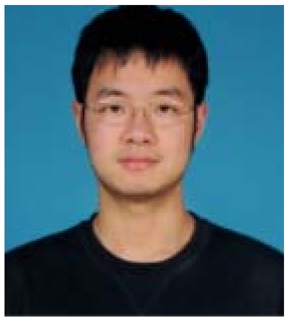
Correlation-driven topological phases in 1/4 doped electronic systems on the honeycomb lattice



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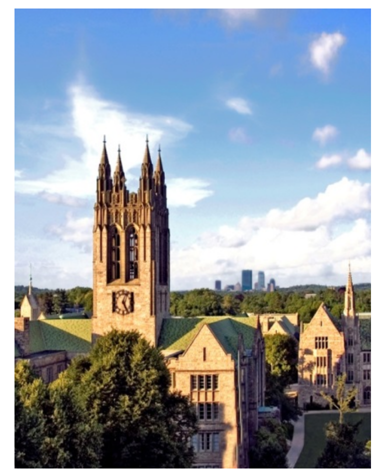

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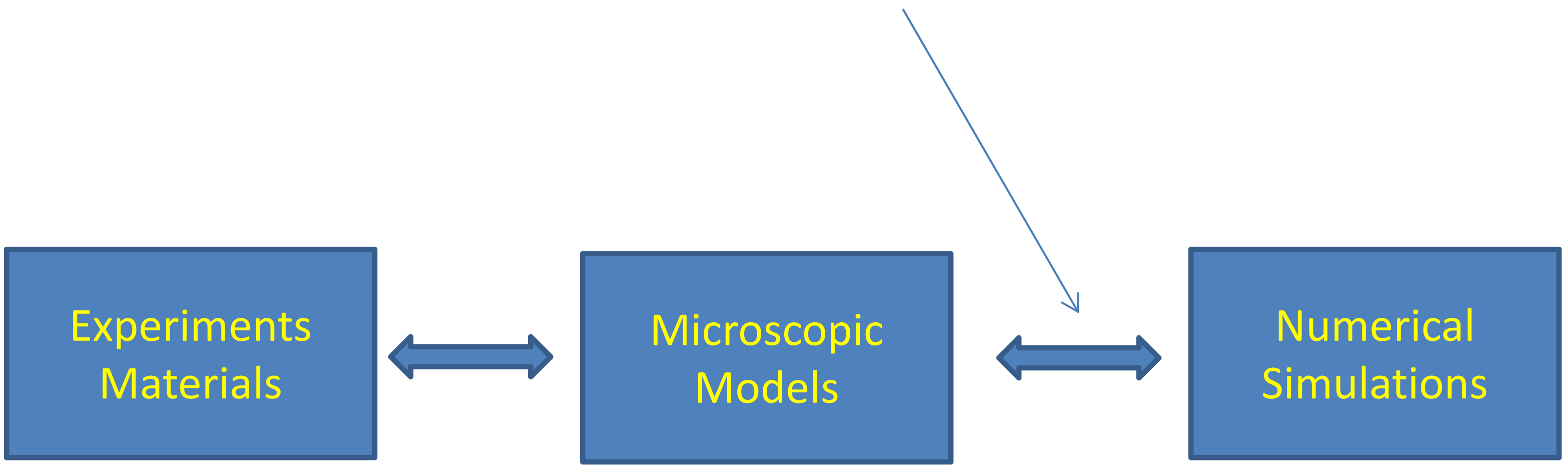
Motivation

- Developing new methods to reliably simulate phase diagrams of correlated electronic models in the presence of doping.

----can one say something sharp?



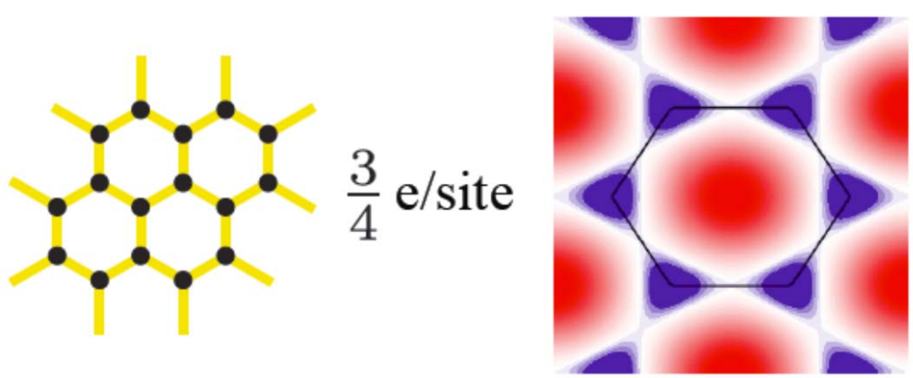
Motivation

- Developing new methods to reliably simulate phase diagrams of correlated electronic models in the presence of doping.

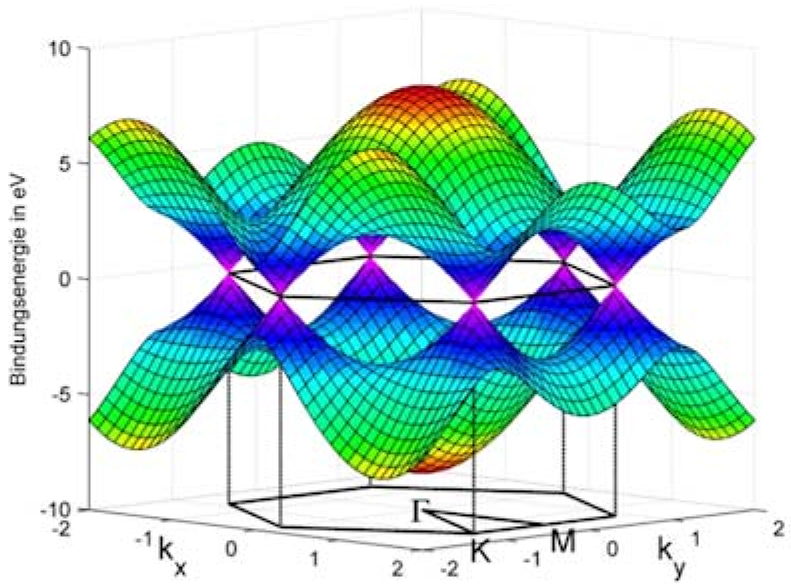
  ----can one say something sharp?

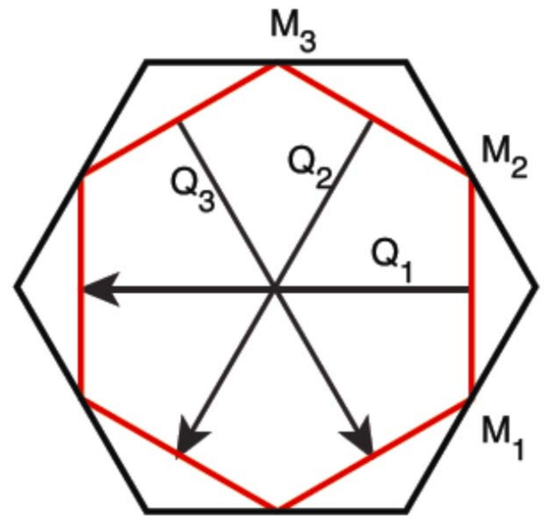
- Searching for correlation-driven topological phases in simple realistic models (Hubbard, t-J)

- What would happen if graphene is doped to ¼ ?



Graphene at ¼ doping





Hexagonal Fermi surface

Two features:

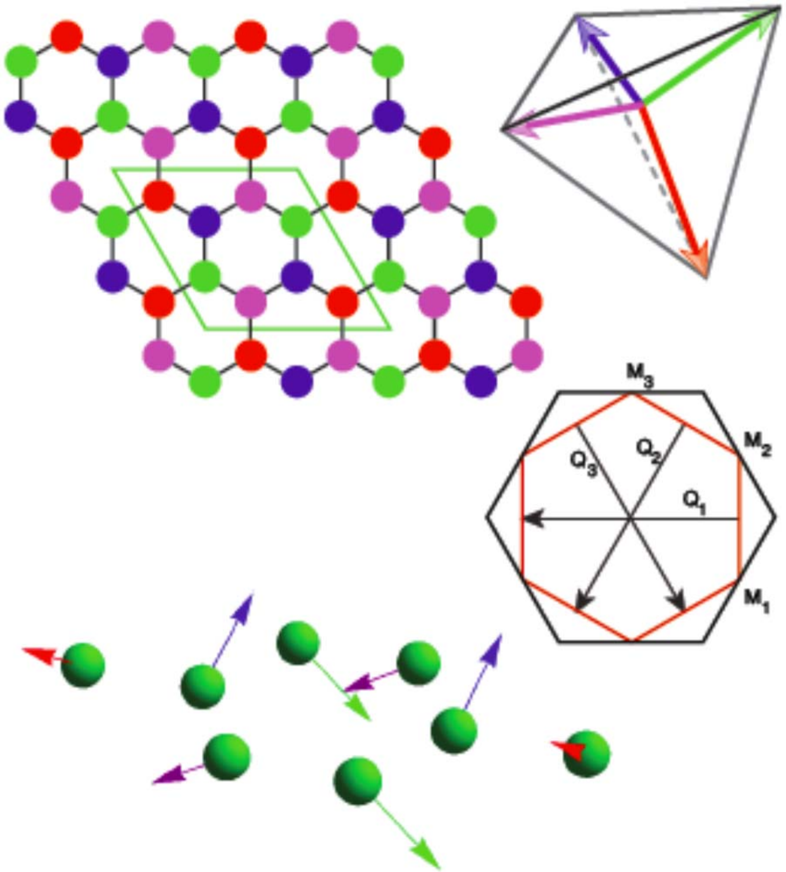
- Nested Fermi surface
- Van Hove singularity at three M-points

One expects instability even in the presence of weak interaction.

Graphene is intermediately correlated: short-range part U/t = 2 ~3 --- What will happen?

Previous proposals:

1) Chiral SDW (cSDW)



Raghu *et al* (2010); Li (2012)
Nandkishore *et al* (2012)
Wang *et al* (2012); Kiesel *et al* (2012)

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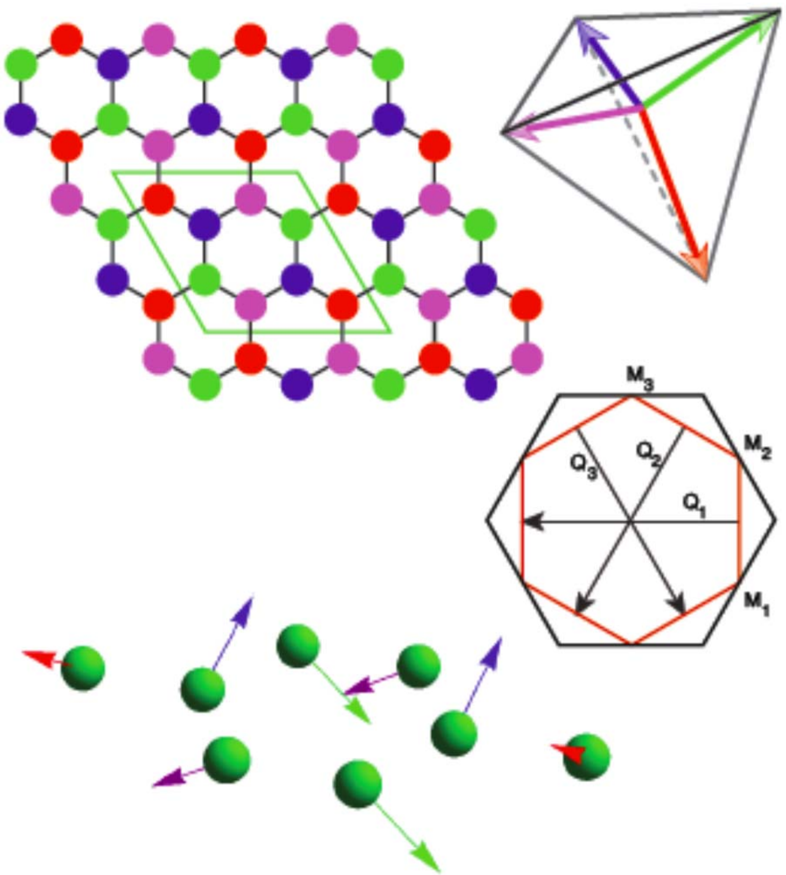
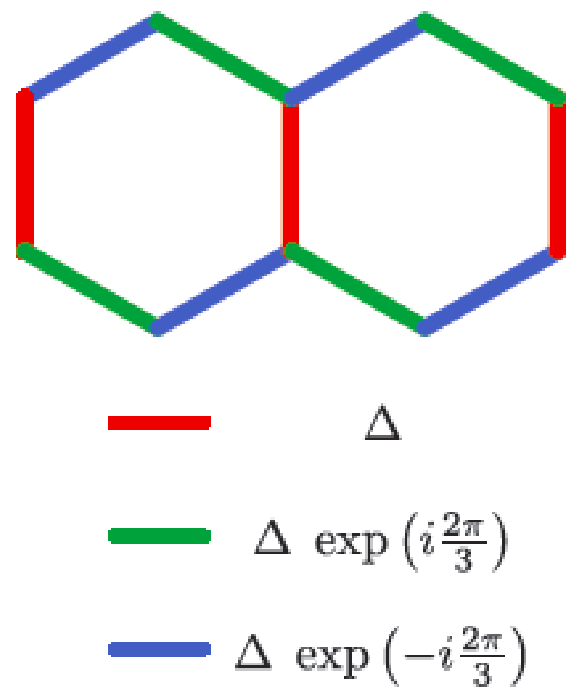


2) d+id superconductor



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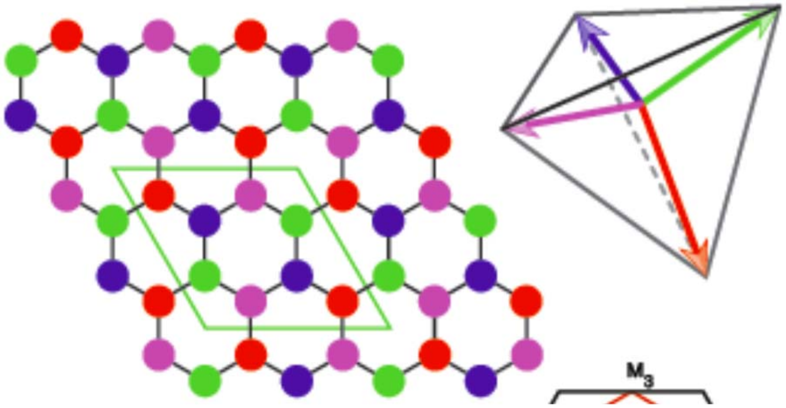
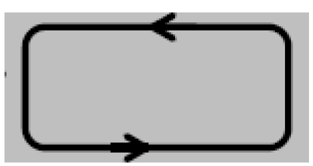
Interestingly, both proposed phases are correlation-driven topological phases

1) Chiral SDW (cSDW)

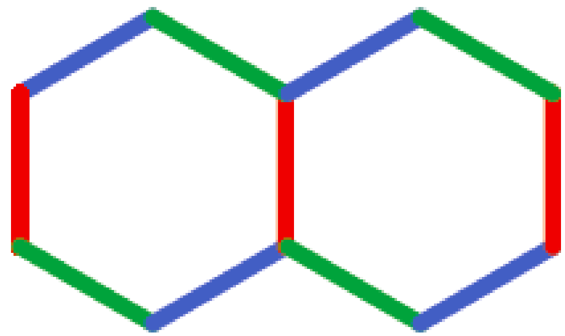


Quantum anomalous hall effect:
(Handane 1988, Nagaosa, Niu, Qi, Dai, Fang, Zhang…, Xue's group 2013)

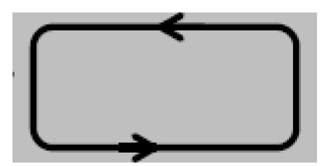


$$\sigma_{xy} = e^2/h$$

2) d+id superconductor



Spin quantum hall effect (NOT quantum spin hall):
(Senthil et.al, 1999)



$$j_x^z = \sigma_{xy}^s \left(-\frac{dB^z(y)}{dy}\right) \qquad \sigma_{xy}^s = \frac{\hbar}{4\pi}$$

- This motivates us to carefully study the phase diagrams of correlated models on the honeycomb lattice at ¼ doping, from intermediate to strong correlation strength

Hubbard model:

$$H_H = -t \sum_{<ij>,\alpha} (c^\dagger_{i\alpha} c_{j\alpha} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

t-J model:

$$H_{tJ} = P_G \sum_{<ij>,\alpha} -t(c^\dagger_{i\alpha} c_{j\alpha} + h.c.) P_G + P_G \sum_{<ij>} J(\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i \cdot n_j) P_G.$$

Limitation of previous studies:

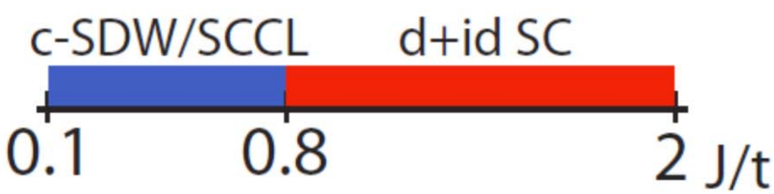
- Mean-field type studies: biased
- RG type studies: reliable for weak-couplings.

Our results:

- Using a combination of analytical construction of wavefunctions and various numerical simulations (ED, DMRG, VMC…):

Phase diagrams:

t-J model



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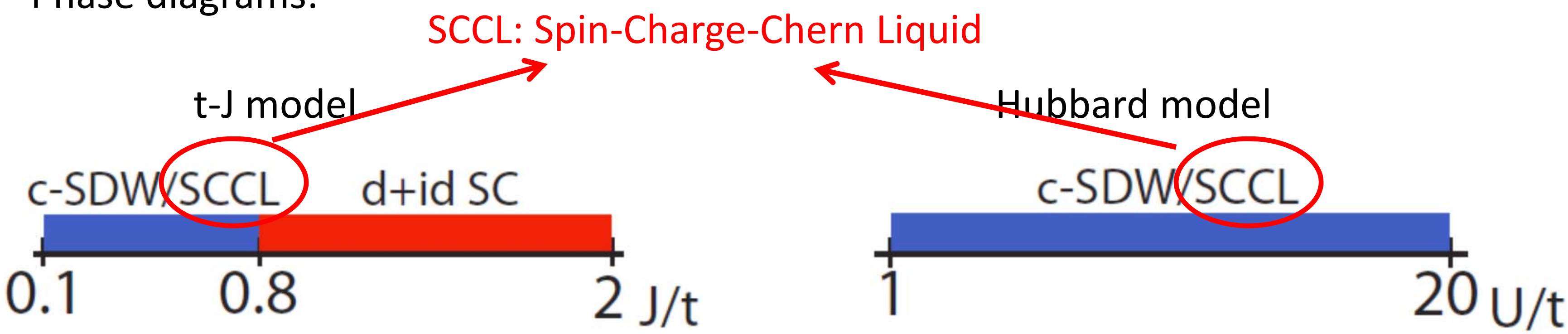

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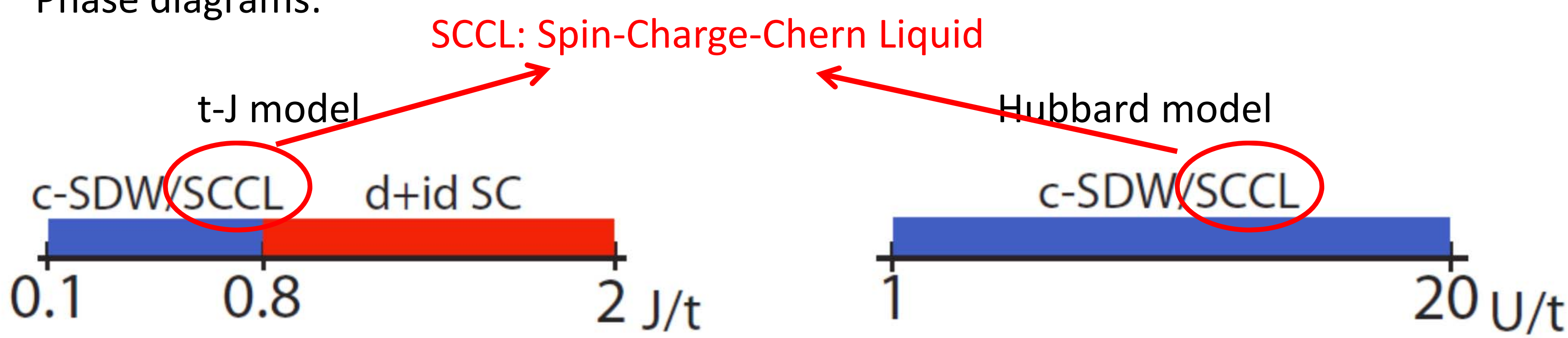

SCCL is a new type of topologically ordered phase: featuring charge-1/2 $\pi/4$-anyon excitations.

SCCL is the resulting phase after magnetic order in c-SDW is quantum melted.
In our finite-size numerical simulations, we cannot sharply distinguish SCCL from c-SDW.

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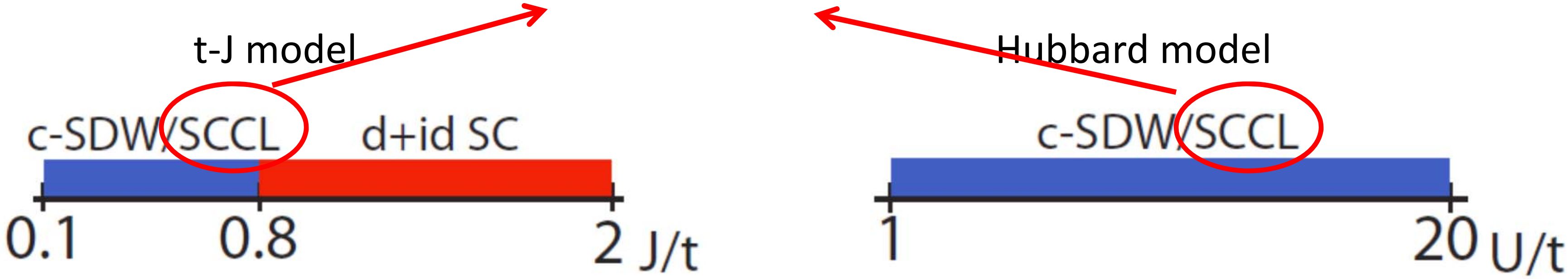
Phase diagrams:

SCCL: Spin-Charge-Chern Liquid



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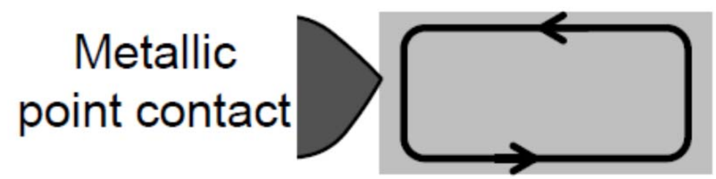
SCCL is the resulting phase after magnetic order in c-SDW is quantum melted.
In our finite-size numerical simulations, we cannot sharply distinguish SCCL from c-SDW.
Nevertheless they can be sharply distinguished in tunneling conductance experiment:

At low temperatures,
(assuming $G \ll e^2/h$)

SCCL: $G(T) \sim T^4$
c-SDW: $G(T) \sim$ Const.



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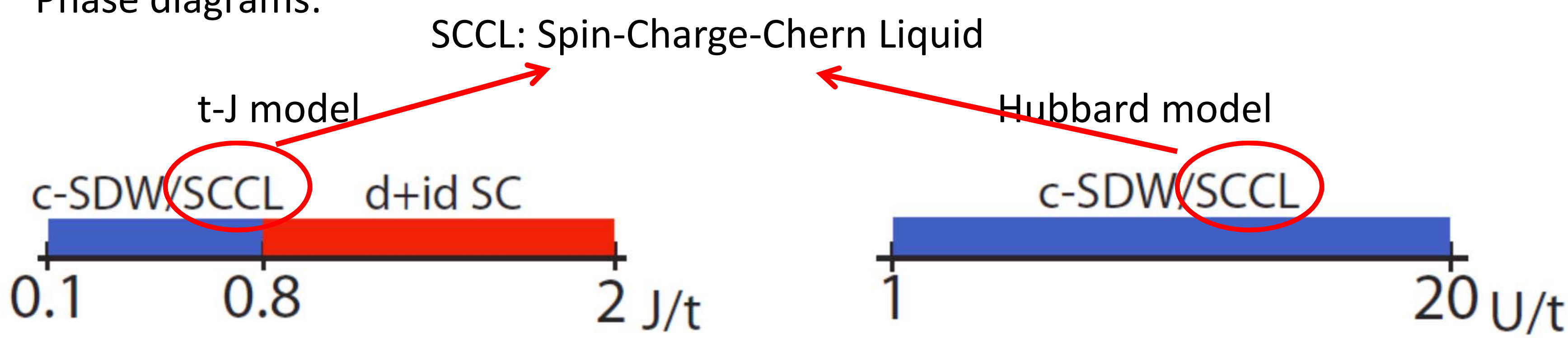


What we learned from these results:

- ¼ -doped graphene is likely *not* a d+id superconductor
- d+id superconductor is realized in a regime in the t-J model: $J/t>0.8$
- Proposed a new state of matter: SCCL with exotic anyon excitations, possibly realized in practical materials, and has characteristic transport experiment signature! $G(T) \sim T^4$

My plan

(1) Explain our method to reliably simulate doped correlated electronic systems.

It allows us to sharply distinguish d+id superconductor from the c-SDW/SCCL phase.

(2) More details on the new phase: SCCL

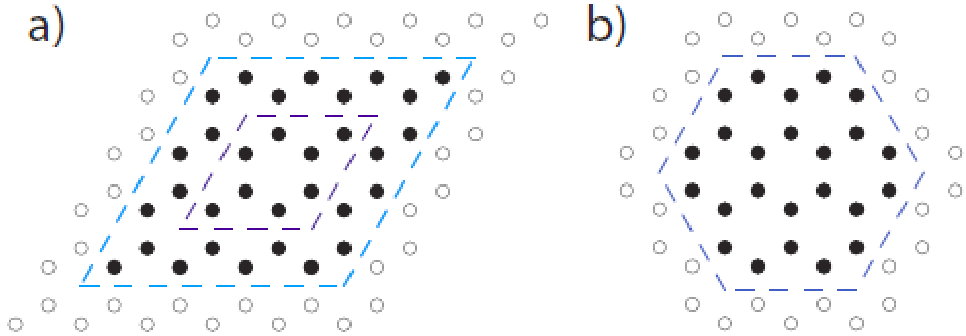
How did we obtain the phase diagrams?

- Historically, writing down quantum wavefunctions are known to be useful.

How did we obtain the phase diagrams?

- We analytically constructed symmetric quantum wavefunctions for the c-SDW/SCCL phase and the d+id SC phase on lattice,

using slave-particle methods.



Examples of symmetric lattices:
(a) 8-site,32-site (b) 24-site

why we bother to use these highly technical methods?

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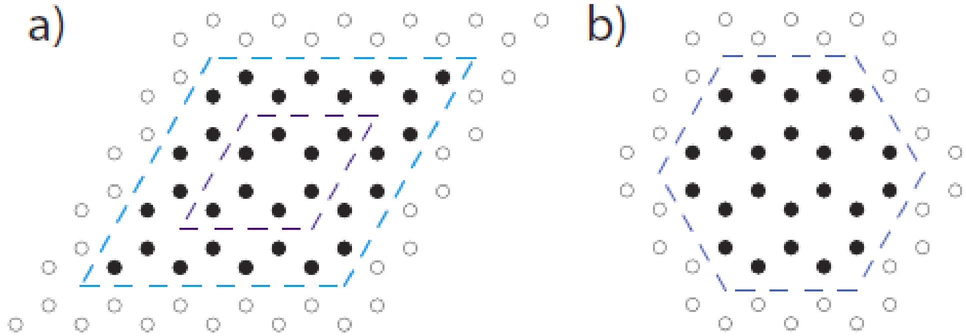


Note: no symmetry breaking on finite lattices. Wavefunctions should be symmetric (e.g, spin singlet).

(This is also why c-SDW and SCCL cannot be sharply distinguished on finite lattices)

Examples of symmetric lattices:
(a) 8-site,32-site (b) 24-site

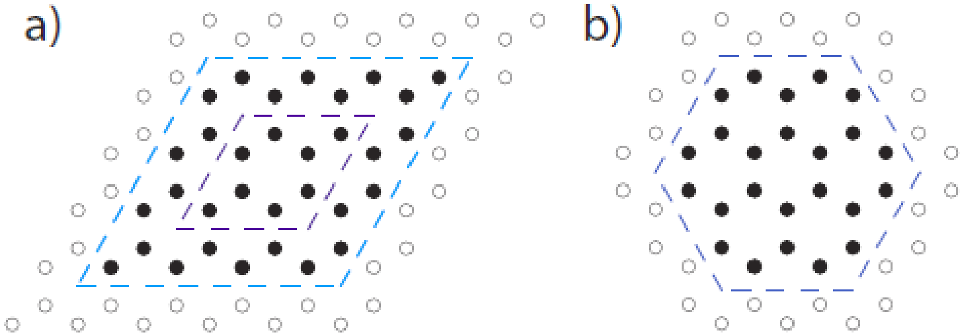
The slave-particle methods allow one to construct fully symmetric wavefunctions on symmetric finite-size lattice.

How did we obtain the phase diagrams?

- We analytically constructed symmetric quantum wavefunctions for the c-SDW/SCCL phase and the d+id SC phase on lattice, using slave-particle methods.

Then we study the ground state on symmetric lattices.

--- always two-fold irrep.



(we always use periodic boundary condition)

: on $4N \times 4N \times 2$ lattices

Sym.	c-SDW or SCCL	d+id SC
Lattice mom.	Γ	Γ
$60°$-rot. C_6	$\begin{pmatrix} e^{-\pi i/3} & 0 \\ 0 & e^{\pi i/3} \end{pmatrix}$	$\begin{pmatrix} e^{2\pi i/3} & 0 \\ 0 & e^{-2\pi i/3} \end{pmatrix}$
Mirror σ	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Time-Reveral	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Inversion(C_6^3)	-1	1

: on $(4N+2) \times (4N+2) \times 2$ lattices and Fig. (b)

Sym.	c-SDW or SCCL or d+id SC
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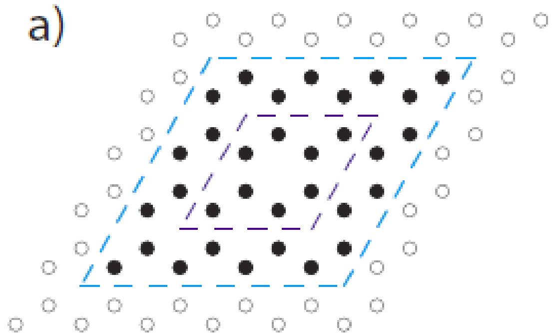

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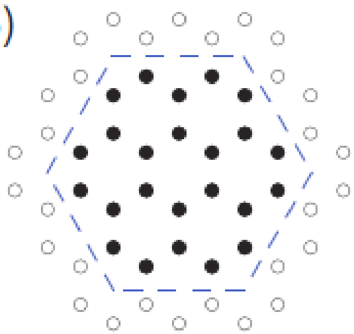

Then we study the ground state on symmetric lattices.

--- always two-fold irrep.

lattice distinguishing c-SDW/SCCL from d+id:

32-site sample

: on $4N \times 4N \times 2$ lattices

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Lattice mom.	Γ	Γ
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Mirror σ	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Time-Reveral	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Inversion(C_6^3)	1

Some detail: Constructing c-SDW/SCCL wavefunctions

$$c_{i\alpha} = b_{i\alpha} f_i^\dagger$$

$$H^{MF}_{c-SDW/SCCL}(b) = \sum_{ij} \left(B_{ij} b^\dagger_{i\alpha} b_{j\alpha} + A_{ij} b_{i\alpha} b_{j\beta} \epsilon_{\alpha\beta} + h.c.\right) - \mu_b \sum_i b^\dagger_{i\alpha} b_{i\alpha},$$

$$H^{MF}_{c-SDW/SCCL}(f) = \sum_{ij} \left(\chi_{ij} f_i^\dagger f_j + h.c.\right) - \mu_f \sum_i f_i^\dagger f_i,$$

$$|s_1, s_2, ...s_N\rangle \equiv \prod_{s_{i_a}=\uparrow} b^\dagger_{i_a,\uparrow} \prod_{s_{i_b}=\downarrow} b^\dagger_{i_b,\downarrow} \prod_{s_{i_c}=0} f^\dagger_{i_c} |0\rangle$$

Definition of wavefunction:

$$\begin{aligned} &\langle s_1, s_2, ...s_N | \Psi_{c-SDW/SCCL}\rangle \\ =&\langle 0 | [\prod_{s_{i_a}=\uparrow} b^\dagger_{i_a,\uparrow} \prod_{s_{i_b}=\downarrow} b^\dagger_{i_b,\downarrow}]^\dagger |\Psi_b^{MF}\rangle \\ &\cdot \langle 0 | [\prod_{s_{i_c}=0} f^\dagger_{i_c}]^\dagger |\Psi_f^{MF}\rangle; \end{aligned}$$

A product of permanent and determinant

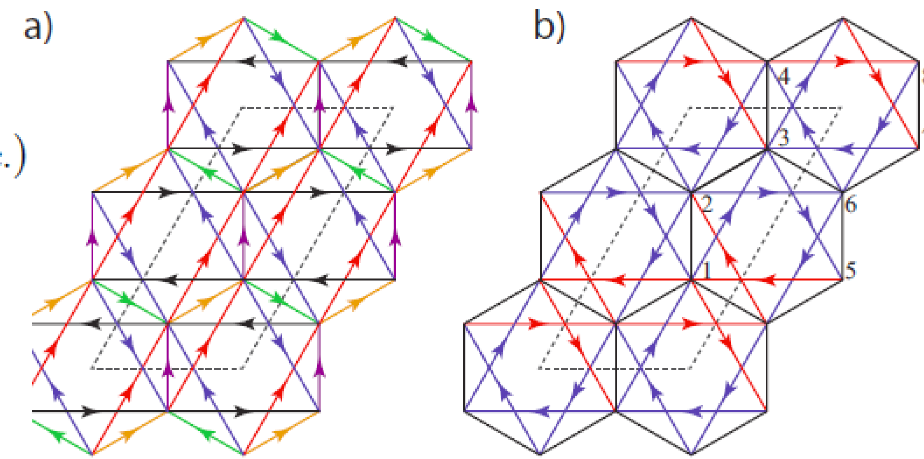

FIG. 4. (color online) The real space pattern of the slave-fermion amplitudes describing the c-SDW/SCCL phases. The dashed line encircles the doubled unit-cell. (a): The nearest neighbor(NN) and next nearest neighbor(NNN) boson pairing amplitudes A_{ij} are directional (labeled by arrows) since $A_{ij} = -A_{ji}$. A_{ij} on the NN(NNN) bonds have the same magnitude respectively. Their different phases are represented by different colors. Black: 1; Violet: $e^{i\pi/2}$; Green: $e^{i5\pi/6}$; Orange: $e^{i\pi/6}$; Red: $e^{i\pi/3}$; Blue: $e^{i2\pi/3}$. (b): The NN(NNN) boson/fermion hopping amplitudes B_{ij}/χ_{ij} also have uniform magnitudes respectively. When they are complex, the amplitudes are directional $B_{ij} = B^*_{ji}, \chi_{ij} = \chi^*_{ji}$ (labeled by arrows). The phases are illustrated by colors. Black: ± 1; Blue: $e^{i\phi}$; Red: $-e^{i\phi}$. Here the real number $\phi = \phi_b$ for bosons and $\phi = \phi_f$ for fermions. ϕ_b and ϕ_f can be viewed as two varia-

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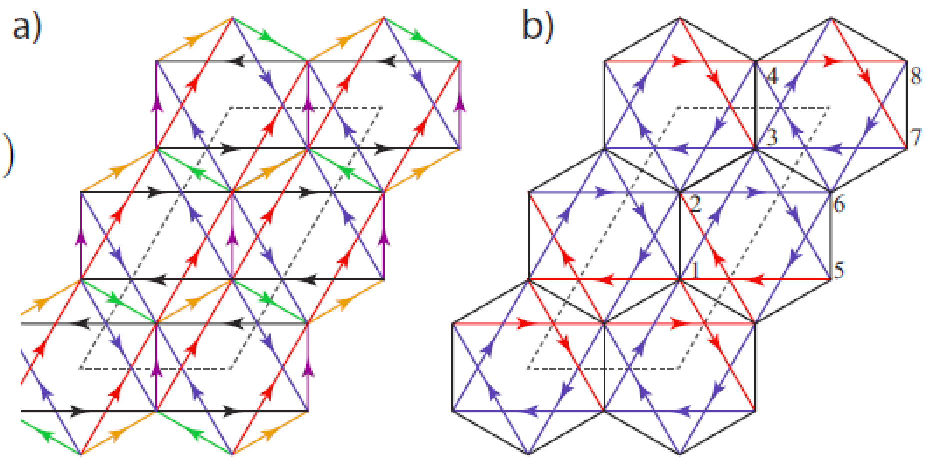

The complicated pattern is to ensure:

(1) Full lattice symmetry
(2) Tetrahedral magnetic pattern

Definition of wavefunction:

$$\begin{aligned} &\langle s_1, s_2, \ldots s_N | \Psi_{c-SDW/SCCL} \rangle \\ =& \langle 0 | [\prod_{s_{i_a}=\uparrow} b^\dagger_{i_a,\uparrow} \prod_{s_{i_b}=\downarrow} b^\dagger_{i_b,\downarrow}]^\dagger | \Psi^{MF}_b \rangle \\ &\cdot \langle 0 | [\prod_{s_{i_c}=0} f^\dagger_{i_c}]^\dagger | \Psi^{MF}_f \rangle; \end{aligned}$$

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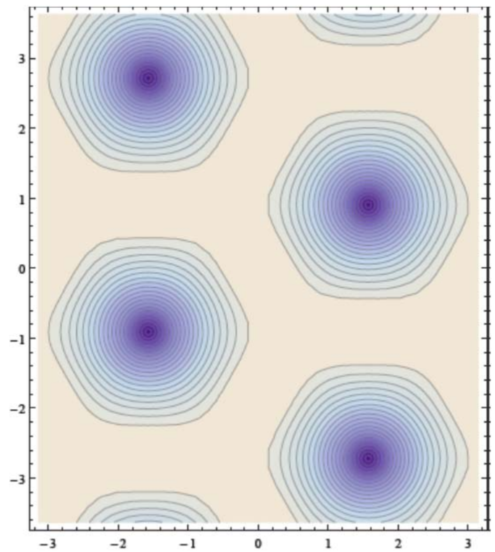
Mean-field picture:

- Fermionic holon fills a Chern band.
- Bosonic spinon band structure:



Boson band minima touch zero?

Yes: (possible only in thermodynamic limit)
Boson condensation → long-range c-SDW

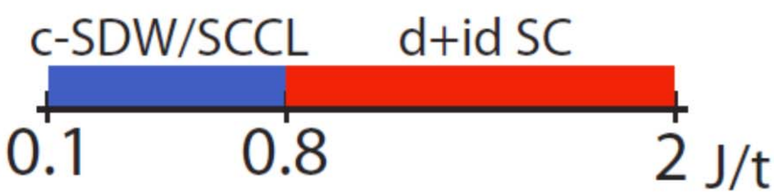
No: SCCL (fully gapped in bulk)

How did we obtain the phase diagrams?

- 32-site is still too large for exact diagonalization, so we performed the DMRG(density matrix RG) simulation on 32-site.

DMRG results: (using Itensor software: itensor.org)

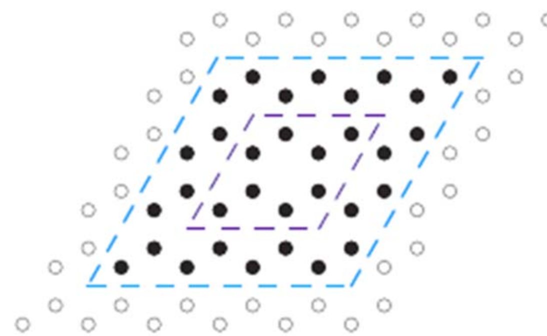
t-J model



Hubbard model

Blue: lattice quantum number matches c-SDW/SCCL

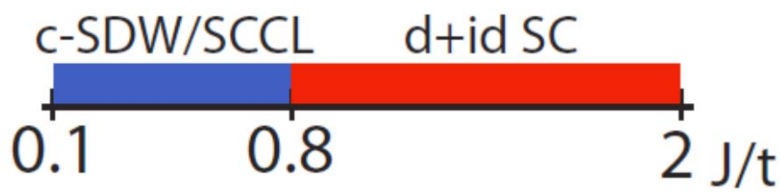
Red: lattice quantum number matches d+id



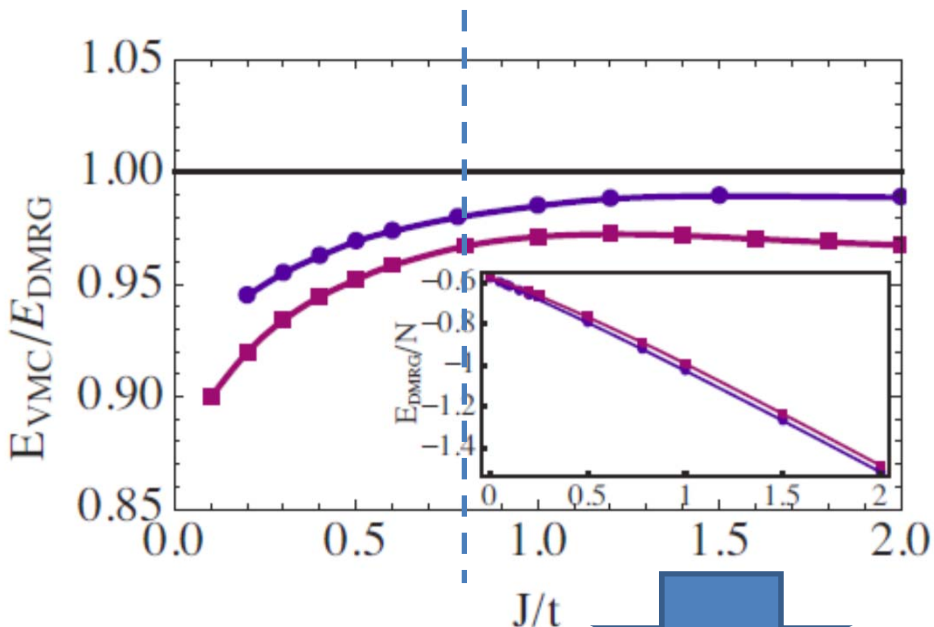
But can there be other phases?

How did we obtain the phase diagrams?

- Can there be other phases?



First check: comparing with variational Monte Carlo results for the d+id wavefunction.



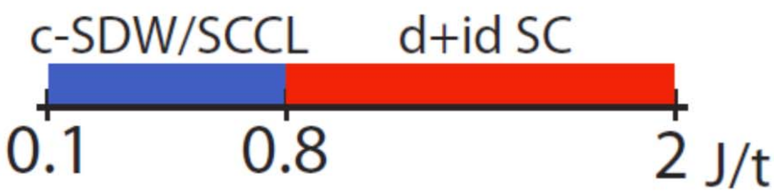
Blue: 24-site sample

Red: 32-site sample

Single-parameter d+id variational wavefunction captures ~97-99% ground state energy

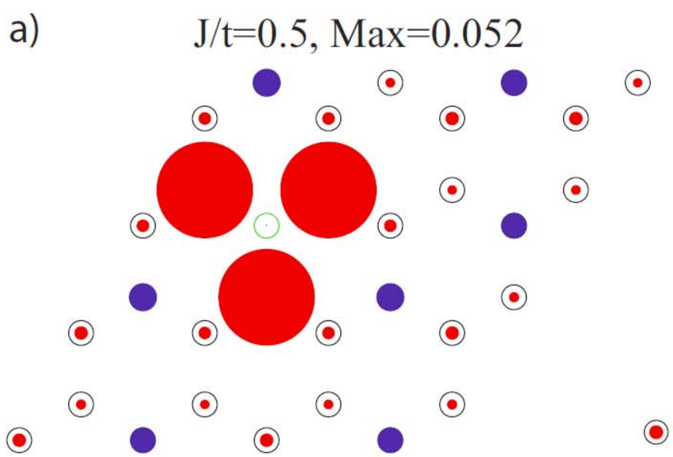
How did we obtain the phase diagrams?

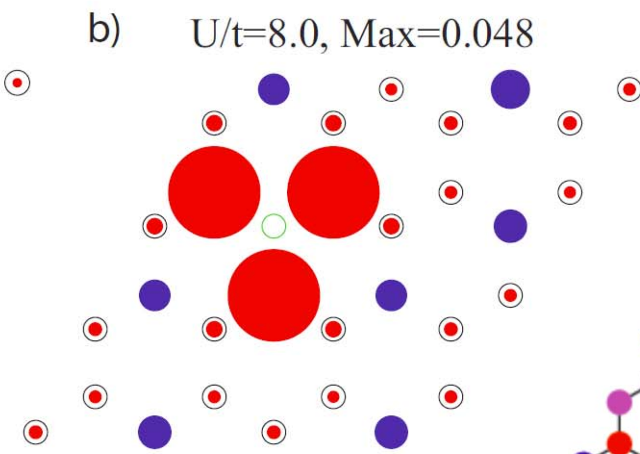
- Can there be other phases?

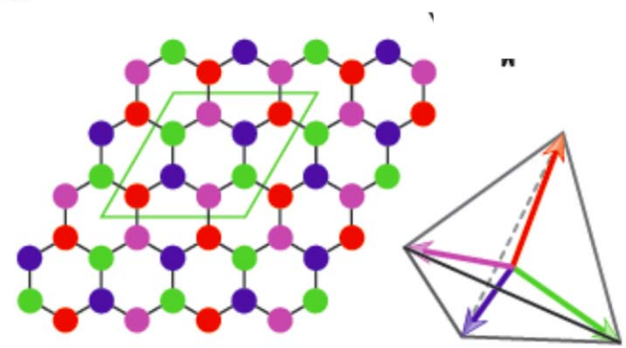


Spin-Spin correlation function in the c-SDW/SCCL regimes:

Red: S(green-site)·S <0,
Blue: S(green-site)·S >0 exactly matches the tetrahedral pattern.

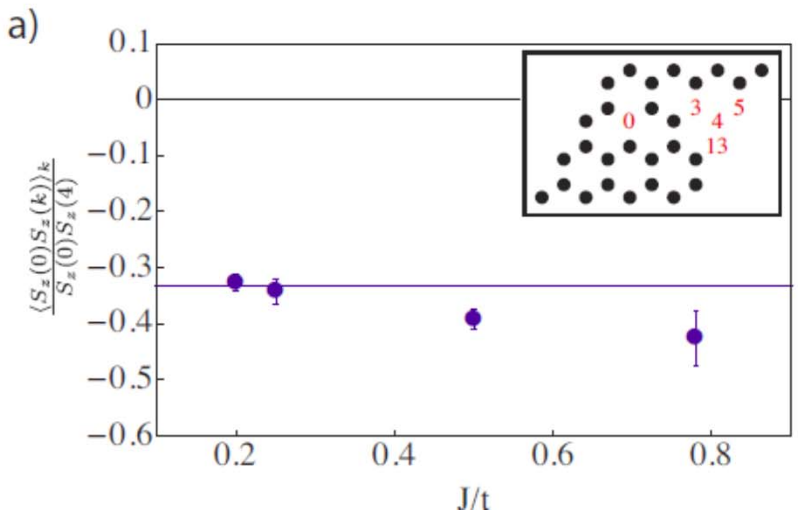


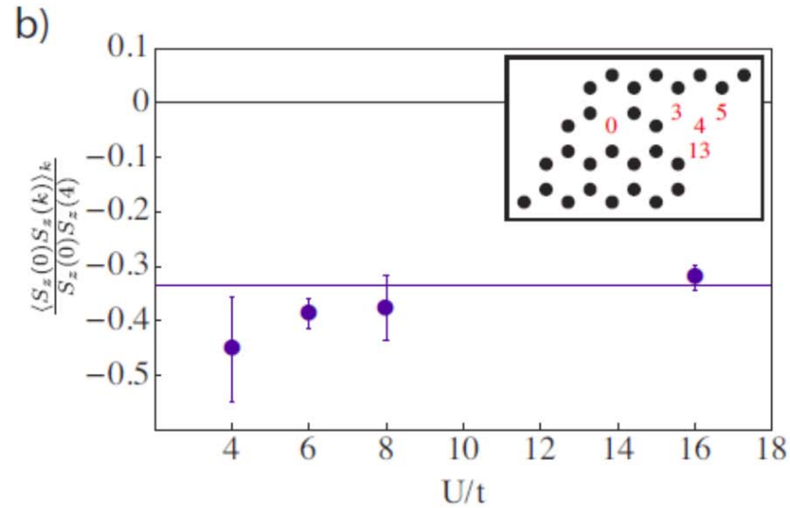


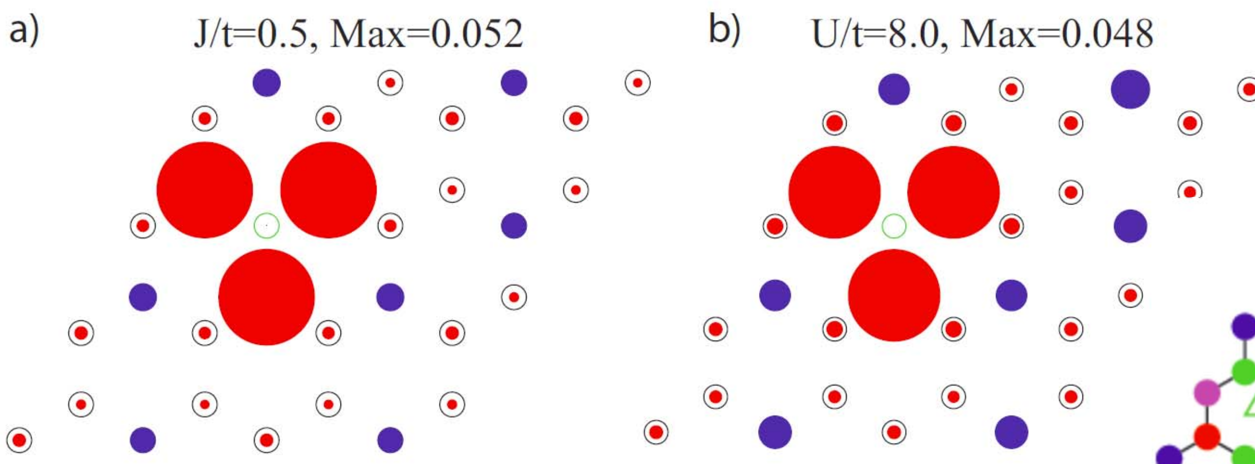


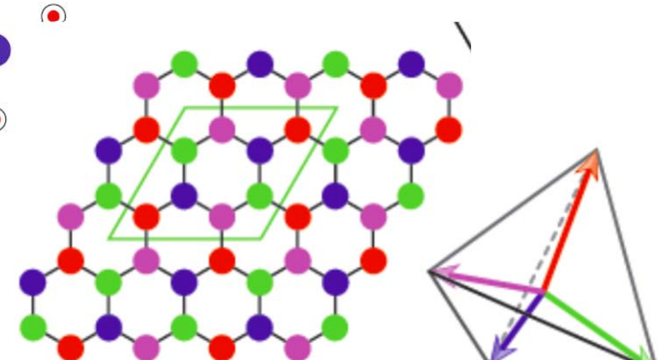
$<S_i \cdot S_j>$ in the DMRG ground state

How did we obtain the phase diagrams?





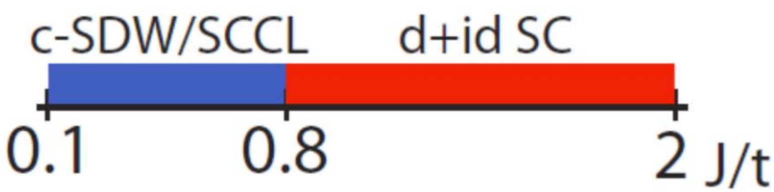




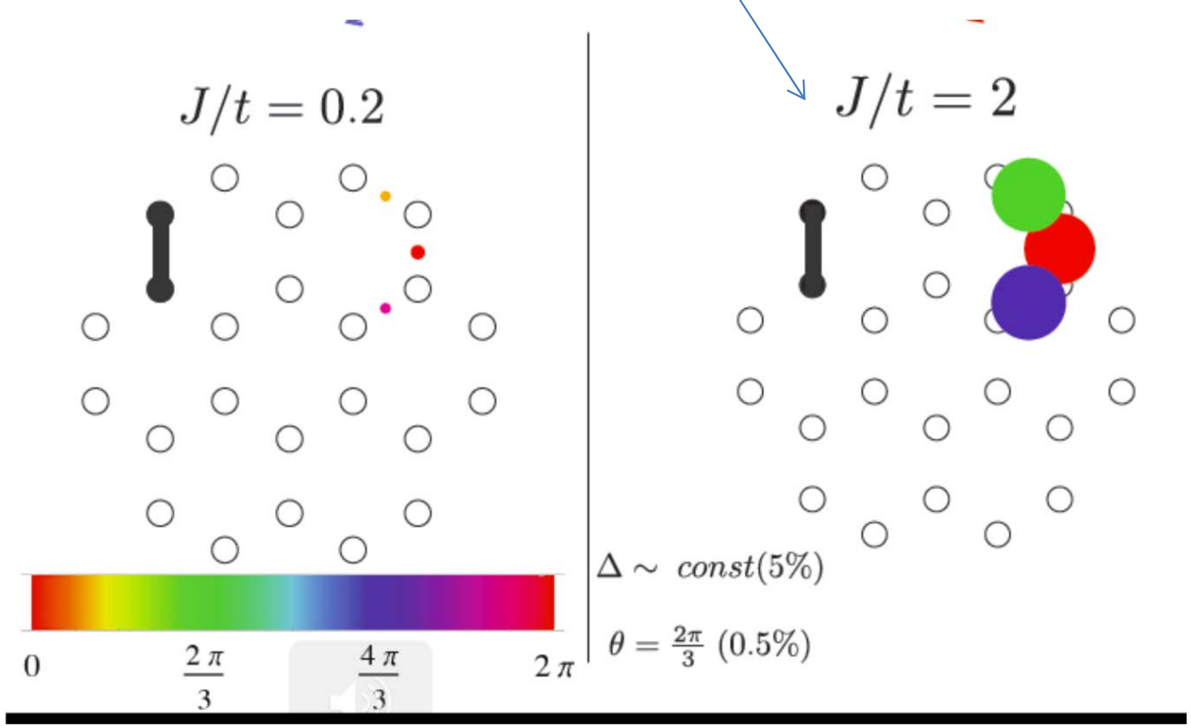
<Si·Sj > in the DMRG ground state

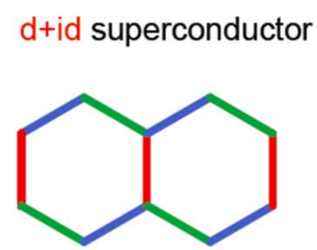
How did we obtain the phase diagrams?

- Can there be other phases?



Pair-pair correlation function in the d+id regimes: exactly matches the d+id pattern





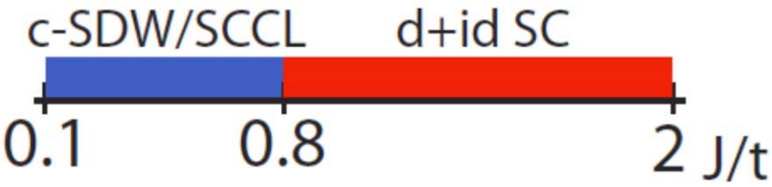
My plan

(1) Explain our method to reliably simulate doped correlated electronic systems.

A combination of analytical and reliable numerical methods shows strong evidences supporting these phase diagrams.

(2) More details on the new phase: SCCL

My plan



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The Spin-Charge-Chern Liquid

- SCCL can be viewed as the resulting phase after the magnetic order in the c-SDW is quantum melted. Fully gapped in the bulk.

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- Two comments:

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(2) SCCL may be easier to be detected, comparing with QSL.

SCCL has characteristic electric transport signature! $G(T) \sim T^4$

The Spin-Charge-Chern Liquid: intuitive understanding

- In the bulk, the spin-neutral charge-1/2 π/4-anyon (visons) can be viewed as the counterpart of the Z_2 vortex in the c-SDW.

Note: c-SDW order parameter manifold= SO(3), SU(2)/SO(3)=Z_2

Z_2 vortex carries π-Berry's phase, coupling to a Chern-band, giving charge-1/2.

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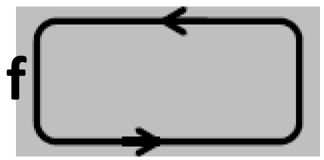
- On the edge, the chiral electron mode in the c-SDW lost spin-coherence, and became change-1, spin-neutral chiral holon mode.

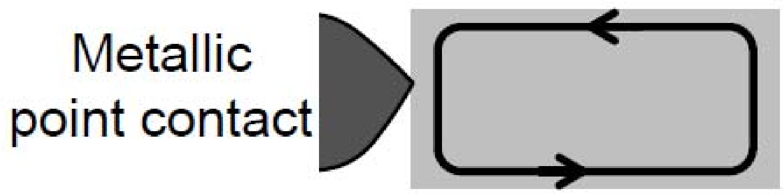


Holon **f** carries charge-1 but spin-0.

The Spin-Charge-Chern Liquid: exp. signatures

Bulk: QAH E&M response: $j_x = \sigma_{xy} E_y$ where $\sigma_{xy} = e^2/h$

Boundary: Gapless chiral holon f $\Longrightarrow$ insulating

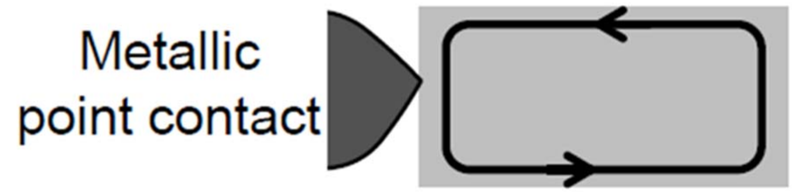


Pair-tunneling $\Longrightarrow$ $G \sim T^4$ (c-SDW has $G \sim const$)

(assuming weak-tunneling regime: $G \ll e^2/h$)

Some detail: tunneling conductance

- Point junction



(assuming weak-tunneling regime: G<< e^2/h)

Consider SCCL: pair-tunneling into edge is allowed due to the **bosonic spinon pairing**:

$$H_{tunn} = [t f^\dagger(x=\xi) f^\dagger(x=0)\psi_{M,\uparrow}(x=0)\psi_{M,\downarrow}(x=0) + h.c.]$$

f: holon at SCCL edge ψ: electron in the metal lead

Perturbative RG: dimension analysis

$$[f]=[\psi]=1/2, f(x=0)f(x=\xi) \sim f\partial_x f$$

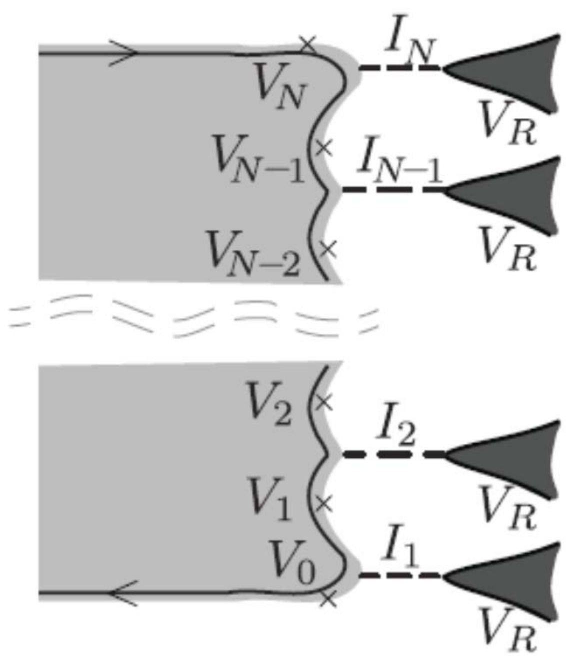
$$t_{eff}(T) \sim T^{(2+1)-1} = T^2 \quad \text{conductance: } G \sim t_{eff}(T)^2 \sim T^4$$

Some detail: tunneling conductance

- Line junction: (the usual experiment setup)

  can be modeled as an irregular array of point junctions

  (assuming each point contact is in weak-tunneling regime: G<< e^2/h)



We find in SCCL: $$G(T)=\frac{e^2}{h}\left[1-e^{-\frac{T^4}{T_K^4}}\right]$$

Even for the SCCL phase, universal conductance $G=e^2/h$ can be reached in regime $T>T_K$.

T_K is non-universal energy scale determined by the microscopic details of the line-junction.

Note: $G=e^2/h$ has been viewed as one signature of the QAH edge mode in c-SDW.
This calculation shows that $G=e^2/h$ CANNOT distinguish c-SDW and SCCL: need $G<<e^2/h$ regime

The Spin-Charge-Chern Liquid: full effective theory

- SCCL has an unusual Z_2 topological order.

The low energy effective theory:

multi-component Chern-Simons theory (X-G Wen…)

$$L_{eff} = \frac{\epsilon_{\mu\nu\lambda}}{4\pi} \sum_{I,J} a_\mu^I K_{I,J} \partial_\nu a_\lambda^J$$

$$\mathbf{K} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ -1 & 2 & 0 \end{pmatrix}$$

charge vector $\mathbf{t}_c = (1, 0, 0)$

S_z vector $\mathbf{t}_{S_z} = (1/2, -1, 0)$

Quasiparticle	Charge	Spin	Statistics
Spinon	0	½	0
Vison	½	0	π/4
Bound SV	½	½	5π/4

Summary

- Simple realistic models realizing correlation-driven topological phases. Relevant for graphene and other correlated solid-state or cold-atom systems on the honeycomb lattice.

- We propose a new topologically ordered phase: SCCL, with anyon excitations in the bulk and characteristic transport signatures. SCCL may be realized in practical materials.

An advertisement --- symmetric tensor networks (arXiv:1505.03171)

- One major challenge of numerical simulations:

Variational wavefunctions → optimize energy --- a local property

Quantum phases --- generally need thermodynamic limit to define

To fully determine phase diagram:

require scaling to larger sample sizes ---often very challenging

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NO:

candidate phases could have distinct quantum numbers on finite size samples.

trivial example: ferromagnet vs. antiferromagnet

In this talk: c-SDW/SCCL vs. d+id SC

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Quantum phases --- generally need thermodynamic limit to define

- But do we always need large sample sizes to distinguish candidate phases?

NO:

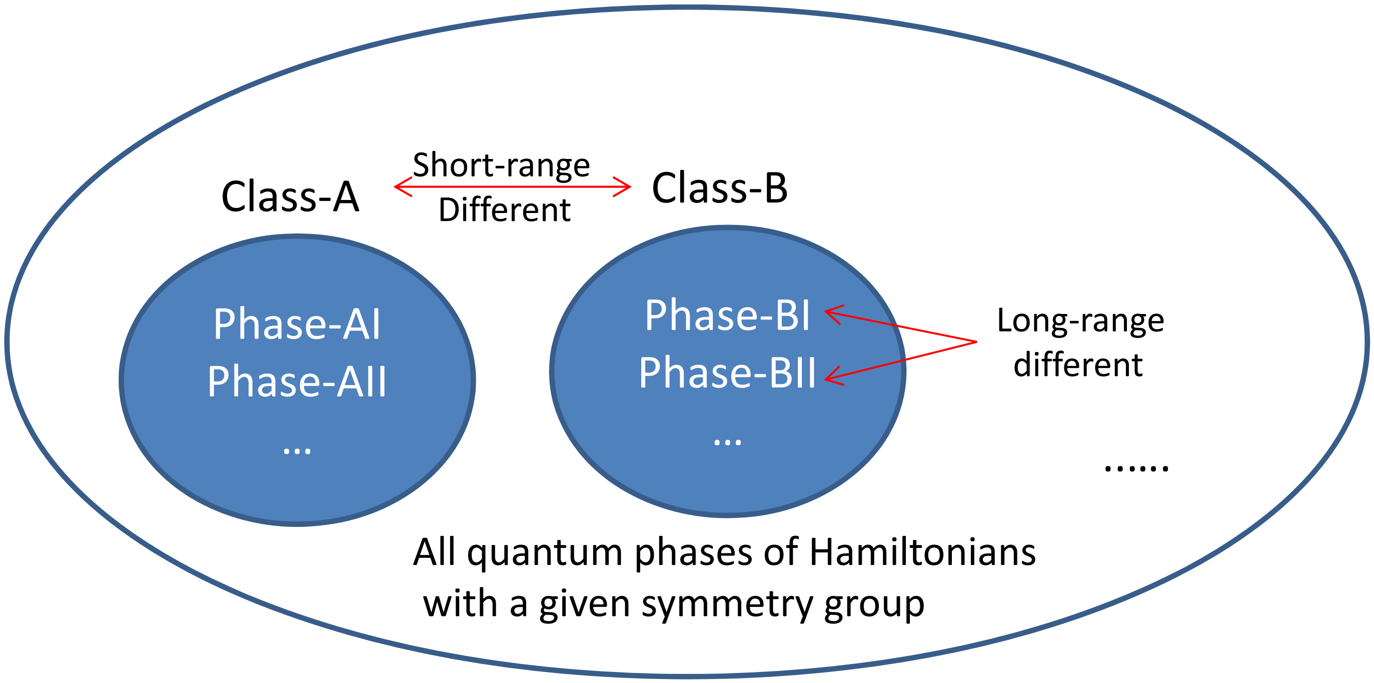
candidate phases could have distinct quantum numbers on finite size samples.

These candidate phases have completely different short-range physics. And distinguishing them should be much easier.

An advertisement --- symmetric tensor networks (arXiv:1505.03171)

- This discussion motivates the following intuitive picture:

--- a crude classification of quantum phases



- Different phases in different classes are distinguished by short-range physics (how symmetry is implemented in local patches of the wavefunction is different.)
- Different Phases in the same class are distinguished by long-range physics(symmetry breaking)

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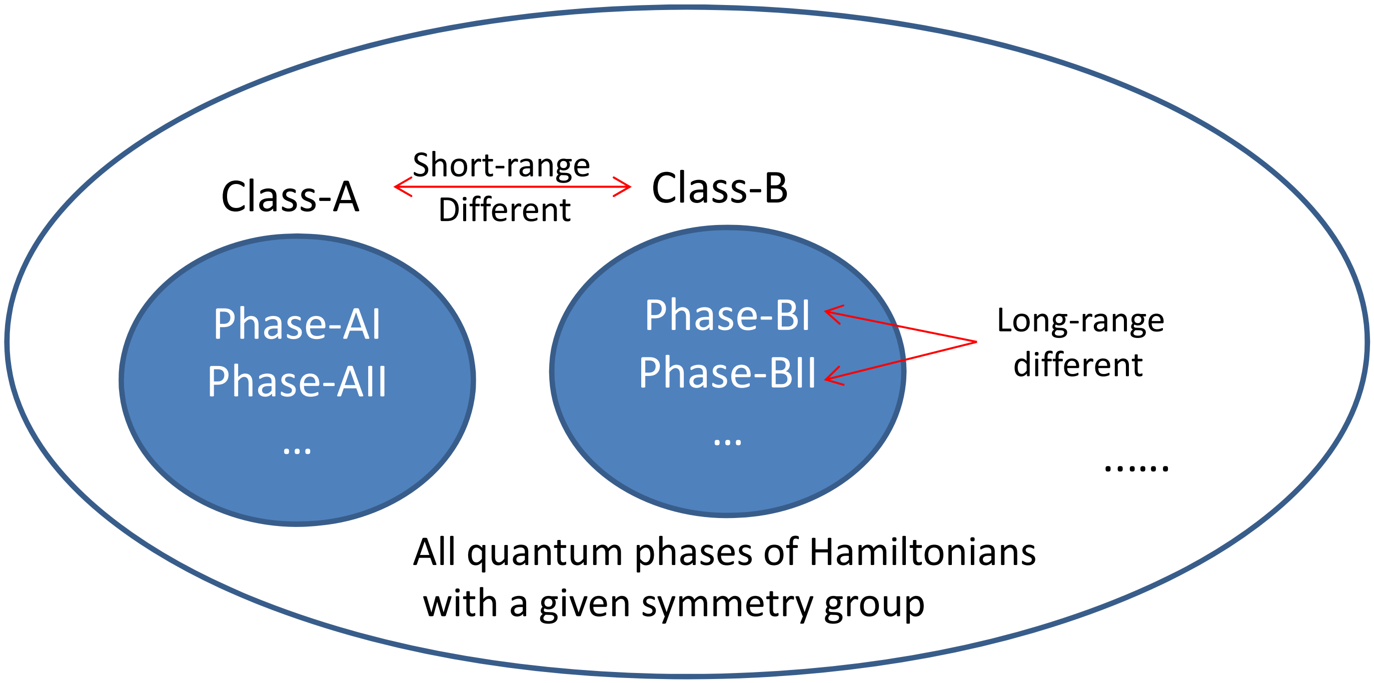

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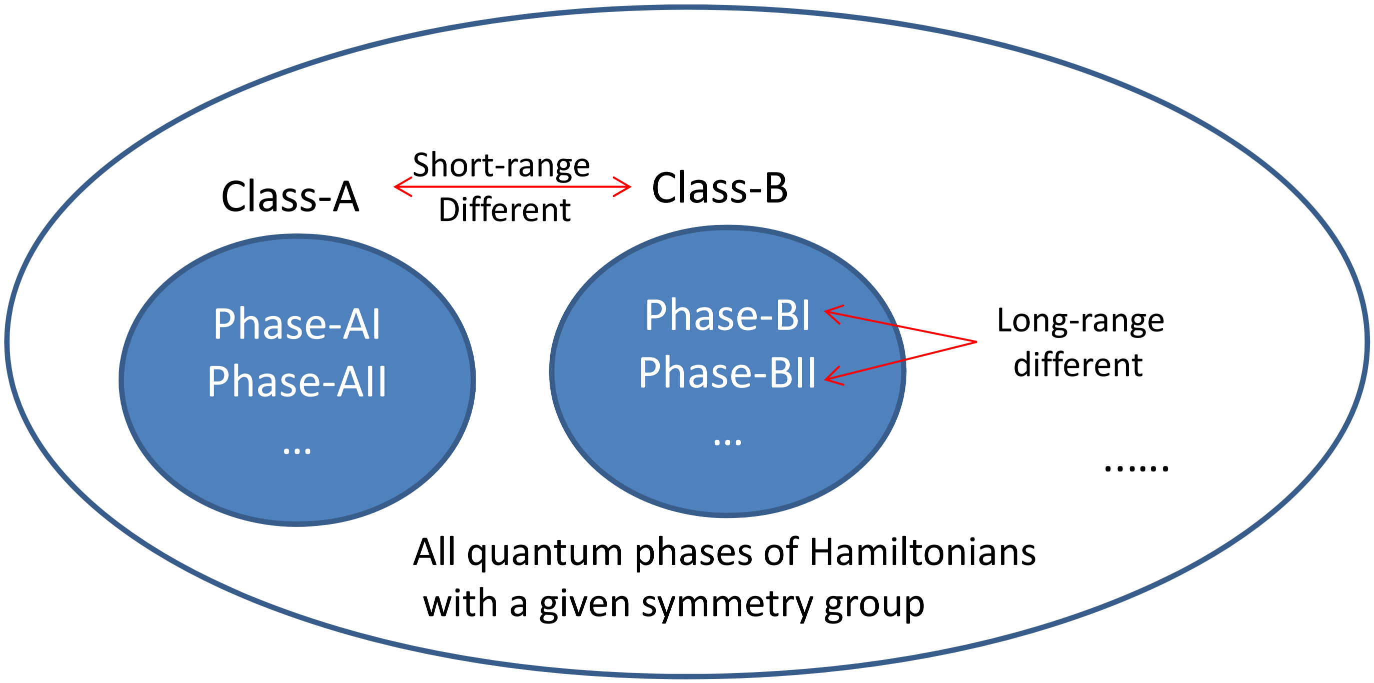

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--These are important questions:
solutions would lead to a systematic numerical method to perform the "short-range" part of the simulation task, which is also very useful for the "long-range" part.

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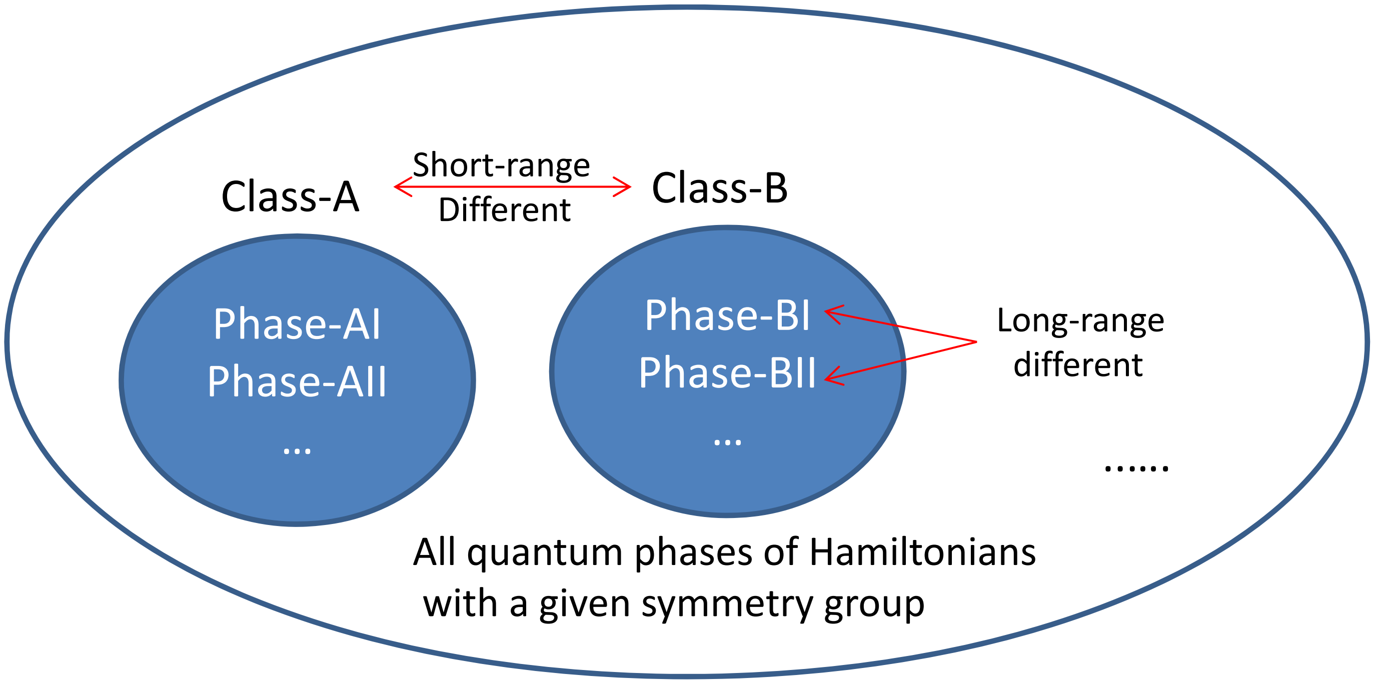

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--These are important questions:

Recall in the system studied in this talk, we are lucky --- we have a good guess of what are the candidate phases. But answers to these questions solve the general problems.

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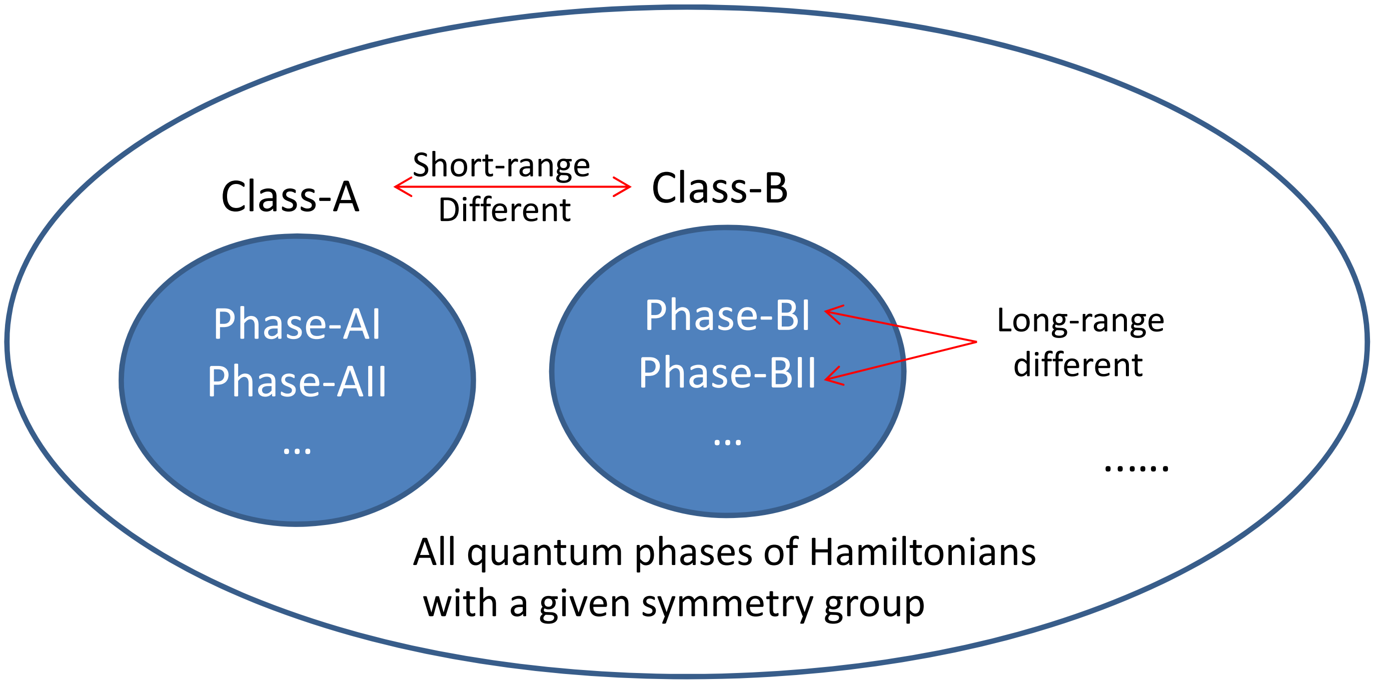

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Our work arXiv:1505.03171 is an partial answer for these questions using tensor networks:
We develop a general machinery: If symmetry and microscopic d.o.f are specified, our machinery classifies crude classes/constructs generic wavefunctions for each class.

An advertisement --- symmetric tensor networks (arXiv:1505.03171)

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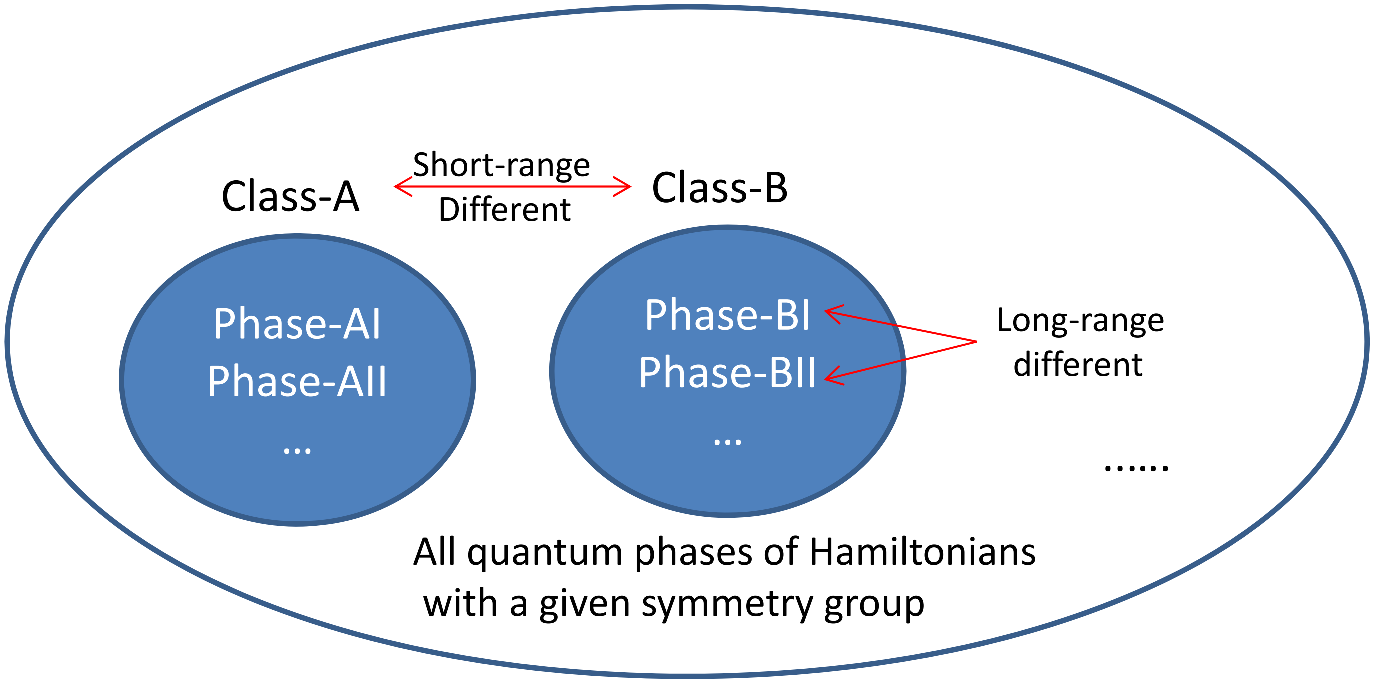

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Our work arXiv:1505.03171 is an partial answer for these questions using tensor networks: e.g. for half-integer spin systems on the kagome lattice, under natural assumptions, **32** crude classes are constructed with sharp knowledge on member phases in each class.

Thank you!