# CONSIDERATIONS DETERMINING PAIRING Tc FOR SUPERCONDUCTIVITY THROUGH E.E. INTERACTIONS

## Momentum and Frequency Dependence of effective electron-electron Interactions for High Tc?

Based on (ArXiv.org), RMP (colloq. ?).

## What is the Physical Basis for the Following?

Electron-phonon based supercond. $Maximum\ T_c/\theta_D\ is\ O(10^{-1})$
Liquid He(3) $T_c/E_f\ is\ \ O(10^{-3})$
Heavy Fermions $O(10^{-1})\ to\ O(10^{-2})$
Cuprates $O(10^{-2})$
Pnictides $O(10^{-2})$
Cold Fermions (Low Density Fermions with variable Interactions) $Maximum\ near\ unitarity : O(10^{-1})$

**Chandra Varma, UC Riverside**

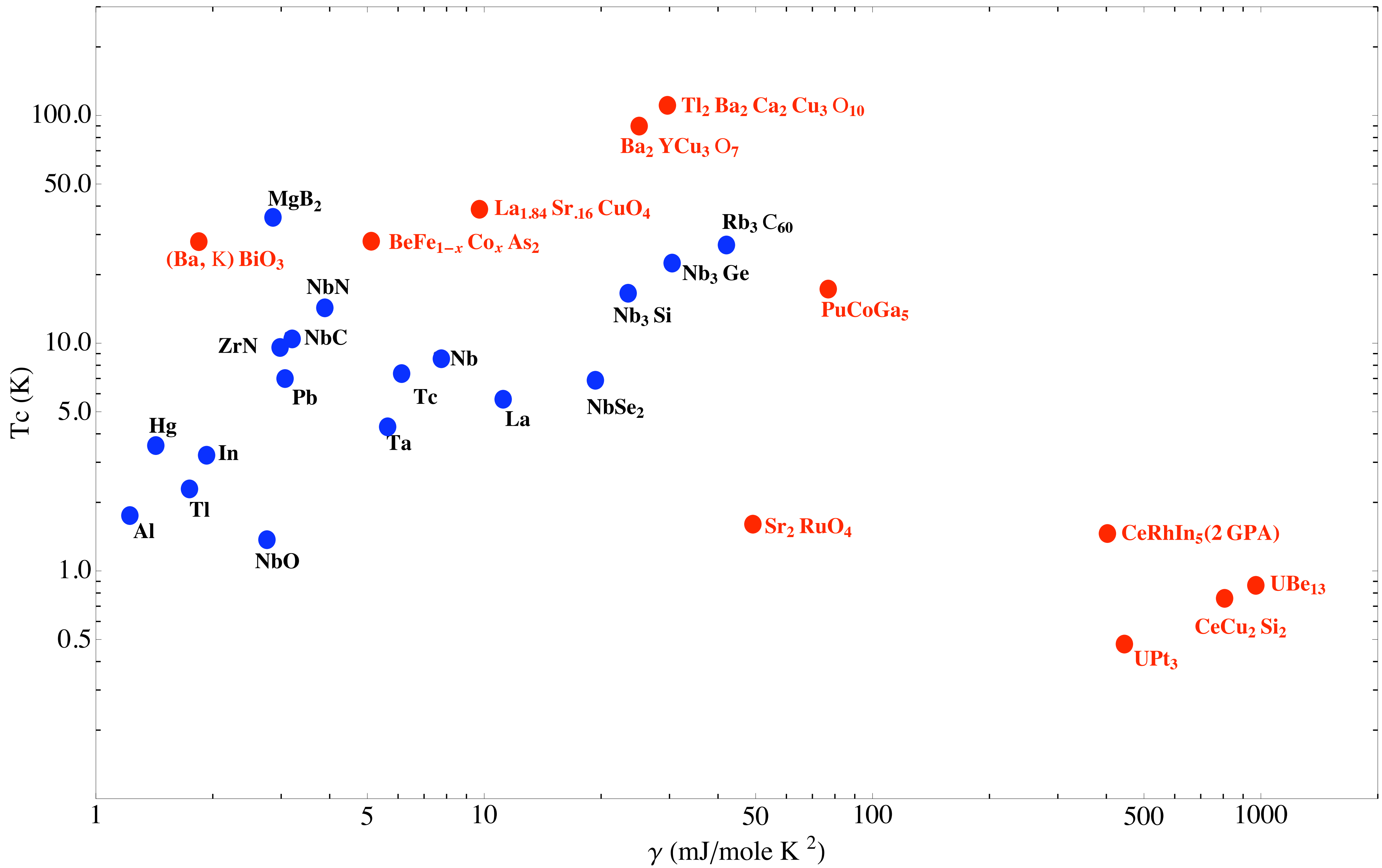
**KITP Material Design Conf, Feb 9, 2010**

## Related Questions Encountered:

Why is electronically induced pairing invariably found near QCP's? Do these QCP's have to be of a special nature?

Is Higher Temperature Superconductivity Possible?
And if so, is there any theoretical guidance as to where one should look for it?

# Field of Action:



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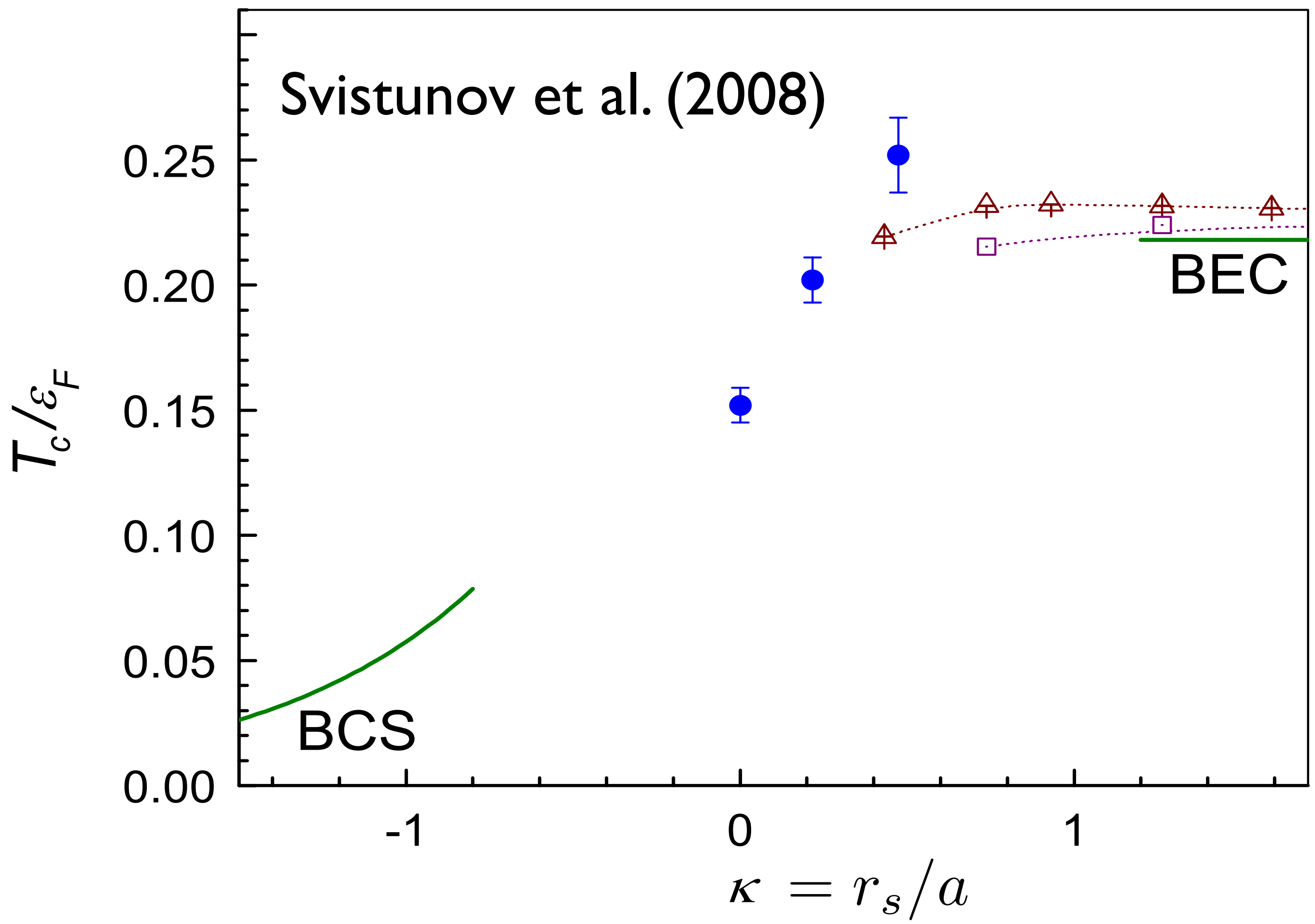


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## Empirical Relations:

$$McMillan: \; N(0) < I^2 > \approx \; constant \; within \; a \; class$$

$$Varma \; and \; Dynes(1975) : < I^2 > /M < \omega^2 > \quad also \; constant$$

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Several Derivations: Simplest by Friedel et al. in tight binding representation of el-ph. interactions:

$$N(0) < I^2 > \approx N(0) < (\partial t/\partial R)^2 > \approx E_c/r_0^2$$

Varma and Dynes : (Following Friedel's reasoning)

$$< I^2 > /M < \omega^2 > \approx < (\partial t/\partial R)^2 > / < \partial^2 t/\partial R^2 >_{renorm} \approx E_c/r_0^2$$

Lessons: parameters determining Tc are gross averages and they are inter-related.
This is equally true for e.e. induced superconductivity, especially for problems where fermi-liquid theory can be applied.

There is one clear example of a fermi-liquid superconductor: liquid He(3), (possibly also Sr(2) Ru O(4)).

# Maximum Tc from El-Ph Interactions

Using the observed empirical relation and their approx. derivation:

$$(Tc)_{max} \approx E_c/Mr_0^2 \exp(-3/2) \approx \omega_{unren.} \exp(-3/2)$$

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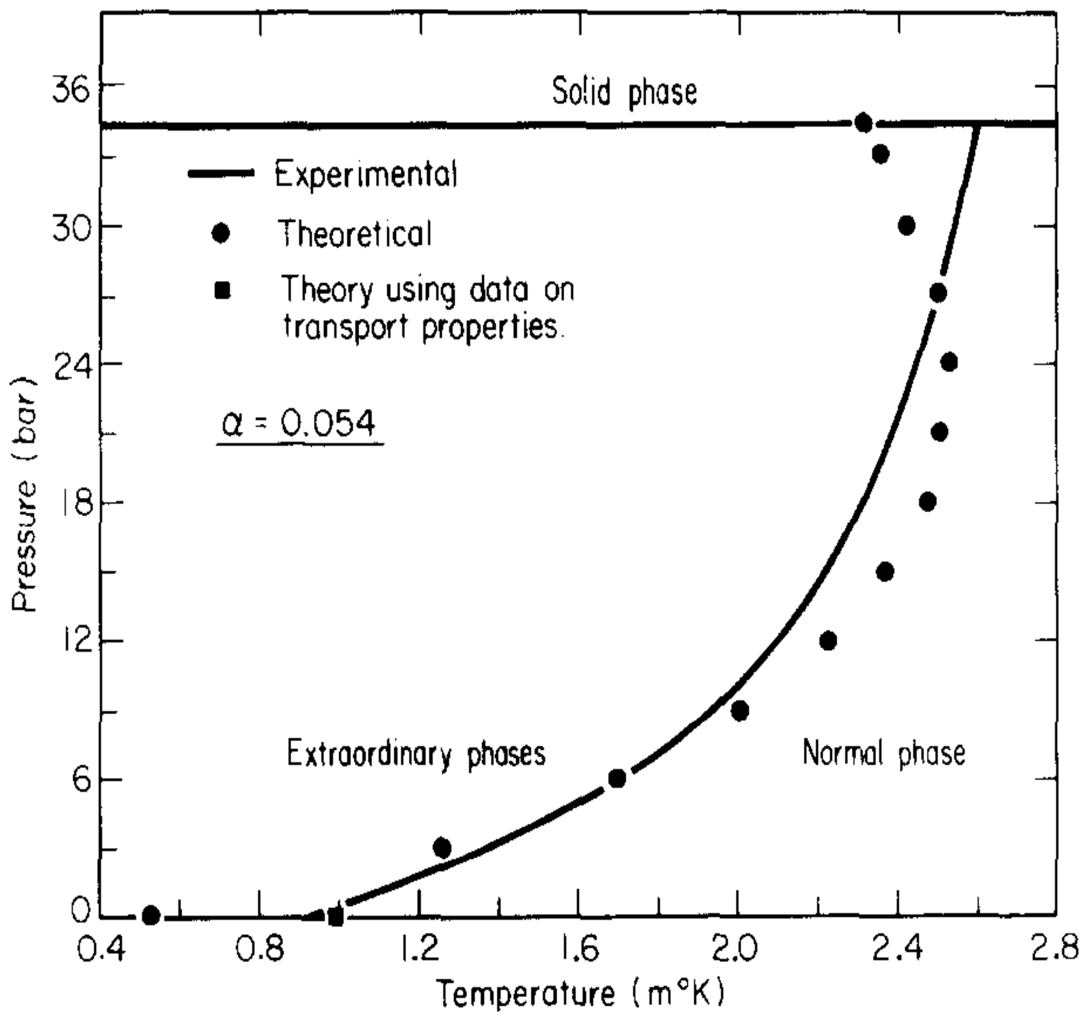

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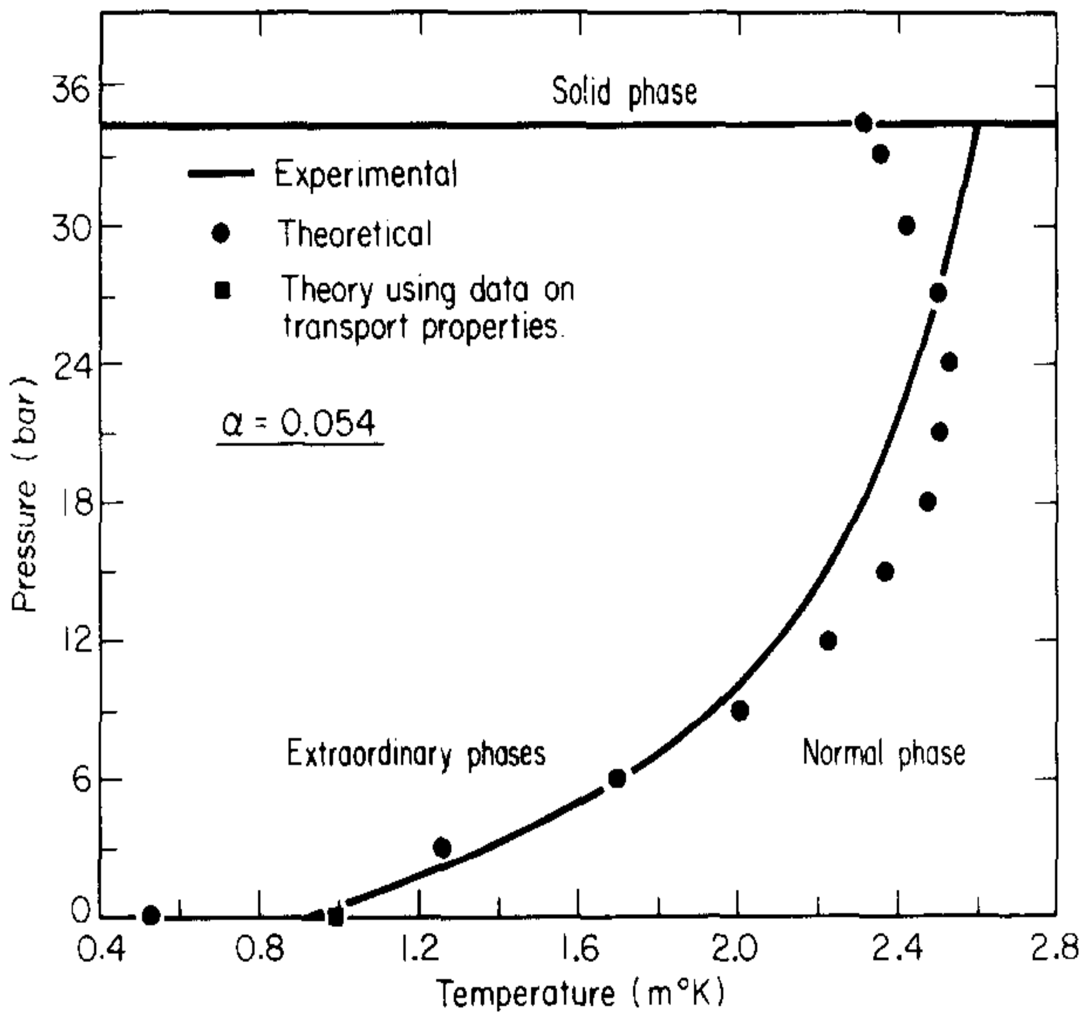


Pfinzner and Woelfle: Variation of Tc with P understood systematically by calculating interactions parameters constraining them by measured Landau parameters but with a renormalization of pre-factor downwards by about 1/10.

## Things to remember when thinking of fermion interaction induced pairing:

1. The actual superconductivity interactions parameters continually connected (and actually close) to Landau's A parameters, which are always less than 1,
due to cancellation of self-energy and vertices.

2. Pre-factor: Do not forget the self-energy.

S-Waves: $T_c \approx \omega_c e^{-(1+\lambda_0)/\lambda_0}$

$\lambda_0$ : s-wave interaction parameter.

D-Waves: $T_c \approx \omega_c e^{-(1+\lambda_0)/\lambda_2}$

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SELF ENERGY ALWAYS MORE DELETERIOUS FOR FINITE ANG. MOMENTUM PAIRING

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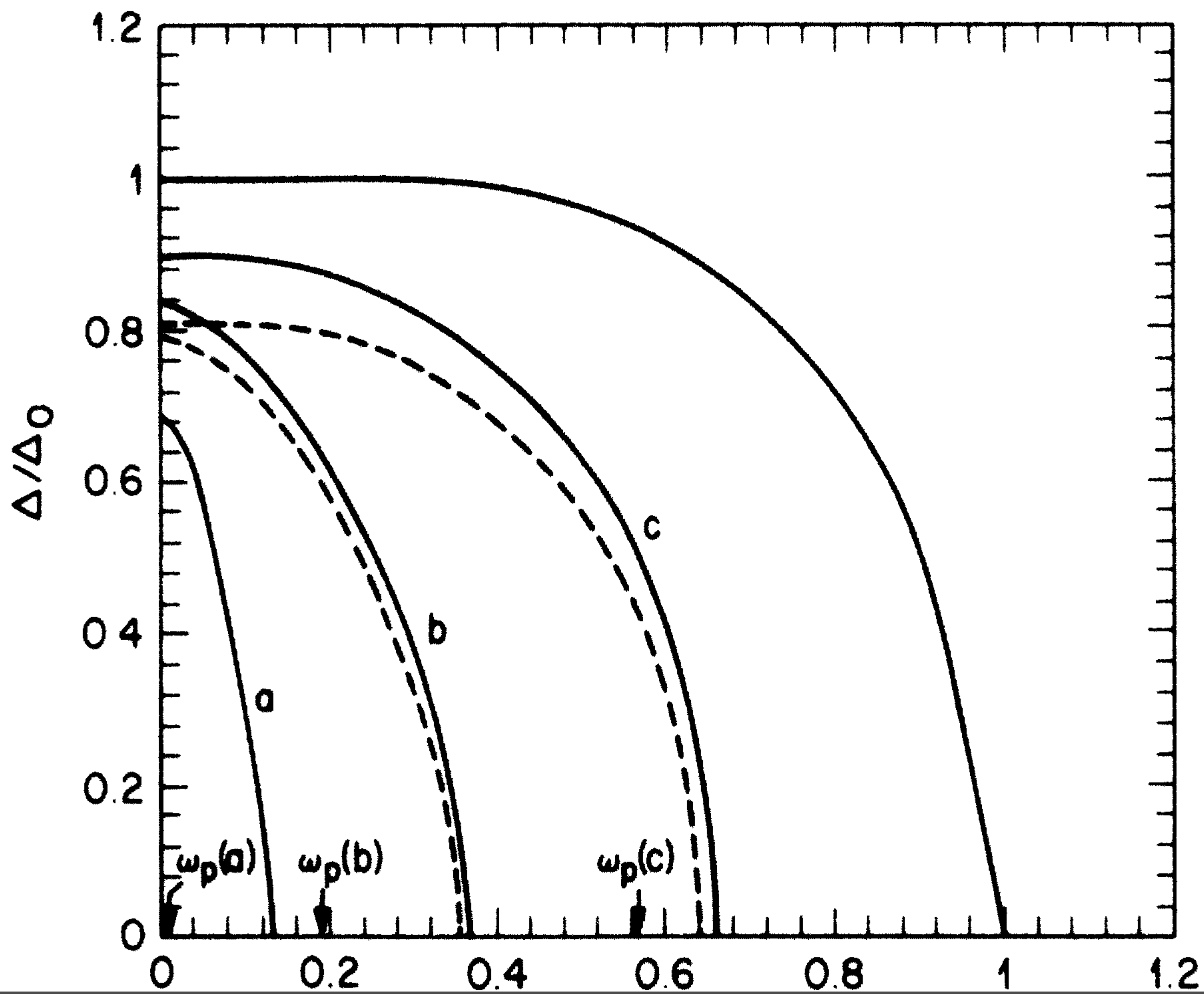
3. Nothing much for superconductivity by exchange of incoherent fluctuations (Luttinger-Kohn).
Must stick to collective modes but as opposed to phonons, they have only a fraction of the spectral weight.

4. INELASTIC SCATTERING LIMITS Tc WHEN Tc IS HIGH FOR D-WAVE SCATTERING WHILE IT IS HARMLESS FOR S-WAVE SCATTERING.

Inelastic Scattering or ‘real’ Scattering may be regarded for this purpose as elastic scattering from excitations with $\omega$ up to O(T).

How bad is it?: Millis, Sachdev, CV (1988).




D-wave pairing solution from Eliashberg Eqns. with successively increasing spectral weight of low-energy excitations with total weight kept constant.

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This is harmless for s-wave pairing: Cancellation of self-energy and vertex. But it is deleterious for finite ang. momentum pairing.

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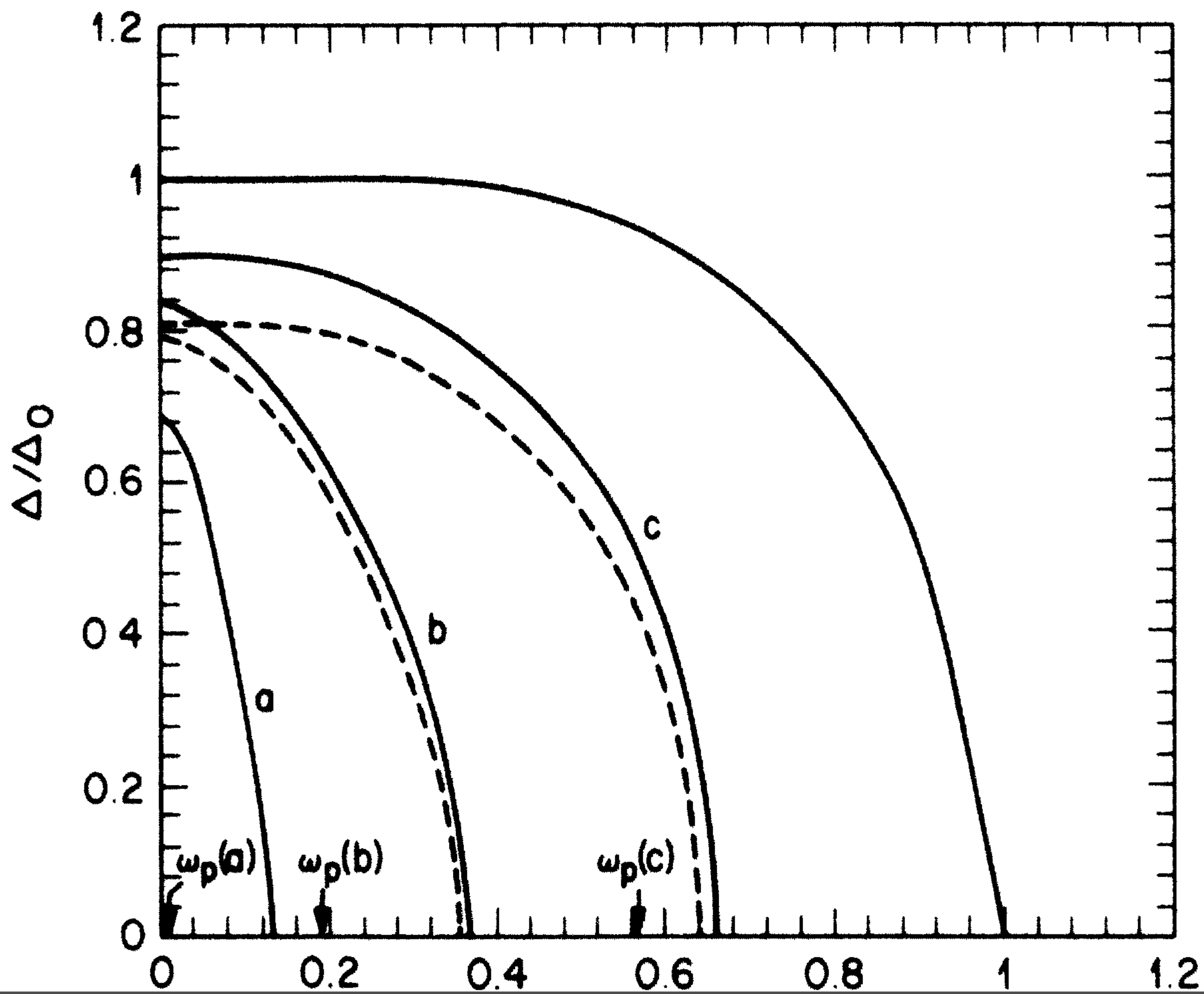


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## One Immediate Conclusion:

Ordinary or "Gaussian" Quantum-Critical Points are bad for Tc.

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$$T_c \approx \omega_c e^{-(1+\lambda_0)/\lambda_2}$$

1. Weight of Fluctuations goes towards zero frequency at such critical points so 'prefactor' goes to 0 at the critical point.

2. Not captured by the above expression: the role of inelastic scattering because it introduces Imaginary part in the gap-function similar to the effect of magnetic impurities in s-wave case.

# Some High Tc Electronically induced d-wave Superconductors

Heavy fermions: mass renormalizations of O(100)

Tc/Ef is only about 1/100

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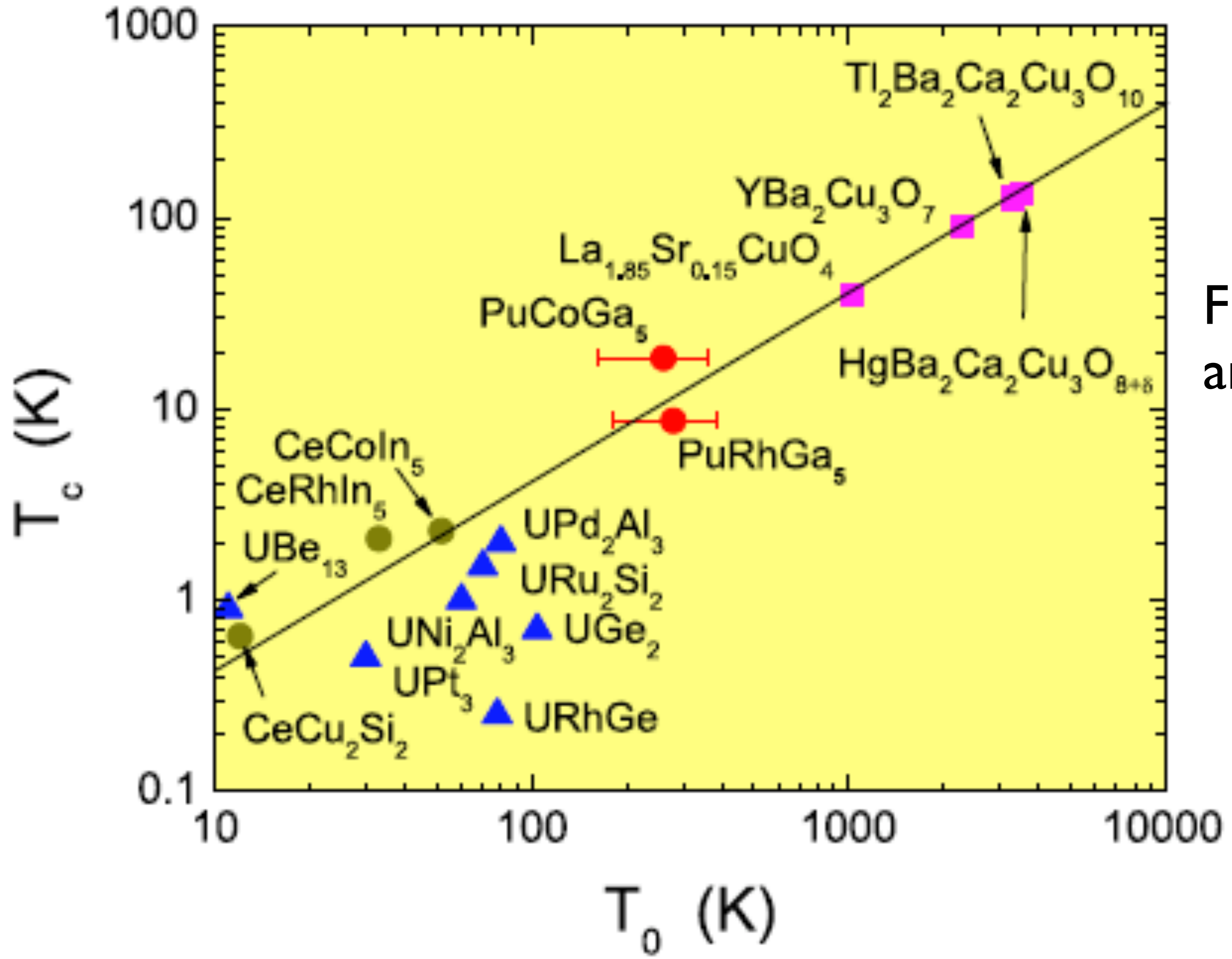


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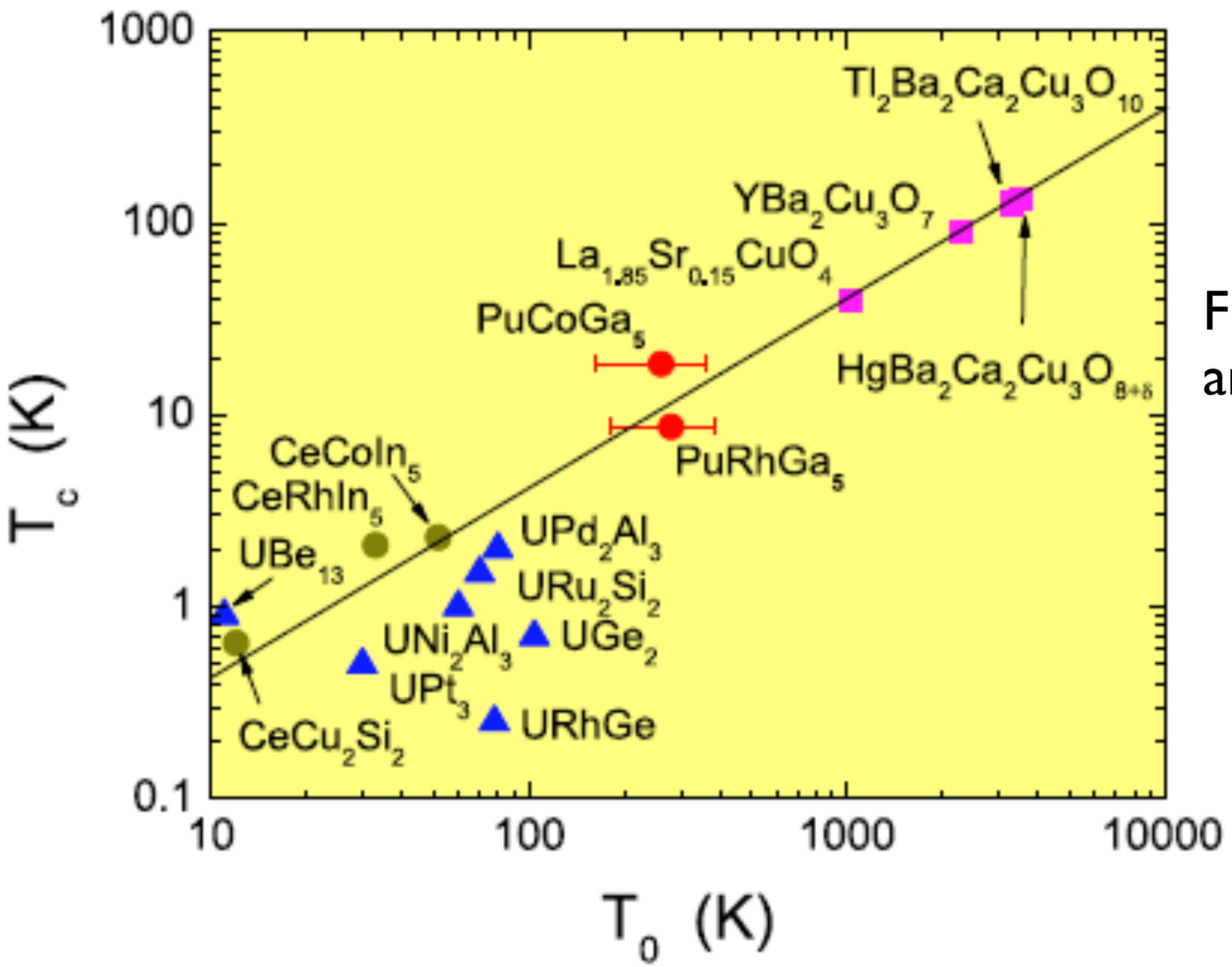


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Gained a factor of 10 in the dimensionless ratio, but lost by a factor of 100 in the heavy fermions in absolute magnitude. But no such loss in Cuprates.

# Heavy Fermion Superconductivity

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First suggestion (1982) following the discovery in CeCu(2) Si(2) and UBe(13) that heavy fermion superconductivity could not be electron-phonon induced but induced by AFM spin fluctuations and analysis of Expts to show d-wave pairing.
(Miyake, Schmitt-Rink, Varma (1986)

Why is Tc so low in heavy Fermions?

(1) Even though repulsion is large, the pair energy has a cut-off of the spin-fluctuation Energy which is the same order as the effective fermi-energy which is very small.

(2) Large Inelastic scattering unhelpful for high Tc.

(3) Also, s-wave scattering always larger than d-wave scattering leading to large self-energy effects which are bad for Tc.

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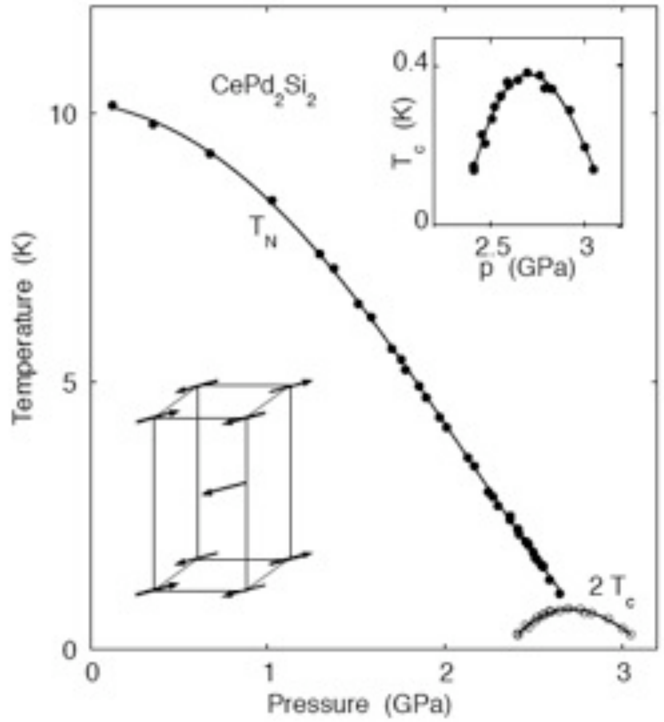


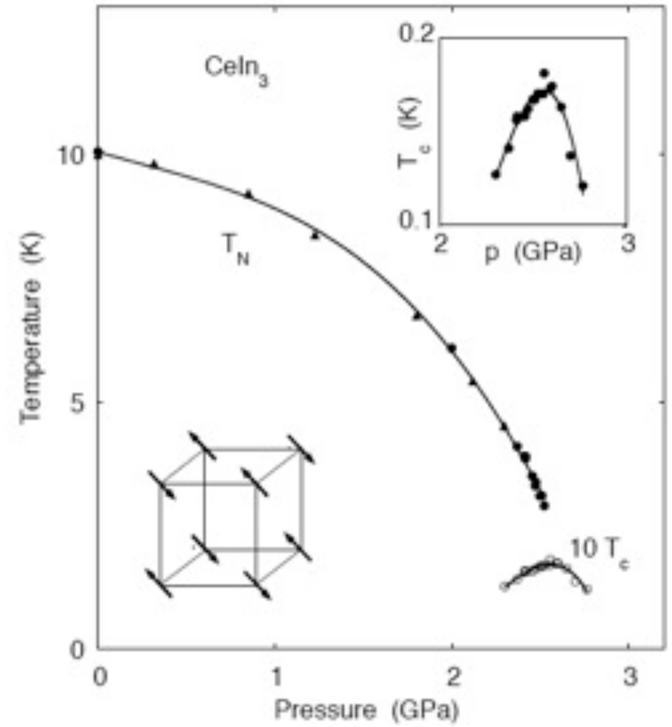

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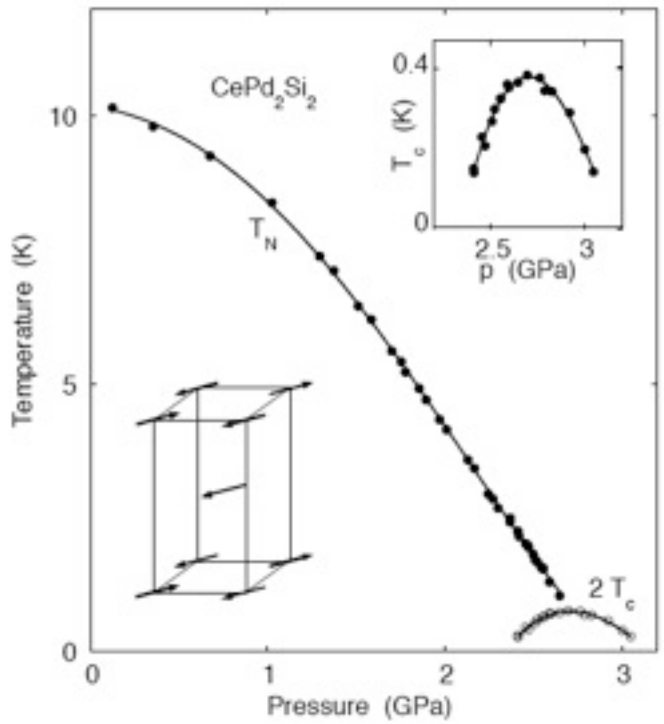
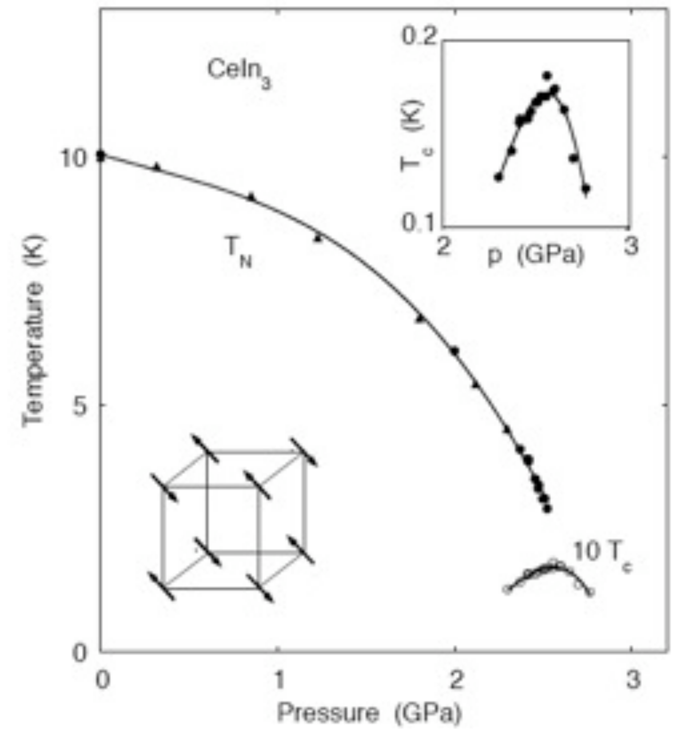
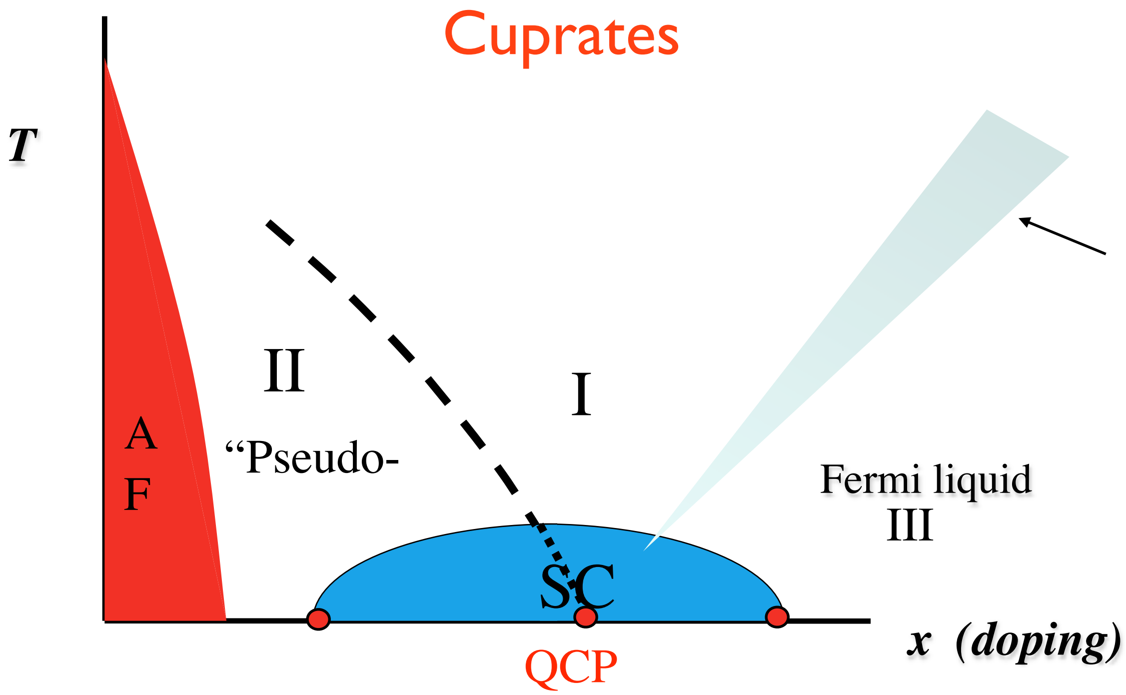
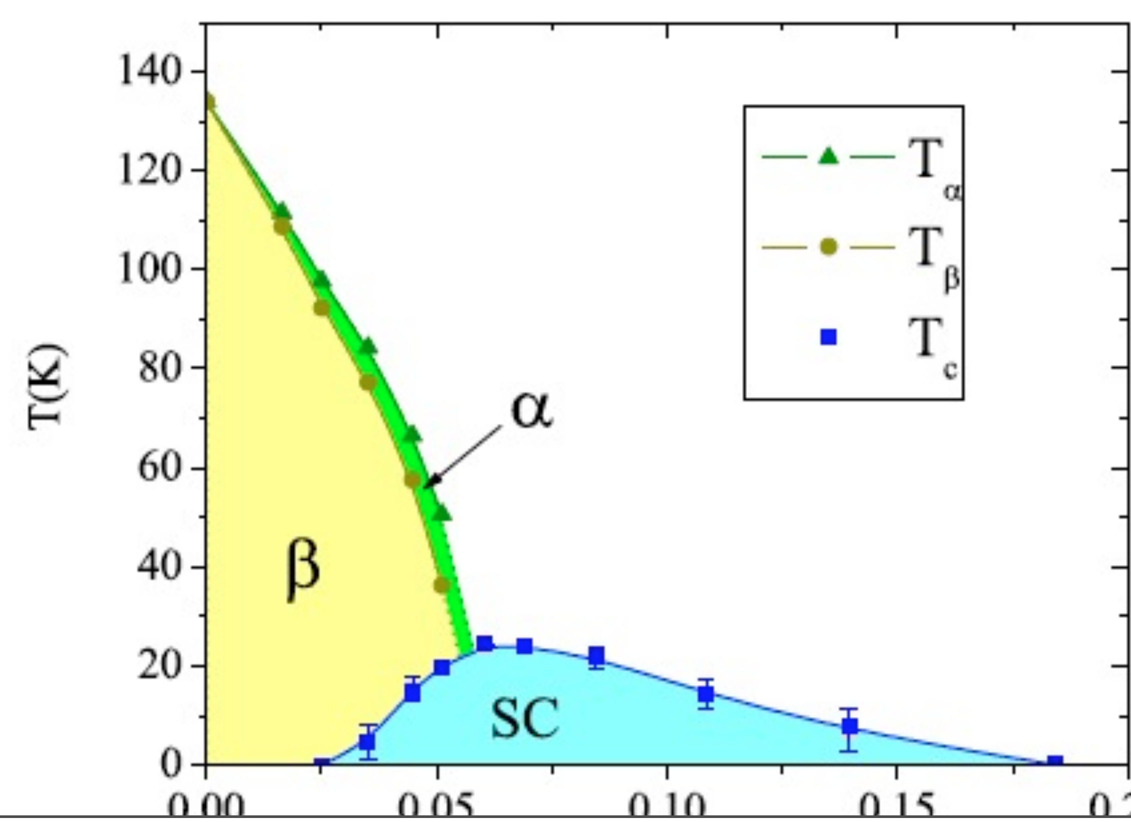



## Cuprates




## Ba[Fe(1-x)Co(x)As(2)



# Experimental Evidence for Spectra of Quantum-critical Form in Cuprates

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Single-particle Spectra measured in ARPES

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Linewidth proportional to $\omega$ for $\omega \lesssim \omega_c$ and constant beyond
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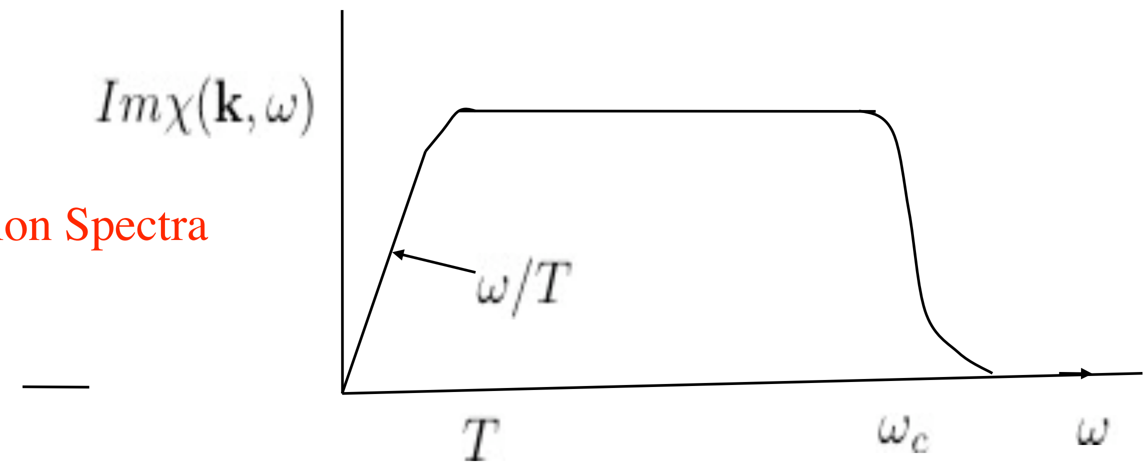
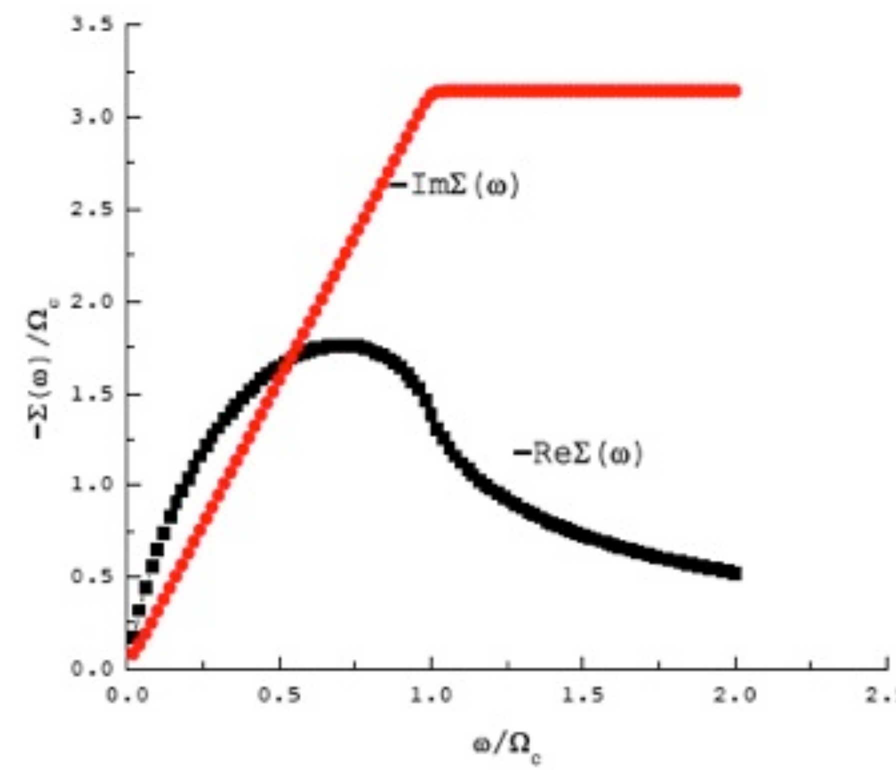
Excitation Spectra




Predicted single-particle Linewidth



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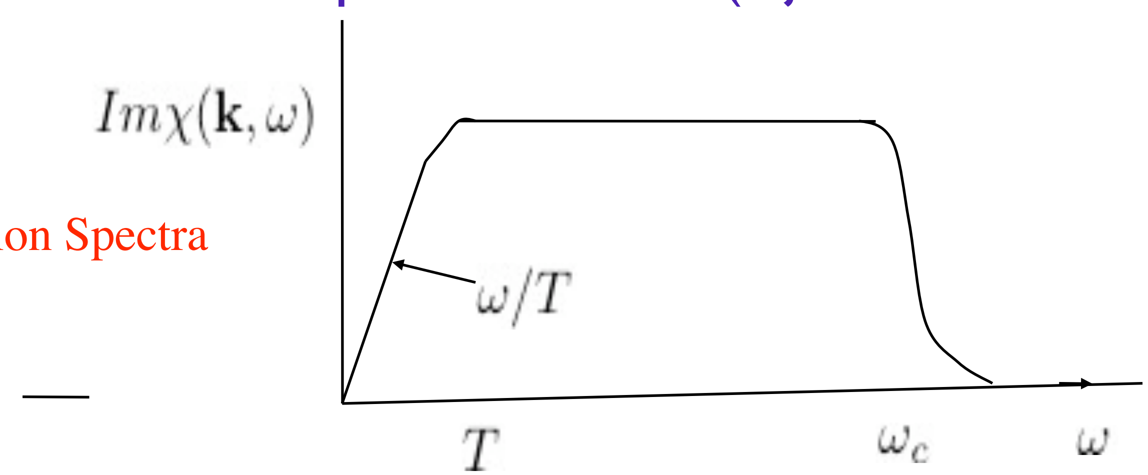


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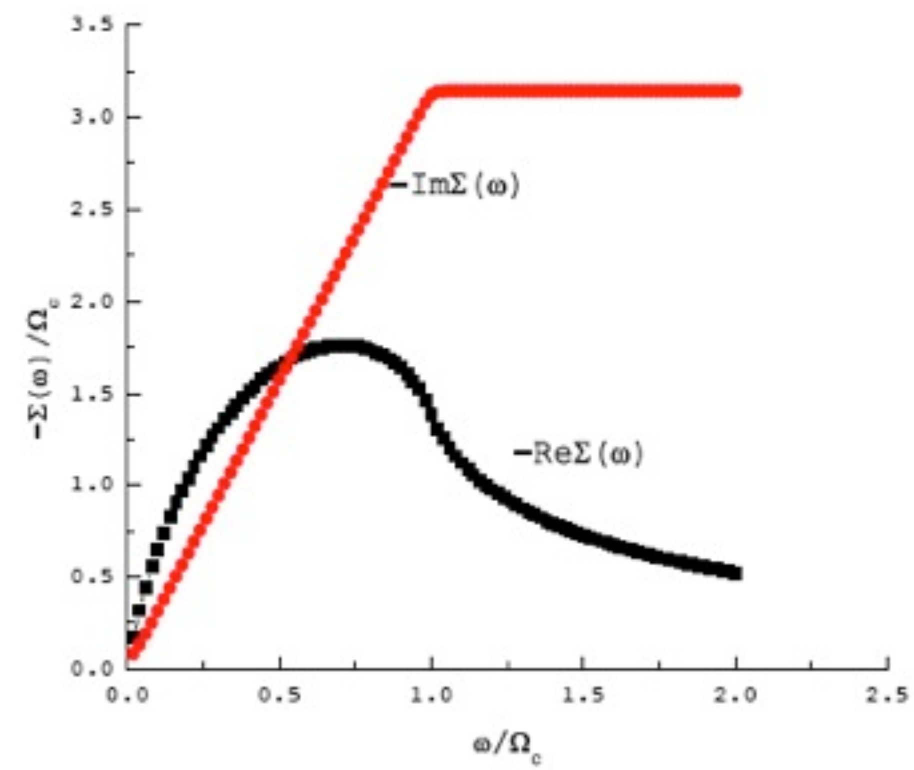


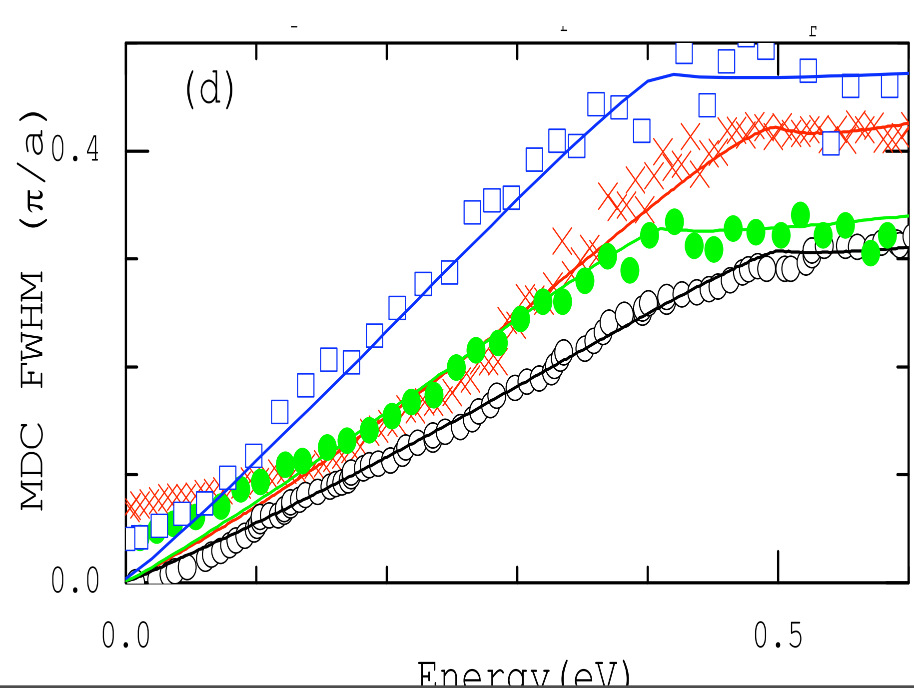


- O OP Bi2201 Nodal , Meevasana et al.
- X OP-Bi2212 Nodal, Lanzara et al.
- ● LSCO OP, Nodal , Chang et al.
- ■ LSCO Nodal underdoped, Chang et al.

This spectrum is ideal for high Tc because:

1. Locality implies least value of $\lambda_0/\lambda_2$
2. Least inelastic scattering imaginable.
3. Large Upper cut-off.
4. Fermi-liquid renormalizations are of O(1).

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## Quantitative estimates of $T_c,\ \Delta/T_c$

### Why is Tc quite high?

From single-particle spectra, $\omega_c \approx 0.4eV$

Coupling parameter for single-particle scattering $\lambda_s$ is about 1.0

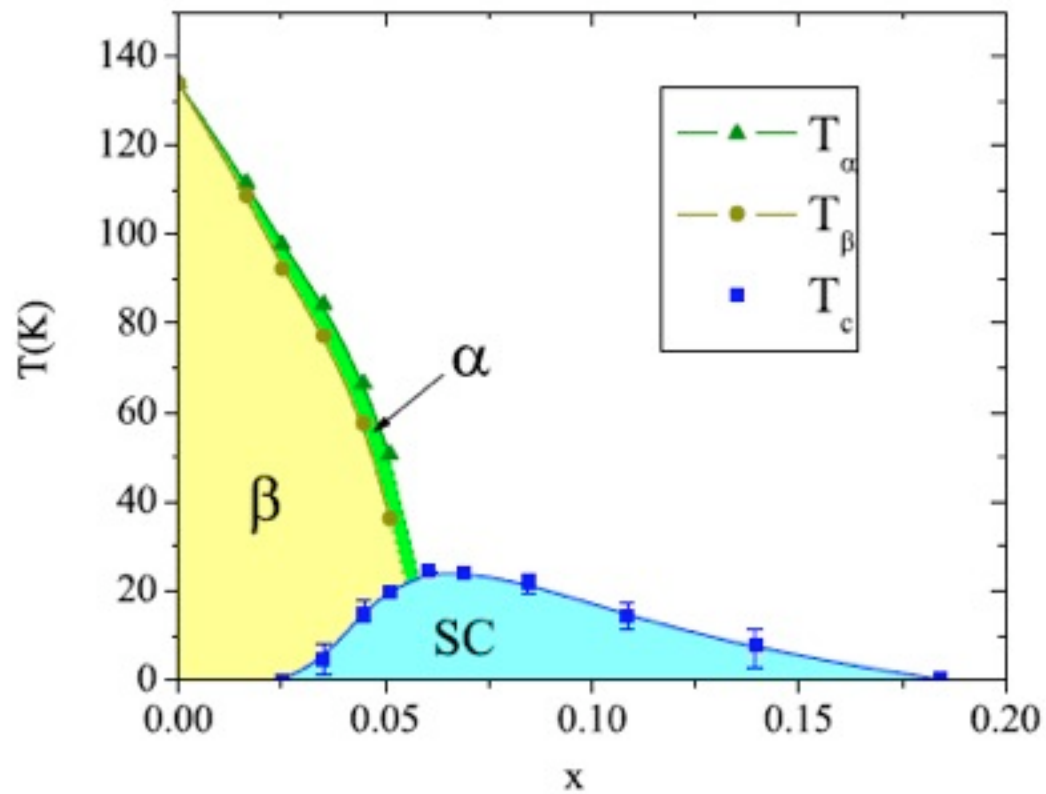
Transition temperature for d-wave superconductors approximately given by

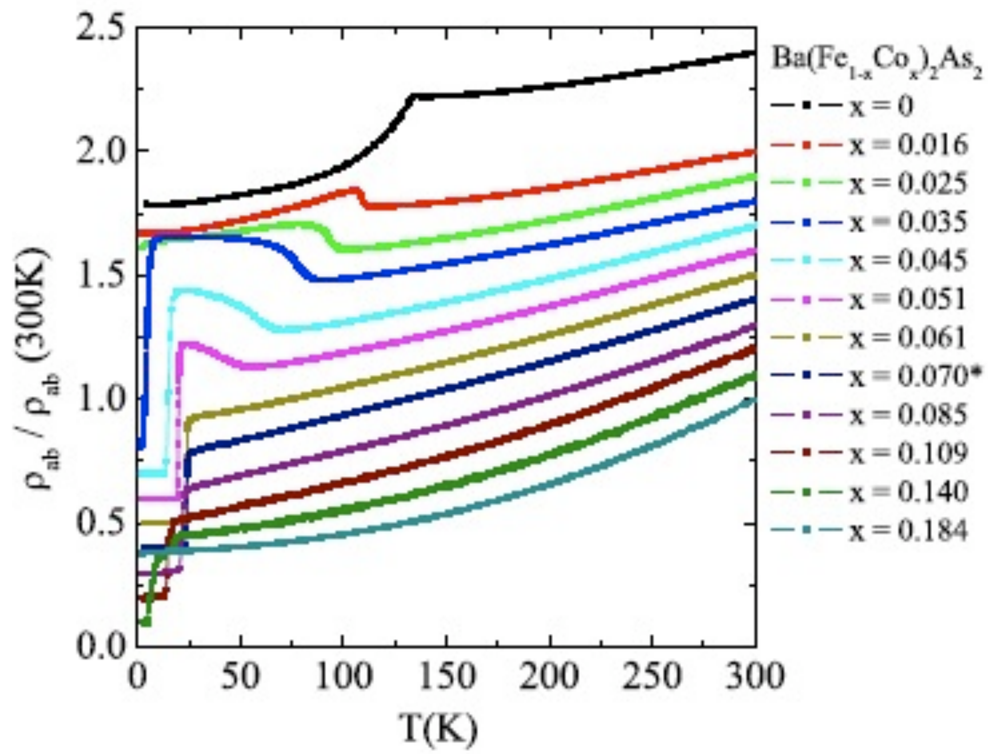
$$T_c \approx \omega_c \exp(-(1+|\lambda_s|)/|\lambda_d|)$$

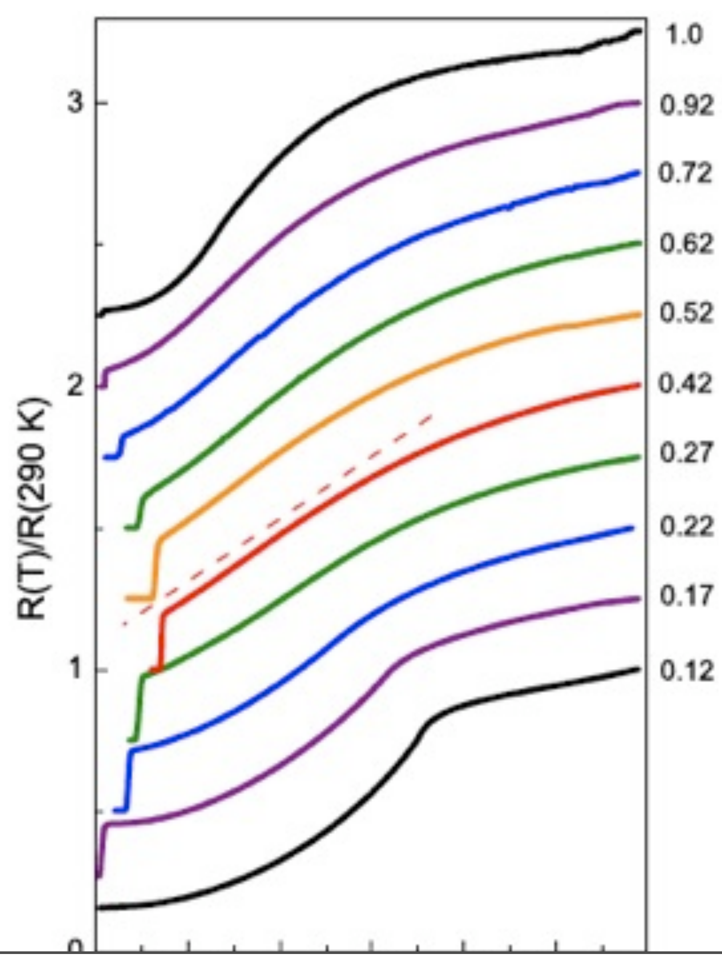
↑ ↑

single-particle, d-wave coupling constants

Ba[Fe(1-x)Co(x)As(2)  Chu et al. (Stanford), Canfield (Ames)




## $K_xSr_{1-x}Fe_2As_2$:

Melissa Gooch[1], Bing Lv[2], Bernd Lorenz[1], Arnold M. Guloy[2], and Ching-Wu Chu[1,3,

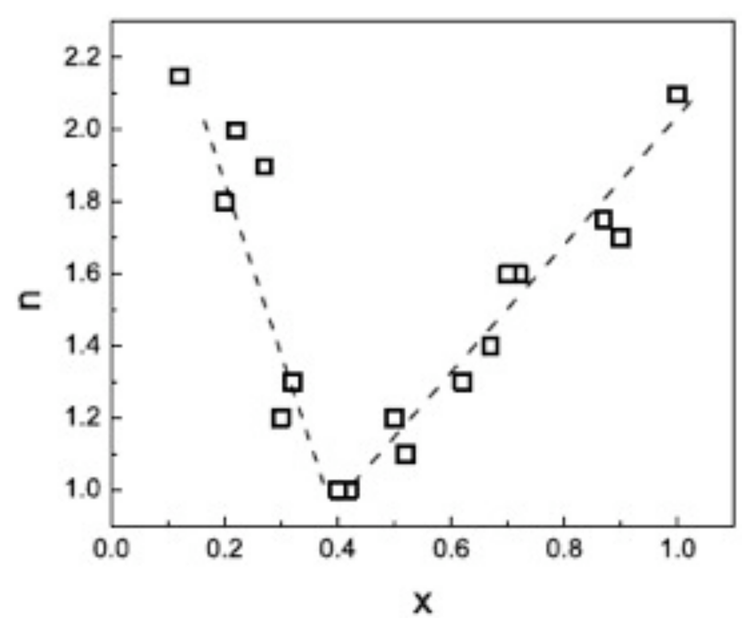
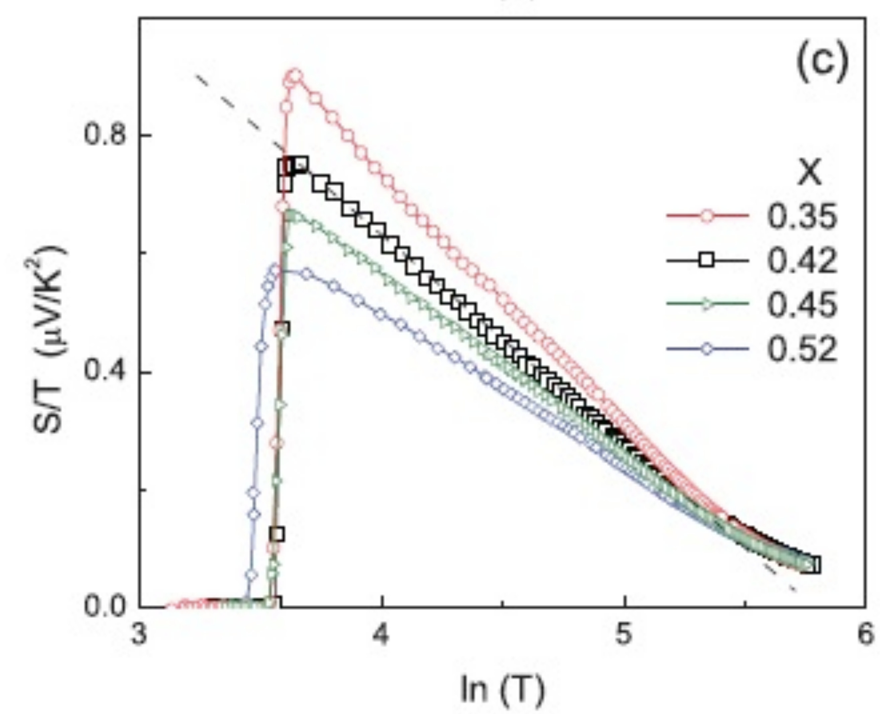



FIG. 2: The resistivity exponent n as a function of x



# How to get much Higher Tc.

Electronically induced pairing to get high upper cut-off.
“Fermi-liquid” renormalizations only of O(1): avoid the sad experience of heavy fermions and liquid He(3).

Topological Quantum critical point so that criticality does not renormalize scale downward and inelastic scattering does not hurt.

S-wave pairing

Is electronically induced s-wave pairing near topological quantum critical point possible?

# Case of BaBiO(3) doped with K?

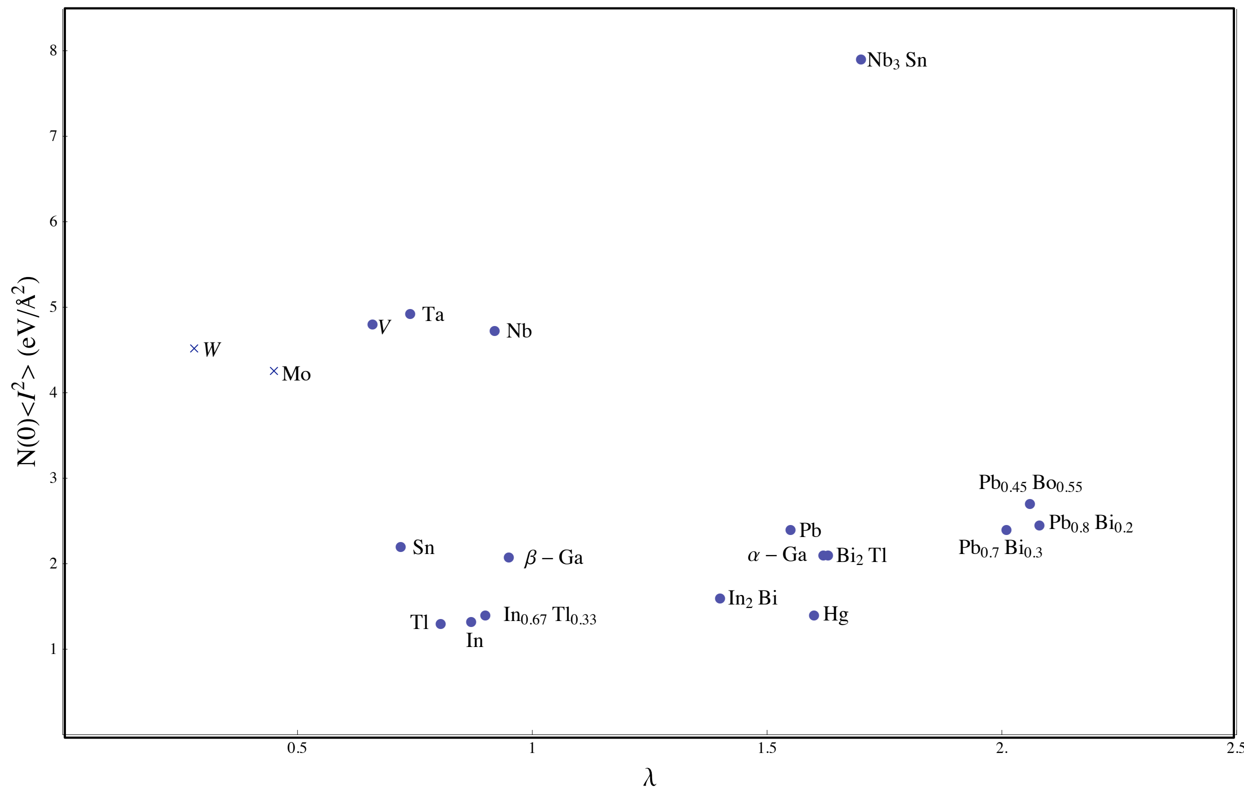
# Summary

In terms of (Tc/cut-off energy) finite ang.
Momentum pairing will always give much smaller Tc than s-wave.
Because of normal self-energy and inelastic scattering.

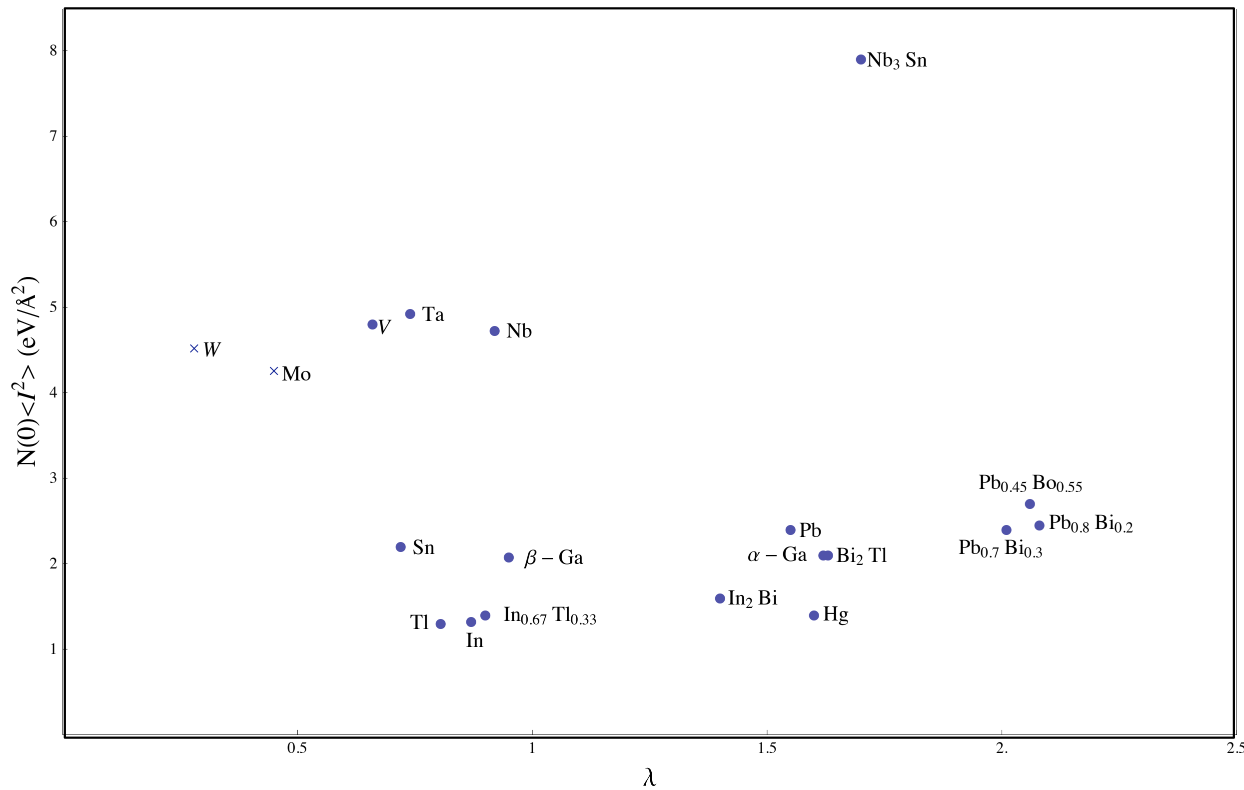
Strong Interactions reduced cut-off energy as well.
At Gaussian criticality, cut-off scale is sharply reduced which hurts.

Topological or local quantum criticality with w/T scaling of spectra maintaining large cut-off is ideal and the secret of high Tc in Cuprates and possibly pnictides.

To get significantly higher Tc, need s-wave electronically induced pairing with similar criticality.
This may not be impossible.



Nb3 Sn
W
Mo
V
Ta
Nb
Pb0.45 Bo0.55
Pb0.8 Bi0.2
Pb0.7 Bi0.3
Pb
α − Ga
Bi2 Tl
Sn
β − Ga
In2 Bi
Hg
Tl
In
In0.67 Tl0.33
N(0)<I2> (eV/Å2)
λ



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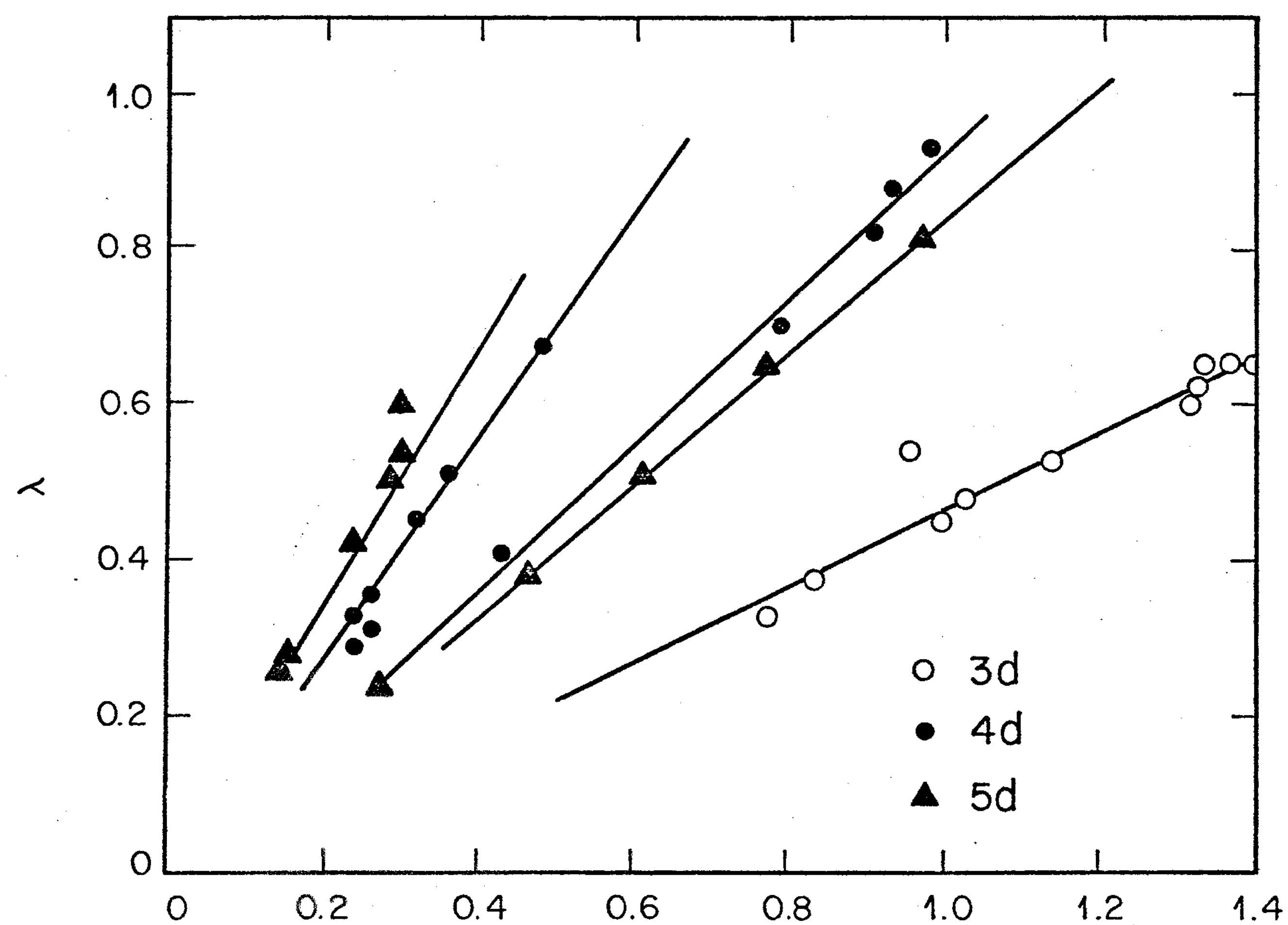
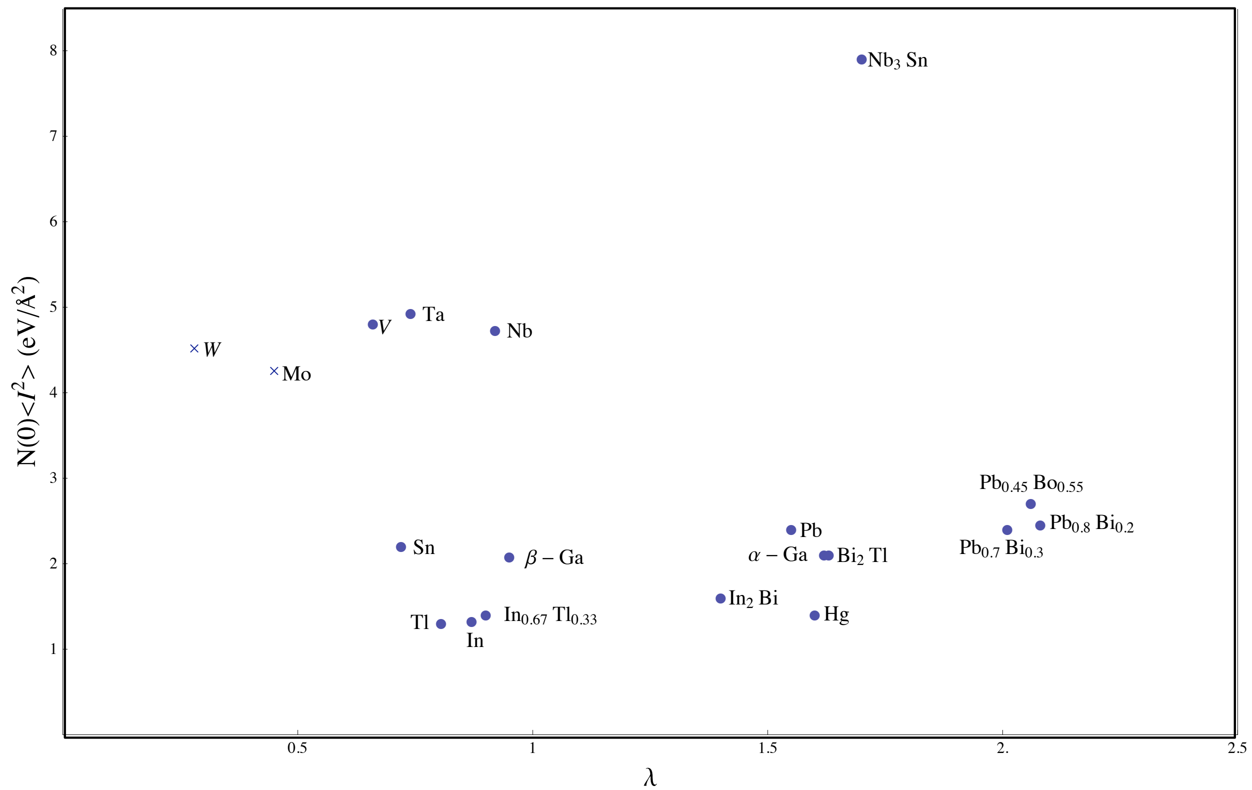


λ
3d
4d
5d




Friedel et al. (1975)

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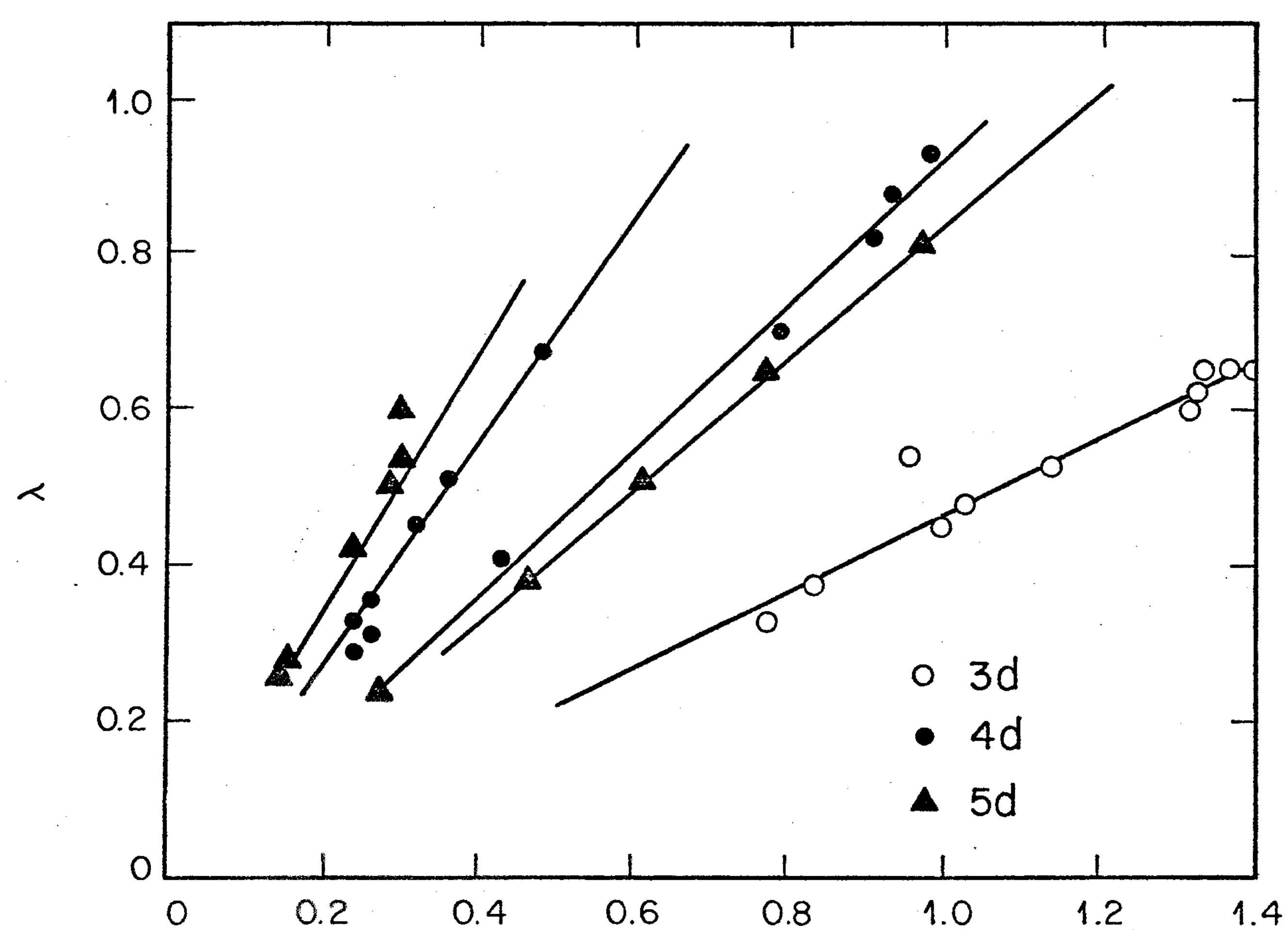

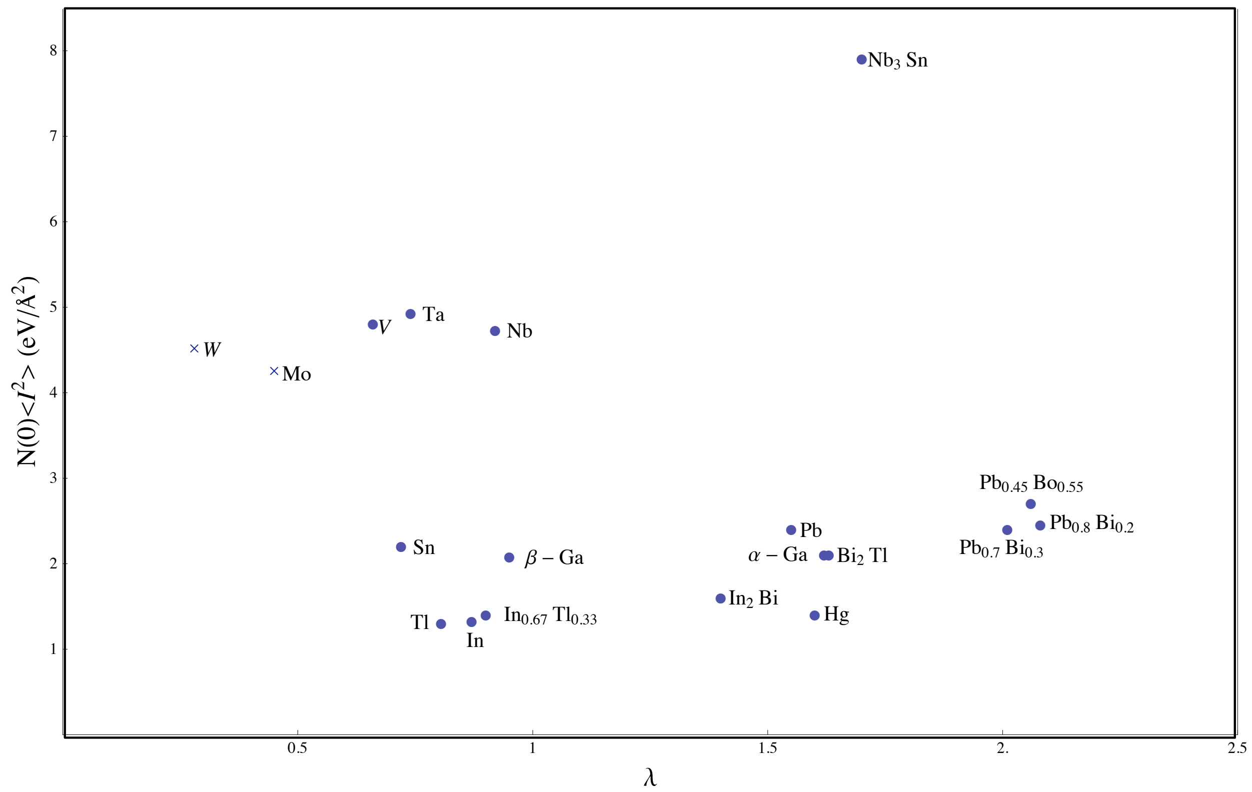


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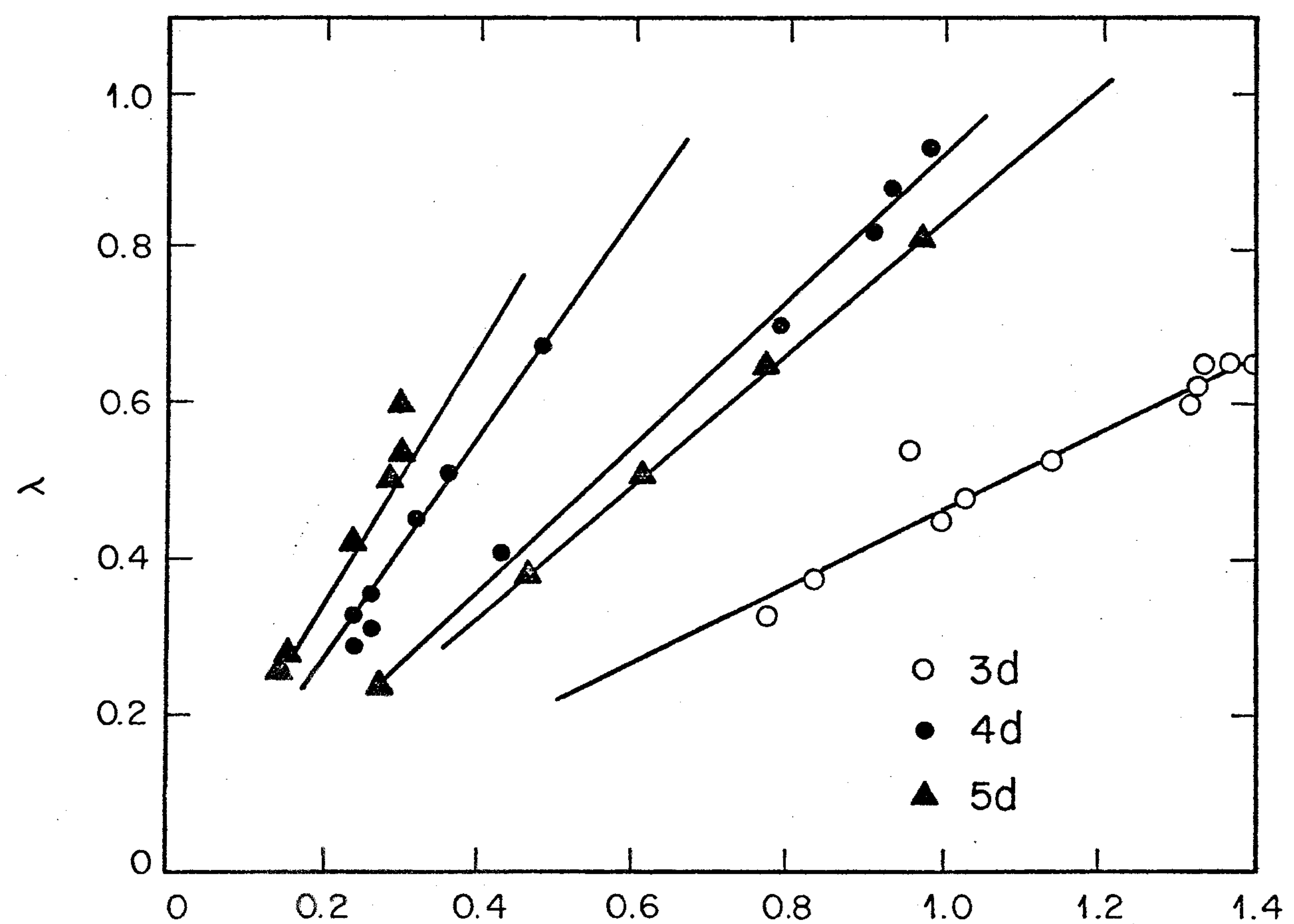


Proven by cmv-Dynes (1977).