Quasiparticles of topological states

D.-H. Lee, U.C.Berkeley

Seidel, Fu, Leinaas, Moore, DHL, Phys. Rev. Lett. **95**, 266405 (2005).

Seidel and DHL, Phys. Rev. Lett. **97**, 056804 (2006).

DHL, Zhang and Xiang PRL, **99**, 196805 (2007).

Tewari, Das Sarma, DHL, PRL, 99, 037001 (2007).

Alex Seidel
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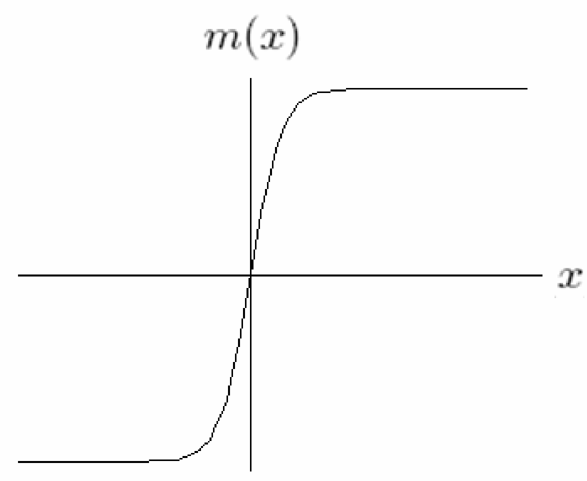
Outline

• The Jackiw-Rebbi and Su-Schrieffer-Heeger soliton.
• Abelian quantum Hall state on a thin torus and center-of - mass conserving Hamiltonian.
• Bosonic pfaffian state on a thin torus.
• A generalization of Jackiw-Rebbi's index theorem and the Majorana zero mode in the vortex of a p+ip superconductor.

Jackiw-Rebbi half-charged soliton

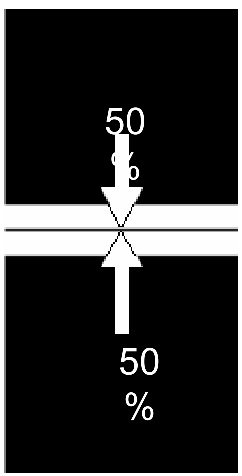
R. Jackiw and C. Rebbi, Phys. Rev. D 13, 3398 (1976)

Position-dependent mass



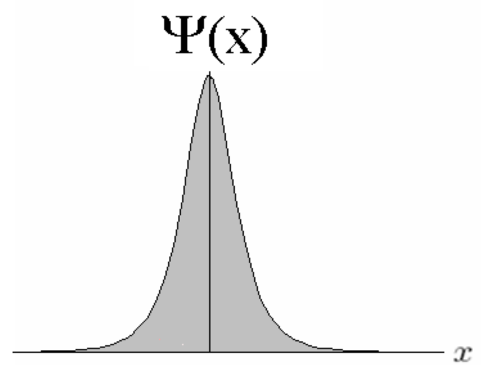
$$H = \int dx \Big[-i\psi^\dagger \sigma_z \partial_x \psi + m(x)\psi^\dagger \sigma_x \psi \Big]$$

Massive Dirac Eq. in 1D of spinless electrons



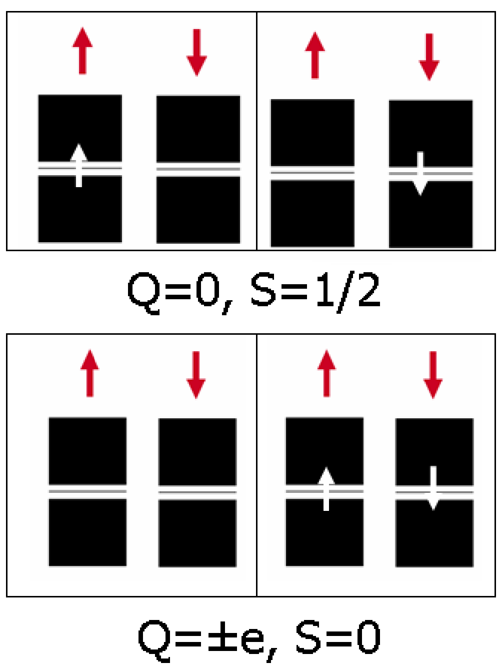
Occupy: Q= -e/2

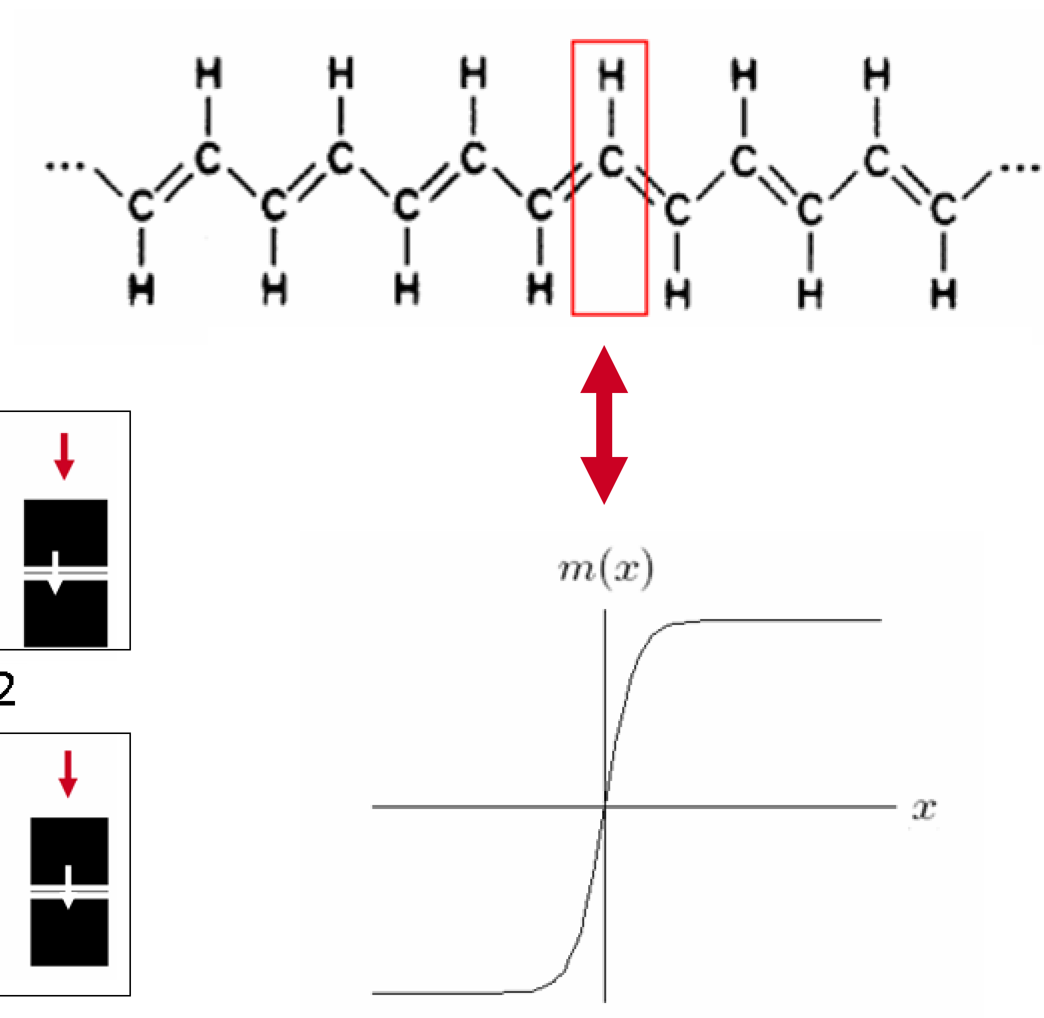
Empty: Q= e/2



The Su-Schrieffer-Hegger domain wall

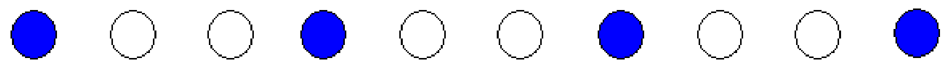
W. P. Su, J. R. Schrieffer, A. J. Heeger, PRB 22, 2099 (1980)

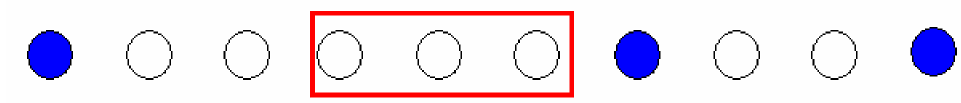




The Su-Schrieffer counting argument

Fractional charged domain wall in 1D charge density wave





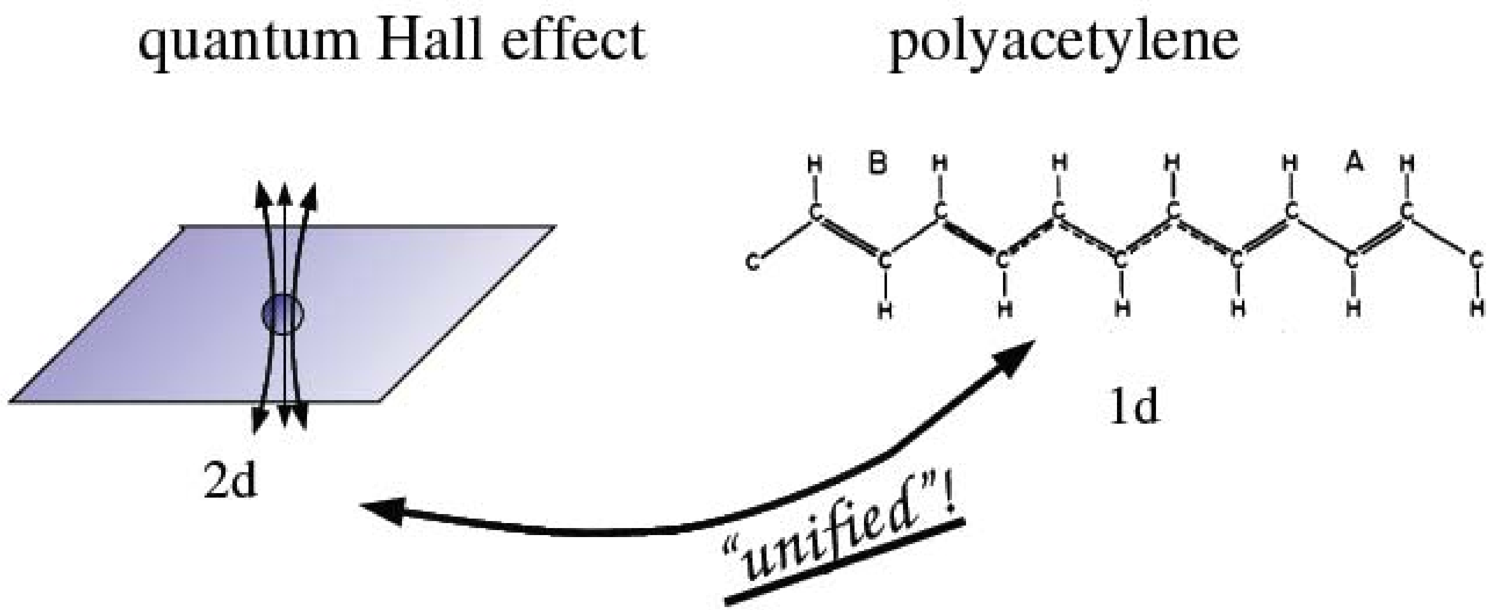
3 domain walls cause $\Delta Q = +e$ → one domain wall causes $\Delta Q = +e/3$.

Abelian quantum Hall state on a thin torus

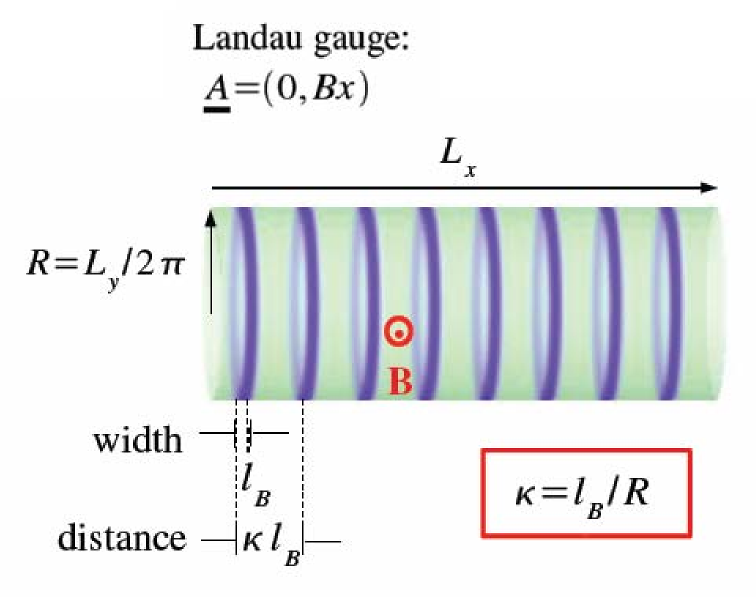
A. Seidel, H. Fu, D.-H. Lee, J. M. Leinaas, and J. E. Moore Phys. Rev. Lett. **95**, 266405 (2005).

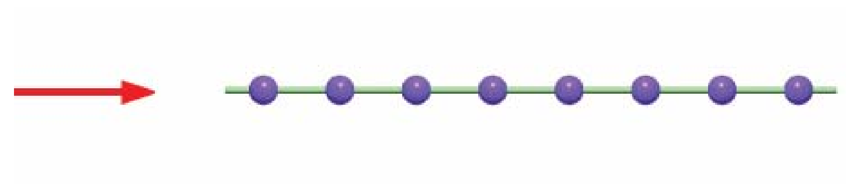
E.J. Bergholtz and A. Karlhede, J. Stat. Mech. (2006) L04001.

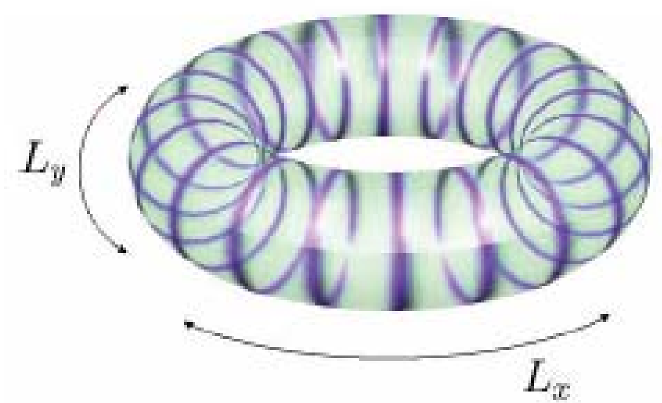
A. Seidel and K. Yang, Phys. Rev. Lett **101**, 036804 (2008).



The Lowest Landau level and 1D lattice



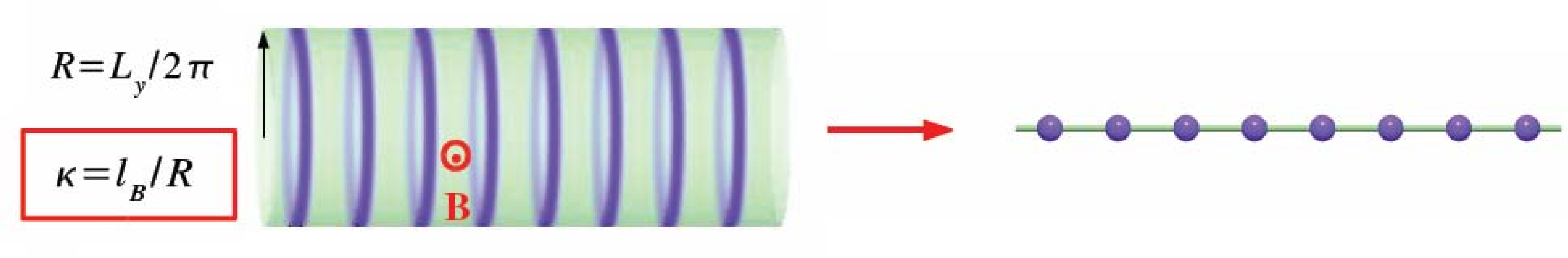




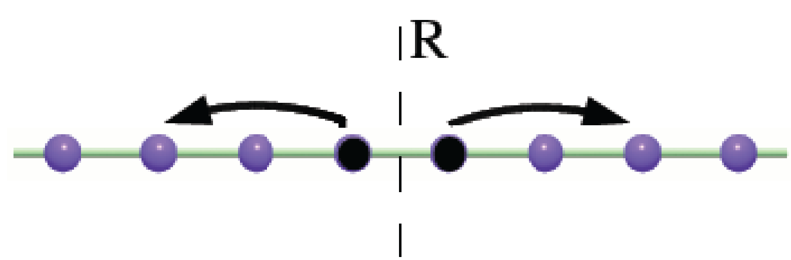
# of Dirac flux quanta → # of sites

$$H = \int d^2r d^2r' \nabla^2 \delta(\mathbf{r} - \mathbf{r}') \psi^+(\mathbf{r}) \psi^+(\mathbf{r}') \psi(\mathbf{r}') \psi(\mathbf{r})$$

S. A. Trugman and S. Kivelson, Phys. Rev. B **31**, 5280 (1985).



$$H = \kappa^3 \sum_{R,x,y} \left(x e^{-\kappa^2 x^2} \right) \left(y e^{-\kappa^2 y^2} \right) C^+_{R+x} C^+_{R-x} C_{R-y} C_{R+y}$$



<u>Simultaneously conserves C.M. momentum and position !</u>

Real space – momentum space duality

$$H = \sum_R Q_R^+ Q_R$$

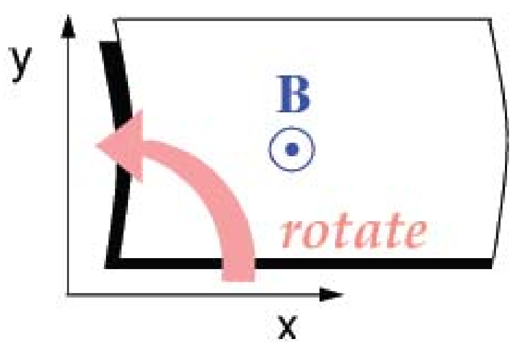
$$= \sum_R \Big[\sum_x f_\kappa(x)^* \, C_{R+x}^+ C_{R-x}^+ \Big] \Big[\sum_y f_\kappa(y) \, C_{R-y} C_{R+y} \Big]$$

Fourier transform

$$= \sum_{\boldsymbol{P}} \Big[\sum_k \tilde{f}_\kappa(2k)^* \, \tilde{C}_{\boldsymbol{P}+k}^+ \tilde{C}_{\boldsymbol{P}-k}^+ \Big] \Big[\sum_q \tilde{f}_\kappa(2q) \, \tilde{C}_{\boldsymbol{P}-q} \tilde{C}_{\boldsymbol{P}+q} \Big]$$

$$= \sum_{\boldsymbol{P}} \tilde{Q}_{\boldsymbol{P}}^+ \tilde{Q}_{\boldsymbol{P}}$$

Hamiltonian looks the ***same*** in real and momentum space,
up to a trivial shift of parameter:



$$\kappa \longrightarrow \frac{2\pi}{\kappa L}$$

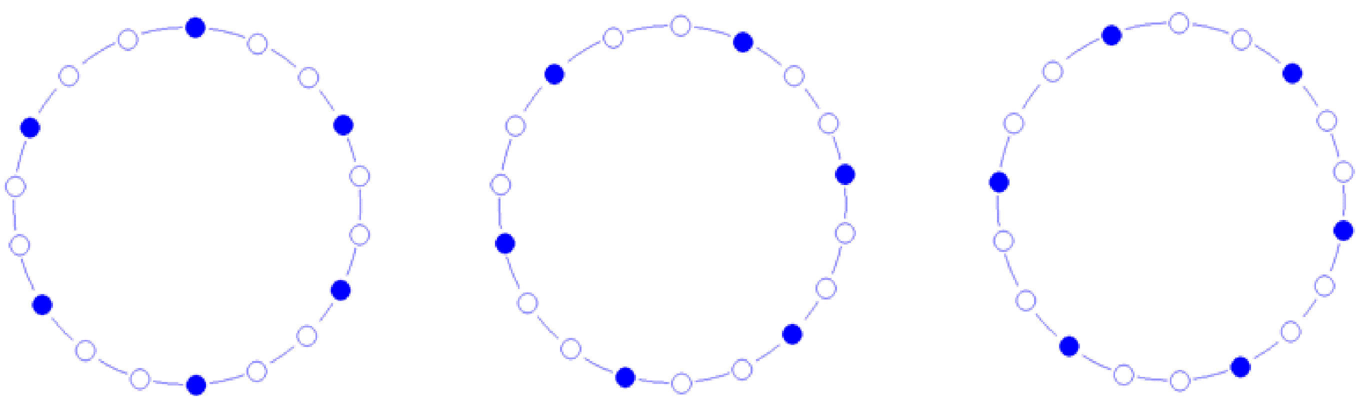
duality relation

$$\boldsymbol{f(x)}$$

$$\|$$

$$H = \kappa^3 \sum_{R,x,y} \left(x e^{-\kappa^2 x^2}\right)\left(y e^{-\kappa^2 y^2}\right) C^+_{R+x} C^+_{R-x} C_{R-y} C_{R+y}$$

$$\xrightarrow{\kappa >> 1} \quad H = \sum_{i=1}^{L} \left[f(1/2)^2 \; C^+_{i+1} C_{i+1} C^+_i C_i + f(1)^2 \; C^+_{i+2} C_{i+2} C^+_i C_i \right]$$



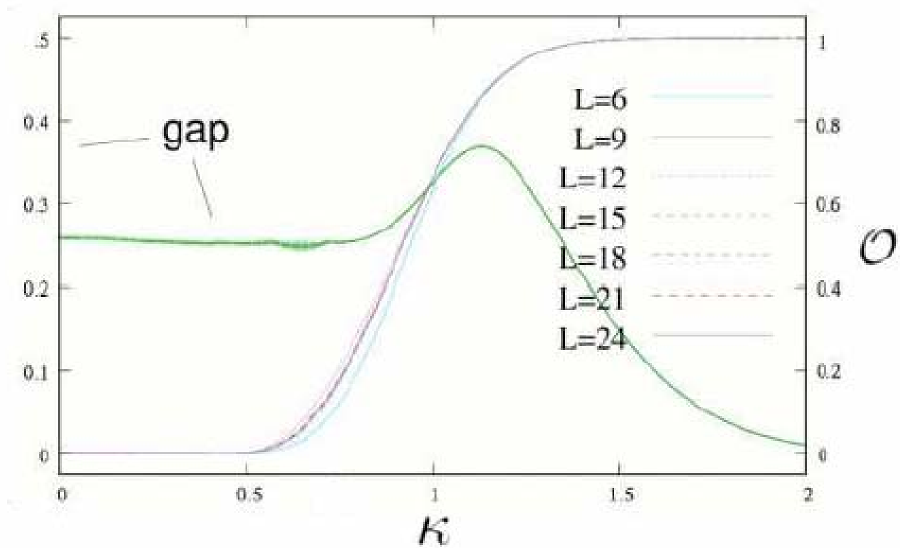
Three degenerate ground states correspond
to three different center-of-mass position

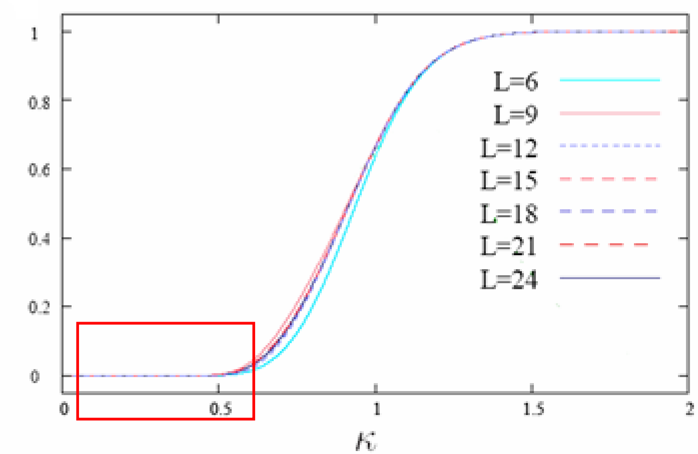
As κ decreases pair hopping range increases, but the center-of-mass position never changes → degeneracy is preserved.

What happens as we reduce κ ?

Will there be a phase transition?

$$\mathcal{O} = \frac{1}{N}\sum_{j=1}^{L} e^{i\frac{2\pi}{3}j} < C_j^{+} C_j >$$





Exp($-\alpha/\kappa^2$)

Seidel *et al*, PRL (2005)

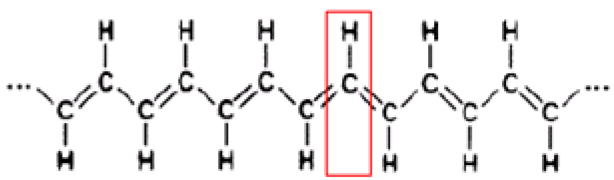
E. H. Rezayi and F. D. M. Haldane, Phys. Rev. B **50**, 17 199 (1994).

Small and large κ limit are adiabatically connected !!!

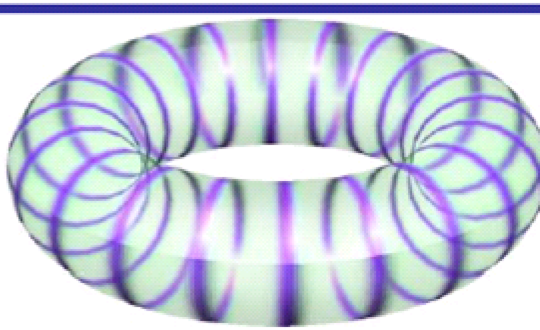
Topological degeneracy ⟷ symmetry breaking degeneracy

Niu & Wen (1990)

Laughlin quasiparticle ⟷ Domain wall



Fractional charged domain wall in polyacetylene



Fractional charged quasiparticles in quantum Hall liquid

Unified

Center of mass conserving models are good candidates for exhibiting topological ordered ground states

<u>Theorem (Oshikawa, Hastings):</u>

A system at fractional filling factor $\nu=p/q$ with an energy gap must at least *q*-fold ground state degeneracy!

M. Oshikawa, Phys. Rev. Lett. **84**, 1535 (2000).
M. B. Hastings, Phys. Rev. B **69**, 104431 (2004).

Mechanisms to achieve this degeneracy:

Symmetry breaking	center-of-mass conserving dynamics
usually the case, but "boring"	*guarantees* q-fold degeneracy, independent of symmetry breaking

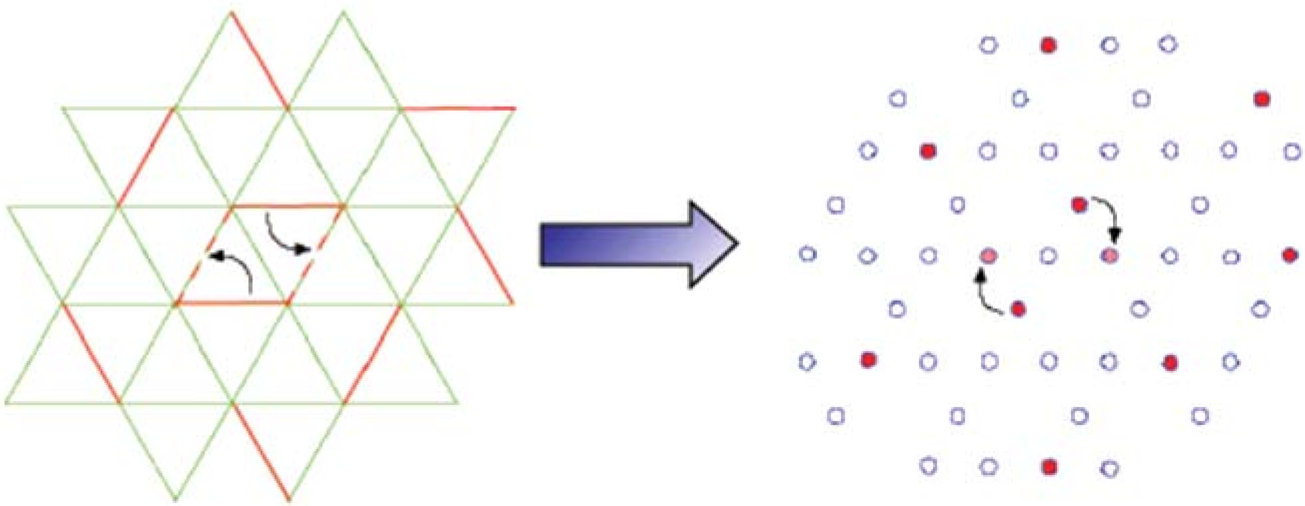
A. Seidel, H. Fu, D.-H. Lee, J. M. Leinaas, and J. E. Moore, Phys. Rev. Lett. **95**, 266405 (2005).

H. Tasaki, cond-mat/0407616.

1D gapped systems with local interaction has local order parameters

2D example

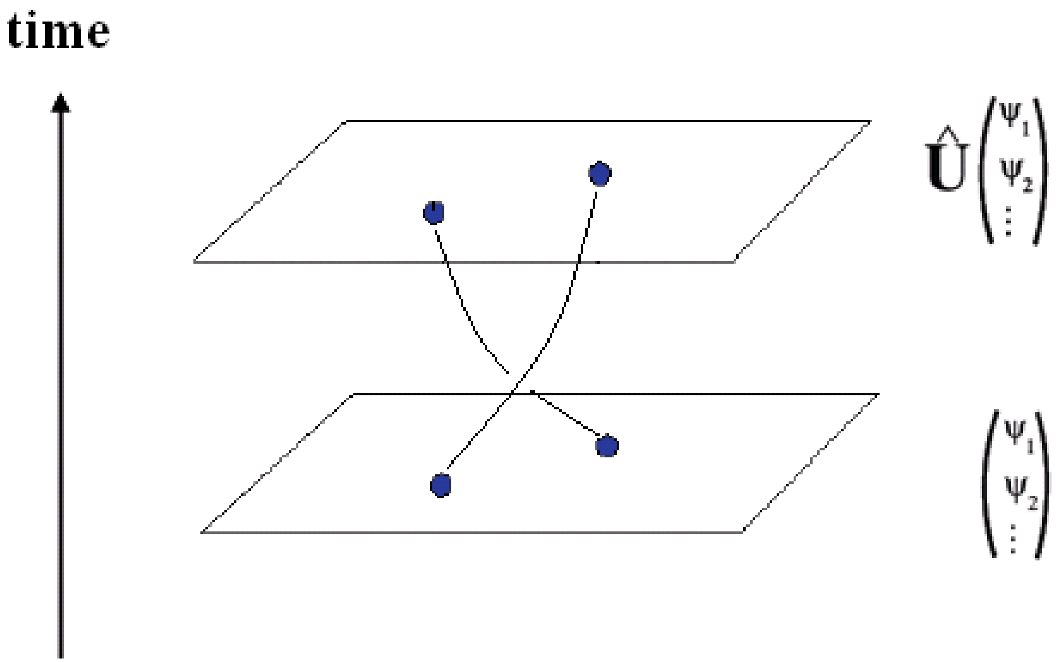
D. S. Rokhsar and S. A. Kivelson, Phys. Rev. Lett. **61**, 2376 (1988).
R. Moessner and S. L. Sondhi, Phys. Rev. Lett. **86**, 1881 (2001).



$$H = \sum_R Q_R^+ Q_R \qquad\qquad Q_R|\psi>=0 \ \ \forall R$$

Pfaffian state on a thin torus

Bergholtz, Kailasvuori, Wikberg ,and Hansson PRB 74, 081308 (2006).
Seidel and DHL, PRL **97**, 056804 (2006).



Pfaffian facts

ground state wavefunction (disc geometry):

$$\Psi = Pf\Big[\frac{1}{z_i - z_j}\Big] \prod_{(ij)} (z_i - z_j)^q \exp[-\sum_k |z_k|^2/4]$$

filling factor: $\nu=1/q$

Moore and Read, Nucl. Phys. B (1991).

Quasi particle charge: $Q=1/(2q)$

degeneracy of 2n quasi-hole state: 2^{n-1}

ground state degeneracy on torus: 3q

Now, specialize to $\nu=q=1$ (bosons):

pseudo potential Hamiltonian:

$$H = \sum_{(ijk)} \delta(z_i - z_j)\delta(z_i - z_k)$$

lattice version:

$$H = \sum_R Q_R^+ Q_R$$

$$Q_R = \sum_{m+n+p=3R \bmod N} f(R-m, R-n, R-p)\, c_m c_n c_p$$

The thin torus limit for ν=1 Pfaffian

Again, take $\kappa \to \infty$ limit :

$$H \approx \kappa^2 \sum_n [(c_n^\dagger)^3 (c_n)^3 + \exp(-\kappa^2/3)(c_n^\dagger)^2 c_{n\pm 1}^\dagger (c_n)^2 c_{n\pm 1}]$$

=> No more than two particles may occupy two adjacent sites!

$$H \xrightarrow{\kappa >> 1} \kappa^2 \sum_n [(c_n^\dagger)^3 (c_n)^3 + \exp(-\kappa^2/3)(c_n^\dagger)^2 c_{n\pm1}^\dagger (c_n)^2 c_{n\pm1}]$$

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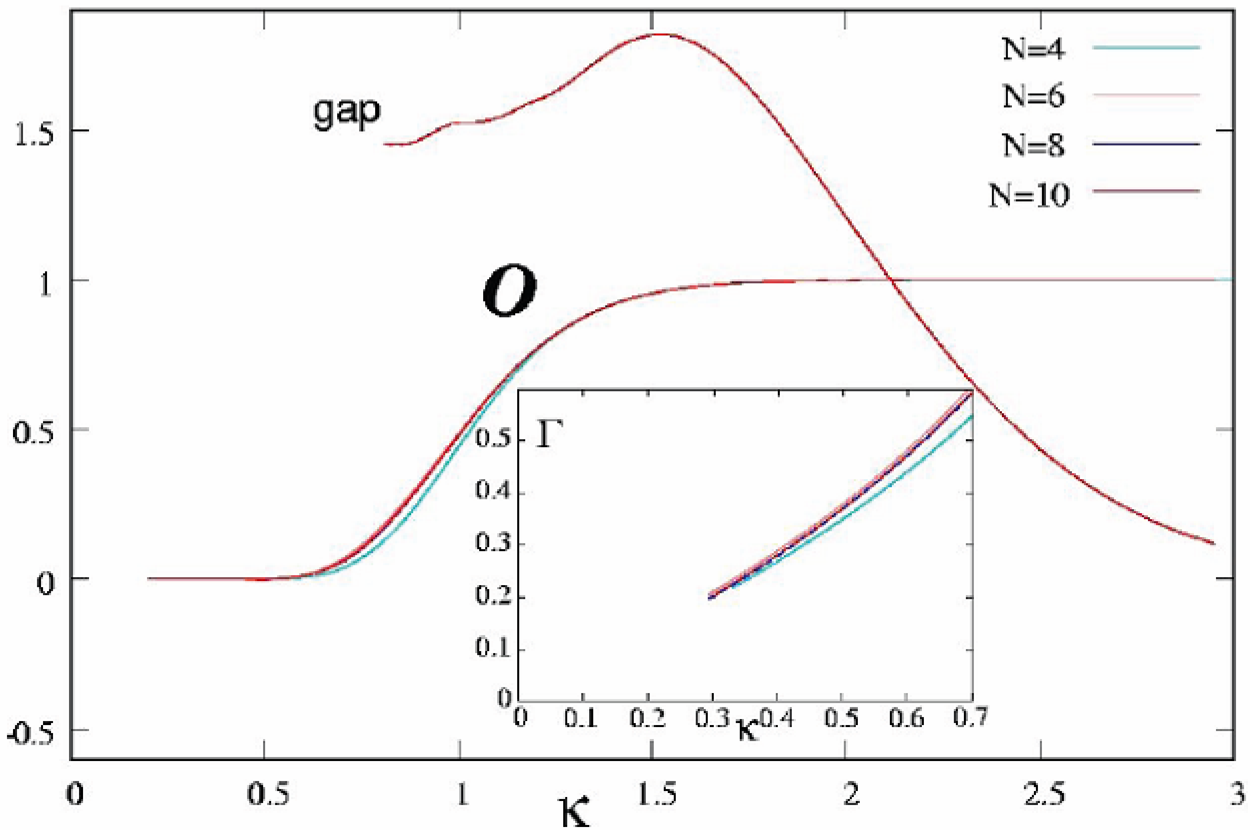


$$O = \exp\left(-1/\Gamma(\kappa)^2\right)$$

What do we gain from adiabatic continuity?

Simple, intuitive picture for:

- degeneracies on torus ✓
- fractional charge of quasi-particle/holes ✓

Convenient way to organize, label Hilbert space of quasi-particles/holes:

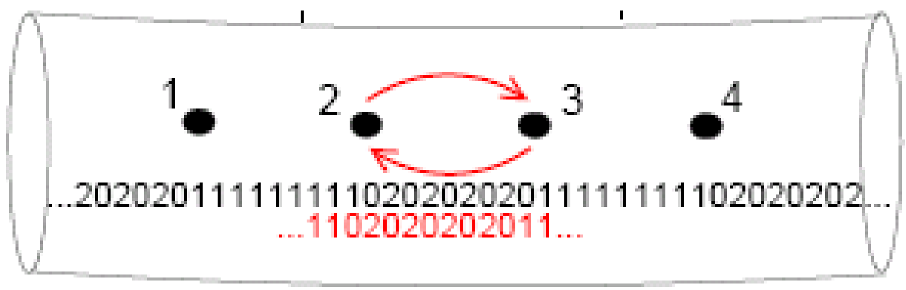
$$\mathcal{H}_{2\,\mathrm{hole}} = \{ \hat{S}_{\kappa} |020201111102020\rangle , ... \}$$

Counting the number of n-hole states

F. D. M. Haldane, Bull. Am. Phys. Soc. **51**, 633 (2006).
N. Read, Phys. Rev. B **73**, 245334 (2006).

Understanding abelian and non-abelian statistics from the thin torus point of view

A. Seidel and DHL, PRB, 76, 155101 (2007)
A. Seidel, PRL 101, 196802 (2008)



Relation to Jack polynomials

X.-G. Wen and Z. Wang, Phys. Rev. B **77**, 235108 (2008).

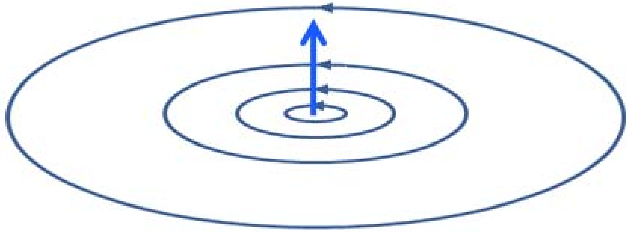
B. A. Bernevig and F. D. M. Haldane, arXiv:0707.3637 (unpublished).

B. A. Bernevig and F. D. M. Haldane, arXiv:0711.3062 (unpublished).

An index theorem for the Majorana zero mode

Tewari, Das Sarma and DHL, PRL, 99, 037001 (2007).

N. Read and D. Green, Phys. Rev. B **61**, 10267 (2000)



$$c_{\mathbf{k}} = \frac{1}{\sqrt{2\pi k}} \sum_{m=-\infty}^{\infty} c_{m,k} e^{im\theta_{\mathbf{k}}}$$

Perform angular momentum decomposition

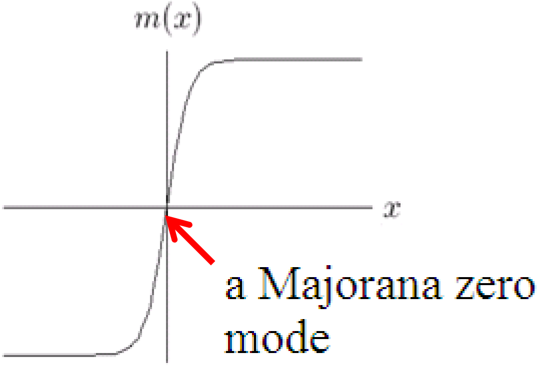
$$H_M = \int dx \Big[-iv_F \chi^\dagger \sigma_z \partial_x \chi + m(x) \chi^\dagger \sigma_x \chi \Big]$$

$$\chi^\dagger(x) = \big(c^\dagger(x),\ c(-x)\big)$$

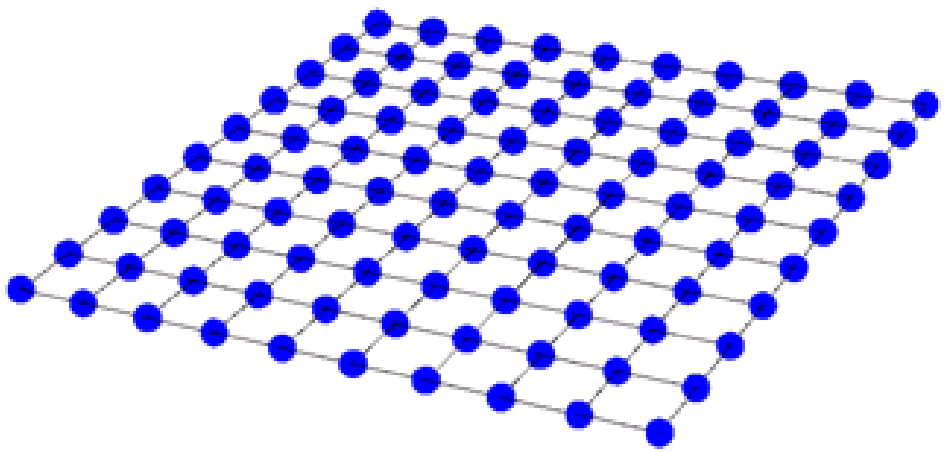
Jackiw-Rebbi

$$H_D = \int dx \Big[-iv_F \psi^\dagger \sigma_z \partial_x \psi + m(x) \psi^\dagger \sigma_x \psi \Big]$$

$$\psi^\dagger(x) = \big(f_1^\dagger(x),\ f_2^\dagger(x)\big)$$

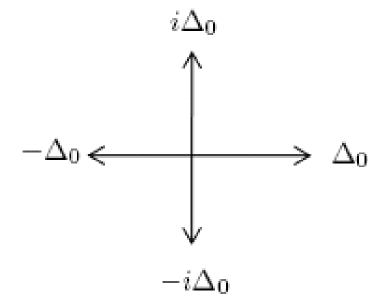


P+iP Bogoliubov-de Gennes Hamiltonian on square lattice

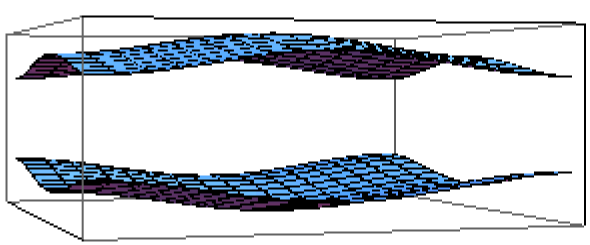


$$H = \sum_{i,j} (c_i^\dagger, c_i) \cdot U_{ij} \cdot \begin{pmatrix} c_j \\ c_j^\dagger \end{pmatrix}$$

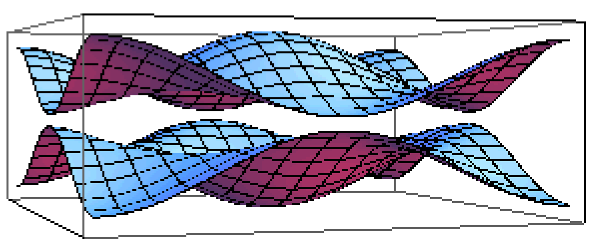
$$U_{ij} = \begin{pmatrix} -t-\mu & \Delta_{ij} \\ -\Delta_{ij}^* & t+\mu \end{pmatrix}$$



Molecule limit



BCS limit



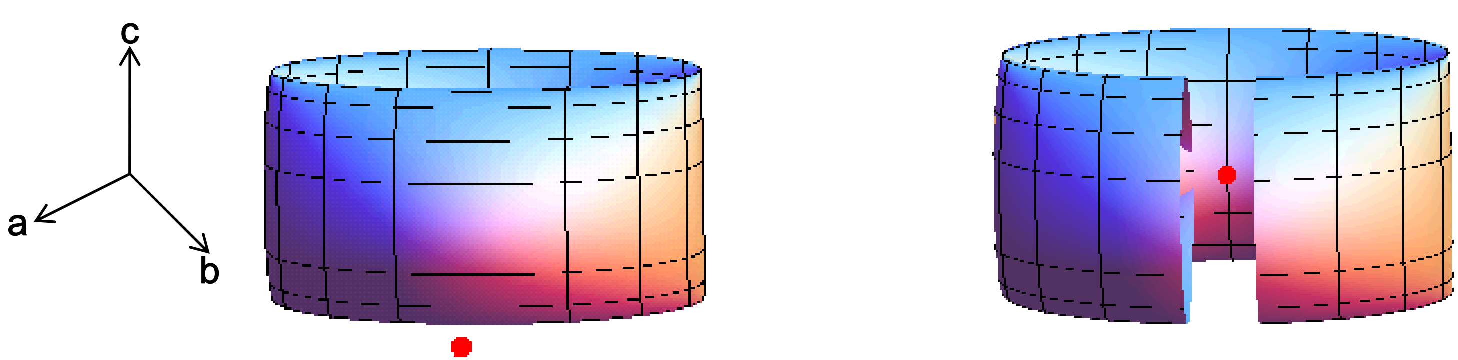
2x2 matrix

$$H = \sum_{ij} \Psi_i^\dagger h_{ij} \Psi_j \qquad \Psi_i = \begin{pmatrix} c_i \\ c_i^\dagger \end{pmatrix}$$

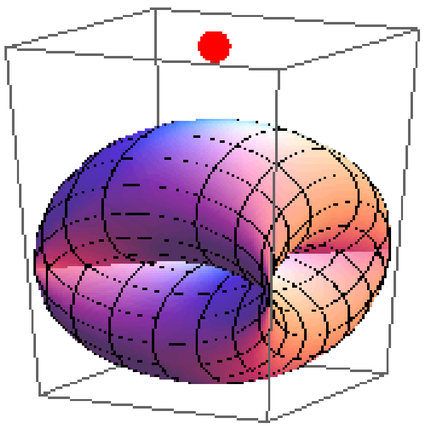
$$h(\vec{k}) = a(\vec{k})\sigma_x + b(\vec{k})\sigma_y + c(\vec{k})\sigma_z$$

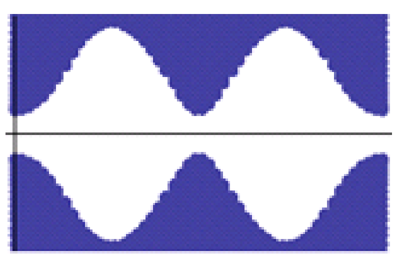
$$(k_x, k_y) \to (a(\vec{k}), b(\vec{k}), c(\vec{k}))$$

$$E(\vec{k}) = \pm\sqrt{a(\vec{k})^2 + b(\vec{k})^2 + c(\vec{k})^2}$$

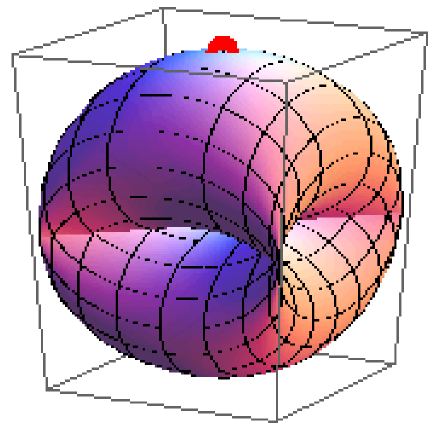


Molecular





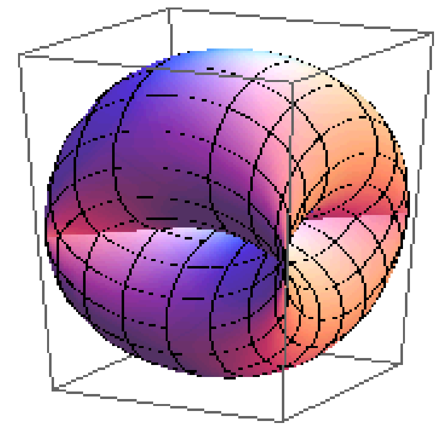
Gap closing transition

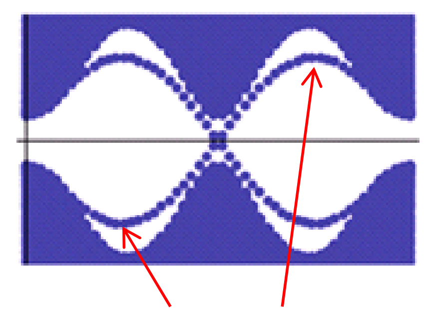


$$H_M = \int dx - i v_F \chi^\dagger \sigma_z \partial_x \chi$$

$$\chi^\dagger(x) = \left(c^\dagger(x),\ c(-x)\right)$$

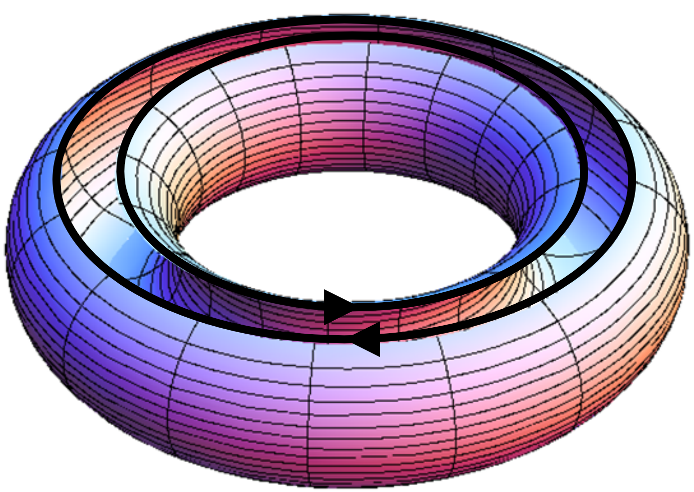
BCS

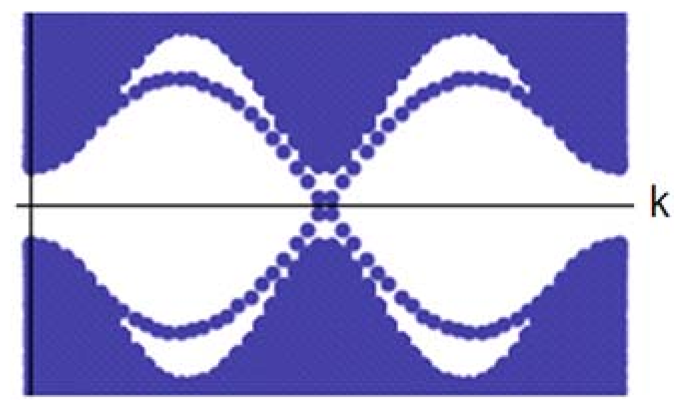




Majorana zero mode as edge soliton

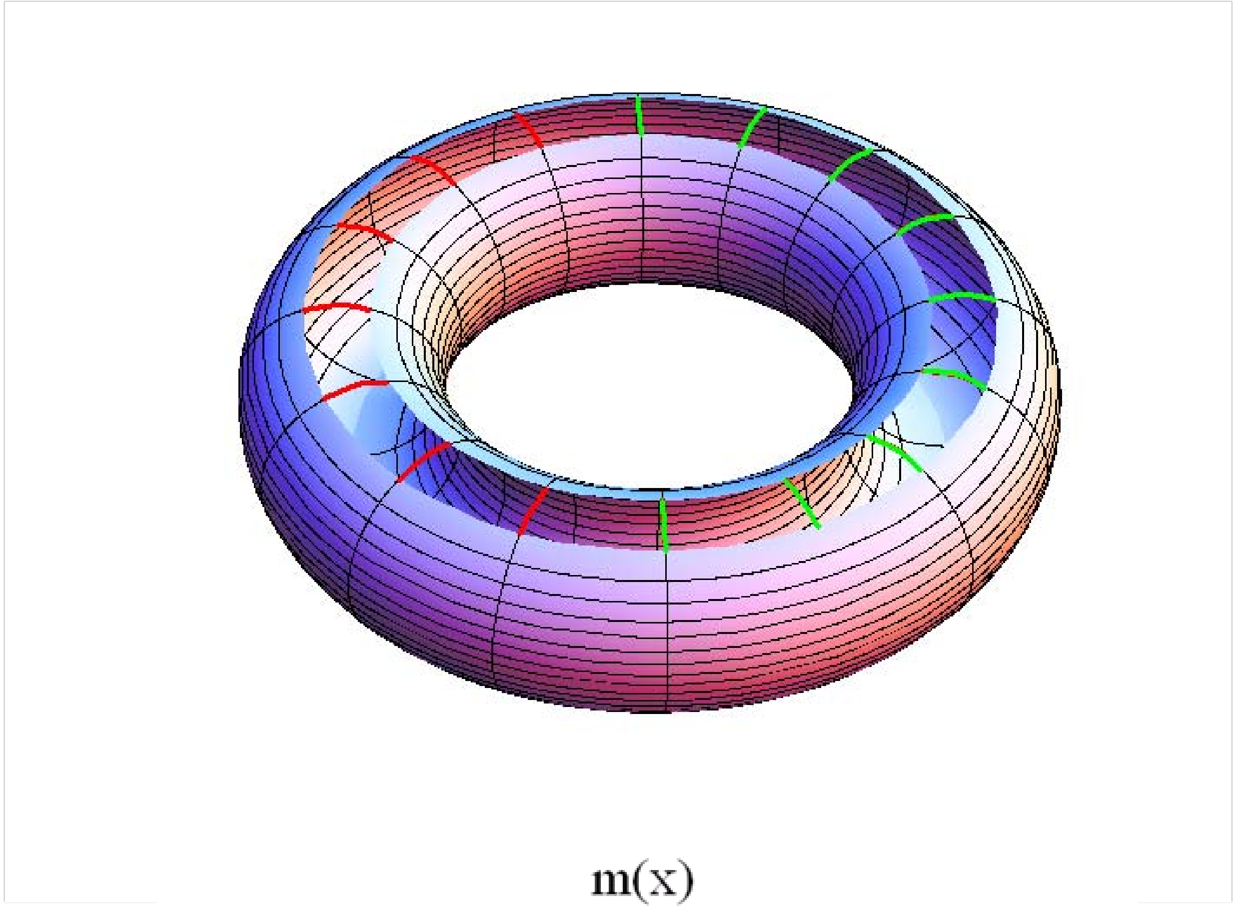
DHL, G-M Zhang, T Xiang PRL 99, 196805 (2007)

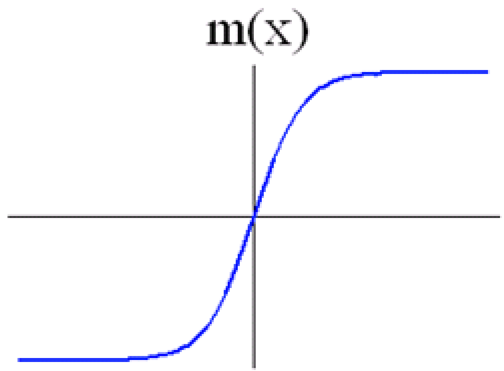




$$H_M = \int dx - i v_F \chi^\dagger \sigma_z \partial_x \chi$$

$$\chi^\dagger(x) = \left(c^\dagger(x),\ c(-x)\right)$$





Majorana
fermion
mode

$$H_M = \int dx \Big[-iv_F \chi^\dagger \sigma_z \partial_x \chi + m(x) \chi^\dagger \sigma_x \chi \Big]$$

Conclusion

- The Jackiw-Rebbi-Su-Schrieffer-Heeger soliton.
- Viewing the abelian and non-abelian quantum Hall states on a thin torus. Quasiparticle and the JRSSH soliton.
- Center-of-mass conserving Hamiltonian and topological order.
- A generalization of Jackiw Rebbi index theorem
- Majorana fermion zero mode as edge soliton