Interaction and disorder effects in graphene

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and
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Collaborators

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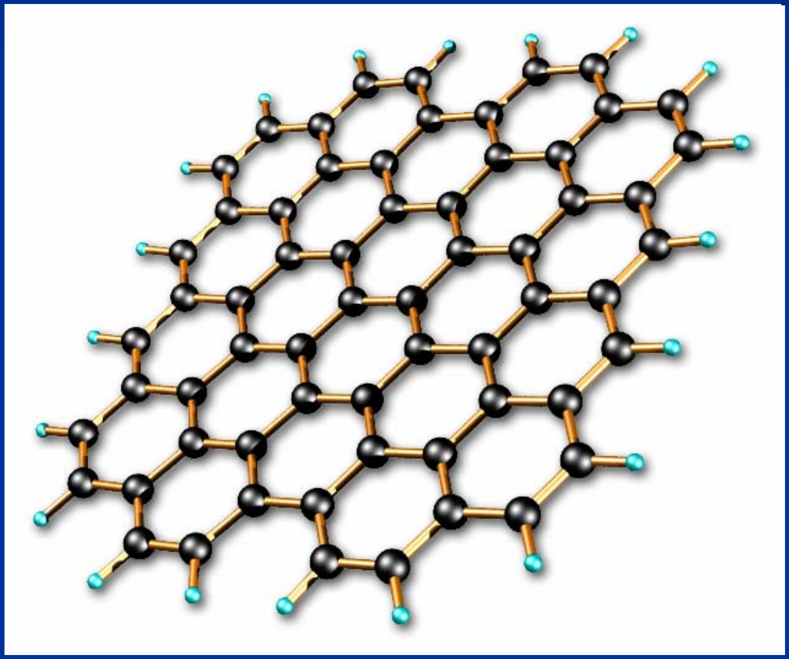
This talk is based on:

OV arXiv.0810.3697

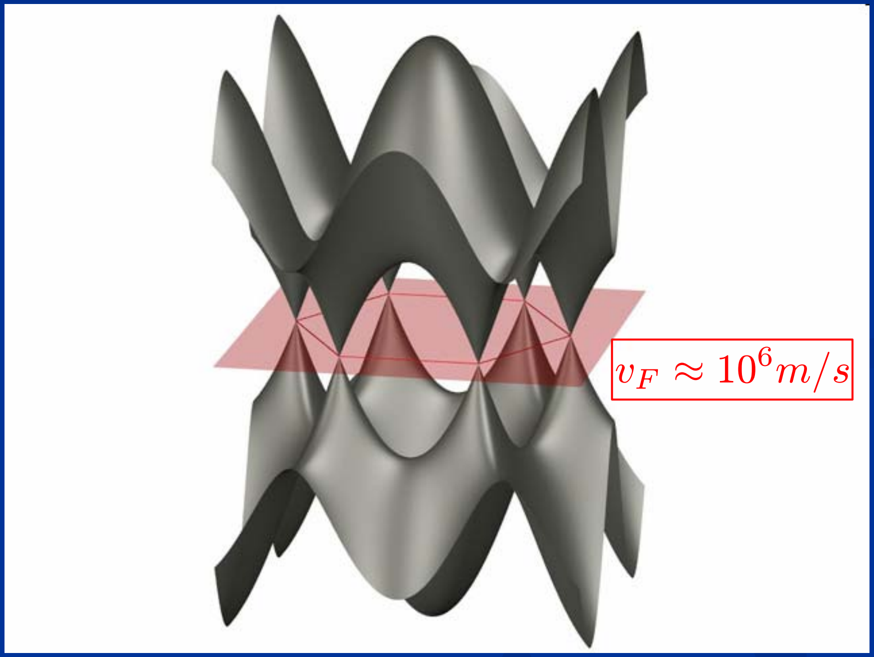
*I.F. Herbut, V. Juricic, OV, PRL **100**, 046403 (2008).*
*OV and M.J. Case, PRB **77**, 033410 (2008).*
*OV PRL**98**, 216401 (2007).*
*OV PRL**97**, 266406 (2006).*

Outline

- *Dirac Fermions and marginal irrelevance of the 1/r Coulomb interactions*
- *Rippling and the resulting random strain coupling to the massless Dirac particles:*
  random vector Δ_A + scalar Δ_ϕ potentials
- *Combined effect of interactions and disorder:*
  Infra-red stable line of fixed points
- *Minimal conductivity as a critical amplitude:*
  ω-dependence from the flow towards the fixed line

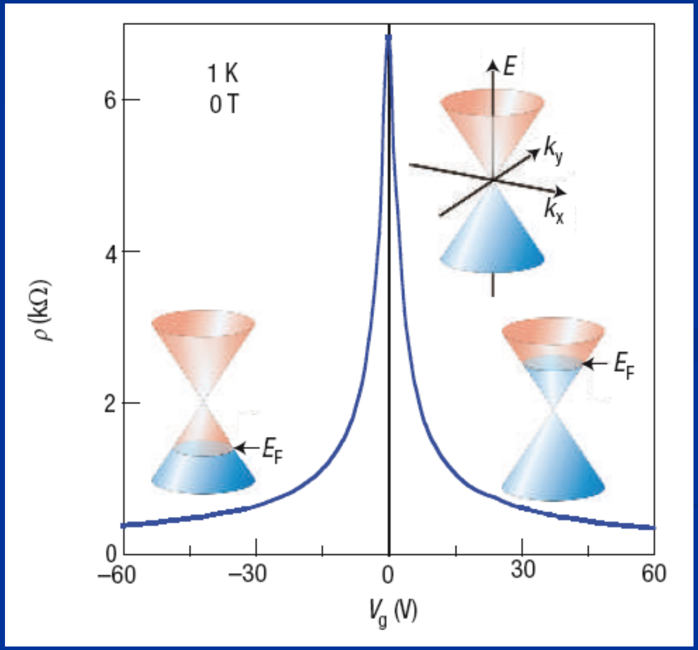


$\Longrightarrow$



Minimal Conductivity: Experimental status

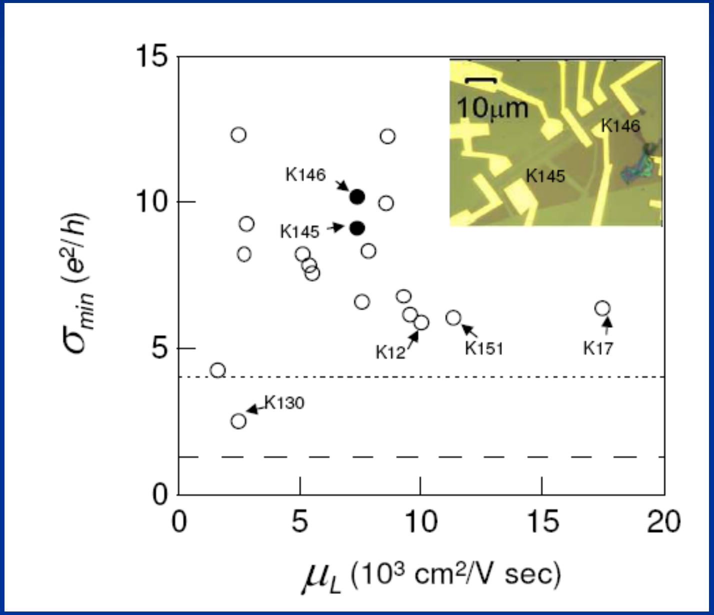
$r \sim h/e^2$ What sets the proportionality constant?



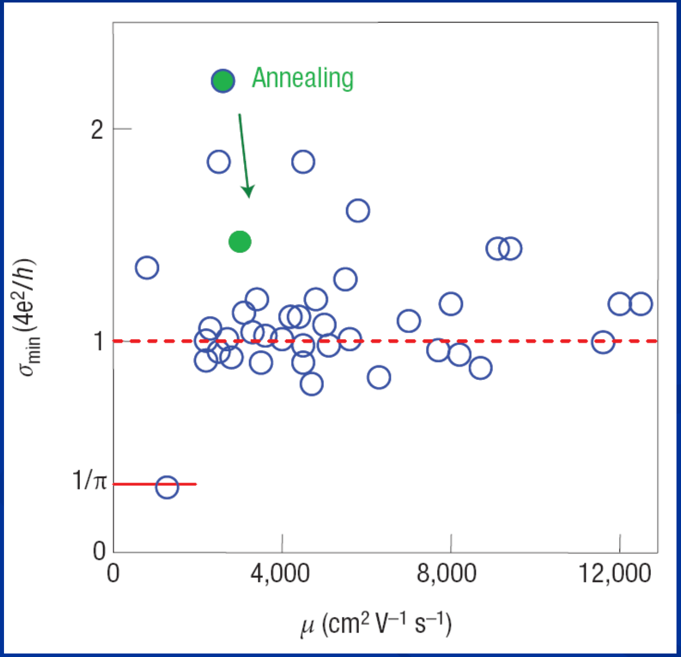
From: Geim and Novoselov Nature Materials 2007

Minimum conductivity: Experimental status

Collision dominated regime ω<<T (d.c. conductivity)



Y.-W. Tan et.al. PRL 2007

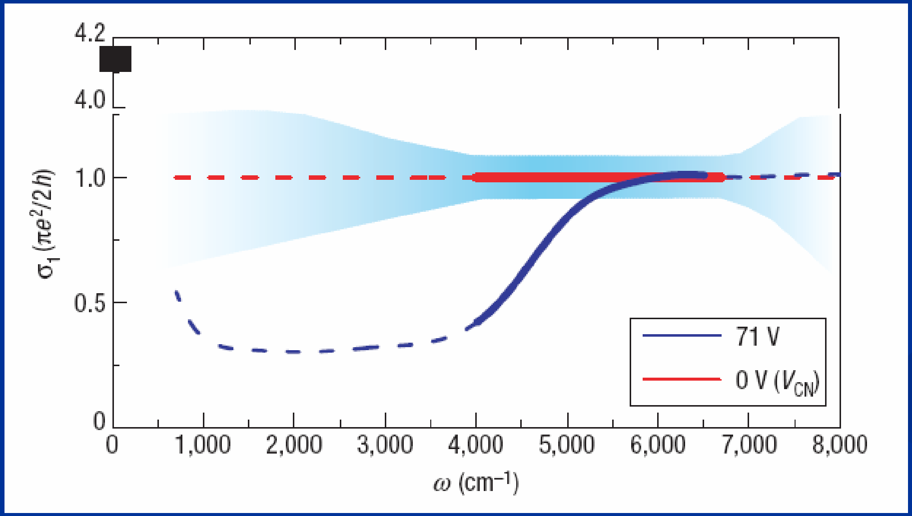


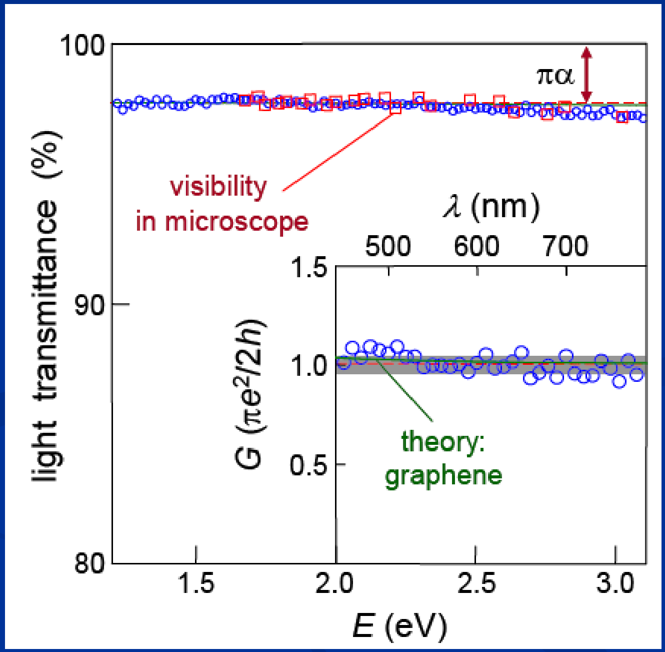
Geim and Novoselov Nature Materials 2007

$$\sigma_{min}(\omega < Hz, T \sim 10-100K) = (non-universal\#) \times \frac{e^2}{h}$$

Minimum conductivity: Experimental status

"collisionless" regime ω>>T (a.c. conductivity)



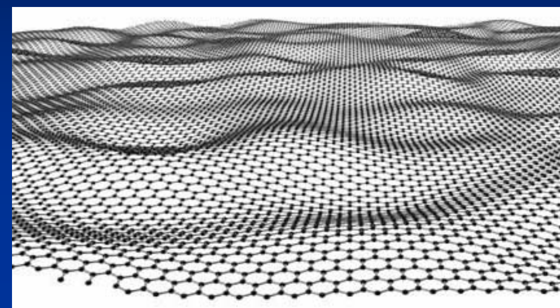


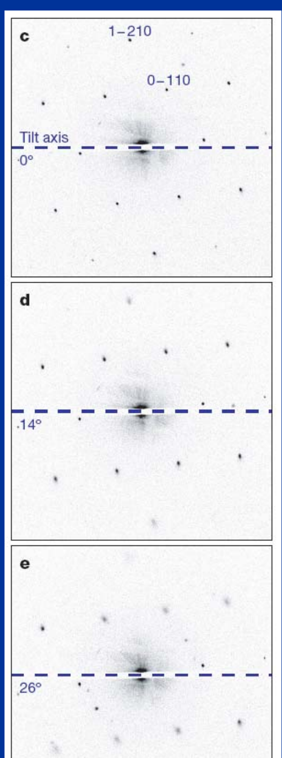
Z. Q. Li et.al. Nature Physics 2008

R. R. Nair et.al. Science 2008
**likely slightly away from neutrality*

$$\sigma_{min}(\omega \sim eV, T \sim 10-100K) \approx \frac{\pi}{2} \times \frac{e^2}{h}$$

Statement of the problem



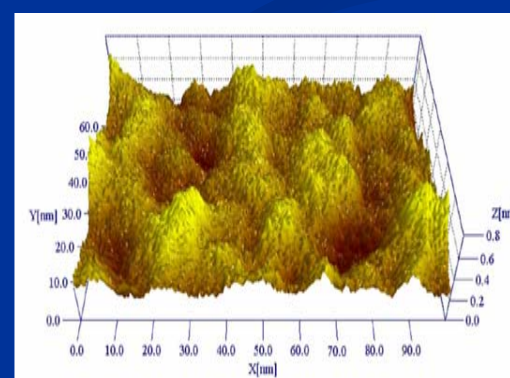


Electron diffraction from a <u>suspended</u> graphene monolayer under different incidence angles.

J. Meyer et. al.

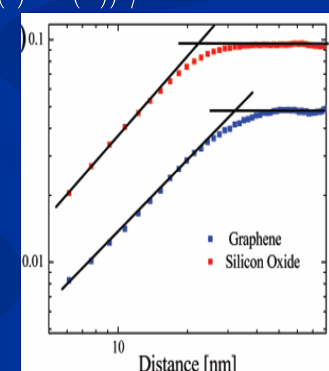
Vol 446 | 1 March 2007 | **doi:10.1038/nature05545**

Ripples are ~0.5nm in height w/ typical size ~5nm laterally.



STM image of a single layer graphene <u>on a silicon oxide</u>: Stolyarova et al., PNAS, 104, 9209 (2007).

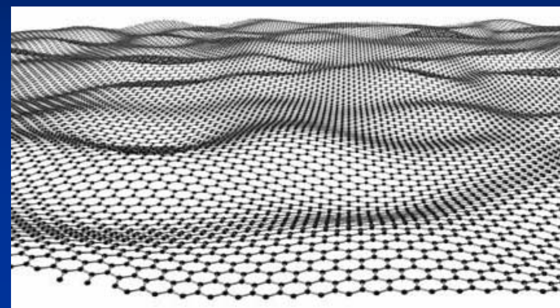
$\langle (h(\mathbf{r}) - h(\mathbf{r}'))^2 \rangle$



Ishigami et al., Nano Lett., 7, 1643 (2007).

Statement of the problem

What is the combined effect of the rippling and the Coulomb interactions on the Dirac fermions?



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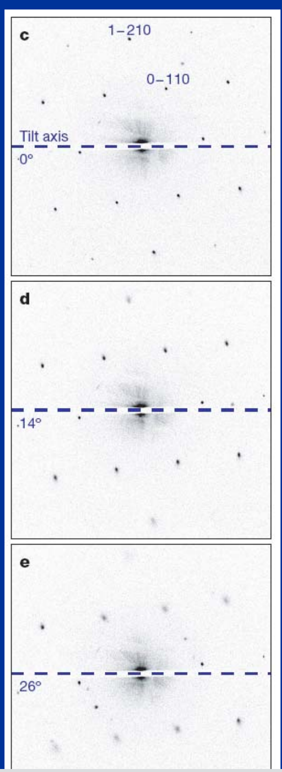

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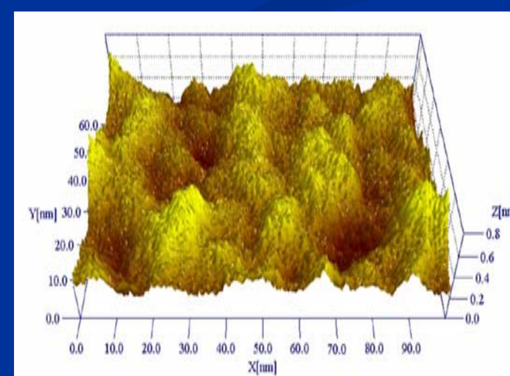

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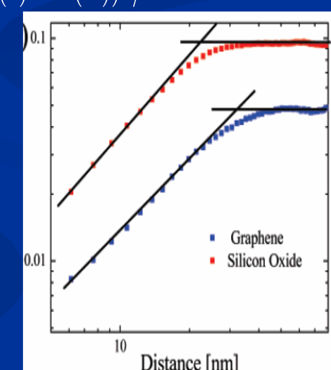

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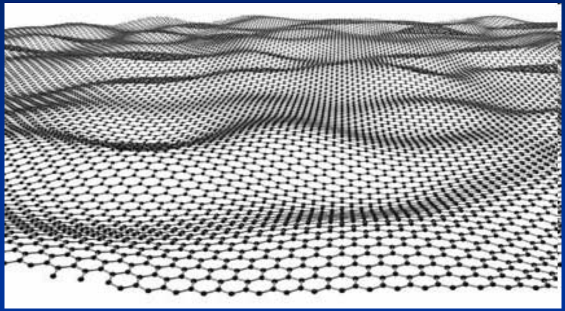
Formulation of the problem

What is the combined effect of the rippling and the Coulomb interactions on the Dirac fermions?



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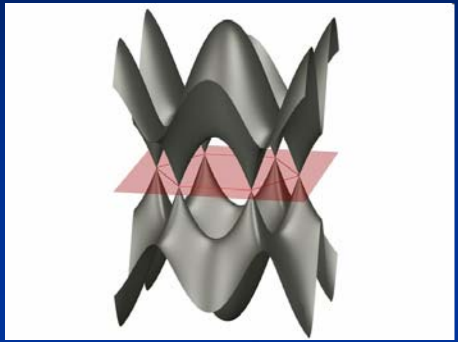
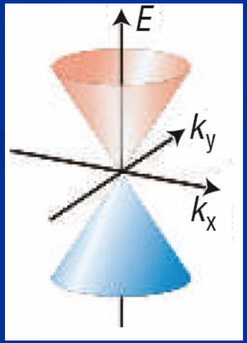


Rippling introduces strain u_{ij} which couples to the Dirac Femions through scalar and vector potentials



$$\mathcal{H}_0 = \sum_{j=1}^{N} \int d^2\mathbf{r} \left[\psi_j^\dagger(\mathbf{r}) v_F \mathbf{p} \cdot \sigma \psi_j(\mathbf{r}) \right].$$

$$\mathcal{H}_{disorder} = \sum_{j=1}^{N} \int d^2\mathbf{r} \left[\psi_j^\dagger(\mathbf{r}) \left(\phi(\mathbf{r}) + (-1)^j v_F \mathbf{a}(\mathbf{r}) \cdot \sigma \right) \psi_j(\mathbf{r}) \right].$$

$\phi = g(u_{xx} + u_{yy})$, $a_x = b(u_{yy} - u_{xx})$, $a_y = 2bu_{xy}$, and $g \approx 20 - 30eV$, $b \approx A^{-1}$

H. Suzura, T. Ando al., PRB (2002), Mariani, von Oppen PRL (2008), Guinea et.al. PRB (2008)

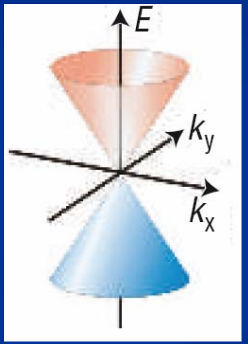
Formulation of the problem

Strain + Coulomb interaction effects:

$$\mathcal{H}_0 = \sum_{j=1}^{N} \int d^2\mathbf{r} \left[\psi_j^\dagger(\mathbf{r}) v_F \mathbf{p}\cdot\sigma\psi_j(\mathbf{r})\right].$$

$$\mathcal{H}_{disorder} = \sum_{j=1}^{N} \int d^2\mathbf{r} \left[\psi_j^\dagger(\mathbf{r}) \left(\phi(\mathbf{r}) + v_F \mathbf{a}(\mathbf{r})\cdot\sigma\right)\psi_j(\mathbf{r})\right].$$

$$\hat{V}_{Coulomb} = \frac{1}{2}\int d^2\mathbf{r} d^2\mathbf{r}' \left[\delta\hat{n}(\mathbf{r})\frac{e^2}{\epsilon}\frac{1}{|\mathbf{r}-\mathbf{r}'|}\delta\hat{n}(\mathbf{r}')\right].$$



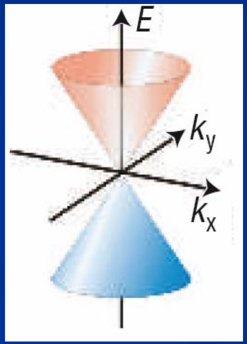
$$\langle\phi(\mathbf{k})\rangle = 0; \quad \langle\phi_\mu(\mathbf{k})\phi(\mathbf{k}')\rangle = \Delta_\phi\delta_{\mu\nu}(2\pi)^2\delta(\mathbf{k}-\mathbf{k}')$$

$$\langle a_\mu(\mathbf{k})\rangle = 0; \quad \langle a_\mu(\mathbf{k}) a_\nu(\mathbf{k}')\rangle = \Delta_A\delta_{\mu\nu}(2\pi)^2\delta(\mathbf{k}-\mathbf{k}')$$

OV arXiv.0810.3697

Formulation of the problem

Replica field theory: $\psi \to \psi^i;\ i = 1, 2 \ldots, n$



$$\langle Z^n \rangle_{dis} = \int D\psi^\dagger \psi e^{-(S_0 + S_\phi + S_A + S_{int})}$$

$$S_0 = \int_0^\beta d\tau \int d^2 r \psi^\dagger (\partial_\tau + v_F \sigma \cdot \mathbf{p})$$

$$S_\phi = -\frac{1}{2} v_F^2 \Delta_\phi \int_0^\beta d\tau d\tau' \int d^2\mathbf{r} \psi^{i\dagger} \psi^i(r,\tau) \psi^{j\dagger} \psi^j(r,\tau')$$

$$S_A = -\frac{1}{2} v_F^2 \Delta_A \int_0^\beta d\tau d\tau' \int d^2\mathbf{r} \psi^{i\dagger} \sigma^a \psi^i(r,\tau) \psi^{j\dagger} \sigma^a \psi^j(r,\tau')$$

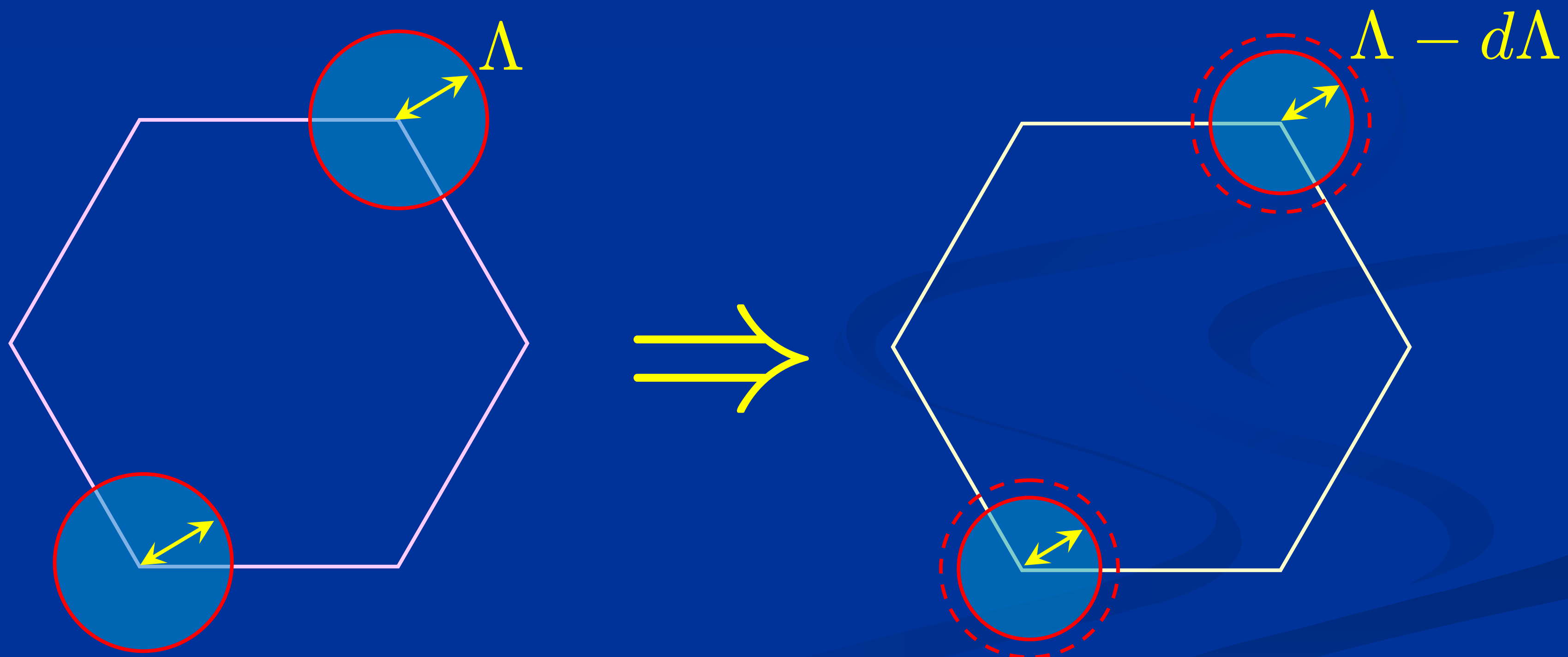
$$S_{int} = \frac{1}{2} \int_0^\beta d\tau \int d^2\mathbf{r} d^2\mathbf{r}' \psi^{i\dagger} \psi^i(r,\tau) V(|r - r'|) \psi^{i\dagger} \psi^i(r',\tau')$$

$$\alpha \equiv \frac{e^2}{\epsilon v_F}$$

OV arXiv.0810.3697

Renormalization group approach

The degrees of freedom of interest reside near the K and K' points in the reciprocal space.

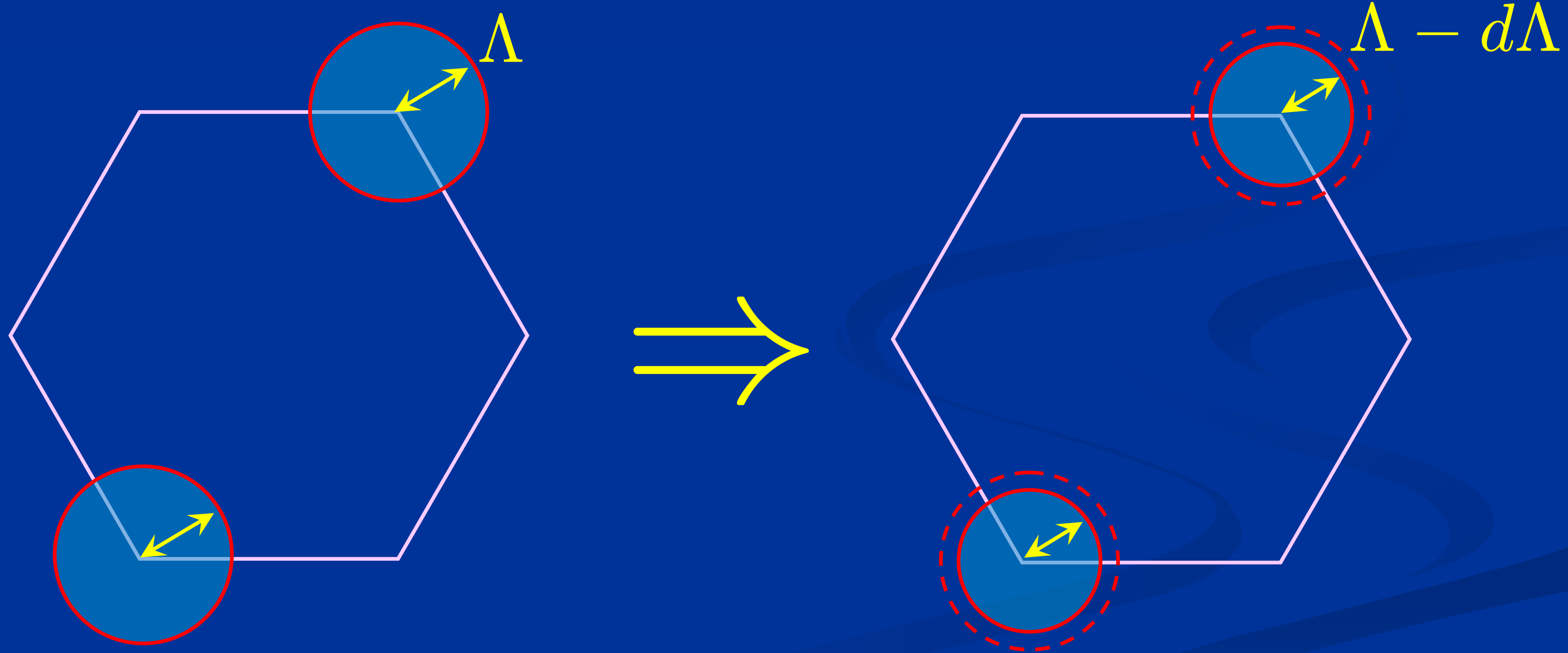


$$\frac{d}{d\Lambda}\langle \mathcal{O} \rangle = 0 \Rightarrow$$

Observables must be independent of the arbitrary cutoff Λ upon proper rescaling and adjustment of the coupling constants

Renormalization group approach

Analyze the fate of the "coupling constants": $e^2, v_F, \Delta_A, \Delta_\phi$



OV arXiv.0810.3697

Minimal conductivity: RG perspective

$$\sigma_{dc}\left(\{g_i(\Lambda)\}\right) = \sigma_{dc}\left(\{g_i(\Lambda')\}\right).$$

• the conductivity calculated from the original couplings in the theory, σ*({g_i(Λ)})*, with the cutoff set at Λ or from the new couplings σ*({g(Λ')})* with the cutoff set at Λ', is the same. Therefore, *the conductivity is a function of the universal fixed point couplings only and is itself universal*.

• the universality of conductivity here means dependence on the *fixed point couplings only*. If instead of a single point there is actually a line of such fixed points, then the conductivity depends on the precise position along such line, which is typically experimentally uncontrollable, giving rise to an appearance of non-universality.

• *the electrical conductivity measurement at the neutrality point is therefore a direct probe of the highly non-trivial physics emerging at the end of the renormalization group trajectory.*

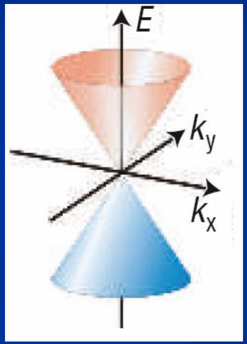
See e.g M.-C. Cha et. al. PRB 1991, Sondhi et.al. RMP 1997

Special cases I

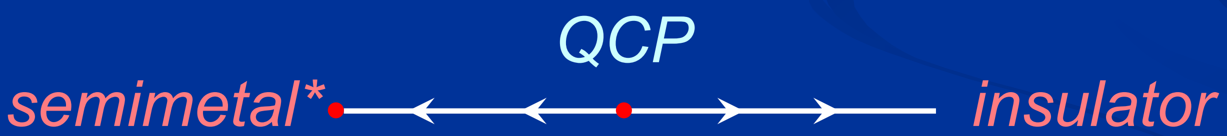
Coulomb interaction only

$$\frac{de^2}{d\ln\Lambda} = 0$$
$$\frac{dv_F}{d\ln\Lambda} = -\frac{e^2}{4\epsilon} + \mathcal{C}\frac{e^4}{\epsilon^2 v_F} + \mathcal{O}(e^6)$$

$\mathcal{C} \approx 0.3 > 0$



OV and M.J. Case, PRB ***77****, 033410 (2008)*



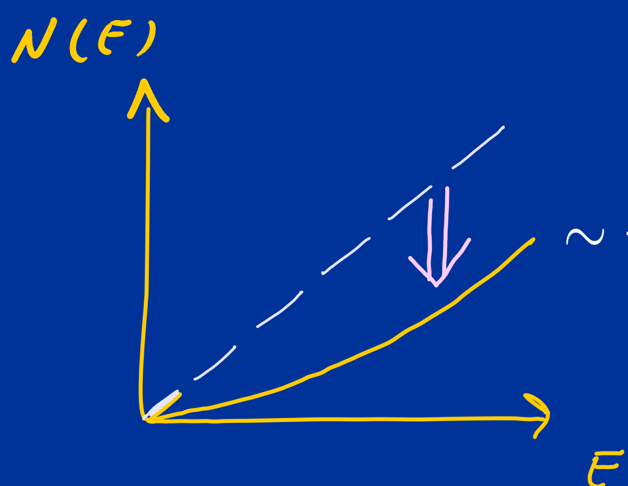
$\frac{e^2}{\epsilon v_F} = 0$ $\frac{e^2}{\epsilon v_F} \approx 0.833$ *(underestimate)*

**Monte Carlo gives :* 1.11 ± 0.06

Drut and Lahde PRL ***102****, 026802 (2009)*

Effects of Coulomb interactions on (clean) Dirac fermions

$$\frac{e^2}{\epsilon v_F} = 0 \qquad \frac{e^2}{\epsilon v_F} \approx 0.833$$ *(underestimate)*



$$\sim \frac{E}{\ln^2 E}$$

- specific heat suppressed

$$c_V \sim \frac{T^2}{\ln^2 T}$$

*OV PRL **98**, 216401 (2007).*

Electrical transport at the clean Dirac point: Coulomb interactions

At $T = 0$ the free system is metallic, with conductivity $\sigma_0(\omega) = N \frac{\pi}{8} \frac{e^2}{h}$ (note, ω independent due to $N(\omega) \sim \omega$)
We might expect, that Coulomb interactions would drive it

insulating since the density of states is suppressed. Not so! There is a cancellation between density of states suppression and vertex enhancement. With Coulomb interactions:

$$\sigma(\omega) = N \frac{\pi}{8} \frac{e^2}{h} \left(1 + c \frac{e^2}{\hbar\epsilon \left(v_F + \frac{e^2}{\hbar\epsilon} \ln \frac{\Lambda}{\omega} \right)} \right).$$

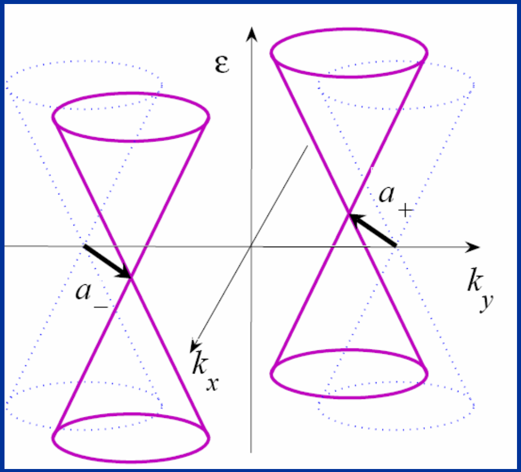
$$c = (25/3 - 2\pi) \approx 2.05$$

Interactions enhance a.c. conductivity!

*I.F. Herbut, V. Juricic, OV, PRL **100**, 046403 (2008)*

Rippling induced strains and Dirac Fermions

The smooth corrugations tend to introduce spatial modulation of the hopping amplitudes. At long wavelengths, this results in local shifts of the Dirac points.



$$\mathcal{H}_{Dirac} = \begin{pmatrix} v_F \sigma \cdot (\mathbf{p} - \mathbf{a}(\mathbf{r})) & 0 \\ 0 & v_F \sigma \cdot (\mathbf{p} + \mathbf{a}(\mathbf{r})) \end{pmatrix}.$$

- *For any given realization of the "vector" potential, the overall time reversal symmetry is preserved.*
- *Still, in the vicinity of the nodal points, the varying hopping appears to induce varying magnetic field **H**. This field changes direction for the two nodes.*

Iordanskii and Koshelev, JETP Lett. 41, 574 (1985).
Kane and Mele PRL (1997), Morozov et.al. PRL (2006),
Morpurgo and Guinea PRL (2006), Abanin et. al. PRL (2007) …

Free Dirac Fermions and random vector potential

$$\mathcal{H}_0 = \sum_{j=1}^{\frac{N}{2}} \int d^2\mathbf{r} \left[\psi^\dagger_{j\pm}(\mathbf{r}) v_F \left(\mathbf{p} \pm \mathbf{a}(\mathbf{r}) \right) \cdot \sigma \psi_{j\pm}(\mathbf{r}) \right]$$

$$\langle a_\mu(\mathbf{k}) \rangle = 0; \quad \langle a_\mu(\mathbf{k}) a_\nu(\mathbf{k}') \rangle = \Delta_A \delta_{\mu\nu} (2\pi)^2 \delta(\mathbf{k} - \mathbf{k}')$$

$$\frac{dv_F}{d\ln\kappa} = v_F \frac{\Delta_A}{\pi}$$

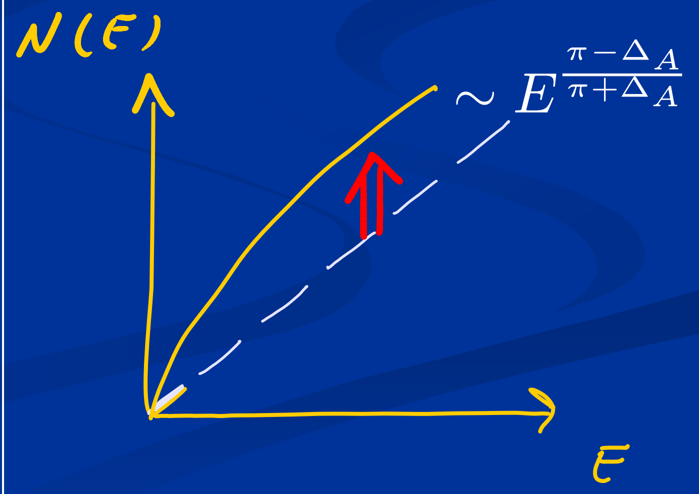
$$\frac{d\Delta_A}{d\ln\kappa} = 0$$

- *Random vector potential is an exactly marginal perturbation; can write down an exact zero energy eigenstate.*
- *Disorder dependence of the "dynamical critical exponent" z are known exactly*

$$\omega \sim k^z \Rightarrow N(E) \sim E^{-1+\frac{2}{z}}$$

$$z = 1 + \frac{\Delta_A}{\pi} \text{ for } \Delta_A \leq 2\pi$$

$$z = 4\sqrt{\frac{\Delta_A}{2\pi}} - 1 \text{ for } \Delta_A > 2\pi$$



Ludwig et.al. PRB 1994; Motrunich et.al. PRB 2002

Free Dirac Fermions and random vector potential: Numerical Check

$$\mathcal{H}_{lattice} = \sum_{\langle rr' \rangle} t_{\mathbf{r},\mathbf{r}'} \left[c^\dagger_{\mathbf{r}} c_{\mathbf{r}'} + h.c. \right]; \qquad t_{\mathbf{r},\mathbf{r}'} = -t e^{-\phi_{\mathbf{r}} + \phi_{\mathbf{r}'}}$$

$$\mathcal{P}[\{\phi_{\mathbf{r}}\}] \sim e^{-\frac{1}{2\Delta_A} \sum_{\langle \mathbf{r}\mathbf{r}' \rangle} (\phi_{\mathbf{r}} - \phi_{\mathbf{r}'})^2}$$

- *We take periodic boundary conditions*
- *Then generate the random configurations of ϕ using Metropolis algorithm*
- *Calculate the density of states and average over configurations*
- *Expect:*

$$N(E) \sim E^{-1+\frac{2}{z}}; \qquad z = 1 + \sqrt{3}\frac{\Delta_A}{\pi}$$

Motrunich et.al. PRB 2002

Free Dirac Fermions and random vector potential: Numerical Check

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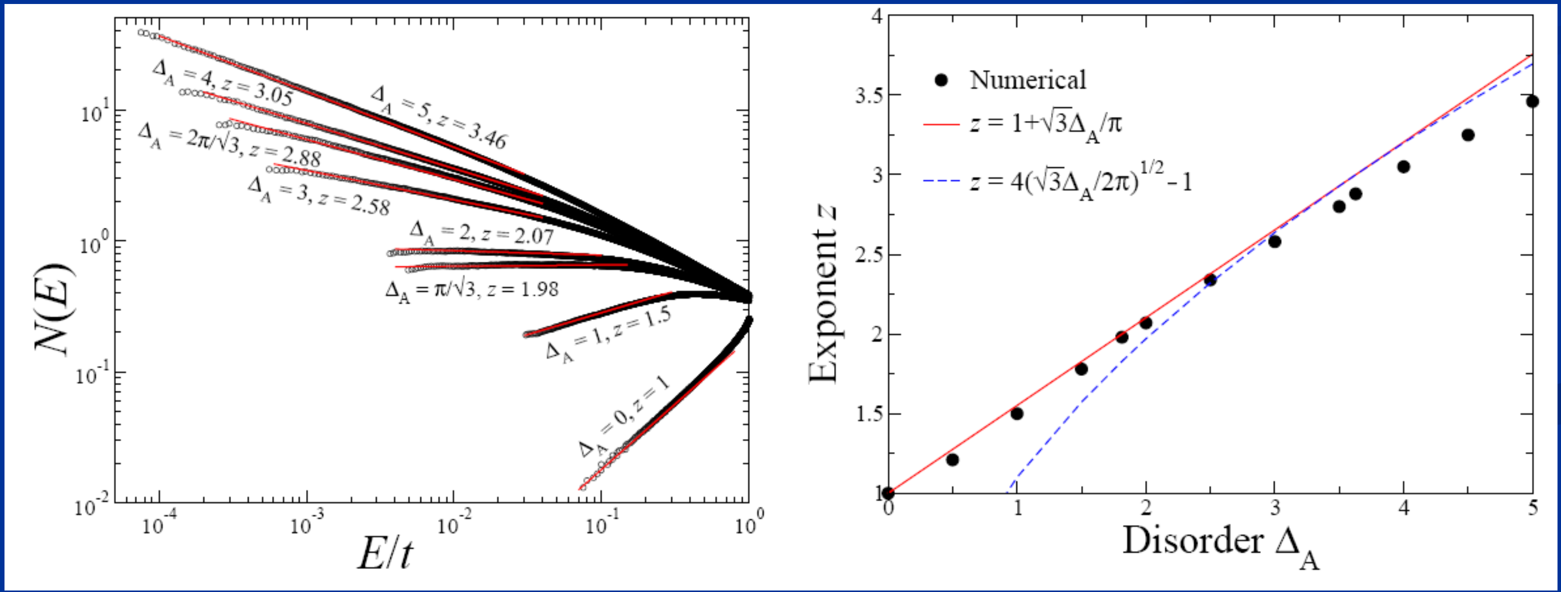

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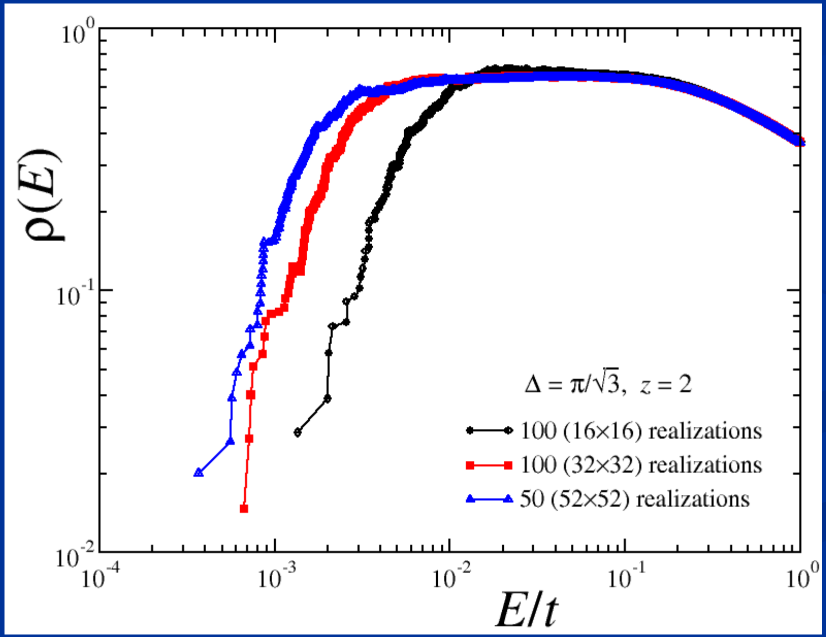
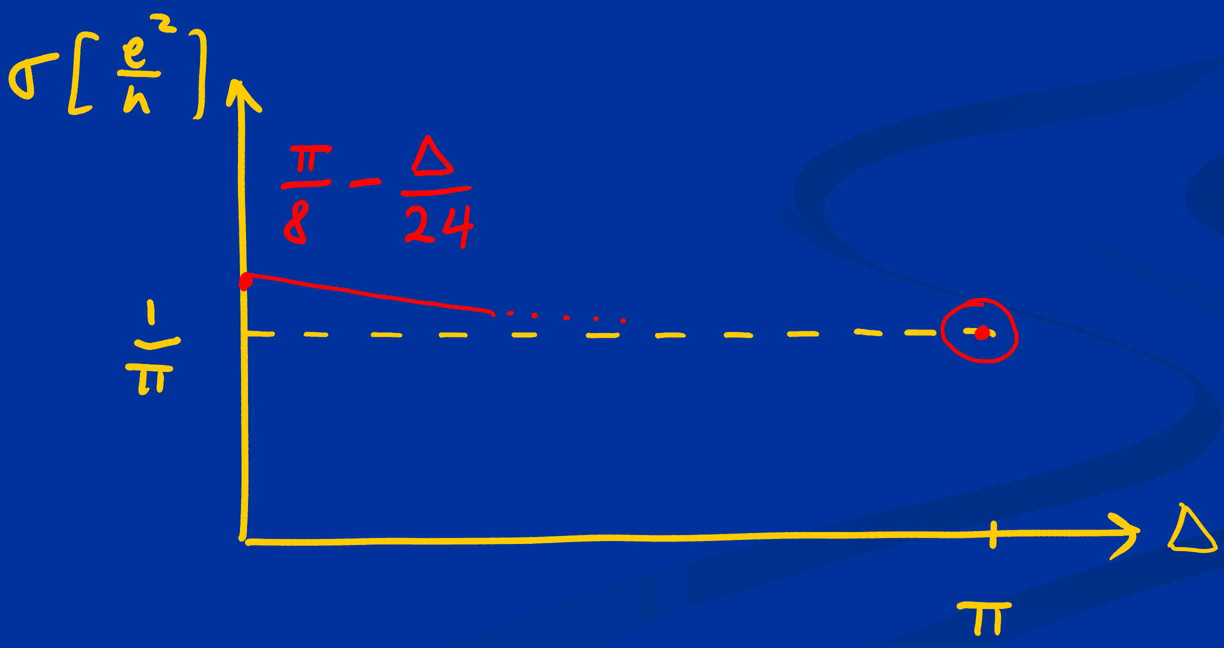
$$\mathcal{P}[\{\phi_{\mathbf{r}}\}] \sim e^{-\frac{1}{2\Delta_A}\sum_{\langle \mathbf{r}\mathbf{r}' \rangle}(\phi_{\mathbf{r}}-\phi_{\mathbf{r}'})^2}$$



Free Dirac Fermions and random vector potential

- *Conductivity along the non-interacting fixed line is non-universal*
- *Ludwig et. al. (1994) hypothesis: for z=2,* $\sigma=1/\pi\,(e^2/h)$
- *In combination with the perturbative result it is reasonable to conjecture that the conductivity is monotonic in* Δ_A *up to* $\Delta_A=\pi$.



Special cases II

Coulomb interactions and random vector potential only

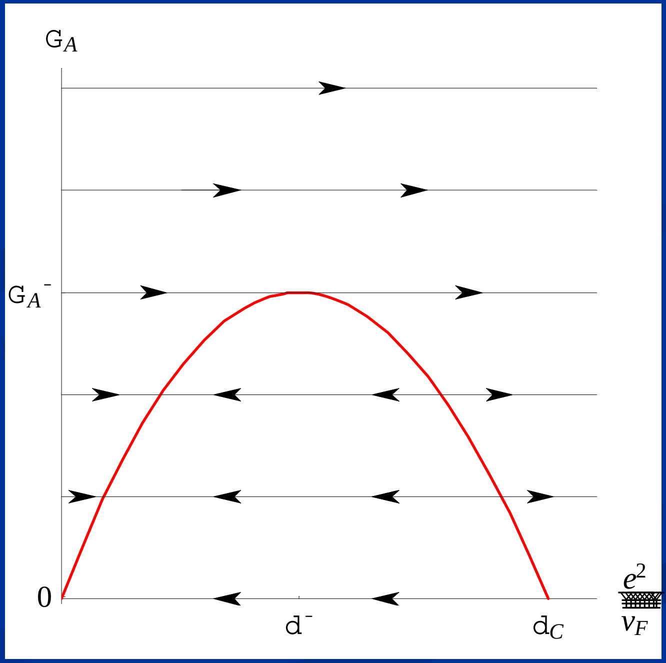
$$\frac{dv_F}{d\ln\Lambda} = v_F\frac{\Delta_A}{\pi} - \frac{e^2}{4\epsilon} + \mathcal{A}v_F\Delta_A^2 + \mathcal{B}\Delta_A\frac{e^2}{\epsilon} + \mathcal{C}\frac{e^4}{\epsilon^2 v_F} + \mathcal{O}(e^6)$$

$$\frac{de^2}{d\ln\Lambda} = 0$$

$$\frac{d\Delta_A}{d\ln\Lambda} = 0$$ ⟸ *marginal(!)*

$$\mathcal{A} = 0,\quad \mathcal{B} = \frac{1}{8\pi},\quad \mathcal{C} = \frac{N}{12} - \frac{103}{96} + \frac{3}{2}\ln 2.$$

$$\sigma(\omega) = \frac{e^2}{h}\frac{\pi}{2}\left(1 + \left(\frac{8}{\pi} - 2\right)\Delta_A\right).$$



*I.F. Herbut, V. Juricic, OV, PRL **100**, 046403 (2008).*
*OV and M.J. Case, PRB **77**, 033410 (2008).*
Stauber et.al. PRB (2005); Foster and Aleiner PRB (2008)

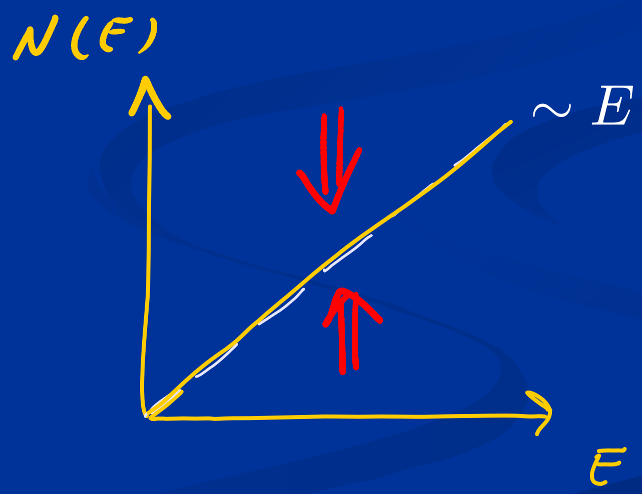
Combined effect of interactions and disorder: infra-red (locally) stable line of fixed points

Competition between logarithmic depletion of density of states and logarithmic increase, balances along a fixed line

$$\frac{dv_F}{d\ln\kappa} = v_F\frac{\Delta_A}{\pi} - \frac{e^2}{4\epsilon} + \mathcal{A}v_F\Delta_A^2 + \mathcal{B}\Delta_A\frac{e^2}{\epsilon} + \mathcal{C}\frac{e^4}{\epsilon^2 v_F} + \mathcal{O}(e^6)$$

$$\frac{de^2}{d\ln\kappa} = 0$$

$$\frac{d\Delta_A}{d\ln\kappa} = 0$$



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*OV and M.J. Case, PRB **77**, 033410 (2008).*

The general case

$$\beta_\alpha = \frac{\partial \alpha}{\partial \ln \Lambda} = -\alpha\left(\frac{\Delta_A}{\pi} + \frac{\Delta_\phi}{2\pi} - \frac{\alpha}{4}\right)$$

$$\beta_{\Delta_\phi} = \frac{\partial \Delta_A}{\partial \ln \Lambda} = 0$$

$$\beta_{\Delta_\phi} = \frac{\partial \Delta_\phi}{\partial \ln \Lambda} = -2\Delta_\phi\left(\frac{\Delta_A}{\pi} + \frac{\Delta_\phi}{2\pi} - \frac{\alpha}{4}\right).$$

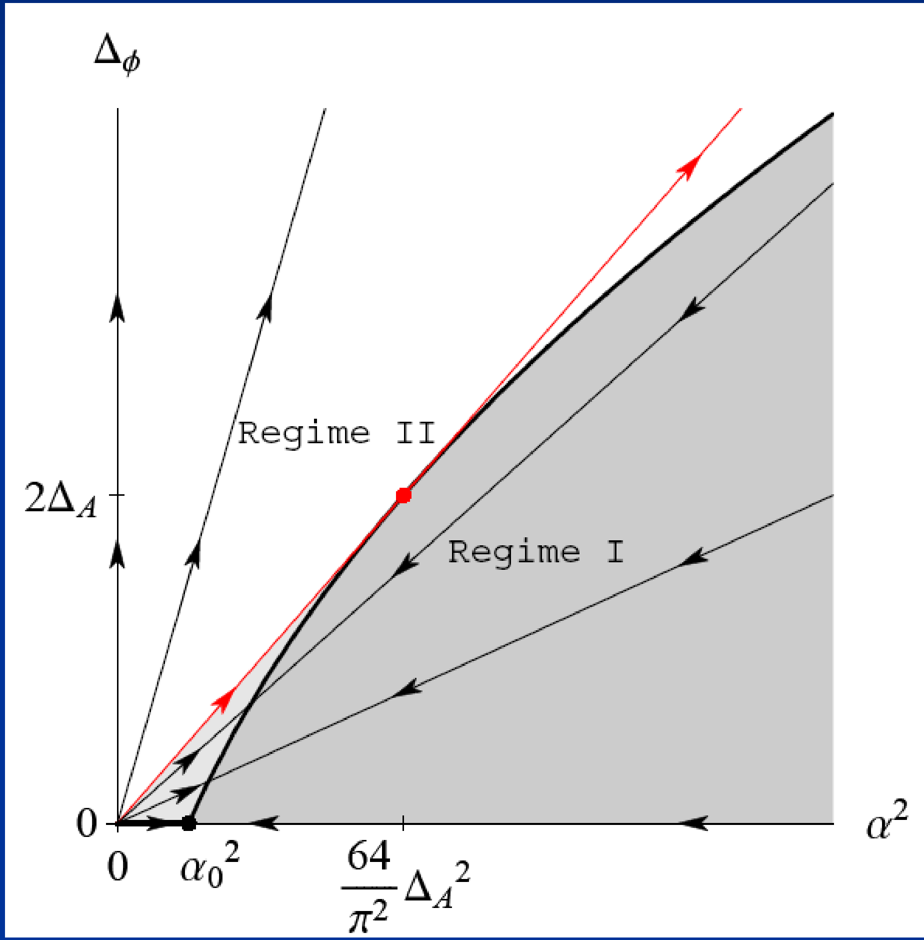
$$\frac{d}{d\ln\Lambda}\left[\Delta_A\right] = 0$$

$$\frac{d}{d\ln\Lambda}\left[\frac{\Delta_\phi}{\alpha^2}\right] = 0$$

2 marginal parameters!

OV arXiv.0810.3697
Foster and Aleiner PRB (2008)

The general case



Regime I:

Flow to the perturbatively accessible IR stable fixed line at nonzero(!) Δ_ϕ , Δ_A , e^2/v_F.

The scalar disorder (puddles) -> effectively screened. Dirac point is a useful starting point

Regime II:

Runaway flows, no perturbative control. Thinking about the puddles is likely more useful.

OV arXiv.0810.3697

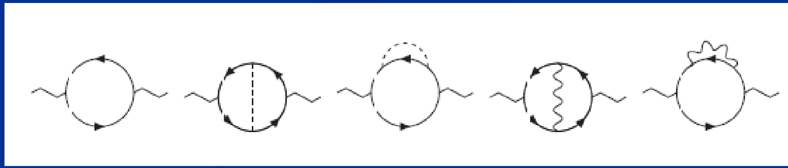
The general case: conductivity from RG

Since the conductivity does not develop an anomalous dimension:

$$\Rightarrow \left(\frac{\partial}{\partial \ln \Lambda} + \beta_{\Delta_\phi} \frac{\partial}{\partial \Delta_\phi} + \beta_{\Delta_A} \frac{\partial}{\partial \Delta_A} + \beta_\alpha \frac{\partial}{\partial \alpha} \right) \sigma \left(\omega, \Lambda, \Delta_\phi, \Delta_A, \alpha = \frac{e^2}{v_F} \right) = 0$$

$$\Rightarrow \sigma\left(\omega, \Lambda, \Delta_\phi[\Lambda], \Delta_A[\Lambda], \alpha[\Lambda]\right) = \sigma\left(\omega, \rho\Lambda, \Delta_\phi[\rho\Lambda], \Delta_A[\rho\Lambda], \alpha[\rho\Lambda]\right)$$

A pedestrian perturbation theory gives:



$$\sigma\left(\omega, \Lambda, \Delta_\phi, \Delta_A, \alpha\right) = 4\frac{e^2}{h}\left(\frac{\pi}{8} - \frac{\Delta_\phi}{24} - \frac{\Delta_A}{24} + \alpha\frac{\pi}{16}\left(\frac{25}{6} - \pi\right)\right).$$

To make this result consistent with the scaling law

$$\sigma\left(\omega, \Lambda, \Delta_\phi, \Delta_A, \alpha\right) = 4\frac{e^2}{h}\left(\frac{\pi}{8} - \frac{1}{24}\frac{\Delta_\phi}{\alpha^2}\alpha^2(\omega/\Lambda) - \frac{\Delta_A}{24} + \alpha(\omega/\Lambda)\frac{\pi}{16}\left(\frac{25}{6} - \pi\right)\right).$$

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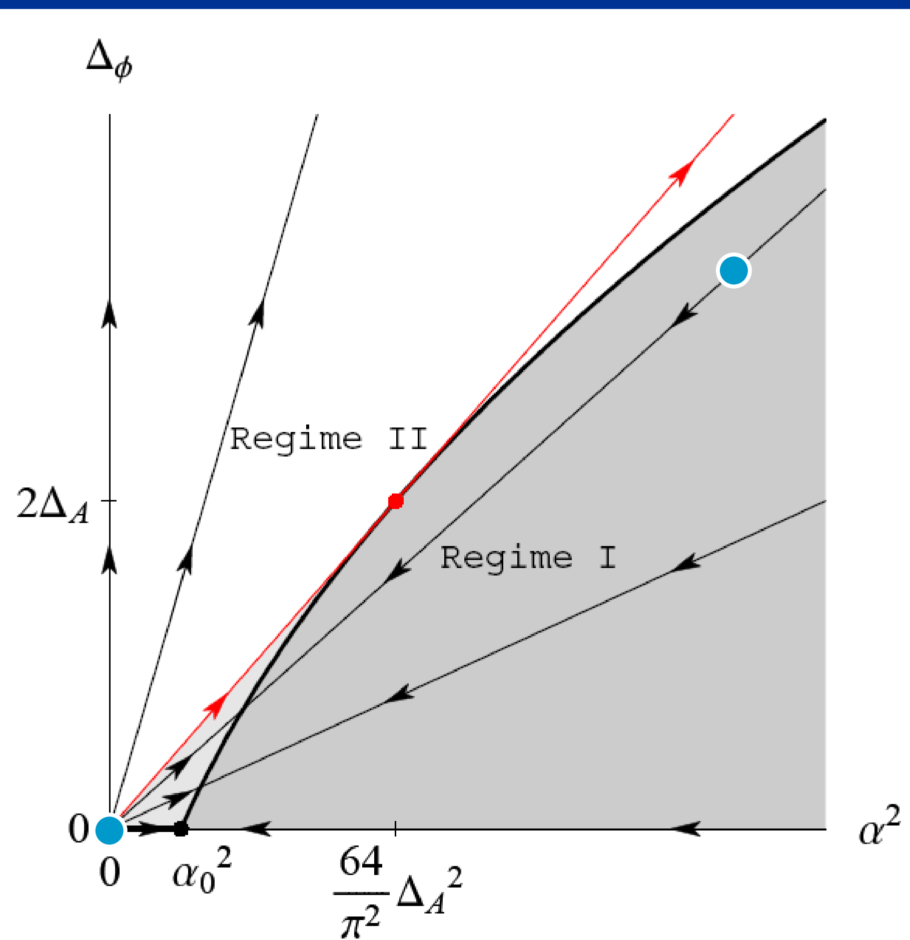
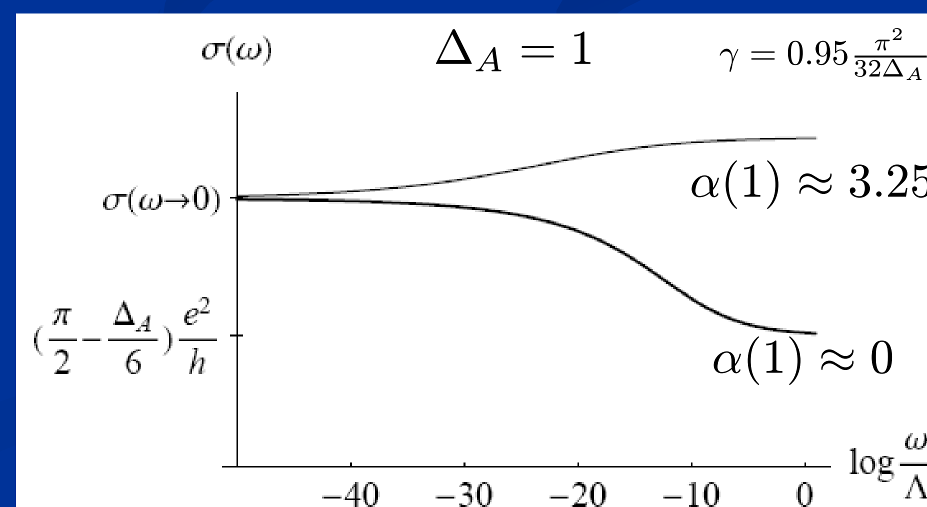
RG calculation of conductivity

$$\sigma\left(\omega,\Lambda,\Delta_\phi,\Delta_A,\alpha\right)=4\frac{e^2}{h}\left(\frac{\pi}{8}-\frac{1}{24}\left[\frac{\Delta_\phi}{\alpha^2}\right]\alpha^2(\omega/\Lambda)-\frac{\Delta_A}{24}+\alpha(\omega/\Lambda)\frac{\pi}{16}\left(\frac{25}{6}-\pi\right)\right).$$



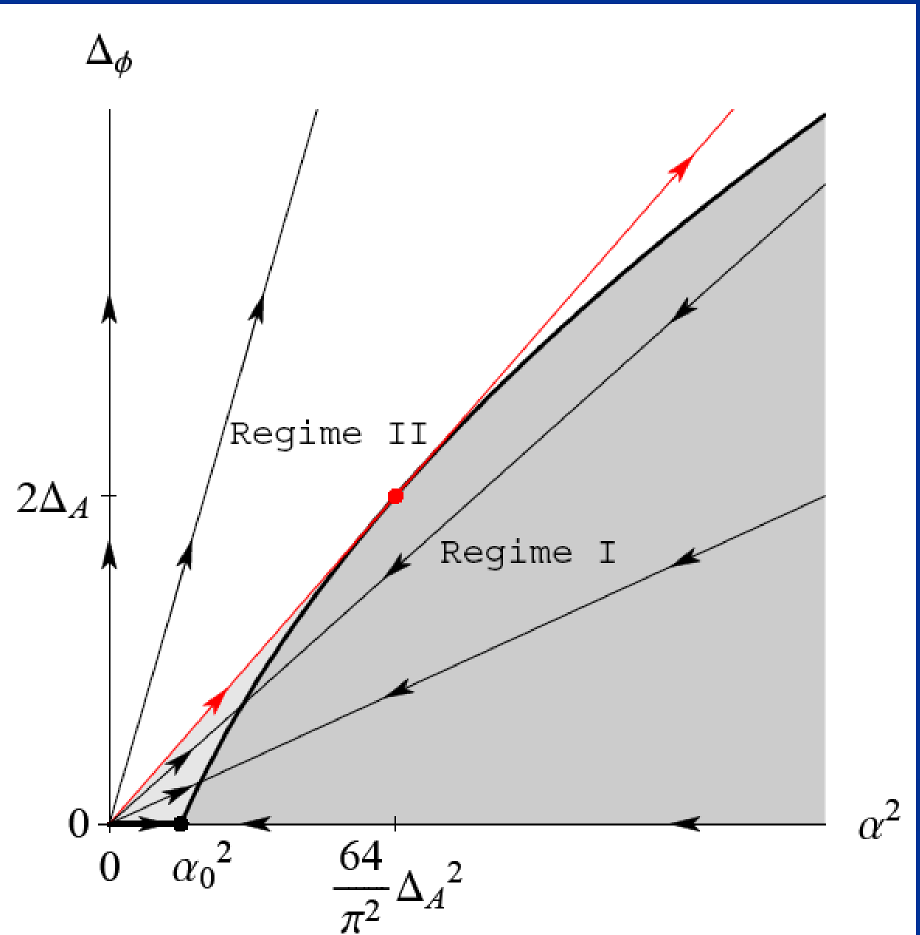
$$\frac{d\alpha}{d\ln\rho}=-\alpha\left(\frac{\gamma}{2\pi}\alpha^2-\frac{\alpha}{4}+\frac{\Delta_A}{\pi}\right).$$

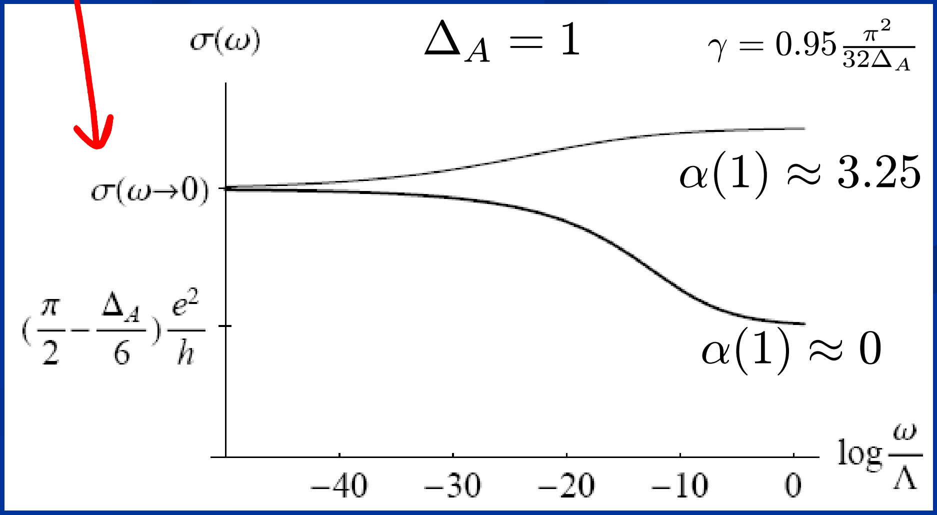
$$\alpha(1)=\alpha$$



RG calculation of conductivity

$$\sigma \to \frac{e^2}{h}\left(\frac{\pi}{2}+\frac{\Delta_A}{6}+\frac{(23-6\pi)\pi^2}{96\gamma}\left(1-\sqrt{1-\frac{32}{\pi^2}\gamma\Delta_A}\right)\right).$$

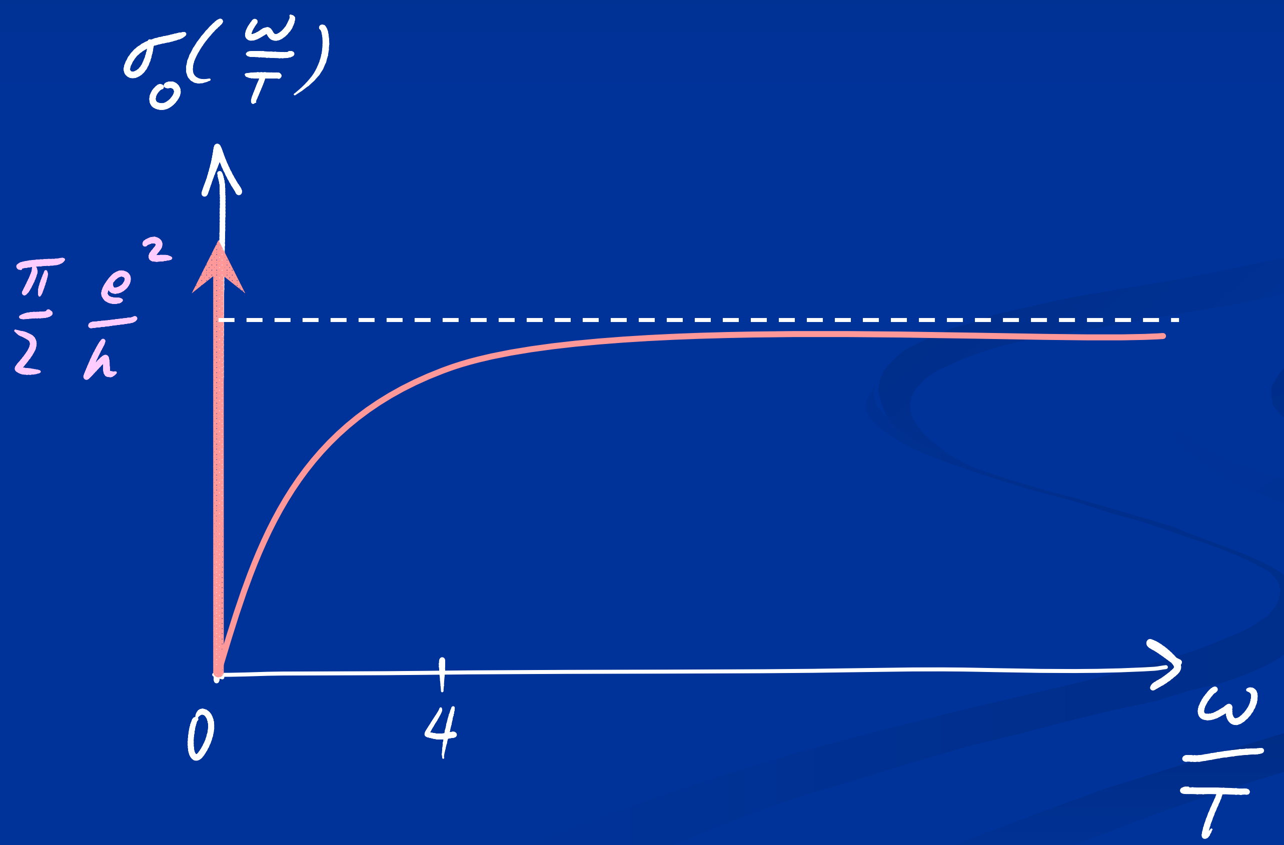




Conductivity scaling function

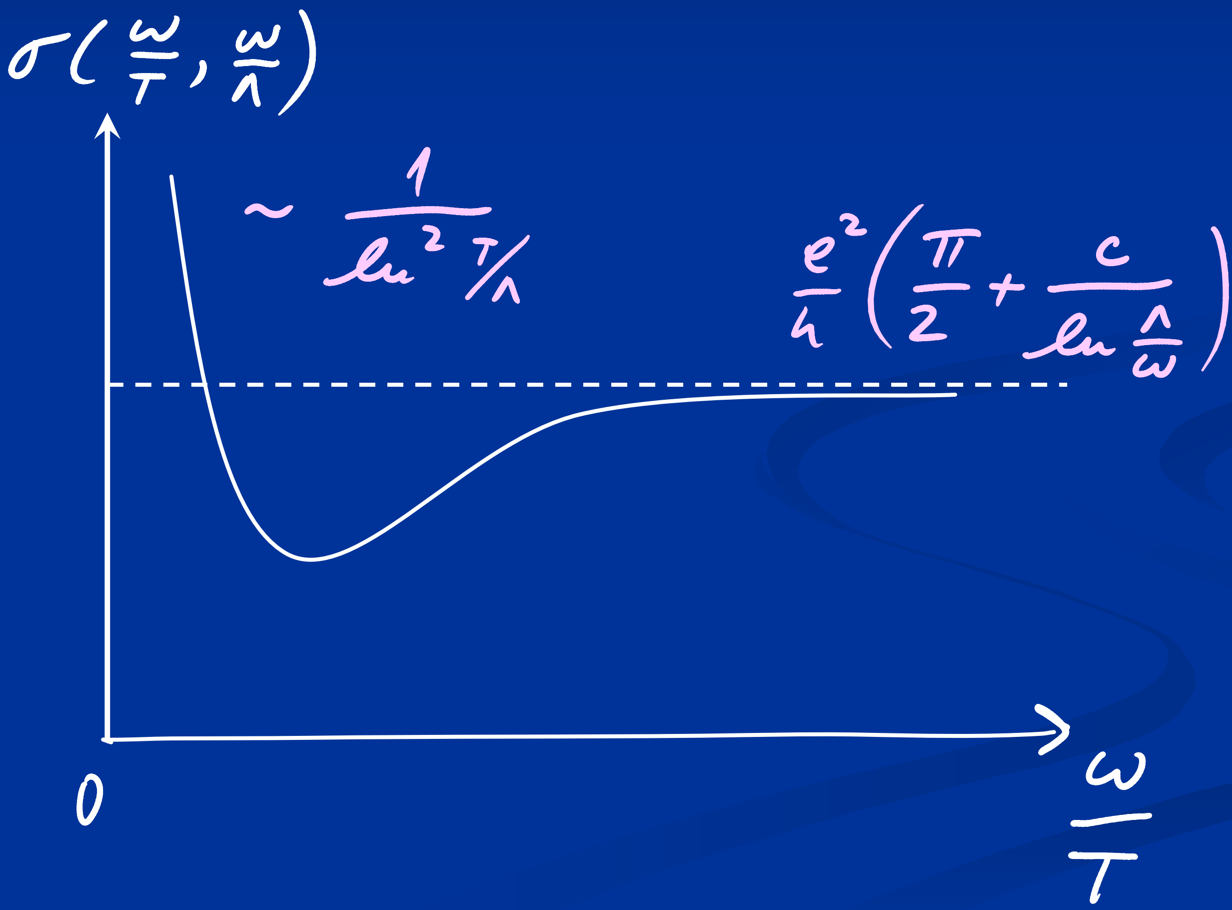
For non-interacting Dirac particles

$$\sigma_0(\omega, T) = \frac{e^2}{h}\left(\frac{\pi}{2}\tanh\left(\frac{\omega}{4T}\right) + 4\pi T \ln 2\delta(\omega)\right).$$



Conductivity scaling function

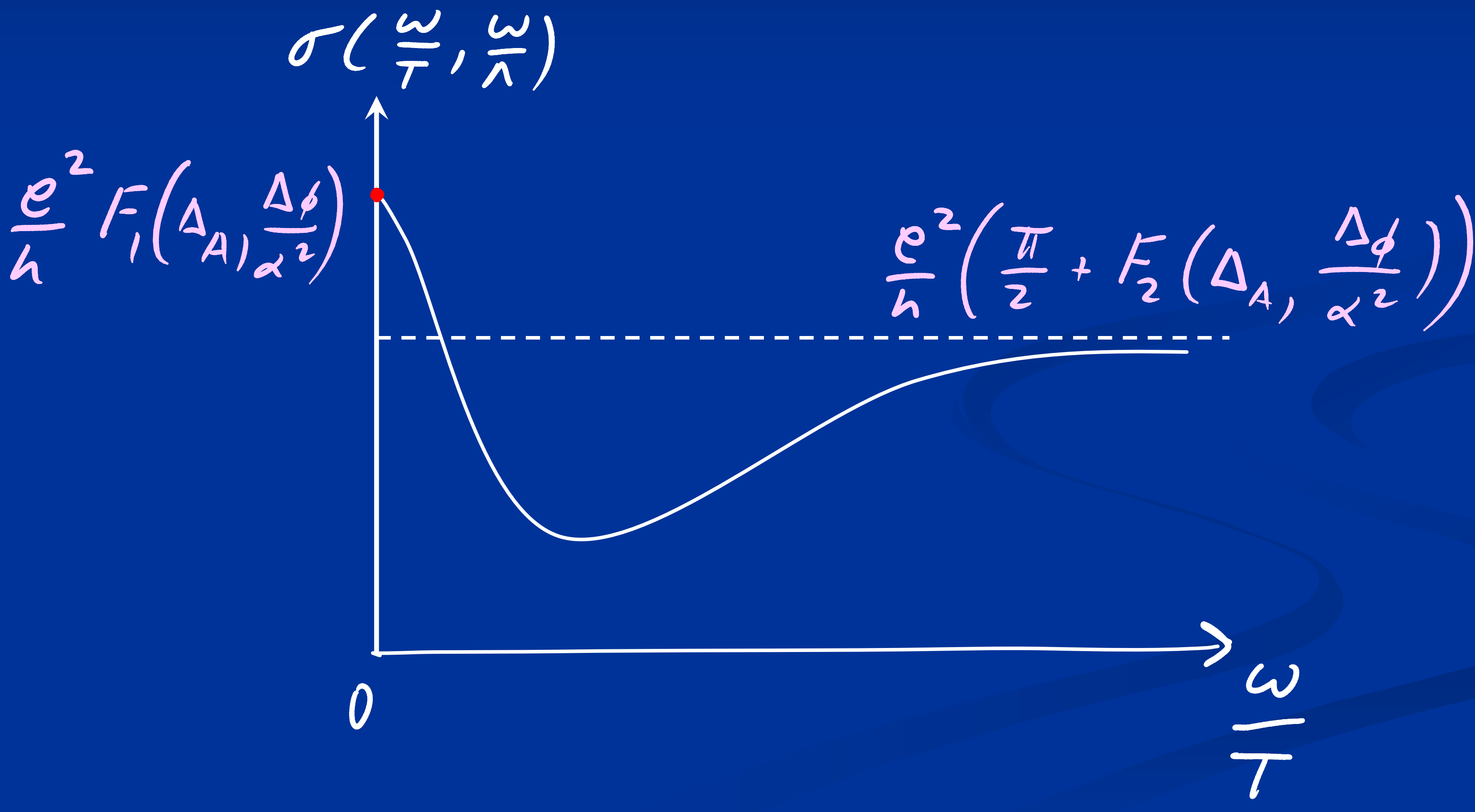
For (weakly) Coulomb-interacting clean Dirac particles



L. Fritz et. al. PRB 2008, A. Kashuba PRB 2008

Conductivity scaling function

For (weakly) Coulomb-interacting Dirac particles with disorder



Conclusions

- *The combined effect of Coulomb interactions and rippling disorder can lead to a (locally) infra-red stable line of fixed points with linear density of states.*
- *Minimal conductivity along such fixed line is non-universal, disorder and interaction dependent.*

OV arXiv.0810.3697