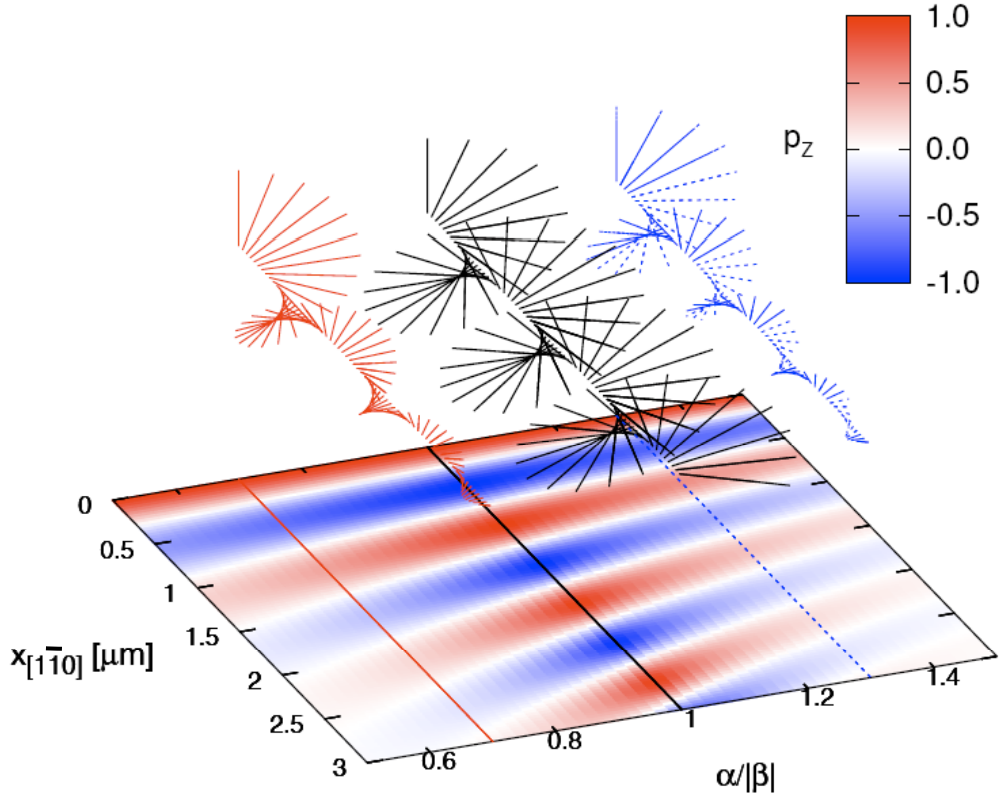
Steady state spin transport in diffusive regime

Steady state solution for the spin-polarization component if propagating along the [1-10] orientation

$$S_{z/x-}(x_{[1\bar{1}0]}) = S^{0}_{z/x-}\exp[q\,x_{[1\bar{1}0]}]$$

$$(Dq^2+T_2)(Dq^2+T_1+T_2)-C_2^2q^2=0$$

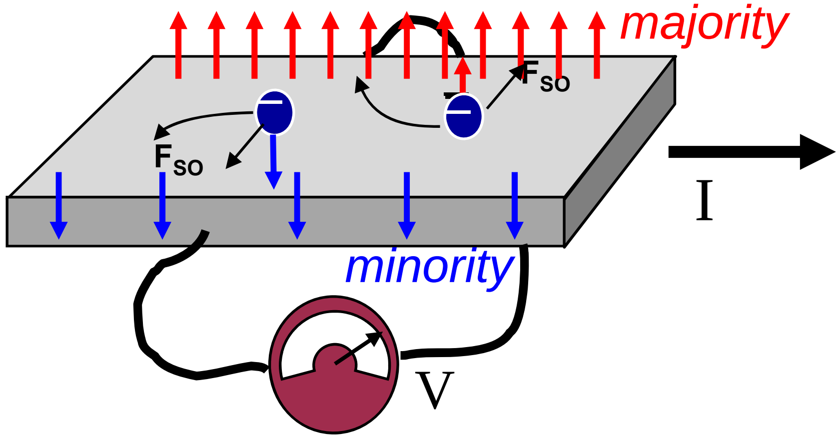
$$q=|q|\exp(i\theta),\ |q|=(\tilde{L}_1^{\,2}\tilde{L}_2^{\,2}+\tilde{L}_2^{\,4})^{1/4},\ \theta=\tfrac{1}{2}\arctan\left(\frac{\sqrt{\tilde{L}_1^{\,2}\tilde{L}_2^{\,2}-\tilde{L}_1^{\,4}/4}}{\tilde{L}_2^{\,2}-\tilde{L}_1^{\,2}/2}\right)\qquad \tilde{L}_{1/2}=2m|\alpha\pm\beta|/\hbar^2$$



Spatial variation scale consistent with the one observed in SIHE

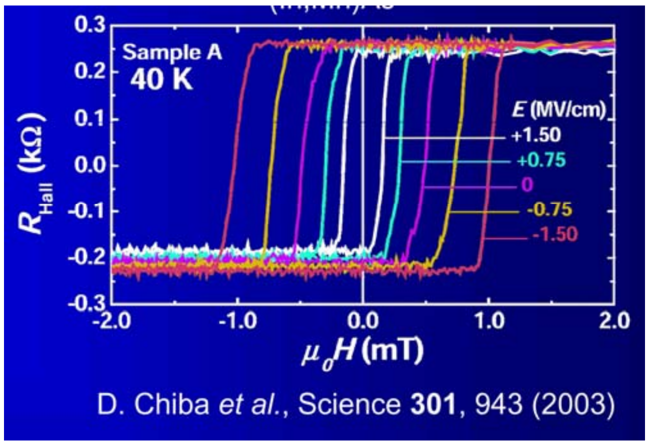
Understanding the Hall signal of the SIHE: Anomalous Hall effect

*Spin dependent "force" deflects **like-spin** particles*



$$\rho_H = R_0 B_\perp + 4\pi R_s M_\perp$$

$$R_0 << R_s$$



Simple electrical measurement of out of plane magnetization

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{xx} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} \approx \frac{1}{\sigma_{xx}}$$

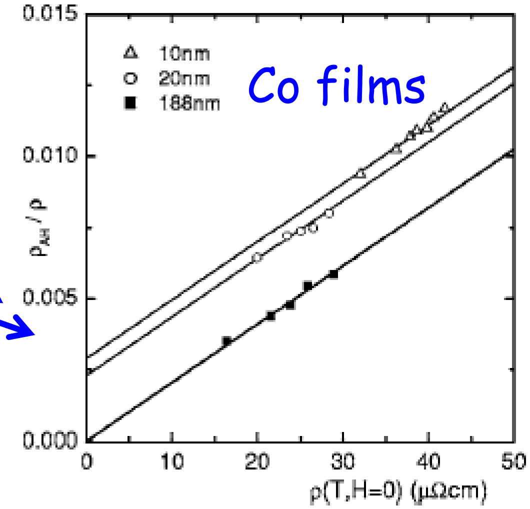
$$\rho_{xy} = \frac{-\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2} \approx \frac{-\sigma_{xy}}{\sigma_{xx}^2} \approx -\sigma_{xy}\rho_{xx}^2 \approx -A\rho_{xx} - B\rho_{xx}^2$$

$$\sigma_{xy} \approx B + A\sigma_{xx}$$

Anomalous Hall effect (scaling with ρ)

$$\rho_{xy} = -A\rho_{xx} - B\rho_{xx}^2$$

$$\sigma_{xy} \approx B + A\sigma_{xx}$$



Kotzler and Gil PRB 2005

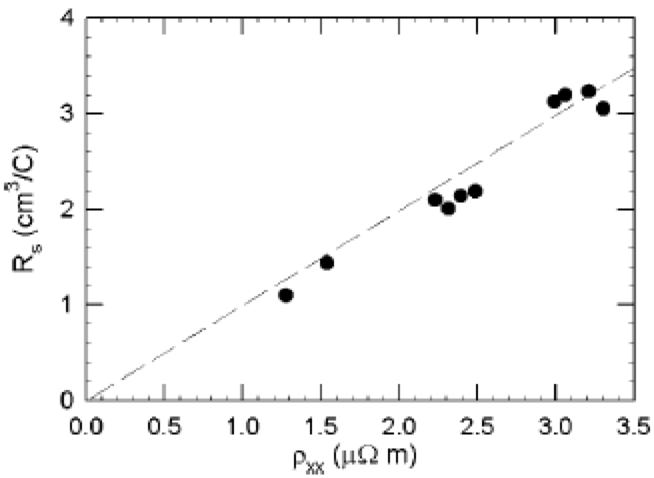


FIG. 5. Anomalous Hall coefficient R_S as a function of electrical resistivity ρ for $Sb_{2-x}Cr_xTe_3$. Data are taken from all samples with $x \geqslant 0.031$ and at temperatures ranging from 2 K up to the respective Curie temperatures. The dashed line illustrates the relation $R_S = c\rho^1$, which is consistent with AHE due to skew scattering.

Dyck et al PRB 2005

Weak SO coupled regime

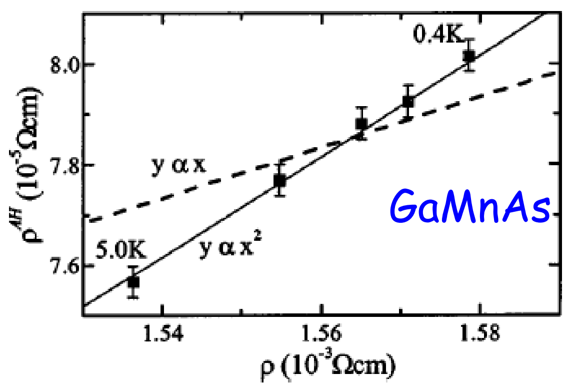


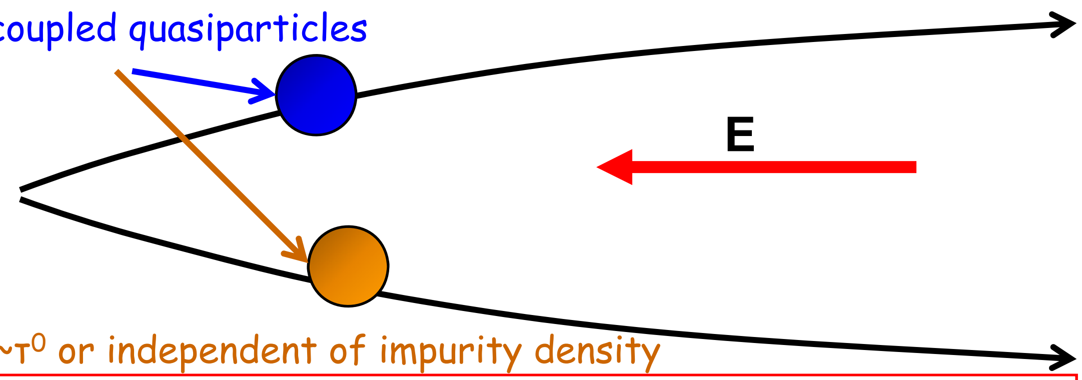
FIG. 3. Anomalous Hall resistivity ρ_{xy}^{AH} vs longitudinal resistivity ρ at fixed $B = 10$ T and varying T between 0.4 and 5.0 K for the $x = 0.06$ sample (squares); best fit lines assuming $R^{AH} \propto \rho$ (broken line) and $R^{AH} \propto \rho^2$ (full line).

Edmonds et al APL 2003

Strong SO coupled regime

STRONG SPIN-ORBIT COUPLED REGIME ($\Delta_{so} > \hbar/\tau$)

Intrinsic deflection



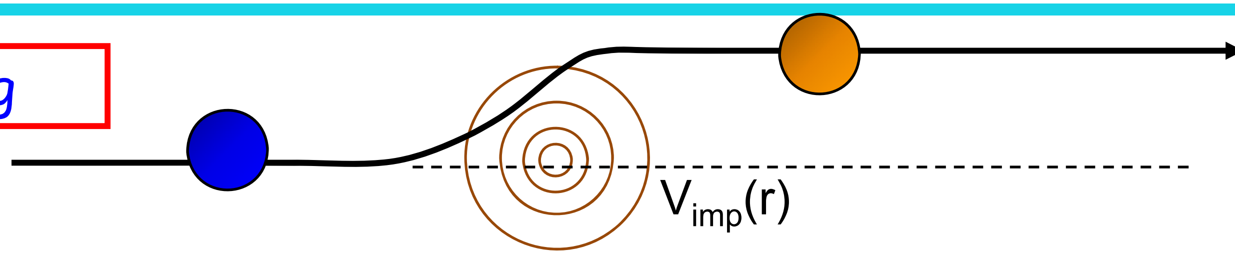
Electrons deflect to the right or to the left as they are accelerated by an electric field ONLY because of the spin-orbit coupling in the periodic potential (electronics structure)

$$\dot{x}_c = \frac{\partial \epsilon}{\hbar \partial \vec{k}} + (e/\hbar)\vec{E} \times \vec{\Omega}.$$

$\sim\tau^0$ or independent of impurity density

Electrons have an "anomalous" velocity perpendicular to the electric field related to their Berry's phase curvature which is nonzero when they have spin-orbit coupling.

Side jump scattering

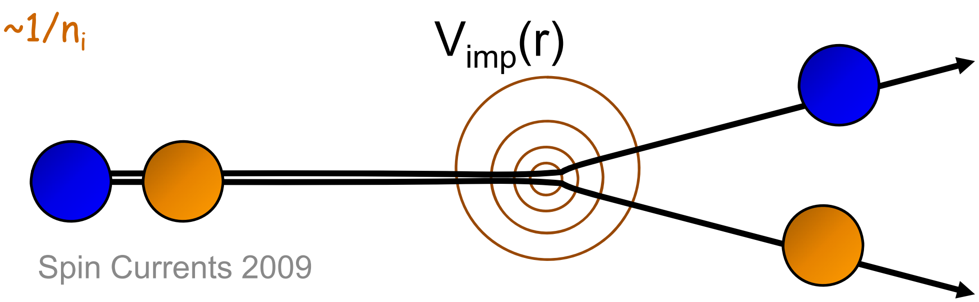


independent of impurity density

Electrons deflect first to one side due to the field created by the impurity and deflect back when they leave the impurity since the field is opposite resulting in a side step. They however come out in a different band so this gives rise to an anomalous velocity through scattering rates times side jump.

Skew scattering

$\sim 1/n_i$



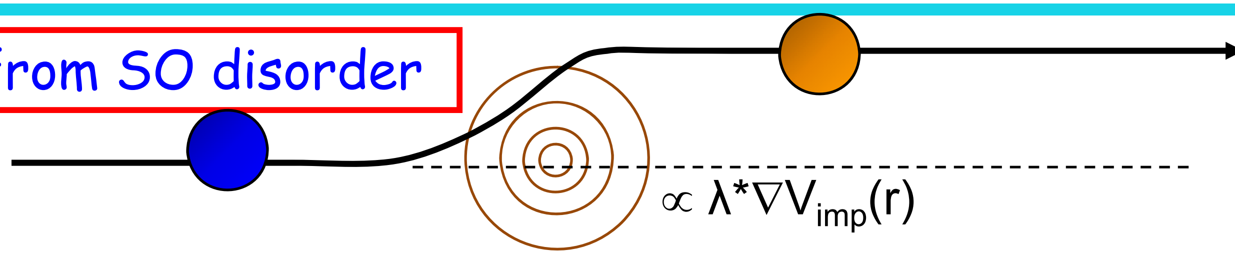
Asymmetric scattering due to the spin-orbit coupling of the electron or the impurity. This is also known as Mott scattering used to polarize beams of particles in accelerators.

WEAK SPIN-ORBIT COUPLED REGIME ($\Delta_{so}<\hbar/\tau$)

Better understood than the strongly SO couple regime

The terms/contributions dominant in the strong SO couple regime are strongly reduced (quasiparticles not well defined due to strong disorder broadening). Other terms, originating from the interaction of the quasiparticles with the SO-coupled part of the disorder potential dominate.

Side jump scattering from SO disorder



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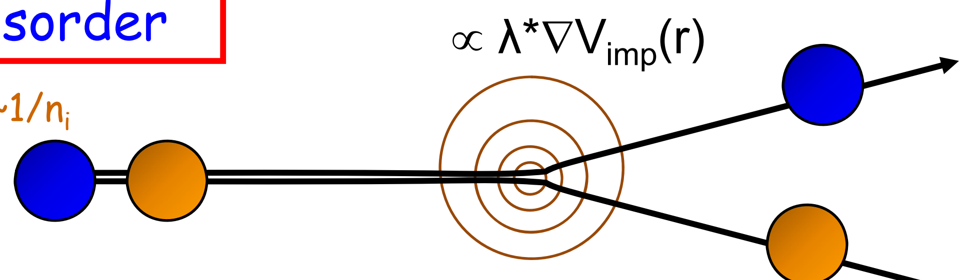


AHE contribution

$$H_{\text{2DEG}} = \frac{\hbar^2 k^2}{2m} + \alpha\left(k_y\sigma_x - k_x\sigma_y\right) + \beta\left(k_x\sigma_x - k_y\sigma_y\right) + \lambda^*\vec{\sigma}\cdot(\vec{k}\times\nabla \mathrm{V}_{\text{dis}}(\vec{r}))$$

Two types of contributions:

i) S.O. from band structure interacting with the field (external and internal)

ii) Bloch electrons interacting with S.O. part of the disorder

Type (i) contribution much smaller in the weak SO coupled regime where the SO-coupled bands are not resolved, dominant contribution from type (ii)

$$\left|\sigma_{xy}\right|^{\text{skew}} = \frac{2\pi e^2\lambda^*}{\hbar^2} V_0 \tau \; n\,(n_\uparrow - n_\downarrow)$$

$$\left|\sigma_{xy}\right|^{\text{side-jump}} = \frac{2e^2\lambda^*}{\hbar}(n_\uparrow - n_\downarrow)$$

Crepieux et al PRB 01
Nozier et al J. Phys. 79

$$\left|\alpha_H\right|^{\text{side-jump}} \approx 5.3\times 10^{-4}$$

$$\alpha_H(x_{[1\bar{1}0]}) = 2\pi\lambda^* \sqrt{\frac{e}{\hbar n_i \mu}}\; n\, p_z(x_{[1\bar{1}0]}) \approx 1.1\times 10^{-3}\, p_z$$

Lower bound estimate of skew scatt. contribution

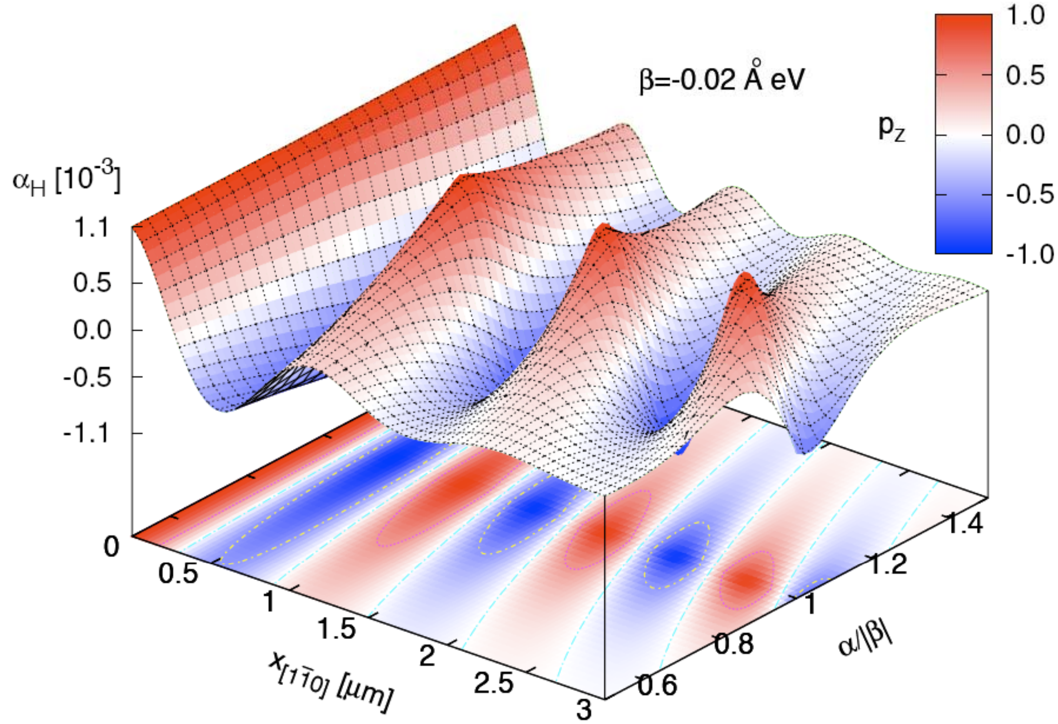
Spin injection Hall effect: Theoretical consideration

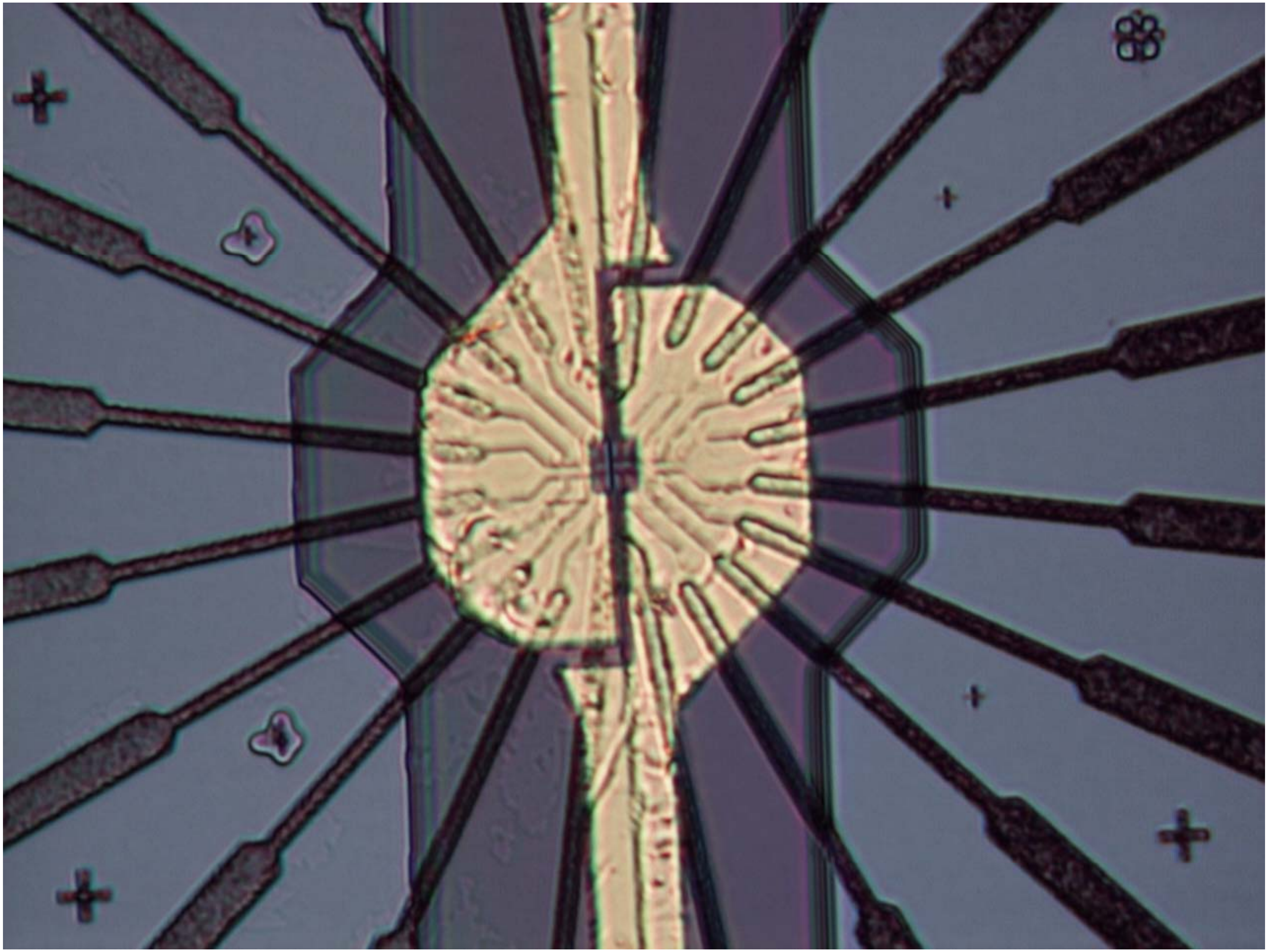
Local spin polarization → calculation of the Hall signal

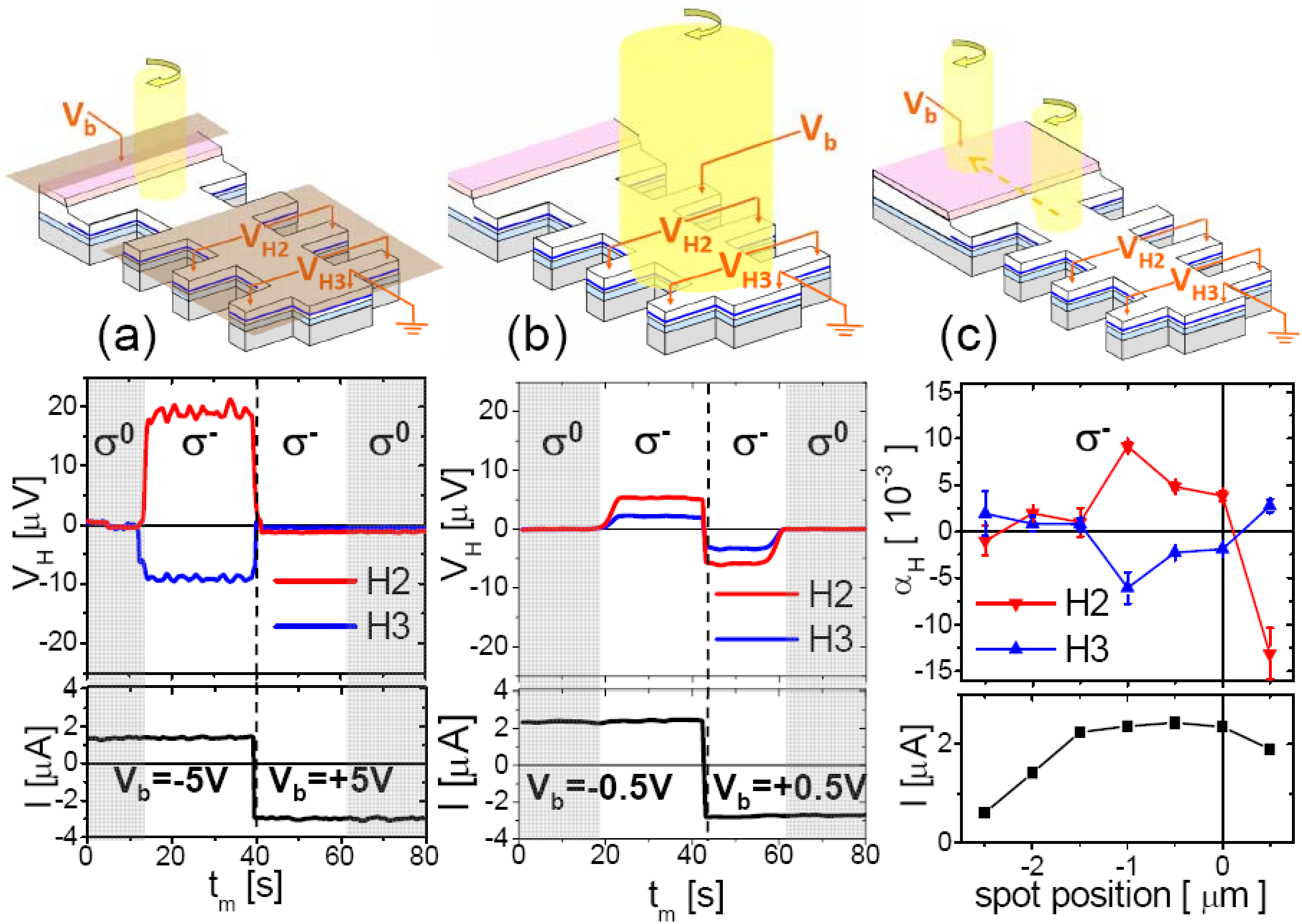
Weak SO coupling regime → extrinsic skew-scattering term is dominant

$$\alpha_H(x_{[1\bar{1}0]}) = 2\pi\lambda^* \sqrt{\frac{e}{\hbar n_i \mu}}\; n\, p_z(x_{[1\bar{1}0]})$$

Lower bound estimate







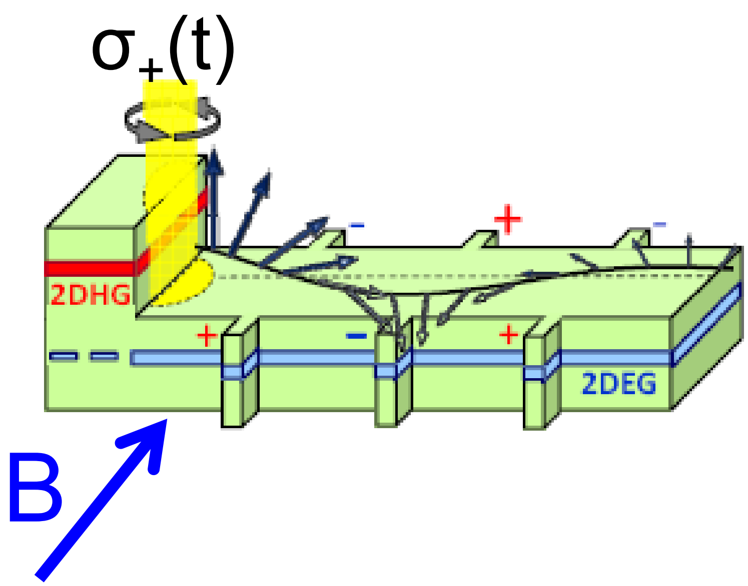
V_b
V_H2
V_H3
(a)
(b)
(c)
σ0
σ-
V_H [μV]
H2
H3
I [μA]
V_b=-5V
V_b=+5V
t_m [s]
V_b=-0.5V
V_b=+0.5V
α_H [10^-3]
spot position [μm]

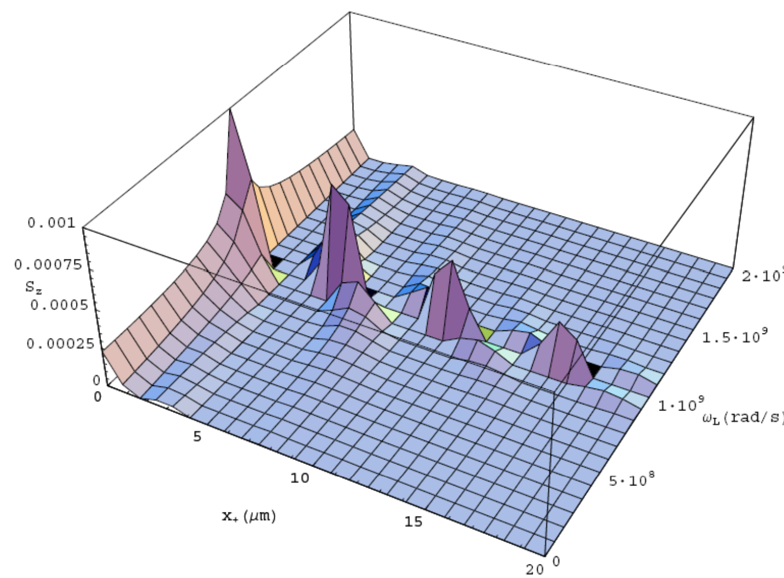
Drift-Diffusion eqs. with magnetic field perpendicular to 110 and time varying spin-injection

$$\partial_t S_{x_+} = D\partial^2_{x_+} S_{x_+} - C\partial_{x_+} S_z - T_1 S_{x_+} + \mu E \partial_{x_+} S_{x_+} - \mu E Q S_z + \omega_L S_z$$
$$\partial_t S_z = D\partial^2_{x_+} S_z + C\partial_{x_+} S_{x_+} - (T_1 + T_2) S_z + \mu E \partial_{x_+} S_z + \mu E Q S_{x_+} - \omega_L S_{x_+} + I_z(t)\delta(x_+)$$

Jing Wang, Rundong Li, SC Zhang, et al

Similar to steady state B=0 case, solve above equations with appropriate boundary conditions: **resonant behavior around ω_L and small shift of oscillation period**





Semiclassical Monte Carlo of SIHE

Numerical solution of Boltzmann equation

$$\frac{\partial f}{\partial t} + \dot{\mathbf{r}}\partial_{\mathbf{r}} f + \dot{\mathbf{k}}\partial_{\mathbf{k}} f = \left(\frac{\partial f}{\partial t}\right)_c$$

Spin-independent scattering:

$$W(\vec{k}\sigma, \vec{k}'\sigma')$$

Spin-dependent scattering:

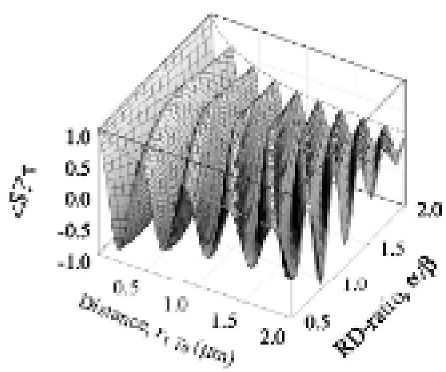
- phonons,
- remote impurities,
- interface roughness, etc.

- side-jump, skew scattering.

$\Longrightarrow$ AHE

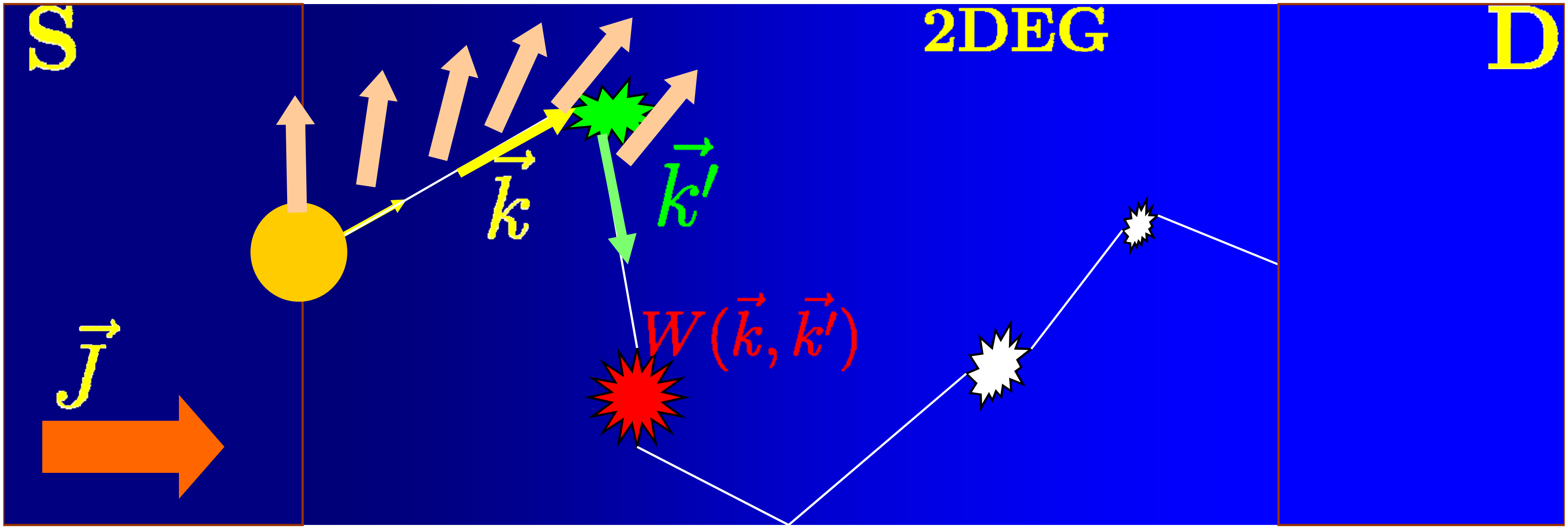
Persistent spin helix

PRB 77, 045323 (2008)



- Realistic system sizes (μm).
- Less computationally intensive than other methods (e.g. NEGF).

Single Particle Monte Carlo



Spin-Dependent Semiclassical Monte Carlo

- Temperature effects, disorder, nonlinear effects, transient regimes.
- Transparent inclusion of relevant microscopic mechanisms affecting spin transport (impurities, phonons, AHE contributions, etc.).
- Less computationally intensive than other methods(NEGF).
- Realistic size devices.

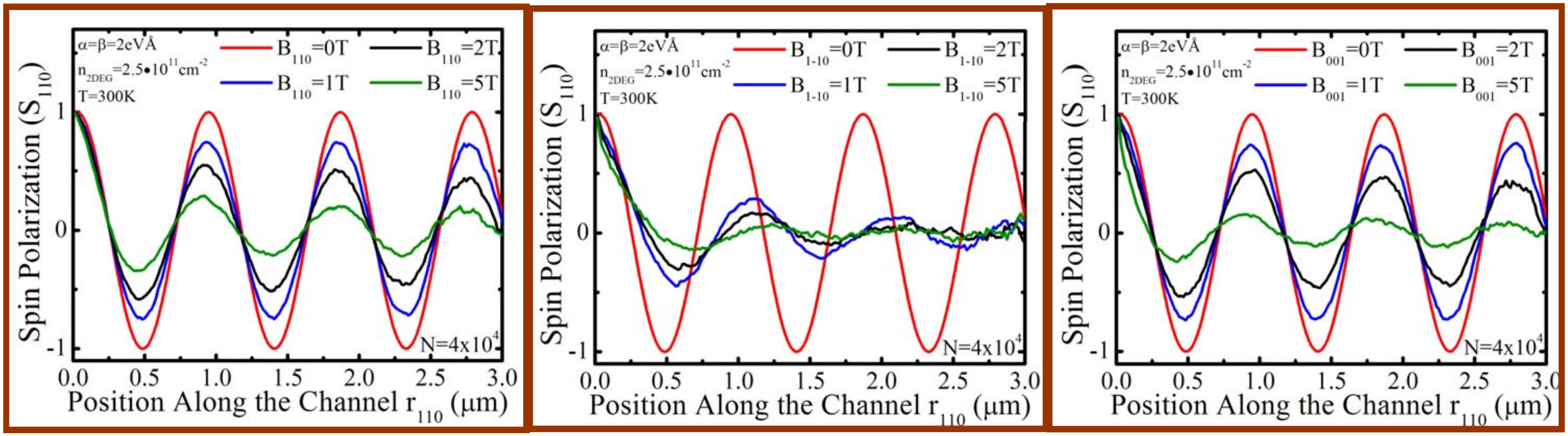
Effects of B field: current set-up

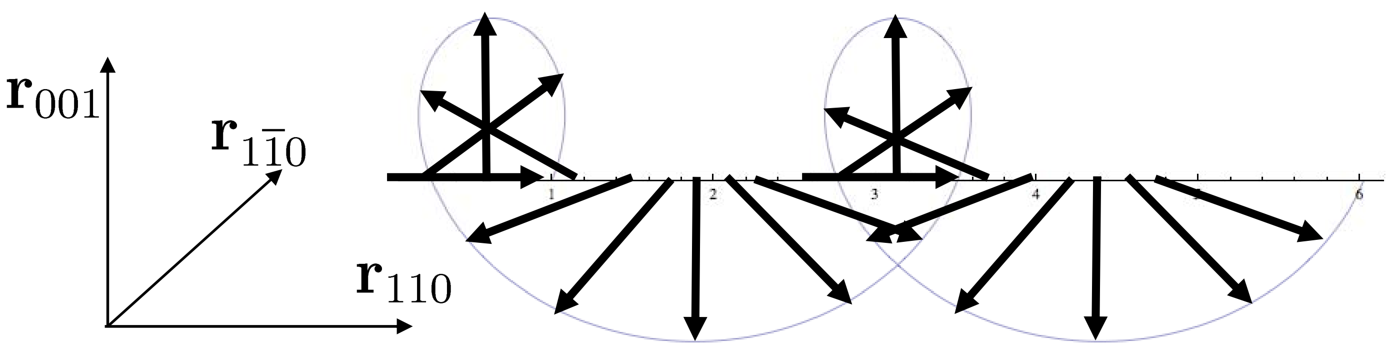
$\alpha = \beta$

In-Plane magnetic field

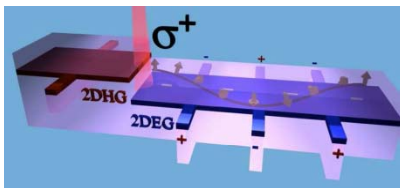
Out-of plane magnetic field

B_{110} $B_{1\bar{1}0}$ B_{001}





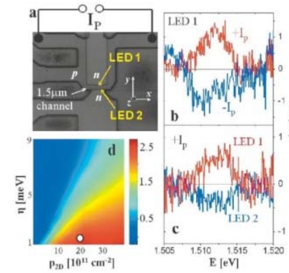
The family of spintronics Hall effects



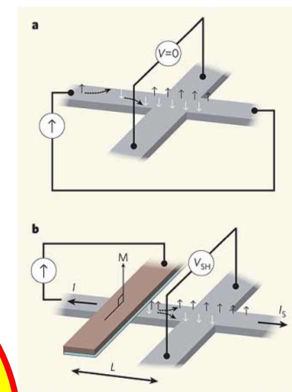
SIHE
B=0
Optical injected polarized current gives charge current

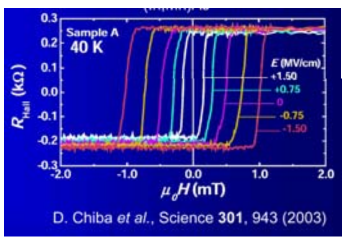
Electrical detection

SHE
B=0
charge current gives spin current



Optical detection





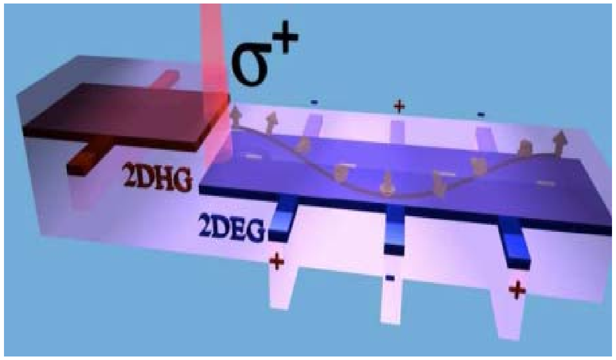
AHE
B=0
polarized charge current gives charge-spin current

Electrical detection

SHE^{-1}
B=0
spin current gives charge current

Electrical detection

SIHE: a new tool to explore spintronics



•nondestructive electric probing tool of spin propagation without magnetic elements

•all electrical spin-polarimeter in the optical range

•Gating (tunes α/β ratio) allows for FET type devices (high T operation)

•New tool to explore the AHE in the strong SO coupled regime