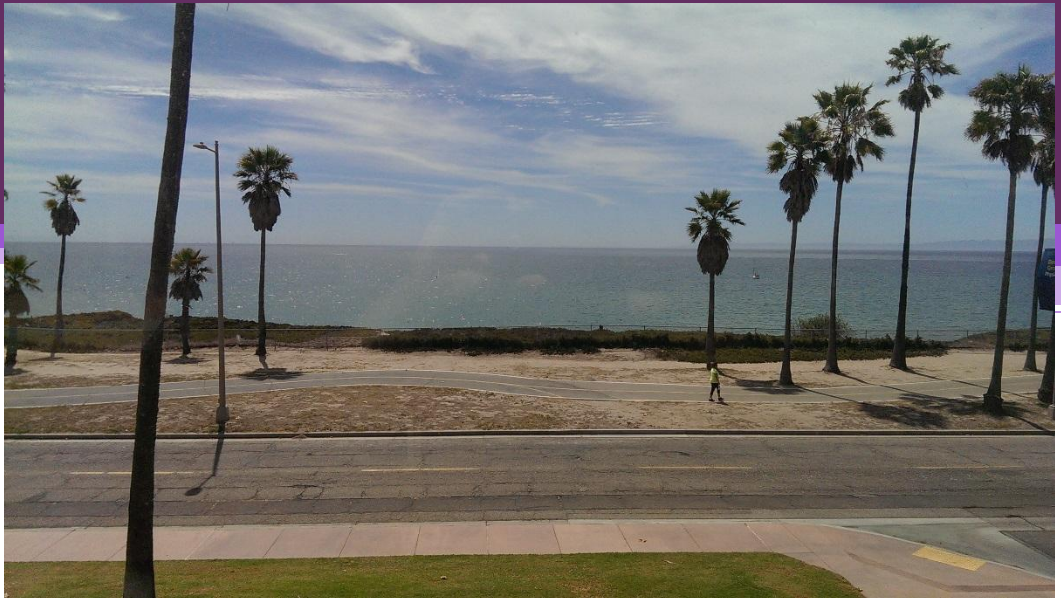
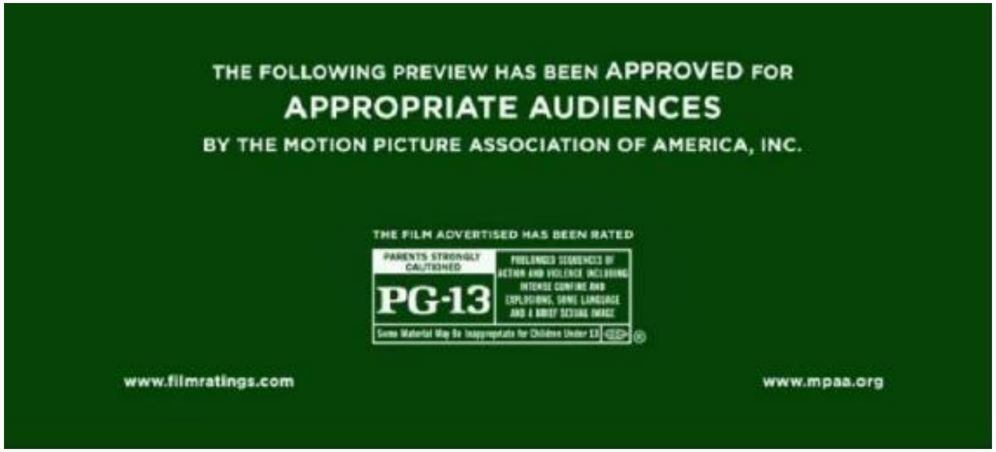
From coupled wires to 3D Fractional Topological Insulators

Yuval Oreg (Weizmann)



Santa Barbara 2015

Before we begin



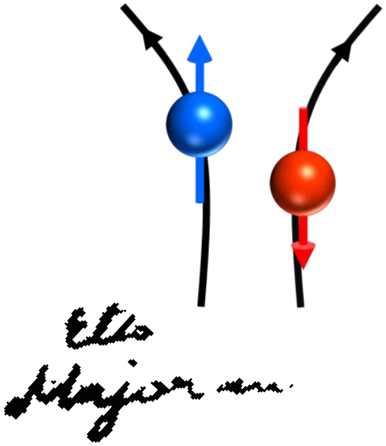
Signatures of Majorana zero modes in spin-resolved current correlations and in charge sensing

Arbel Haim, Erez Berg, Felix von Oppen, and YO

arXiv: 1411.0673->PRL

Gilad Ben-Shach, Arbel Haim, Ian Appelbaum, YO, Amir Yacoby, Bertrand I. Halperin

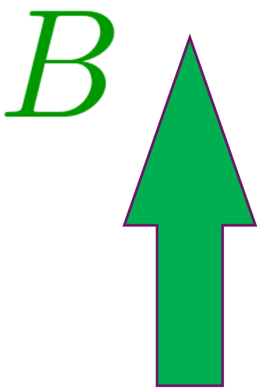
arXiv:1406.5172->PRB

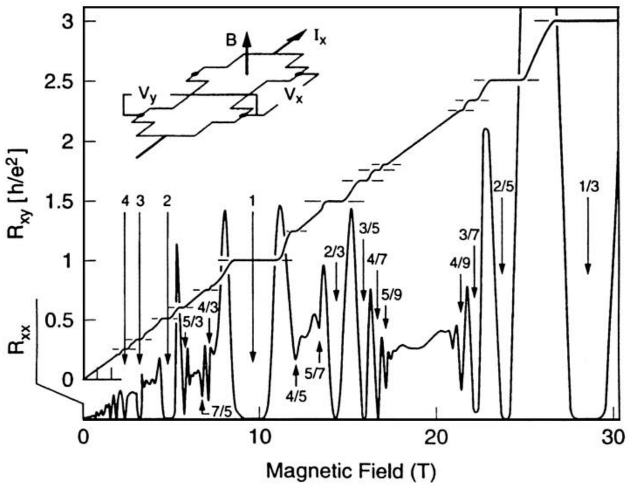


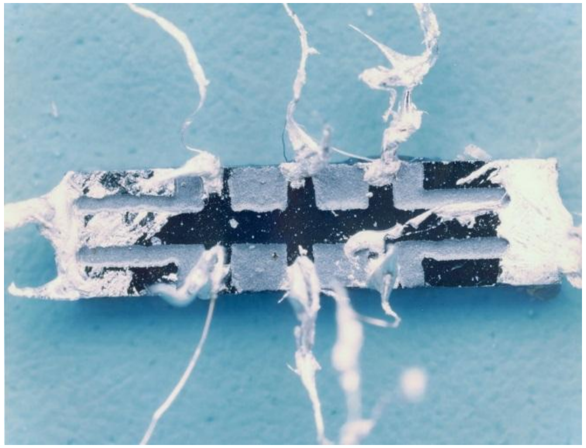
Outline

- (Fractional) Quantum Hall effect from wires
- Spin orbit analog and fractional Majoranas
  (with Eran Sela and Ady Stern).
- Fractional Chern insulator and Fractional topological insulator from wires (with Eran Sagi)
  (see also Neupert, Chamon Mudry &Thomale)
- Fractional (6π) Josephson effect
  (with Eran Sagi, Ady Stern and Bert Halperin)
- 3D Fractoinal topological insulator
  (with Eran Sagi)

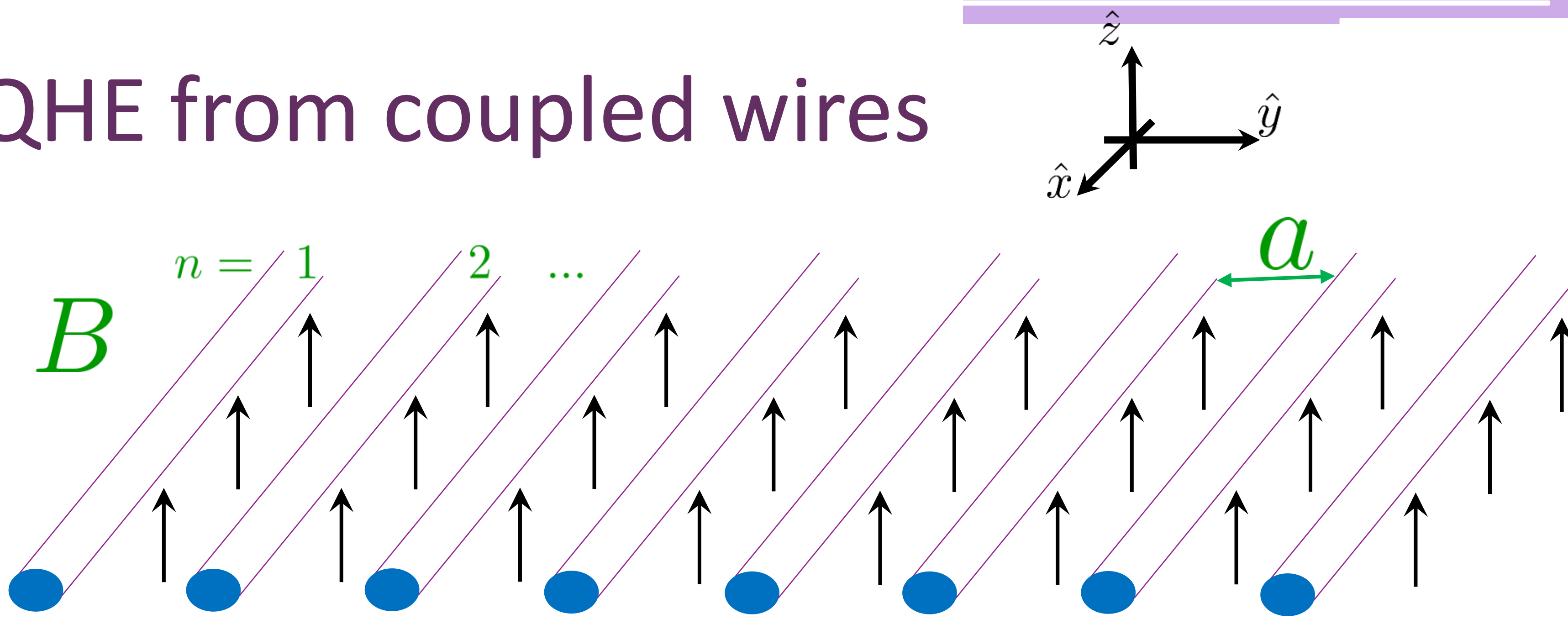
IQHE from coupled wires







IQHE from coupled wires

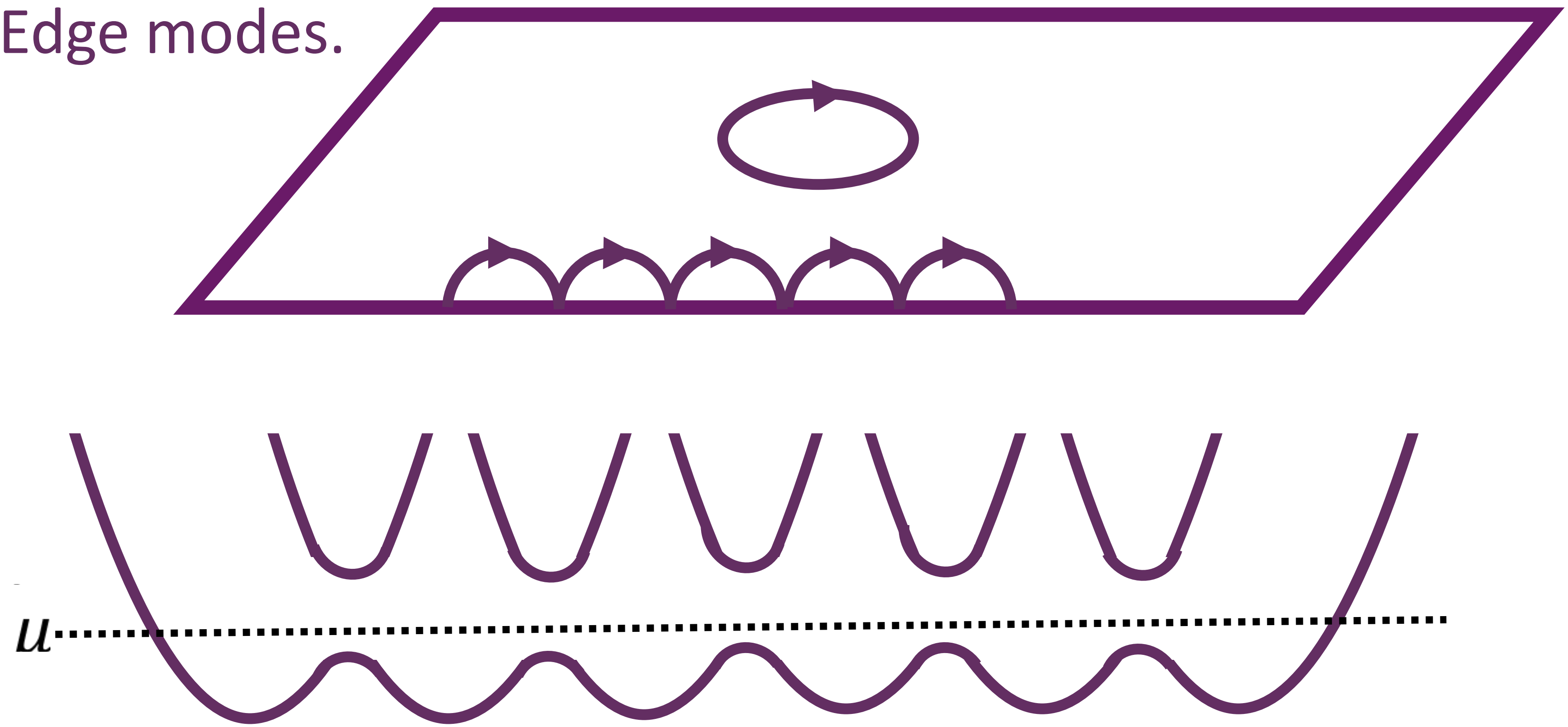


- In the Landau gauge: For each wire $\frac{1}{2m}(p_x - eA_x)^2$; $A_x = Hy = Han$

$$\mathcal{H} = \sum_n \frac{1}{2m}(p_n - eBan)^2$$

IQHE from coupled wires

- Small tunneling between the wires opens a bulk gap.
- Edge modes.

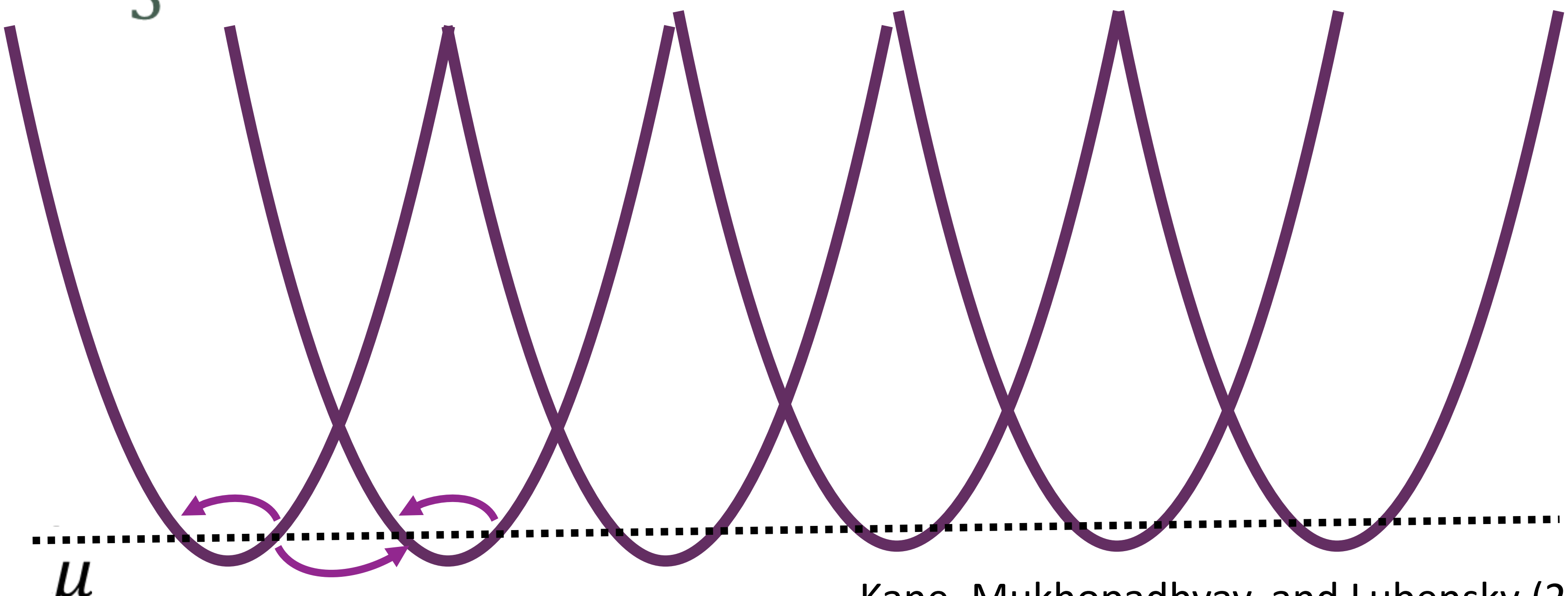


Yakovenko (1991), Sondhi & Yang (2001)

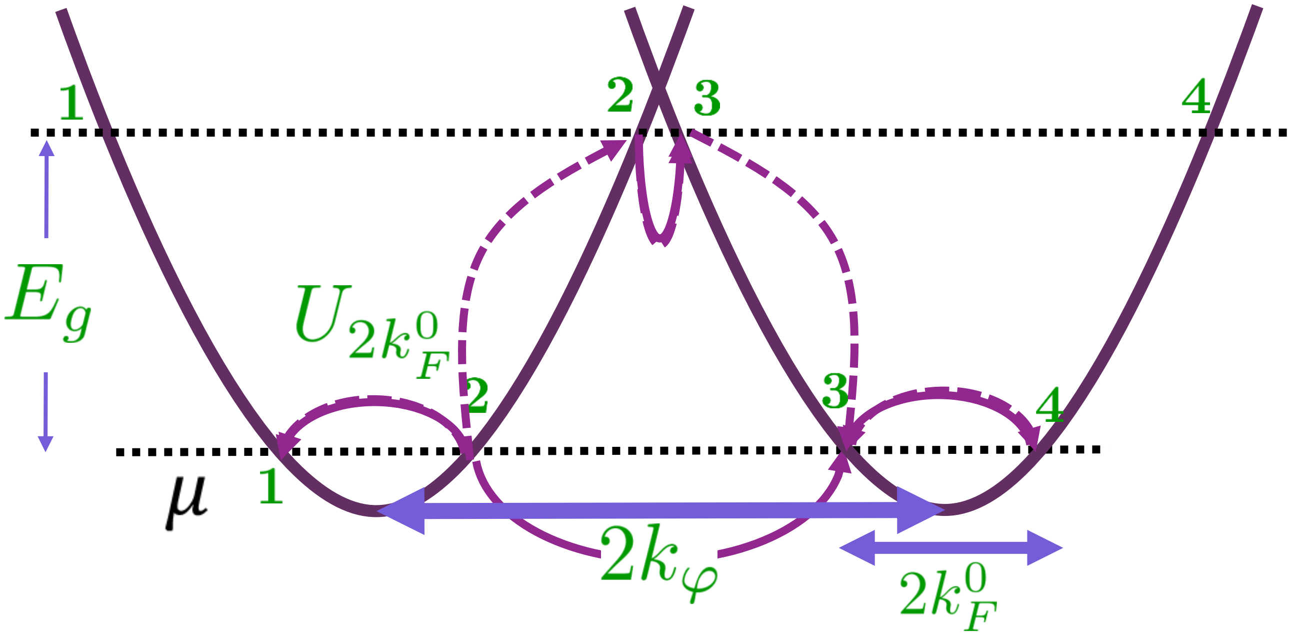
FQHE from coupled wires

- Interactions are now essential.
- Can study Q1D systems using bosonization.

$\nu = \dfrac{1}{3}$:



Kane, Mukhopadhyay, and Lubensky (2002)



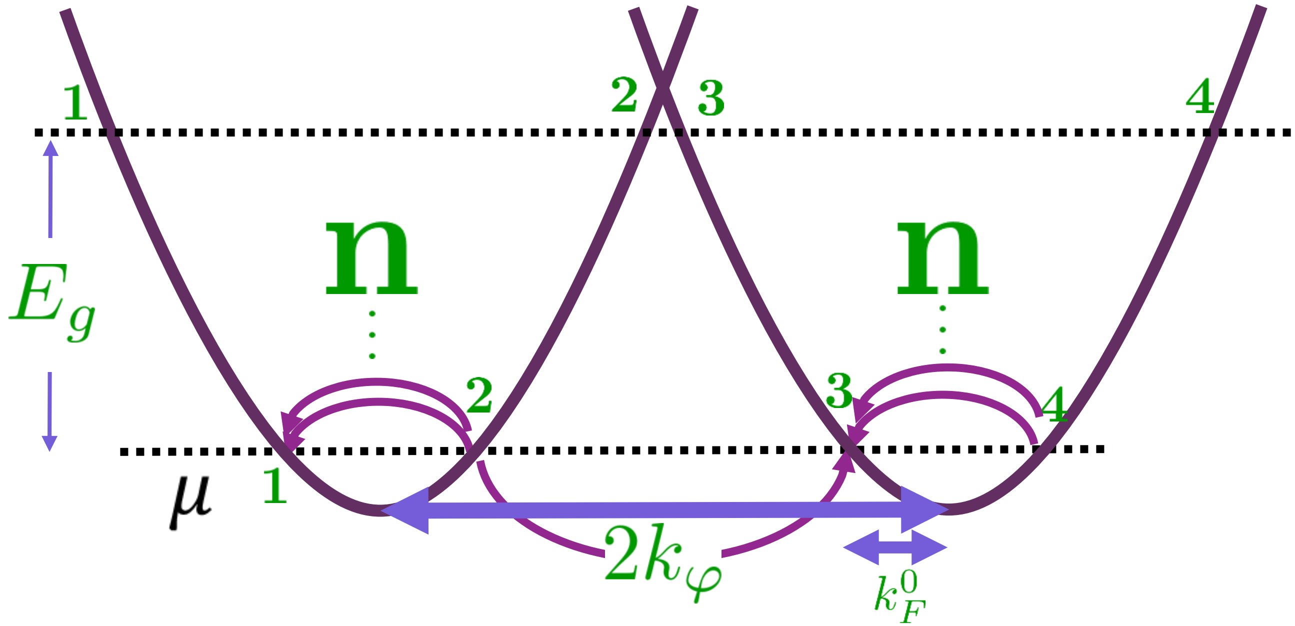
$$\mathcal{O}_B^0 = g_B \psi_2 \psi_3^\dagger = e^{i(\phi_3-\phi_2)+i(k_3-k_2)x}$$

$$\mathcal{O}_B^1 = g_B \frac{U^2}{E_g^2}\left[(\psi_1^\dagger\psi_2)\psi_2\right]\left[\psi_3^\dagger(\psi_3^\dagger\psi_4)\right]$$

$$= e^{i((2\phi_3-\phi_4)-(2\phi_2-\phi_1))+i((2k_3-k_4)-(2k_2-k_1))x}$$

$$= e^{i(\eta_3-\eta_2)+i(q_3-q_2)x}$$

$$q_3 - q_2 = 0 \Rightarrow 6k_F^0 = 2k_\varphi \Rightarrow \nu = \frac{k_F^0}{k_\varphi} = \frac{1}{3}$$



$$\mathcal{O}_B^0 = g_B \psi_2 \psi_3^\dagger = e^{i(\phi_3-\phi_2)+i(k_3-k_2)x}$$

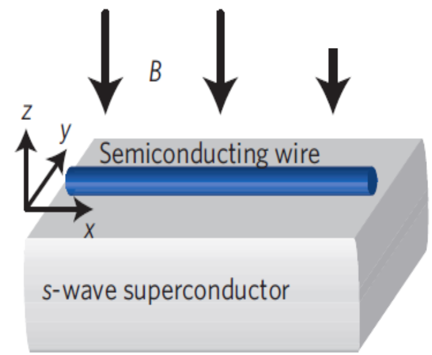
$$\mathcal{O}_B^n = g_B \frac{U^{2n}}{E_g^{2n}} \left[(\psi_1^\dagger \psi_2)^n \psi_2\right] \left[\psi_3^\dagger (\psi_3^\dagger \psi_4)^n\right]$$

$$= e^{i(\eta_3-\eta_2)+i(q_3-q_2)x}$$

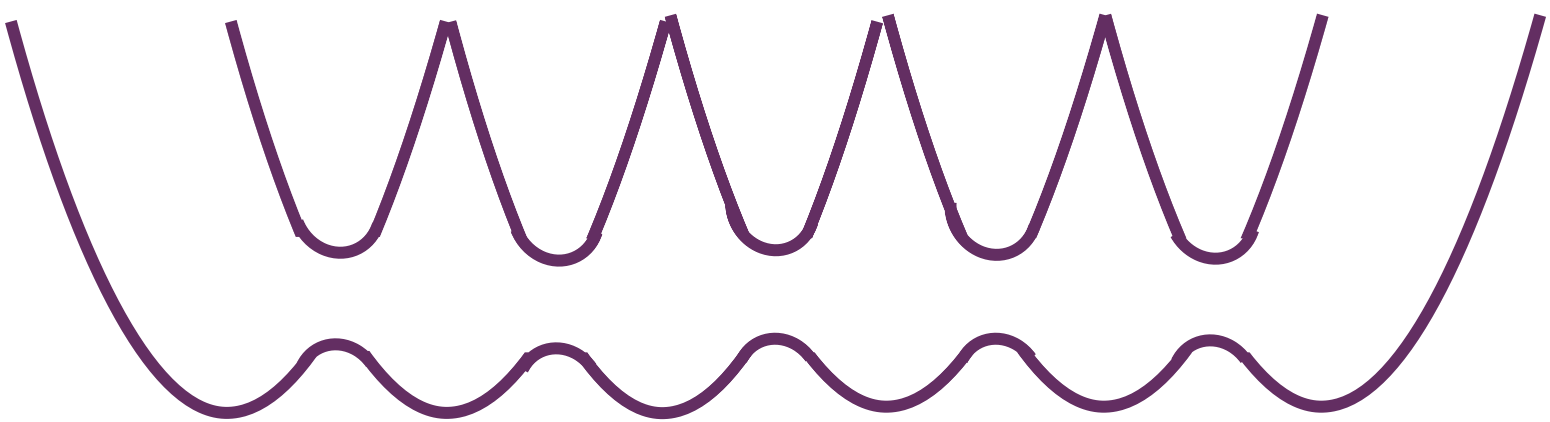
$$q_3 - q_2 = 0 \Rightarrow 2(2n+1)k_F^0 = 2k_\varphi \Rightarrow \nu = \frac{k_F^0}{k_\varphi} = \frac{1}{2n+1}$$

IQHE and Majorana modes

$$H = \int \Psi^\dagger(x)\mathcal{H}\Psi(x)dx; \quad \Psi^\dagger = \left(\psi_\uparrow^\dagger, \psi_\downarrow^\dagger, \psi_\downarrow, -\psi_\uparrow\right)$$



$$\mathcal{H} = \left[\frac{p^2}{2m} - \mu(x)\right]\tau_z + u(x)p\sigma_z\tau_z + B(x)\sigma_x + \Delta(x)\tau_x$$



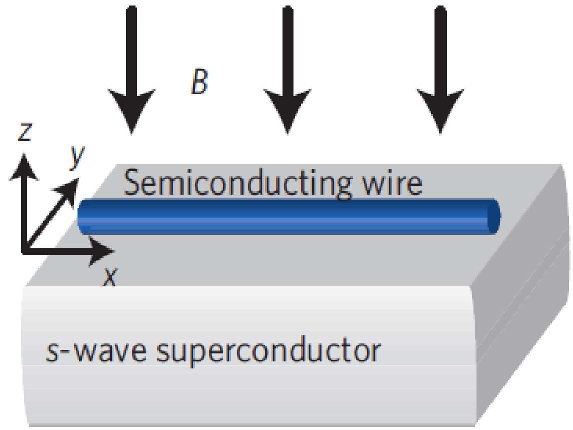
YO, Sela, Stern (2014)

Fractional Majorana

(Z_6 Parafermions zero energy states)

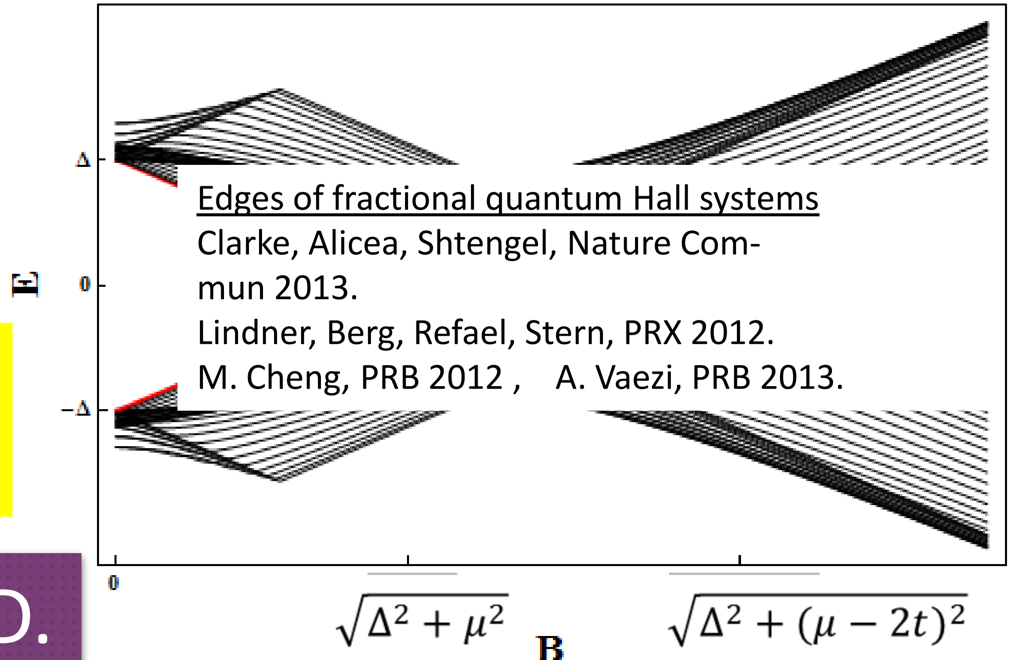
$$g_B \cos(\eta_3 - \eta_2) \qquad g_\Delta \cos(\eta_1 + \eta_4)$$

$$g_B \propto B \frac{U_{2k_F}^2}{E_g^2} \qquad g_\Delta \propto \Delta \frac{U_{2k_F}^2}{E_g^2}$$



Is there a ground state degeneracy (D)?

n=0: diagonalize fermionic model→

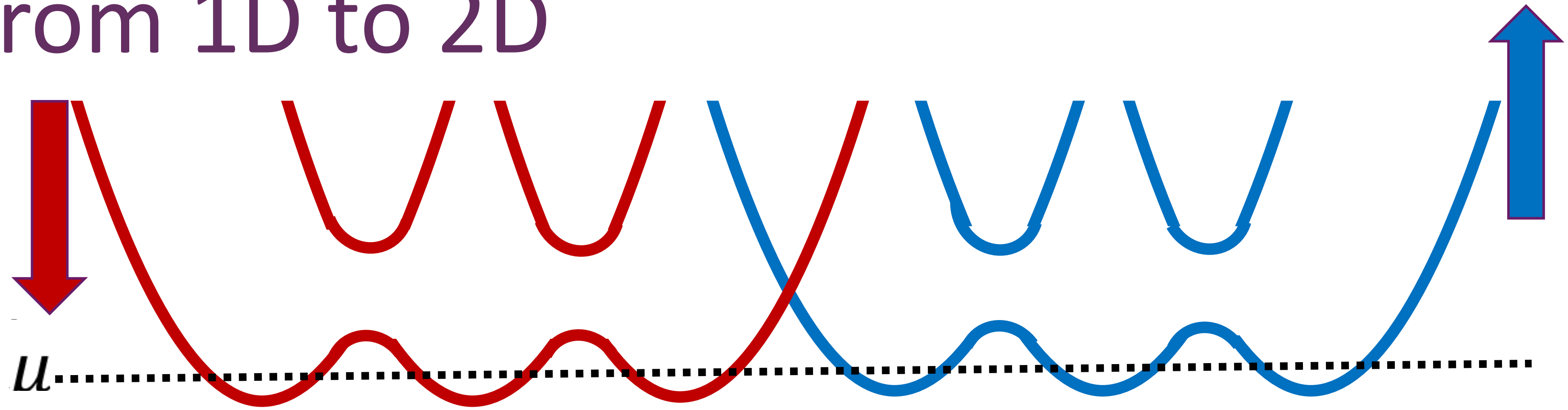


Edges of fractional quantum Hall systems
Clarke, Alicea, Shtengel, Nature Commun 2013.
Lindner, Berg, Refael, Stern, PRX 2012.
M. Cheng, PRB 2012 , A. Vaezi, PRB 2013.

result:
clean wire: D =2(2n+1)=2,6,10…
dirty wire: D=2

Disorder Effects in Q1D.

From 1D to 2D



$$\mathcal{H} = \sum_n \frac{1}{2m}(p_n - eBan)^2$$

$$\mathcal{H} = \sum_n \frac{1}{2m}p_n^2 - \alpha p_n n \sigma_z$$

- Essentially like edges of TI in 2D but requires SO increasing with the wire index.
- Can we create other topological states (CI, FCI, TI, FTI) using similar wires constructions?

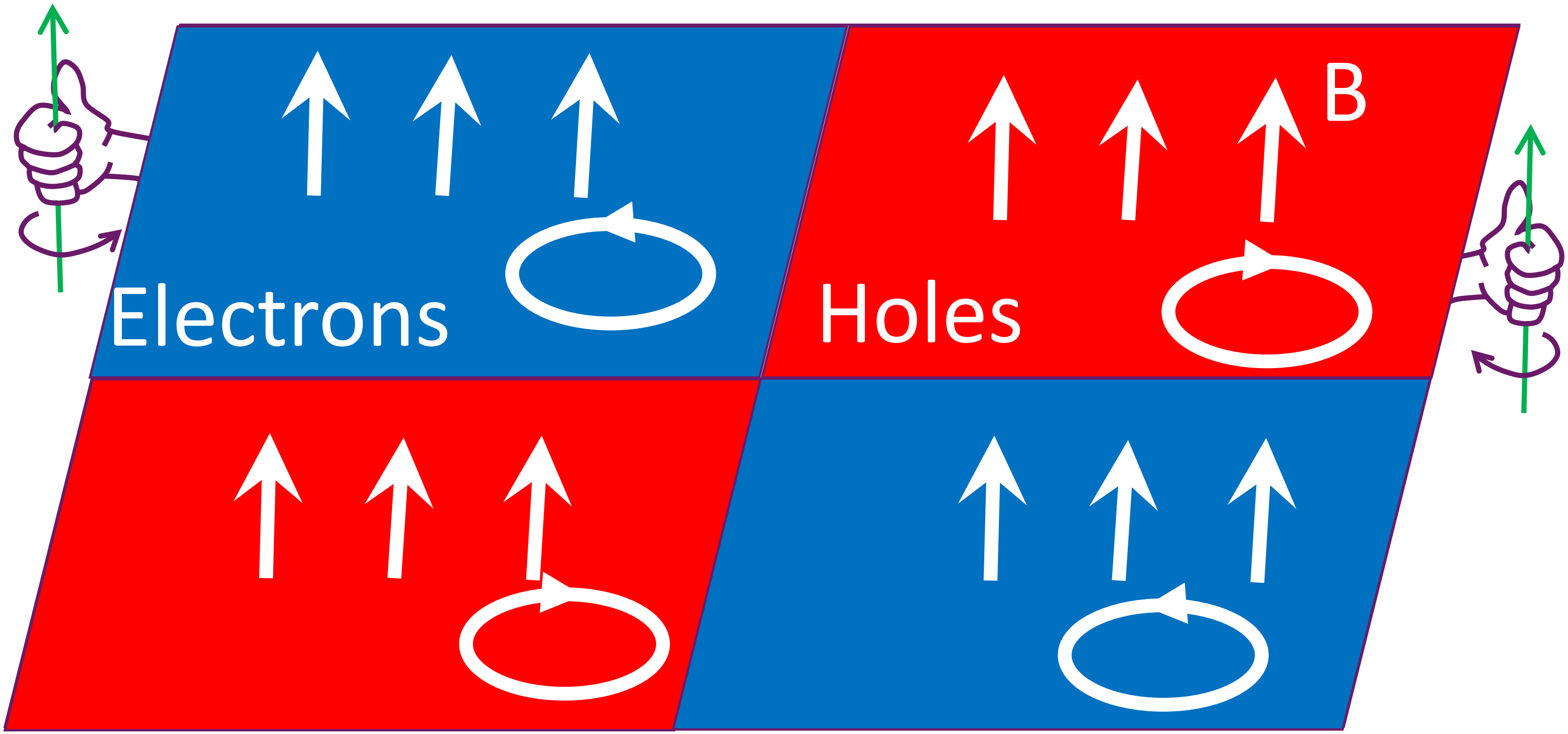
FQHE from coupled wires

- T&K: FQH states (including non Abelian) can be identified in the Q1D limit.
- Can we create other topological states using similar wires constructions?
- Chern insulator? Topological insulator?
- Fractional Chern insulator? Fractional topological insulator? So far mainly numerical and or flat band models, (Bernevig, Regnault, Maciejko, Qi, Barkeshli, Parameswaran, Roy, Sondhi, Lu, Neupert, Thomale, Santos, Ryu, Chamon, Mudry, Haldane …)

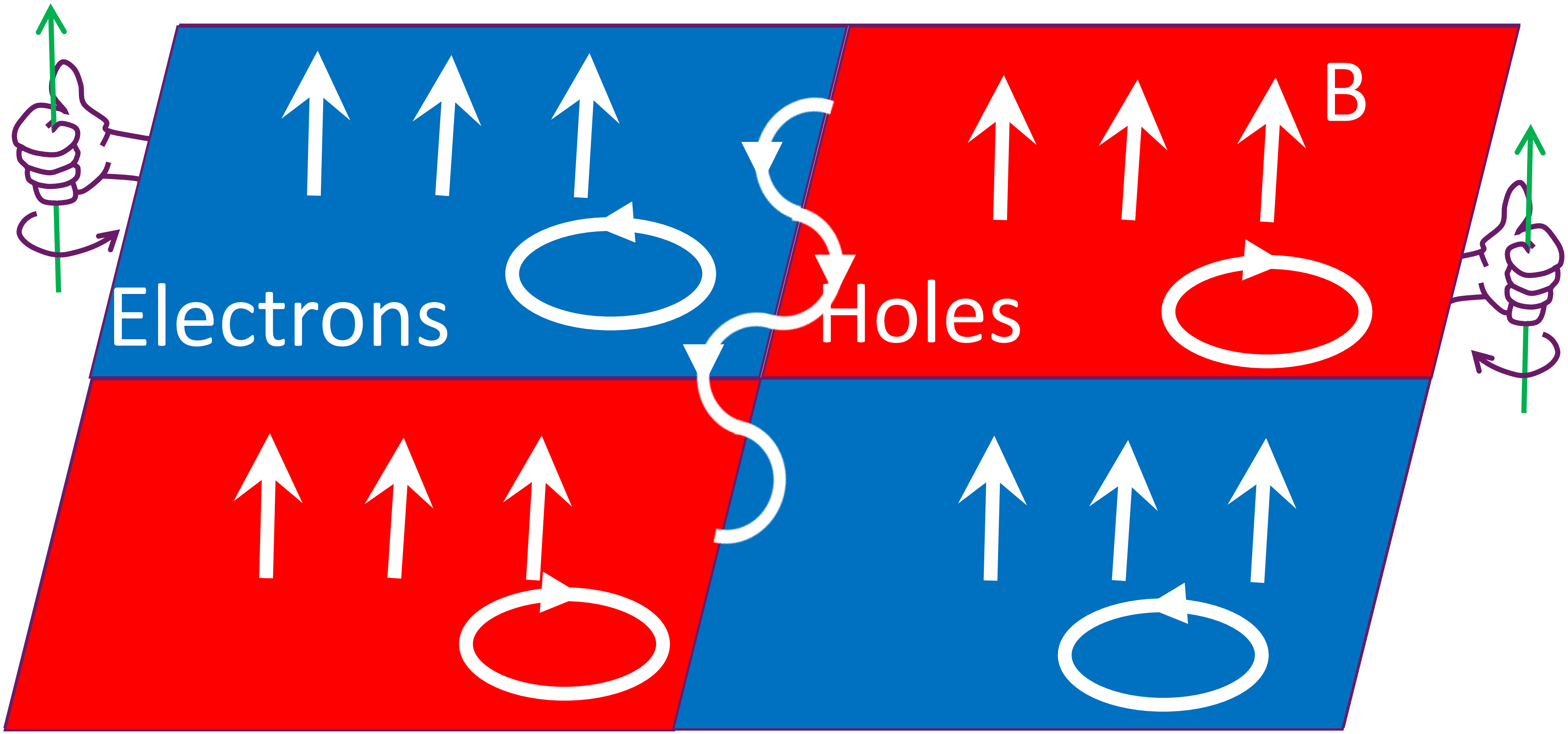
Teo and Kane (2011)

Fractional Chern Insulator From coupled wires

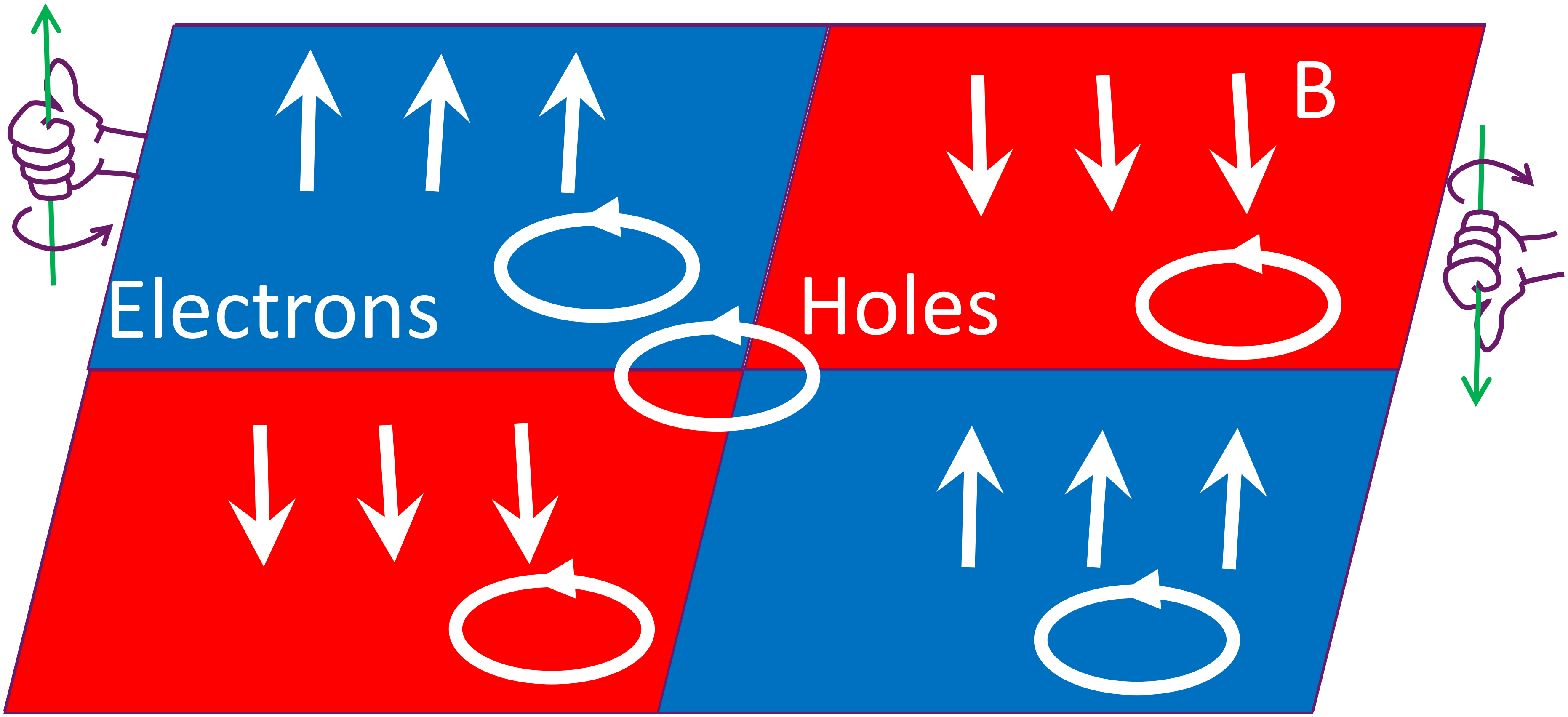
A Semi-classical picture of a Chern insulator



A Semi-classical picture for a Chern insulator

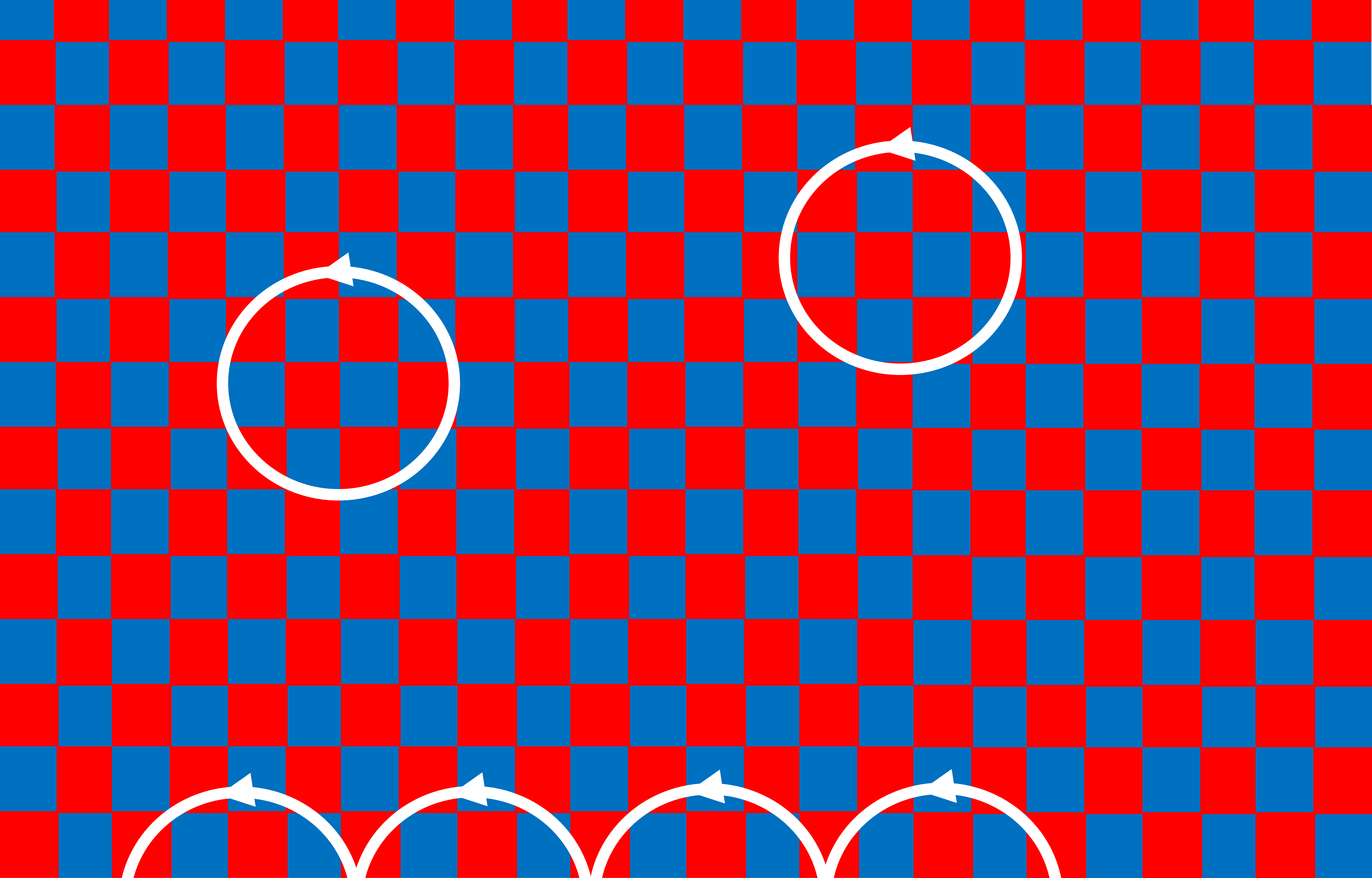


A Semi-classical picture for a Chern insulator

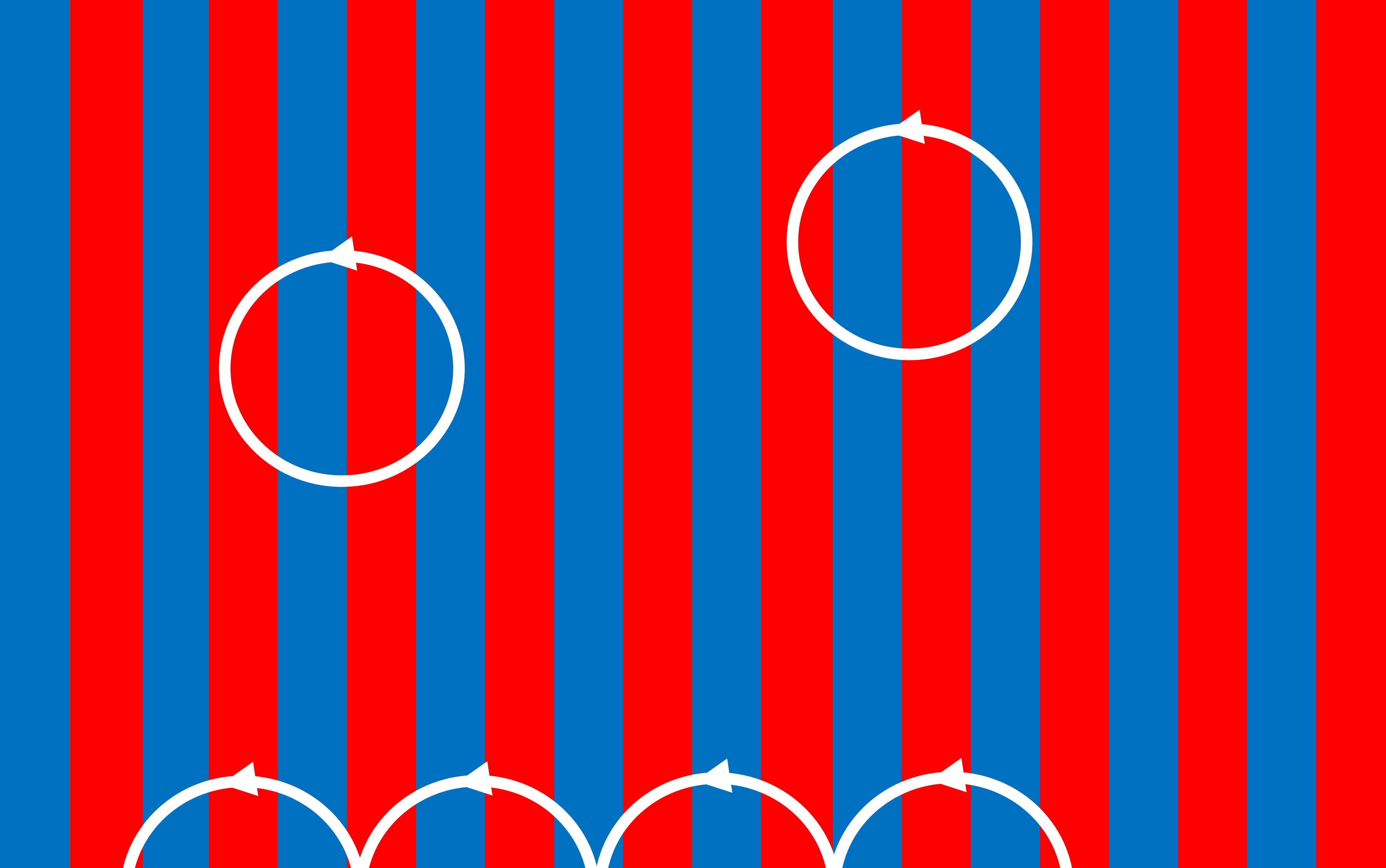


Haldane 88'

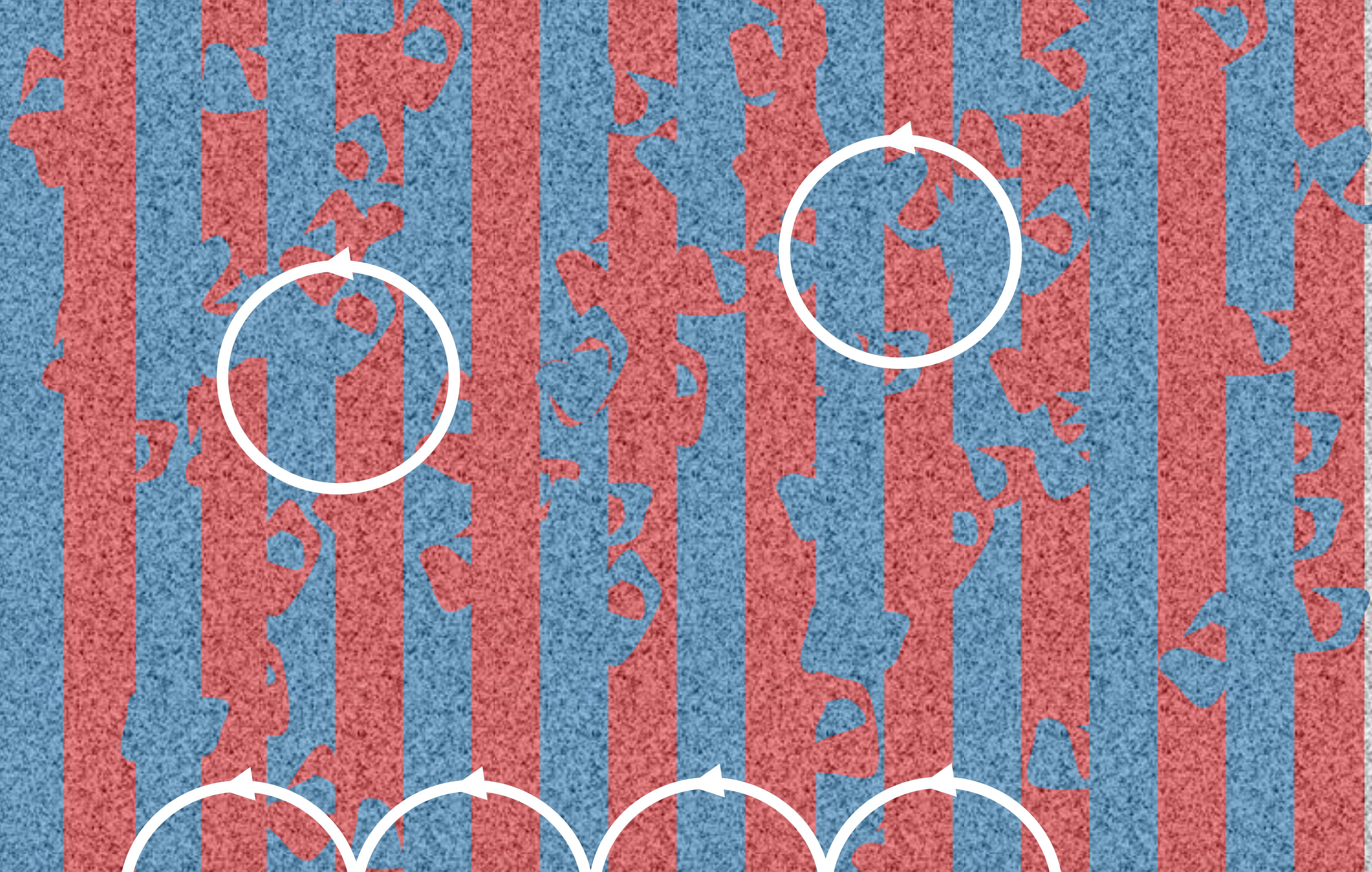
Chess Board



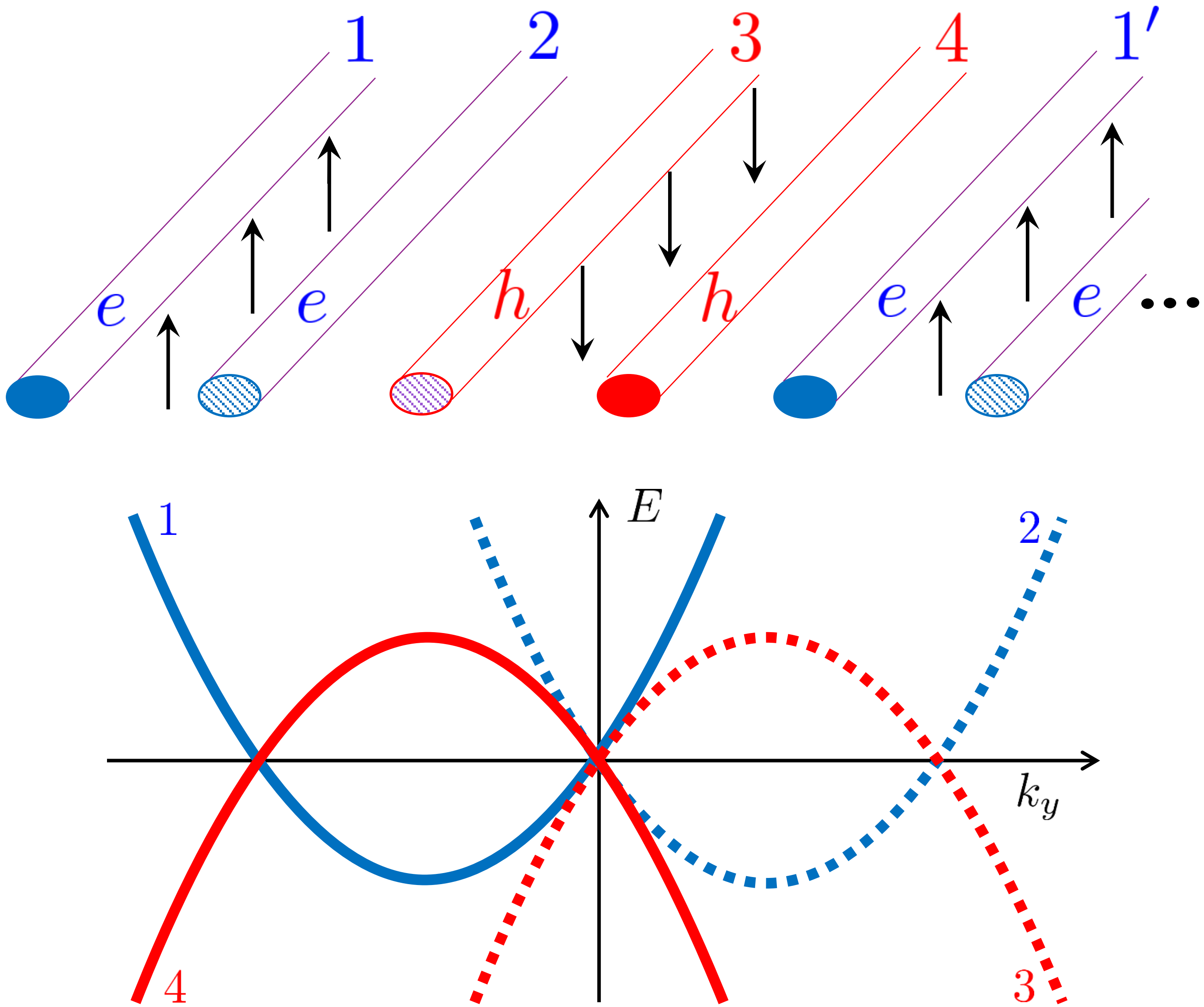
Striped Pyjamas



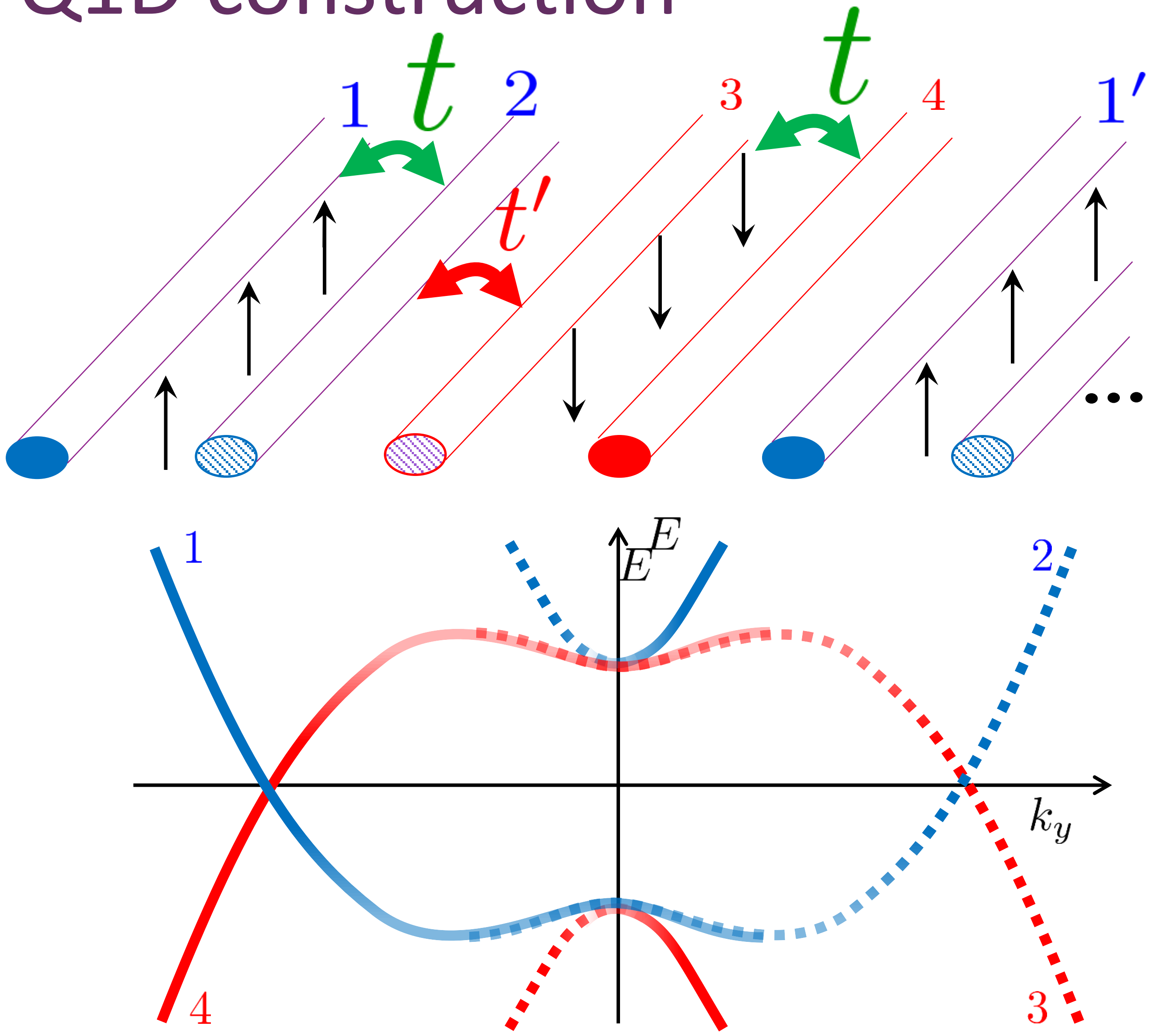
Striped Pyjamas- Disorder



Q1D construction - Chern Insulator

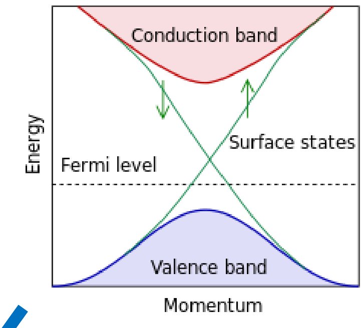


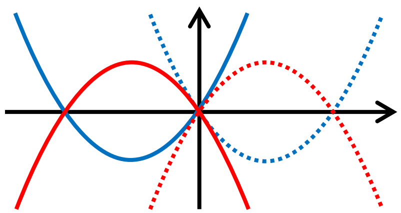
Our Q1D construction

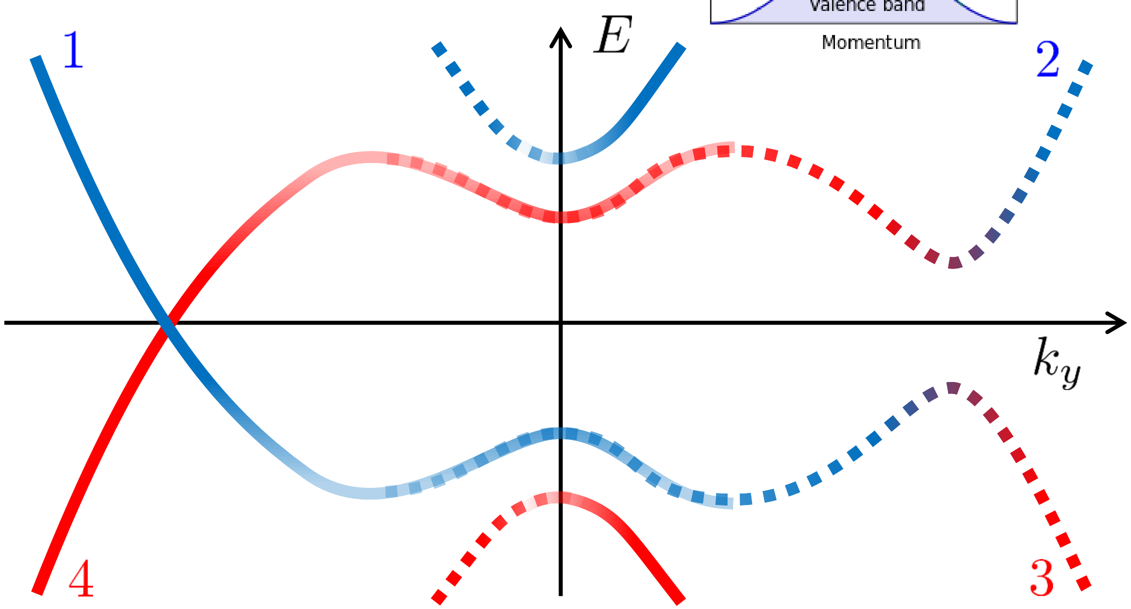


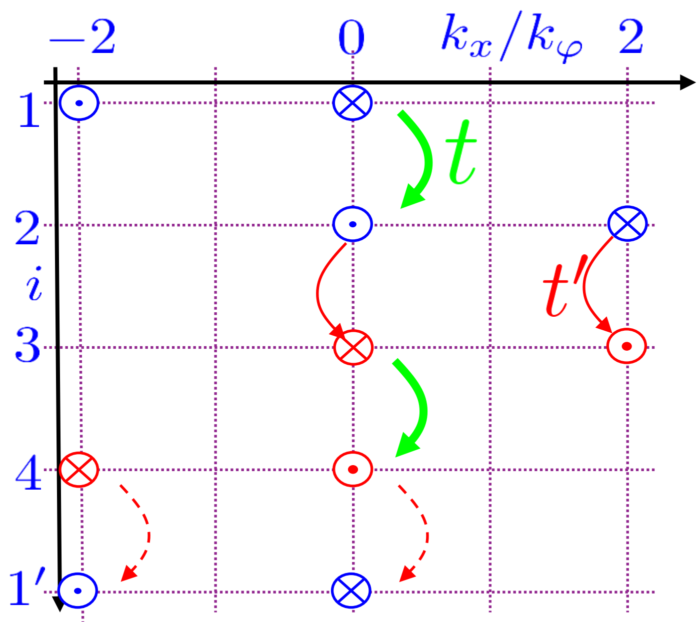
Our Q1D construction

- Small tunneling between the wires opens a bulk gap.
- Edge modes.





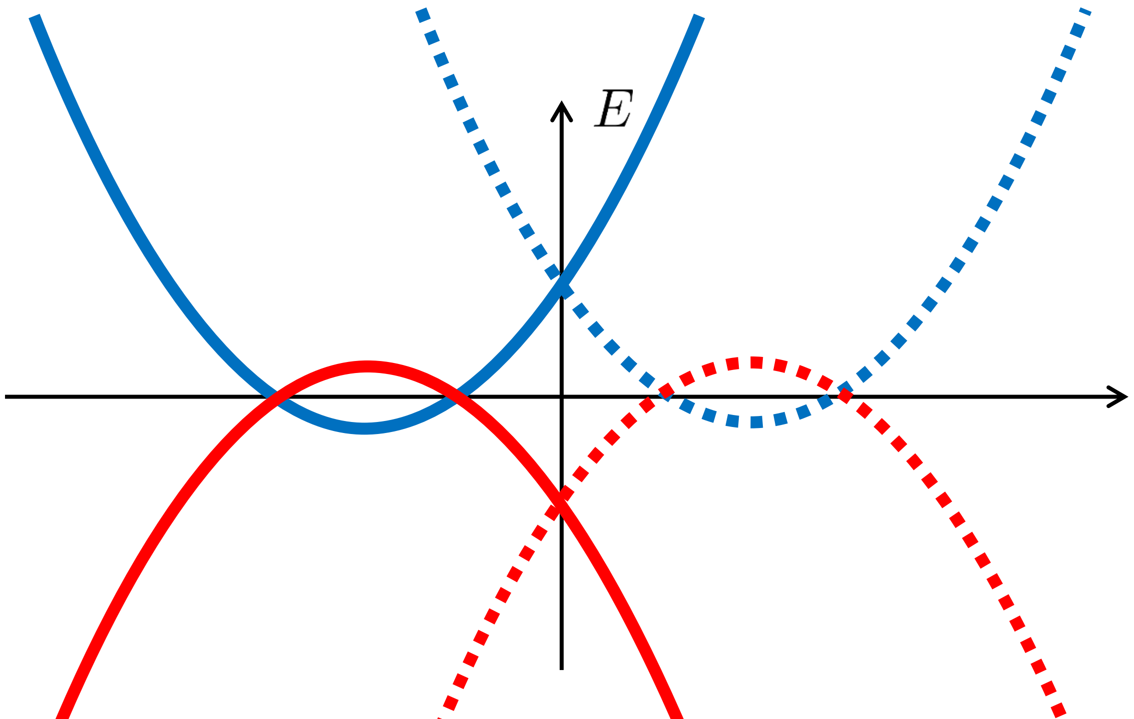




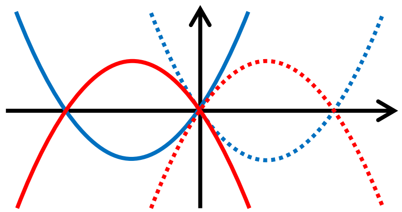
Tight Binding Model

Fractional Chern insulator (Abelian, $\nu = 1/3$)

- Interactions are required.
- We linearize the spectrum, and define the chiral bosonic fields, $\phi_i^{R/L}$, such that $\psi_i^{R/L} \propto e^{i\left(\phi_i^{R/L} + k_i^{R/L} x\right)}$.



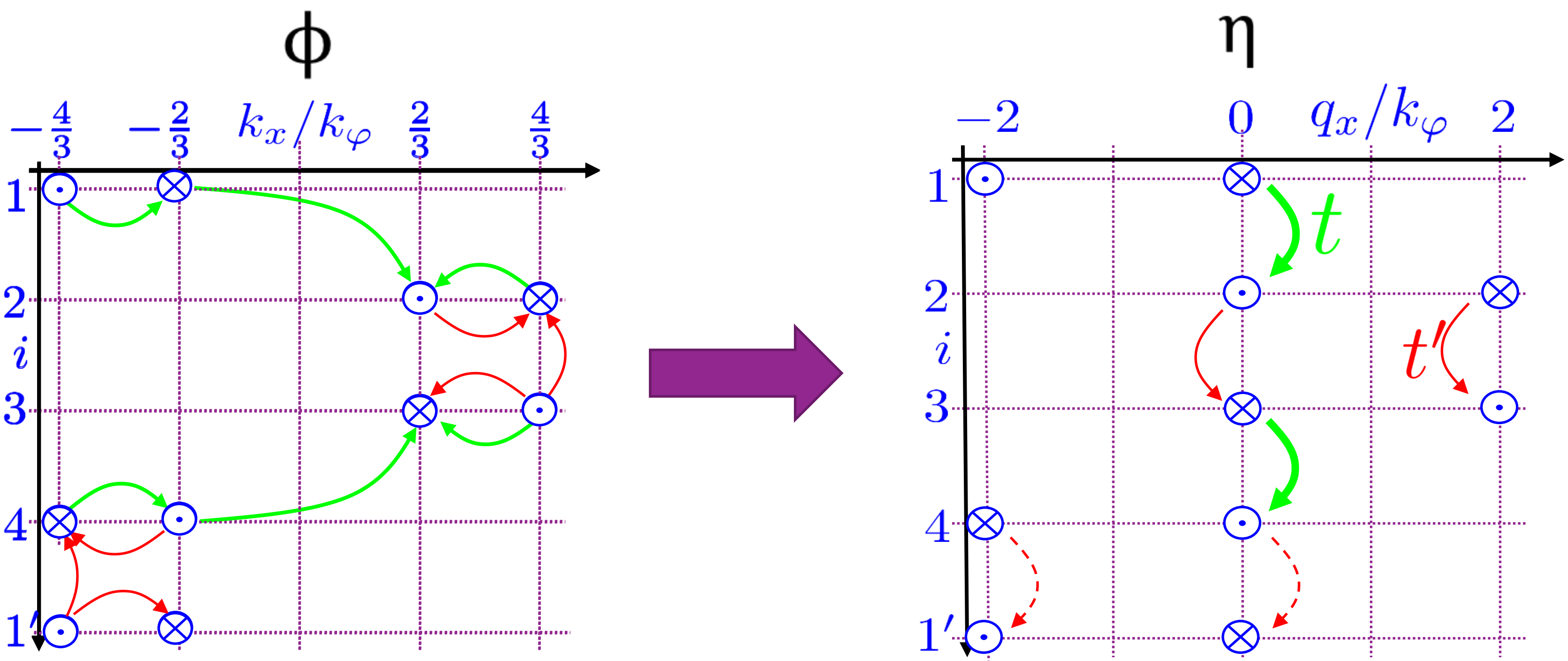
$\nu = 1$



Fractional Chern insulator

- Useful to define new composite fields:

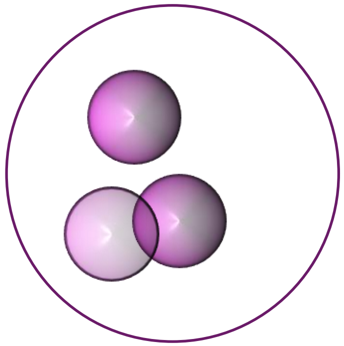
$$\phi_i^{R/L}, k_i^{R/L} \rightarrow \eta_i^{R/L} = 2\phi_i^{R/L} - \phi_i^{L/R}, q_i^{R/L} = 2k_i^{R/L} - k_i^{L/R}$$



Novel metallic state at t=t'

Fractional Dirac Liquid?

$$e^{i\eta} \sim \psi^\dagger(\psi^\dagger\psi)$$



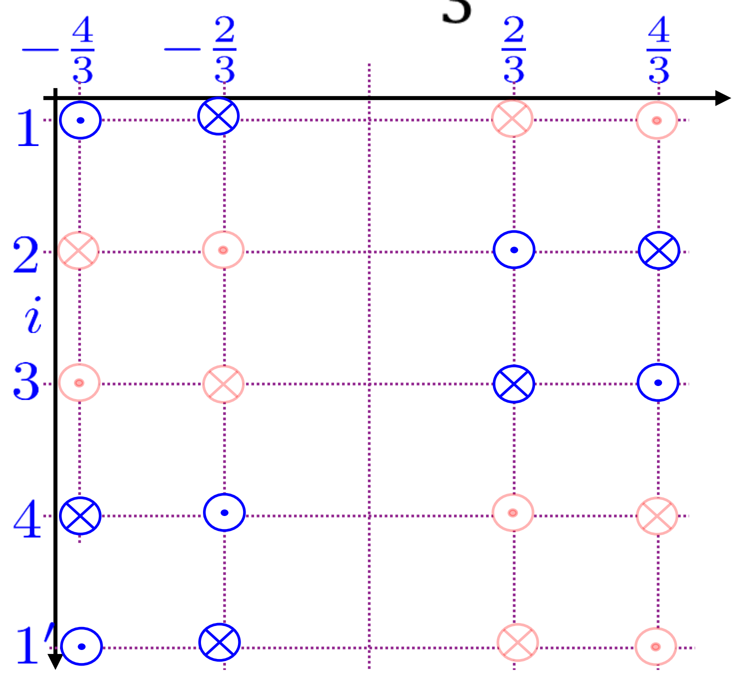
Generalization to a topological insulator

- Spinful electrons.
- Alternating spin-orbit coupling (in the $\hat{z}$ direction).
- We can generate all the above states with spin-conserving processes.

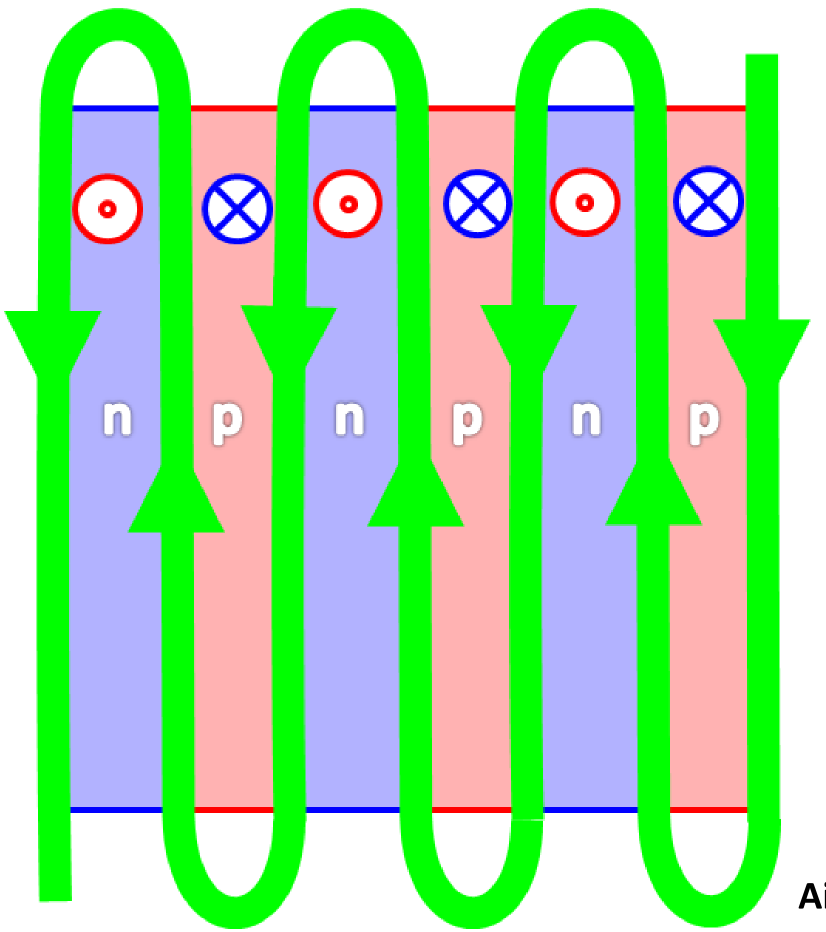
⬇

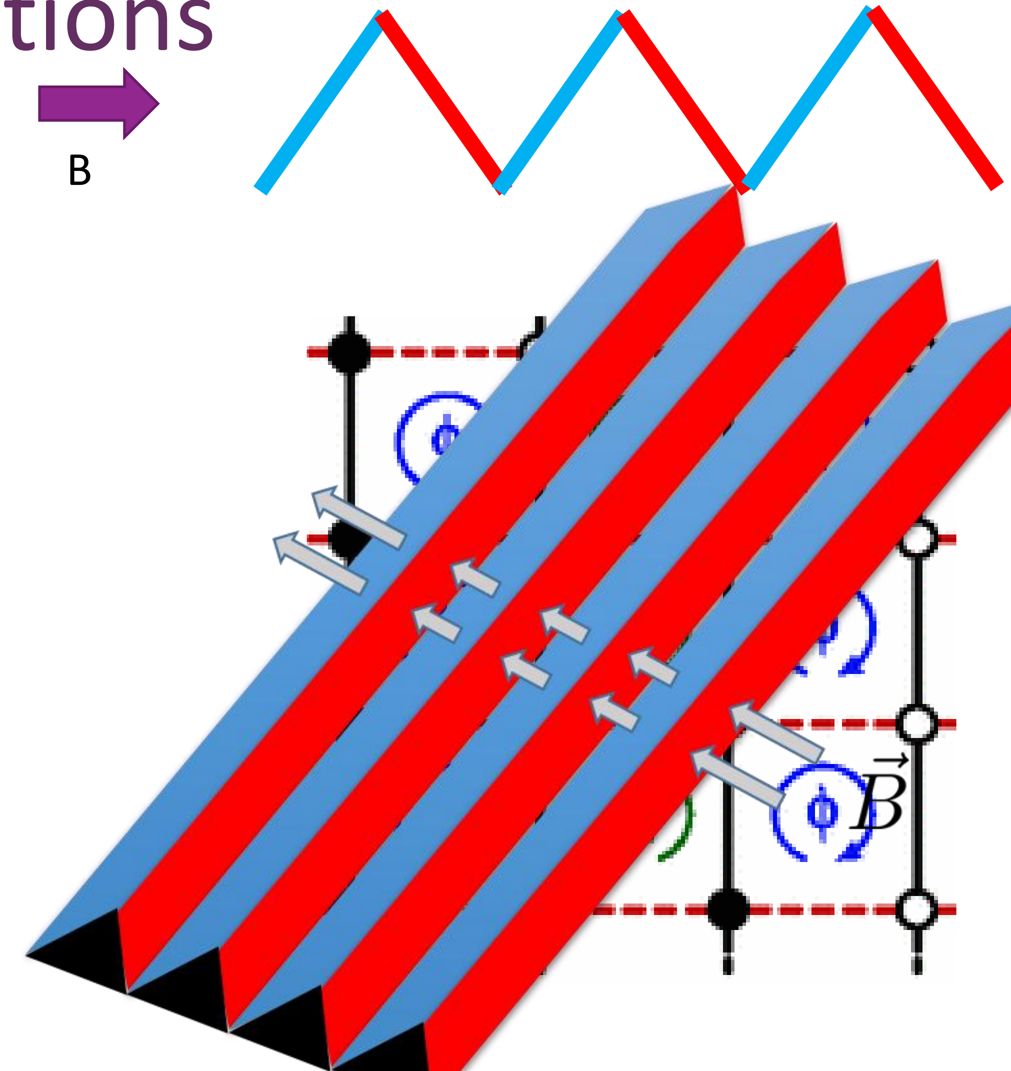
- Fractional topological insulators.

$$\nu = \frac{1}{3}$$



Experimental realizations



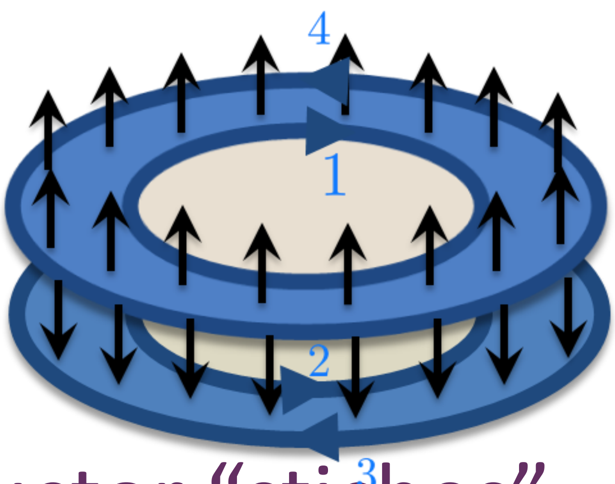


Aidelsburger, Atala, Nascimbene, Trotzky, Chen, and Bloch (2011)

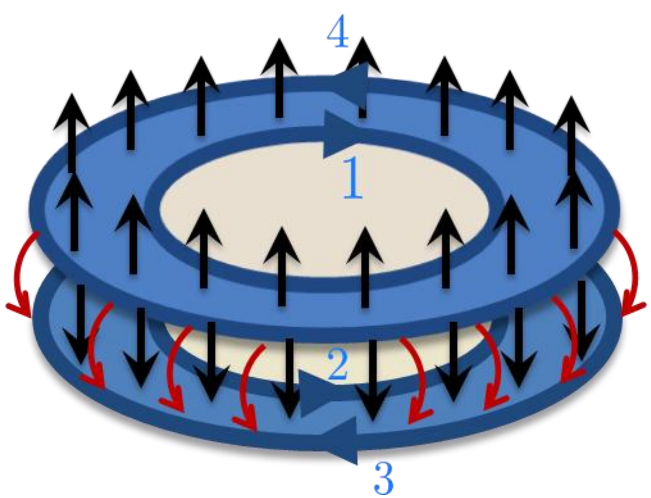
Degeneracy on a Torus and a Fractional Josephson Effect

- Sagi, YO, Stern Halperin PRB

Double layer quantum Hall system where the two layers experience opposite magnetic fields (= TI).

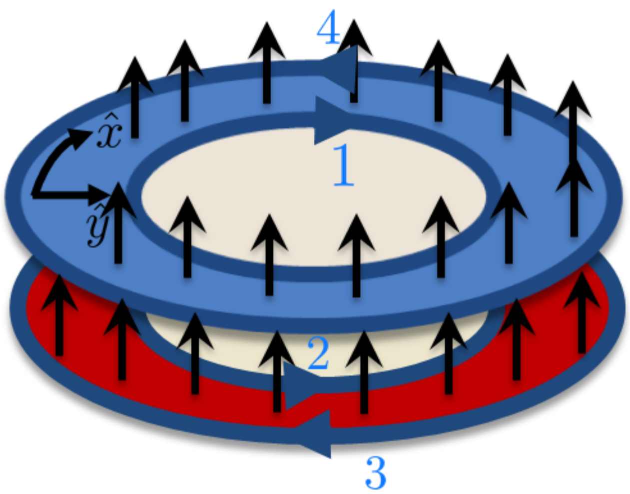


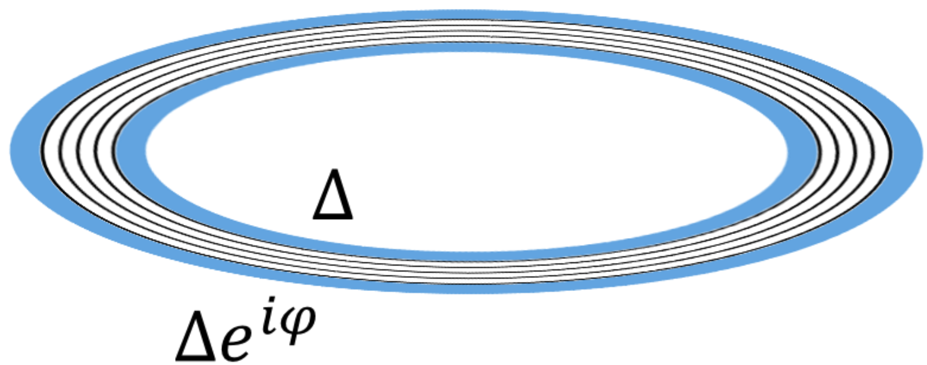
- Backscattering or Superconductor "stiches" the double layer system into a torus.



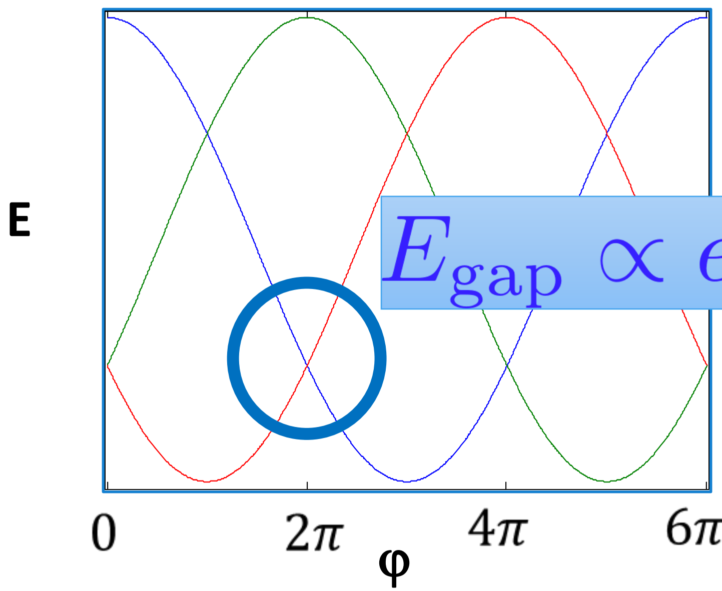
$\cong$

6 π Josephson Effect



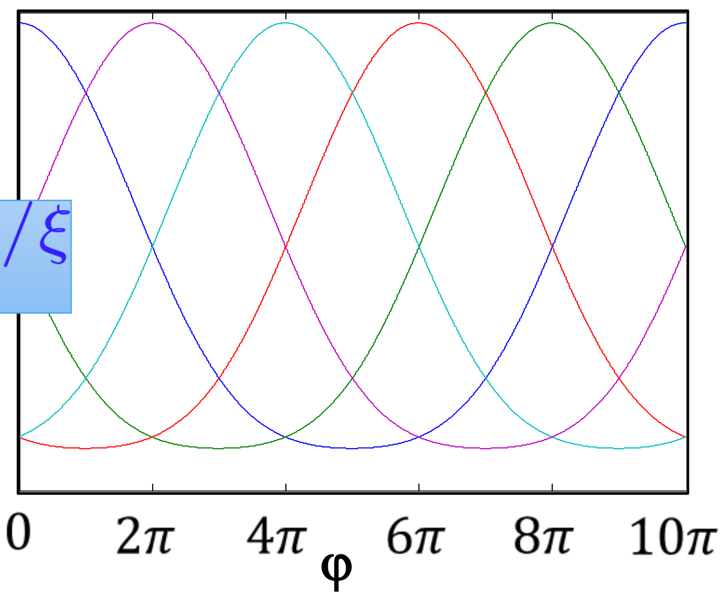


$\boldsymbol{\nu = {}^{1}/_{3}}$



$$E_{\mathrm{gap}} \propto e^{-L/\xi}$$

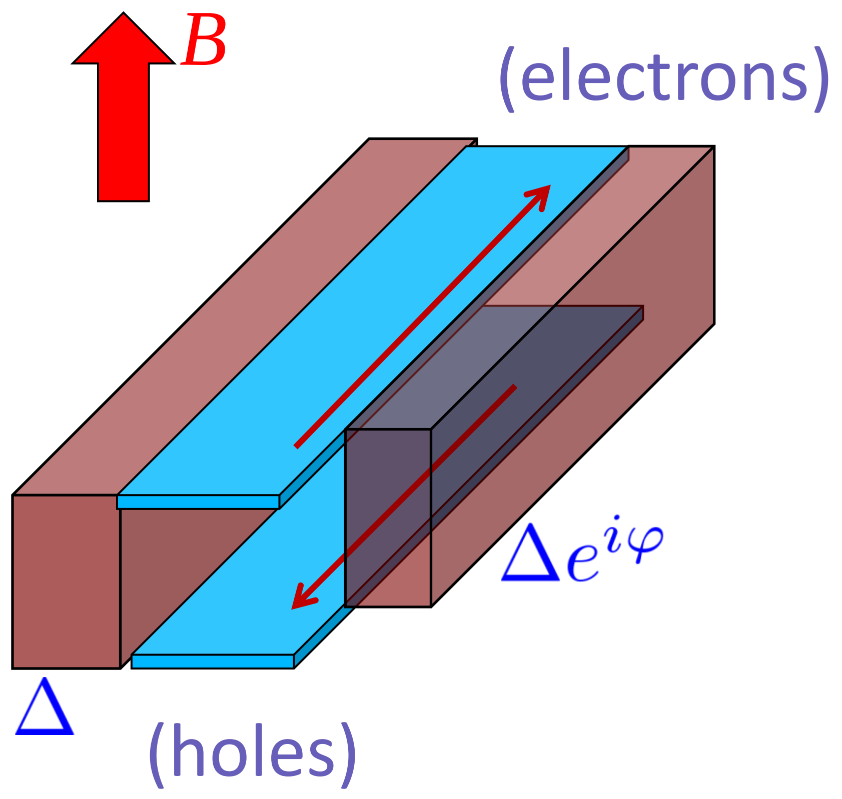
$\boldsymbol{\nu = {}^{2}/_{5}}$



Bi-layer graphene electron-hole system

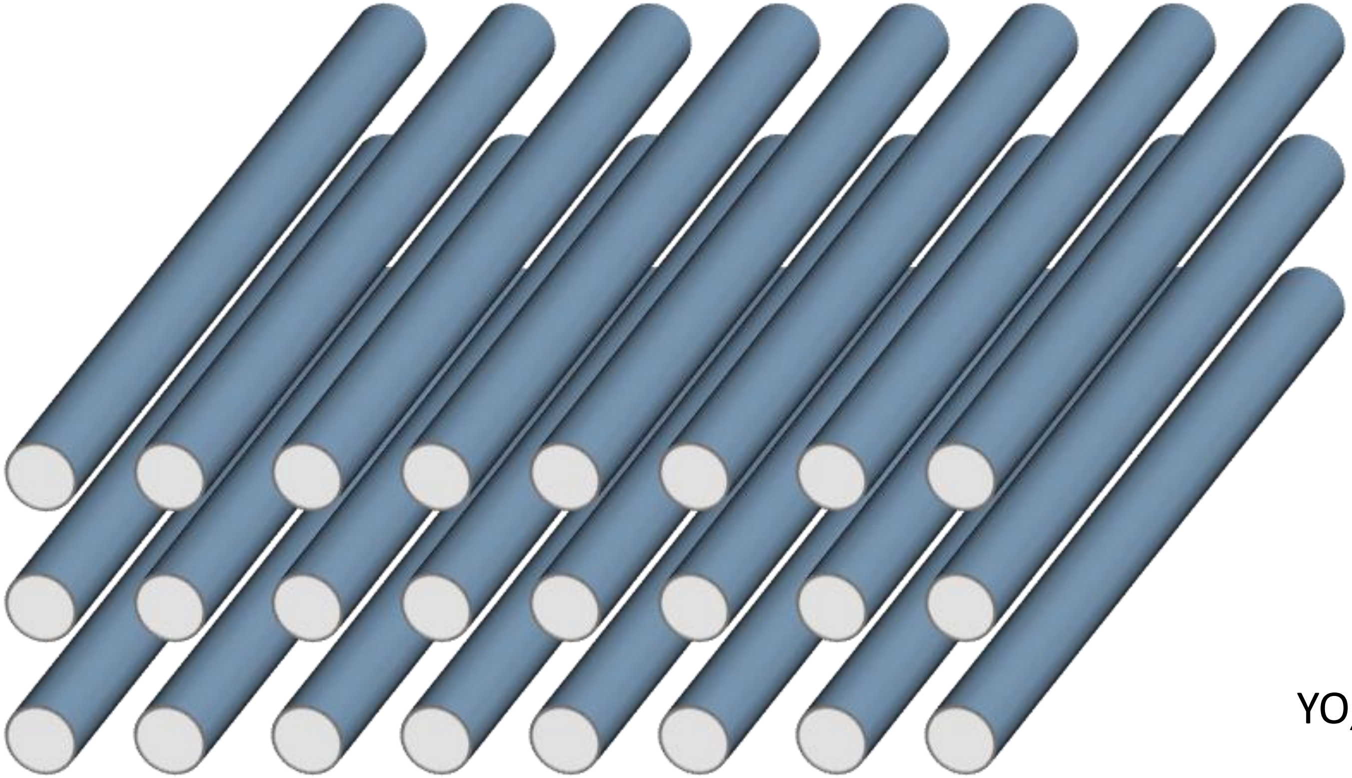
$$\nu = 1/m$$

$$\nu = -1/m$$

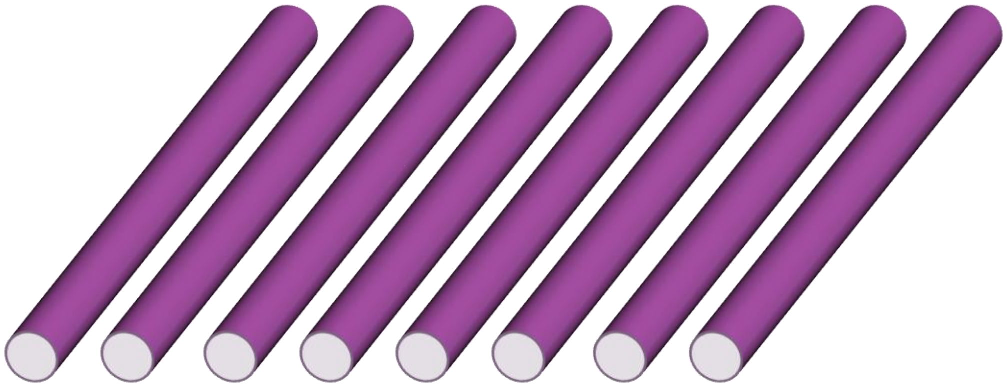


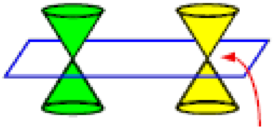
3D Fractional Topological insulator

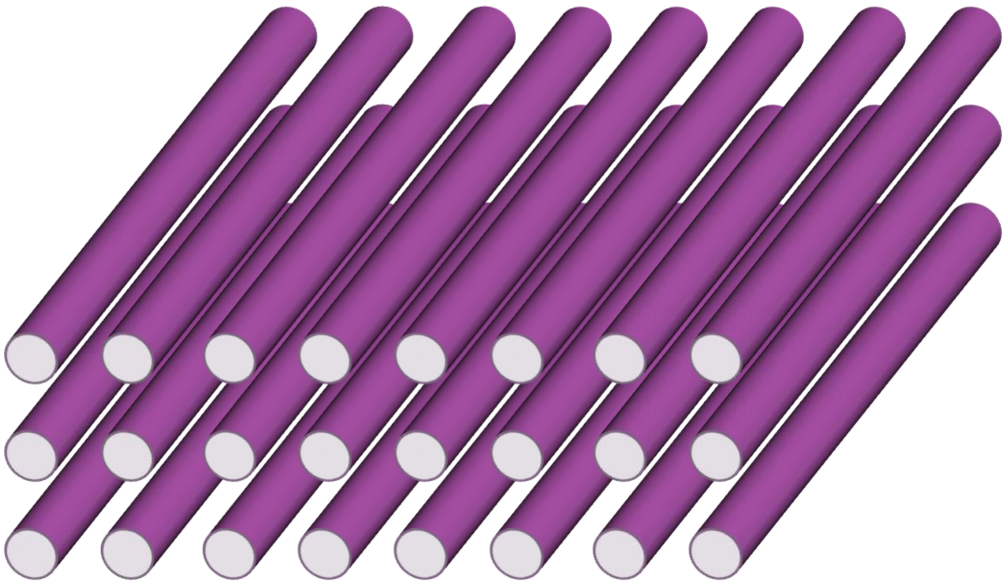
$$\phi \Rightarrow \eta$$



YO, Eran Sagi



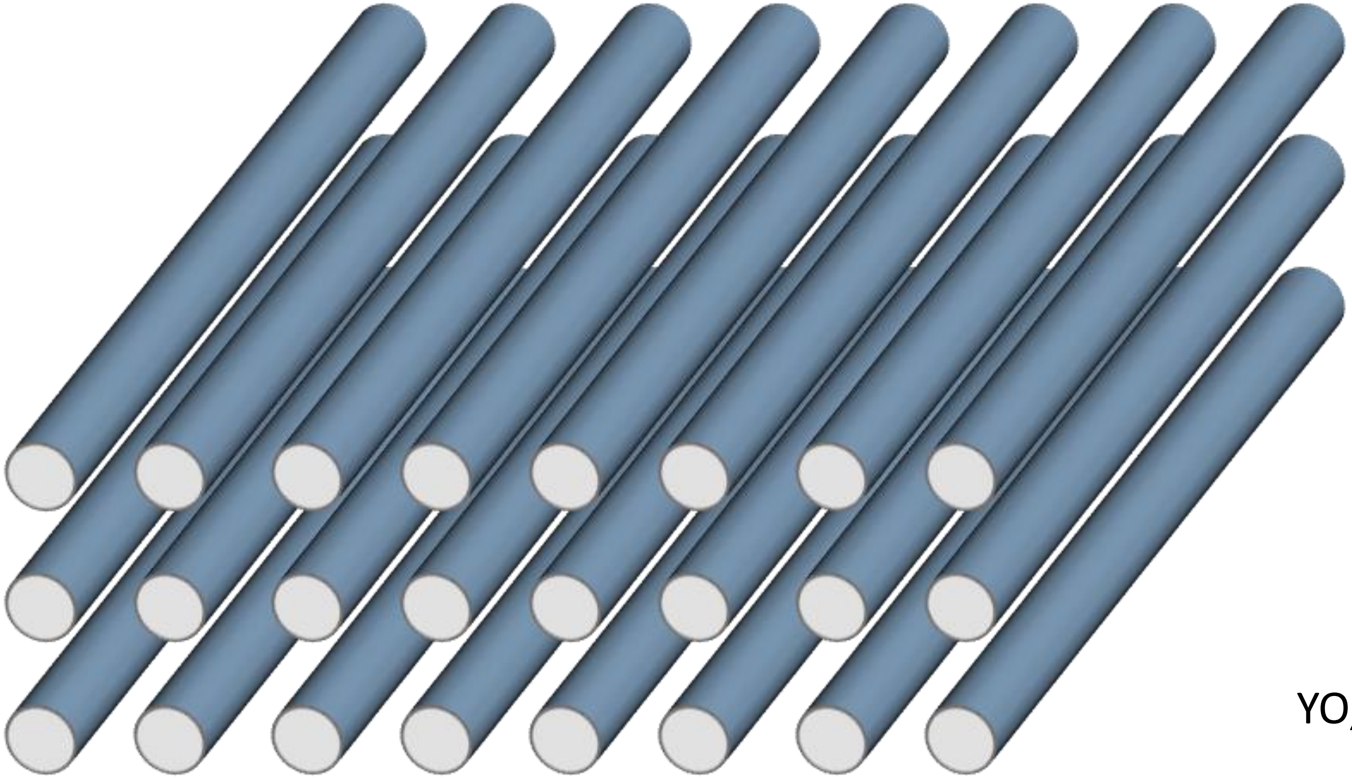




Hosur, Ryu, Vishwanath PRB 81, 045120 2010

3D Fractional Topological insulator

$$\phi \Rightarrow \eta$$



YO, Eran Sagi

Fractional Dirac Liquid?

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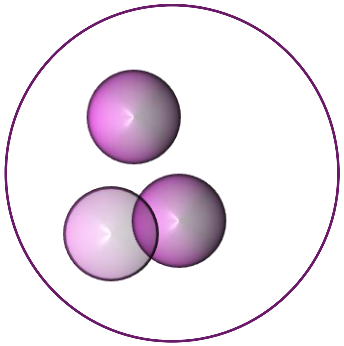

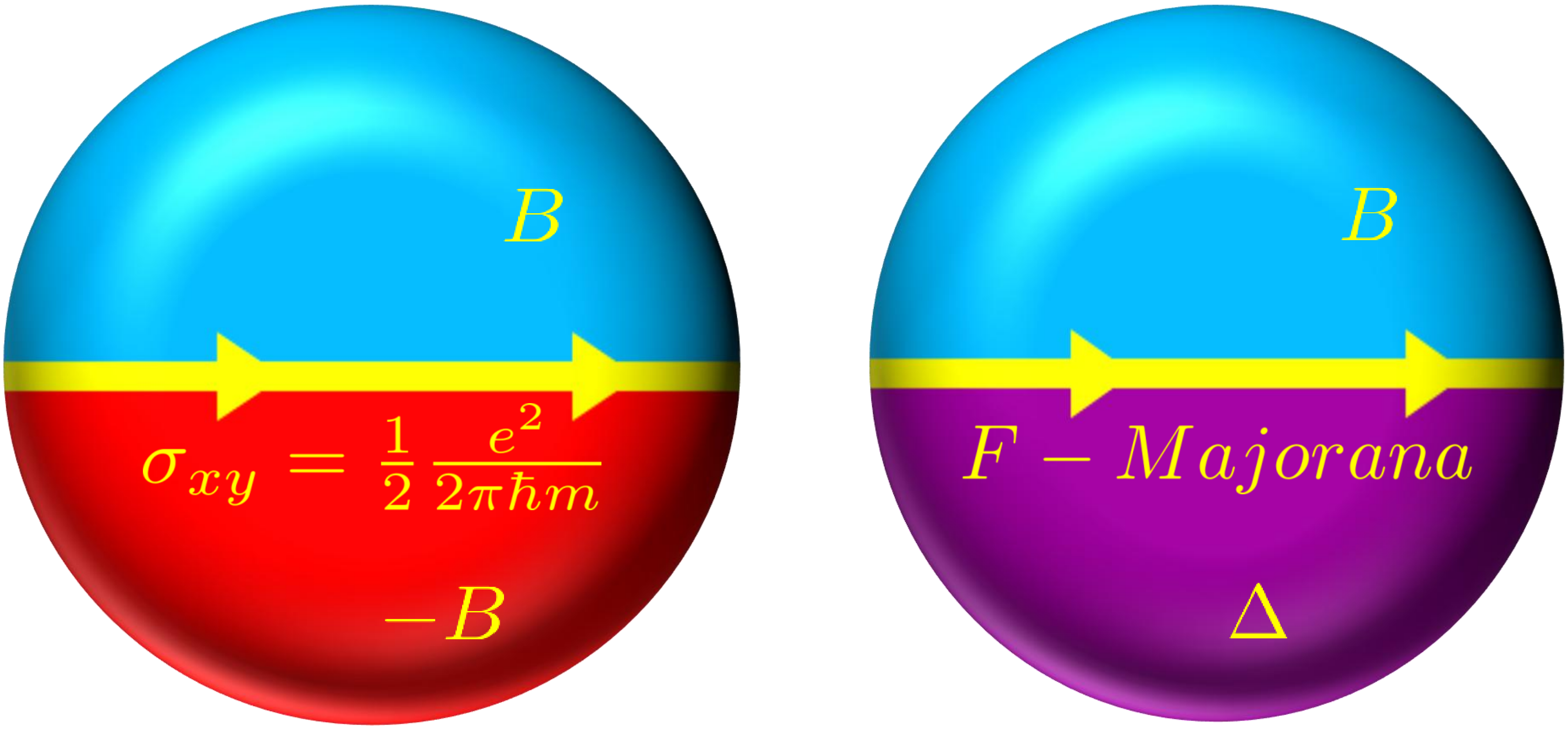

$$dI/dV \propto V^{m-1}$$

Summary

- Fractional Chern ins. and Fractional topological ins. from wires.
- Simple realization of Chern Ins.
- Frac. (6π) Josephson effect
- 3D Frac. Topological Ins.
- Frac. Majoranas modes, Frac. Metal

THANK YOU!