Interacting surface states of 3D topological insulators

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UNIVERSITY OF ALBERTA

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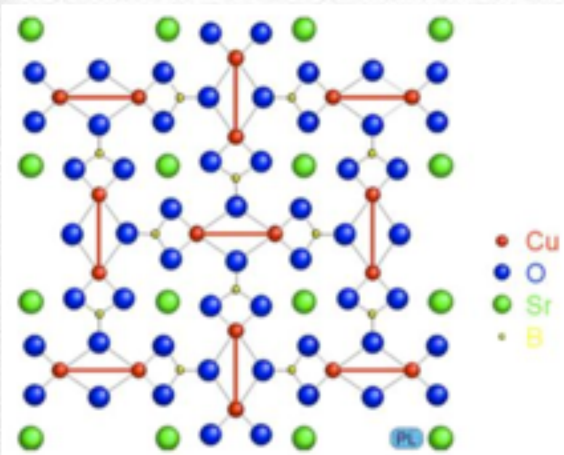
"We are really really good at solving problems of noninteracting electrons."

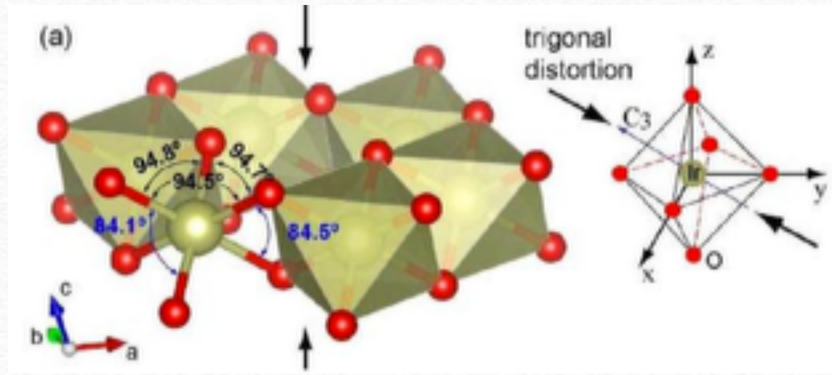
–Leon Balents

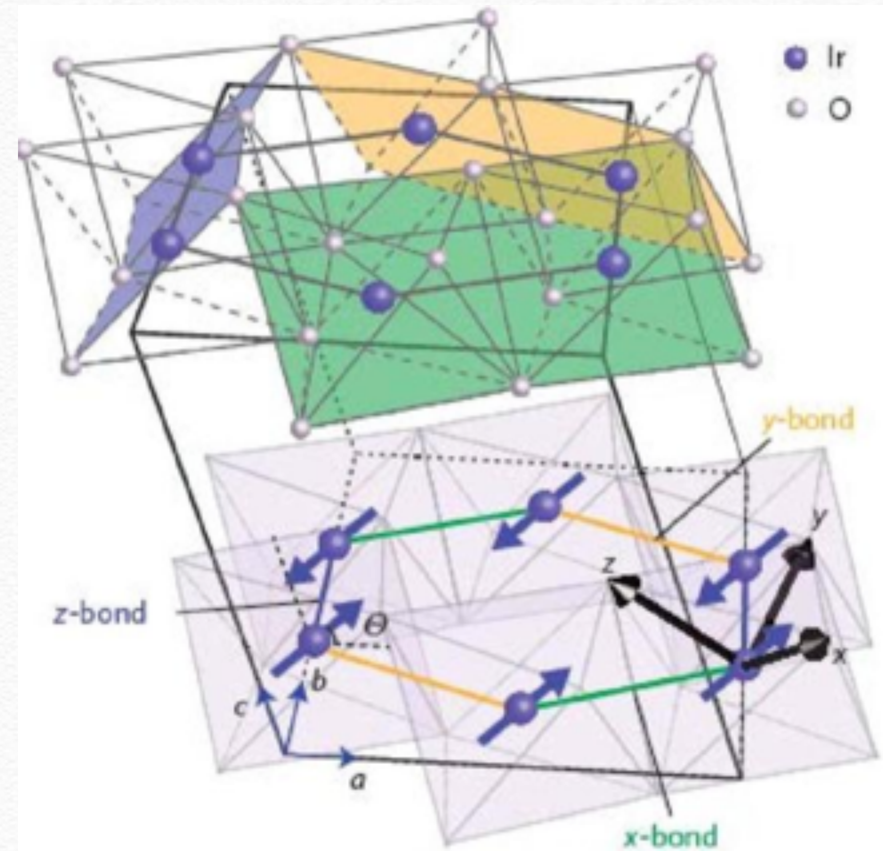
What about interactions?

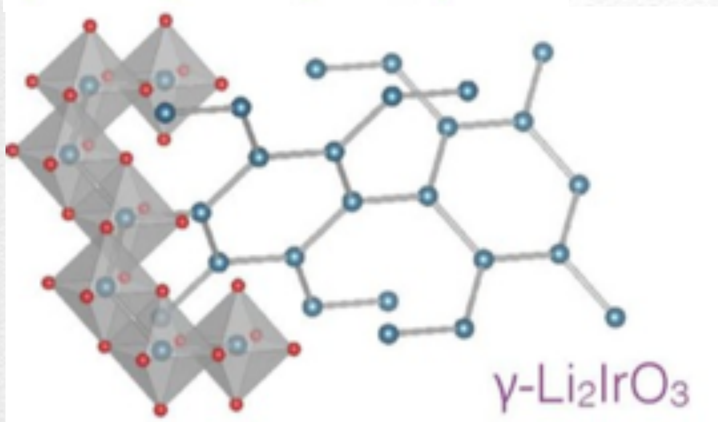
- Problems of interacting electrons are hard...

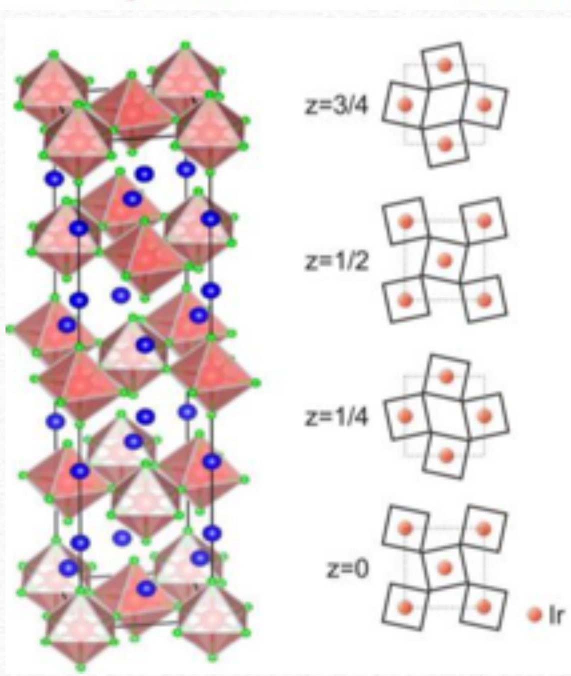
What about interactions?

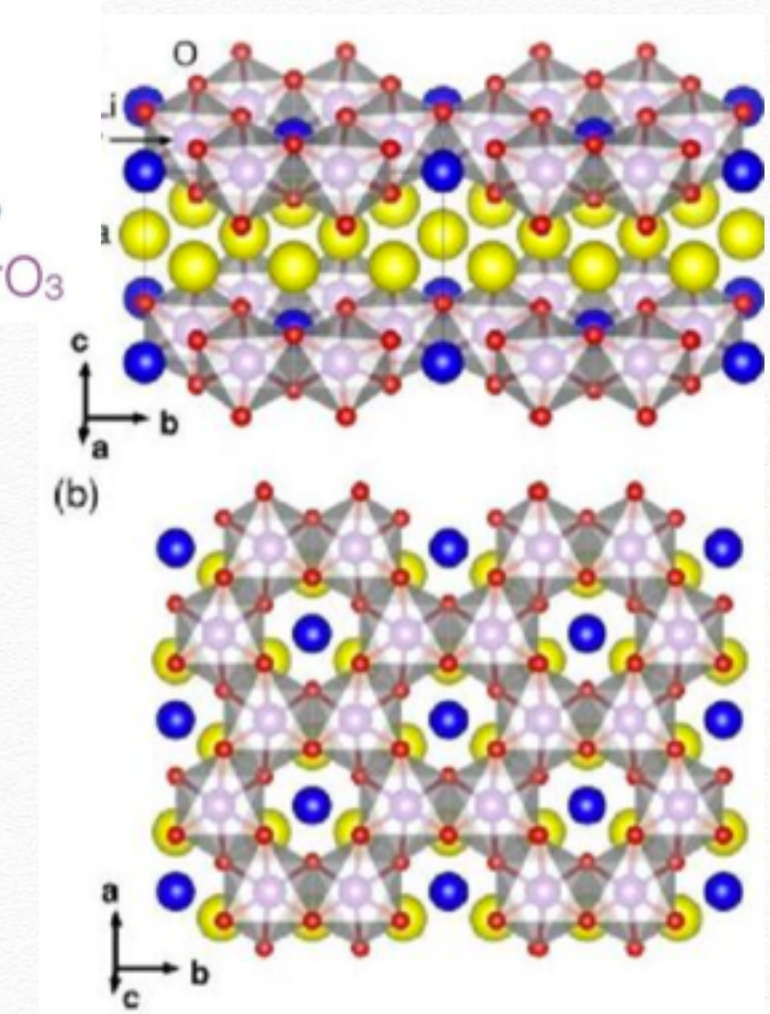












- A lot depends on microscopics: chemistry, lattices, ...

Topological phases

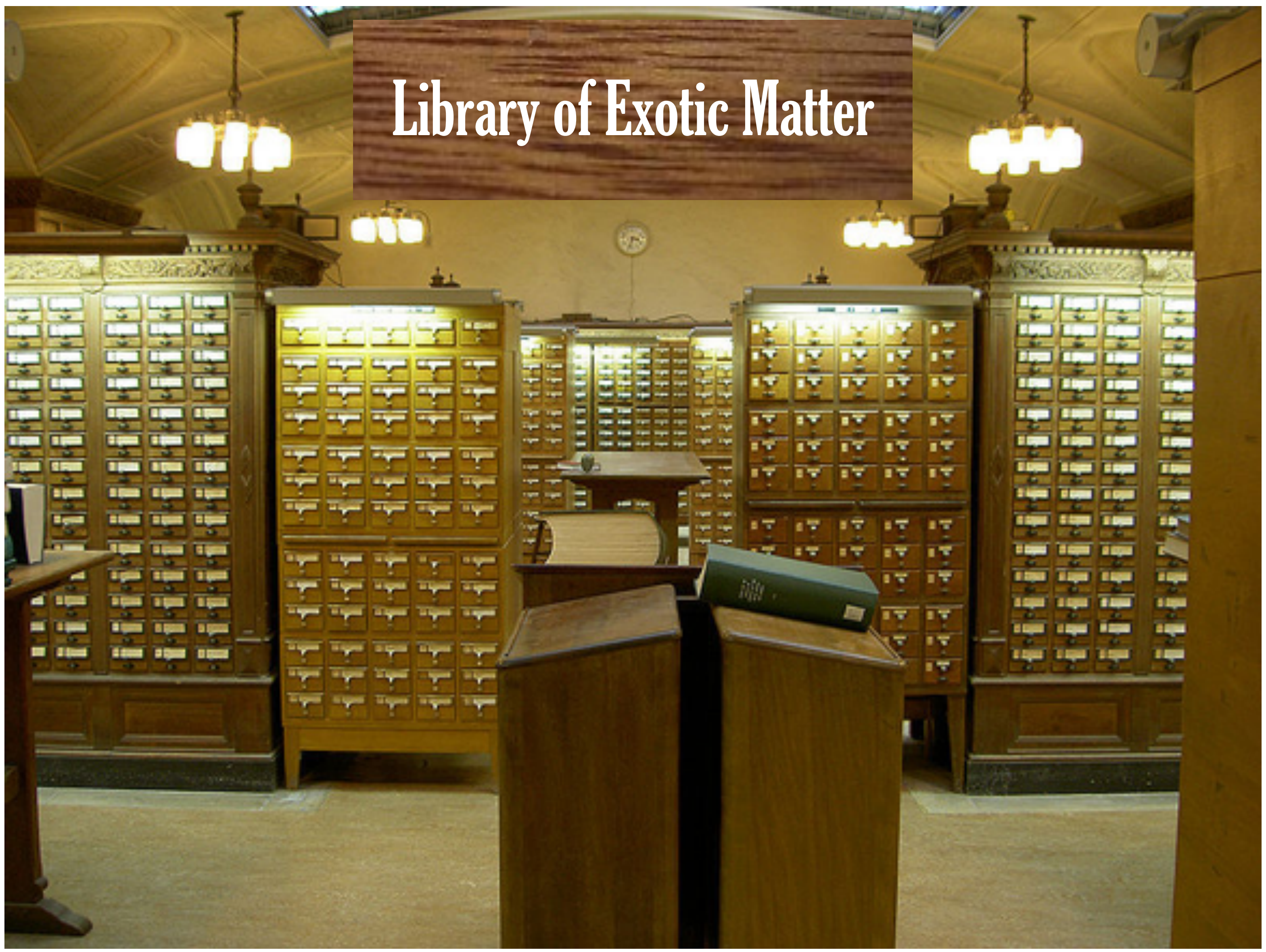
- Topological phases of matter are nice, because their long-wavelength properties are universal
- Bulk: quantized response, emergent gauge and/or matter d.o.f.
- Surface: robust gapless d.o.f.

Field theory of topological phases

- Universality lets us be lazy, ignore microscopic details, and do field theory
- Bulk: topological field theory
- Surface: field theory

(L. Balents, EQPCM 2013)

Classification



cohomology	K-matrix	General Subject
000, 040, 080	AC	Z_2 TI
010, 020, 090	Z	fibonacci
030	AE	IQHE
050	AP	ASL
060	AS	E8
070	PN	$SO(6)_3$
100	B-BJ	Philosophy (Gen.)
110-120	BD	Speculative Philosophy
130, 150	BF	Psychology
140, 180, 190	B	Philosophy (Gen.)
160	BC	Logic
170	BJ	Ethics
200, 210, 290	BL	Religions. Mythology
220	BS	The Bible
230	BT	Doctrinal Theology
240, 250	BV	Practical Theology
260, 270	BR	Christianity
280	BX	Christian Denominations
300	H	Soc. Sci. (General)
310	HA	Statistics
320	J	Gen. Legislative papers
330	HB	Economic Theory
340	K	Law
350	JF-JS	Political Institutions
360	HN, HV	Social History, Soc. Pathology
370	L	Education (General)
380	HD	Industries. Land Use. Labor
390	GT	Manners and customs

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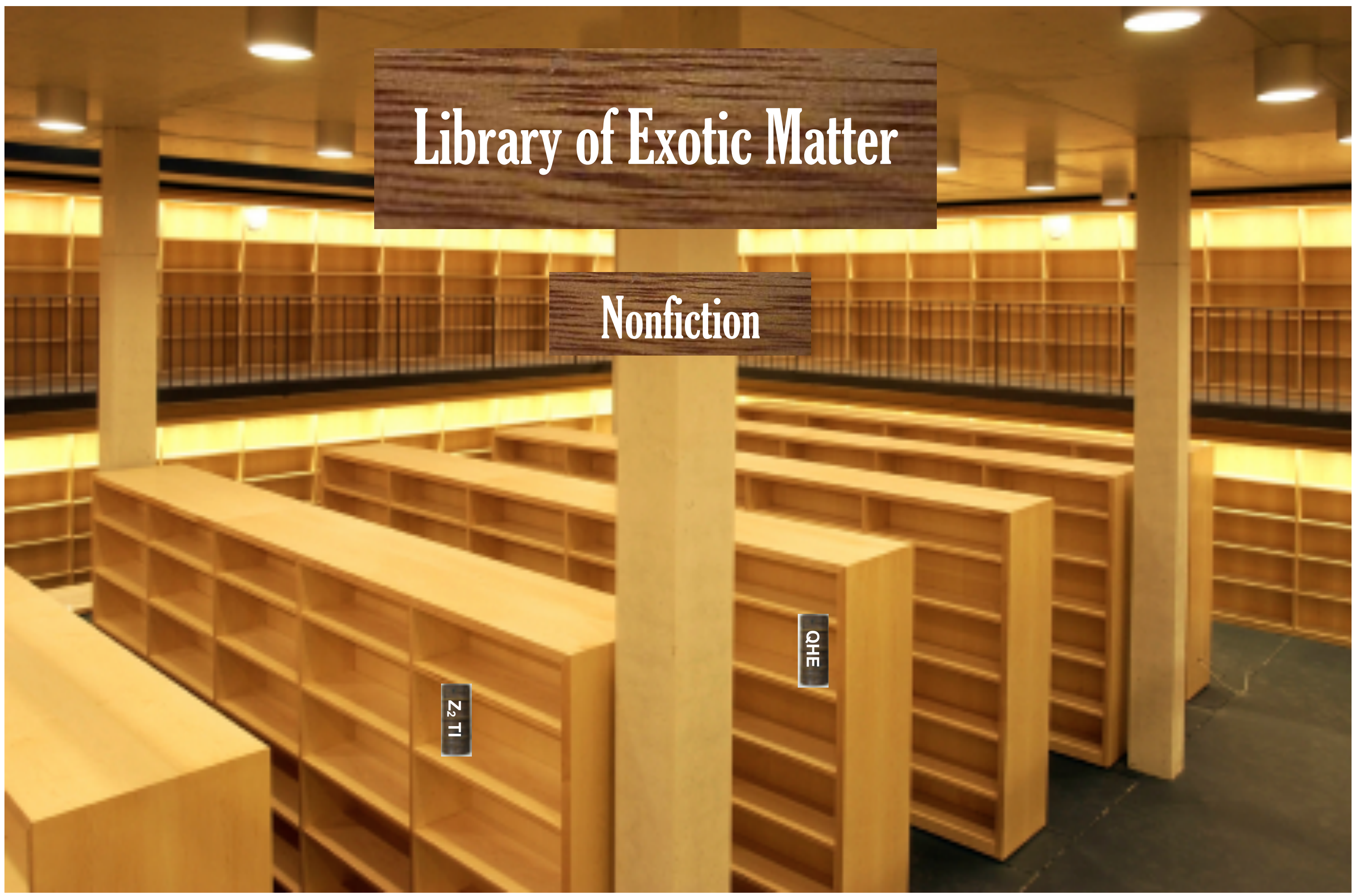

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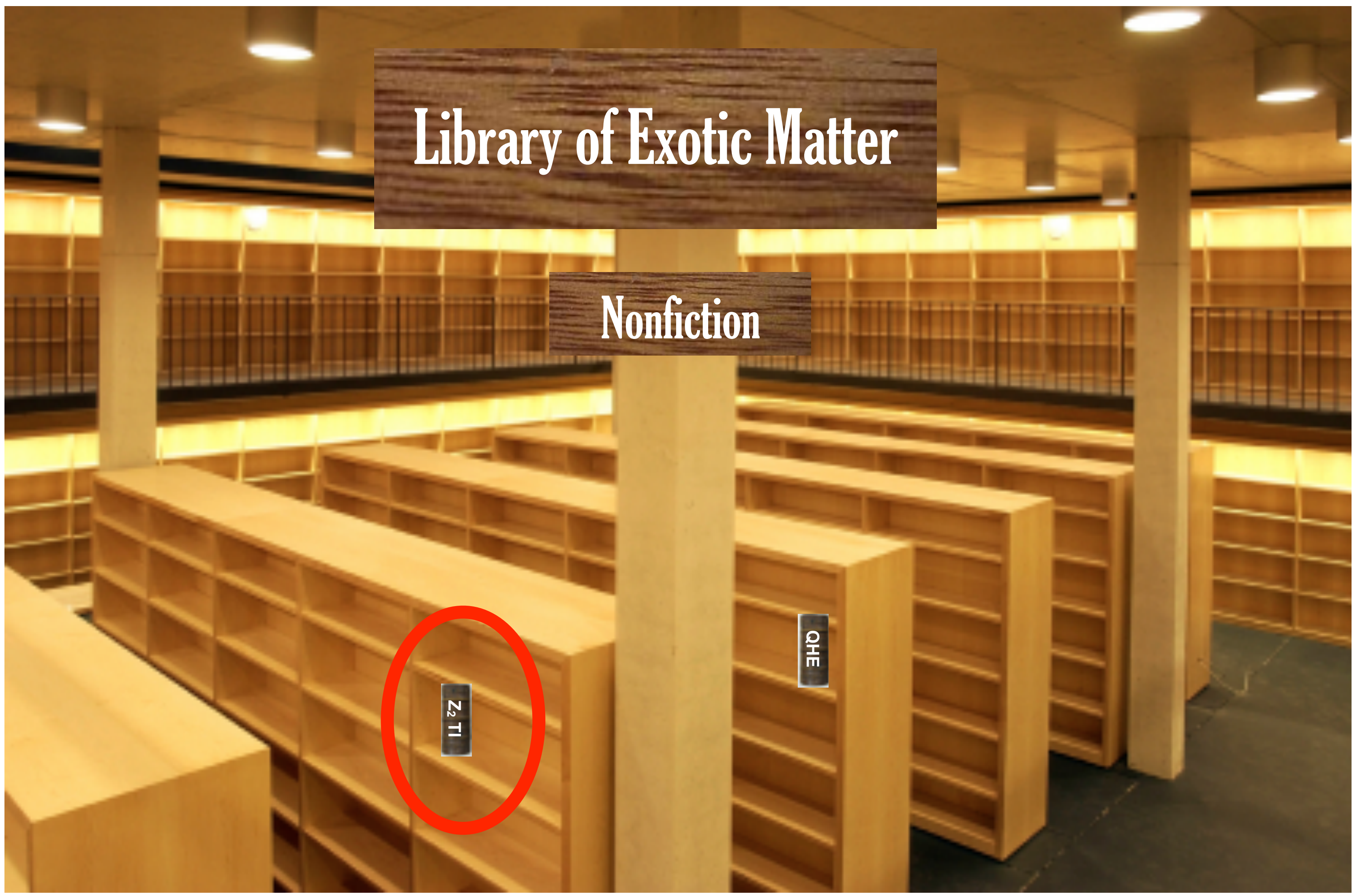

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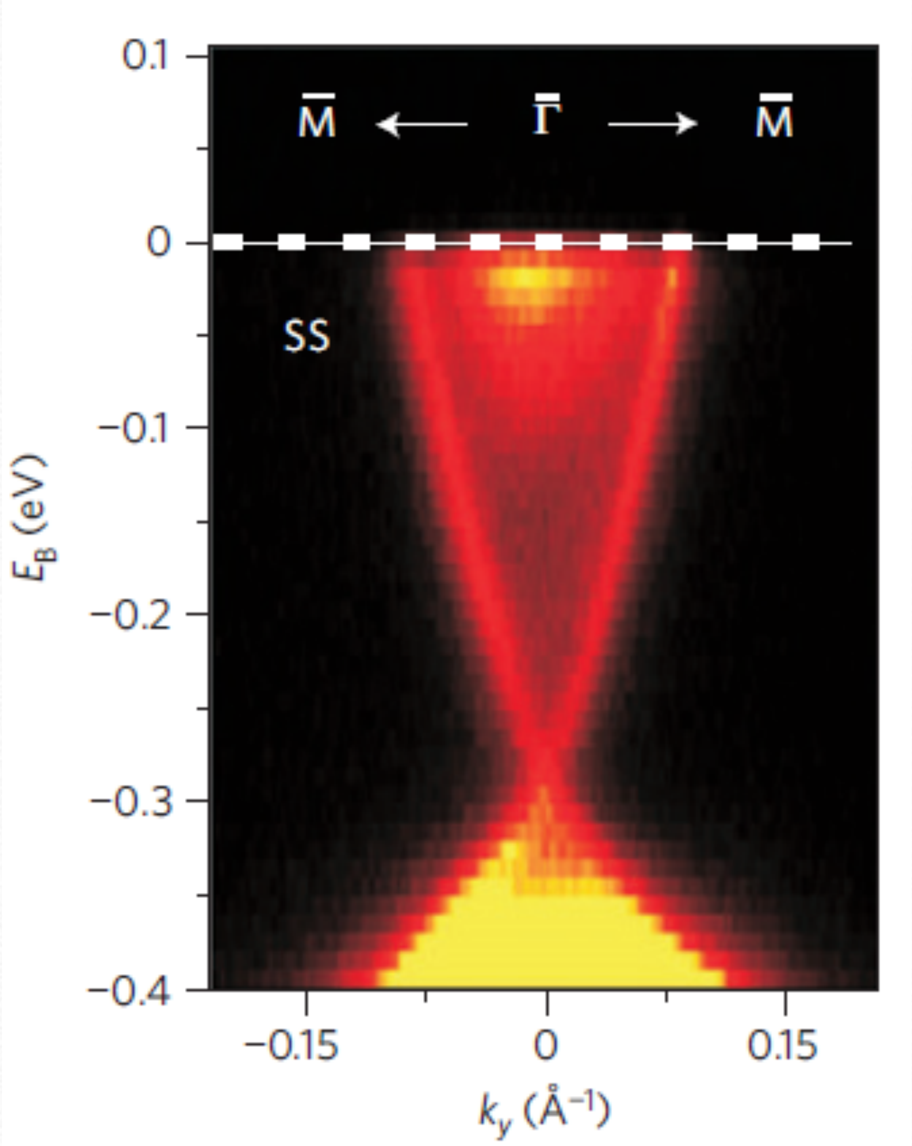
3D topological insulators

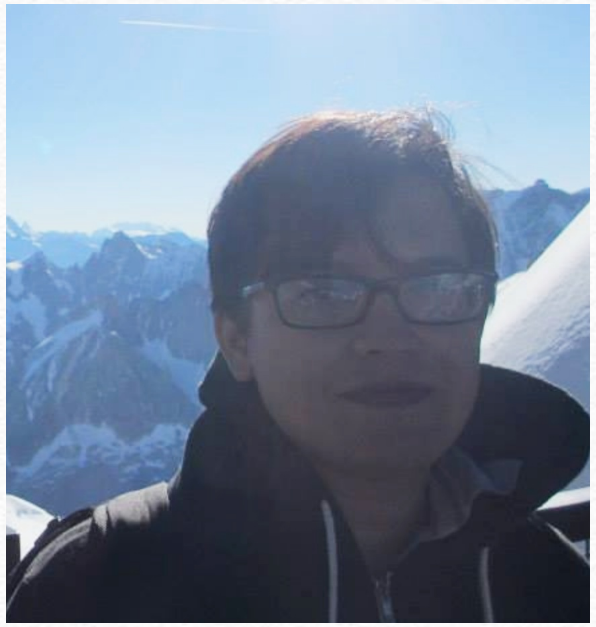
- Surface state = 2D Dirac fermion
- Goal: universal (materials-independent) description of surface state interactions & instabilities



Bi_2Se_3

(Xia et al., Nat. Phys. 2009)

Collaborators



R. Lundgren
(UT Austin)

W. Witczak-Krempa
(Harvard)

Outline

- Weak correlations: Landau theory of helical Fermi liquids

R. Lundgren and JM, PRL **115**, 066401 (2015)

- Strong correlations: Universal conductivity at semimetal-superconductor QCP

W. Witczak-Krempa and JM, arXiv:1510.XXXXX

Landau Fermi liquid theory

- Fundamental paradigm of many-body physics (Landau 1956; Abrikosov, Khalatnikov 1957)

- Adiabatic continuity between energy levels of free & interacting systems: QP with momentum **k**, spin σ, distribution function $n_{\mathbf{k}\sigma}$

Landau Fermi liquid theory

- Landau functional: energy of many-body excited state (configuration of QPs) relative to GS

$$\delta E[\delta n] = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} \delta n_{\mathbf{k}\sigma} + \frac{1}{2V} \sum_{\mathbf{k},\mathbf{k}',\sigma,\sigma'} f_{\sigma\sigma'}(\mathbf{k},\mathbf{k}') \delta n_{\mathbf{k}\sigma} \delta n_{\mathbf{k}'\sigma'}$$

$$\delta n_{\mathbf{k}\sigma} = n_{\mathbf{k}\sigma} - n_{\mathbf{k}\sigma}^0$$

Landau parameters

- Interactions between QPs near the FS: Landau parameters F_l^s, F_l^a
- Most general symmetry-allowed short-range interaction: TRS, spatial SO(3) rotations, spin SU(2) rotations

$$f_{\sigma\sigma'}(\mathbf{k},\mathbf{k}') = f_{\sigma\sigma'}(\mathbf{k}_F,\mathbf{k}'_F) = f^s(\theta) + \sigma\sigma' f^a(\theta)$$

$$f_l^{s,a} = (2l+1)\int_0^\pi \frac{d\Omega}{4\pi} f^{s,a}(\theta) P_l(\cos\theta)$$

$$F_l^{s,a} = 2N^*(0) f_l^{s,a}$$

Landau parameters

- (Finite) renormalization of physical properties due to interactions

effective mass	$\frac{m^*}{m} = 1 + \frac{1}{3}F_1^s$	Galilean invariance
specific heat ($c_V = \gamma T$)	$\frac{\gamma}{\gamma_0} = \frac{m^*}{m}$	
compressibility	$\frac{\kappa}{\kappa_0} = \frac{m^*}{m}\frac{1}{1+F_0^s}$	
spin susceptibility	$\frac{\chi}{\chi_0} = \frac{m^*}{m}\frac{1}{1+F_0^a}$	

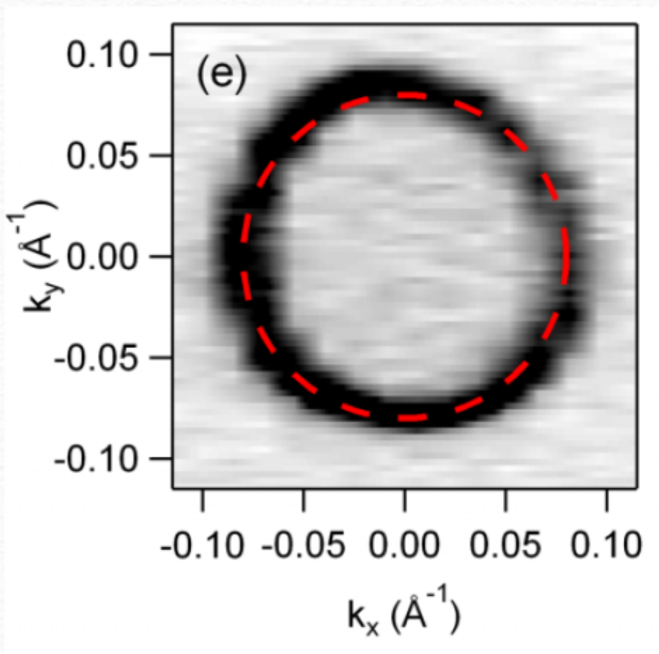
A theory of helical Fermi liquids?

- Phenomenological Landau theory for the 3D TI surface state?
- Qualitative differences from ordinary FL theory due to SOC
- FL theory for non-topological Rashba 2DEG recently constructed (Ashrafi, Rashba, Maslov, PRB 2013), but complicated due to 2 FS
- Here, topology => single FS: effectively spinless FL theory

Symmetries of the helical FL

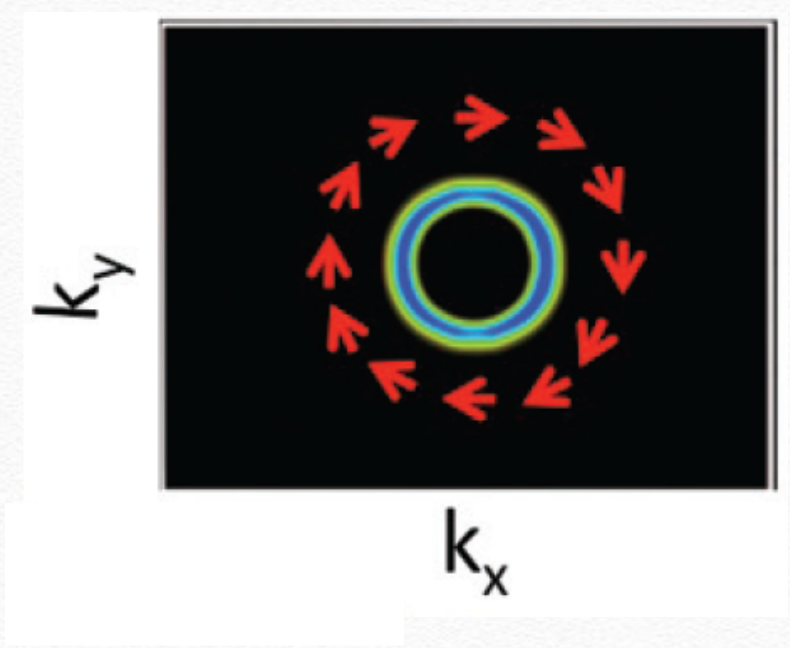
- TRS = protecting symmetry of 3D TI
- Rotation symmetry: focus on materials with (almost) perfectly circular FS

Bi_2Se_3



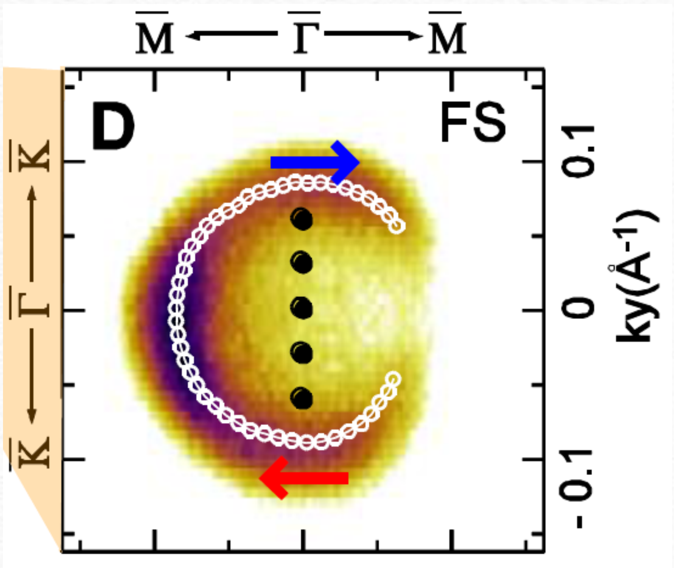
Pan et al., PRL 2011

Bi_2Te_2Se



Neupane et al., PRB 2013

$TlBiSe_2$

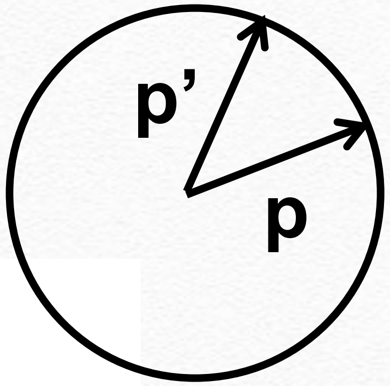


Kuroda et al., PRB 2015

Landau functional

- SOC: QP distribution function is 2x2 matrix

$$\delta n_{\boldsymbol{p}}^{\alpha\beta} \equiv n_{\boldsymbol{p}}^{\alpha\beta} - n_{\boldsymbol{p}}^{(0)\alpha\beta}$$



- Landau functional

$$\delta E[\delta n_{\boldsymbol{p}}] = \int đp\, h_{\alpha\beta}(\boldsymbol{p})\delta n_{\boldsymbol{p}}^{\alpha\beta} + \frac{1}{2}\int đp\, đp'\, V_{\alpha\beta;\gamma\delta}(\hat{\mathbf{p}},\hat{\mathbf{p}}')\delta n_{\boldsymbol{p}}^{\alpha\beta}\delta n_{\boldsymbol{p}'}^{\gamma\delta}$$

$$h(\boldsymbol{p}) = v_F\,\hat{\boldsymbol{z}}\cdot(\boldsymbol{\sigma}\times\boldsymbol{p})$$

Spin & charge densities

$$\delta\rho_{\boldsymbol{p}} = \sigma^0_{\alpha\beta}\delta n^{\alpha\beta}_{\boldsymbol{p}} = \delta_{\alpha\beta}\delta n^{\alpha\beta}_{\boldsymbol{p}}$$

$$\delta s^i_{\boldsymbol{p}} = \frac{1}{2}\sigma^i_{\alpha\beta}\delta n^{\alpha\beta}_{\boldsymbol{p}}$$

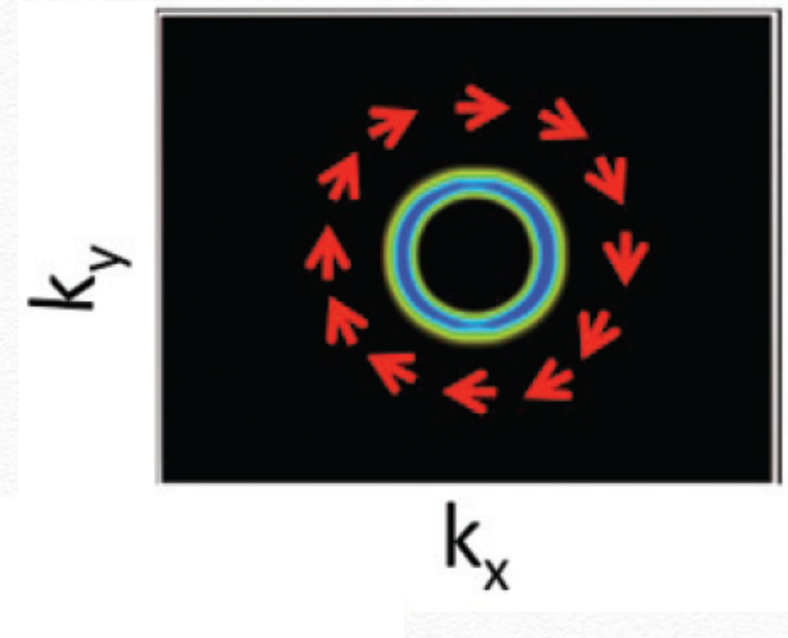
Spin-orbit rotation symmetry

- L_z and S_z not good quantum numbers, only $J_z = L_z + S_z$ is

$$h(\boldsymbol{p}) = v_F \hat{\boldsymbol{z}} \cdot (\boldsymbol{\sigma} \times \boldsymbol{p})$$

$$[J_z, h(\boldsymbol{p})] = 0$$

$$J_z = -i\frac{\partial}{\partial \theta_{\boldsymbol{p}}} + \frac{1}{2}\sigma^z$$



- Determine most general interaction invariant under J_z rotations and TRS

Charge-charge interactions

$$\delta V_{cc} = \frac{1}{2} \sum_{l=0}^{\infty} \int đp\, đp'\, f_l^{cc} \cos l\theta_{\boldsymbol{pp'}} \delta\rho_{\boldsymbol{p}} \delta\rho_{\boldsymbol{p'}}$$

- Identical to spinless 2D FL theory

Spin-spin interactions

$$\delta V_{ss} = \tfrac{1}{2} \sum_{l=0}^{\infty} \int đp\, đp' \Big\{ \cos l\theta_{\boldsymbol{pp'}}$$

$$\Big(f_l^{ss,1} (\delta s^x_{\boldsymbol{p}} \delta s^x_{\boldsymbol{p'}} + \delta s^y_{\boldsymbol{p}} \delta s^y_{\boldsymbol{p'}}) + f_l^{ss,2} \delta s^z_{\boldsymbol{p}} \delta s^z_{\boldsymbol{p'}} \Big)$$

$$+ f_l^{ss,3} \sin l\theta_{\boldsymbol{pp'}} \delta \boldsymbol{s_p} \times \delta \boldsymbol{s_{p'}}$$

$$+ \cos l\theta_{\boldsymbol{pp'}}$$

$$\Big(f_l^{ss,4} \left[(\hat{\mathbf{p}} \cdot \delta \boldsymbol{s_p}) (\hat{\mathbf{p}}' \times \delta \boldsymbol{s_{p'}}) + (\hat{\mathbf{p}} \times \delta \boldsymbol{s_p}) (\hat{\mathbf{p}}' \cdot \delta \boldsymbol{s_{p'}}) \right]$$

$$+ f_l^{ss,5} \left[(\hat{\mathbf{p}} \cdot \delta \boldsymbol{s_p}) (\hat{\mathbf{p}}' \cdot \delta \boldsymbol{s_{p'}}) - (\hat{\mathbf{p}} \times \delta \boldsymbol{s_p}) (\hat{\mathbf{p}}' \times \delta \boldsymbol{s_{p'}}) \right] \Big) \Big\}$$

Spin-spin interactions

$$\delta V_{ss} = \frac{1}{2} \sum_{l=0}^{\infty} \int đp\, đp' \Big\{ \cos l\theta_{\boldsymbol{p}\boldsymbol{p}'}$$

XXZ

$$\Big(f_l^{ss,1} (\delta s^x_{\mathbf{p}} \delta s^x_{\mathbf{p}'} + \delta s^y_{\mathbf{p}} \delta s^y_{\mathbf{p}'}) + f_l^{ss,2} \delta s^z_{\mathbf{p}} \delta s^z_{\mathbf{p}'} \Big)$$

$$+ f_l^{ss,3} \sin l\theta_{\boldsymbol{p}\boldsymbol{p}'} \delta \boldsymbol{s}_{\mathbf{p}} \times \delta \boldsymbol{s}_{\mathbf{p}'}$$

$$+ \cos l\theta_{\boldsymbol{p}\boldsymbol{p}'}$$

$$\Big(f_l^{ss,4} \left[(\hat{\mathbf{p}} \cdot \delta \boldsymbol{s}_{\mathbf{p}}) (\hat{\mathbf{p}}' \times \delta \boldsymbol{s}_{\mathbf{p}'}) + (\hat{\mathbf{p}} \times \delta \boldsymbol{s}_{\mathbf{p}}) (\hat{\mathbf{p}}' \cdot \delta \boldsymbol{s}_{\mathbf{p}'}) \right]$$

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XXZ

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DM

$$+ \cos l\theta_{\boldsymbol{p}\boldsymbol{p}'}$$

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Spin-spin interactions

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XXZ

$$\left(f_l^{ss,1}(\delta s^x_{\mathbf{p}}\delta s^x_{\mathbf{p}'} + \delta s^y_{\mathbf{p}}\delta s^y_{\mathbf{p}'}) + f_l^{ss,2}\delta s^z_{\mathbf{p}}\delta s^z_{\mathbf{p}'} \right)$$

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"compass model"

Spin-charge interactions

$$\delta V_{sc} = \sum_{l=0}^{\infty} \int đp\, đp' \times \Big[(f_l^{sc,1} \cos l\theta_{\boldsymbol{pp}'} + f_l^{sc,2} \sin l\theta_{\boldsymbol{pp}'}) \delta\rho_{\boldsymbol{p}} \hat{\mathbf{p}}' \cdot \delta \boldsymbol{s}_{\mathbf{p}'} + (f_l^{sc,3} \cos l\theta_{\boldsymbol{pp}'} + f_l^{sc,4} \sin l\theta_{\boldsymbol{pp}'}) \delta\rho_{\boldsymbol{p}} \hat{\mathbf{p}}' \times \delta \boldsymbol{s}_{\mathbf{p}'} \Big]$$

- New feature of SO coupled systems: direct spin-charge interactions

Landau parameters

- 10 Landau parameters (per angular momentum):

$$f_l^{cc}$$

$$f_l^{ss,1}, \ldots, f_l^{ss,5}$$

$$f_l^{sc,1}, \ldots, f_l^{sc,4}$$

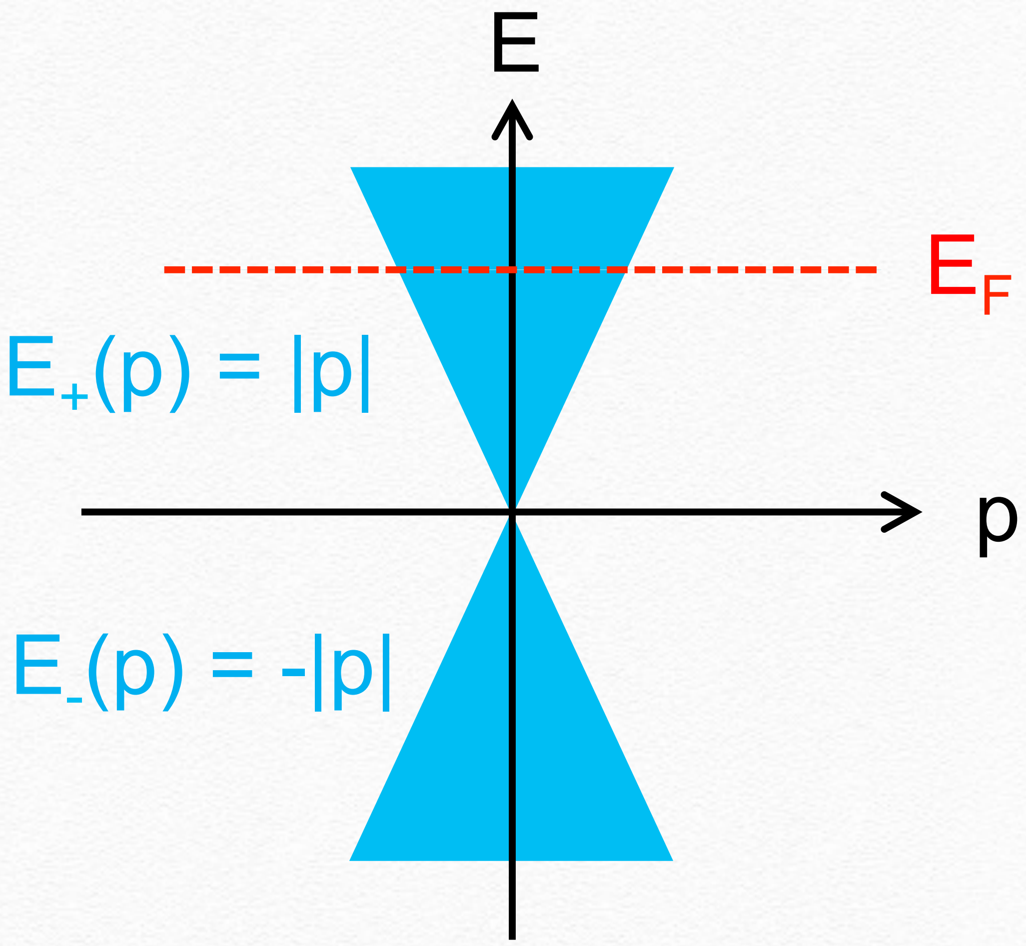
- Compared to 2 for standard FL theory

Projected Fermi liquid theory

- FL theory: only keep d.o.f. near FS

$$c_{\boldsymbol{p}\uparrow} = \frac{ie^{-i\theta_{\boldsymbol{p}}}}{\sqrt{2}}(\psi_{\boldsymbol{p}+} + \psi_{\boldsymbol{p}-})$$

$$c_{\boldsymbol{p}\downarrow} = \frac{1}{\sqrt{2}}(\psi_{\boldsymbol{p}+} - \psi_{\boldsymbol{p}-})$$



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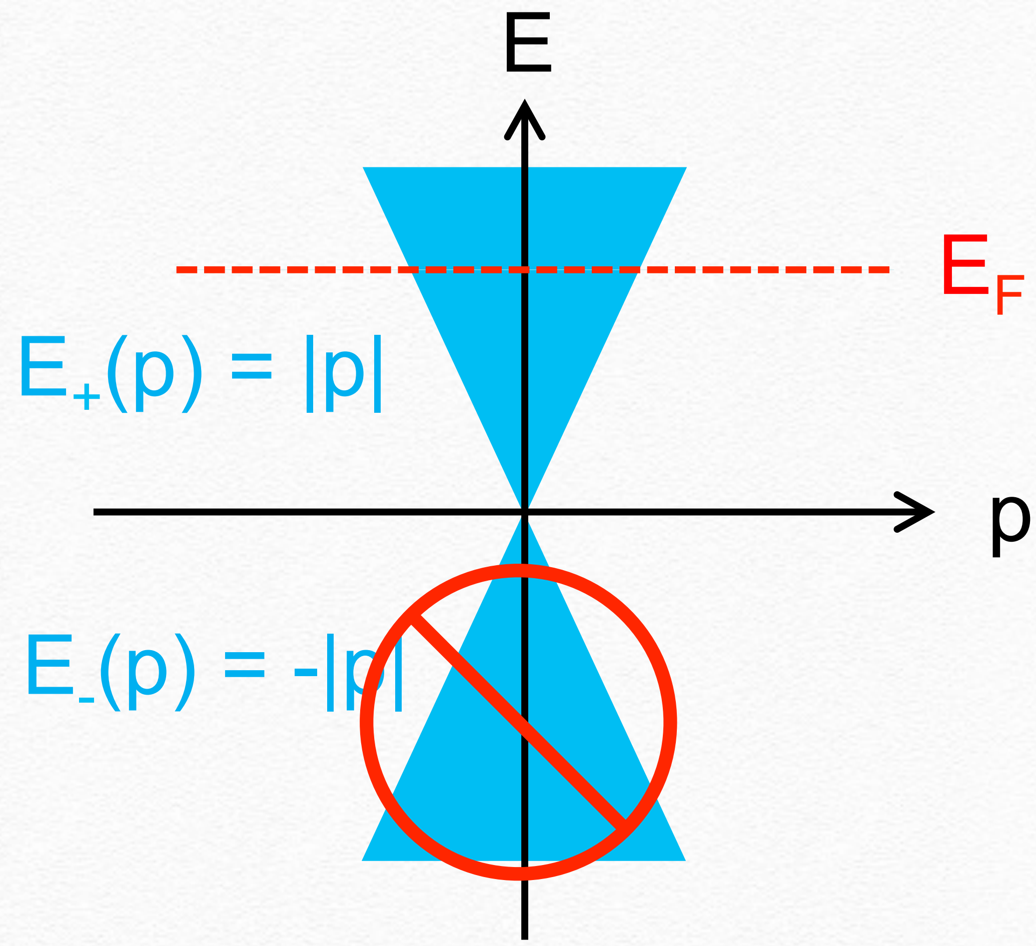


Projected Fermi liquid theory

- Projection to "+" helicity band = effectively spinless theory

$$\delta \bar{E}[\delta \bar{n}_{\boldsymbol{p}}] = \int đp\, \epsilon^0_{\boldsymbol{p}} \delta \bar{n}_{\boldsymbol{p}} + \frac{1}{2} \sum_{l=0}^{\infty} \int đp\, đp'\, \bar{f}_l \cos l\theta_{\boldsymbol{p}\boldsymbol{p}'} \delta \bar{n}_{\boldsymbol{p}} \delta \bar{n}_{\boldsymbol{p}'}$$

$$\epsilon^0_{\boldsymbol{p}} = v_F |\boldsymbol{p}|$$

$$\delta \bar{n}_{\boldsymbol{p}} = \bar{n}_{\boldsymbol{p}} - \bar{n}^{(0)}_{\boldsymbol{p}}$$

$$\bar{n}_{\boldsymbol{p}} \equiv \langle \psi^\dagger_{\boldsymbol{p}+} \psi_{\boldsymbol{p}+} \rangle$$

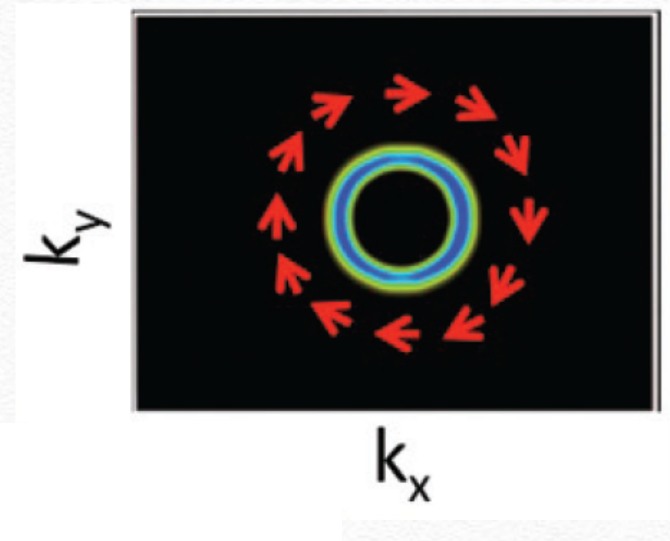
Projected Fermi liquid theory

- Physical spin & charge densities

$$\delta\rho_{\boldsymbol{p}} = \delta\bar{n}_{\boldsymbol{p}},$$

$$\delta s^i_{\boldsymbol{p}} = \frac{1}{2}\epsilon_{ij}\hat{p}_j\delta\bar{n}_{\boldsymbol{p}},\ i = x, y,$$

$$\delta s^z_{\boldsymbol{p}} = 0$$



Projected Fermi liquid theory

- Projected Landau parameters

$$\bar{f}_l = f_l^{cc} - f_l^{sc,3} - \frac{1}{4} f_l^{ss,5} + \frac{1}{8}\left(f_{l-1}^{ss,1} - f_{l-1}^{ss,3} + f_{l+1}^{ss,1} + f_{l+1}^{ss,3}\right)$$

- Projection to helical FS can effectively raise/lower angular momentum of the interaction (cf. Fu, Kane, PRL 2008)

$$\bar{f}_1 = f_1^{cc} - f_1^{sc,3} - \frac{1}{4} f_1^{ss,5} + \frac{1}{8}\left(f_0^{ss,1} - f_0^{ss,3} + f_2^{ss,1} + f_2^{ss,3}\right)$$

p-wave

s-wave

Physical properties

$$\frac{v_F^0}{v_F} = 1 + \bar{F}_1$$

but no Galilean invariance!

$$\frac{\gamma}{\gamma_0} = \left(\frac{v_F^0}{v_F}\right)^2$$

$$\frac{\kappa}{\kappa_0} = \left(\frac{v_F^0}{v_F}\right)^2 \frac{1}{1 + \bar{F}_0}$$

Spin susceptibility

- Not purely a FS property in the helical FL!
- Free Dirac surface state:

$$\chi_{xx} = \tfrac{1}{8} g^2 \mu_B^2 \rho(\Lambda), \qquad \chi_{zz} = \tfrac{1}{4} g^2 \mu_B^2 [\rho(\Lambda) - \rho(\epsilon_F)]$$

- Projected FL theory:

$$\chi_{xx} = \tfrac{1}{8} g^2 \mu_B^2 \rho(\epsilon_F) \frac{1}{1 + \bar{F}_1}, \qquad \chi_{zz} = 0$$

- Agree in limit of large Fermi energy $E_F \rightarrow \Lambda$

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- Projected FL theory:

$$\chi_{xx} = \tfrac{1}{8} g^2 \mu_B^2 \rho(\epsilon_F) \frac{1}{1 + \bar{F}_1}, \qquad \chi_{zz} = 0$$

- Agree in limit of large Fermi energy $E_F \to \Lambda$

Pomeranchuk instabilities

- Instabilities towards spontaneous distortions of the FS (Pomeranchuk, JETP 1958)

$$p_F(\theta) - p_F = \sum_{l=-\infty}^{\infty} A_l e^{il\theta}$$

$$\delta \bar{E}[\delta \bar{n}_{\boldsymbol{p}}] = \frac{\epsilon_F}{2\pi\hbar^2} \sum_{l=0}^{\infty} (1 + \bar{F}_l)|A_l|^2$$

- Stability of FS requires $\bar{F}_l > -1$

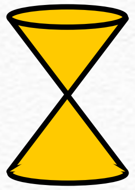
Pomeranchuk instabilities

- l=0: phase separation

$$\frac{\kappa}{\kappa_0} = \left(\frac{v_F^0}{v_F}\right)^2 \frac{1}{1+\bar{F}_0}$$

- l=1: in-plane magnetic order (Xu, PRB 2010)

$$\chi_{xx} = \tfrac{1}{8} g^2 \mu_B^2 \rho(\epsilon_F) \frac{1}{1+\bar{F}_1}$$



Pomeranchuk instabilities

- l=0: phase separation

$$\frac{\kappa}{\kappa_0} = \left(\frac{v_F^0}{v_F}\right)^2 \frac{1}{1+\bar{F}_0}$$

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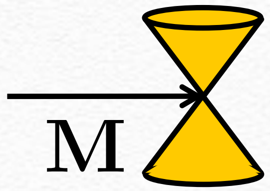


Pomeranchuk instabilities

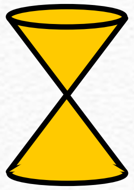
- l=2: nematic instability

$$\cos 2\theta_{\boldsymbol{p}\boldsymbol{p}'}\delta\bar{n}_{\boldsymbol{p}}\delta\bar{n}_{\boldsymbol{p}'} = \frac{1}{2}\,\mathrm{Tr}\,\bar{Q}(\boldsymbol{p})\bar{Q}(\boldsymbol{p}')$$

$$\bar{Q}_{ij}(\boldsymbol{p}) = (2\hat{p}_i\hat{p}_j - \delta_{ij})\delta\bar{n}_{\boldsymbol{p}}$$

- Unprojected theory: quadrupolar "spin-orbital" order parameter (Park, Chung, JM, PRB 2015; Fu, PRL 2015)

$$Q_{ij}(\boldsymbol{p}) = \hat{p}_i\delta s^j_{\boldsymbol{p}} + \hat{p}_j\delta s^i_{\boldsymbol{p}} - \delta_{ij}\hat{\boldsymbol{p}}\cdot\delta\boldsymbol{s}_{\boldsymbol{p}}$$



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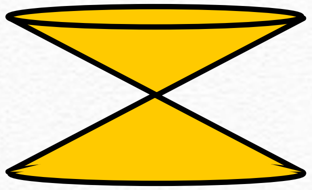
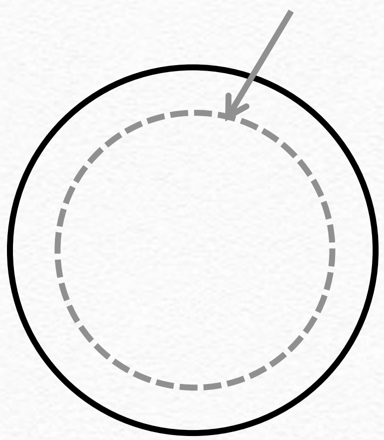


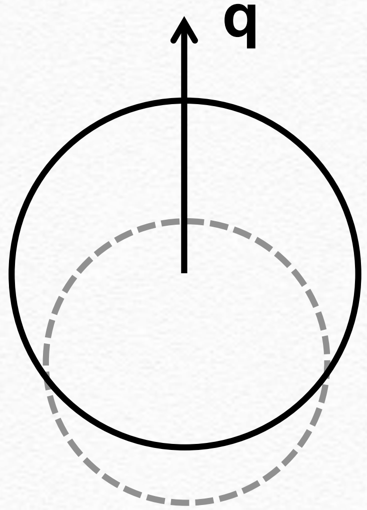
Collective modes

- Hydrodynamic regime $\omega\tau << 1$: first sound, $\omega(q) = c_1 q$



$$c_1 = v_F \sqrt{\tfrac{1}{2}(1+\bar{F}_0)(1+\bar{F}_1)}$$

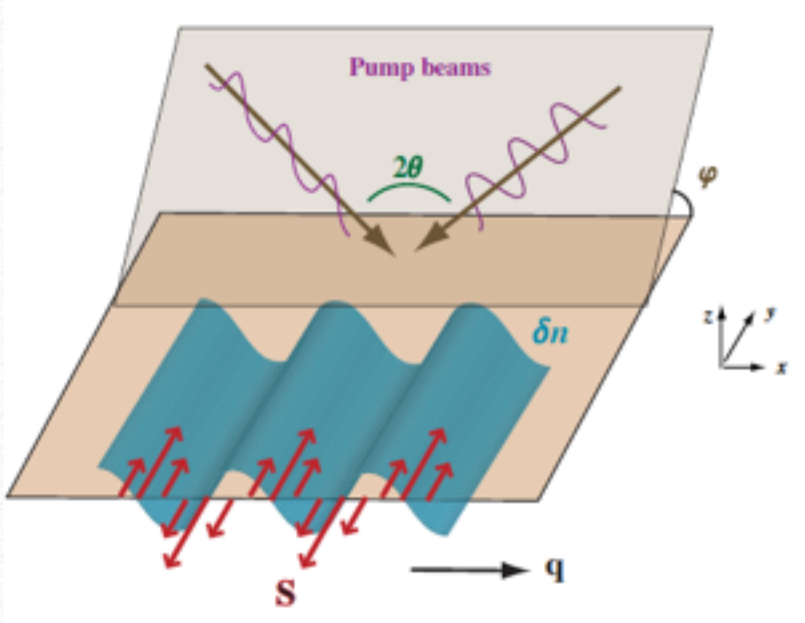
- Collisionless regime $\omega\tau >> 1$: zero sound, $\omega(q) = c_0 q$



$$c_0 \approx v_F \sqrt{\tfrac{1}{2}\bar{F}_0}, \qquad \bar{F}_0 \to \infty,$$
$$c_0 \approx v_F \left(1 + \tfrac{1}{2}\bar{F}_0^2\right), \qquad \bar{F}_0 \to 0.$$

Measuring projected Landau parameters?

- Not obvious: comparing ARPES Fermi velocities with "noninteracting" DFT calculations involves double-counting
- F_0: heat capacity + compressibility measurements
- F_1: transient spin grating experiment



$$\frac{s_q^T}{n_q} = \frac{1}{1+\bar{F}_1}\frac{c_s}{v_F}$$

Raghu et al., PRL 2010

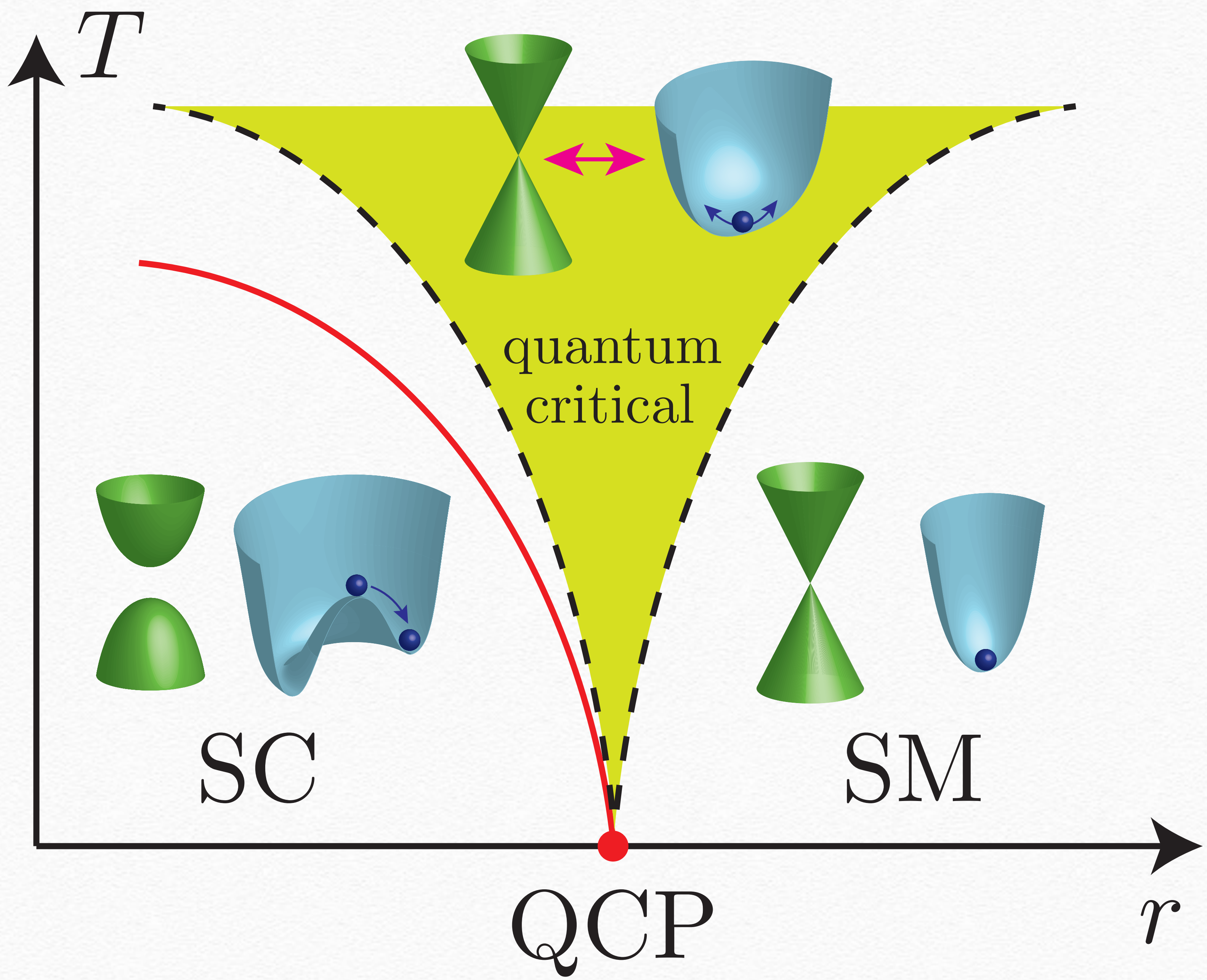
- Strong correlations: Universal conductivity at semimetal-superconductor QCP

W. Witczak-Krempa and JM, arXiv:1510.XXXXX

SC instability of TI surface state

- FL theory: instabilities in particle-hole channel
- Consider pairing instability of Dirac surface state at $\mu = 0$
- Vanishing DOS: finite threshold attraction strength -> QCP

SC instability of TI surface state



SUSY QCP

- QCP has emergent N=2 SUSY! (Grover, Sheng, Vishwanath, Science 2014; Ponte, Lee, NJP 2014)

- Strongly coupled (2+1)D CFT: N=2 Wess-Zumino model

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial_\mu\psi + \frac{1}{2}|\partial_\mu\phi|^2 + \frac{r}{2}|\phi|^2 + \frac{\lambda}{4!}|\phi|^4 + h(\phi^*\psi^T i\gamma_2\psi + \text{c.c.})$$

- Finite $h^2 \propto \lambda$ at the QCP: universality class neither Gaussian nor 3D XY

SUSY QCP

- SUSY fixes exact anomalous dimensions of ψ, ϕ

$$\eta_\phi = \eta_\psi = \frac{1}{3}$$

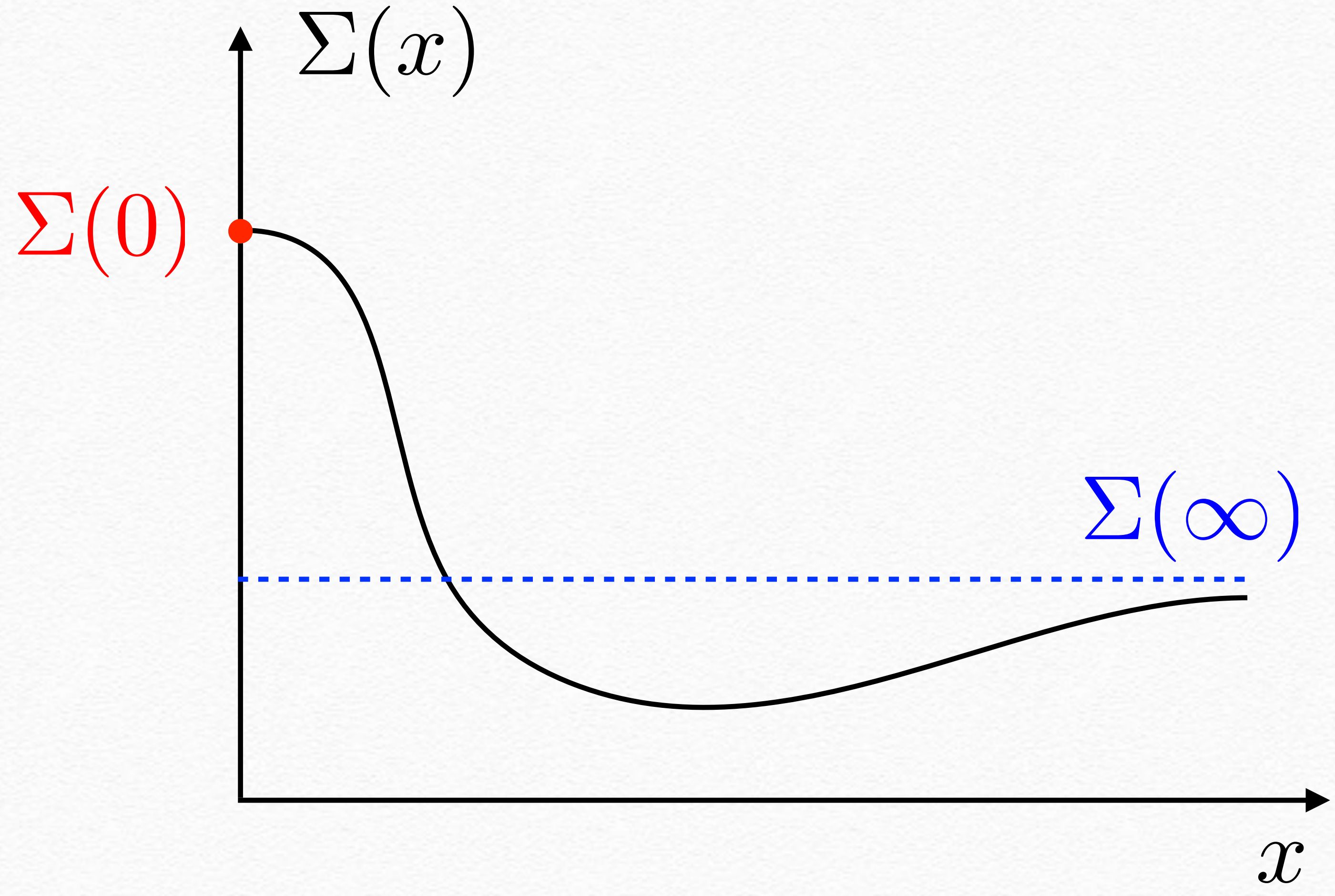
- Correlation length exponent not fixed by SUSY

$$\nu = \frac{1}{2} + \frac{\epsilon}{4} + \mathcal{O}(\epsilon^2)$$

- Can SUSY tell us anything else?

Universal conductivity

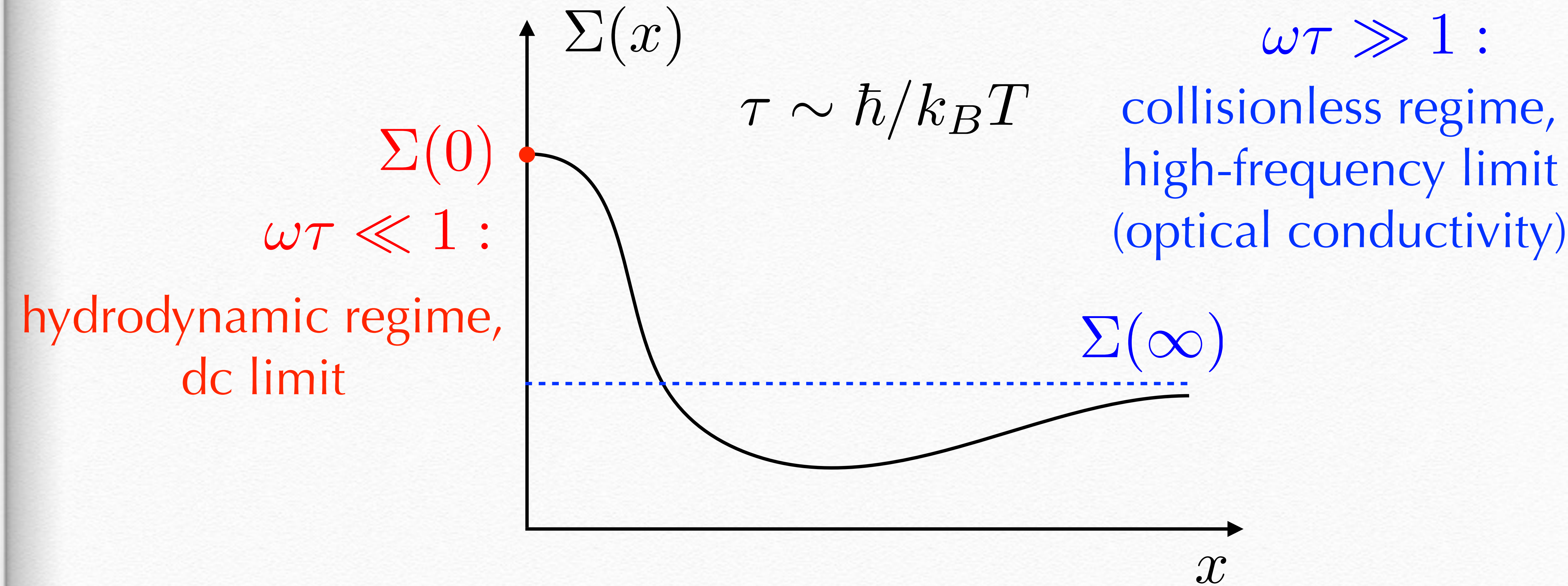
$$\sigma(\omega) = \frac{e^2}{\hbar}\left(\frac{k_B T}{\hbar c}\right)^{(d-2)/z} \Sigma\left(\frac{\hbar\omega}{k_B T}\right)$$



Damle & Sachdev, PRB 1997

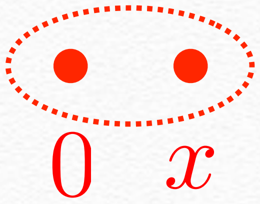
Universal conductivity: (2+1)D

$$\sigma(\omega) = \frac{e^2}{\hbar} \Sigma \left(\frac{\hbar \omega}{k_B T} \right)$$



Damle & Sachdev, PRB 1997

OPE and high-frequency conductivity



$$J_\mu(x) J_\nu(0) \sim \sum_a \frac{C_{\mu\nu a} \mathcal{O}_a(0)}{|x|^{4-\Delta_a}}$$

$$\frac{\sigma(i\omega_n)}{e^2/\hbar} \sim \frac{\langle J_x(\omega_n) J_x(-\omega_n)\rangle_T}{\omega_n} \sim \sum_a C_{xxa} \frac{\langle \mathcal{O}_a \rangle_T}{|\omega_n|^{\Delta_a}}$$

$$\omega_n \gg T$$

Katz et al., PRB 2014

OPE and high-frequency conductivity

$$\frac{\sigma(i\omega_n)}{e^2/\hbar} \sim \sum_a C_{xxa} \frac{\langle \mathcal{O}_a \rangle_T}{|\omega_n|^{\Delta_a}}$$

$$\langle \mathcal{O}_a \rangle_T = c_a T^{\Delta_a}$$

$$\frac{\sigma(\omega)}{e^2/\hbar} = \sum_a b_a \left(\frac{iT}{\omega} \right)^{\Delta_a}$$

Katz et al., PRB 2014

OPE and high-frequency conductivity

$$\frac{\sigma(i\omega_n)}{e^2/\hbar} \sim \sum_a C_{xxa} \frac{\langle \mathcal{O}_a \rangle_T}{|\omega_n|^{\Delta_a}}$$

$$\langle \mathcal{O}_a \rangle_T = c_a T^{\Delta_a}$$

$$\frac{\sigma(\omega)}{e^2/\hbar} = \sigma_\infty + b_{|\phi|^2} \left(\frac{iT}{\omega}\right)^{3-1/\nu} + b_T \left(\frac{iT}{\omega}\right)^3 + \ldots$$

σ_∞ ↑ identity

$b_{|\phi|^2}$ term ↑ $|\phi|^2$

b_T term ↑ stress tensor

Katz et al., PRB 2014

T=0 conductivity

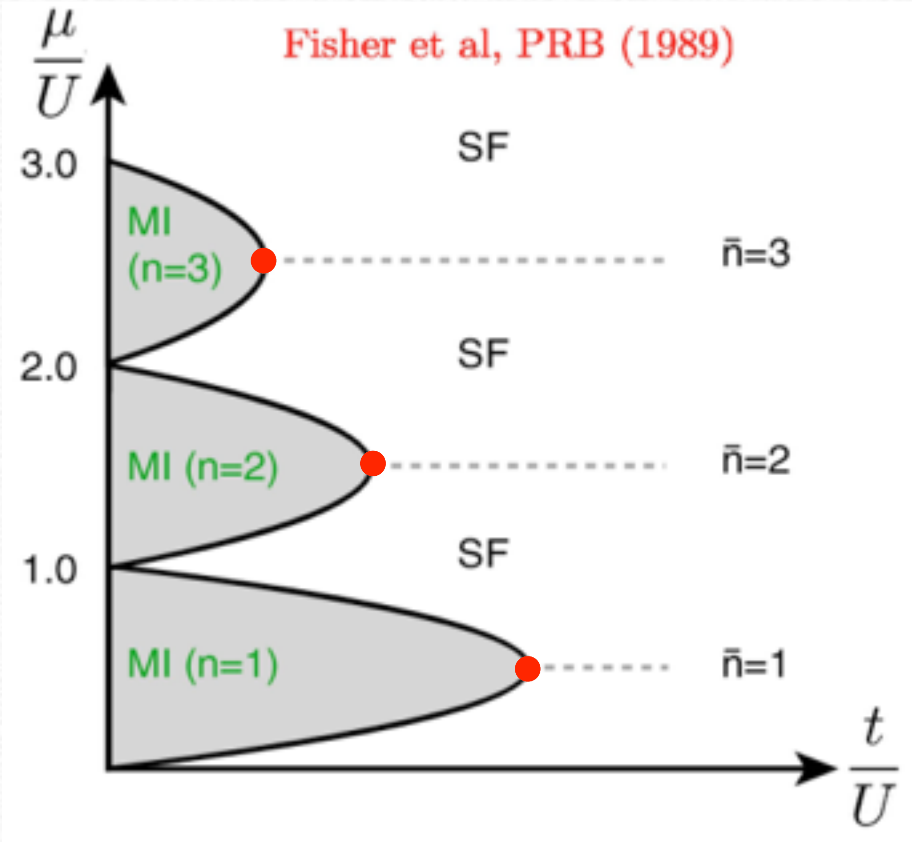
$$\frac{\sigma(\omega)}{e^2/\hbar} = \sigma_\infty + b_{|\phi|^2}\left(\frac{iT}{\omega}\right)^{3-1/\nu} + b_T\left(\frac{iT}{\omega}\right)^3 + \ldots$$

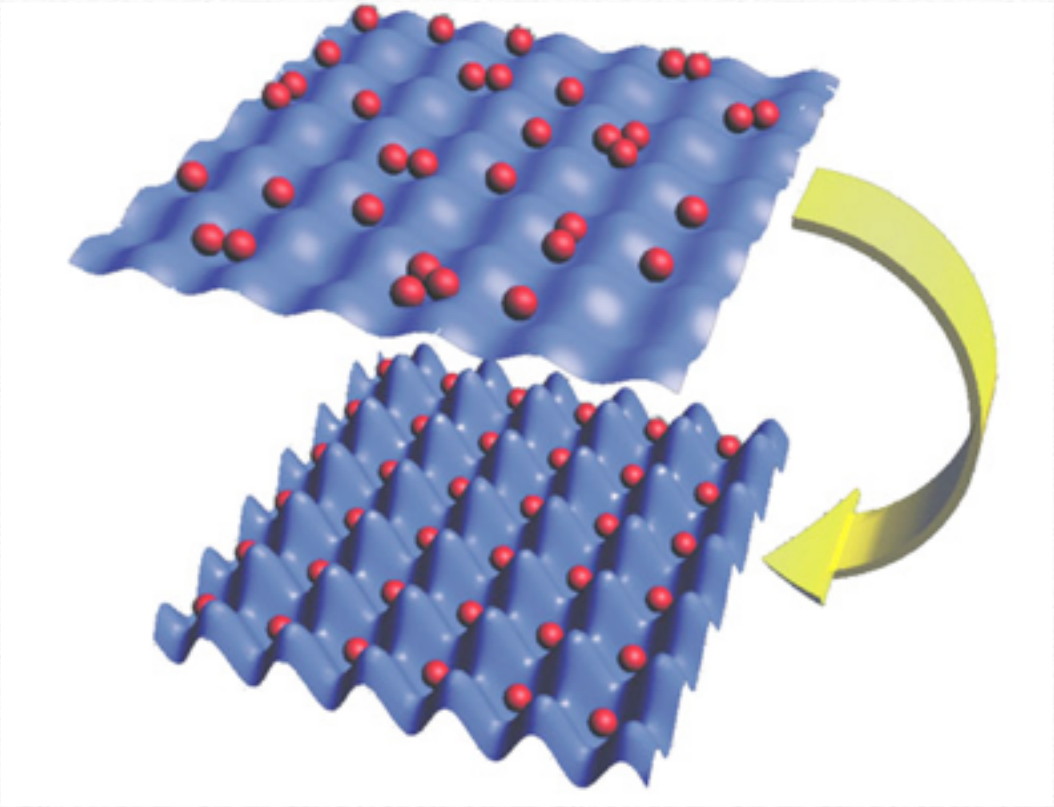
- Ground-state JJ correlation function, constrained by conformal symmetry (Osborn & Petkou, Ann. Phys. 1994)

$$\langle J_\mu(x) J_\nu(0)\rangle = C_J \frac{I_{\mu\nu}(x)}{|x|^4}$$

$$\sigma_\infty = \frac{\pi^2}{2} C_J$$

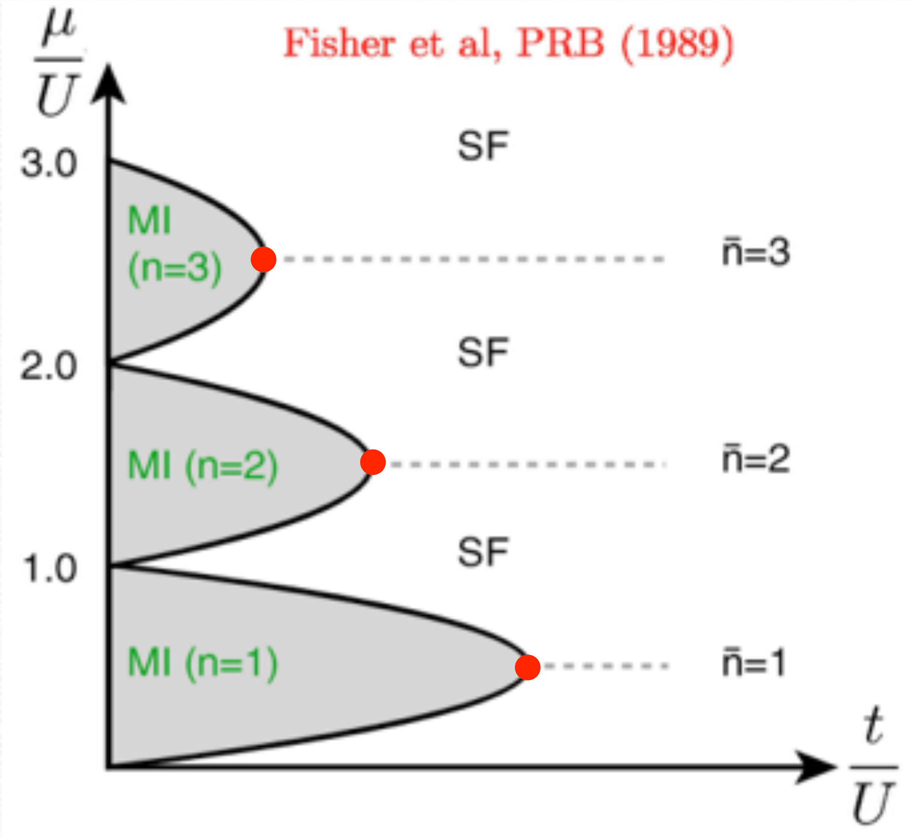
Boson superfluid-insulator QCP

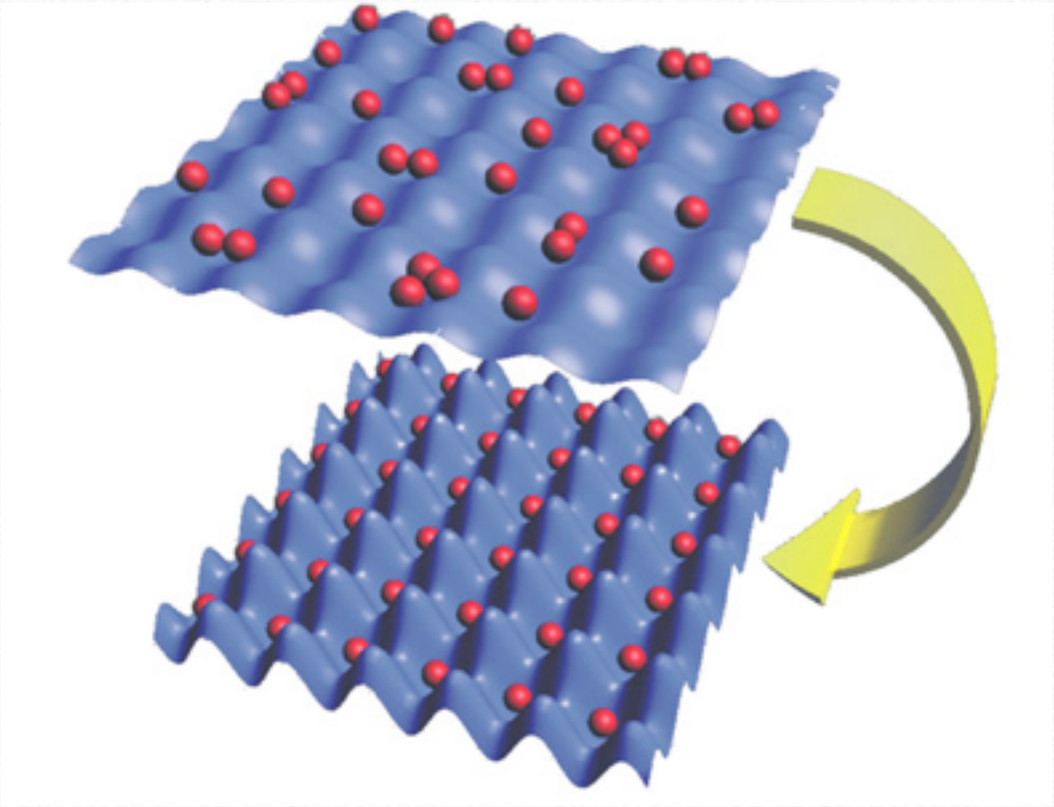




- Universal conductivity σ_{∞}: no exact result, long history (Fisher, Grinstein, Girvin, PRL 1990; Cha et al., PRB 1991; Fazio & Zappalà, PRB 1996; Šmakov & Sørensen, PRL 2005; ...)

Boson superfluid-insulator QCP





- QMC + holography + conformal bootstrap (Katz et al., PRB 2014; Gazit et al., PRB 2013, PRL 2014; Chen et al., PRL 2014; Witczak-Krempa et al., Nat. Phys. 2014; Kos et al., arXiv 2015)

$$\sigma_\infty \simeq 0.057$$

N=2 SCFTs in (2+1)D

- U(1) current and stress tensor are related by SUSY

$$\mathcal{J}_\mu = J_\mu - (\theta\gamma^\nu\bar{\theta})2T_{\nu\mu} + \ldots$$

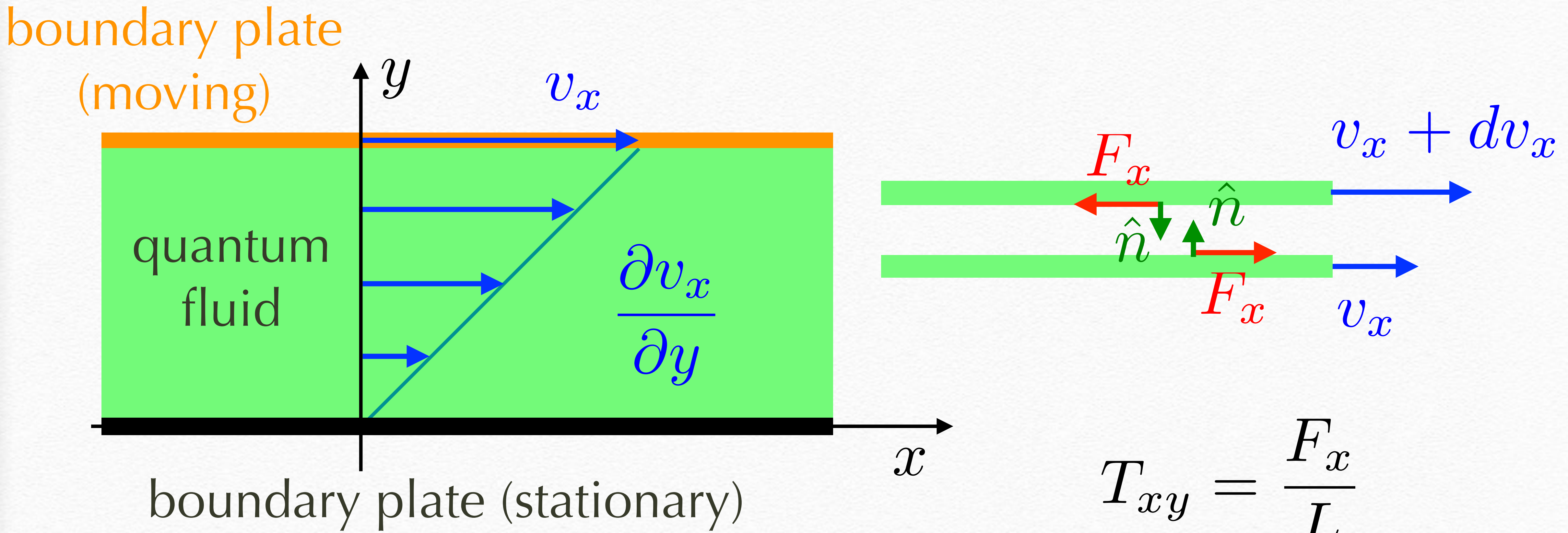
- <JJ> and <TT> are related by SUSY

$$\langle J_\mu(x) J_\nu(0)\rangle = C_J \frac{I_{\mu\nu}(x)}{|x|^4}$$

$$\langle T_{\mu\nu}(x) T_{\rho\sigma}(0)\rangle = C_T \frac{I_{\mu\nu,\rho\sigma}(x)}{|x|^6}$$

$$\frac{C_J}{C_T} = \frac{2}{3}$$

Shear viscosity

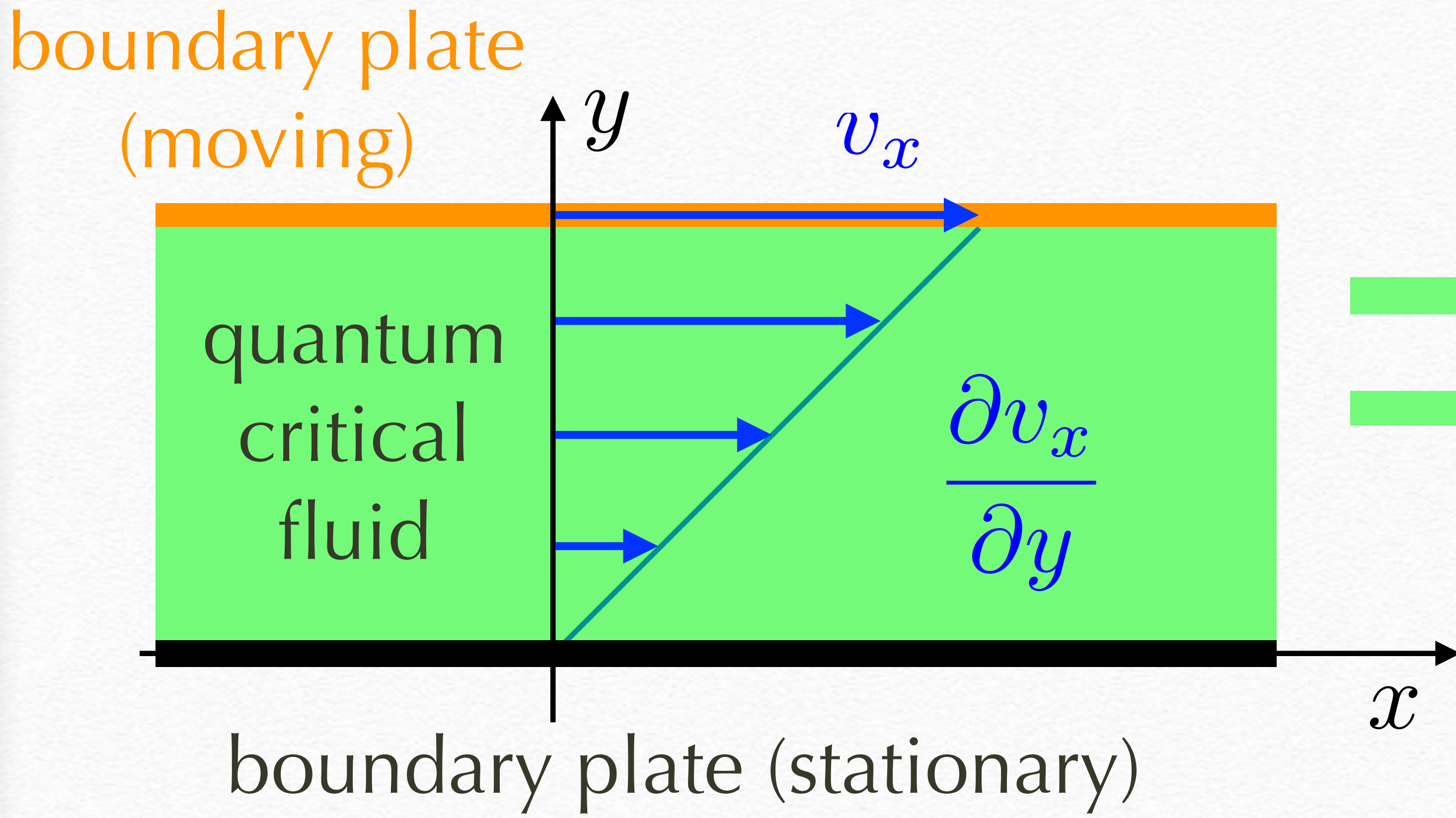


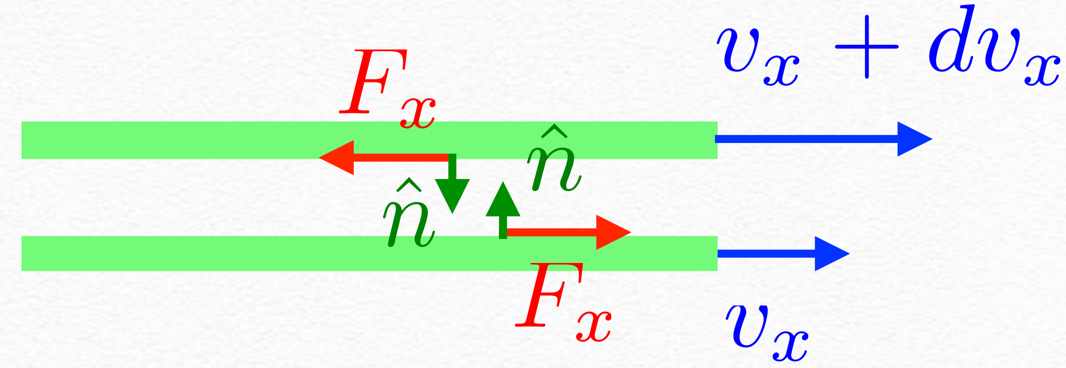
$$T_{xy} = \frac{F_x}{L}$$

shear stress

$$T_{xy} = \eta \frac{\partial v_x}{\partial y} = \eta \delta \dot{g}_{xy}$$

Shear viscosity





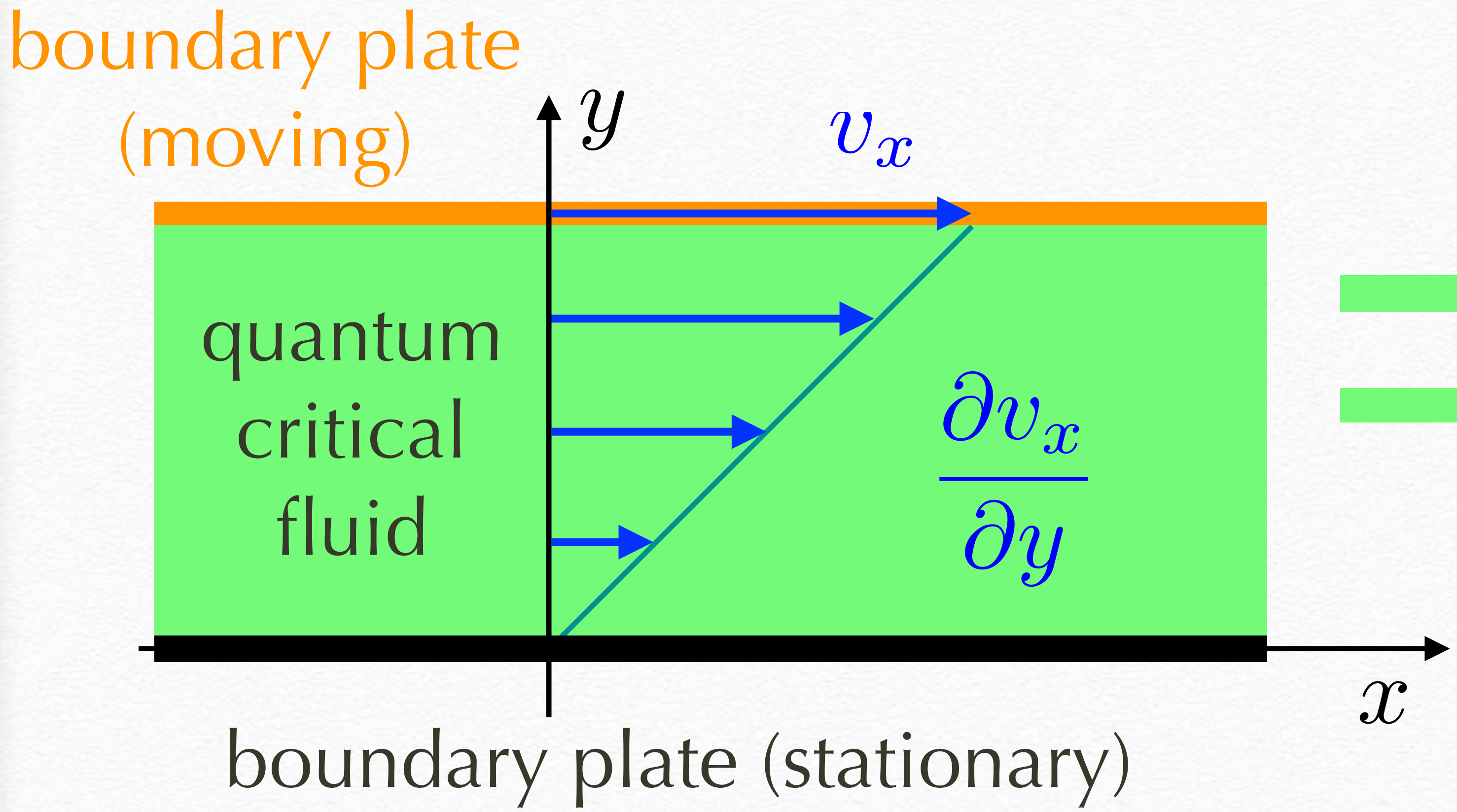
$$T_{xy} = \frac{F_x}{L}$$

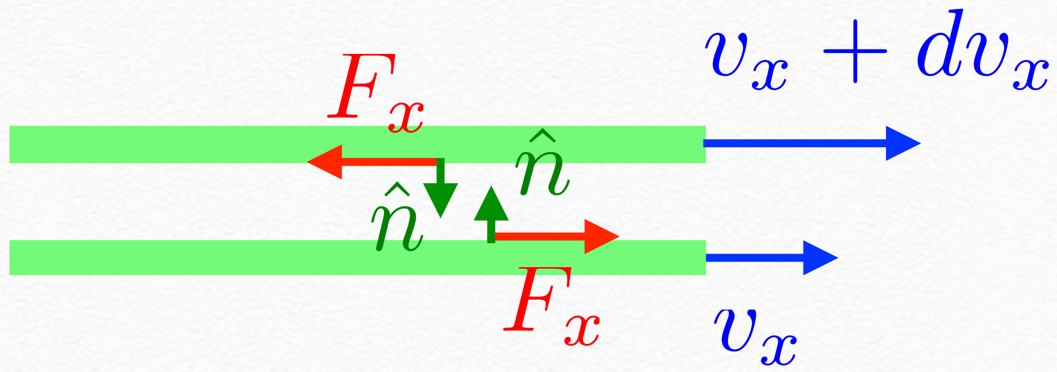
shear stress

$$\eta(i\omega_n) = \frac{1}{\omega_n}\langle T_{xy}(\omega_n) T_{xy}(-\omega_n)\rangle_T = \eta_\infty \omega_n^2 + \ldots$$

(dynamical) shear viscosity

Shear viscosity





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(dynamical) shear viscosity $\eta_\infty = \frac{\pi^2}{48} C_T$

Conductivity vs viscosity

$$\frac{\sigma_\infty}{\eta_\infty} = 16$$

- Exact, "superuniversal" ratio for all (2+1)D QCPs described by a N=2 SCFT

Exact universal conductivity

- C_T can be calculated exactly for the N=2 WZ model by localization on the squashed 3-sphere (Closset et al., JHEP 2013; Nishioka & Yonekura, JHEP 2013)

$$\sigma_\infty = \frac{2(16\pi - 9\sqrt{3})}{243\pi} \simeq 0.0908481$$

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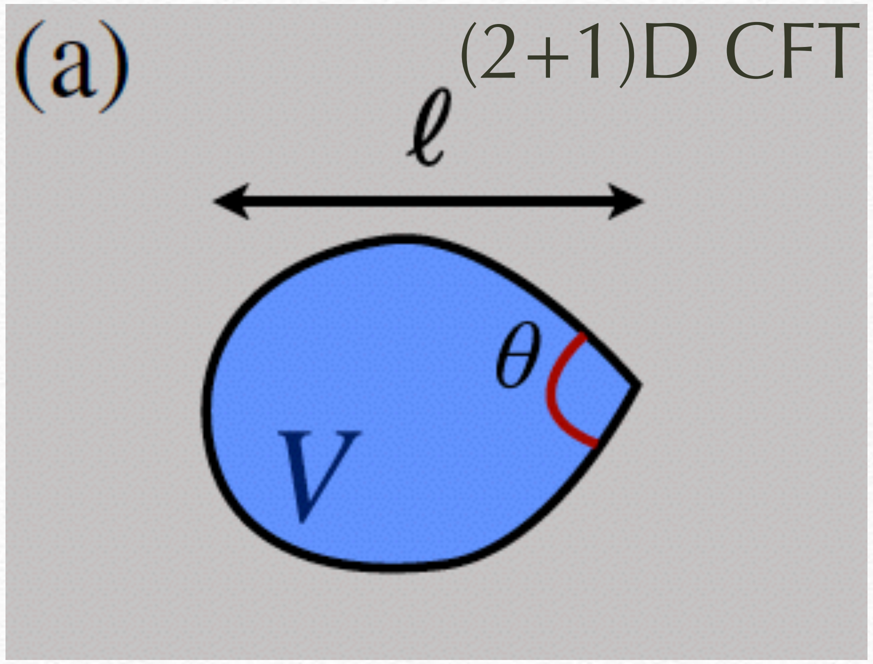
- Exact result for T=0 conductivity (and shear viscosity) of "realistic" strongly coupled quantum fluid in (2+1)D

Corner entanglement entropy



$$S = B\ell/\delta - a(\theta)\ln(\ell/\delta) + \ldots$$

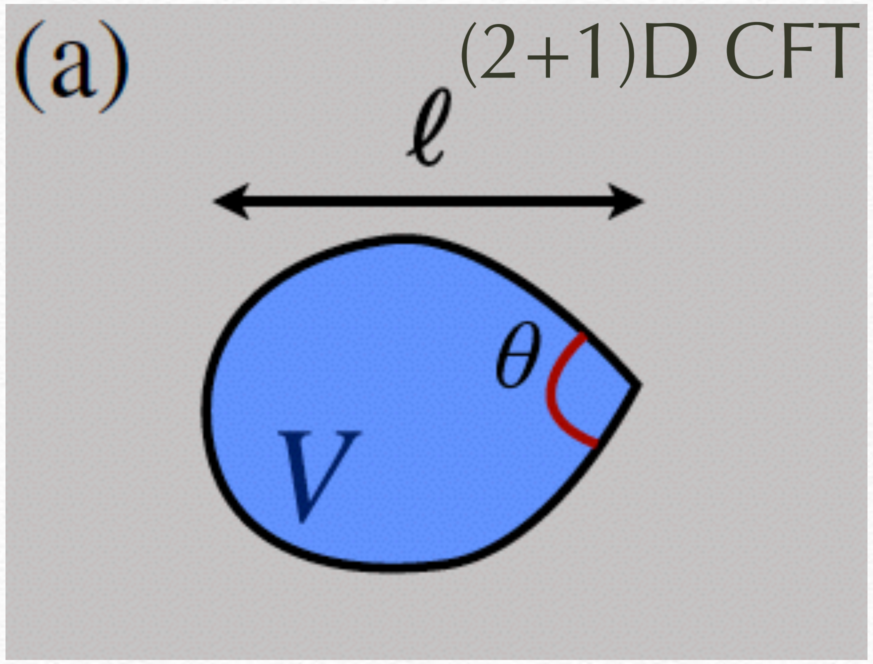
$$a(\theta) \simeq \lambda(\pi - \theta)^2$$

(Casini, Huerta, Leitao, NPB 2009)

- Conjecture (Bueno, Myers, Witczak-Krempa, PRL 2015):

$$\lambda = \pi^2 C_T/24$$

Corner entanglement entropy



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- Conjecture (Bueno, Myers, Witczak-Krempa, PRL 2015):

$$\lambda = \pi^2 C_T/24$$

- Exact result for SM-SC QCP:

$$\lambda = \frac{16\pi - 9\sqrt{3}}{972\pi} \simeq 0.011356$$

Finite temperature?

$$\frac{\sigma(\omega)}{e^2/\hbar} = \sigma_\infty + b_{|\phi|^2}\left(\frac{iT}{\omega}\right)^{3-1/\nu} + b_T\left(\frac{iT}{\omega}\right)^3 + \ldots$$

- Can't say much about $b_{|\phi|^2}$: probably nonzero

Finite temperature?

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- Combine conformal invariance + Ward identities (Osborn & Petkou, Ann. Phys. 1994), and SUSY (Buchbinder, Kuzenko, Samsonov, JHEP 2015):

$$b_T = 0$$

for all (2+1)D QCPs with N=2 SUSY!

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for all (2+1)D QCPs with N=2 SUSY!

- Exact result for finite-T, dynamical response of strongly coupled quantum fluid in (2+1)D

What about the real world?

What about the real world?

ARTICLE

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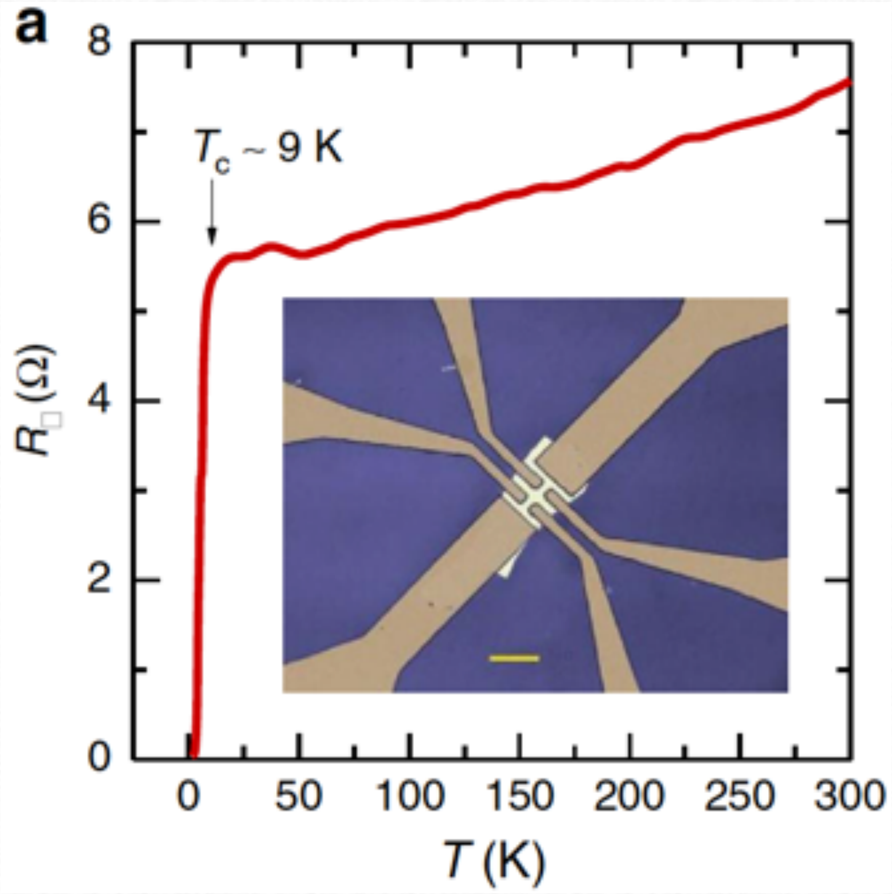
Emergent surface superconductivity in the topological insulator Sb_2Te_3

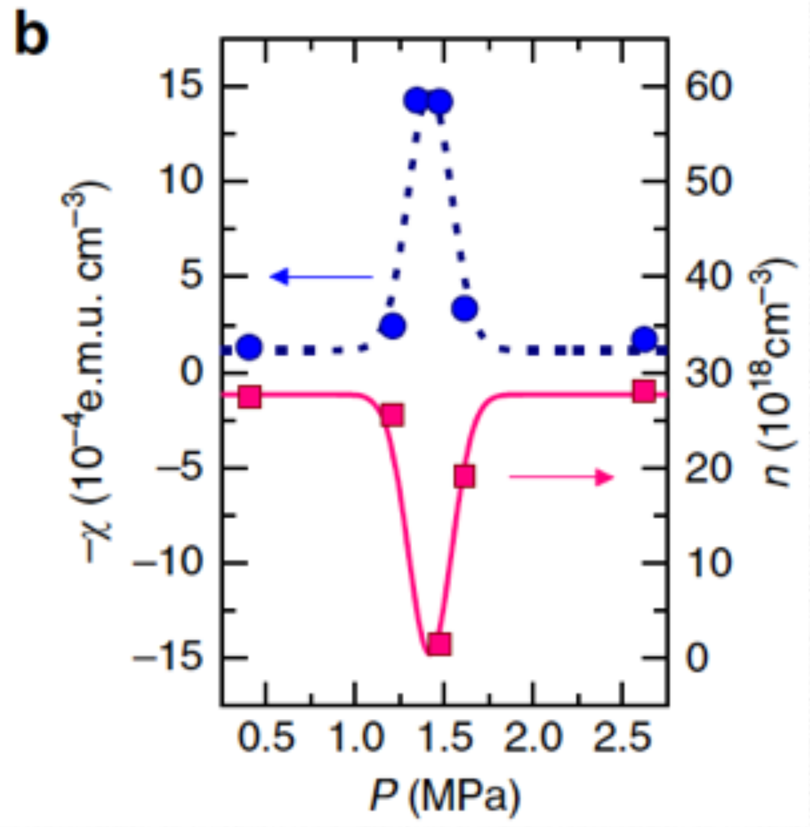
Lukas Zhao[1], Haiming Deng[1], Inna Korzhovska[1], Milan Begliarbekov[1], Zhiyi Chen[1], Erick Andrade[2], Ethan Rosenthal[2], Abhay Pasupathy[2], Vadim Oganesyan[3,4] & Lia Krusin-Elbaum[1,4]

Surface SC in Sb_2Te_3?

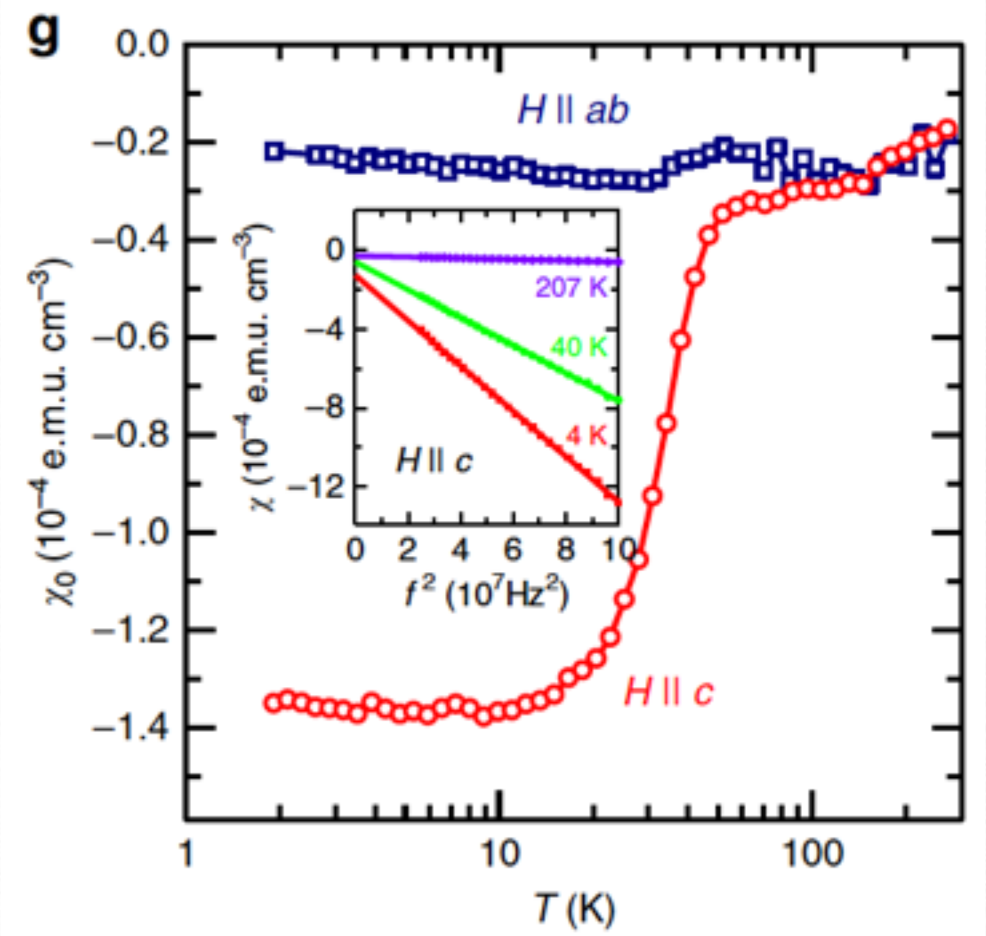


- Resistive transition at T_c = 8.6 K

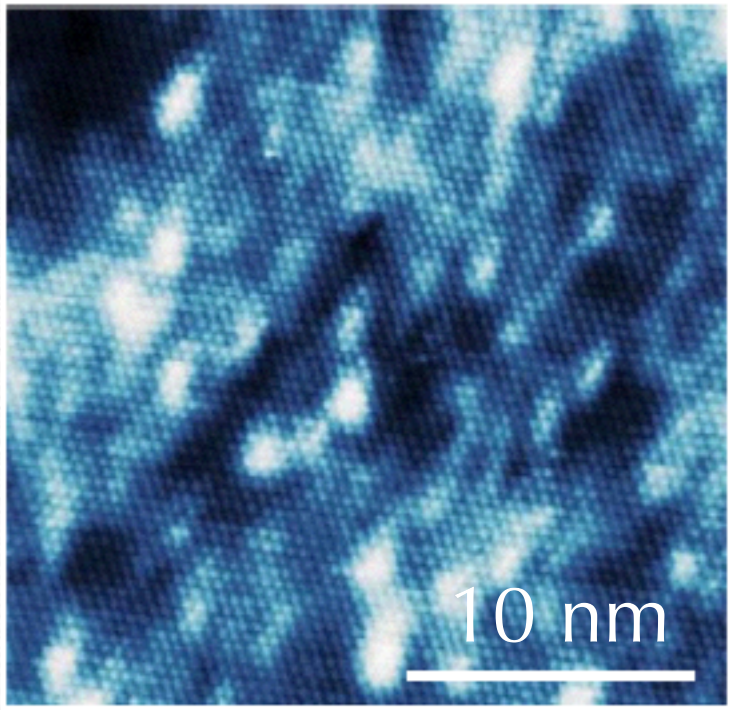
Surface SC in Sb_2Te_3?

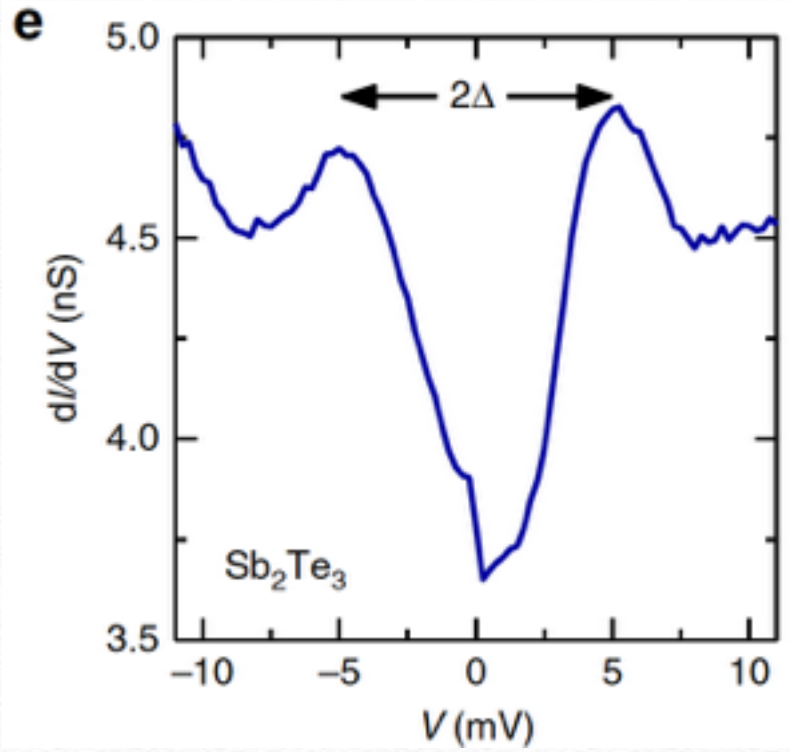


- Resistive transition at $T_c = 8.6$ K
- Anisotropic (2D) diamagnetic screening below T ~ 50 K (~2% of Meissner value)



Surface SC in Sb_2Te_3?





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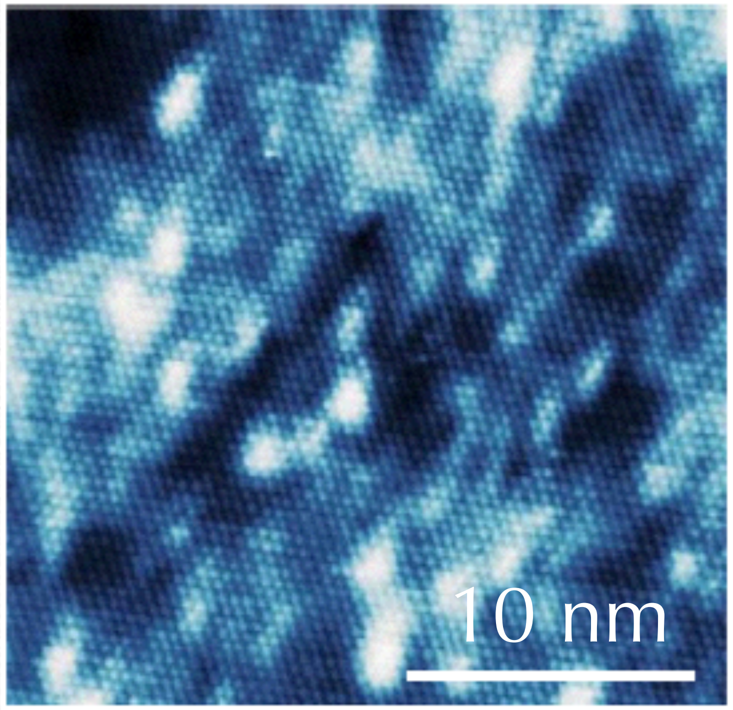
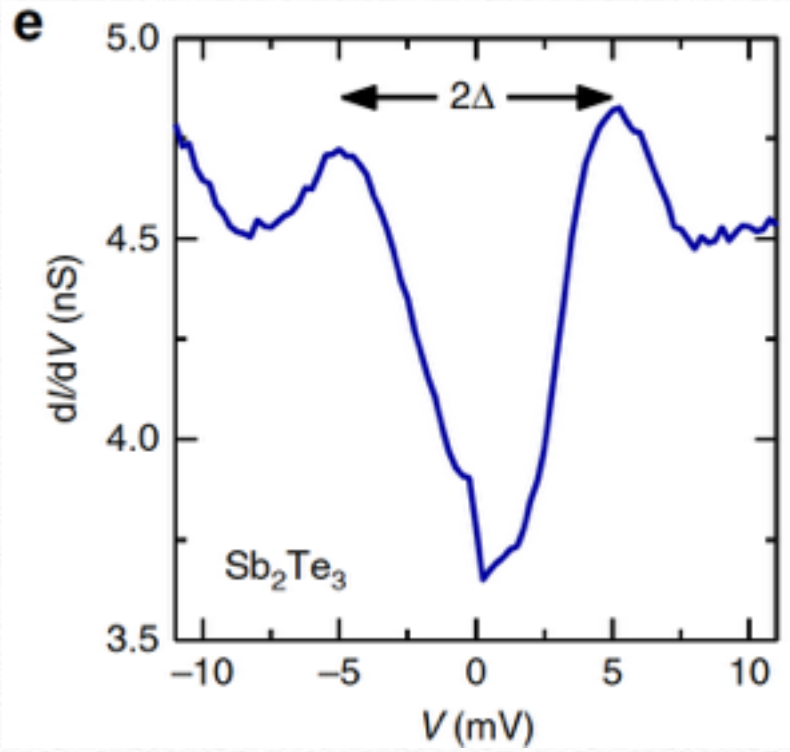
Surface SC in Sb_2Te_3?





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- Anisotropic (2D) diamagnetic screening below T ~ 50 K (~2% of Meissner value)
- Inhomogeneous STM maps, largest local "gap" (~ 20 meV) consistent with local BCS T_c ~ 60 K
- Inhomogeneous BCS pairing in local Dirac "puddles" at T ~ 50-60 K, onset of global phase coherence at T = 8.6 K? (Nandkishore, JM, Huse, Sondhi, PRB 2013)

Surface SC in Sb_2Te_3?

- Far from ideal system... but cleaner materials may lead to desired physics

Summary

- Weakly correlated surface state can be described in a materials-independent way by a effectively spinless, phenomenological "projected" Landau Fermi liquid theory

- SUSY allows us to calculate exactly dynamical response properties (e.g. optical conductivity) at zero and finite temperature for the strongly coupled SM-SC QCP in (2+1)D