

order and excitations in large- S kagomé-lattice antiferromagnets



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Life and Death in Kagomé Flatland



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PHYSICAL REVIEW LETTERS

week ending
5 DECEMBER 2014

Quantum Selection of Order in an XXZ Antiferromagnet on a Kagome Lattice

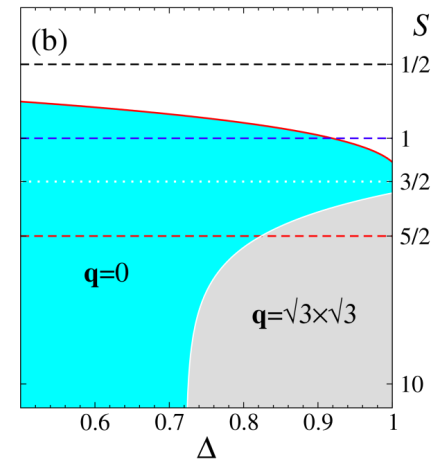
A. L. Chernyshev¹ and M. E. Zhitomirsky²

- ☑ A. L. Chernyshev, arXiv:1507.04738 (→PRB)
- ☑ A. L. Chernyshev and M. E. Zhitomirsky, arXiv:1508.06632

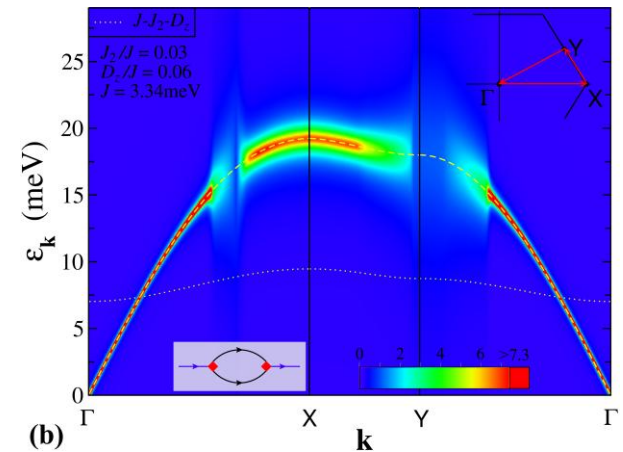


plan

I. quantum selection of the ground state



II. spectra of some real large- S kagomé AFs



history

306

- Ising case: residual entropy at $T=0$, as in the triangular lattice (Wannier, 1950)

Progress of Theoretical Physics, Vol. VI, No. 3, May~June, 1951.

Statistics of Kagomé Lattice

Itiro Syôzi

Department of Physics, Osaka University

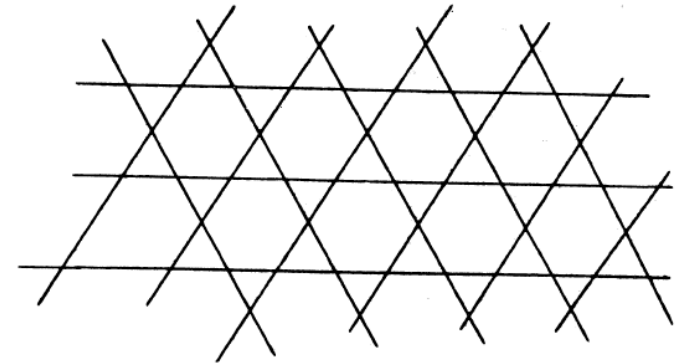


Fig. 1. Kagomé Lattice

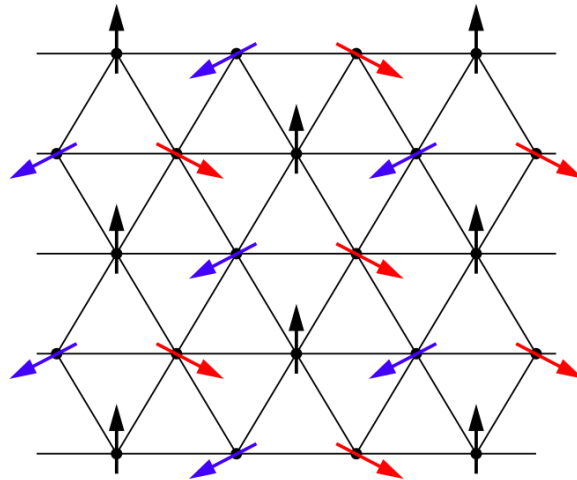
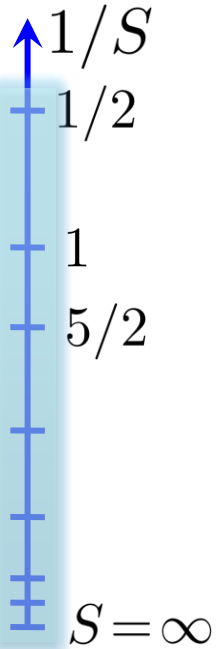
After the work of Onsager,¹⁾ who solved exactly the problem of Ising model for the case of plane square lattice, the same problems for the honeycomb and triangular lattice were treated by several authors.²⁾ Other than these three types of lattices, there is left a lattice, called in Japanese kagomé (woven bamboo pattern), which consists exclusively of equivalent lattice points and equivalent bonds. Since the number of nearest neighbors of a lattice point is as many as



triangular vs kagomé, Heisenberg case

- vector spins: triangular lattice \Rightarrow a unique 120° order for all S

$$\mathcal{H} \Rightarrow \frac{J}{4} \sum_{\Delta} \mathbf{S}_{\Delta}^2 - \frac{3J}{2} \sum_i \mathbf{S}_i^2$$

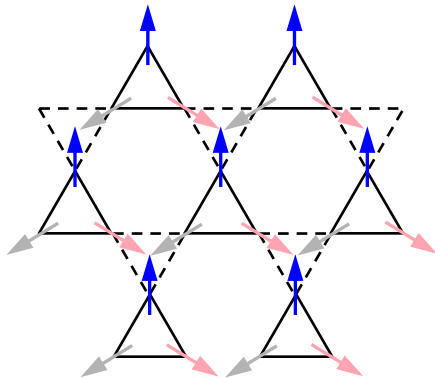


triangular vs kagomé, Heisenberg case

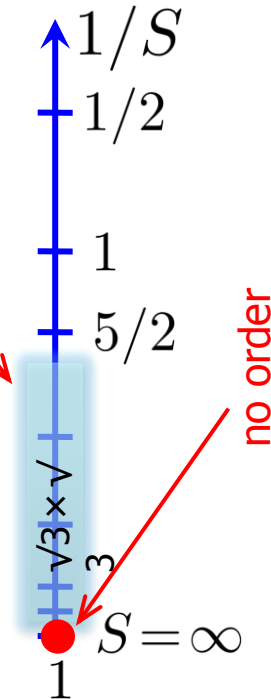
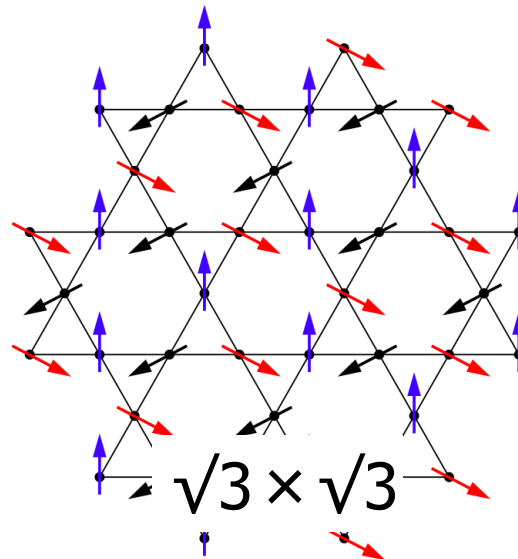
$$\mathcal{H} \Rightarrow \frac{J}{2} \sum_{\Delta} \mathbf{S}_{\Delta}^2 - J \sum_i \mathbf{S}_i^2$$

- kagomé lattice \Rightarrow classical energy is minimized by **any** 120° state with $\mathbf{S}_{\Delta} = 0$
 \Rightarrow massive degeneracy within the 120° coplanar manifold
- fluctuations \Rightarrow order-by-disorder, "island of stability"*

two main contenders
for the ordered state



$\mathbf{q}=0$



no order



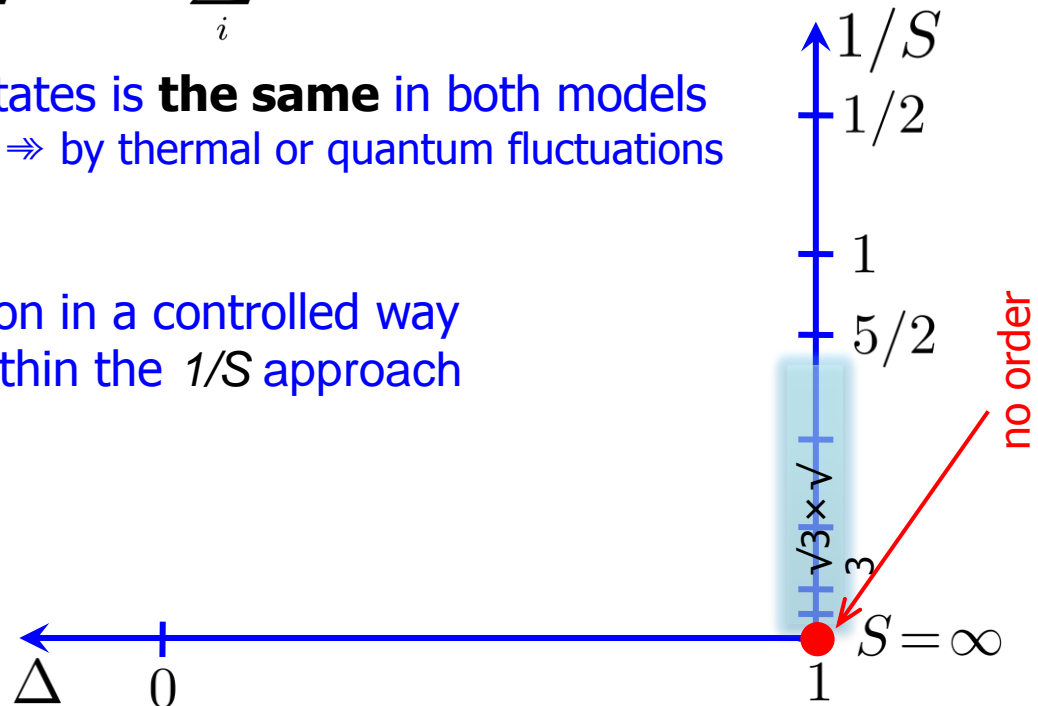
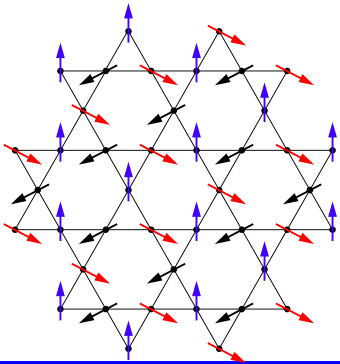
progress by Heisenberg \Rightarrow XXZ

$$\mathcal{H} = J \sum_{\langle ij \rangle} \left(S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z \right)$$

XXZ \Rightarrow analysis of what quantum fluctuations do, why/how

$$\mathcal{H} \Rightarrow \frac{J}{2} \sum_{\Delta} \mathbf{S}_{\Delta}^2 - J \sum_i \mathbf{S}_i^2$$

- degeneracy between **coplanar** states is **the same** in both models
 - *Heisenberg, coplanar structure \Rightarrow by thermal or quantum fluctuations
 - *XXZ \Rightarrow anisotropy
- XXZ \Rightarrow allows to study GS selection in a controlled way (perturbatively) within the $1/S$ approach

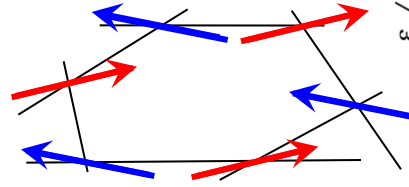


flat mode

- flat mode = local out-of-plane excitations

- Heisenberg limit*:

- flat mode is at $\omega=0$
- $1/S$ -expansion fails ($1/0 = \infty$)

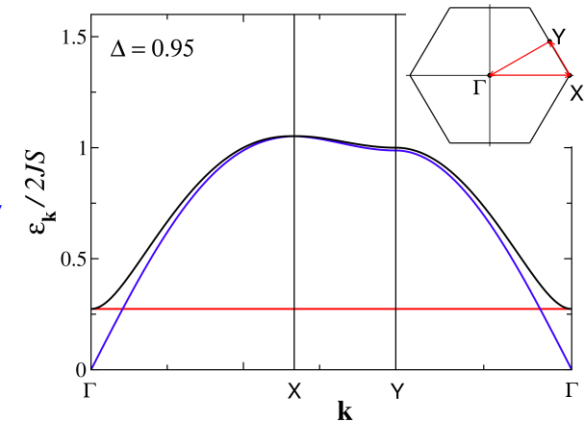
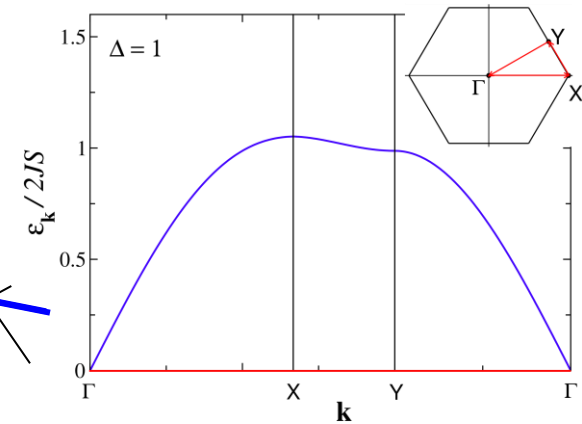
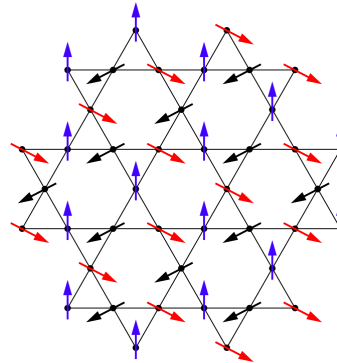


- XXZ case:

- flat mode is gapped, $\omega \propto \sqrt{1 - \Delta}$
- $1/S$ -expansion can be used

- $XXZ \Rightarrow$ degeneracy is not lifted on the harmonic level, providing a more subtle case of order-by-disorder

$$\mathcal{H} = E_{c1} + \mathcal{H}_2 + \dots$$



non-linear terms ...

... are responsible for the GS selection

- in particular, the "cubic" terms

$$\hat{\mathcal{H}} = J \sum_{\langle ij \rangle} \left(\Delta S_i^y S_j^y + \cos \theta_{ij} (S_i^x S_j^x + S_i^z S_j^z) + \sin \theta_{ij} (S_i^z S_j^x - S_i^x S_j^z) \right)$$

[H is in the local reference frame basis of a coplanar structure]

$$S^z = S - a^\dagger a,$$

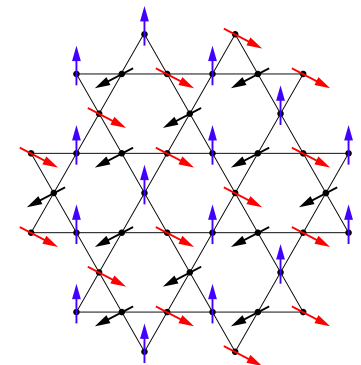
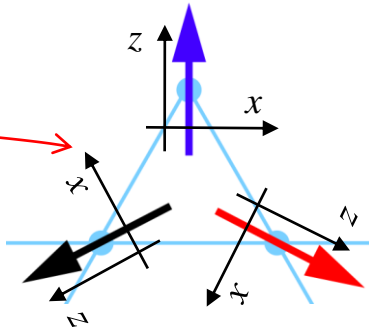
$$S^x \approx \sqrt{S/2} (a^\dagger + a)$$

because for all 120° states all $\cos \theta_{ij} = -1/2$, but $\sin \theta_{ij} = \pm \sqrt{3}/2$

\Rightarrow in the $1/S$ -expansion of the GS $E = E_{\text{cl}} + \langle \mathcal{H}_2 \rangle + \langle \mathcal{H}_4 \rangle + \dots$

only the last term can be responsible for lifting GS degeneracy

$\mathcal{O}(S^0)$



non-linear SWT

Harris et al (92)* (“two step” LSWT diagonalization for kagomé)

- we generalized it to XXZ and for non-linear (cubic) terms [elegant form]



$$\mathcal{H}_3 = \frac{1}{3!} \frac{1}{\sqrt{N}} \sum_{\mathbf{k}, \mathbf{q}} \sum_{\nu\mu\eta} V_{\mathbf{q}\mathbf{k}\mathbf{p}}^{\nu\mu\eta} b_{\nu, \mathbf{q}}^\dagger b_{\mu, \mathbf{k}}^\dagger b_{\eta, -\mathbf{p}}^\dagger + \text{h.c.}, \quad (51)$$

$$+ \frac{1}{2!} \frac{1}{\sqrt{N}} \sum_{\mathbf{k}, \mathbf{q}} \sum_{\nu\mu\eta} \Phi_{\mathbf{q}\mathbf{k}; \mathbf{p}}^{\nu\mu\eta} b_{\nu, \mathbf{q}}^\dagger b_{\mu, \mathbf{k}}^\dagger b_{\eta, \mathbf{p}} + \text{h.c.}, \quad (52)$$

with the vertices for the “source” and the “decay” terms

$$V_{\mathbf{q}\mathbf{k}\mathbf{p}}^{\nu\mu\eta} = -J \sqrt{\frac{3S}{2}} \tilde{V}_{\mathbf{q}\mathbf{k}\mathbf{p}}^{\nu\mu\eta}, \quad \Phi_{\mathbf{q}\mathbf{k}; \mathbf{p}}^{\nu\mu\eta} = -J \sqrt{\frac{3S}{2}} \tilde{\Phi}_{\mathbf{q}\mathbf{k}; \mathbf{p}}^{\nu\mu\eta}, \quad (53)$$

where the symmetrized dimensionless vertices are

$$\begin{aligned} \tilde{V}_{\mathbf{q}\mathbf{k}\mathbf{p}}^{\nu\mu\eta} = & F_{\mathbf{q}\mathbf{k}\mathbf{p}}^{\nu\mu\eta} (u_{\nu\mathbf{q}} + v_{\nu\mathbf{q}})(u_{\mu\mathbf{k}} v_{\eta\mathbf{p}} + v_{\mu\mathbf{k}} u_{\eta\mathbf{p}}) \\ & + F_{\mathbf{k}\mathbf{p}\mathbf{q}}^{\mu\eta\nu} (u_{\mu\mathbf{k}} + v_{\mu\mathbf{k}})(u_{\nu\mathbf{p}} v_{\eta\mathbf{q}} + v_{\nu\mathbf{p}} u_{\eta\mathbf{q}}) \\ & + F_{\mathbf{p}\mathbf{q}\mathbf{k}}^{\eta\nu\mu} (u_{\eta\mathbf{p}} + v_{\eta\mathbf{p}})(u_{\nu\mathbf{q}} v_{\mu\mathbf{k}} + v_{\nu\mathbf{q}} u_{\mu\mathbf{k}}), \end{aligned} \quad (54)$$

and

$$\begin{aligned} \tilde{\Phi}_{\mathbf{q}\mathbf{k}; \mathbf{p}}^{\nu\mu\eta} = & F_{\mathbf{q}\mathbf{k}\mathbf{p}}^{\nu\mu\eta} (u_{\nu\mathbf{q}} + v_{\nu\mathbf{q}})(u_{\mu\mathbf{k}} u_{\eta\mathbf{p}} + v_{\mu\mathbf{k}} v_{\eta\mathbf{p}}) \\ & + F_{\mathbf{k}\mathbf{p}\mathbf{q}}^{\mu\eta\nu} (u_{\mu\mathbf{k}} + v_{\mu\mathbf{k}})(u_{\nu\mathbf{p}} u_{\eta\mathbf{q}} + v_{\nu\mathbf{p}} v_{\eta\mathbf{q}}) \\ & + F_{\mathbf{p}\mathbf{q}\mathbf{k}}^{\eta\nu\mu} (u_{\eta\mathbf{p}} + v_{\eta\mathbf{p}})(u_{\nu\mathbf{q}} v_{\mu\mathbf{k}} + v_{\nu\mathbf{q}} u_{\mu\mathbf{k}}), \end{aligned} \quad (55)$$

These eigenvectors define a unitary transformation of the original Holstein-Primakoff bosons to the new ones

$$a_{\alpha, \mathbf{k}} = \sum_{\nu} w_{\nu, \alpha}(\mathbf{k}) d_{\nu, \mathbf{k}}, \quad d_{\nu, \mathbf{k}} = \sum_{\alpha} w_{\nu, \alpha}(\mathbf{k}) a_{\alpha, \mathbf{k}}, \quad (42)$$

such that the harmonic Hamiltonian $\hat{\mathcal{H}}_2$ in the form of (9) with $\hat{\mathbf{A}}_{\mathbf{k}}$'s and $\hat{\mathbf{B}}_{\mathbf{k}}$'s from (14) with (21), (24), or (29) is turned into three independent Hamiltonians

$$\hat{\mathcal{H}}_2 = 2JS \sum_{\nu, \mathbf{k}} \left[A_{\nu, \mathbf{k}} d_{\nu, \mathbf{k}}^\dagger d_{\nu, \mathbf{k}} - \frac{B_{\nu, \mathbf{k}}}{2} (d_{\nu, \mathbf{k}}^\dagger d_{\nu, -\mathbf{k}}^\dagger + \text{h.c.}) \right].$$



non-linear SWT

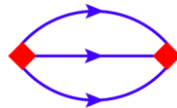
Harris et al (92)* ("two step" LSWT diagonalization for kagomé)

- we generalized it to XXZ and for non-linear (cubic) terms [elegant form]

$$\mathbf{q}=0 \quad F_{\mathbf{q}\mathbf{k}\mathbf{p}}^{\nu\mu\eta} = \sum_{\alpha\beta} \epsilon^{\alpha\beta\gamma} \cos(q_{\beta\alpha}) w_{\nu,\alpha}(\mathbf{q}) w_{\mu,\beta}(\mathbf{k}) w_{\eta,\beta}(\mathbf{p}).$$

$$\sqrt{3} \times \sqrt{3} \quad F_{\mathbf{q}\mathbf{k}\mathbf{p}}^{\nu\mu\eta} = i \sum_{\alpha\beta} \epsilon^{\alpha\beta\gamma} \sin(q_{\beta\alpha}) w_{\nu,\alpha}(\mathbf{q}) w_{\mu,\beta}(\mathbf{k}) w_{\eta,\beta}(\mathbf{p}).$$

- \Rightarrow energy correction

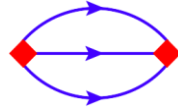


$$\delta E^{(3)} = -\frac{1}{6N} \sum_{\nu\mu\eta} \sum_{\mathbf{q},\mathbf{k}} \frac{|V_{\mathbf{q},\mathbf{k},-\mathbf{k}-\mathbf{q}}^{\nu\mu\eta}|^2}{\epsilon_{\mathbf{q}}^{\nu} + \epsilon_{\mathbf{k}}^{\mu} + \epsilon_{-\mathbf{k}-\mathbf{q}}^{\eta}}$$

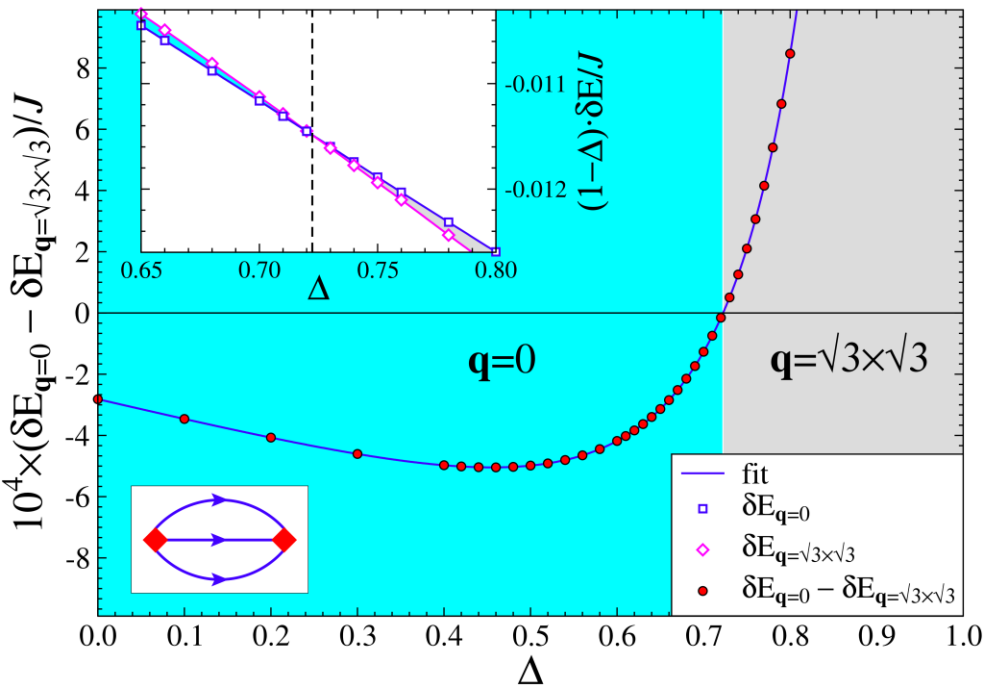


ground-state selection

- ⇒ energy correction*



(*27 terms, 10 distinct)



$$\delta E^{(3)} = -\frac{1}{6N} \sum_{\nu\mu\eta} \sum_{\mathbf{q}, \mathbf{k}} \frac{|V_{\mathbf{q}, \mathbf{k}, -\mathbf{k}-\mathbf{q}}^{\nu\mu\eta}|^2}{\varepsilon_{\mathbf{q}}^{\nu} + \varepsilon_{\mathbf{k}}^{\mu} + \varepsilon_{-\mathbf{k}-\mathbf{q}}^{\eta}}$$

- there is a transition from $\mathbf{q}=0$ to $\sqrt{3} \times \sqrt{3}$ state at $\Delta_c \approx 0.72$
- different from thermal ObD selection
- small $\approx 3 \cdot 10^{-4} J$ per spin
- diverges $\propto 1/(1 - \Delta)$



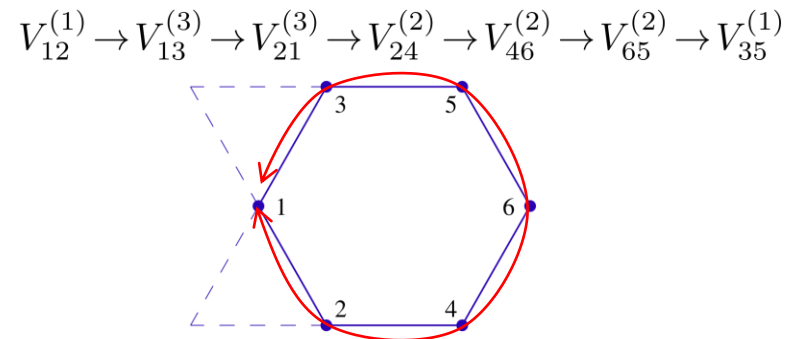
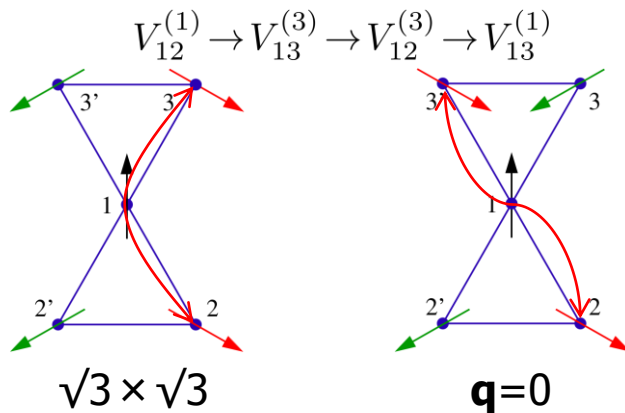
(As far as we know, photo is public domain)



why and how: real-space PT

how do quantum fluctuations select the GS?

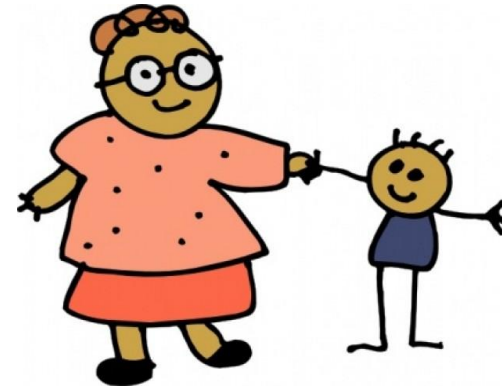
- real-space perturbation theory* $\mathcal{H} \Rightarrow \frac{J}{2} \sum_{\Delta} \mathbf{S}_{\Delta}^2 - J \sum_i \mathbf{S}_i^2$
- expands around a manifold of classical 120° states with $\mathbf{S}_{\Delta} = 0$
- perturbations:
 - $V^{(1)} = -\frac{J}{4} \left(\Delta + \frac{1}{2} \right) \sum_{\langle ij \rangle} (S_i^+ S_j^+ + S_i^- S_j^-)$, double flip
 - $V^{(2)} = \frac{J}{4} \left(\Delta - \frac{1}{2} \right) \sum_{\langle ij \rangle} (S_i^+ S_j^- + S_i^- S_j^+)$, hopping
 - cubic terms $V^{(3)} = \frac{J}{2} \sum_{ij} \sin \theta_{ij} (S - S_i^z) (S_j^+ + S_j^-)$
 - $V^{(4)} = -\frac{J}{2} \sum_{\langle ij \rangle} (S - S_i^z) (S - S_j^z)$
- the lowest-order (4th) correction fails to remove degeneracy (symmetry)
- first graph that does must have a non-trivial **topology** (around hexagon), \Rightarrow 7th (!) order



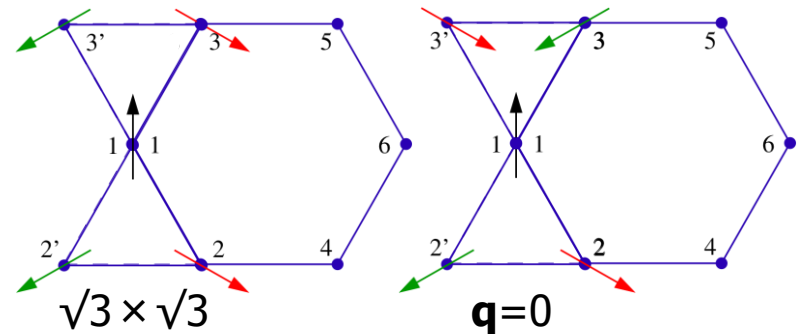
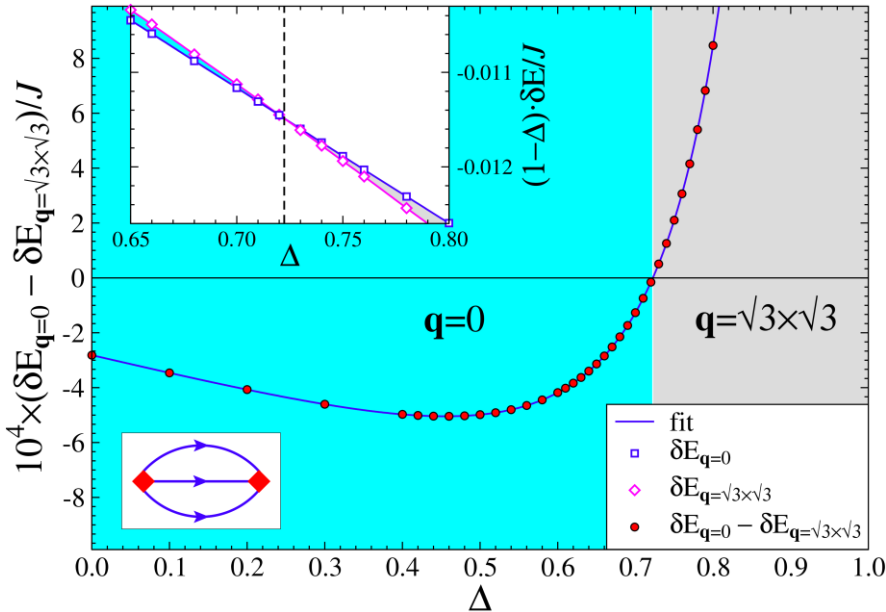
*M. W. Long, J. Phys.: CM **1**, 2857 (1989); M. T. Heinilä and A. S. Oja, PRB **48**, 7227 (1993);
 B. Canals and M. E. Zhitomirsky, J. Phys.: CM **16**, S759 (2004); M.E. Zhitomirsky, J. Phys.: Conf. Ser. **592**, 012110 (2015).
 D. L. Bergman, R. Shindou, G. A. Fiete, and L. Balents, PRB **75**, 094403 (2007); J. Phys.: CM **19**, 145204 (2007)

NLSWT vs RSPT

- ✓ 7th order $\delta E^{(7)} \propto \sin \theta_{12} \sin \theta_{13} \Rightarrow$ 2nd order in cubic terms, same as in NLSWT!
- ✓ coordination number expansion: $\delta E = 8J/z^7 \approx 5 \cdot 10^{-4} J$ per spin \Rightarrow agree
- ✓ for $\Delta < 1/2$ all 7th order processes favor AF $J_2 \Rightarrow \mathbf{q}=0$,
for $\Delta > 1/2$ some switch sign $\Rightarrow \Delta_c > 1/2$ agree



NLSWT RSPT



more RSPT

denominators for ten listed processes

- 1) $\frac{1}{4^3 \cdot 6 \cdot 8^2} \times 2$
- 2) $\frac{1}{4^2 \cdot 6 \cdot 8^2 \cdot 12} \times 6$
- 3) $\frac{1}{4^2 \cdot 6^2 \cdot 8 \cdot 10} \times 2$
- 4) $\frac{1}{4^2 \cdot 6 \cdot 8 \cdot 10 \cdot 12} \times 6$
- 5) $\frac{1}{4^3 \cdot 6 \cdot 8^2} \times 2$
- 6) $\frac{1}{4^2 \cdot 6 \cdot 8^2 \cdot 10} \times 2$
- 7) $\frac{1}{4^2 \cdot 8^2 \cdot 10 \cdot 12} \times 6$
- 8) $\frac{1}{4^3 \cdot 6 \cdot 8^2} \times 2$
- 9) $\frac{1}{4^2 \cdot 6 \cdot 8^2 \cdot 10} \times 2$
- 10) $\frac{1}{4^2 \cdot 8^2 \cdot 10 \cdot 12} \times 6$

$$\Delta E^{(7)} = J \left(\Delta + \frac{1}{2}\right)^5 \sin \theta_{12} \sin \theta_{13} \frac{1}{2^6} \times 2 \times 2 \times \dots$$

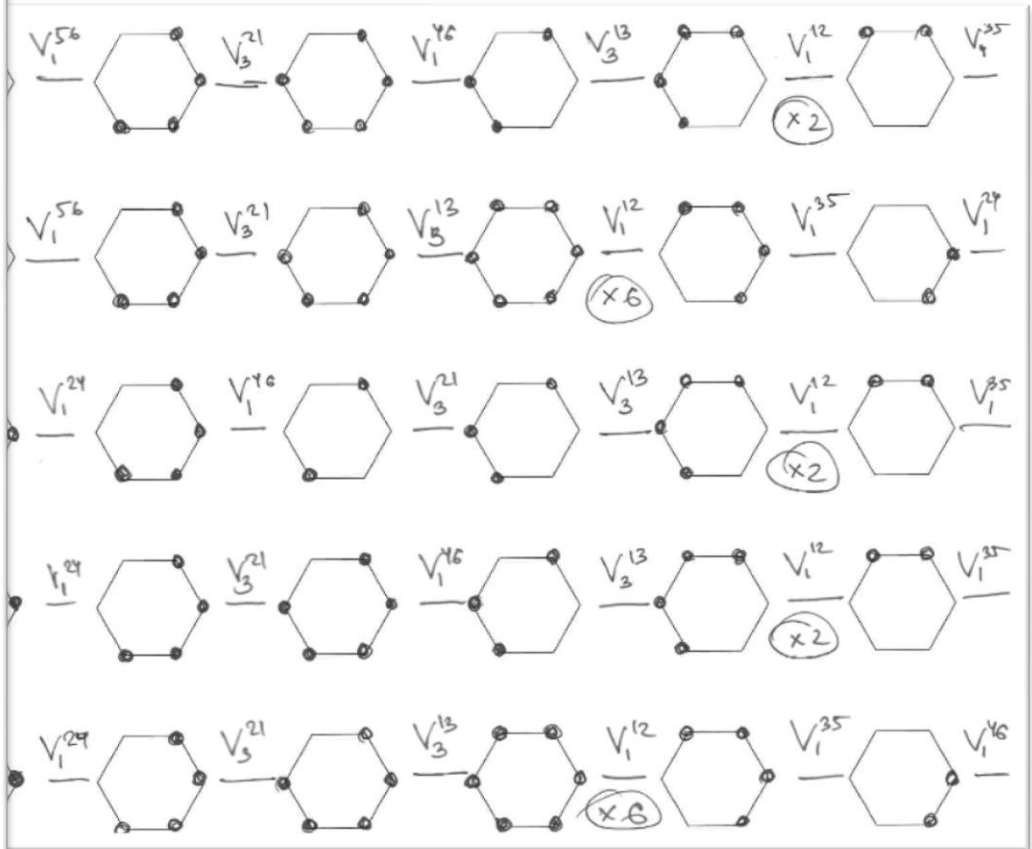
$$\left[\frac{1}{4^3 \cdot 6 \cdot 8^2} \times 6 + \frac{1}{4^2 \cdot 6 \cdot 8^2 \cdot 12} \times 6 + \frac{1}{4^2 \cdot 6^2 \cdot 8 \cdot 10} \times 2 \right. \\ \left. + \frac{1}{4^2 \cdot 6 \cdot 8 \cdot 10 \cdot 12} \times 6 + \frac{1}{4^2 \cdot 6 \cdot 8^2 \cdot 10} \times 4 + \frac{1}{4^2 \cdot 8^2 \cdot 10 \cdot 12} \cdot 12 \right]$$

$$= \frac{11}{3 \cdot 2^{15}} J \sin \theta_{12} \sin \theta_{13}$$

→ difference between ferro and antiferro colors $\left(\frac{13}{2}\right)\left(\frac{13}{2}\right) - \left(\frac{13}{2}\right)\left(-\frac{13}{2}\right) = \frac{13}{2} \times 4 \text{ bands} \times \frac{1}{2} (\text{per spin})$
 → extra factor 3.

$$\Delta E = \frac{11}{3 \cdot 2^{15}} = 1.1189 \cdot 10^{-4} \text{ J}$$

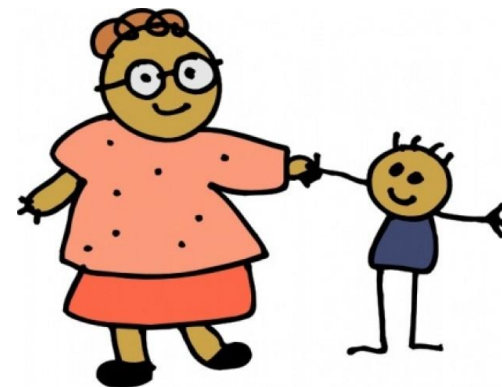
✓ for $\Delta=1/2$ can count **all** 7th order processes



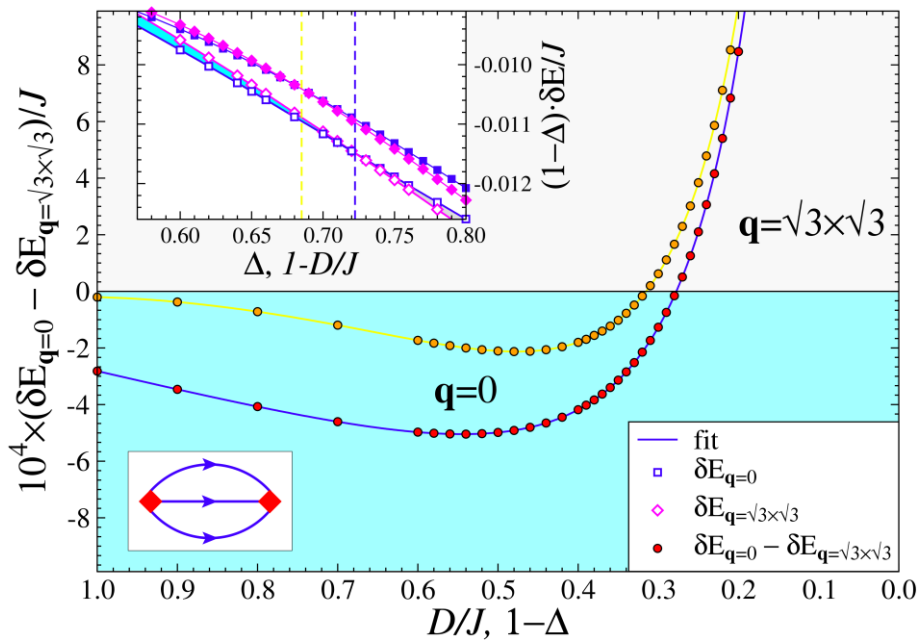
other XXZ's

- ✓ single-ion anisotropy, $D > 0$, same effect, same qualitative GS selection
 - ✓ however, local spin-flips = different, more local, require higher-order process
- coordination number expansion: $\delta E \propto 1/z^8 \Rightarrow$ agree

$$\mathcal{H} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + D \sum_i (S_i^z)^2$$

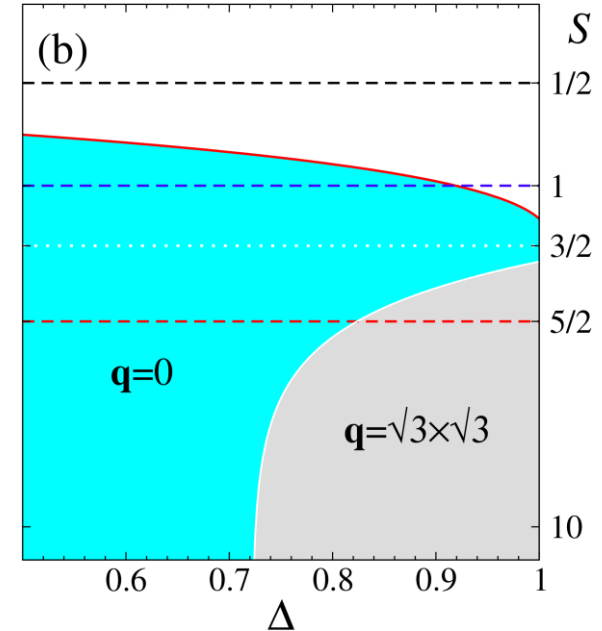
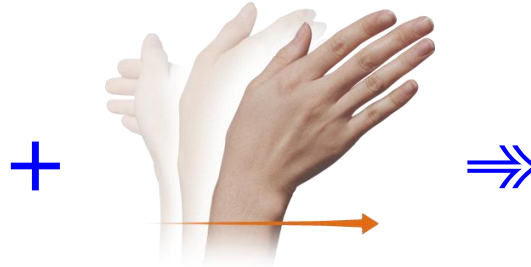
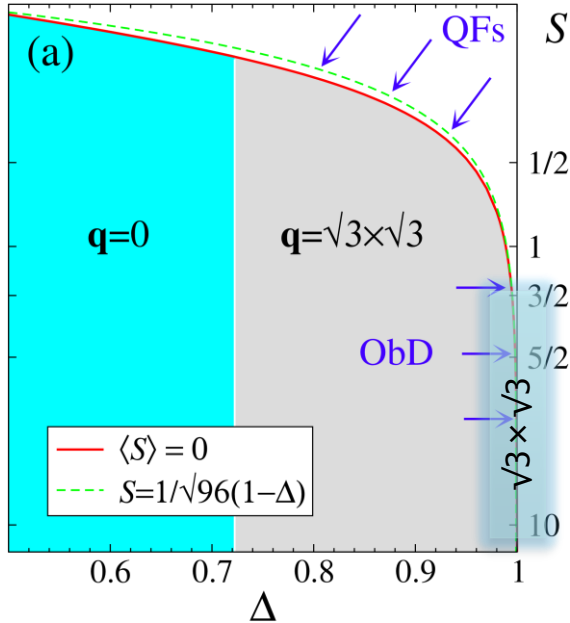


NLSWT RSPT



S-Δ PhD, I

naive, but honest ...



- S - Δ phase diagram from LSWT $\langle S \rangle = 0$, neglects tunneling effects, etc.
- $\mathbf{q}=0$ to $\sqrt{3} \times \sqrt{3}$ transition from NLSWT results

anticipated trends ...

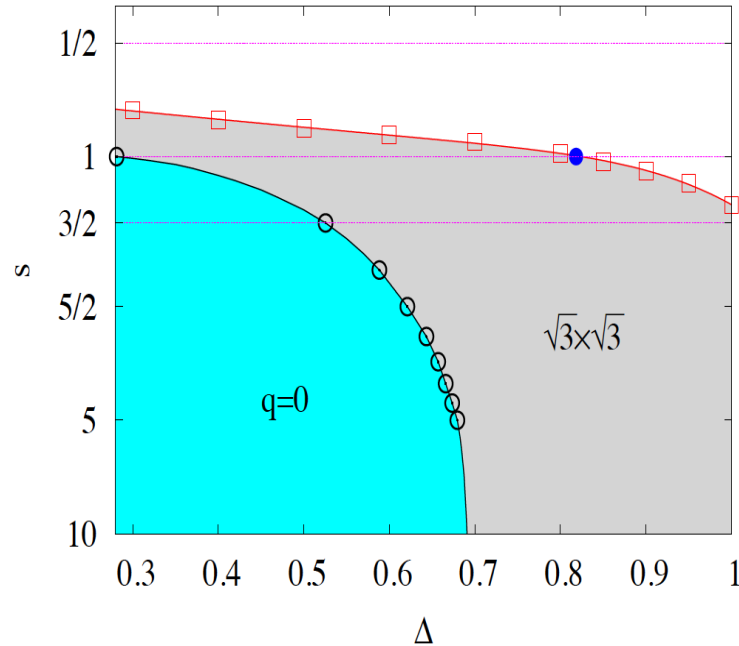
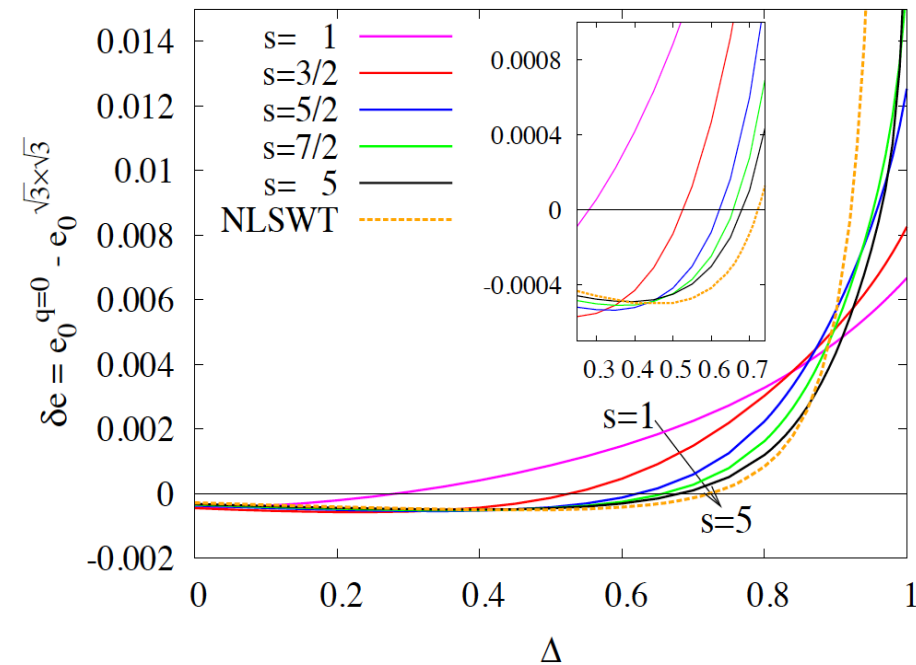
- QFs will suppress ordered region [*DMRG]
- ObD will create "island of stability" [**numerics: \Rightarrow island is big!]
- $\mathbf{q}=0$ to $\sqrt{3} \times \sqrt{3}$ boundary? [***hints from self-consistent SWT]

*Y.-C. He and Y. Chen, PRL **114**, 037201 (2015); A. M. Läuchli; S. R. White, [private communications]

O. Gotze, D. J. J. Farnell, R. F. Bishop, P. H. Y. Li, and J. Richter, PRB **84, 224428 (2011)

***A. V. Chubukov, PRL **69**, 832 (1992)

PhD, II



- numerical method, similar to RSPT, qualitative and quantitative agreement
- different trend for $q=0$ to $\sqrt{3} \times \sqrt{3}$ boundary



spectrum

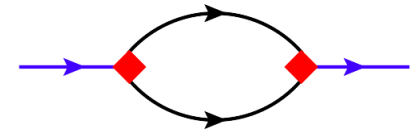


spectrum

what happens with the spectrum due to non-linear terms?

two crucial ingredients:

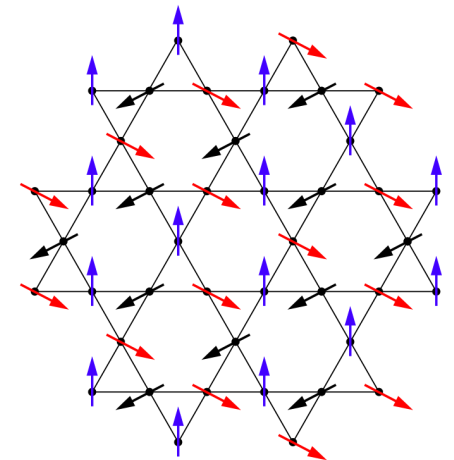
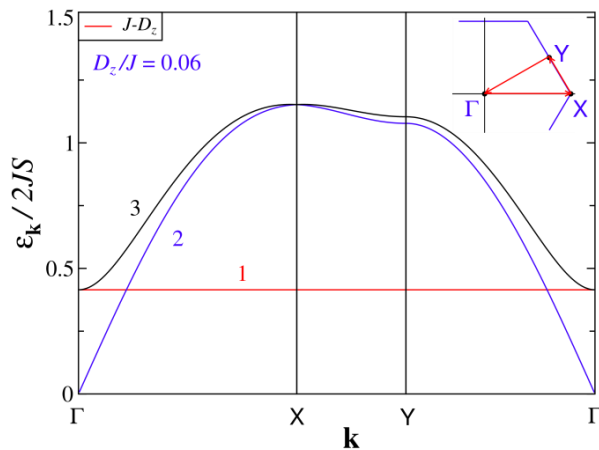
- noncollinear spin structure \Rightarrow 120° state \Rightarrow decays*



$$\hat{\mathcal{H}}_3 = \frac{1}{3!} \frac{1}{\sqrt{N}} \sum_{\mathbf{k}, \mathbf{q}} \sum_{\nu \mu \eta} V_{\mathbf{q}\mathbf{k}\mathbf{p}}^{\nu \mu \eta} b_{\nu, \mathbf{q}}^\dagger b_{\mu, \mathbf{k}}^\dagger b_{\eta, -\mathbf{p}}^\dagger + \text{h.c.},$$

- flat mode(s)

$$+ \frac{1}{2!} \frac{1}{\sqrt{N}} \sum_{\mathbf{k}, \mathbf{q}} \sum_{\nu \mu \eta} \Phi_{\mathbf{q}\mathbf{k}; \mathbf{p}}^{\nu \mu \eta} b_{\nu, \mathbf{q}}^\dagger b_{\mu, \mathbf{k}}^\dagger b_{\eta, \mathbf{p}} + \text{h.c.},$$



1/S or not 1/S ?

what's new when the flat mode is present?

$$S^z = S - a^\dagger a,$$

$$S^x \approx \sqrt{S/2} (a^\dagger + a)$$

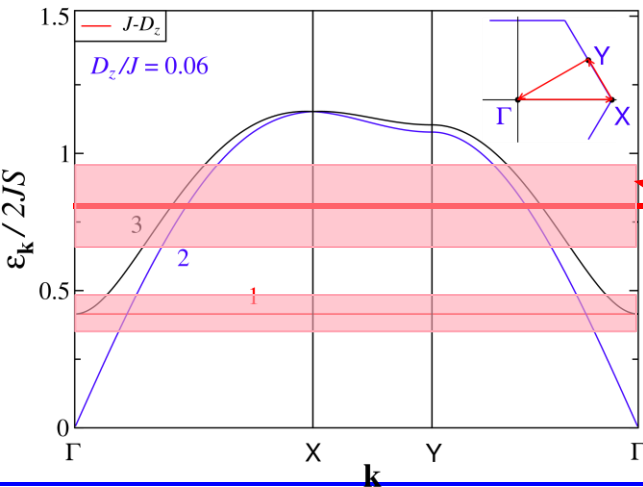
- usually, for decays from higher-energy states to lower ones

$$\Gamma_{\mathbf{k}} \propto J S \sum_{\mathbf{q}} |\Phi_{\mathbf{k},\mathbf{q}}|^2 \delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{q}} - \varepsilon_{\mathbf{k}-\mathbf{q}})$$

$$\Rightarrow \Gamma_{\mathbf{k}} \propto J \Rightarrow \frac{\Gamma_{\mathbf{k}}}{\varepsilon_{\mathbf{k}}} \propto \frac{1}{S}$$

$$\Rightarrow \varepsilon_{\mathbf{q}} \propto J S$$

- flat mode \Rightarrow **resonant-like** decays at twice its energy \Rightarrow singularity



- $1/S$ fluctuations **warp** the flat mode and regularize singularity

- fluctuation-induced dispersion is **spin-independent** $\delta\varepsilon_{\mathbf{k}} \propto J$

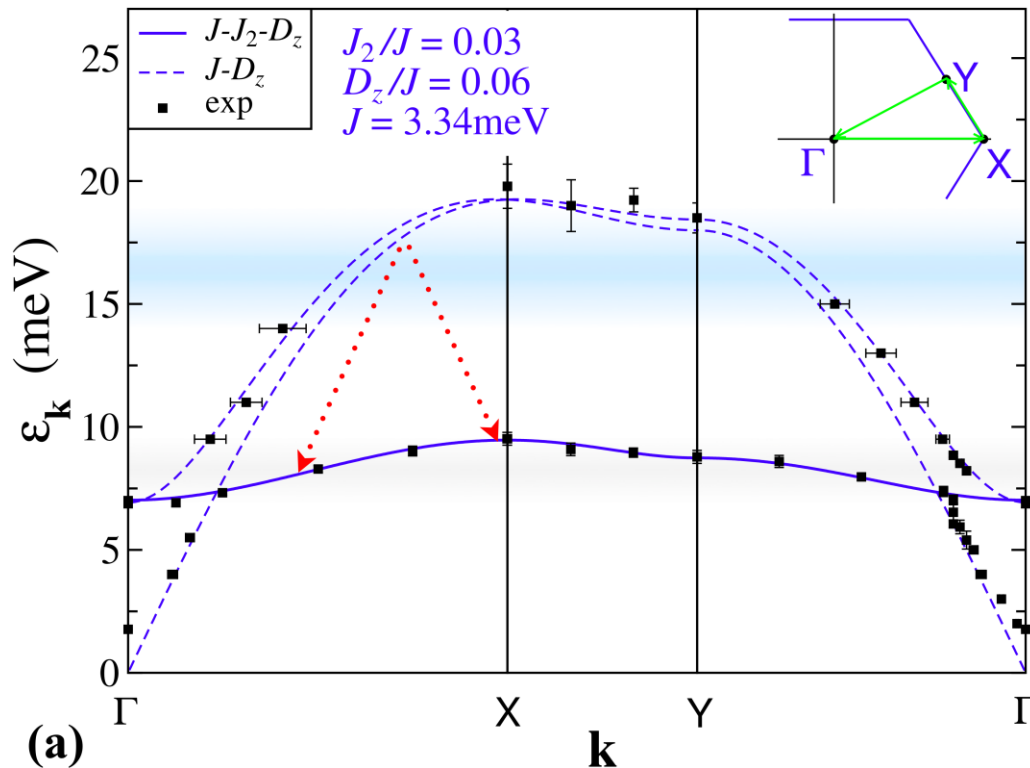
- decay rate $\Gamma_{\mathbf{k}} \propto SJ$ **not** $1/S$ -effect \Rightarrow not small even for large spins!



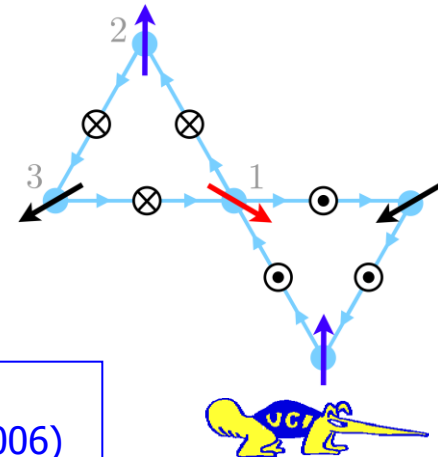
Fe-jarosite

$$\mathcal{H} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \mathbf{D}_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j)$$

almost classical, $S=5/2$, kagome-lattice antiferromagnet

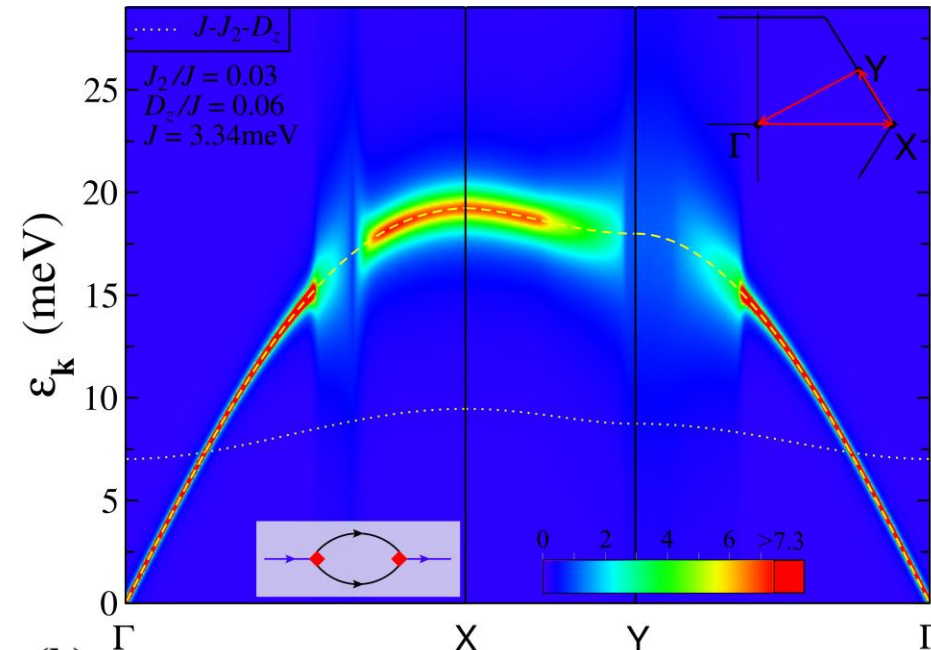
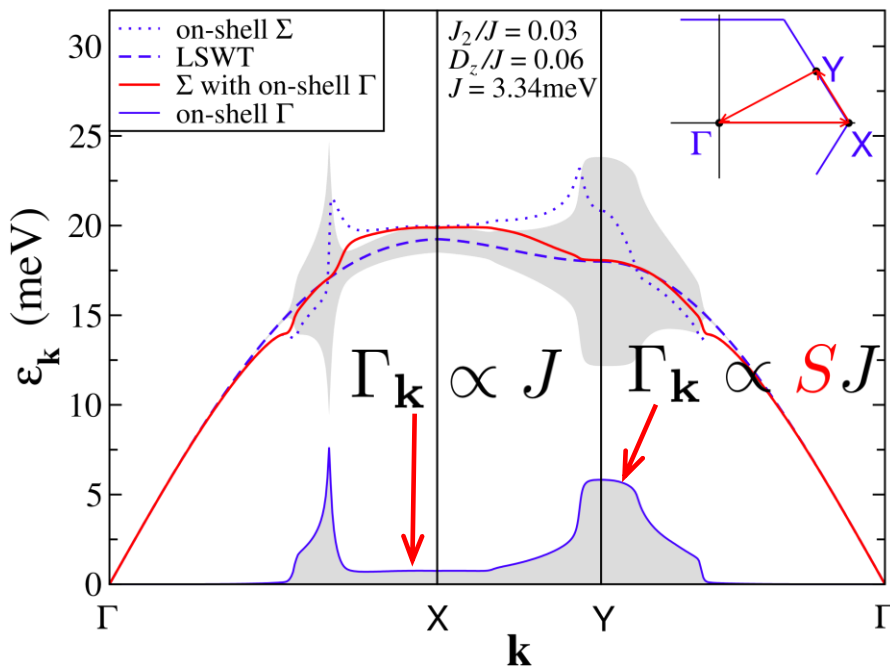


- DM, out-of-plane, gaps flat mode
- dispersion in the flat mode, J_2 ?
- “missing” data at twice flat mode?
- resonant-like decays?



spectral signatures of resonant decays

- outside the resonance region $\Gamma_{\mathbf{k}} \approx 0.2 - 0.3J$
- inside of it, it is about 5 ($=2S$ for FeJ) times larger, in agreement with expectations
- very strong quantum effect in an almost classical antiferromagnet



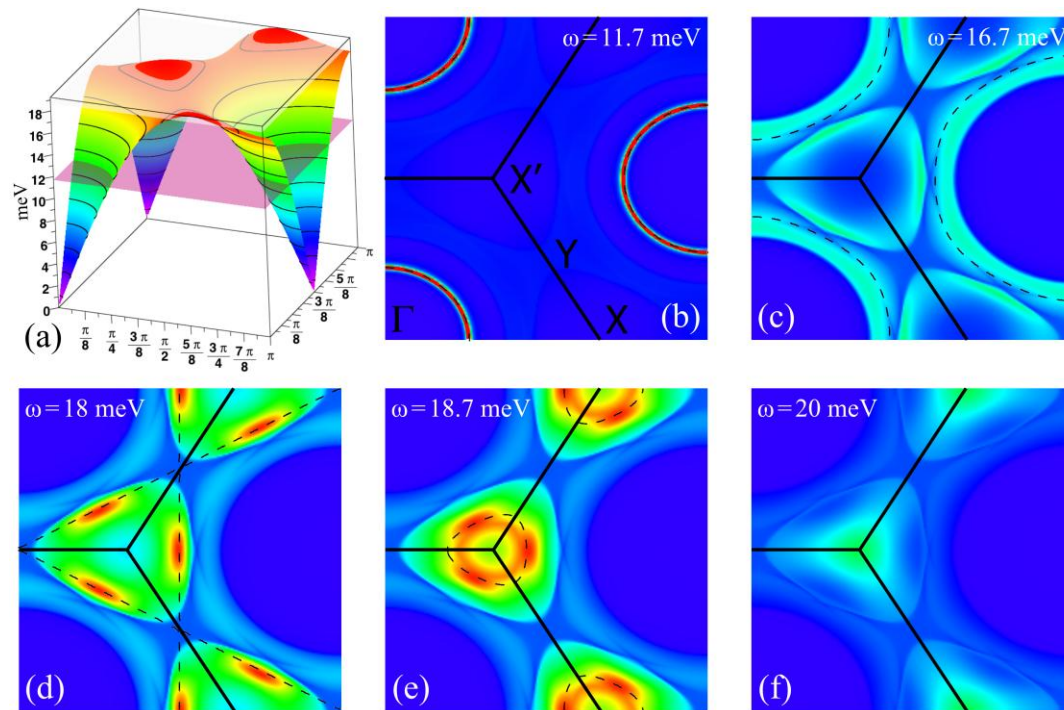
(b)

$$S^{\alpha\alpha}(\mathbf{q}, \omega) \propto \int dt e^{i\omega t} \langle S_{\mathbf{q}}^{\alpha}(t) S_{-\mathbf{q}}^{\alpha} \rangle \propto \sum_{\nu} F_{\nu\mathbf{q}}^{\alpha} A_{\nu}(\mathbf{q}, \omega)$$



constant-energy cuts of $A(q, \omega)$

- wipeout of the (portions of the) spectrum
- strong spectral weight redistribution
- features reminiscent of the quasiparticle breakdown in spin-1/2 systems



“masks” in $S(q, \omega)$

- “kinematic formfactors” are suppressed in one BZ zones and are maximal in others
→ characteristic to the non-Bravais lattices (a la Bragg peak extinction)

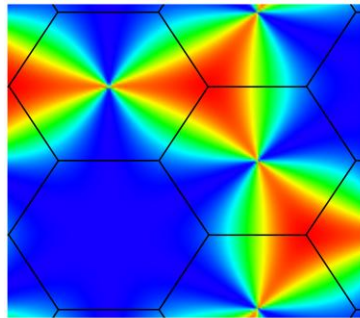
$$S^{\text{in(out)}}(\mathbf{q}, \omega) \approx \sum_{\nu} F_{\nu\mathbf{q}}^{\text{in(out)}} A_{\nu}(\mathbf{q}, \omega),$$

mode $\nu = 1$

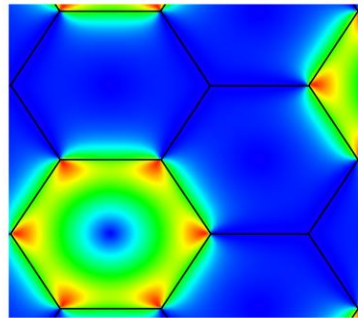
$\nu = 2$

$\nu = 3$

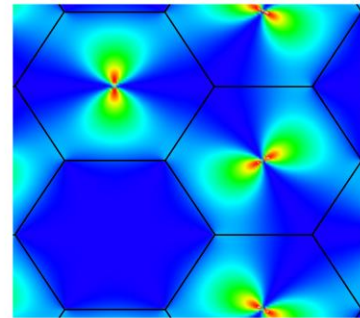
$F_{\nu\mathbf{q}}^{\text{out}}$



(a)

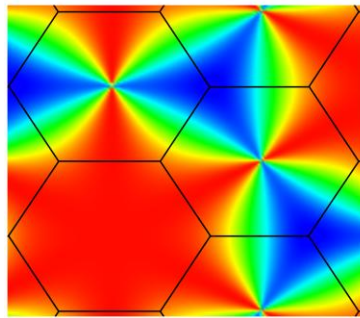


(b)

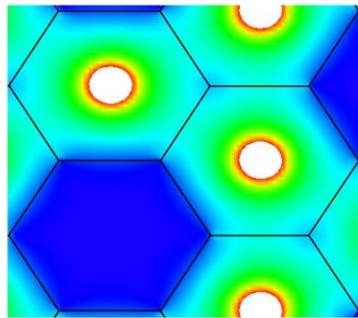


(c)

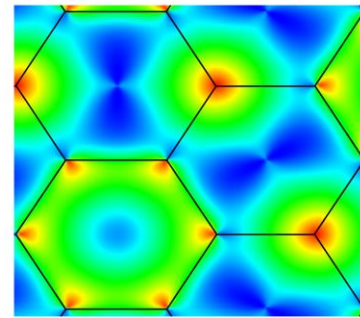
$F_{\nu\mathbf{q}}^{\text{in}}$



(d)



(e)



(f)



conclusions

kagomé-lattice AFs \Rightarrow still many surprises

- ☑ “if you don’t expect the unexpected – you just don’t get it!”
- quantum selection of $\mathbf{q}=0$: a rare case of QObD different from thermal; uses topologically nontrivial tunneling paths to select the ground state
- ☑ very strong anomalous spectral features even in large- S AFs

