Time-Reversal-Invariant Topological Superconductivity

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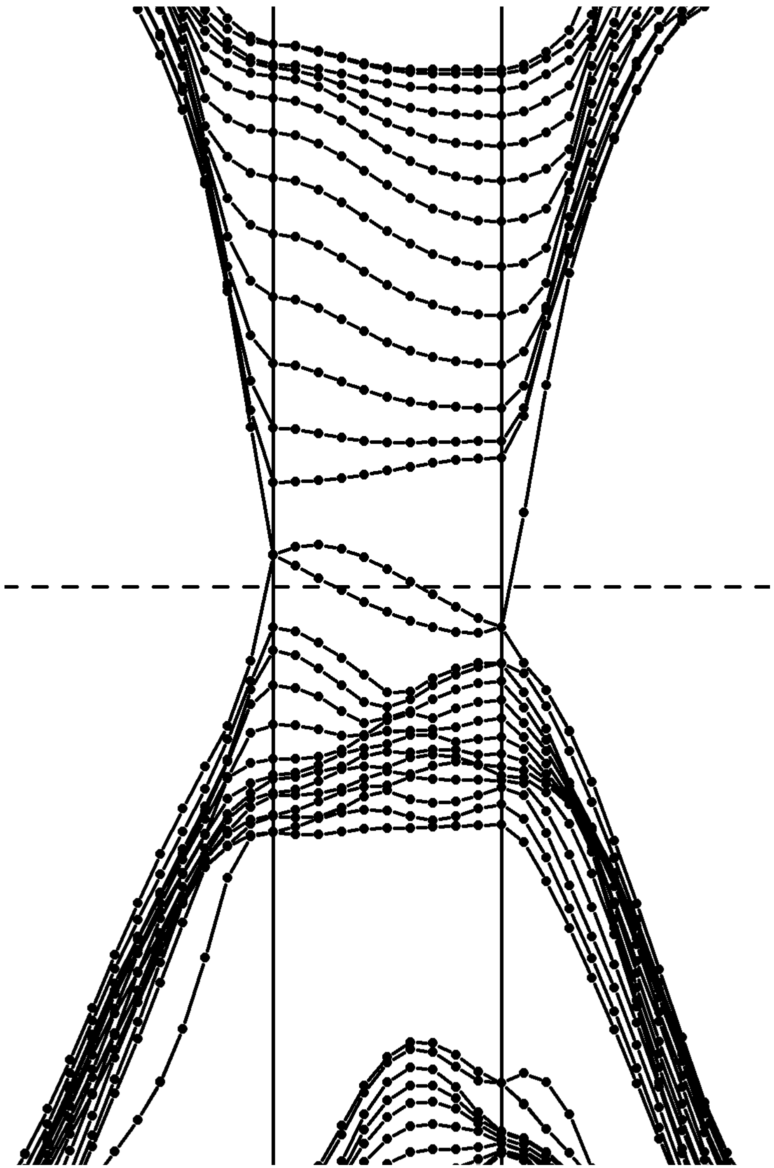


Outline

What **time-reversal symmetry**
can offer for free-fermion systems ?

(1) **Weak** topological insulators and **composite** Weyl semimetals

- **β-Bi_4I_4 and β-Bi_4Br_4** by arXiv:1509.07183 (2015)

(2) **TR-invariant** topological superconductivity

- **1D** topo. S.C. and Majorana Kramers pair
- $Z_2 \times Z_2$ Fractional Josephson effects
- **Periodic table** and Z_4 “parafermions”

Outline

What **time-reversal symmetry**
can offer for free-fermion systems ?

(1) Weak topological insulators and composite Weyl semimetals

with C. -C. Liu (UT-D), J.-J. Zhou (Caltech), Y. Yao (BIT)

(2) TR-invariant topological superconductivity

with C. L. Kane, E. J. Mele, B. J. Wieder (UPenn)

C.-C. Liu, Z.-Q. Bao (UT-D)

CMP 5 v.s. HEP 1

Strong and Weak T.B.I. in 3D

(should be a story in textbook)

Fu-Kane-Mele, PRL 98, 106803 (2007);

Moore-**Balents**, PRB 75, 121306 (2007);

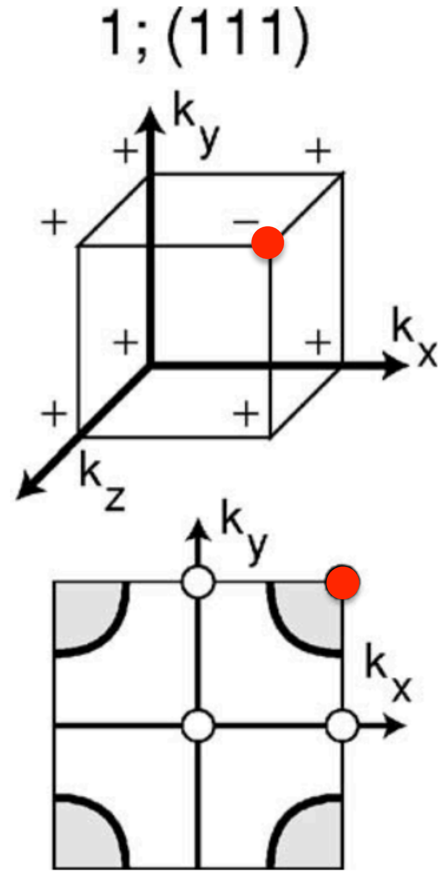
Roy, PRB 79, 195322 (2009).

Z_2 indices: 1; (111)

- odd # of band inversions out of the 8 time-reversal-invariant momenta

- the "$k_x = \pi$ plane" is a "Z_2 QSHI" with odd # of band inversions

strong TI

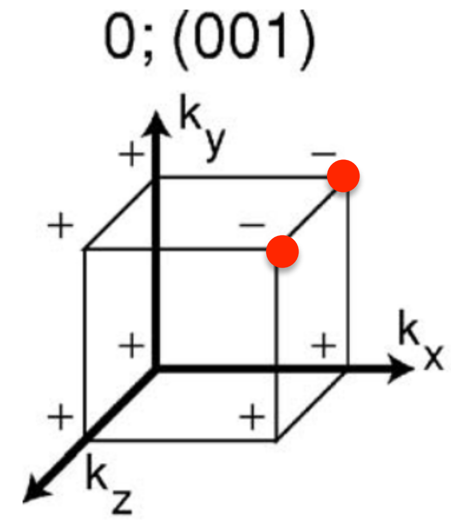


Strong and Weak T.B.I. in 3D

Z_2 indices: 0; (001)

- even number of band inversions
- the "$k_z = \pi$ plane" is a "Z_2 QSHI"

weak TI



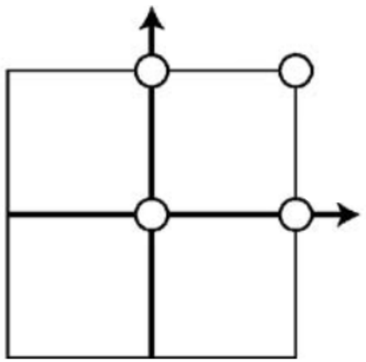
- **Proposal for WTI:**

  as a stack of weakly coupled

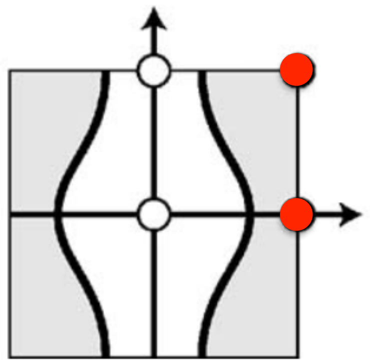
  QSHI layers along the (001) direction

van den Brink group, Nat. Mater. 12, 422 (2013).

(001) surface
no surface states



(100) Surface
with surface states



Weak T.B.I. in Experiment

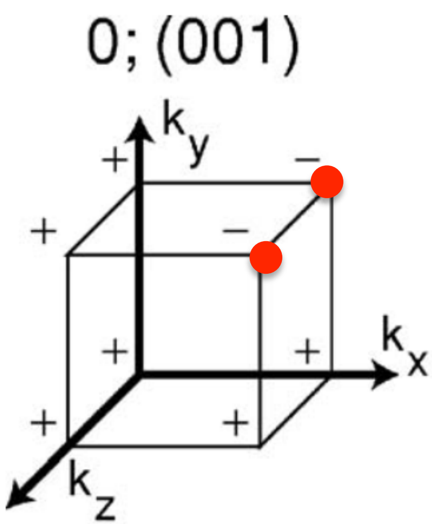
There are now over a dozen documented materials that manifest the physics of STI … The WTI has proven to be an elusive species, there are no confirmed experimental observations.

---- E. J. Mele, Nobel Symposium: Phys. Scr. T164 014004 (2015).

… topological surface states will be present at any crystal face that is not parallel to the (001) plane. Such faces, however, do not correspond to natural cuts of the crystal and the associated surface roughness has prevented us so far from observing these surface states by ARPES on, for instance, (100) surfaces.

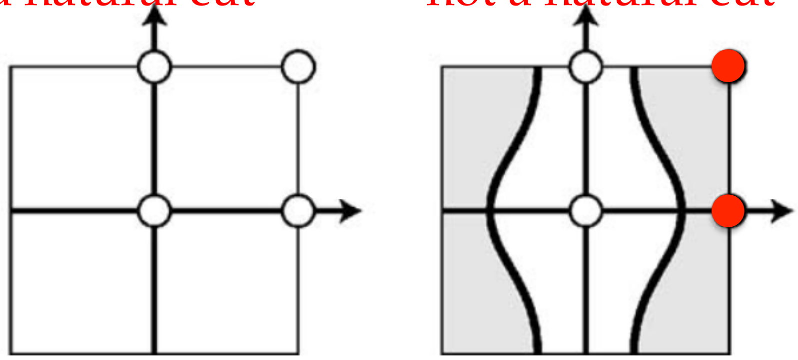
---- van den Brink group, Nat. Mater. 12, 422 (2013).

weak TI



(001) surface
no surface states
a natural cut

(100) Surface
with surface states
not a natural cut

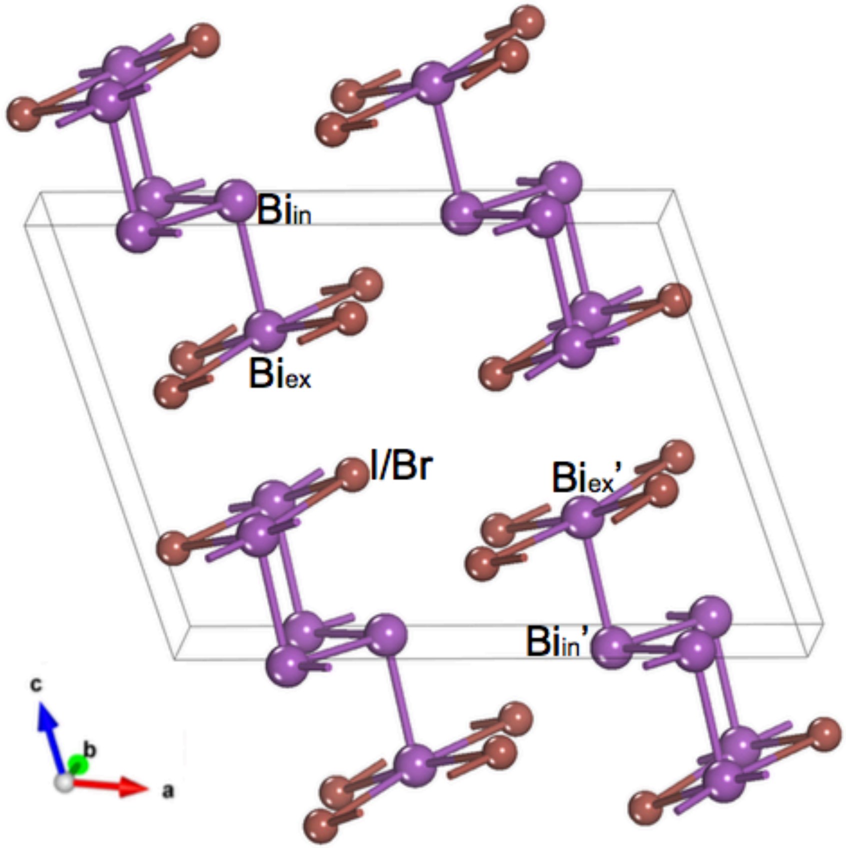


Weak TI : where & how ?

A possible solution: given by arXiv:1509.07183 (2015)

- Look for a 3D material as a stack of 1D atomic chains (y).
- There are two natural cuts: (001) and (100) cleavage surfaces.

β-Bi_4I_4 and β-Bi_4Br_4 (simple chemistry)

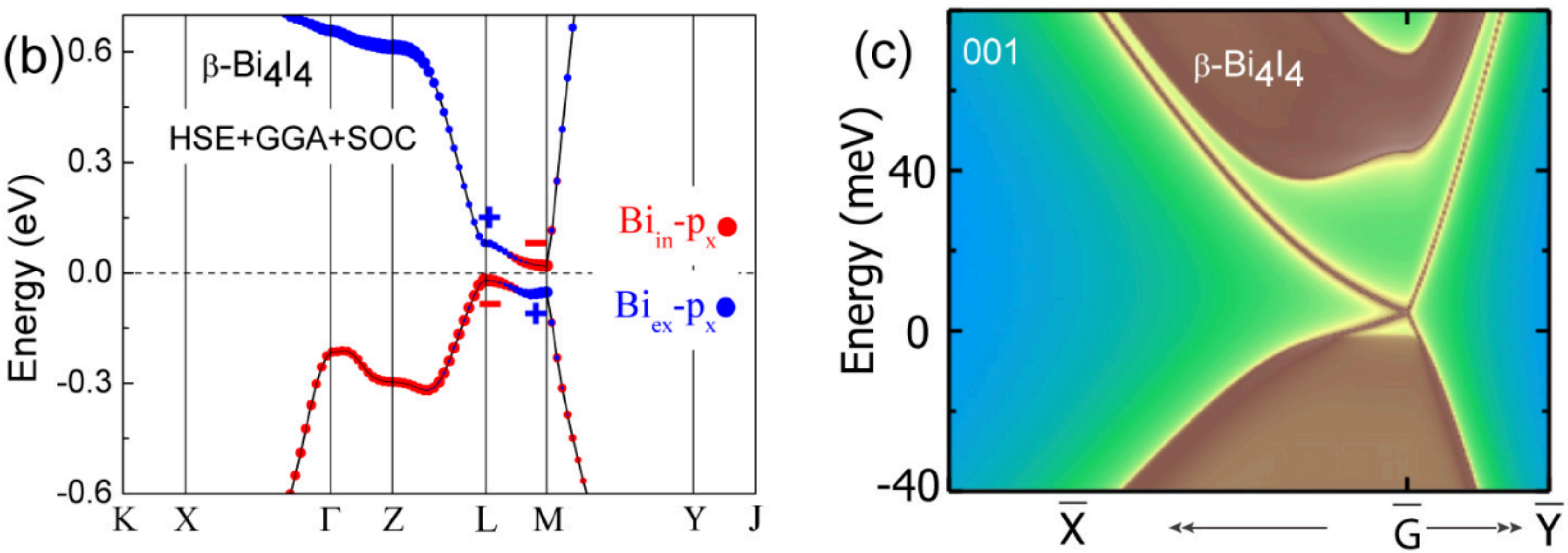


- Inversion symmetry
- Mirror (y) symmetry

- Stack of atomic chains
- (001) and (100) natural cleavage surfaces

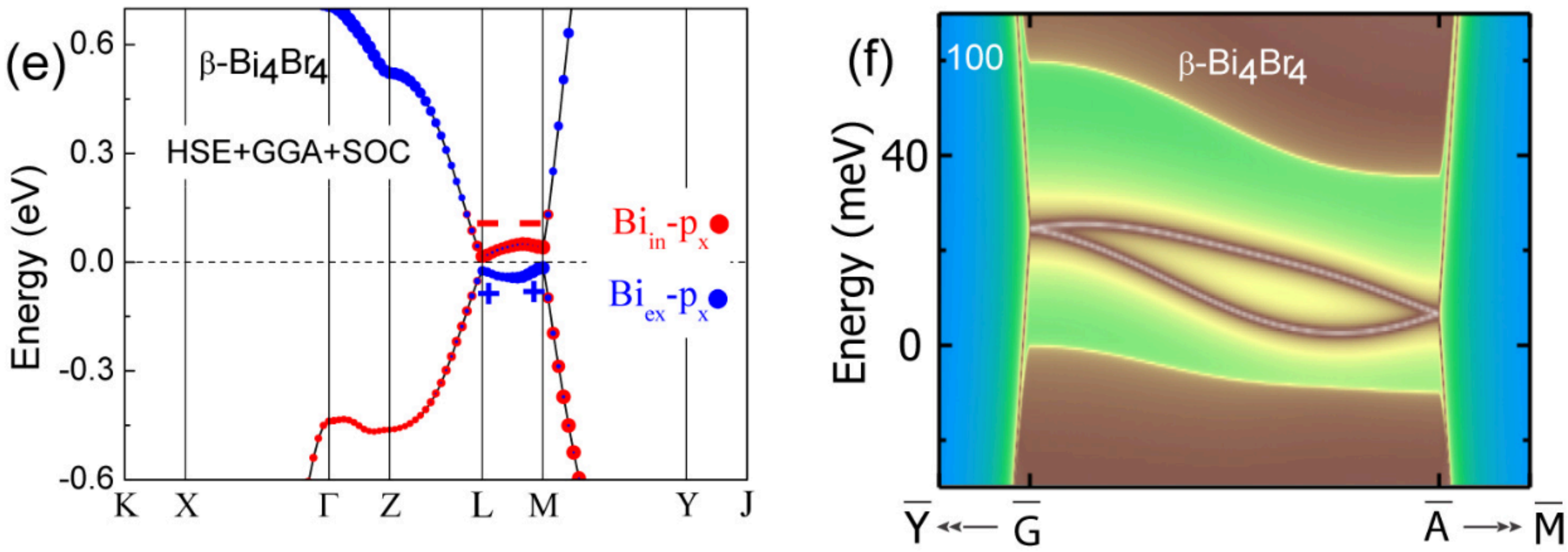
Z. Anorg. Allg. Chem. 438, 37 (1978).
Z. Anorg. Allg. Chem. 438, 53 (1978).

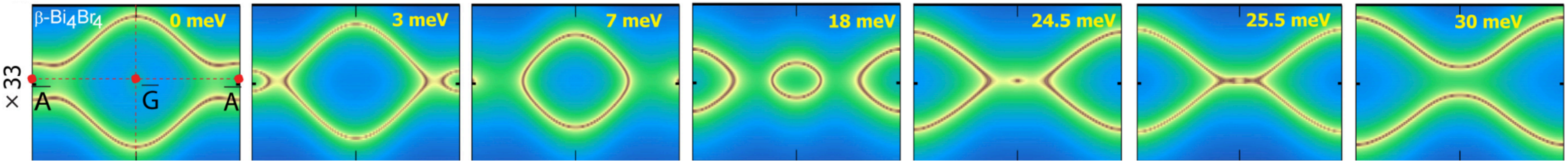
β-Bi_4I_4: a strong T.I.



Liu, Zhou, Yao & **FZ,** arXiv:1509.07183 (2015)

β-Bi_4Br_4: a weak T.I. with 0; (001) indices





Liu, Zhou, Yao & **FZ,** arXiv:1509.07183 (2015)

Uniaxial Strain & Phase Diagram

4-band **effective theory** near the L and M points:

$$\mathcal{H}^i = v_x^i k_x \sigma_y \tau_x + v_y^i k_y \sigma_x \tau_x + v_z^i k_z \tau_y + \boxed{m^i \tau_z} + c^i$$

direct gap

Strain effect: to the lowest order, only change the direct gaps:

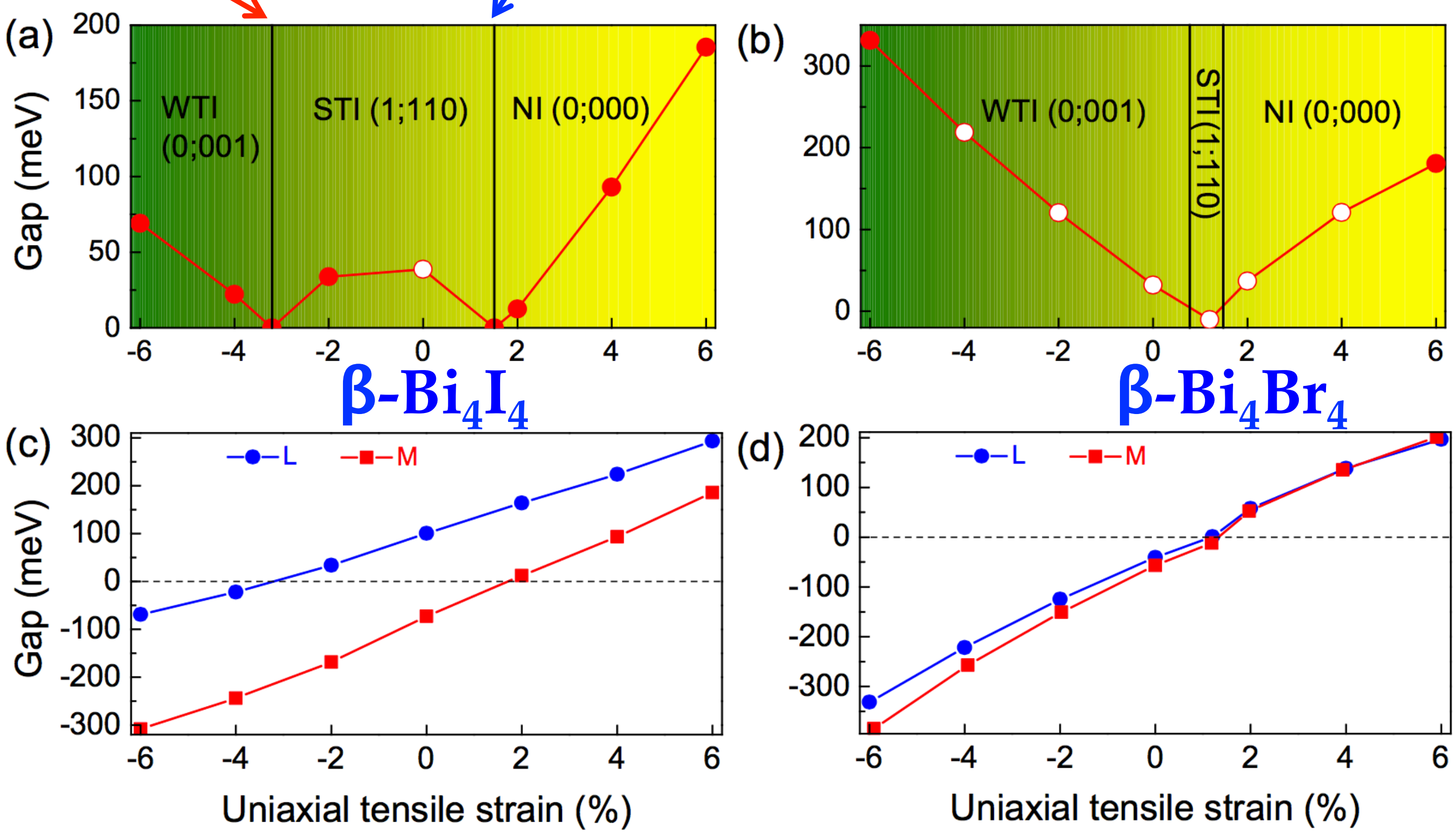
$$\delta\mathcal{H}^i = \delta\mathcal{H}_0^i + \left(\varepsilon_{11}\lambda_{11}^i + \varepsilon_{22}\lambda_{22}^i + \varepsilon_{33}\lambda_{33}^i + \varepsilon_{13}\lambda_{13}^i\right)\tau_z$$

Strain-induced topological phase transitions: NI, STI, WTI

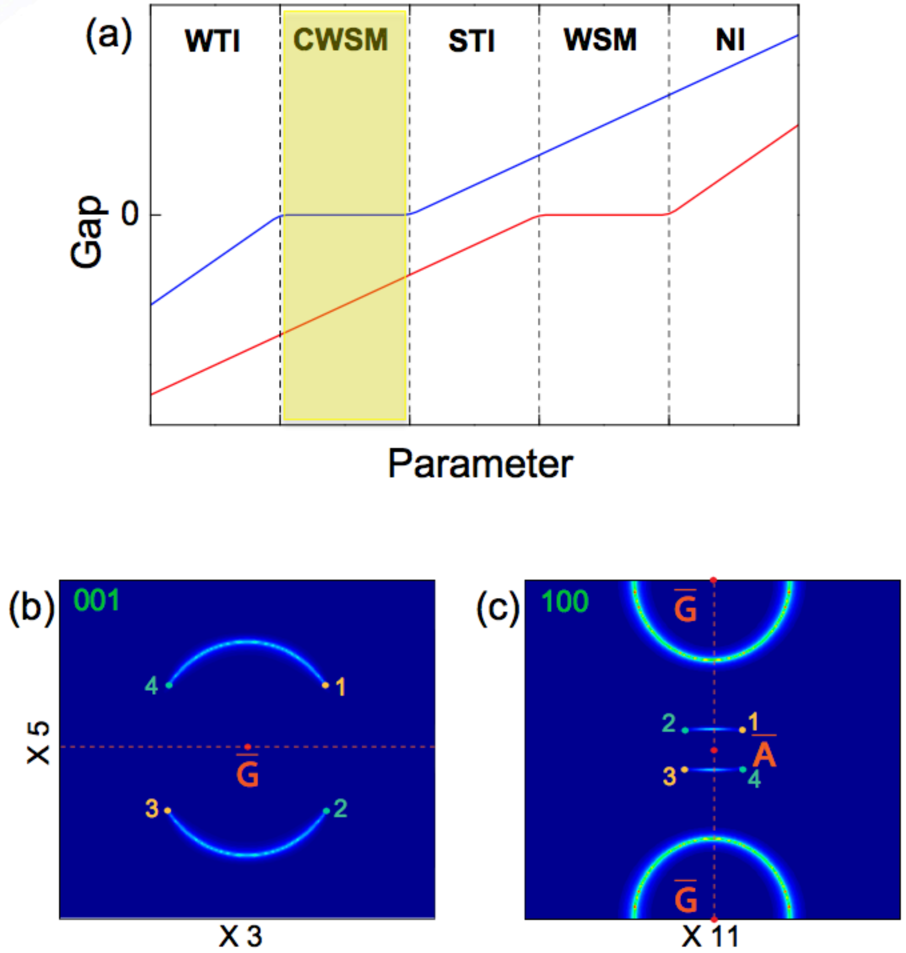
Uniaxial Strain & Phase Diagram

Weyl semimetal (WSM) ?

Weyl semimetal if TR or inversion symmetry is broken, *(by Murakami, Wan, Savrasov, Burkov, Balents, …)*



Composite WSM = WSM (arc) + WTI (circle)



Bulk:

- Fermi surface = Weyl points (L)
- An extra band inversion (M)

Surface:

- Arcs: protected by local Chern #'s
- Circle: by translational symmetry

Liu, Zhou, Yao & **FZ,** arXiv:1509.07183 (2015)

Outline

(1) Weak topological insulators and composite Weyl semimetals

- β-Bi_4I_4 and β-Bi_4Br_4
- T.I. = band inversion + symmetry protection

(2) **TR-invariant** topological superconductivity

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- $\mathbf{Z_2 \times Z_2}$ Fractional Josephson effects
- **Periodic table** and $\mathbf{Z_4}$ "parafermions"

Superconductors Can Be Topological

- Energy gap (for quasi-particles instead of for Cooper pairs)

- Intrinsic particle-hole "symmetry" for BdG Hamiltonians

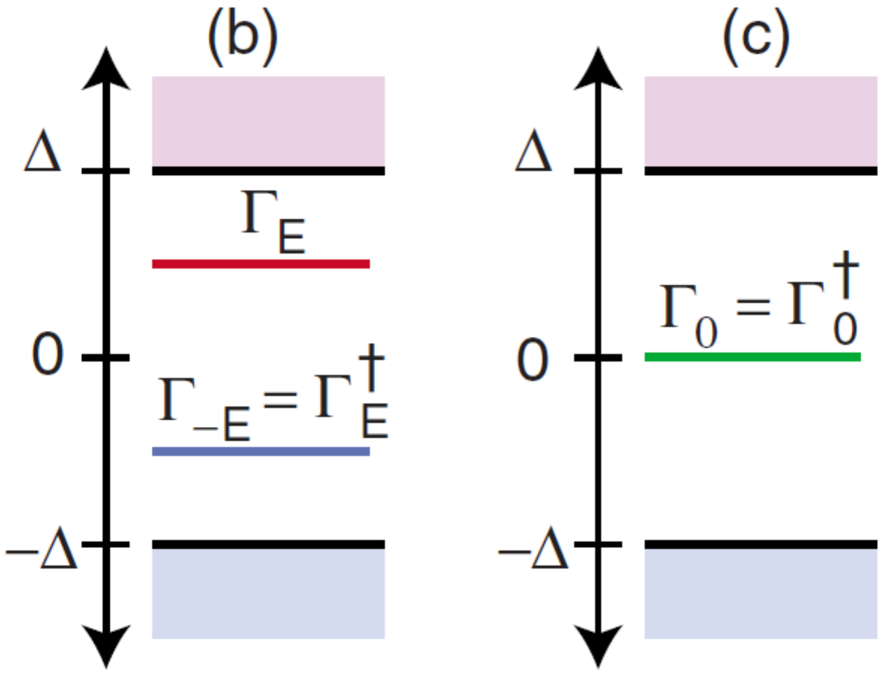
  (e, k) = (e, k) + [(e, -k) + (h, -k)]

  = (2e, 0) + (h, -k)

  = (h, -k)

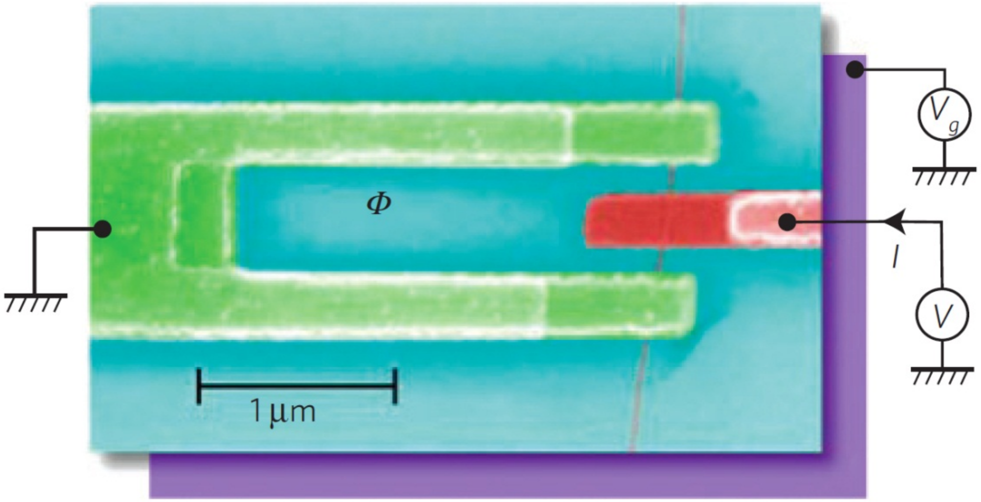
- "Majorana(s)"= zero E mode(s)

  (protected, quantum computing)

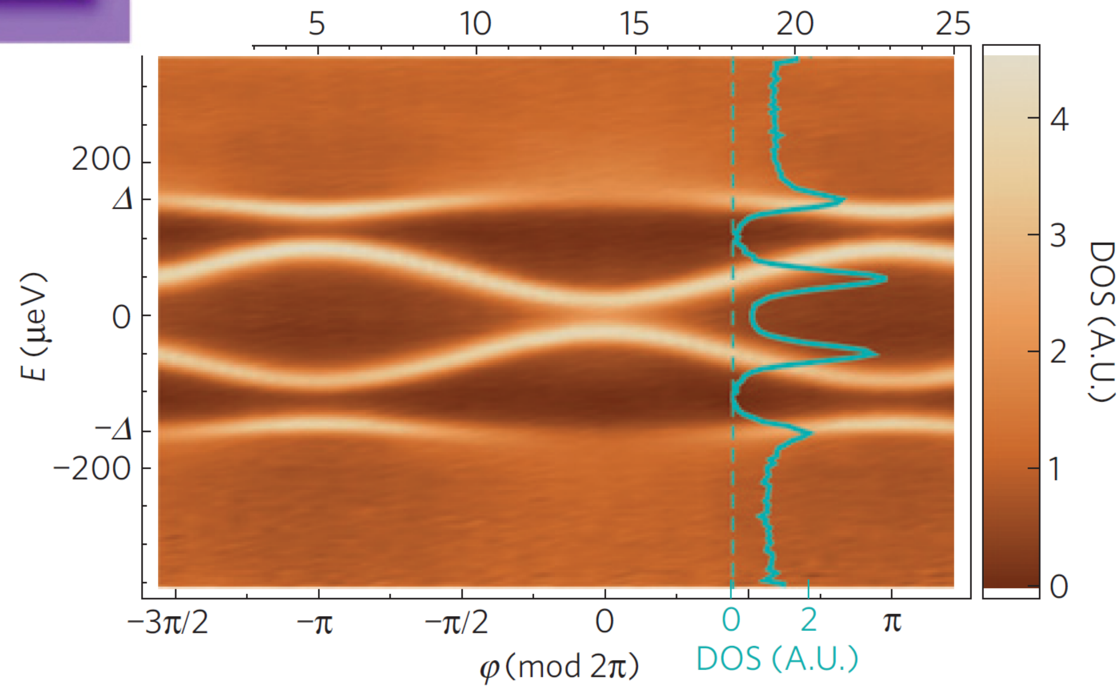


- Time-reversal symmetry (in our case)

Andreev States & Josephson Effects
(special for superconductors)

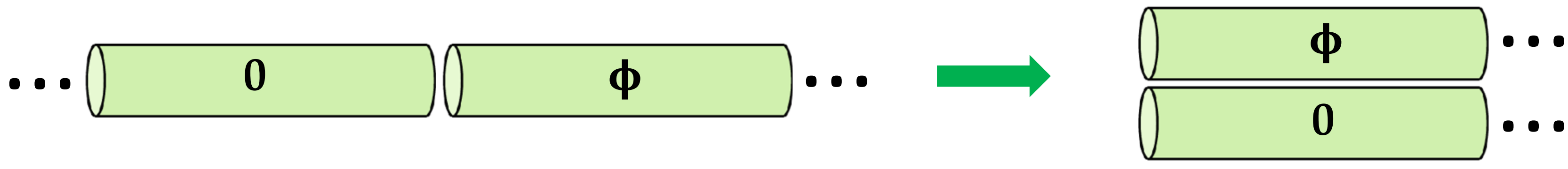


P. Joyez Group
Nature Physics (2010)



- CNT (1D wire)
- STM (tunneling)

Classifying/Understanding Josephson Effects ?



- When folded into each other, the Josephson effects can thus be interpreted as the boundary consequences of the bulk invariant of $H(k,\phi)$;
- $H(k,\phi)$ inherits PH, TR, and chiral symmetry constraints.
- How many topological inequivalent $H(k,\phi)$?

S.C. system modeled by $H(k, r, \phi)$

Chiral symmetry

Symm.	*PHS*	*TRS*	*PHS × TRS*
k	$-$	$-$	$+$
r	$+$	$+$	$+$
$\boldsymbol{\phi}$	$+$	$-$	$-$

Symm.	*PHS*
k	$-$
r	$+$
$\boldsymbol{\phi}$	$+$

when time-reversal symmetry is broken!

topological classification:
$H(k, \phi) = H(k, \mathrm{r})$

The "Omnipotent" Periodic Table
(topological classifications for free fermion systems)

- different dimensions: Dim=dim[k]-dim[r]
- different essential symmetries

(Framework by Kitaev, Schnyder-Ryu-Furusaki-Ludwig, Teo-Kane)

		Symmetry			Dimension ($\boldsymbol{k}$)							
s	AZ	Θ^2	Ξ^2	Π^2	0	1	2	3	4	5	6	7
0	A	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
1	AIII	0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
0	AI	1	0	0	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
1	BDI	1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
2	D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
3	DIII	-1	1	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
4	AII	-1	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
5	CII	-1	-1	1	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
6	C	0	-1	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
7	CI	1	-1	1	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

Anomalous Pumps $H(k, r, \phi)$

Chiral symmetry

Symm.	*PHS*	*TRS*	*PHS* × *TRS*
k	−	−	+
r	+	+	+
$\boldsymbol{\phi}$	+	−	−

- Josephson effects with TRS cannot be understood by the periodic table
- A new class of problems has been formulated

["periodic building" for free-fermion topological phases: 10×8**×8** with the periodic table being its ground floor]

FZ-Kane PRB 90, 020501 (2014)

Time-Reversal-Invariant Topological Superconductivity - modeled by $H(k, \phi)$

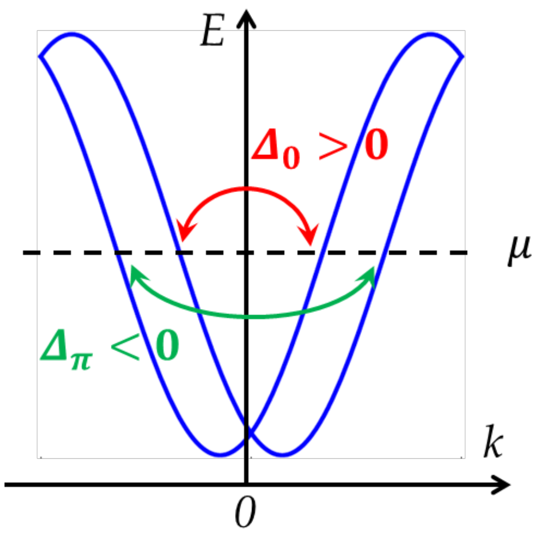
$(d_k - d_r) \bmod 8$		$0,4,5,6$	1	2	3	7
d_ϕ	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	$2\mathbb{Z}$
	1	0	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$\mathbb{Z} \times \mathbb{Z}$	$2\mathbb{Z} \times 2\mathbb{Z}$

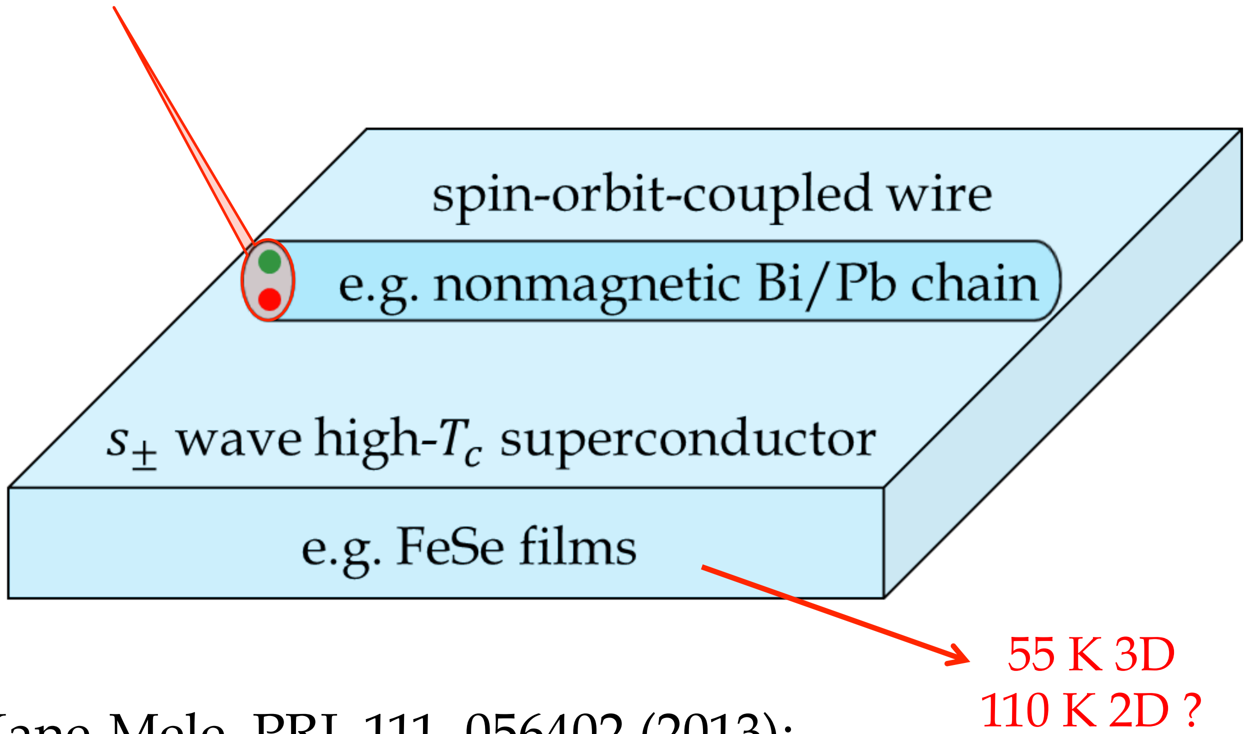
Realizations + Interactions

Similar to ***Moore-Balents*** homotopy theory: **FZ**-Kane PRB 90, 020501 (2014)

1D Z_2 Time-Reversal-Invariant Topological Superconductor

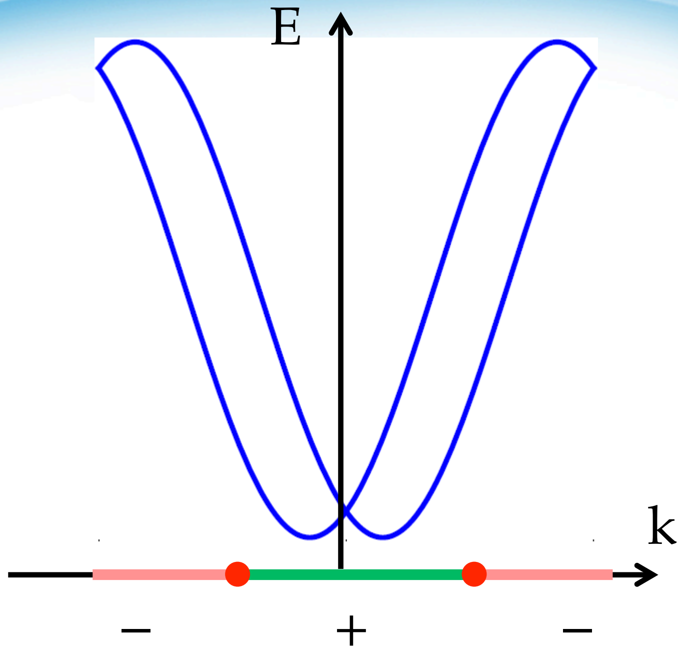
- Majorana Kramers pair

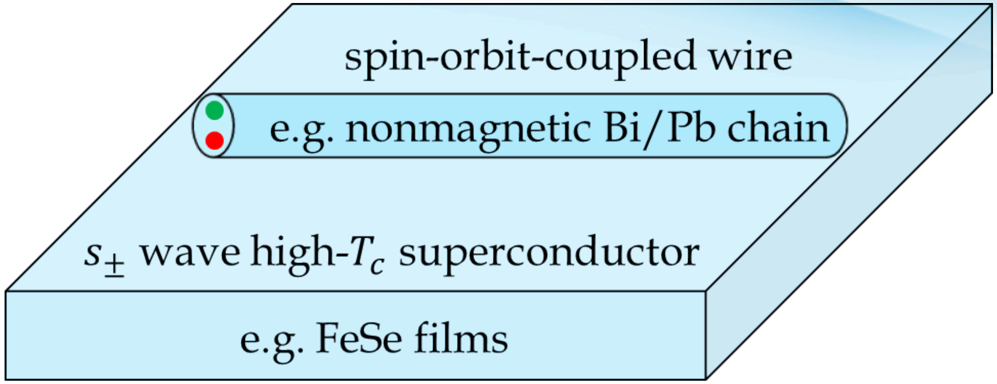


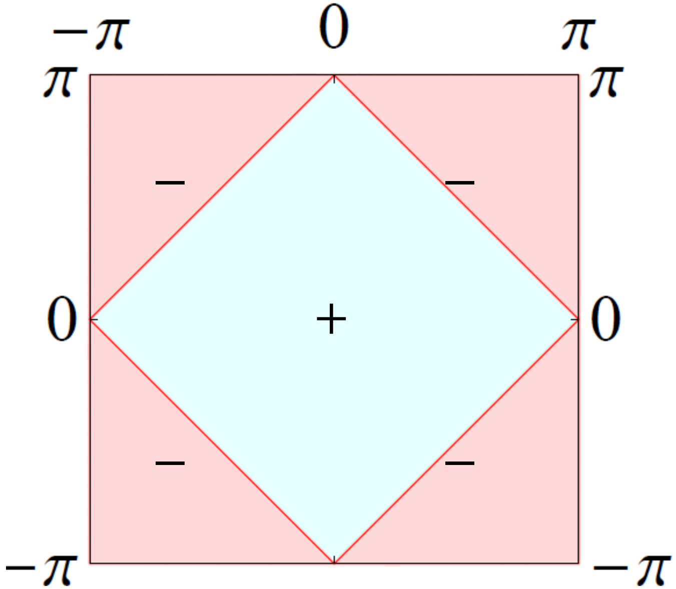


FZ-Kane-Mele, PRL 111, 056402 (2013);
FZ-Kane-Mele, PRL 111, 056403 (2013).

Proximity Effect: 2D Superconductor + 1D Nanowire



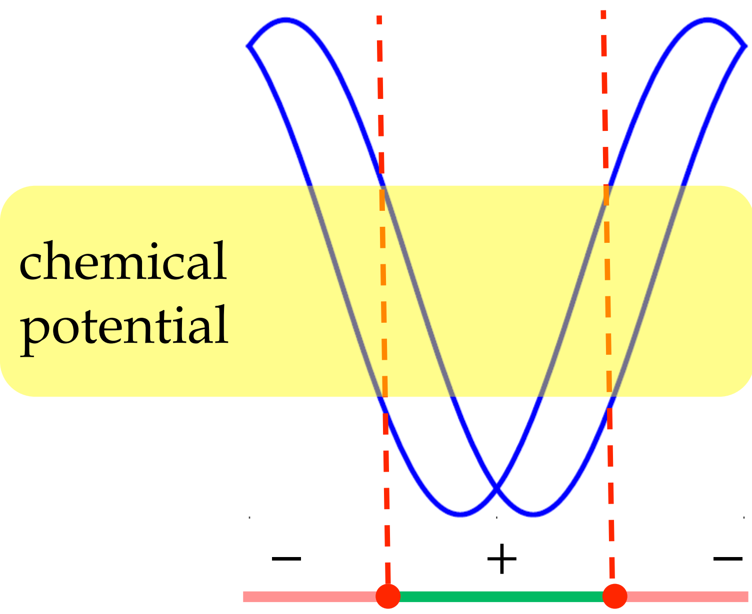


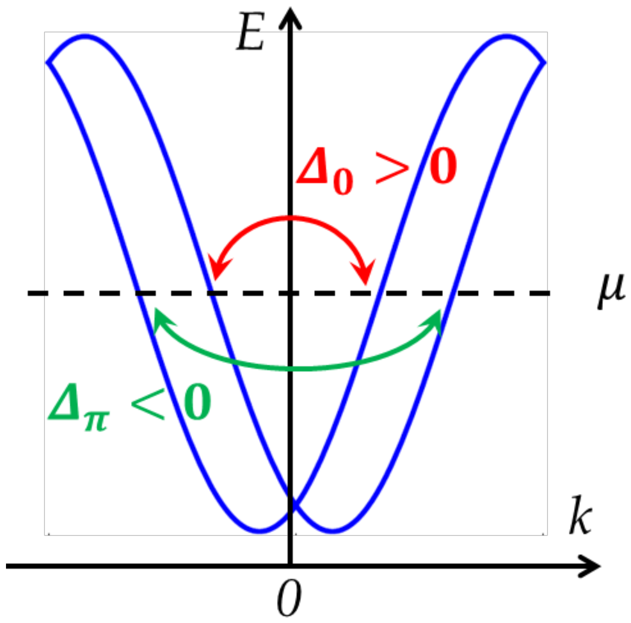


Effects of imperfectness

= shifting the zeros in pairing potential

Time-Reversal-Invariant Topological Superconductivity in 1D





FZ-Kane-Mele, PRL 111, 056402 (2013);
FZ-Kane-Mele, PRL 111, 056403 (2013).

Majorana Kramers Pair (MKP) at 0D Boundary

time-reversal symmetry (TRS)

$\gamma \leftrightarrow \eta$

MKP forms one fermion level

$|1\uparrow\rangle \leftrightarrow |0\downarrow\rangle$

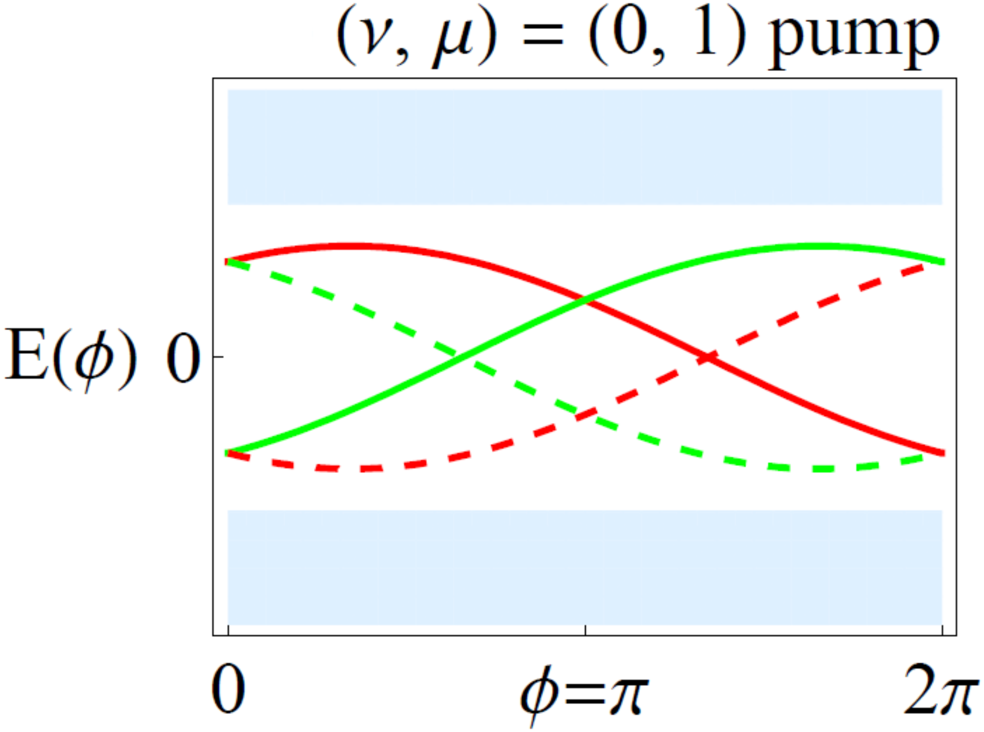
TRS

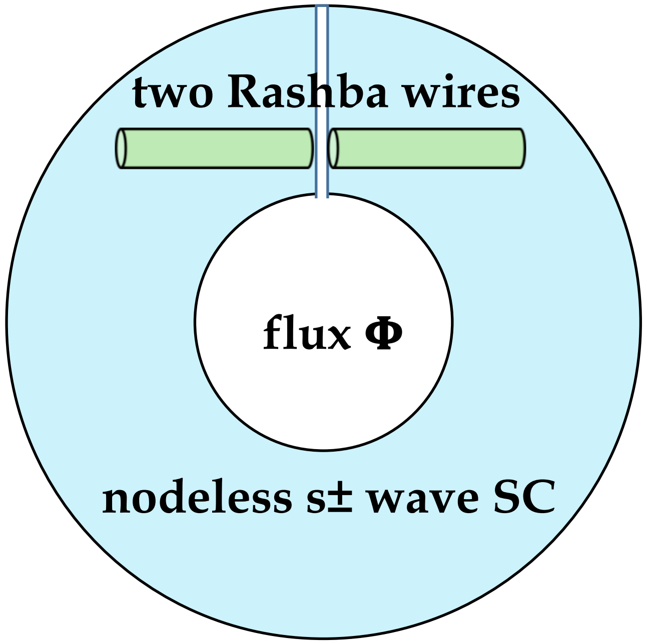
TRS = Super Symmetry ?

$c_\sigma = i c_\sigma^\dagger$

FZ-Kane-Mele, PRL 111, 056402 (2013)

1st Z_2 fractional (4π) Josephson effects





FZ et al., PRL 111, 056402 (2013); PRB 90, 020501 (2014).

Time-Reversal-Invariant Topological Superconductivity
- modeled by $H(k, \phi)$

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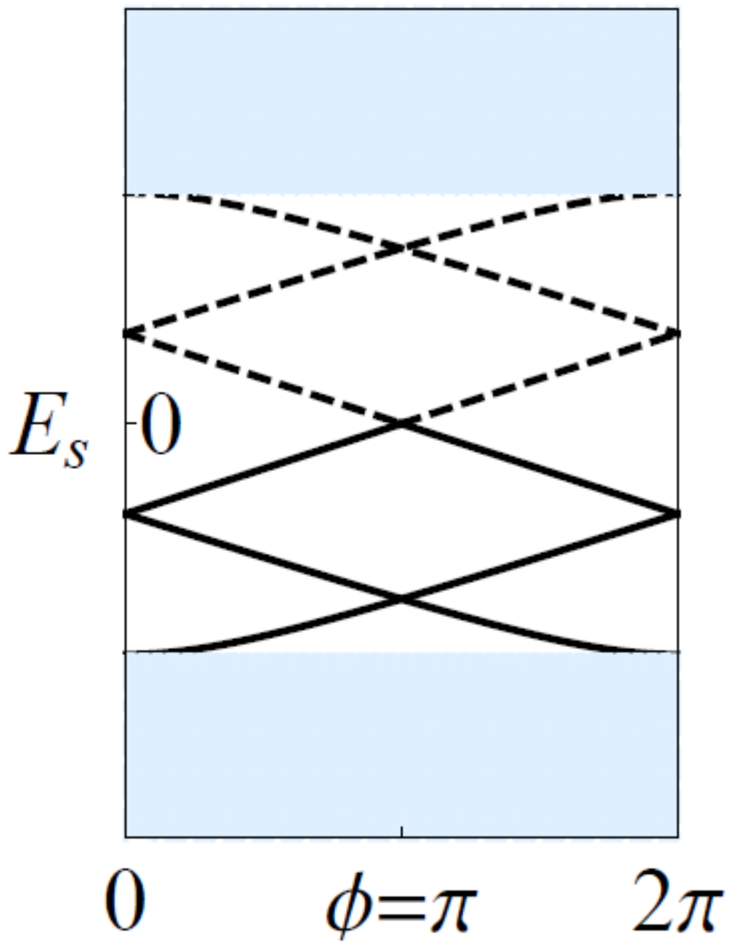
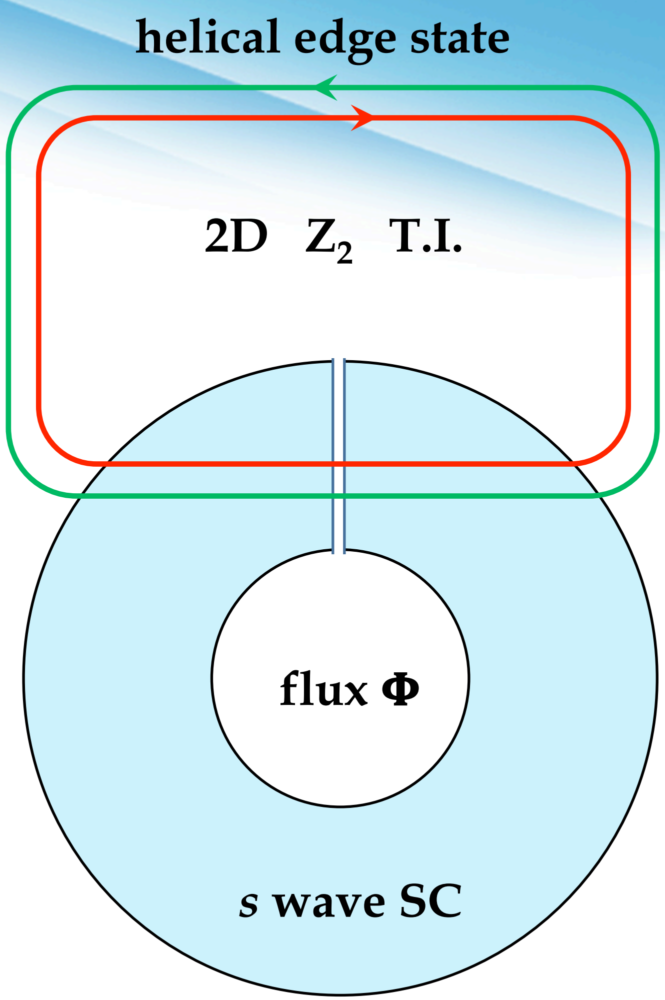
d_ϕ	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	$2\mathbb{Z}$
	1	0	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$\mathbb{Z} \times \mathbb{Z}$	$2\mathbb{Z} \times 2\mathbb{Z}$

Realizations + Interactions

FZ-Kane PRB 90, 020501 (2014)

2^{nd} Z_2 Josephson effects





- Original Fu-Kane proposal
- Look like QSH edge states

- Ongoing experiments:
  L. Molenkamp, A. Yacoby, R. Du, …

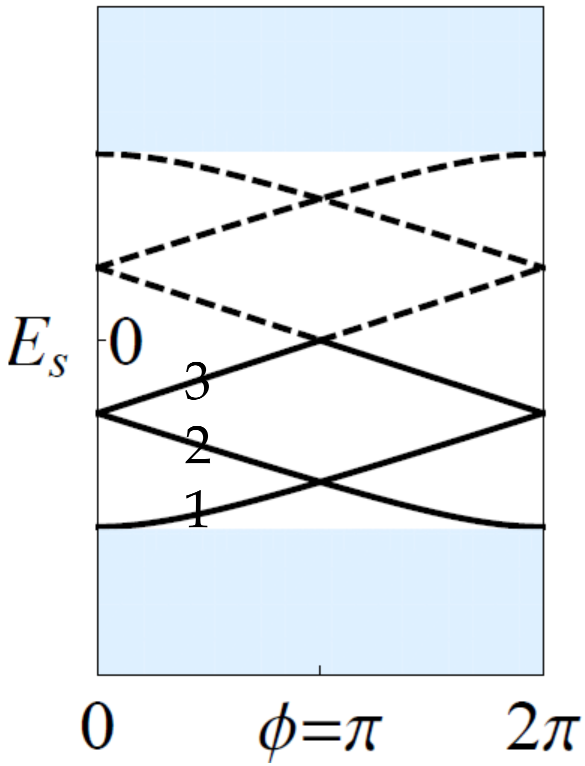
(HgTe/CdTe) (InAs/GaSb)

Many-Body Spectrum

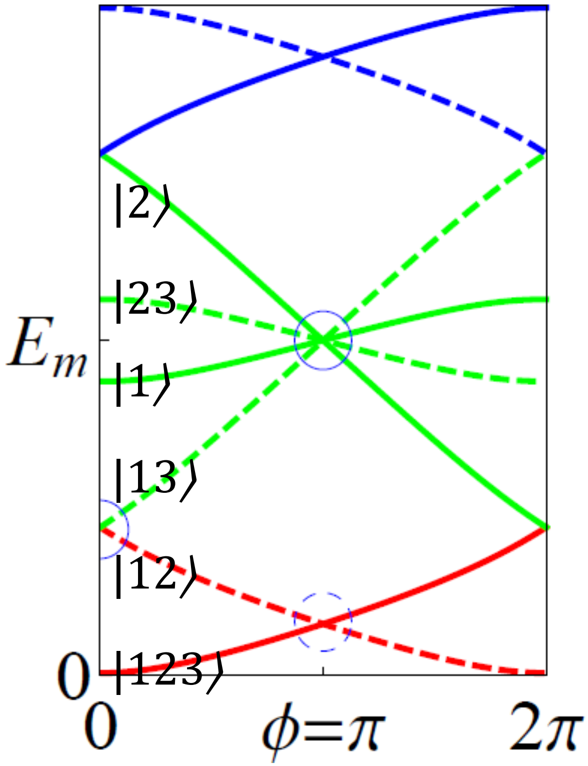
- Add e-e interactions on the junction (respecting time-reversal symmetry)

Result: **the four states split into two Kramers pairs**

single-particle



many-body



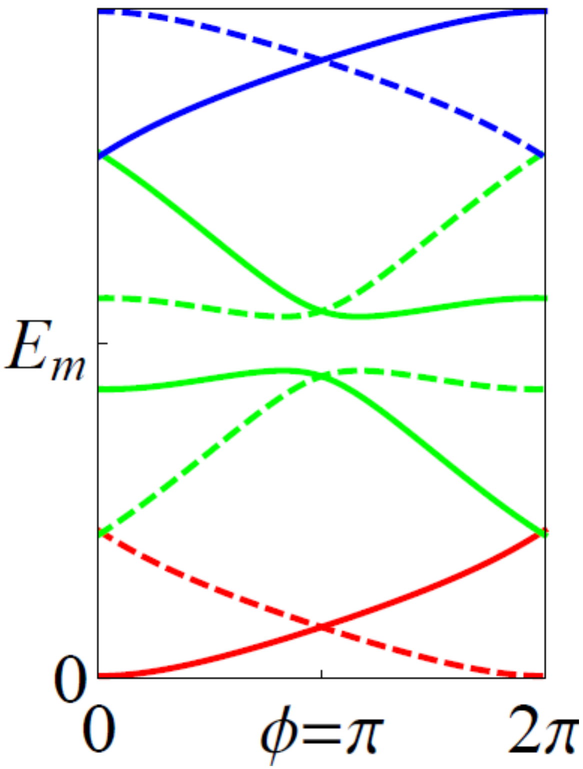
Fourfold degeneracy:

time-reversal symmetry

fermion parity conservation

Dissipation !!!

Z_4 Fractional Josephson Effect



- Dissipationless now
- The periodicity is **8π** [periodicity = (2e/q)2π]

- Charge q = e/2 tunneling between two SCs
- Fractionalization in SC with realistic interactions

- Prediction of topological band theory is modified substantially by weak interactions.
- Bosonization for the strong interactions limit: Z_4 "parafermions" emerge

FZ-Kane, PRL 113, 036401 (2014).

Insulators	Superconductors

- **TRI Topological insulators (e.g. Bi_2Se_3)**
- *Topological crystalline insulators (e.g. SnTe)*
- *2D Weyl/Dirac semimetal (e.g. graphene)*
- *3D Weyl/Dirac semimetals (Na_3Bi, Cd_3As_2)*

- **TRI topological SCs** [PRL 111, 056402 (2013)]
- *Topological Mirror SCs [PRL 111, 056403 (2013)]*
- *2D Weyl/Dirac SCs [e.g. cuprates]*
- *3D Weyl/Dirac SCs [PRL 113, 046401 (2014)]*

Comparison (with TRS)

- **$Z_2 \times Z_2$ Josephson effects**
- **3D "periodic build"**
- **Z_4 fractionalization (e/2)**

[PRB 90, 02050 (2014); PRL 113, 036401 (2014).]

Topological v.s. Trivial   Theory v.s. Experiment