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Interaction effects in a system with localized and delocalized single-electron states

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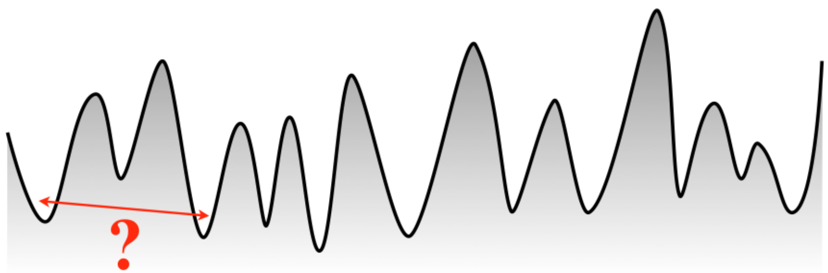
Many-body localization

Disordered (localized) *1d* system + weak interactions



$$H = \hat{T} + W\hat{H}_{\mathrm{dis}} + \lambda\hat{H}_{\mathrm{int}}$$

$W > 0, \lambda = 0$: Anderson insulator

$W > 0, \lambda > 0$: MBL?

Localization ⟷ Eigenstate thermalization

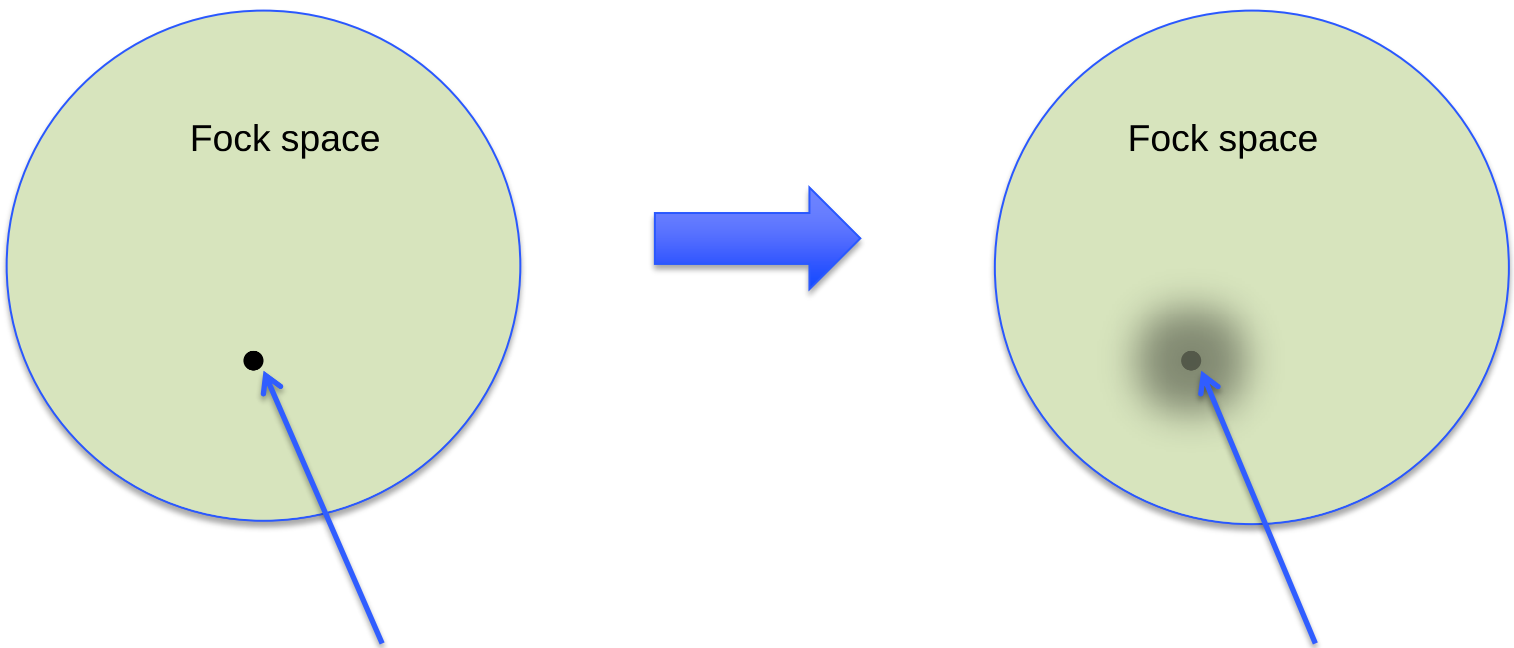
Gornyi, Mirlin & Polyakov 2005; Basko, Aleiner & Altshu

Localization in Fock space

$$H = \hat{T} + W\hat{H}_{\text{dis}} + \lambda\hat{H}_{\text{int}}$$

Non-interacting Anderson insulator

Weakly interacting system

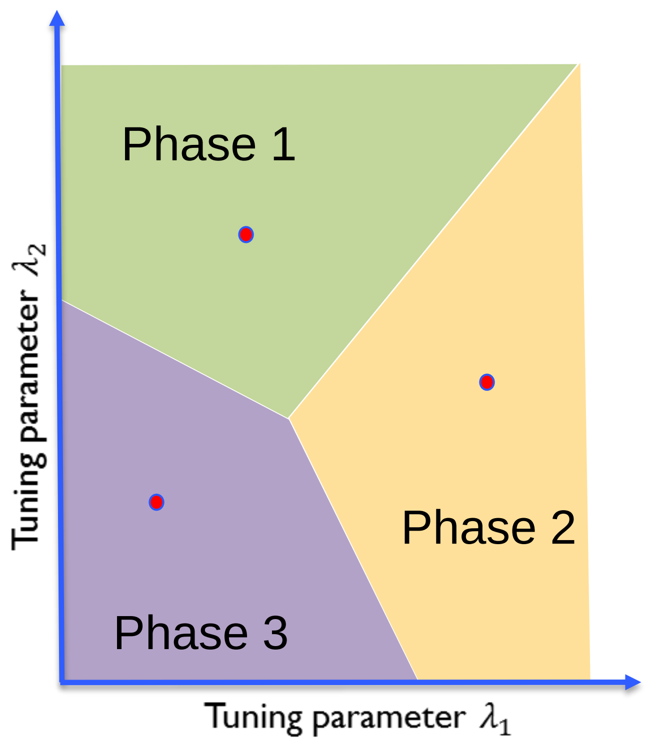


Eigenstate = 1 Slater determinant

Eigenstate = “a few” Slater determinants

Gornyi, Mirlin & Polyakov 2005; Basko, Aleiner & Altshu

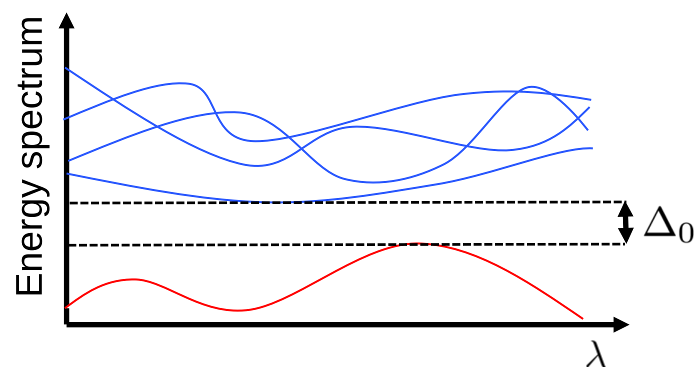
Adiabatic continuity



Many-body ground state
of (trivial) gapped system

Adiabatic path

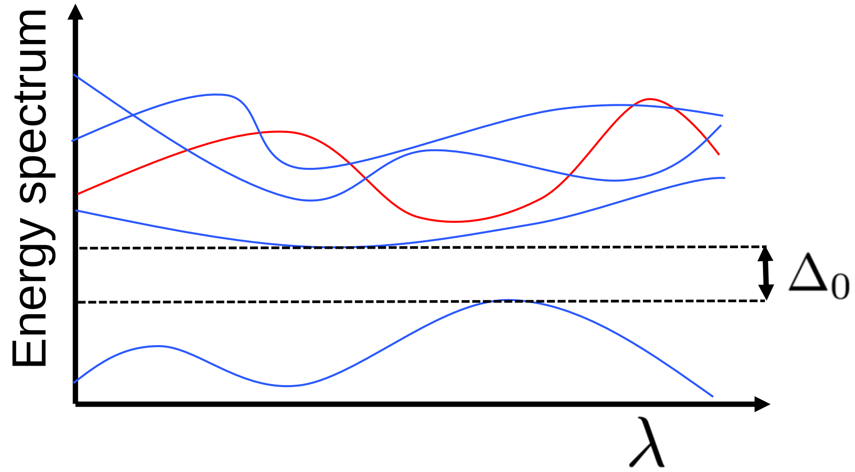
Product states



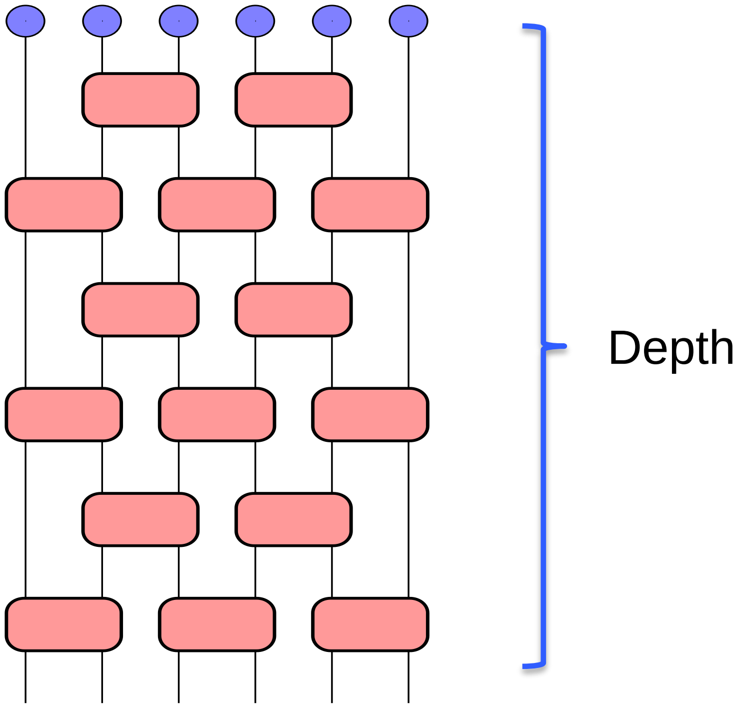
Localized eigenstates

Many-body ground state of (trivial) gapped system ⟷ Product states

Many-body eigenstates of MBL system ⟷ Eigenstates of Anderson insulator



Finite-depth local unitary



Localized eigenstates

Many-body ground state of (trivial) gapped system ⟷ Product states dressed with local fluctuations

Adiabatic path

Many-body eigenstates of MBL system ⟷ Eigenstates of Anderson insulator dressed with local fluctuations

Finite-depth local unitary

BB & C. Nayak, 2013

Localized starting point

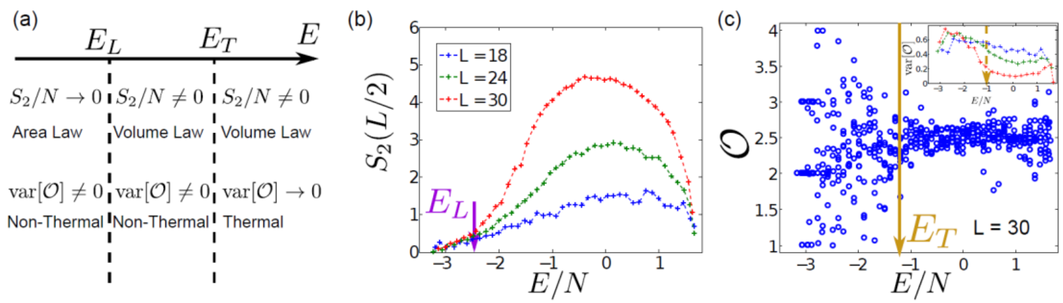
$$H = \hat{T} + W\hat{H}_{\text{dis}} + \lambda\hat{H}_{\text{int}}$$

$\cancel{W > 0, \lambda = 0:}$ ~~Anderson insulator~~

$W > 0, \lambda > 0:$ MBL?

What if the non-interacting limit is not fully localized?

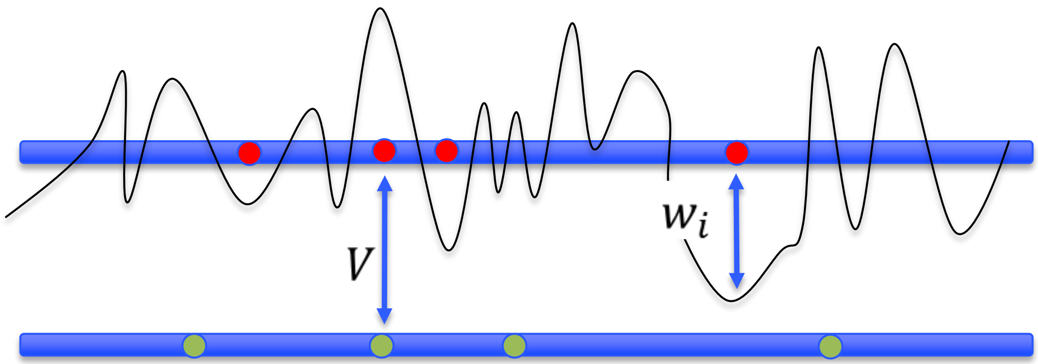
- *Li, Ganeshan, Pixley & Das Sarma, PRL 2015 and Modak & Mukerjee 2015:* Incommensurate potential in $d = 1$ with single-particle mobility edge



- Generic situation in higher dimensions

Ladder model

$$H = -\sum_{\alpha} t_\alpha \sum_{i=1}^{L} \left(\hat{c}^\dagger_{\alpha,i} \hat{c}_{\alpha,i+1} + \text{h.c.} \right) + \sum_{i=1}^{L} w_i \hat{n}_{1,i} + V \sum_{i=1}^{L} \hat{n}_{1,i} \hat{n}_{2,i}$$



- No intra-chain hopping:
  - Particle number preserved on each chain
  - Equivalent to bosons/spins (for OBC)
- Also equivalent: Two-component system

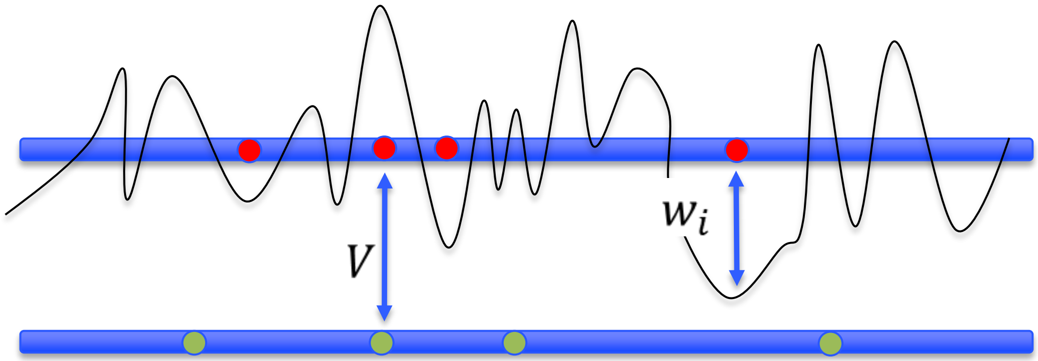
$W > 0, V = 0$: Particles in upper layer localized, lower layer delocalized

$W > 0, V > 0$: ???

K. Hyatt, J. Garrison, BB, to appear; Nandkisho

Ladder model

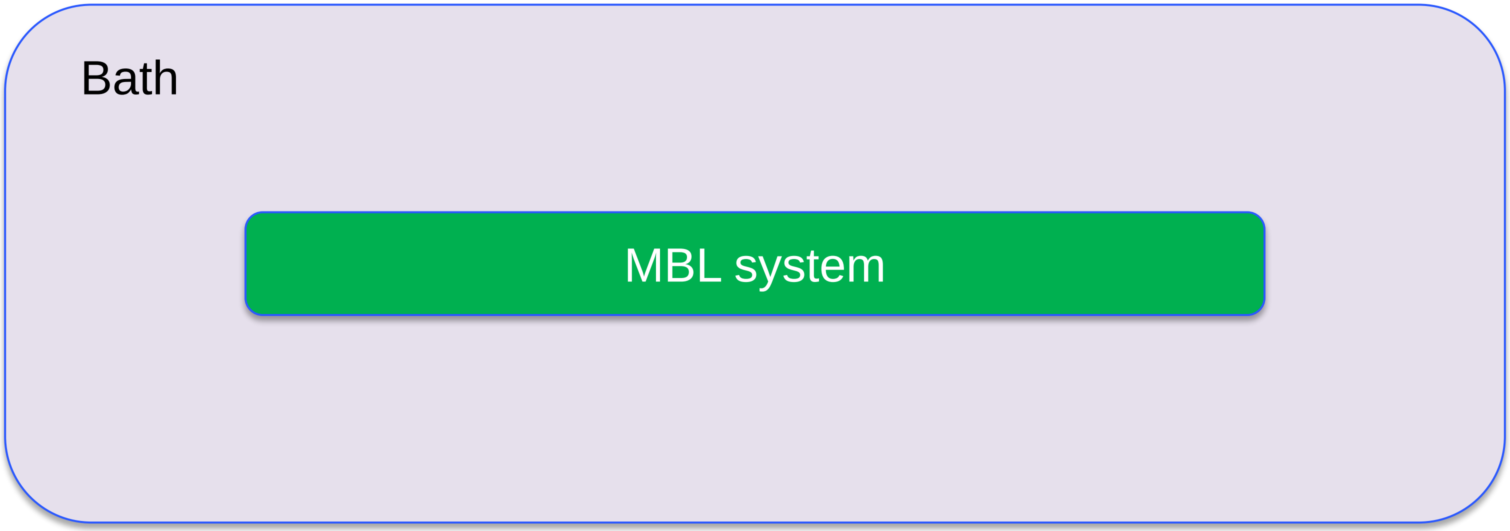
$$H = -\sum_{\alpha} t_\alpha \sum_{i=1}^{L} \left(\hat{c}^\dagger_{\alpha,i} \hat{c}_{\alpha,i+1} + \text{h.c.} \right) + \sum_{i=1}^{L} w_i \hat{n}_{1,i} + V \sum_{i=1}^{L} \hat{n}_{1,i} \hat{n}_{2,i}$$



Localization destroyed	Localization survives
• Delocalized electrons act as bath for localized electrons • Energy transport through lower layer leads to delocalization in upper layer	• Localized layer acts as effective disorder on other layer: $V \sum \hat{n}_{1,i} \hat{n}_{2,i} \to V \sum \langle \hat{n}_{1,i} \rangle \hat{n}_{2,i}$

K. Hyatt, J. Garrison, BB, to appear; Nandkisho

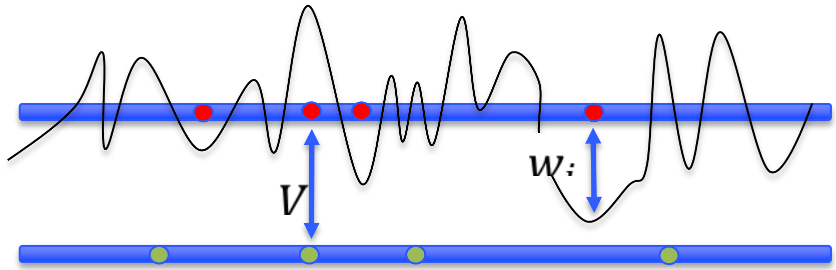
MBL coupled to a bath



$$V_{\mathrm{bath}} \gg V_{\mathrm{sys}}$$

- Weak coupling to bath: Spectral features of MBL phase are broadened (*Nandkishore, Gopalakrishnan & Huse 2014; Johri, Nandkishore & Bhatt 2014*)

MBL coupled to a “small bath”



MBL system

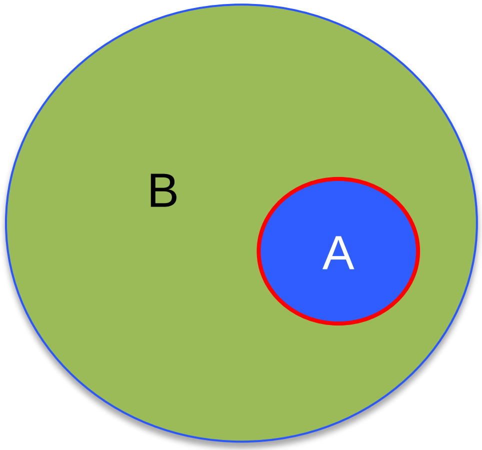
“Bath”

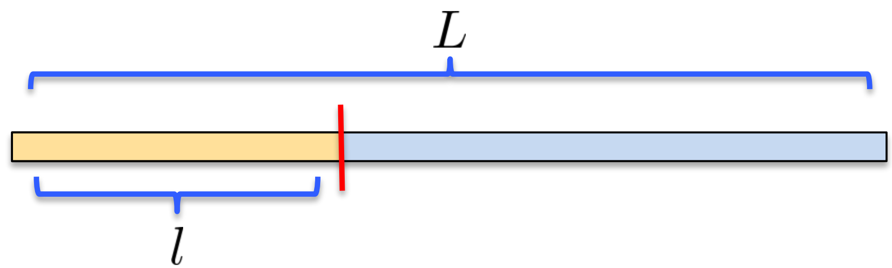
$$V_{\mathrm{bath}} \approx V_{\mathrm{sys}}$$

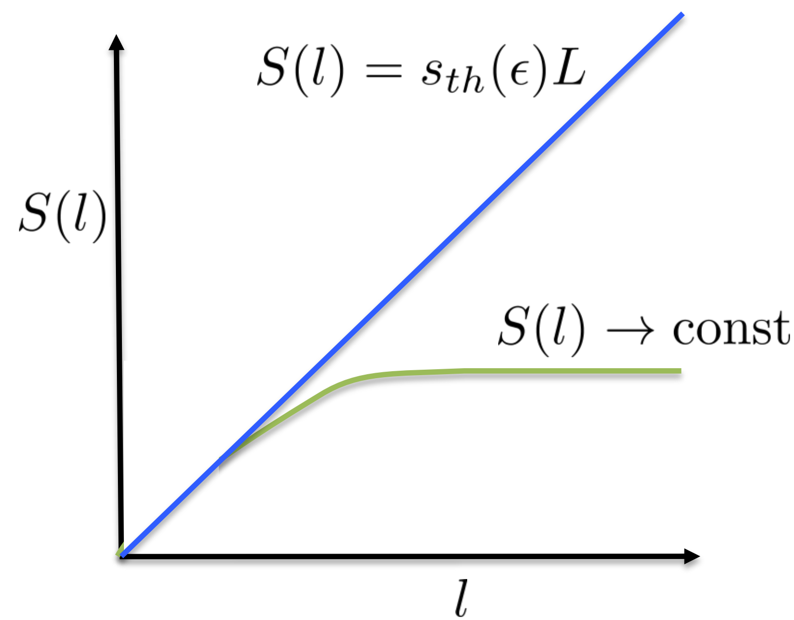
- Potential for back-action: System can localize bath!
- Explore numerically: exact eigenstates using shift-and-invert algorithm (*Luitz et al, PRB 2014*)

Huse et al 2014; Nandkishore 2015; K. Hyatt, J. Garrison, BB,

Entanglement





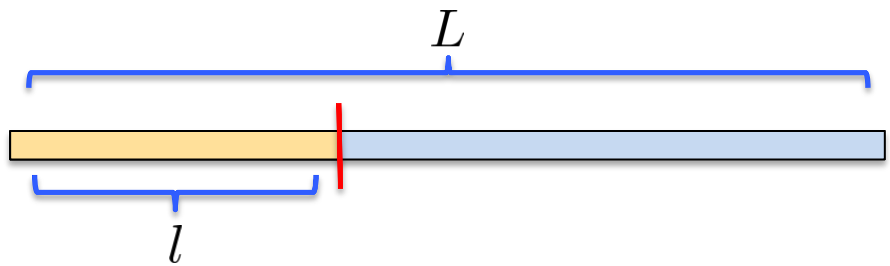


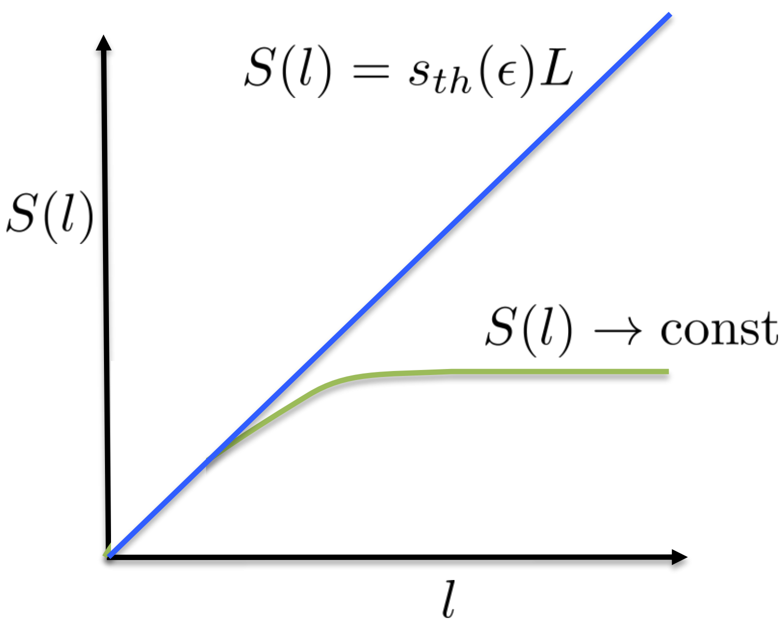
Volume law: $S(\rho_A) \sim \text{vol}(A)$
(generic quantum state,
highly excited states, thermal states)

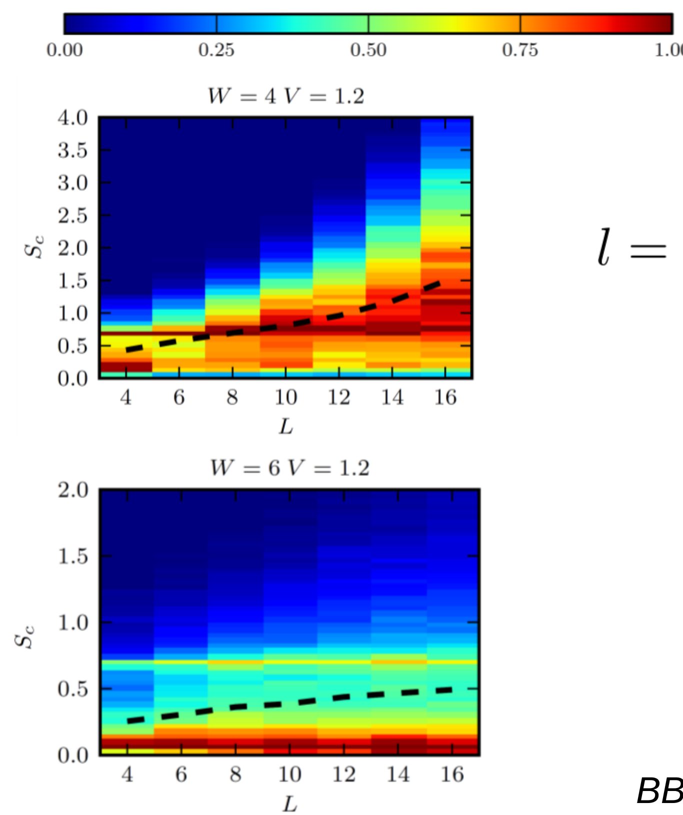
Area law: $S(\rho_A) \sim \partial A$
(ground states of local
Hamiltonians, MBL
eigenstates)

BB & C. Nayak, 2013

Entanglement in MBL



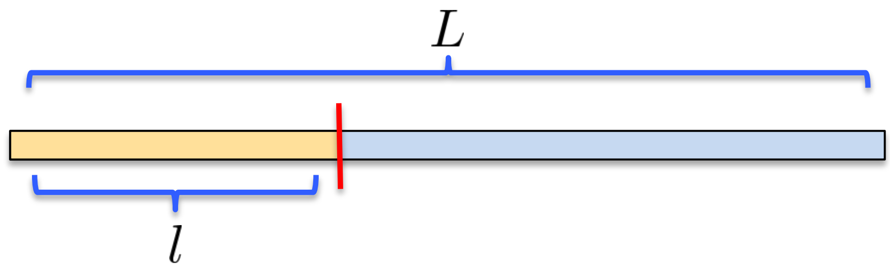


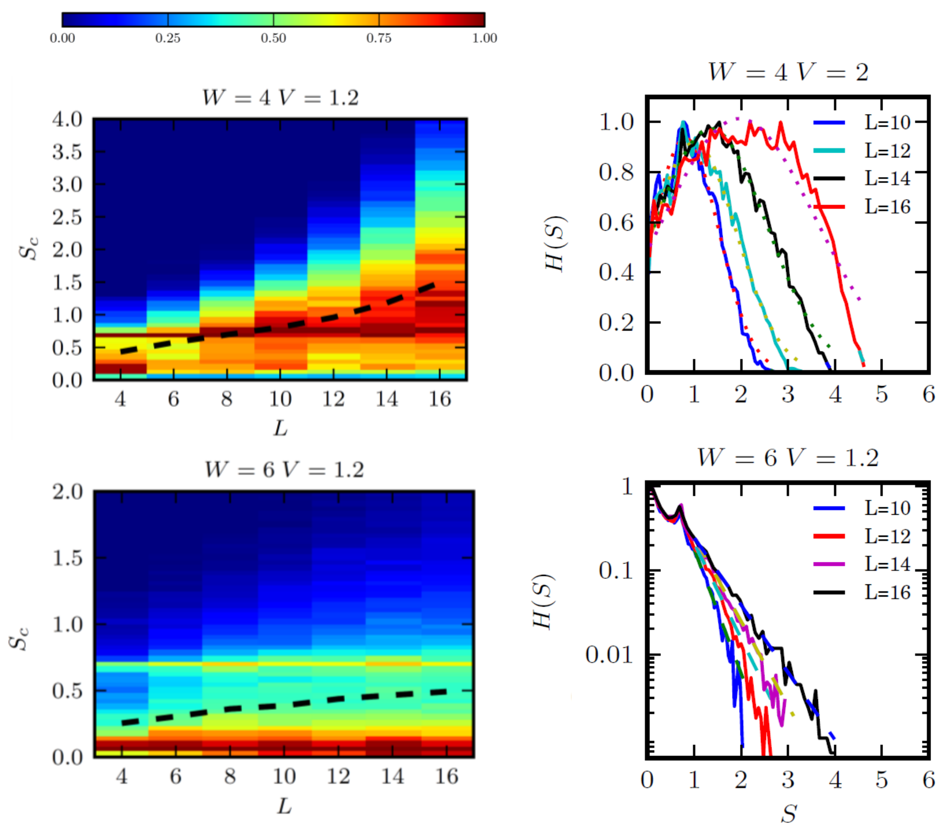


$$l = L/2$$

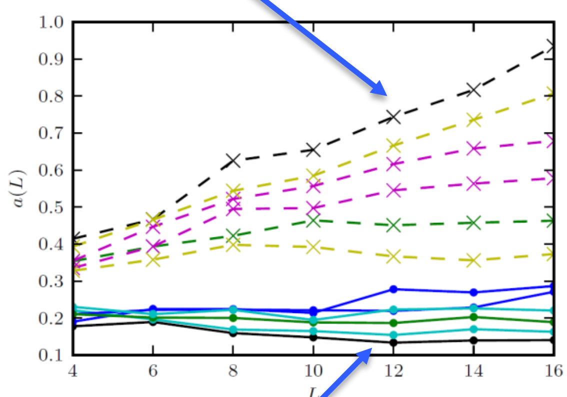
BB & C. Nayak, 2013

Entanglement in MBL





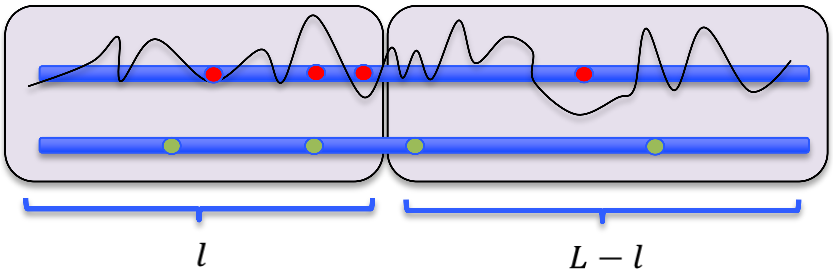
Rare cuts



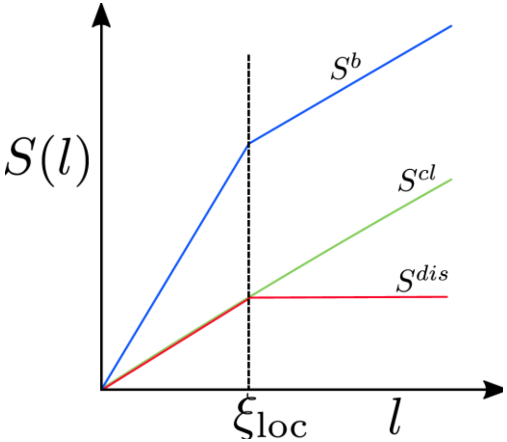
Typical cuts

BB & C. Nayak, 201

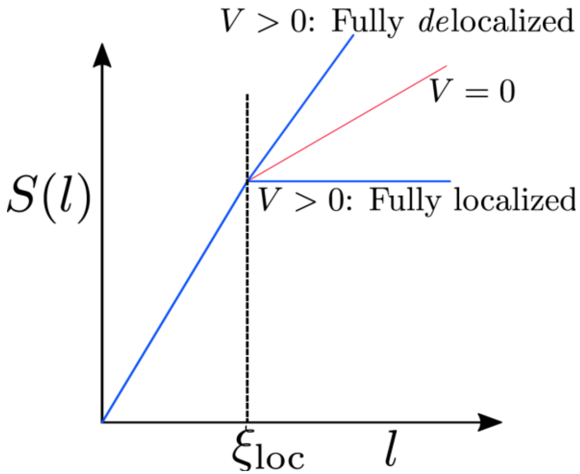
Ladder entropies



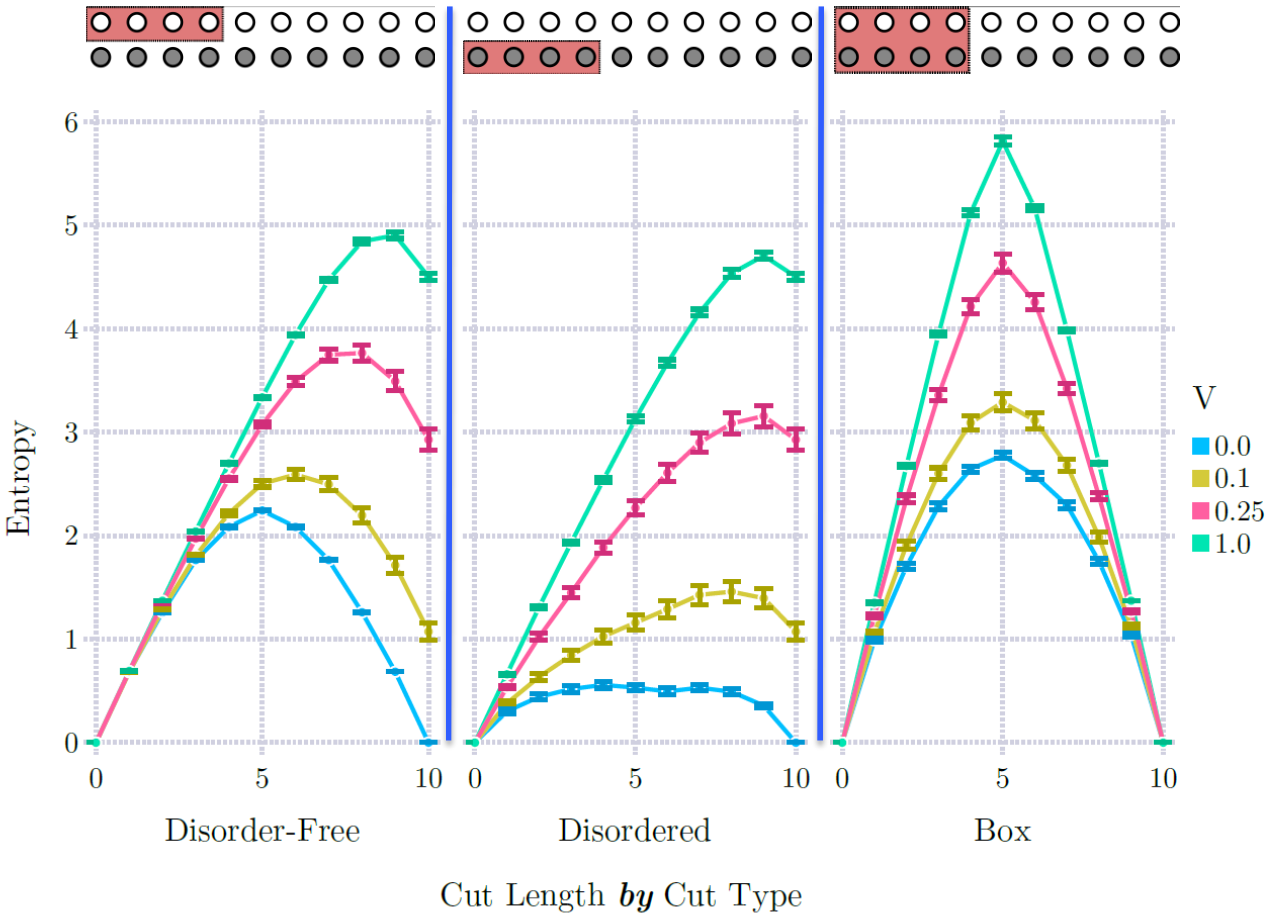
- Decoupled layers: $V = 0$
- One layer localized: $W > 0$



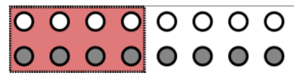
- Coupled layers: $V > 0$
- One layer s.p. localized: $W > 0$

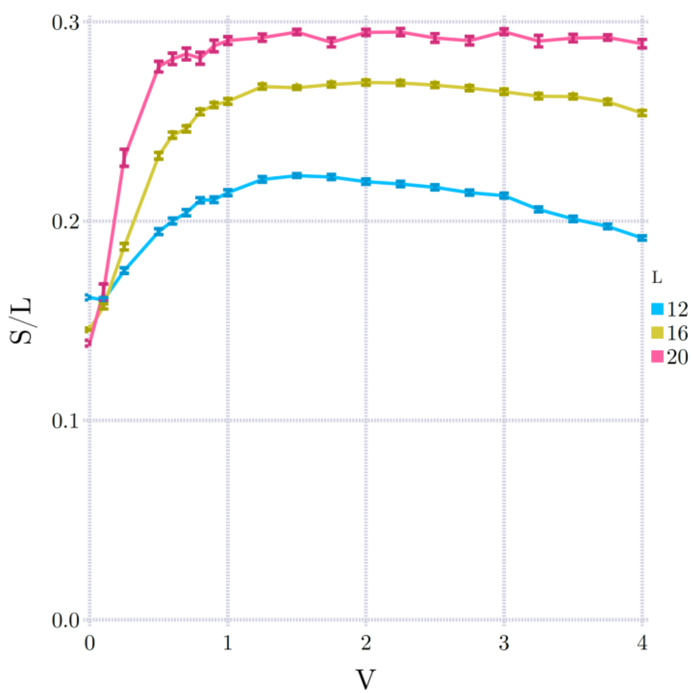


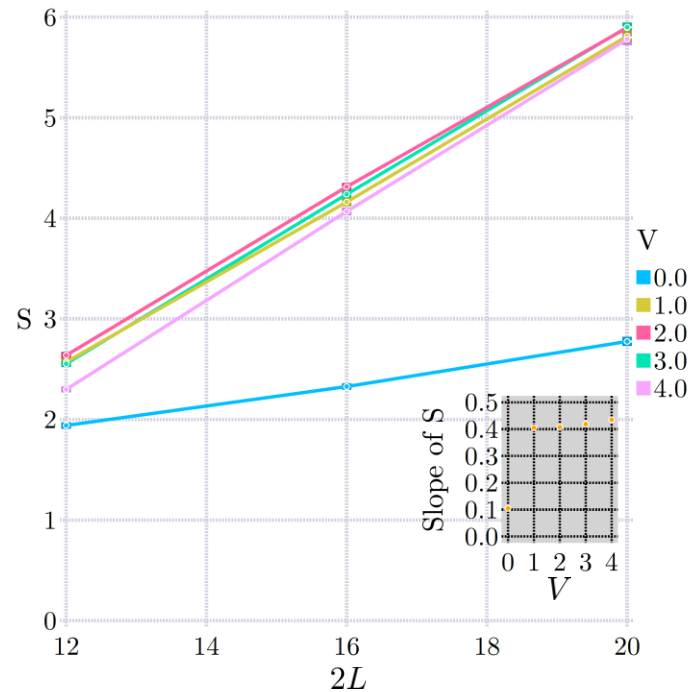
Entropy cuts



Entropy scaling



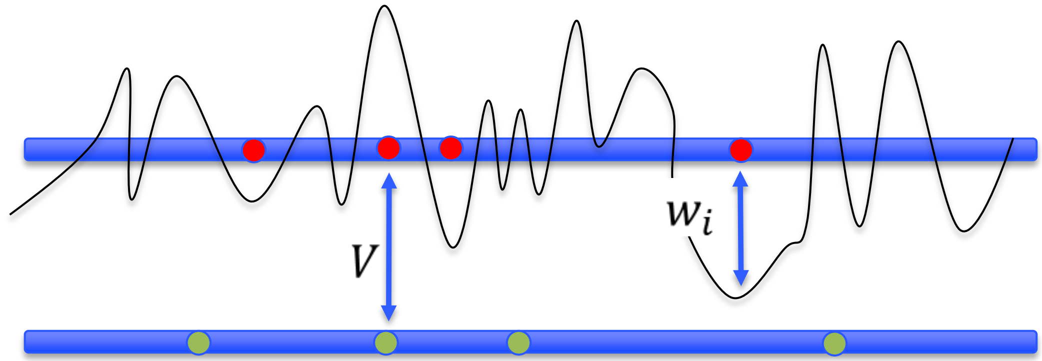




Delocalization of both layers?

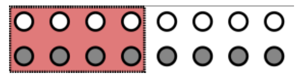
Tune into MBL regime?

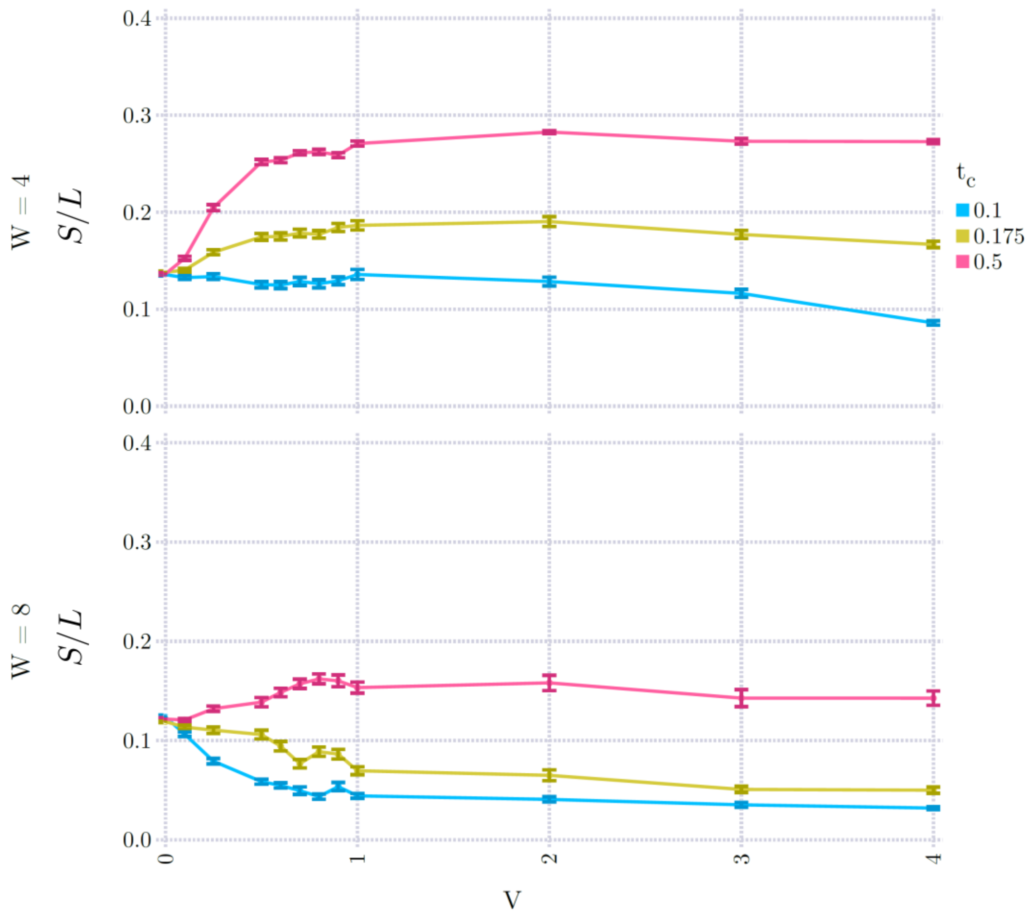
$$H = -\sum_{\alpha} t_\alpha \sum_{i=1}^{L} \left(\hat{c}^\dagger_{\alpha,i} \hat{c}_{\alpha,i+1} + \text{h.c.} \right) + \sum_{i=1}^{L} w_i \hat{n}_{1,i} + V \sum_{i=1}^{L} \hat{n}_{1,i} \hat{n}_{2,i}$$



- Several ways to make bath less effective:
  - Reduce particle density in bath – finite-size corrections?
  - Reduce bandwidth: $t_c \ll t_d$

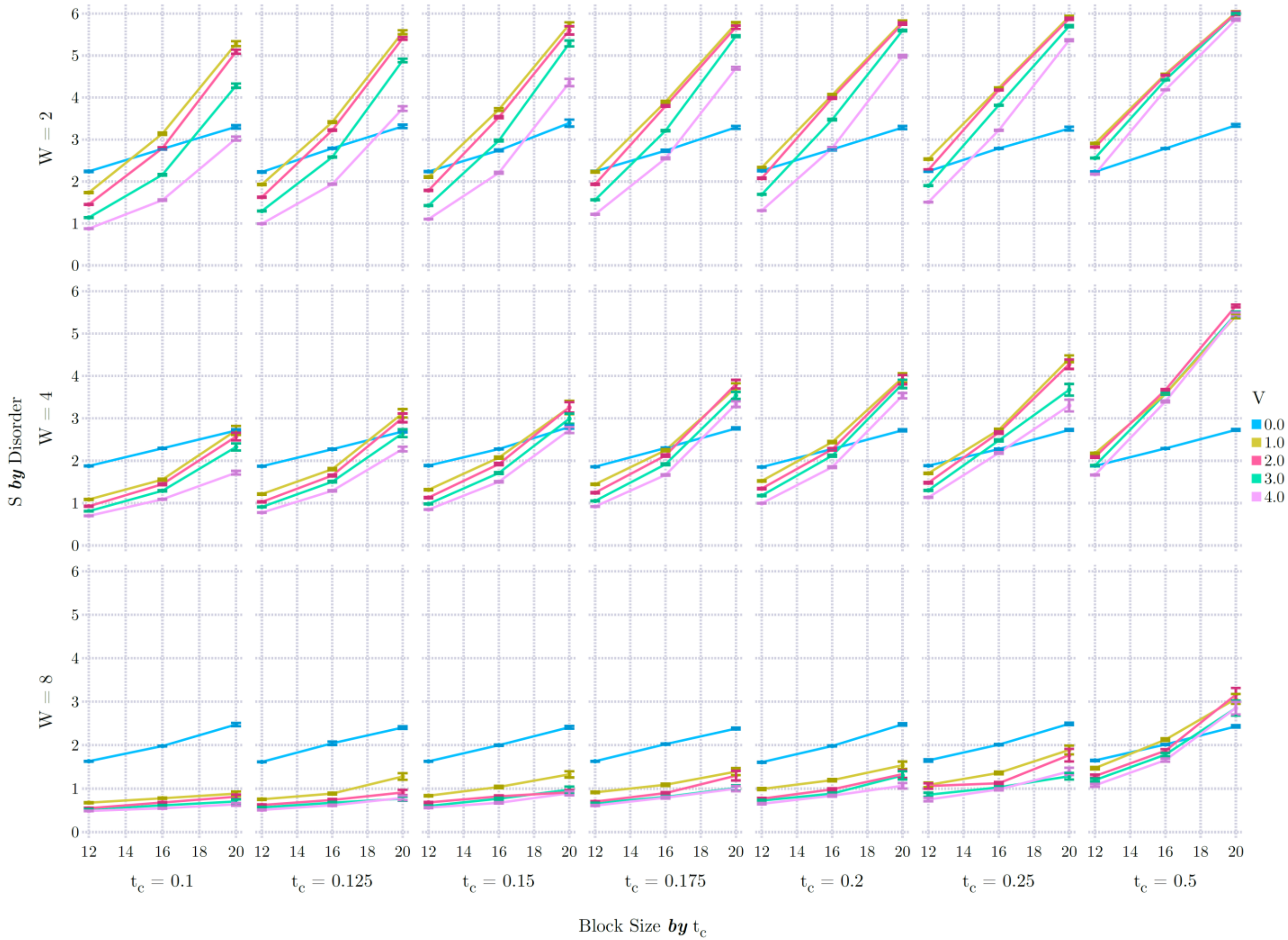
Reduced hopping



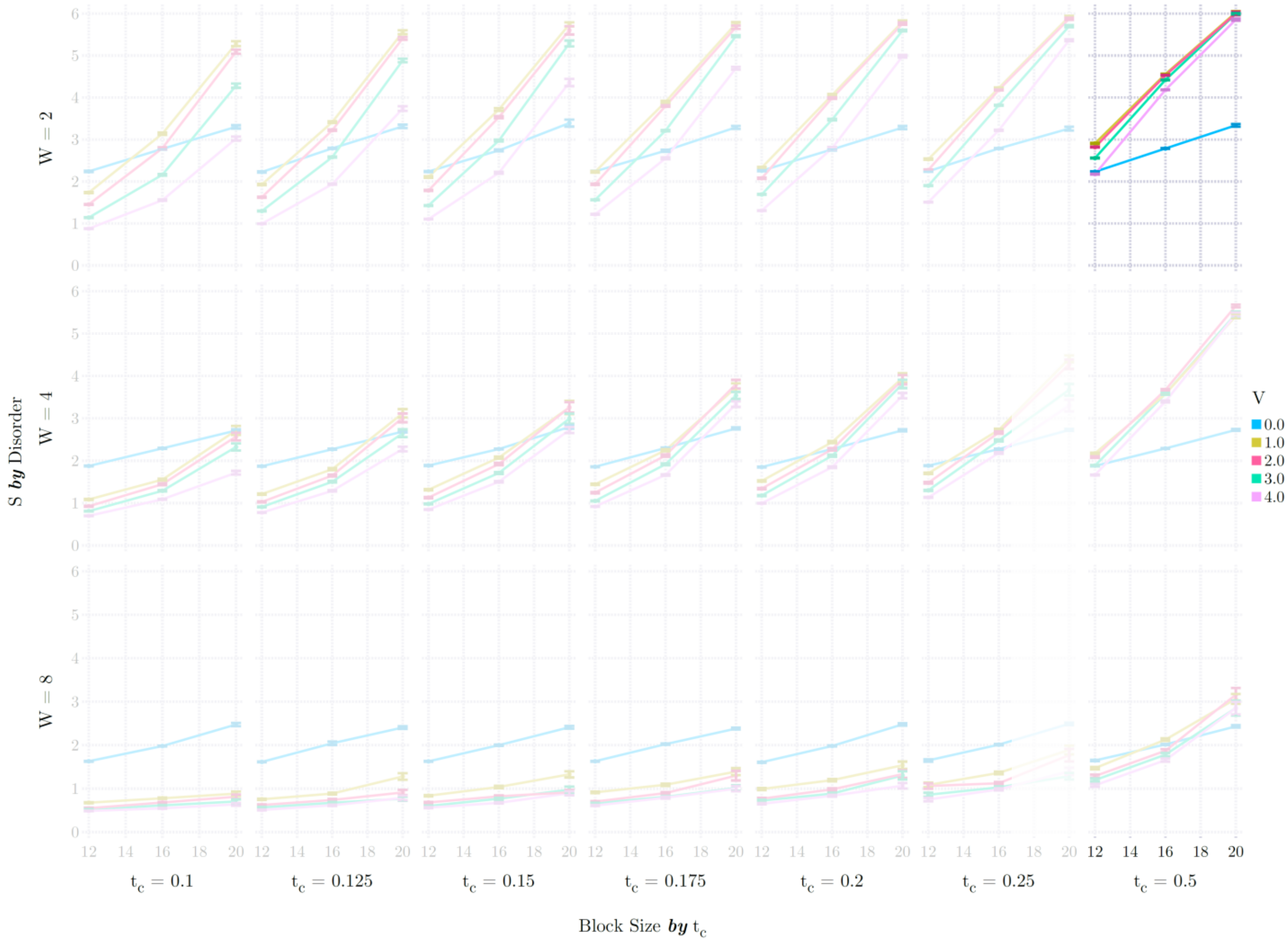


$t_d = 1$

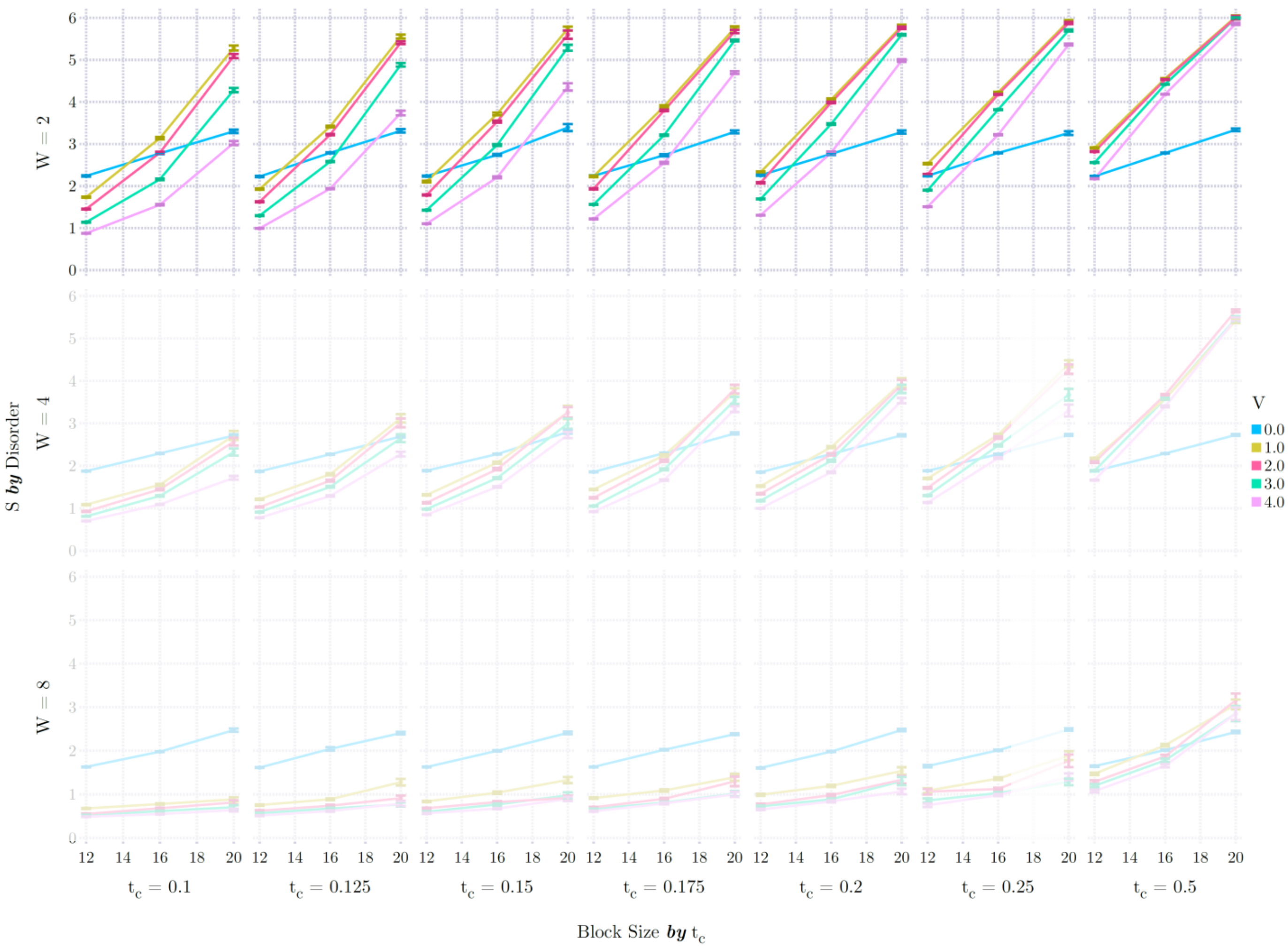
Entropy scaling



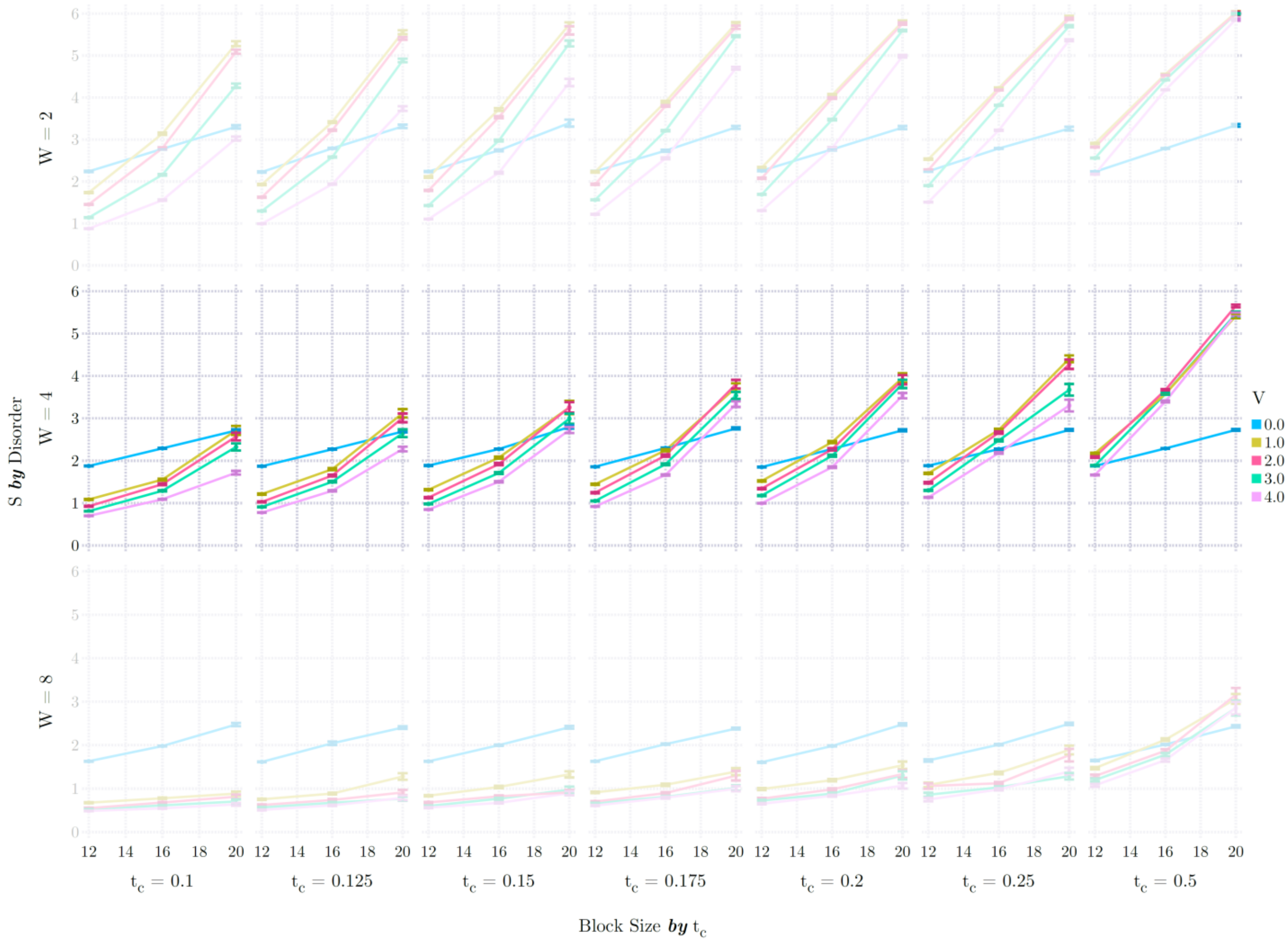
Entropy scaling



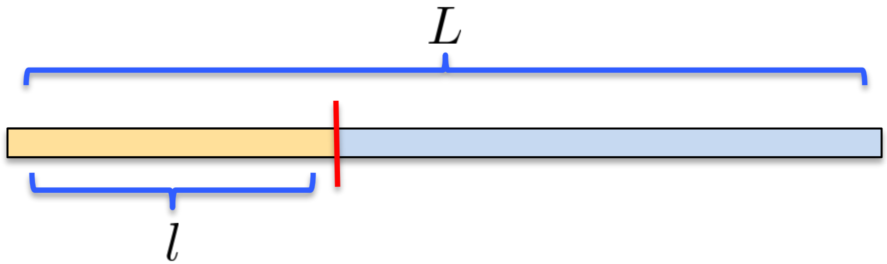
Entropy scaling



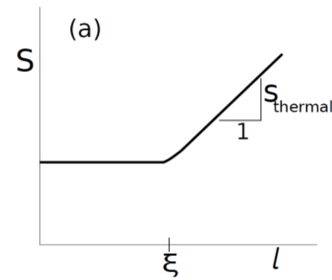
Entropy scaling



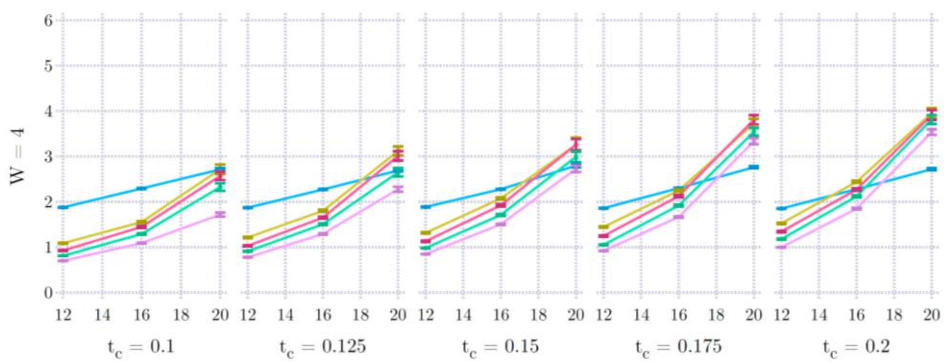
Area law & finite size



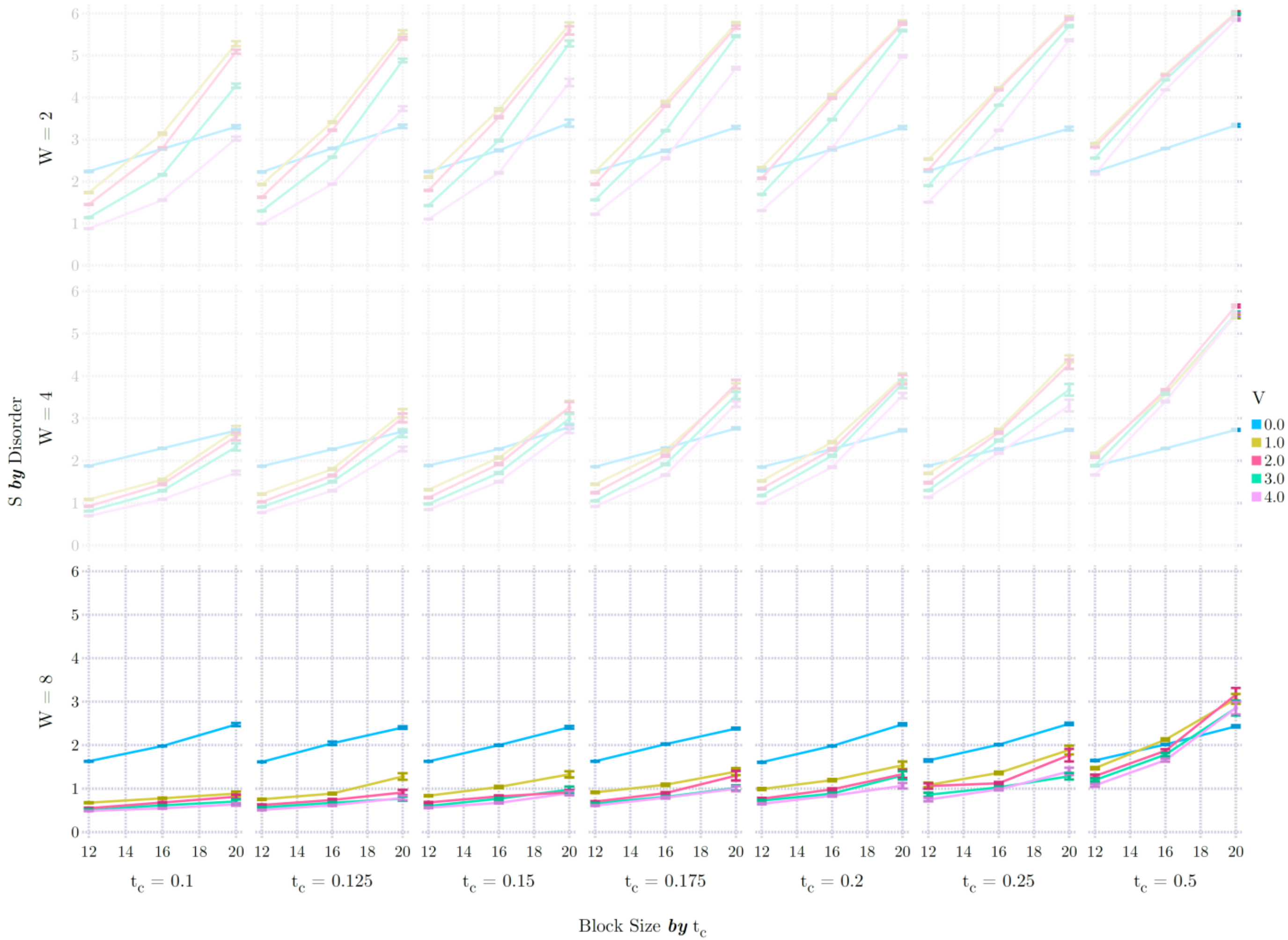
T. Grover 2014: if $l \ll L$,
then $\partial^2 S / \partial l^2 \leq 0$



Would like to achieve
$$\xi \ll l \ll L$$
but in practice
$$\xi_{\text{sp}} < l, l = L/2$$



Entropy scaling

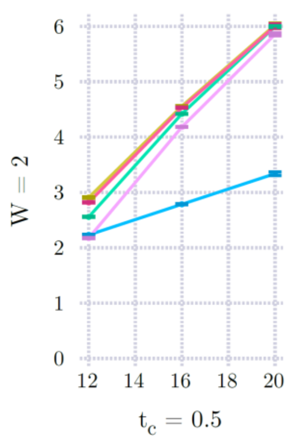


Conclusions

- Ladder model for
  - Many-body localization where non-interacting limit is not fully localized
  - MBL coupled to a small “bath”

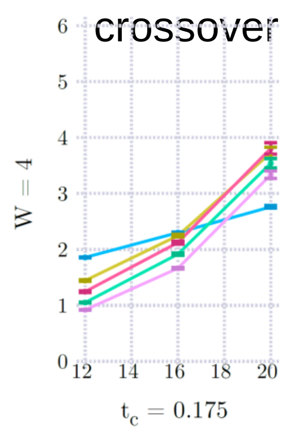
$W > t, t_c \sim t_d$

Delocalizatio
n



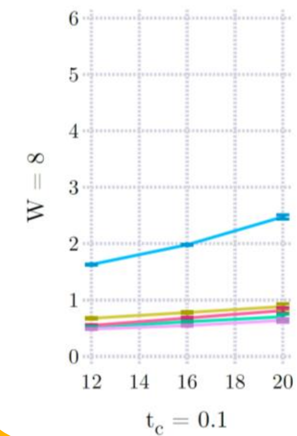
$W > t, t_c < t_d$

Unusual
finite-size
crossover



$W \gg t, t_c \ll t_d$

Localization
of bath?



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Thank you!