Fault tolerant quantum computing with reinforcement learning

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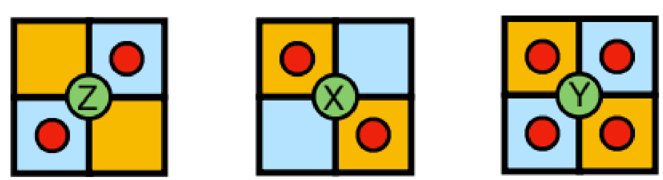
arXiv:1810.07207



Context & Outline

Quantum Error
Correction

Reinforcement Learning

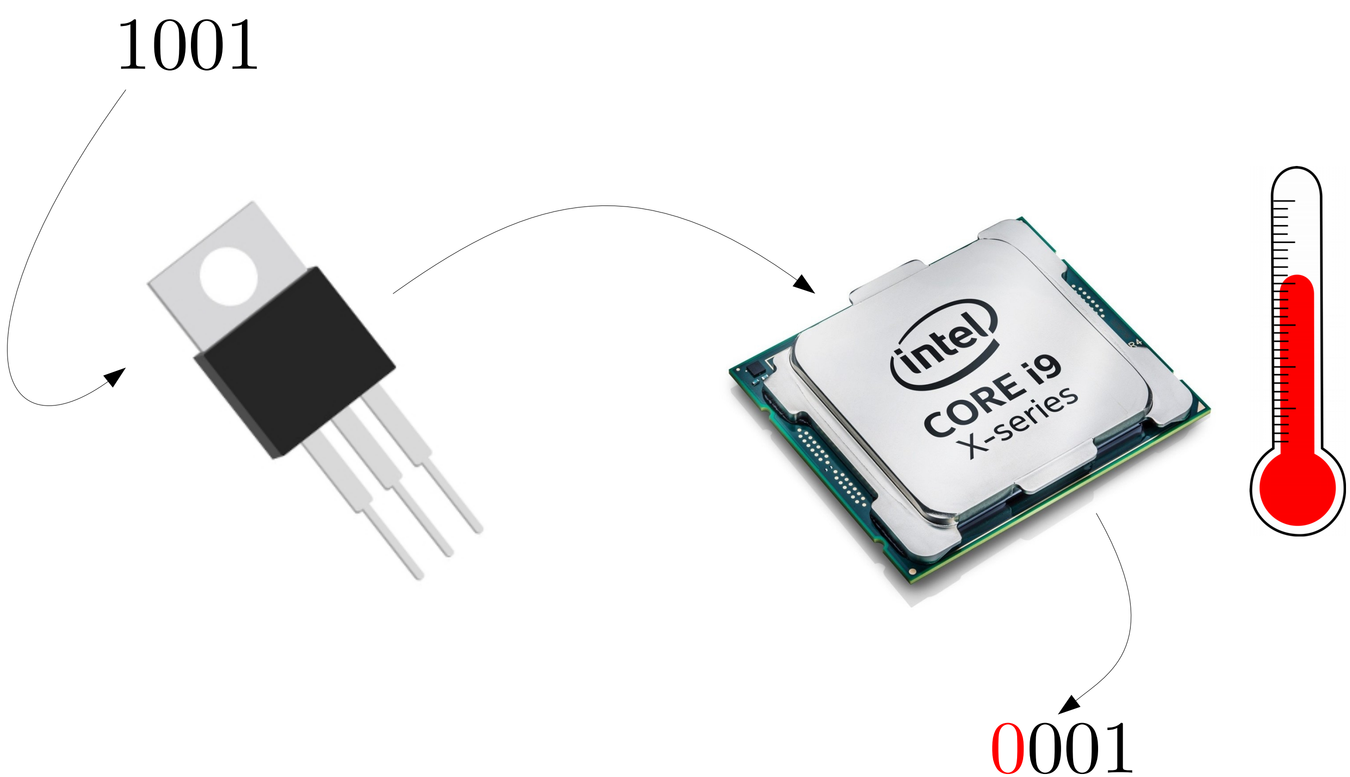
Learning from feedback

Classical Info

A Bit

The unit of information

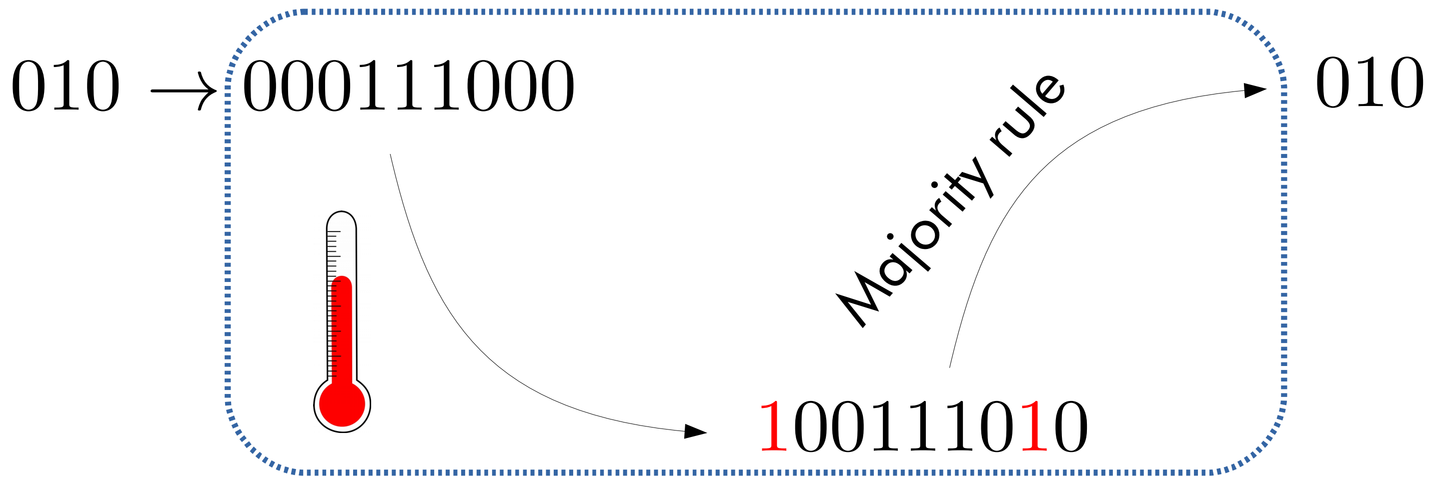
$$b = 0,1$$



Error (Correction) Codes

Redundancy

$$0 \to 000$$
$$1 \to 111$$



Error (Correction) Codes

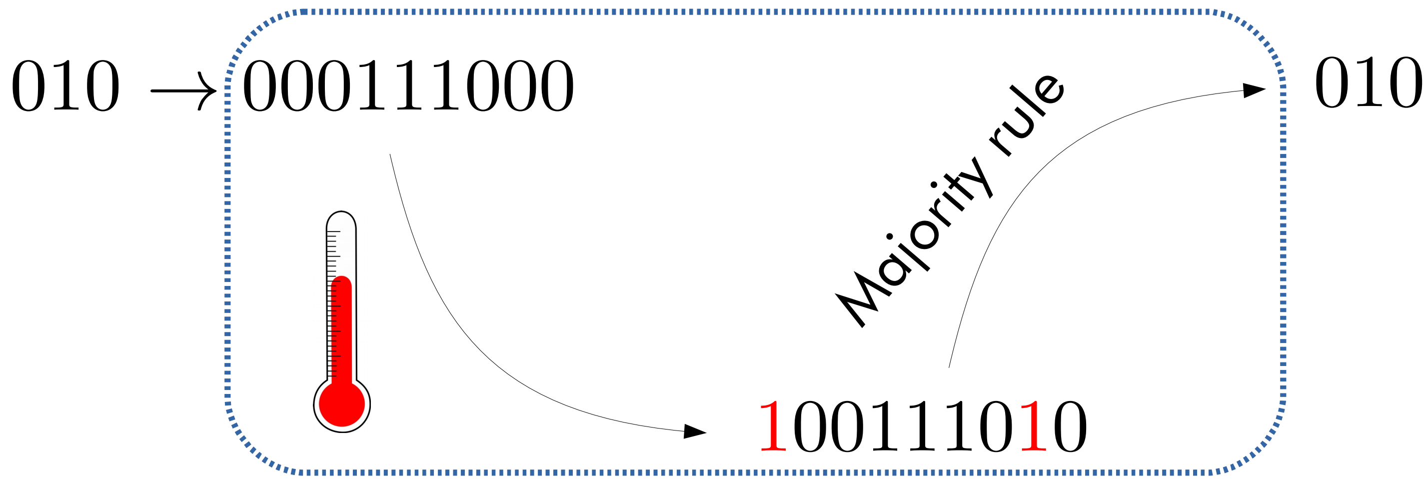
Redundancy

A [3,1,3]-Code

$$0 \to 000$$
$$1 \to 111$$

(Hamming) distance

$$\frac{d-1}{2}$$



Quantum Information

A Qubit (q-bit)

The unit of quantum information

$$|b\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1$$

$$Z|0\rangle = |0\rangle$$
$$Z|1\rangle = -|1\rangle$$

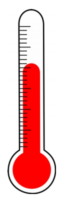
- Superconducting circuits
- Ion traps
- Color centers
- Semiconductor Quantum Dots
- Majorana modes
- Spins of silicon dopants
- ...

Quantum Computation

$$|b\rangle = \alpha|0\rangle + \beta|1\rangle$$

bitflip-noise

$X|b\rangle$



$$\alpha|0\rangle + \beta|1\rangle \rightarrow \beta|0\rangle + \alpha|1\rangle$$

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|0\rangle - \beta|1\rangle$$

phaseflip-noise

$Z|b\rangle$

$$|bbb\rangle$$

No-Cloning Theorem

Encode & Decode & Correct without destroying superpositions!

Quantum Codes

Shor's code

Logical qubit

$$|\tilde{0}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \qquad |\tilde{1}\rangle = \frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)$$

$$|\psi\rangle = \alpha|\tilde{0}\rangle + \beta|\tilde{1}\rangle$$

A [3,1]-code

Quantum Codes

Shor's code

Logical qubit

$$|\tilde{0}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \qquad |\tilde{1}\rangle = \frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)$$

A [3,1]-code

$$|\psi\rangle = \alpha|\tilde{0}\rangle + \beta|\tilde{1}\rangle$$

$$Z_1 Z_2|\tilde{0}\rangle = |\tilde{0}\rangle$$
$$Z_1 Z_2|\tilde{1}\rangle = |\tilde{1}\rangle$$
$$\langle\psi|Z_1 Z_2|\psi\rangle = 1$$
$$\langle\psi|Z_2 Z_3|\psi\rangle = 1$$

Quantum Codes

Shor's code

$|\tilde{0}\rangle, |\tilde{1}\rangle$ are simultaneous +1 eigenstates of both $Z_1 Z_2$ and $Z_2 Z_3$

Single bit-flip error

$$X_1|\tilde{0}\rangle = \frac{1}{\sqrt{2}}\left(|100\rangle + |011\rangle\right)$$
$$X_1|\tilde{1}\rangle = \frac{1}{\sqrt{2}}\left(|100\rangle - |011\rangle\right)$$

$$Z_1 Z_2|\tilde{0}\rangle = Z_1 Z_2|100\rangle + Z_1 Z_2|011\rangle = -|\tilde{0}\rangle$$
$$Z_2 Z_3|\tilde{0}\rangle = Z_2 Z_3|100\rangle + Z_2 Z_3|011\rangle = |\tilde{0}\rangle$$

Quantum Codes

Shor's code

	Z_1Z_2	Z_2Z_3
I	1	1
X_1	-1	1
X_2	-1	-1
X_3	1	-1

Syndrome

Quantum Codes

Shor's code

	Z_1Z_2	Z_2Z_3
I	1	1
X_1	-1	1
X_2	-1	-1
X_3	1	-1

Syndrome

$$\langle\psi|X_1Z_1Z_2X_1|\psi\rangle = \langle\psi| - Z_1Z_2X_1X_1|\psi\rangle = -1$$

$$\{X_1, Z_1\} = 0$$

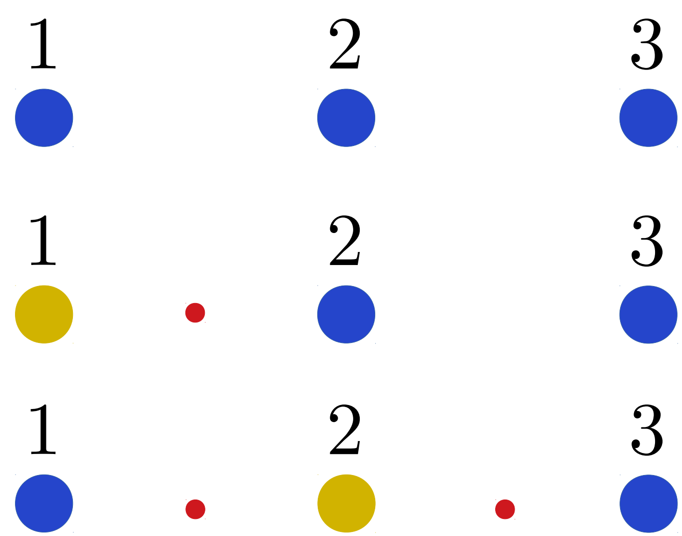
$$X_1^2 = I$$

Quantum Codes

Shor's code

	Z_1Z_2	Z_2Z_3
I	1	1
X_1	-1	1
X_2	-1	-1
X_3	1	-1

Syndrome



Quantum Codes

Shor's code

What about phase flips?

$$|\tilde{0}\rangle = \frac{1}{\sqrt{8}} \left(|000\rangle + |111\rangle\right) \left(|000\rangle + |111\rangle\right) \left(|000\rangle + |111\rangle\right)$$

Quantum Codes

Shor's code

What about phase flips?

$$|\tilde{0}\rangle = \frac{1}{\sqrt{8}}\left(|000\rangle + |111\rangle\right)\left(|000\rangle + |111\rangle\right)\left(|000\rangle + |111\rangle\right)$$

$$X_1X_2X_3X_4X_5X_6|\tilde{0}\rangle = |\tilde{0}\rangle$$
$$X_4X_5X_6X_7X_8X_9|\tilde{0}\rangle = |\tilde{0}\rangle$$

↓ Phaseflip error on qubit 1

$$\langle\tilde{0}|Z_1\,X_1X_2X_3X_4X_5X_6\,Z_1|\tilde{0}\rangle = -1$$
$$\langle\tilde{0}|Z_1X_4X_5X_6X_7X_8X_9Z_1|\tilde{0}\rangle = 1$$

Quantum Codes

Shor's code

A [9, 1]-code

$$|\tilde{0}\rangle = \frac{1}{\sqrt{8}} \left(|000\rangle + |111\rangle\right)\left(|000\rangle + |111\rangle\right)\left(|000\rangle + |111\rangle\right)$$

$$\begin{matrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \\ S_8 \end{matrix} = \begin{matrix} Z_1 & Z_2 & I & I & I & I & I & I & I \\ I & Z_2 & Z_3 & I & I & I & I & I & I \\ I & I & I & Z_4 & Z_5 & I & I & I & I \\ I & I & I & I & Z_5 & Z_6 & I & I & I \\ I & I & I & I & I & I & Z_7 & Z_8 & I \\ I & I & I & I & I & I & I & Z_8 & Z_9 \\ X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & I & I & I \\ I & I & I & X_4 & X_5 & X_6 & X_7 & X_8 & X_9 \end{matrix}$$

Stabilizer Codes

Quantum Codes

A smaller stabilizer example

A [5,1]-code

$$\begin{matrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{matrix} = \begin{matrix} X & Z & Z & X & I \\ I & X & Z & Z & X \\ X & I & X & Z & Z \\ Z & X & I & X & Z \end{matrix}$$

Code space: All simultaneous +1 eigenstates

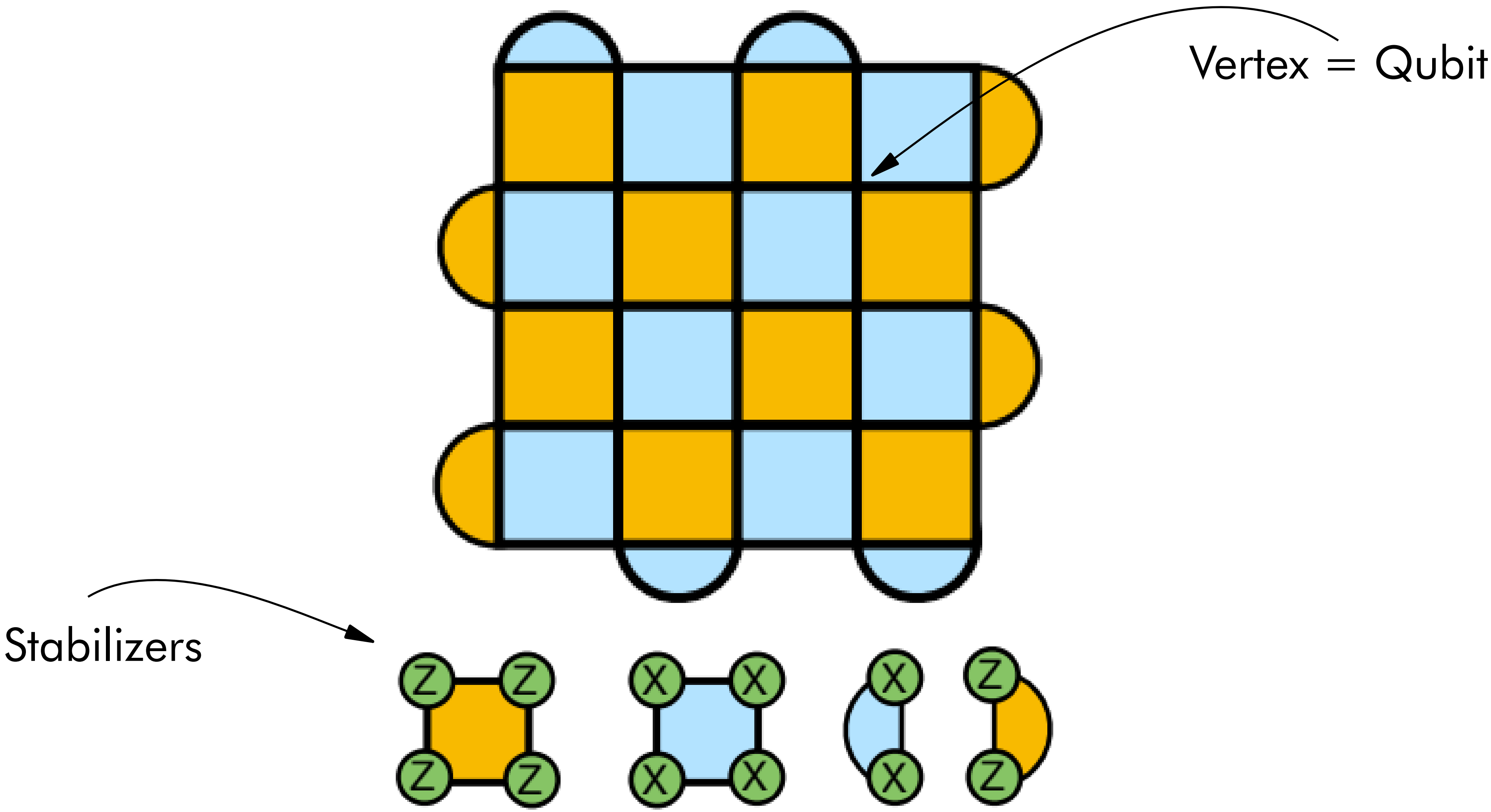
Each error causes a specific -1 measurement **pattern**

Syndrome

Quantum Codes

Topological codes

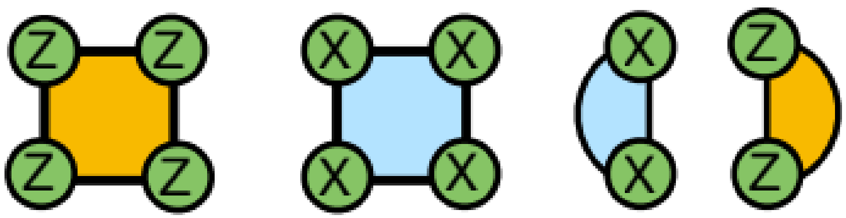
Toric/Surface code

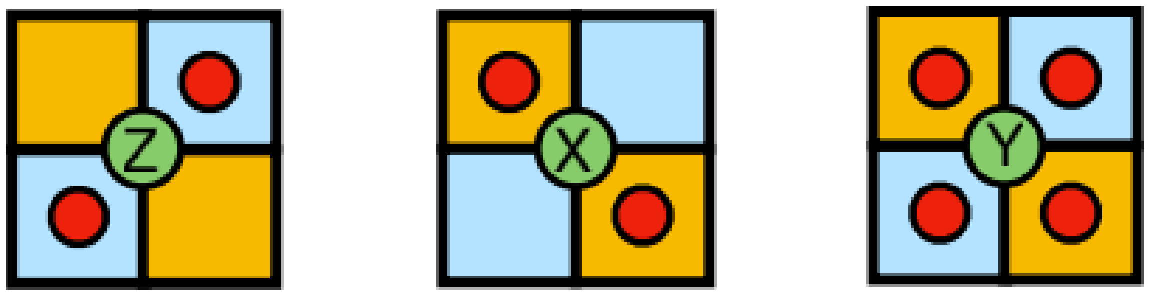


Quantum Codes

Topological codes

Toric/Surface code



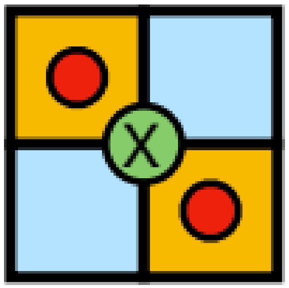


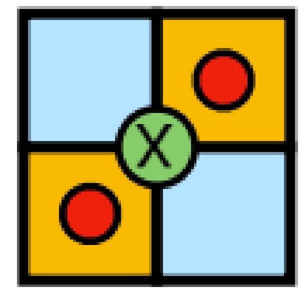
Commute?

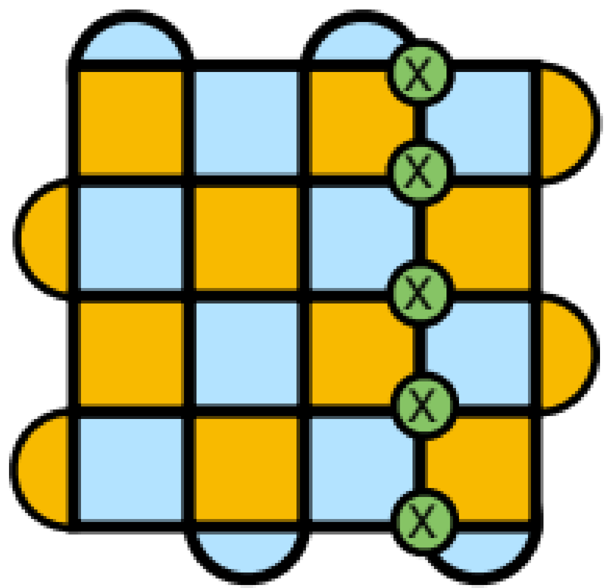
Quantum Codes

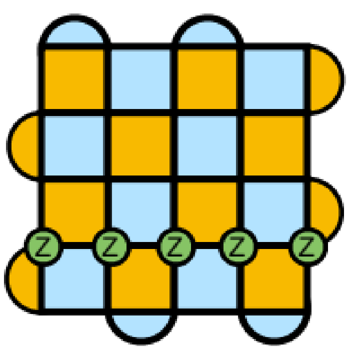
Topological codes

Toric/Surface code









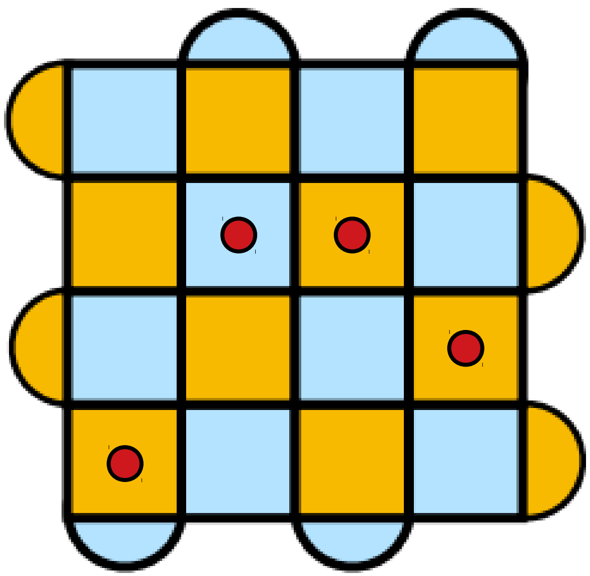
$$|\tilde{0}\rangle \leftrightarrow |\tilde{1}\rangle$$

Quantum Codes

Topological codes

Toric/Surface code

Decoding: Given syndrome, determine which qubits ‘flipped’



Quantum Codes

Topological codes

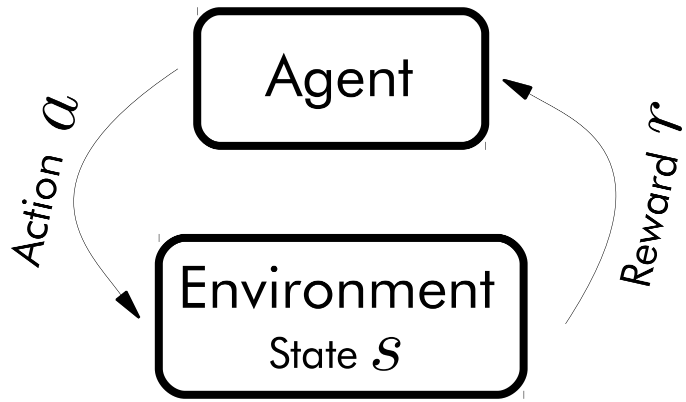
Toric/Surface code

Decoding: Given syndrome, determine which qubits ‘flipped’

Reinforcement Learning

For every state s find the action a that optimizes the expected reward

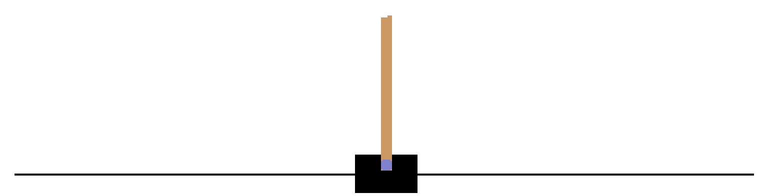
→ *For more RL info, stay tuned for Dries Sels' tutorial session (tomorrow)!*



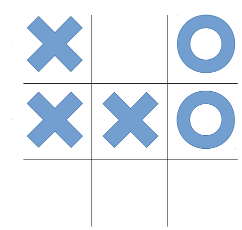
$$s = \{x, v, \theta\}$$

$$a = \{L, R\}$$

$$r = \begin{cases} \pi - \theta \\ T \texttt{ for which } \theta < 5^\circ \end{cases}$$



Hello World: Cartpole



Sutton&Barto v2

OpenAI Gym

Reinforcement Learning

Snippet of "Deepmind Learns Parkour" video

https://www.youtube.com/watch?v=faDKMMwOS2Q

Q-Learning

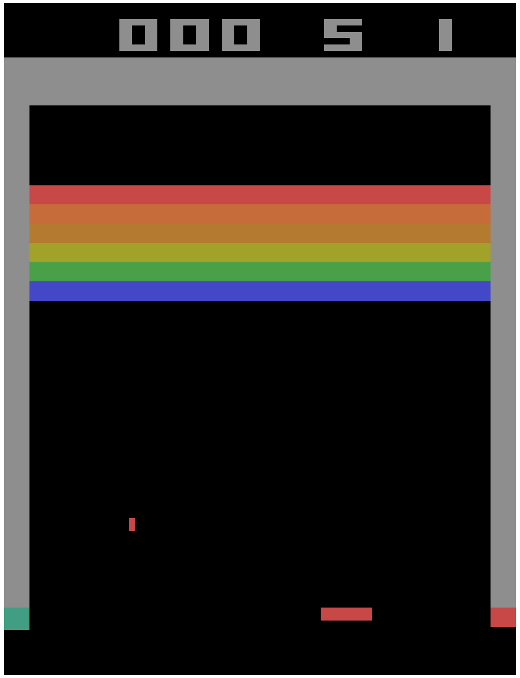
For every state s find the action a that optimizes the expected reward

$$Q(s, a)$$

Defines the strategy (**policy** π) of the agent

$$Q_\pi(s_t, a_t) \sim \mathbb{E}[r + \gamma * Q_\pi(s_{t+1}, a_{t+1})]$$

Max over all actions



Q-Learning

For every state s find the action a that optimizes the expected reward

$$Q(s, a)$$

Probabilities

$$Q(s) = \begin{pmatrix} p(a_1) \\ p(a_2) \\ \vdots \\ p(a_N) \end{pmatrix} = \pi(a|s)$$

DeepQ-Learning

For every state s find the action a that optimizes the expected reward

$$Q(s, a)$$

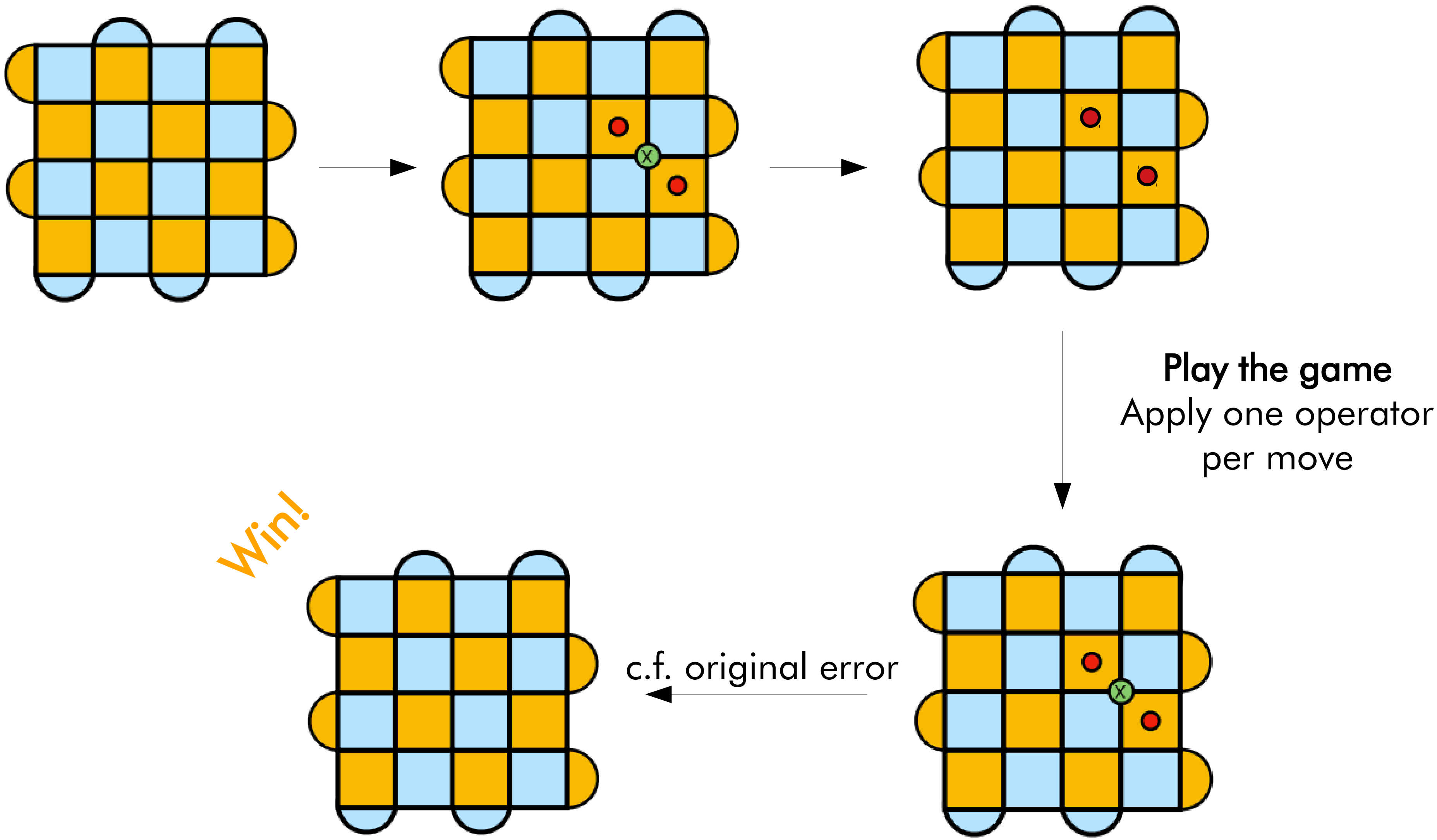
Probabilities

$$Q(s) \sim \begin{pmatrix} p(a_1) \\ p(a_2) \\ \vdots \\ p(a_N) \end{pmatrix} = \pi(a|s)$$

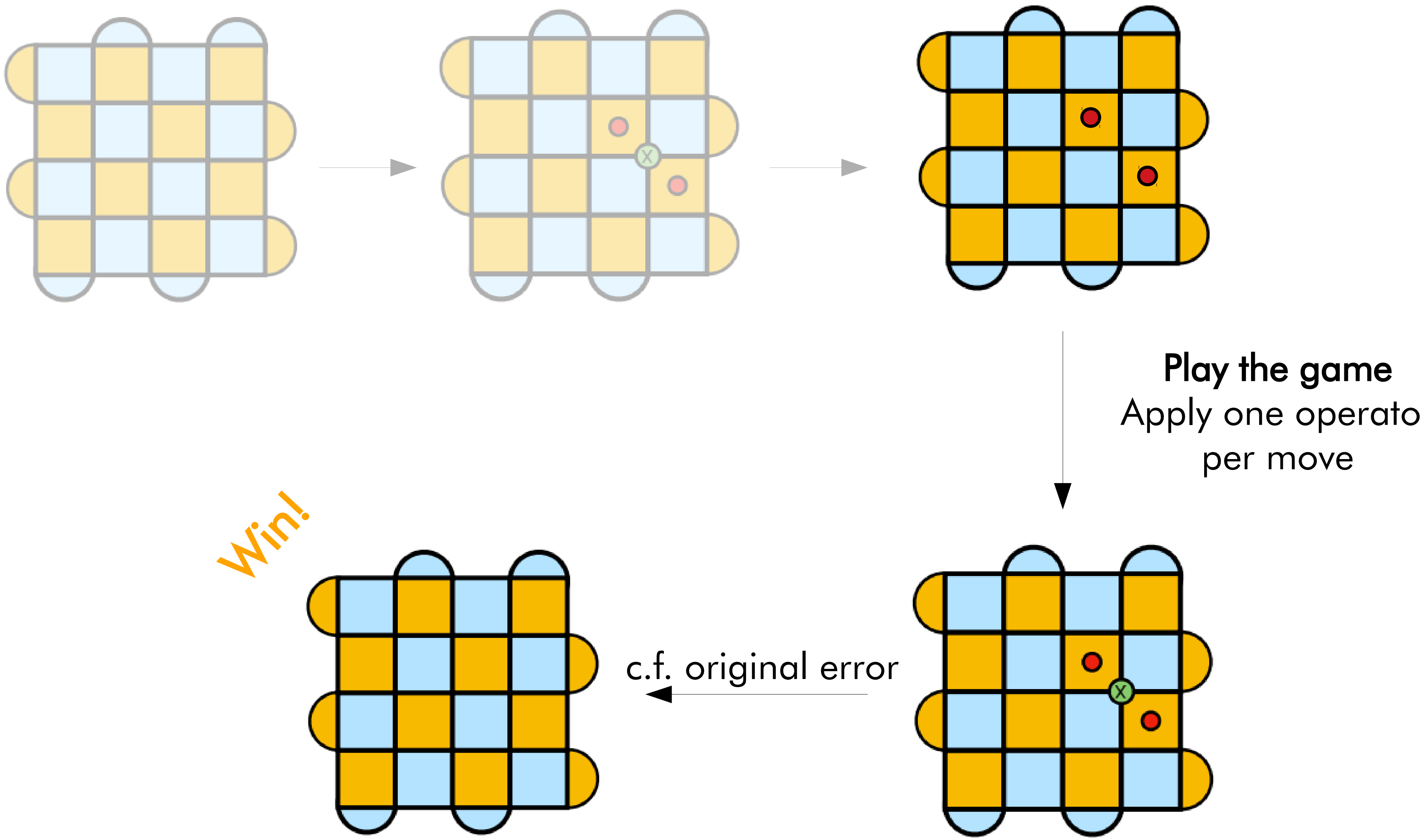
Neural Network

$$Q_{\mathbf{w}}(s)$$

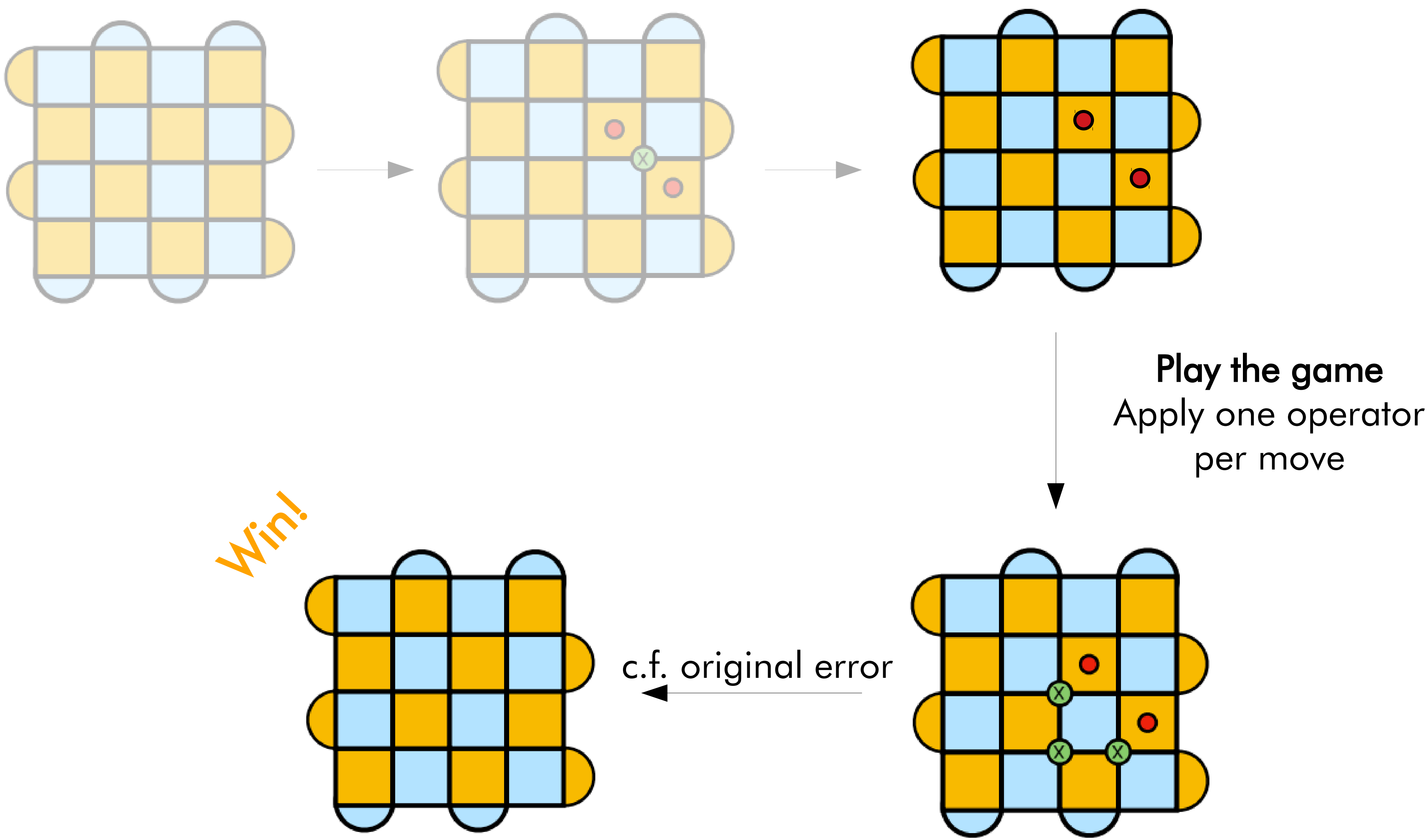
The Error Game



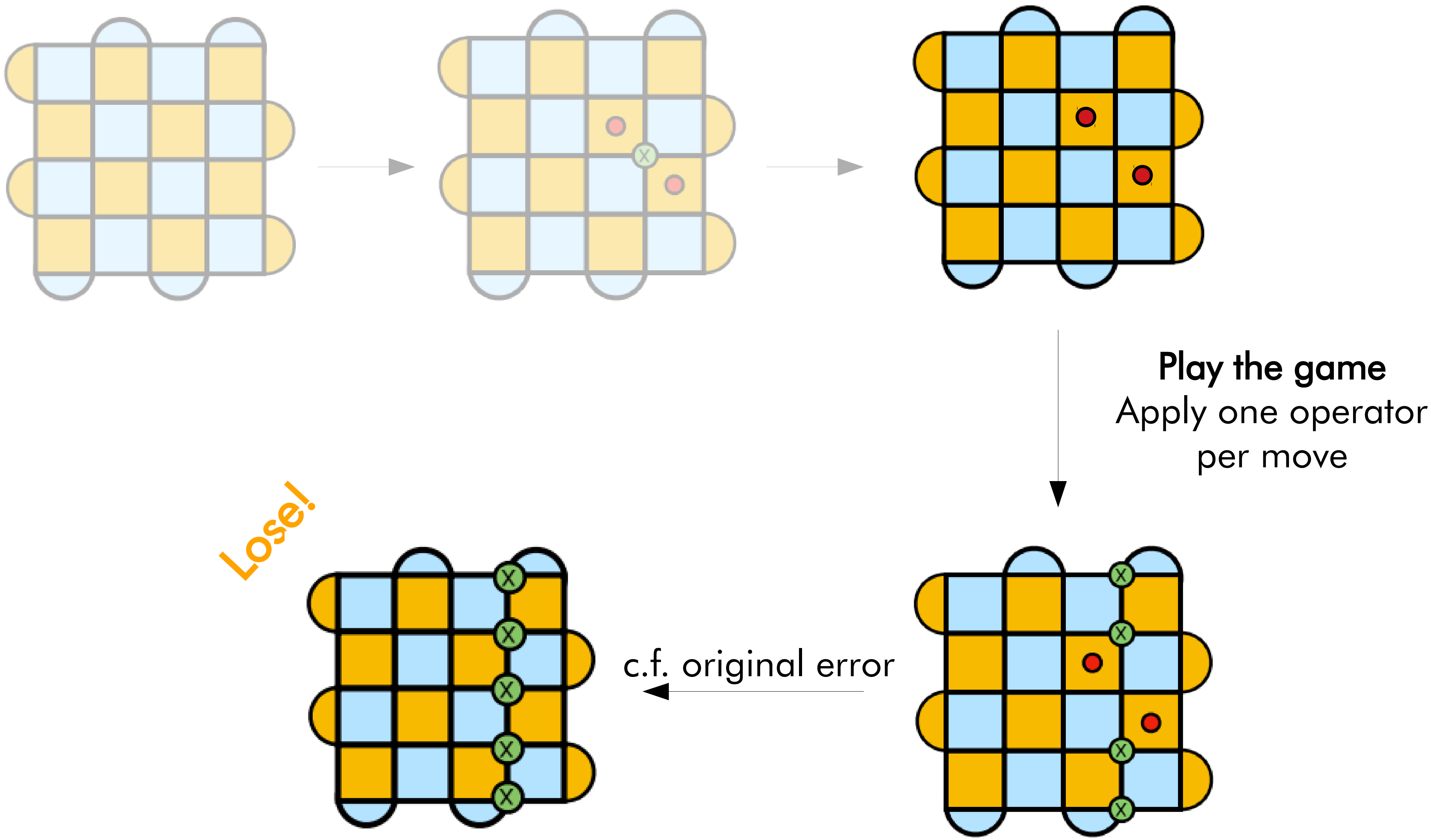
The Error Game



The Error Game

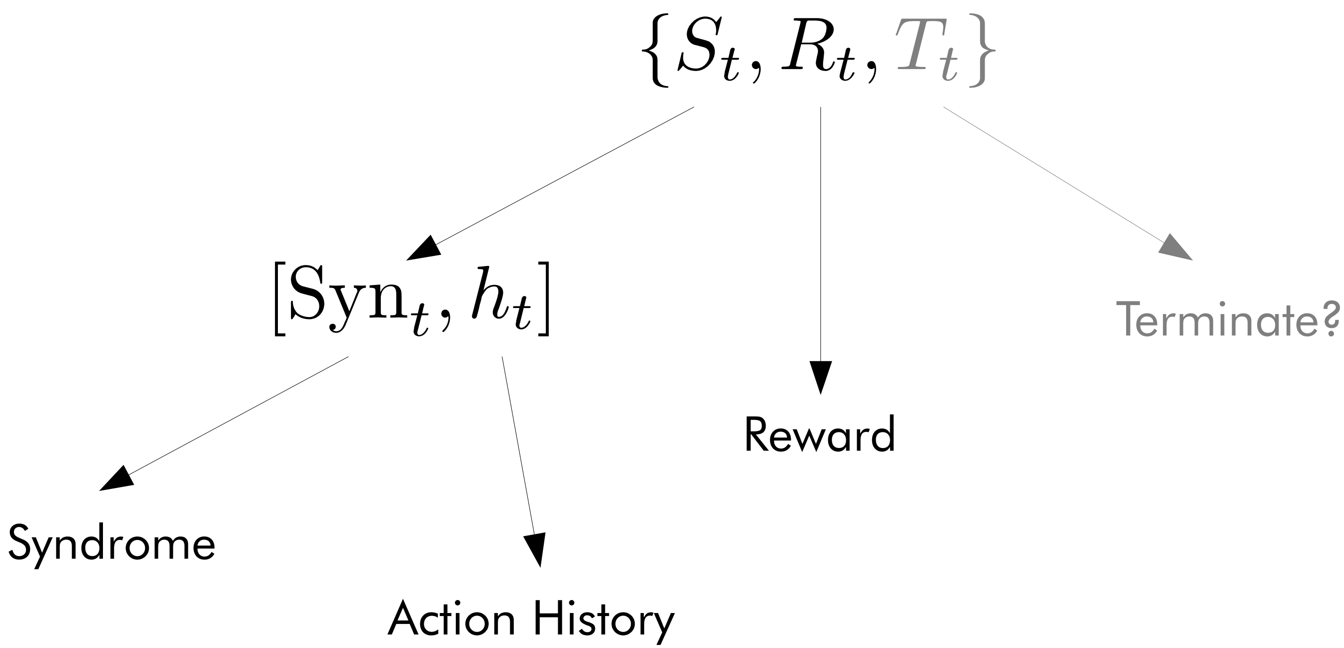


The Error Game



The Error Game

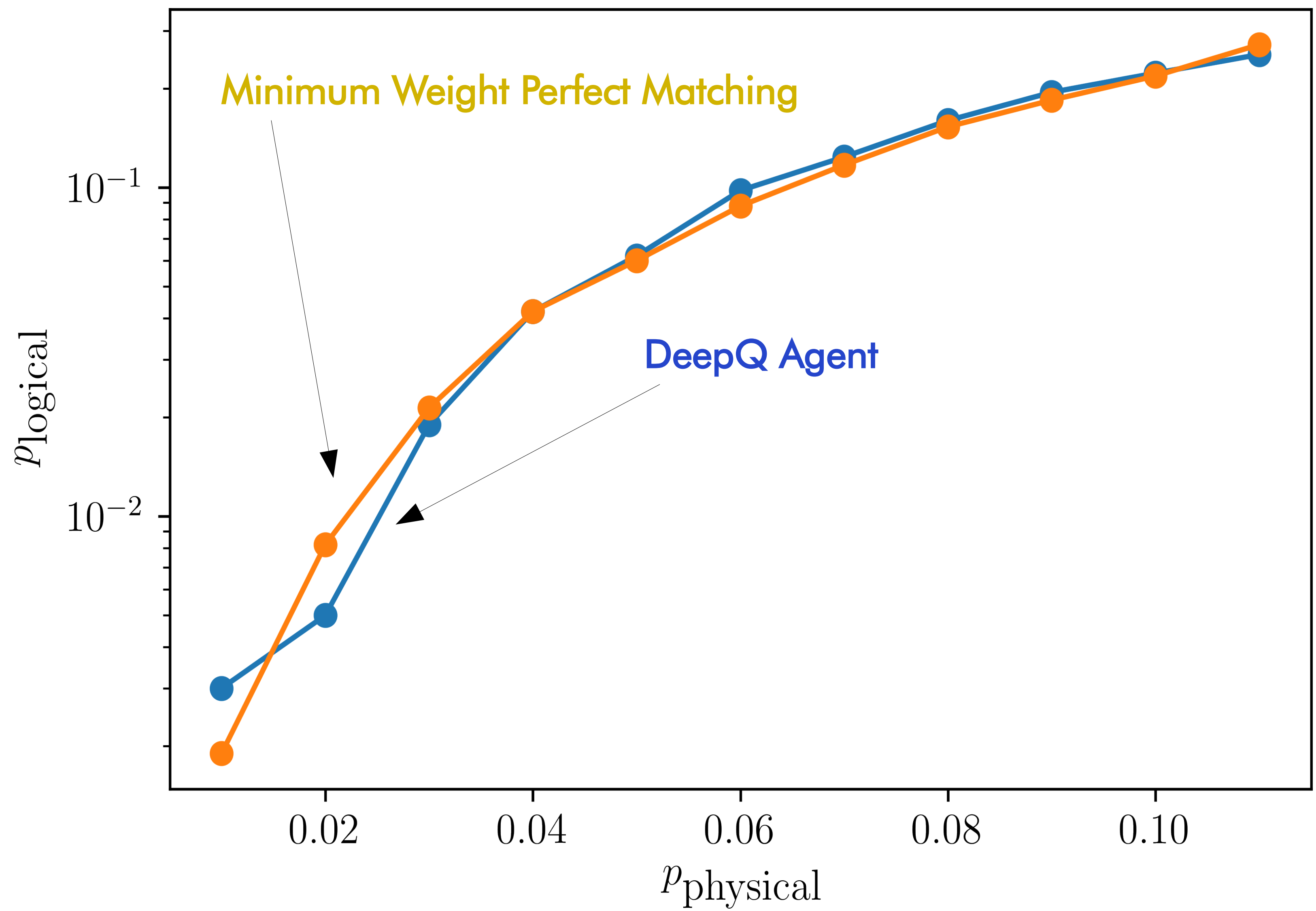
The Game Environment



The Agent's Actions

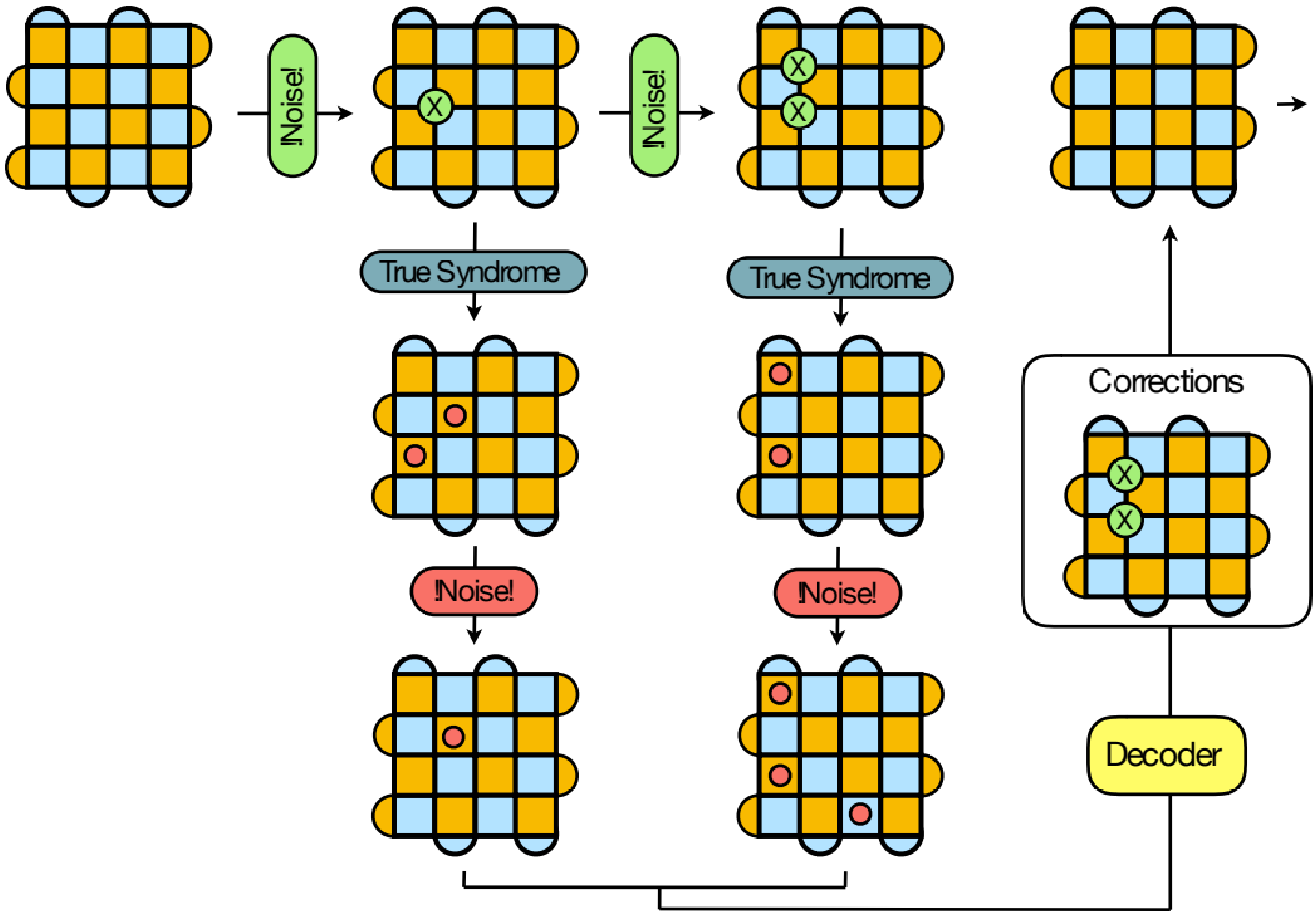
- Flip a qubit
- Request new syndrome

The Benchmark



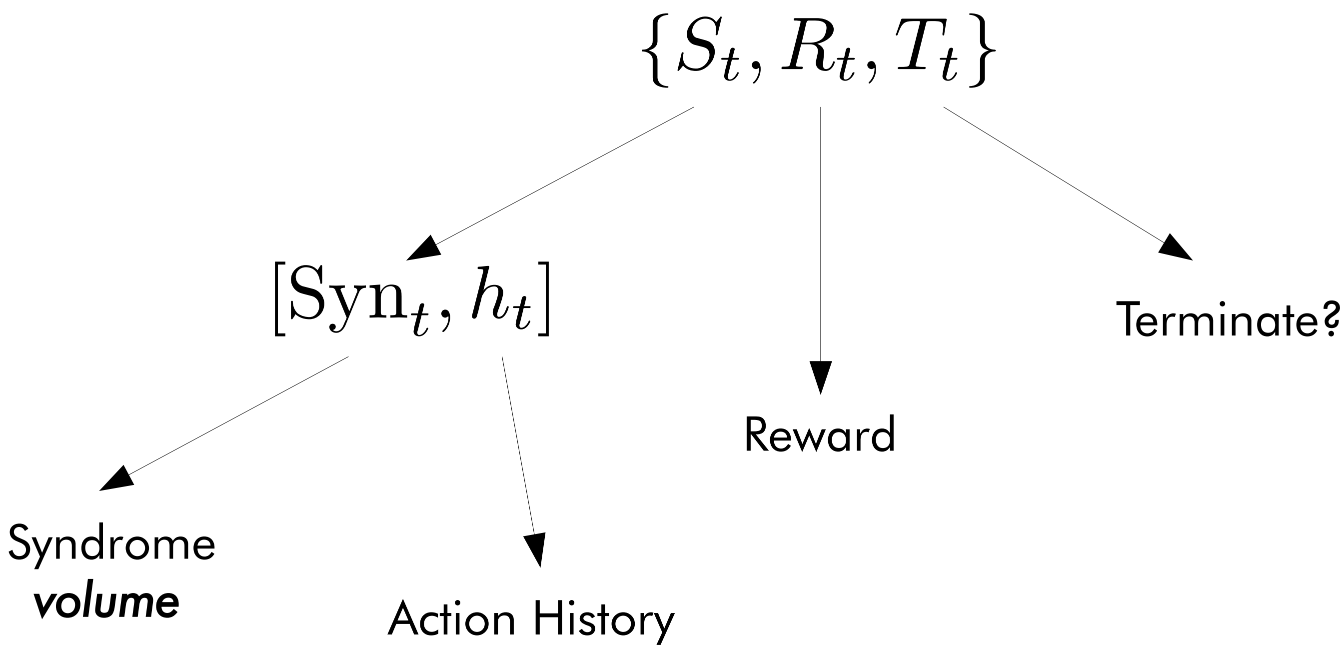
Reward = +1 if win, 0 otherwise

Faulty Measurements



The Error Game

The Game Environment



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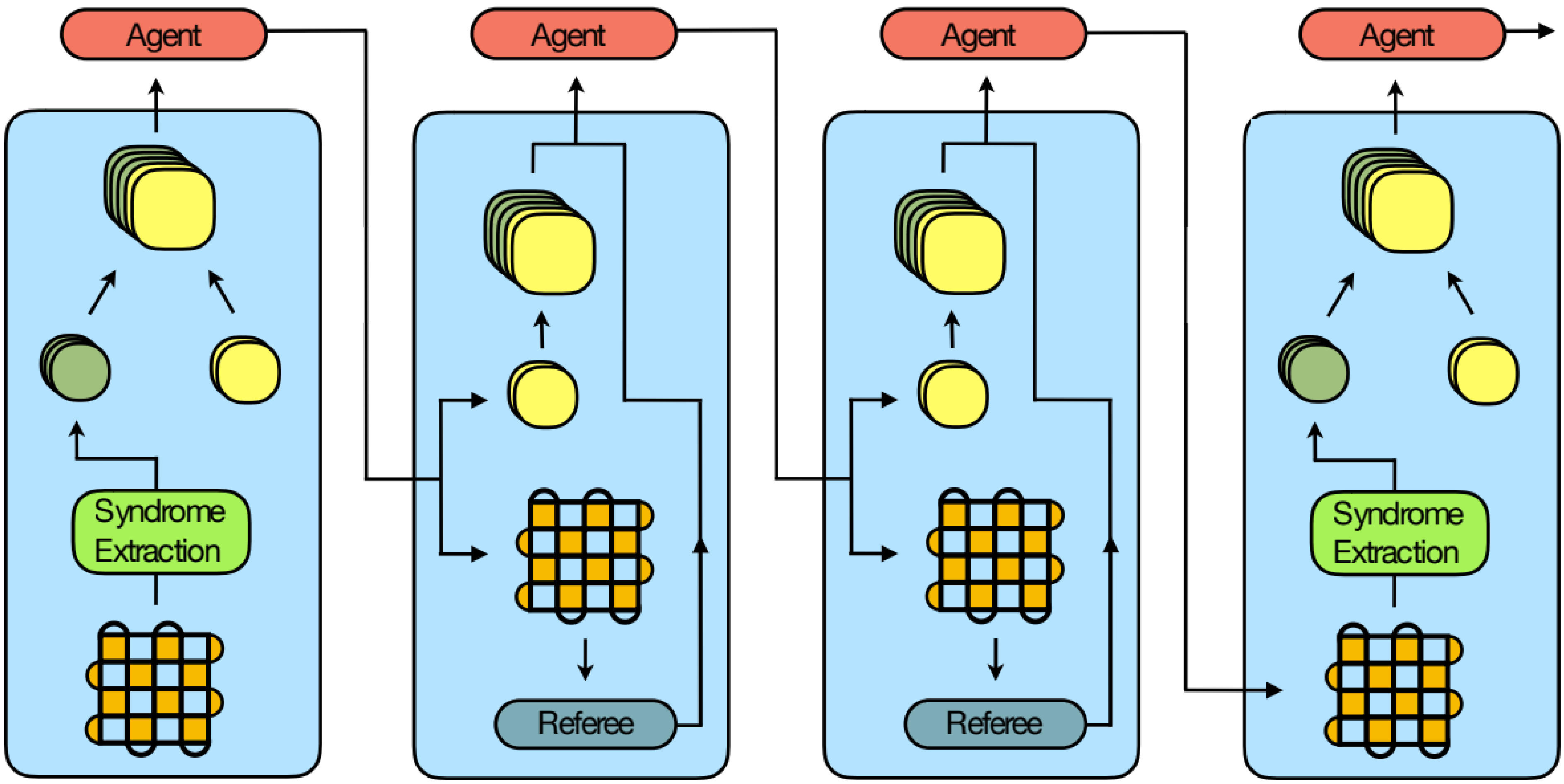
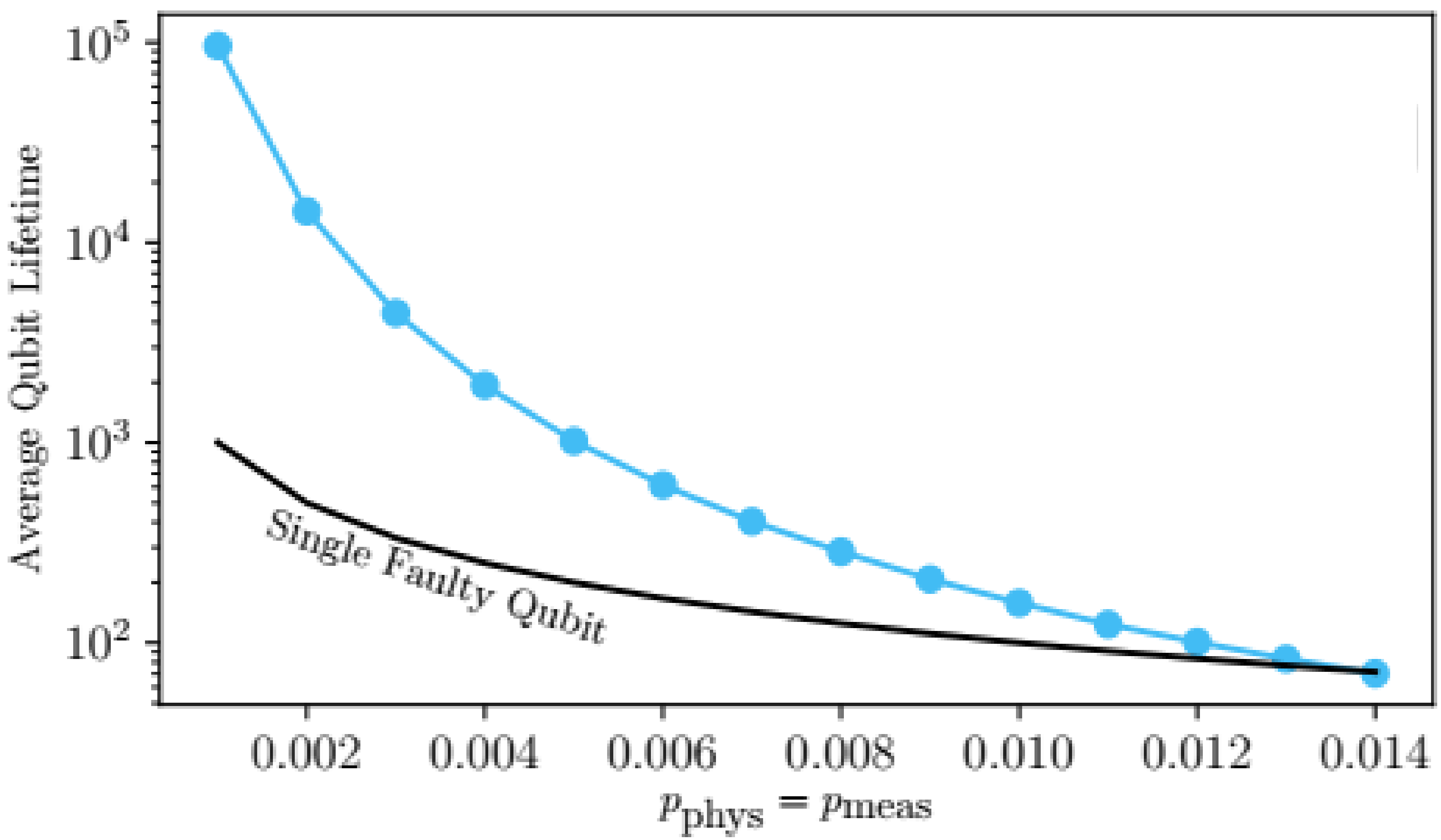
Faulty Measurements

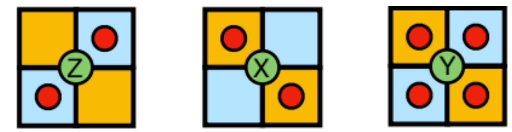


Qubit Lifetimes



Conclusions

Quantum Error
Correction



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Reinforcement Learning

Learning from feedback

- Competitive performance (small codes)
- **Fault tolerant**
- *Implementation specific*
- Correlated errors or non-Abelian code?
- Single-shot?
- Design test-cases to ***extract strategy***?