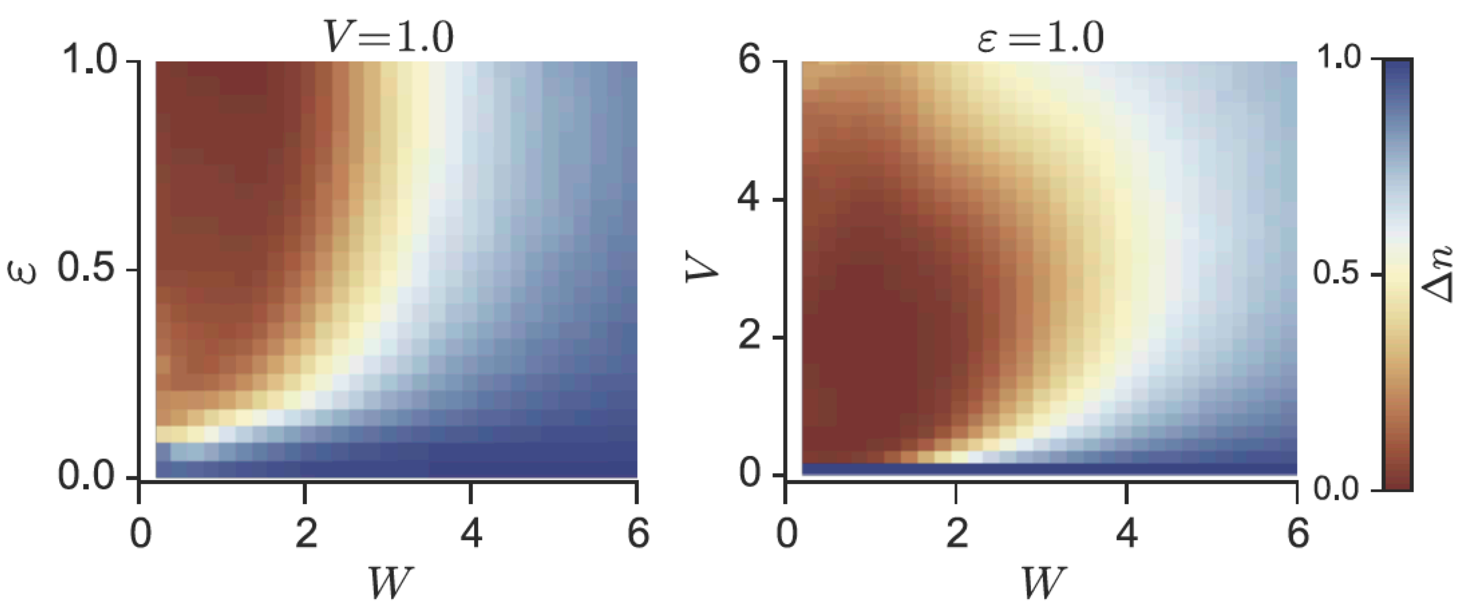
Many-body localization from a one-particle perspective



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Discussions with:
Ehud Altman, Frank Pollmann

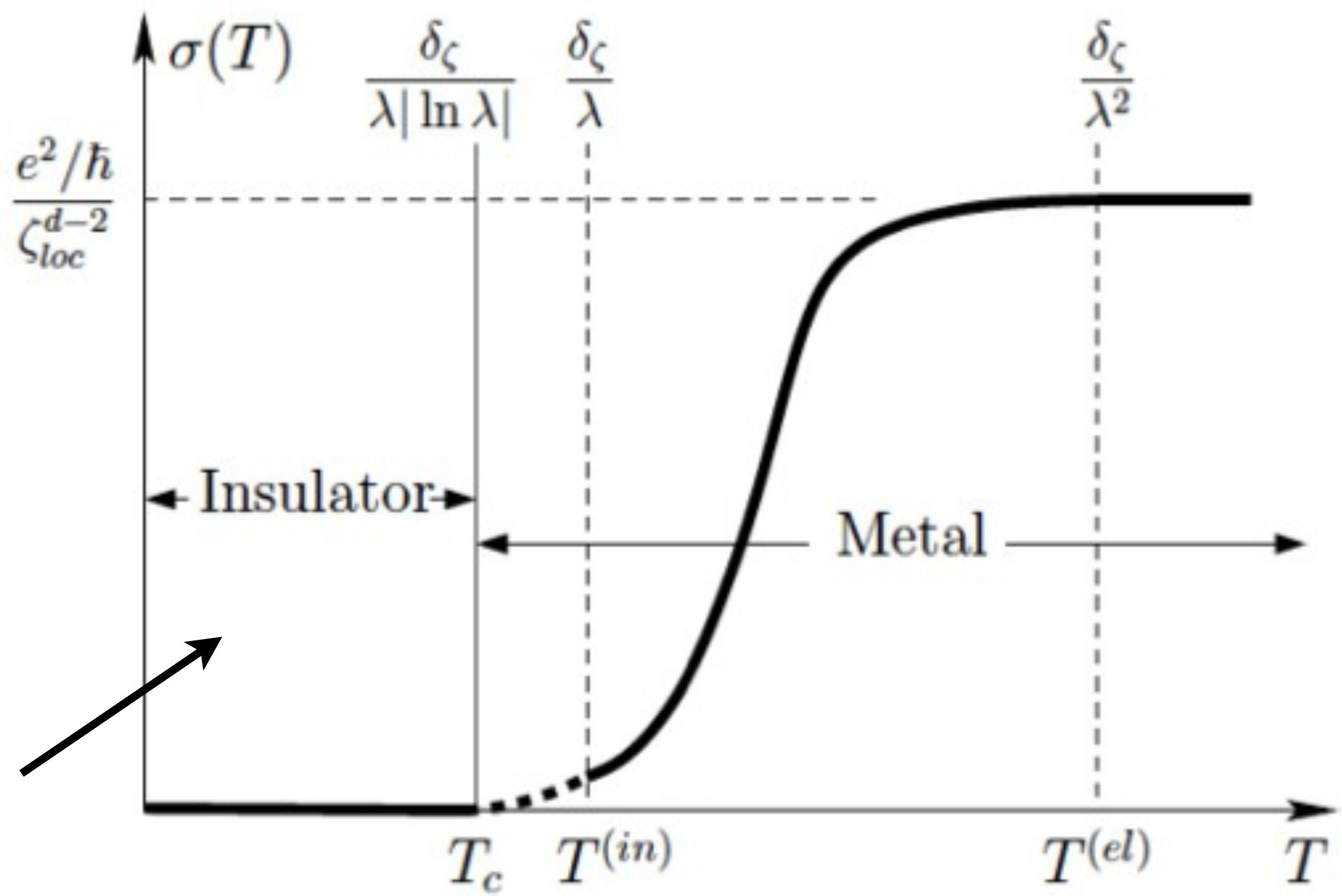
For details: Bera, Schomerus, FHM, Bardarson, Phys. Rev. Lett. 115, 046603 (2015)

Disorder & interactions

Can interactions lead to delocalization?

Is the localized phase stable?

“Many-body” localized



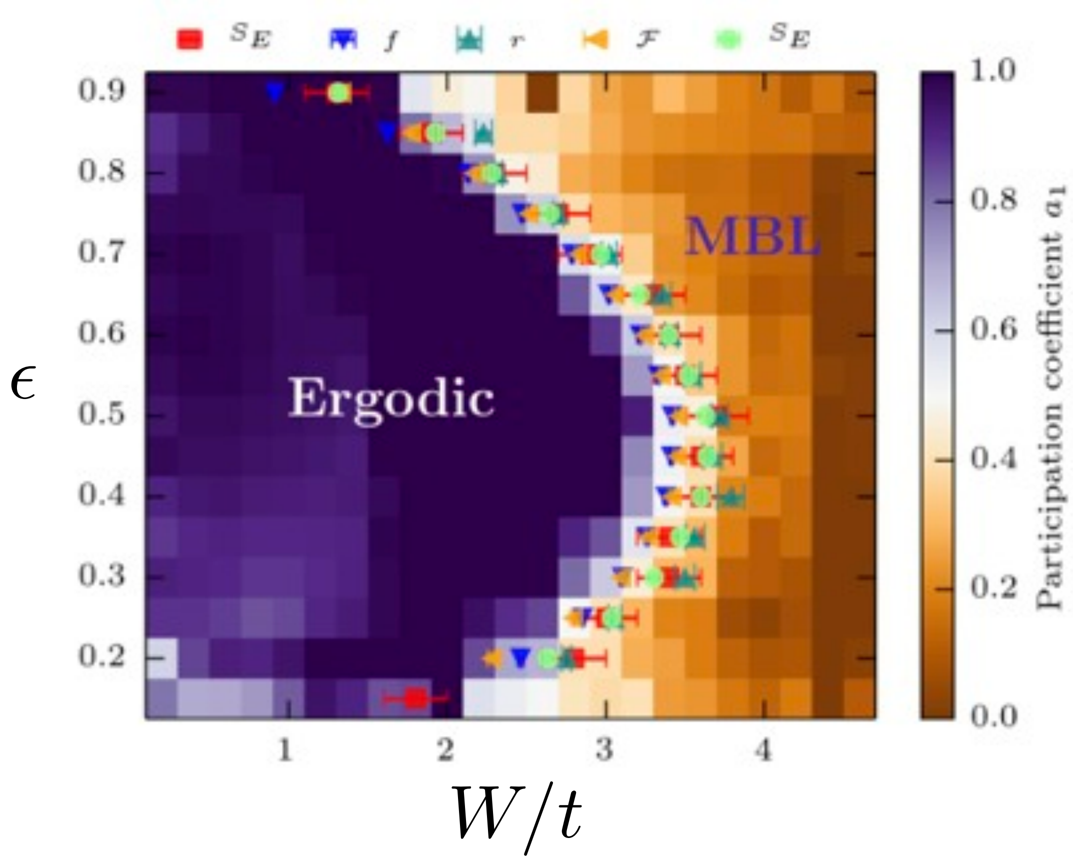
Basko, Aleiner, Altshuler Annals of Physics 321, 1126 (2006)

Standard model of 1D MBL

Spinless fermions in 1D = Spin-1/2 XXZ chain

$$H = \sum_{i=1}^{L}[-\frac{t}{2}(c_{i+1}^{\dagger}c_i + h.c.) + V n_i n_{i+1}] - \sum_i \epsilon_i n_i \qquad \epsilon_i \in [-W, W]$$

Phase diagram



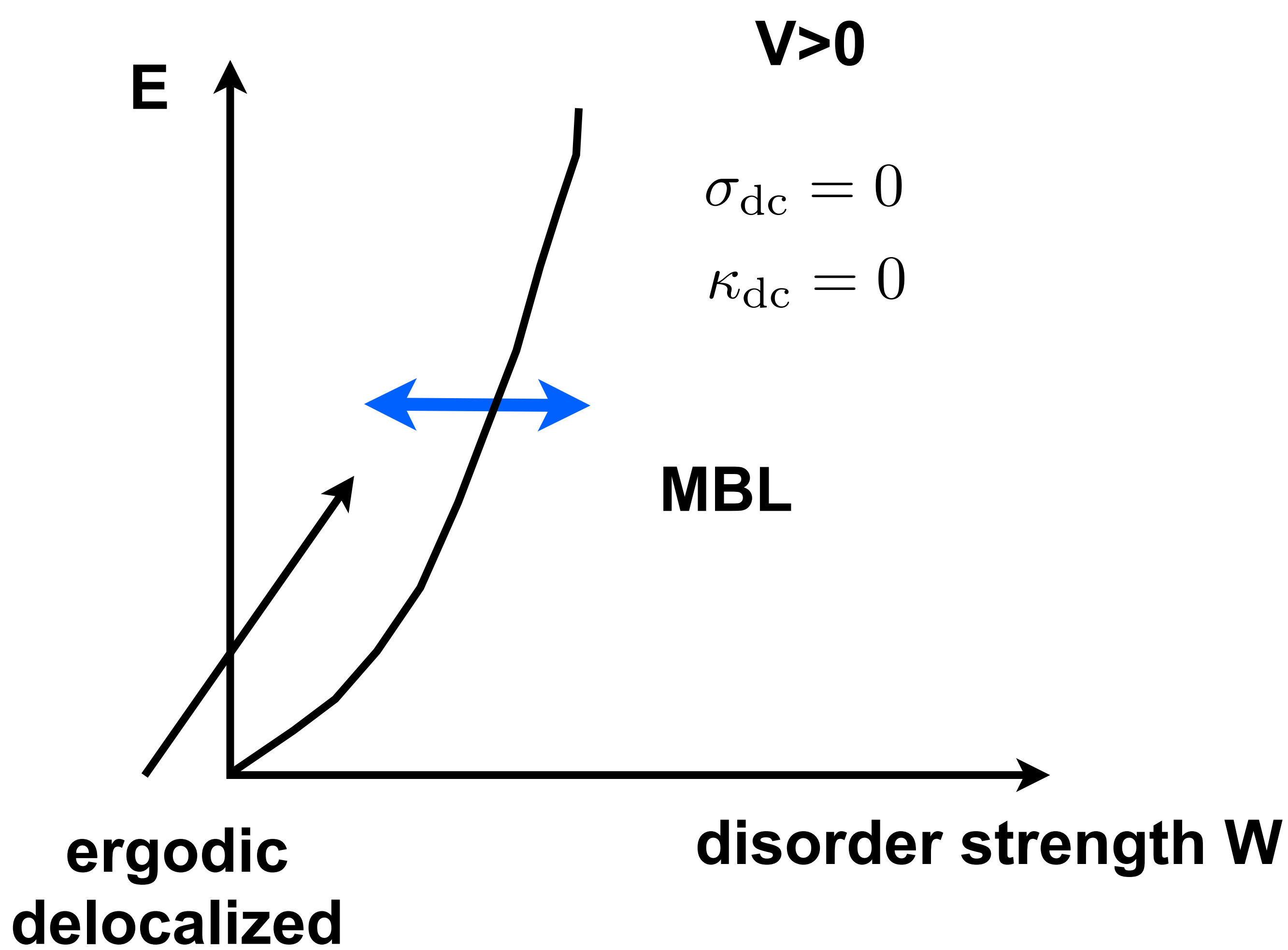
Critical disorder strength at T=∞

$$W_c \sim 3.5t$$

→ **Level spacing**
→ **Entanglement entropy fluctuations**
→ **Participation ratios**
...

Luitz, Laflorencie, Alet Phys. Rev. B 91, 081103(R) (2015)
Oganesyan, Huse, Phys. Rev. B 75, 155111 (2007); Pal, Huse, Phys. Rev. B 82, 174411 (2010)
Bar Lev, Cohen, Reichman Phys. Rev. Lett. 114, 100601 (2015), ...

Properties of the MBL phase & transition



MBL vs AL?
"Visualize" localization length?
Characterization of transition?
2D?

Exciting, because:

Finite E "quantum phase transition"

No signature in thermodynamics

No thermalization, transport

Area law in excited states

Local conserved charges, robust "integrable" system

Log-increase in entanglement

Bardarson, Pollmann, Moore, PRL 2012
Znidaric, Prelovsek, Prosen PRB 2008

Review: Nandkishore, Huse, Annual Rev. CMP 6, 15 (2015)
Altman, Vosk, Annual Rev. CMP 6, 383 (2015)

Anderson localization

Electrons in periodic potential: Bloch states

$$\psi_{\vec{k}}(\vec{r}+\vec{R}) = e^{i\vec{k}\cdot\vec{R}}\psi_{\vec{k}}(\vec{r})$$

$$H = -\sum_{ij} t_{ij}(c_i^\dagger c_j + h.c.)$$

Electrons in the presence of disorder:
full localization of all single-particle eigenstates possible

Asymptotic form of eigenstates:
localization length

$$\psi(r) = f(r)e^{-r/\xi}$$

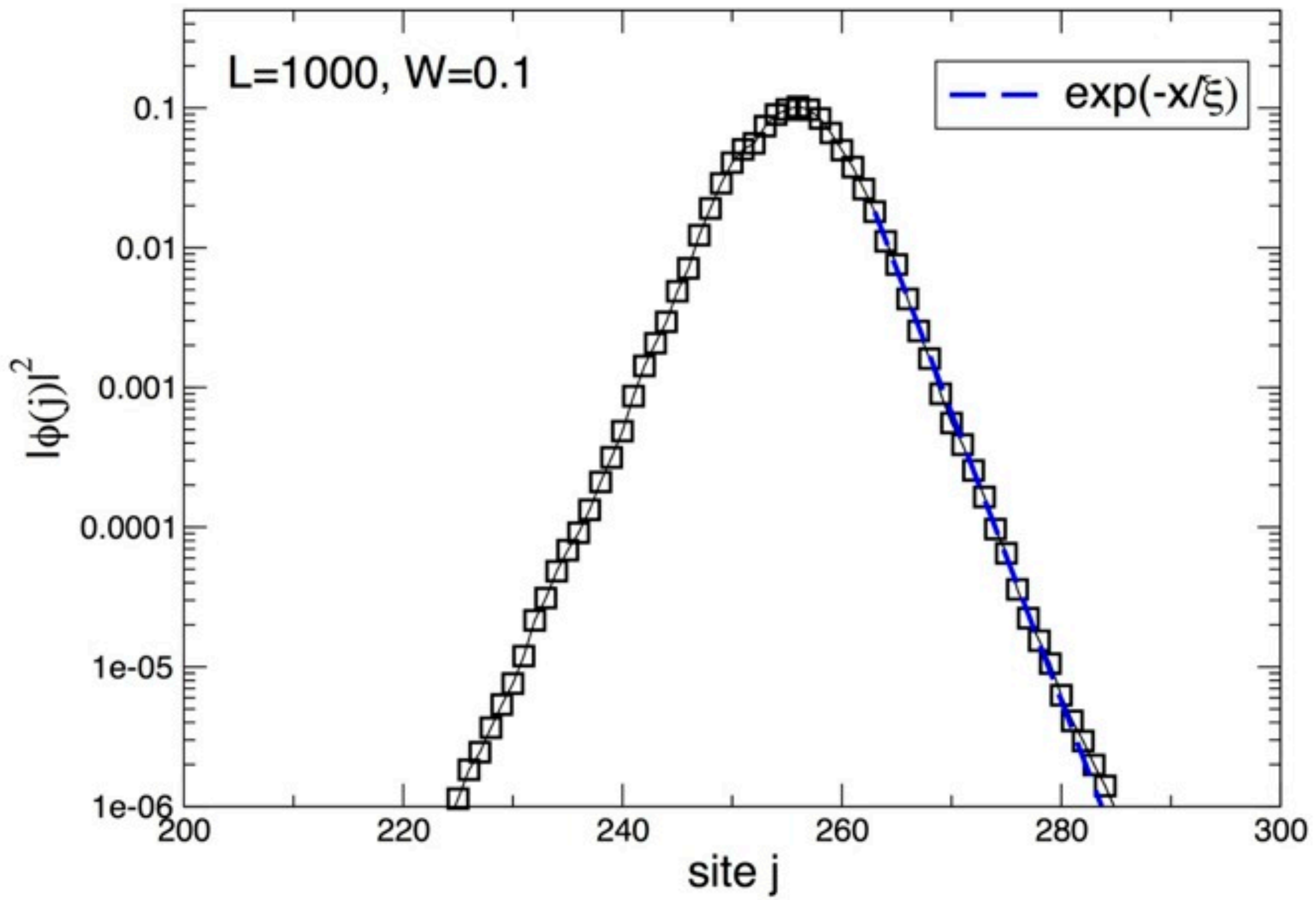
$$H = -\sum_{ij} t_{ij}(c_i^\dagger c_j + h.c.) - \sum_i \epsilon_i n_i$$

$$\epsilon_i \in [-W, W]$$

Anderson Phys. Rev. 109, 1492 (1958)

Kramer, McKinnon Rep. Prog. Phys. 1993, Evers, Mirlin Rev. Mod. Phys. 2008

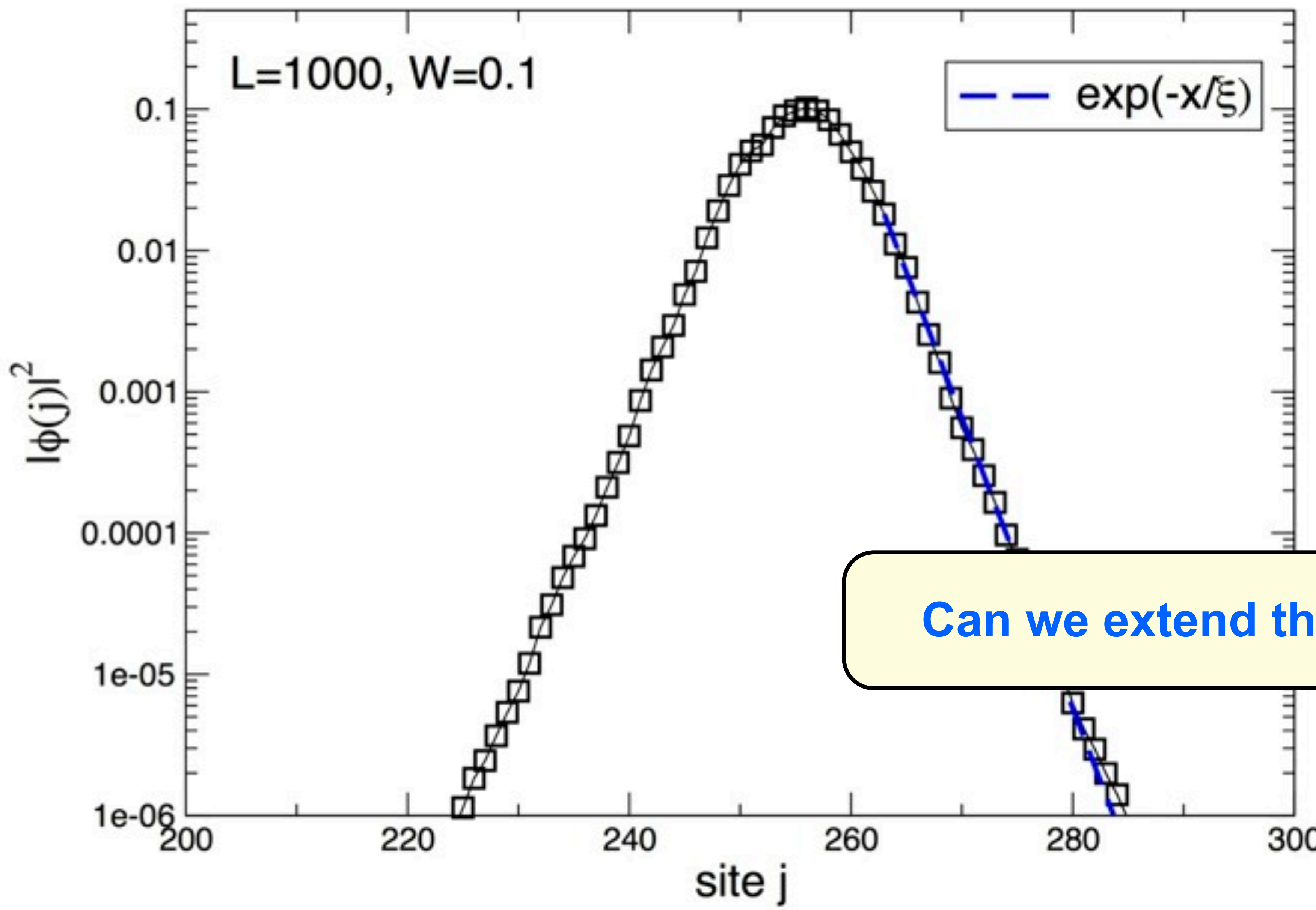
Typical localized single-particle state



Localization length in MBL:
Entanglement entropy, coefficients in l-bit Hamiltonian, ...

Bauer, Nayak J. Stat. Mech. (2013) P09005, Huse, Nandishkore, Oganesyan Phys. Rev. B 90, 174202 (2014)
Serbyn, Papic, and Abanin, Phys. Rev. Lett. 111, 127201 (2013), Vosk, Altman, Phys. Rev. Lett. 110, 067204 (2013), ...

Typical localized single-particle state



Can we extend this to MBL?

Localization length in MBL:
Entanglement entropy, coefficients in l-bit Hamiltonian, ...

Bauer, Nayak J. Stat. Mech. (2013) P09005, Huse, Nandishkore, Oganesyan Phys. Rev. B 90, 174202 (2014)
Serbyn, Papic, and Abanin, Phys. Rev. Lett. 111, 127201 (2013), Vosk, Altman, Phys. Rev. Lett. 110, 067204 (2013), ...

Key results

→ **Model: 1D interacting, spinless fermions**

→ **One-particle density matrix (OPDM): Single-particle description of MBL**

$$\rho_{ij}^{(1)} = \langle \psi_n | c_i^\dagger c_j | \psi_n \rangle$$

$$\rho^{(1)} | \phi_\alpha \rangle = n_\alpha | \phi_\alpha \rangle$$

Natural orbitals

V, realization-dependent set of single-particle states:

Localized in MBL phase
Delocalized in ergodic phase

Occupation spectrum

Fermi-liquid like
Different from Anderson insulator!

One-particle density matrix

Definition

$$\rho^{(1)}(\vec{r},\vec{r}') = N \int d\vec{r}_2^{\,3} \dots \int d\vec{r}_N^{\,3}\, \psi_N^*(\vec{r},\vec{r}_2,\dots,\vec{r}_N)\psi_N(\vec{r}\,',\vec{r}_2,\dots,\vec{r}_N)$$

Eigenstates (natural orbitals) & values (=occupations)

$$\int d\vec{r}\,'\rho^{(1)}(\vec{r},\vec{r}\,')\phi_\alpha(\vec{r}\,') = n_\alpha \phi_\alpha(\vec{r})$$

Bose-Einstein condensation in many-body systems

$$\rho^{(1)}(\vec{r},\vec{r}\,') = n_0\phi_0^*(\vec{r})\phi_0(\vec{r}\,') + \sum_{\alpha\neq 0} n_\alpha \phi_\alpha^*(\vec{r})\phi_\alpha(\vec{r}\,')$$

$$n_0 \sim \mathcal{O}(N)$$

Penrose, Onsager, C.H. Yang

Bloch theorem in many-body systems

$$\phi_\alpha(\vec{r}+\vec{R}) = e^{i\vec{k}\cdot\vec{R}}\phi_\alpha(\vec{r})$$

Koch, Goedecker, Solid State Communications

Single-particle description of MBL

What we do, using exact diagonalization

→ **Fixed energy E**

→ **Obtain many-body eigenstates with E_n ~ E** $H|\psi_n\rangle = E_n|\psi_n\rangle; \quad E_n \approx E$

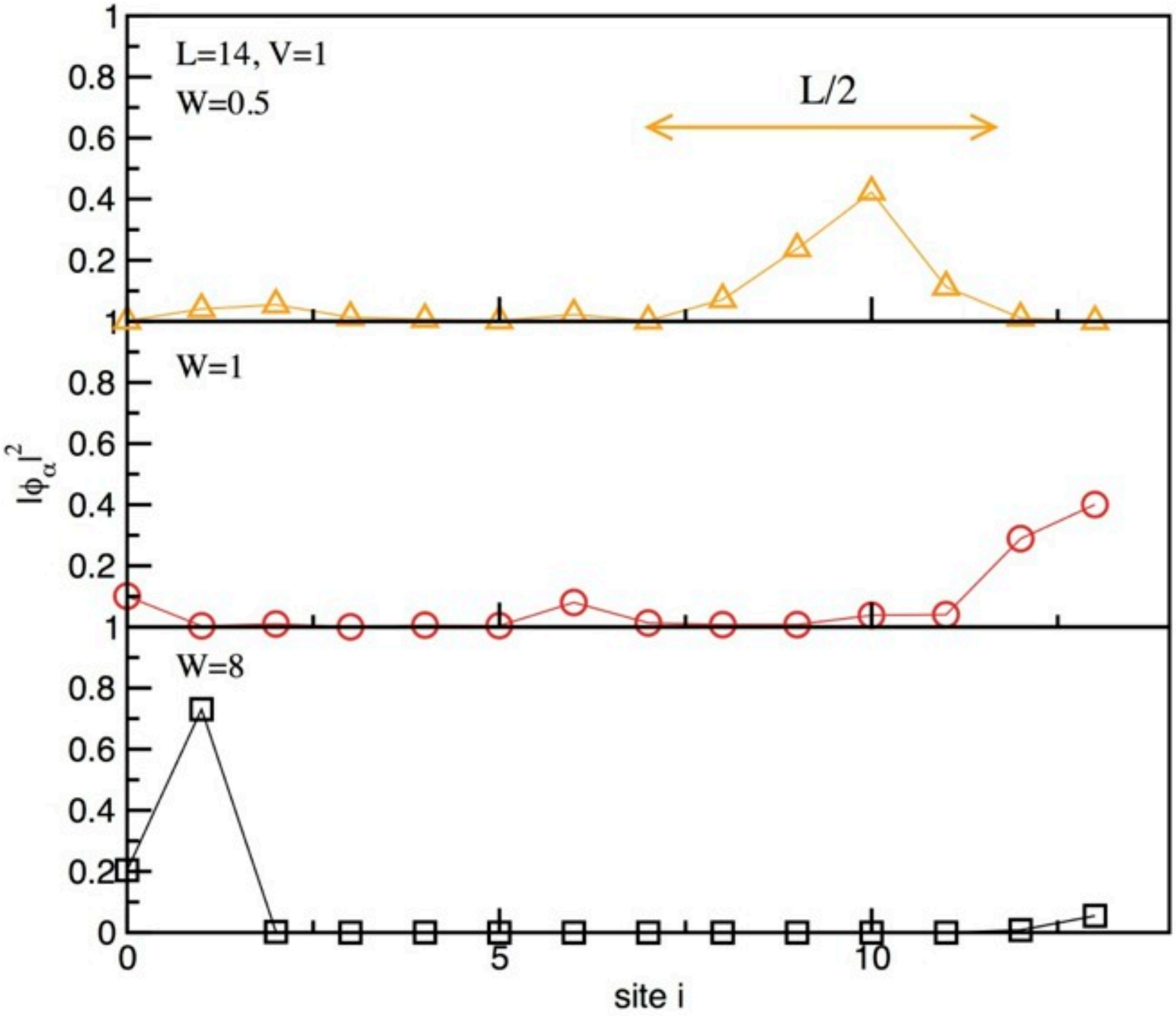
→ **Compute OPDM** $\rho^{(1)}_{ij} = \langle\psi_n|c^\dagger_i c_j|\psi_n\rangle$

→ **Diagonalize it !** $\rho^{(1)}|\phi_\alpha\rangle = n_\alpha|\phi_\alpha\rangle; \quad c^\dagger_\alpha = U_{\alpha j} c^\dagger_j$

→ **Average over impurity configurations**

Two previous studies of OPDM eigenstates of HCBs with disorder (no connection to MBL transition):
Nessi, Iucci Phys. Rev. A 84, 063614 (2011); Gramsch, Rigol, Phys. Rev. A 86, 053615 (2012)

Typical OPDM eigenstates



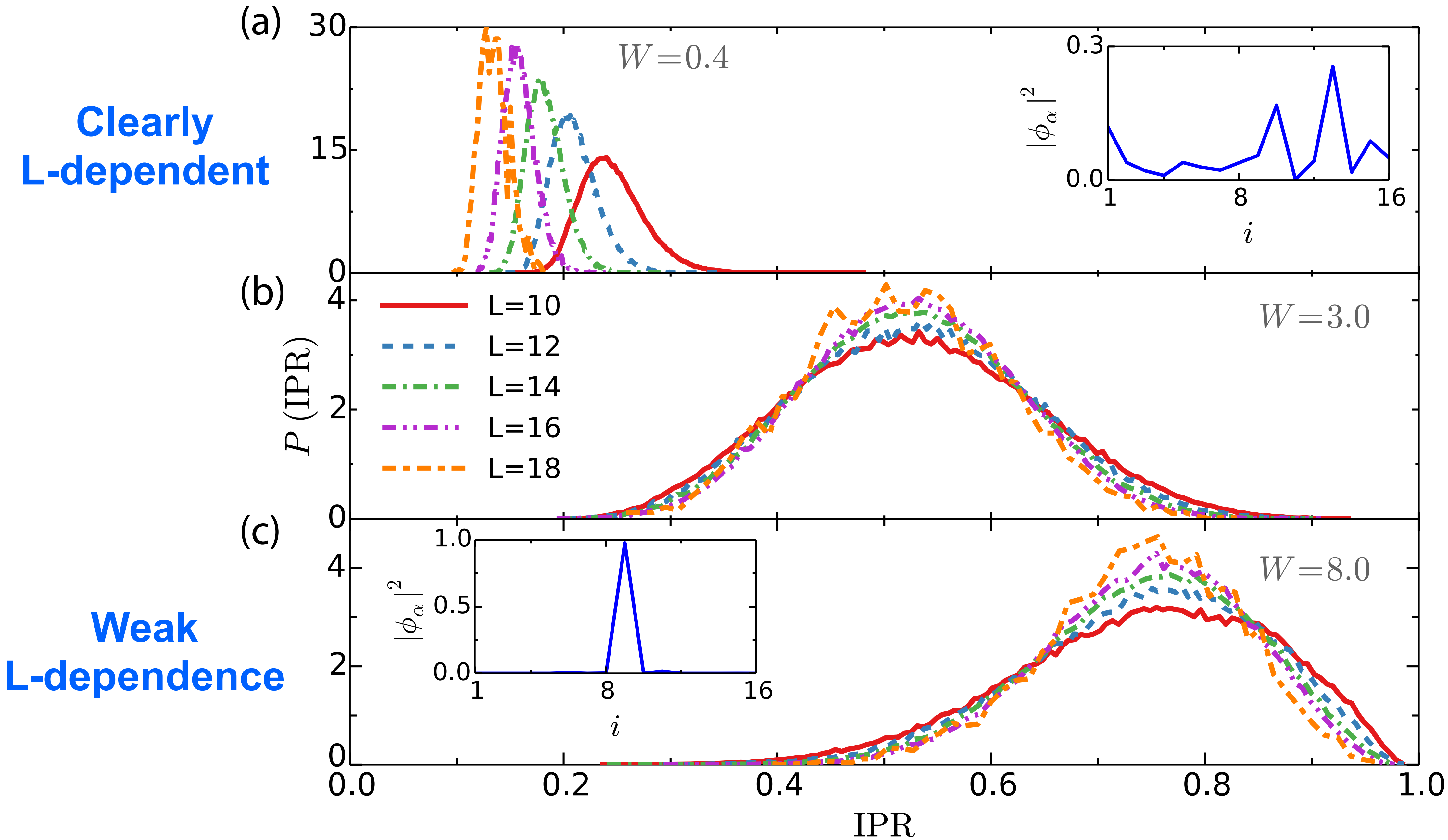
delocalized

several localization centers

localized

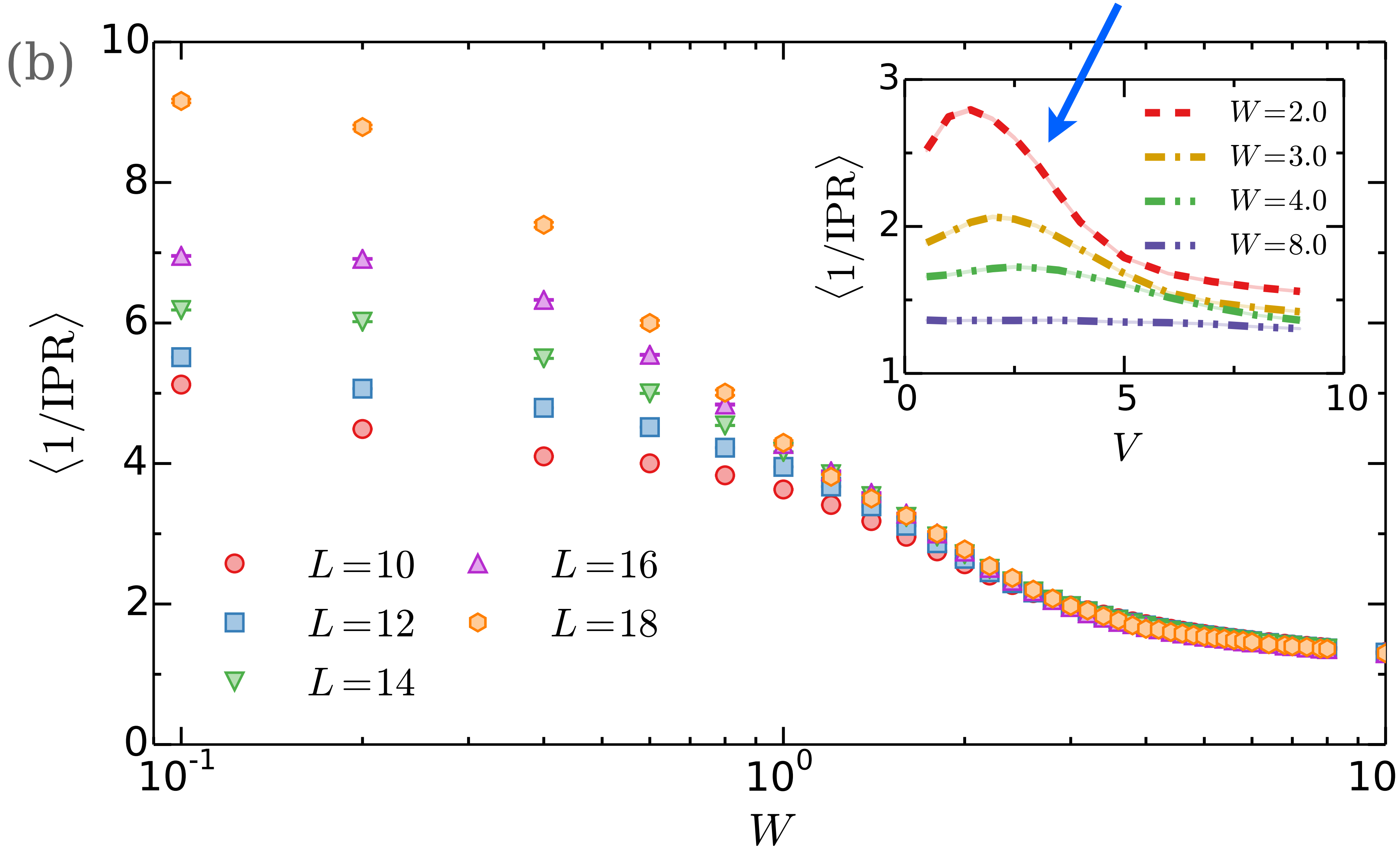
Inverse participation ratio

$$IPR = \frac{1}{N}\sum_{\alpha=1}^{L} n_\alpha \sum_{i=1}^{L} |\phi_\alpha(i)|^4$$



Inverse participation ratio

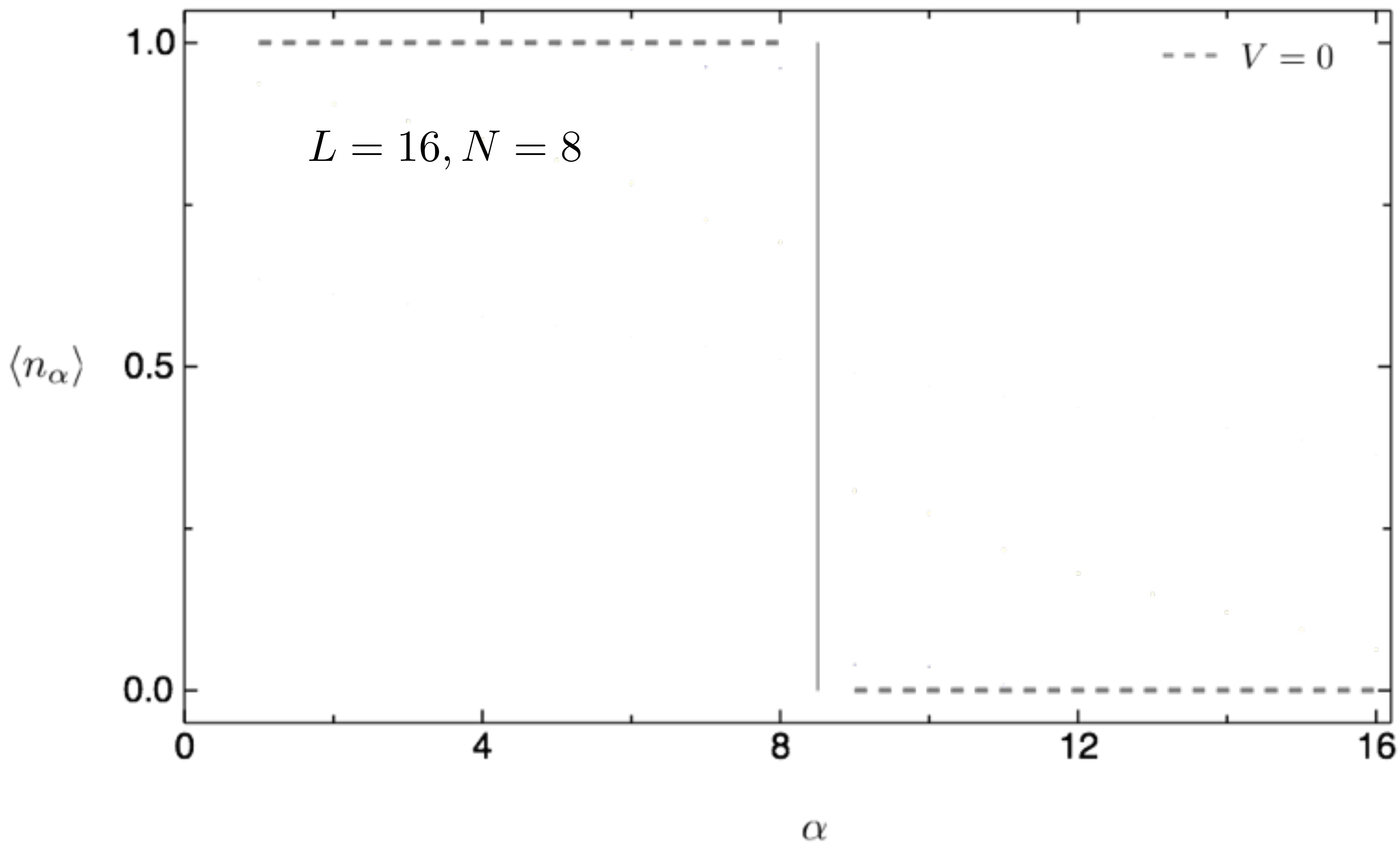
Correct dependence on V



Increases with L in ergodic phase: "ballistic regime"
L-independent in MBL phase
No clear L-dependence at the transition

OPDM occupations

Clean system: $n_\alpha = n_k = \langle c_k^\dagger c_k \rangle$

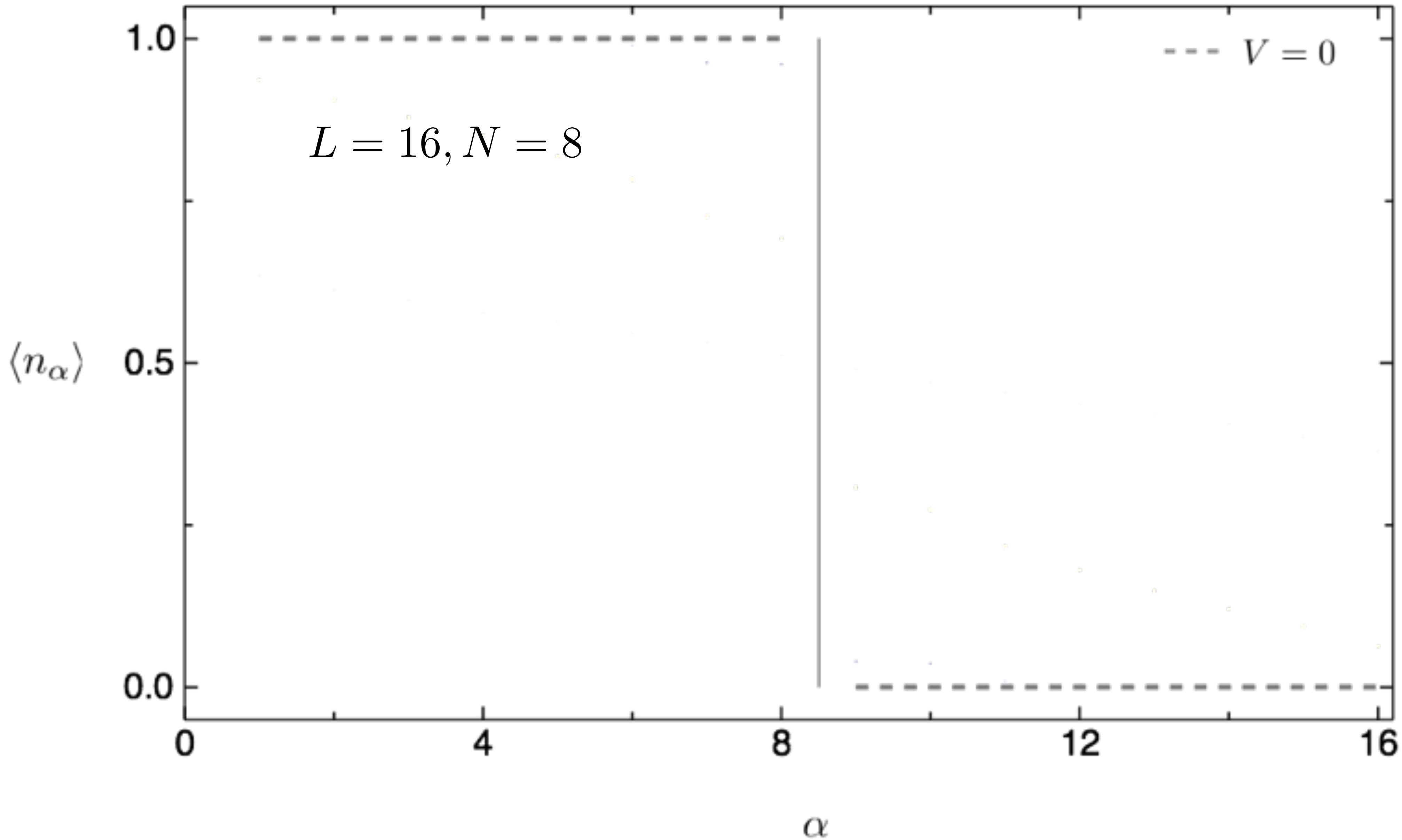


$$H = -t \sum_i (c_i^\dagger c_{i+1} + h.c.) - \sum_i \epsilon_i n_i$$

Anderson insulator

For each many-body state:
Slater determinant
independent of disorder: step function

OPDM occupations

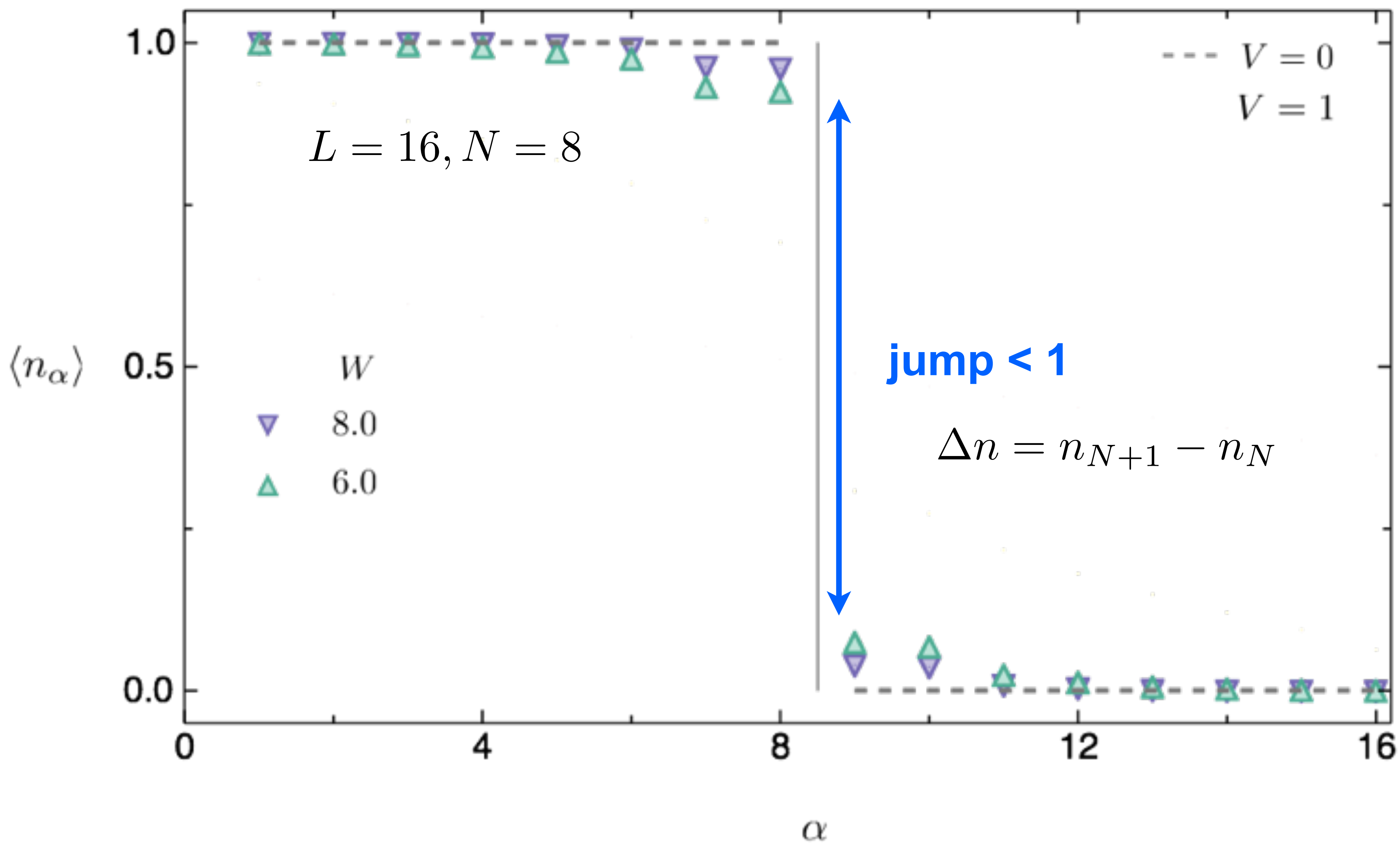


$$W \gg t, V: \quad H = \sum_i \epsilon_i n_i$$

Deep in MBL phase

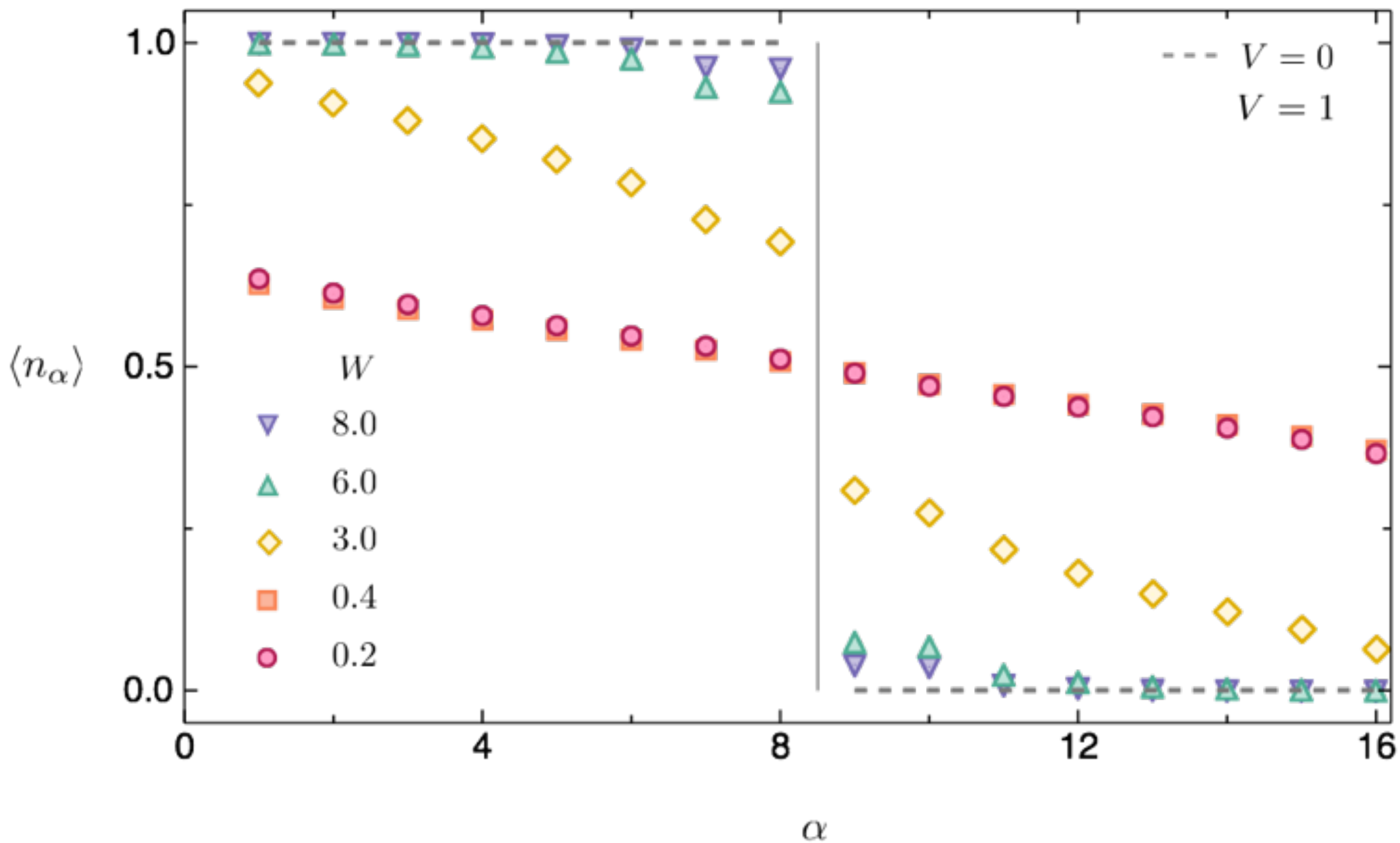
For each many-body state: Slater determinant

OPDM occupations



Deep in the MBL phase: Slater determinant
Finite jump survives - Similar to Fermi-liquid

OPDM occupations

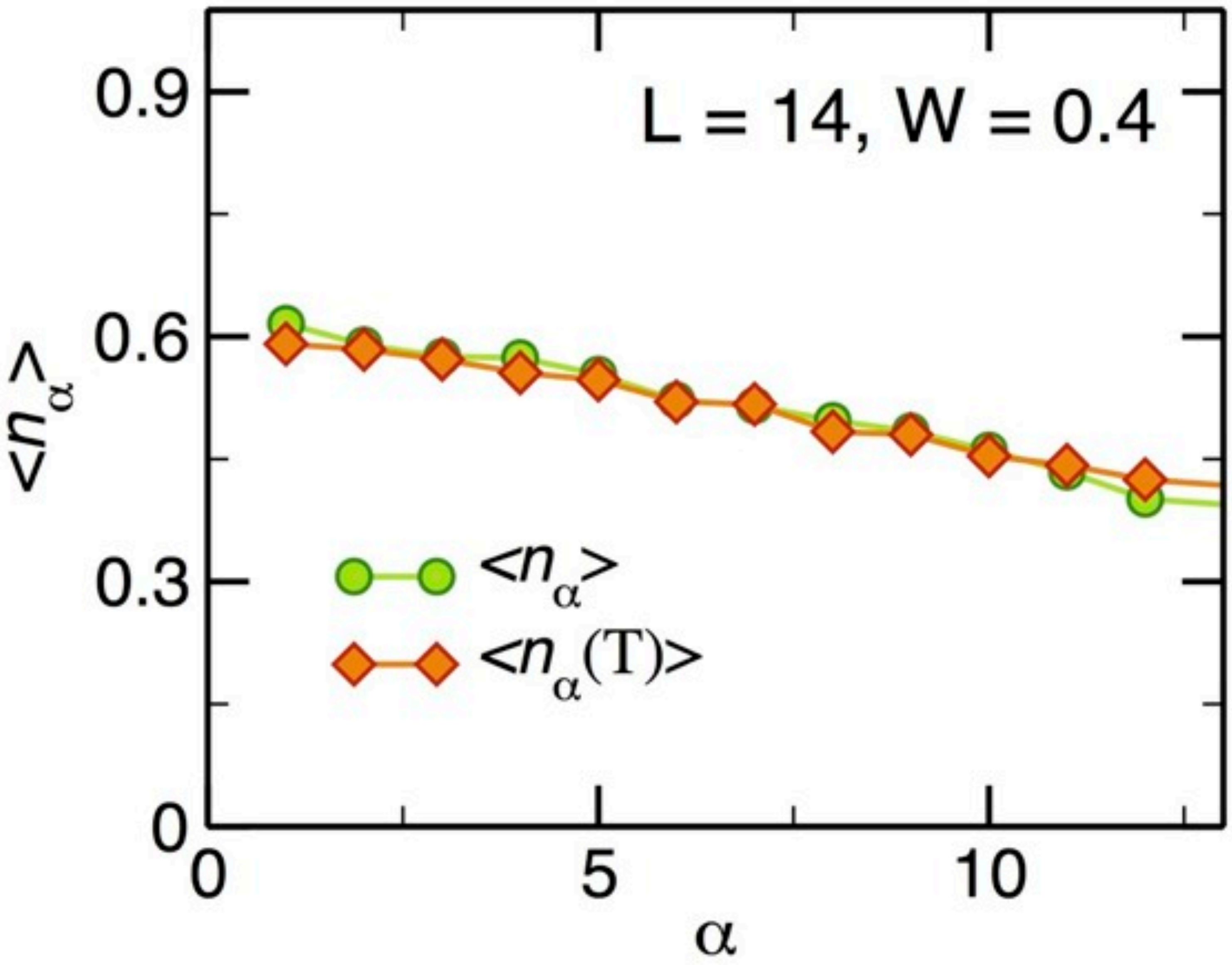


thermal!

MBL: discontinuity, Fermi-liquid-like
Transition is delocalization in Fock space
Thermal distribution in ergodic phase

Basko, Aleiner, Altshuler
2006

OPDM occupations in delocalized phase

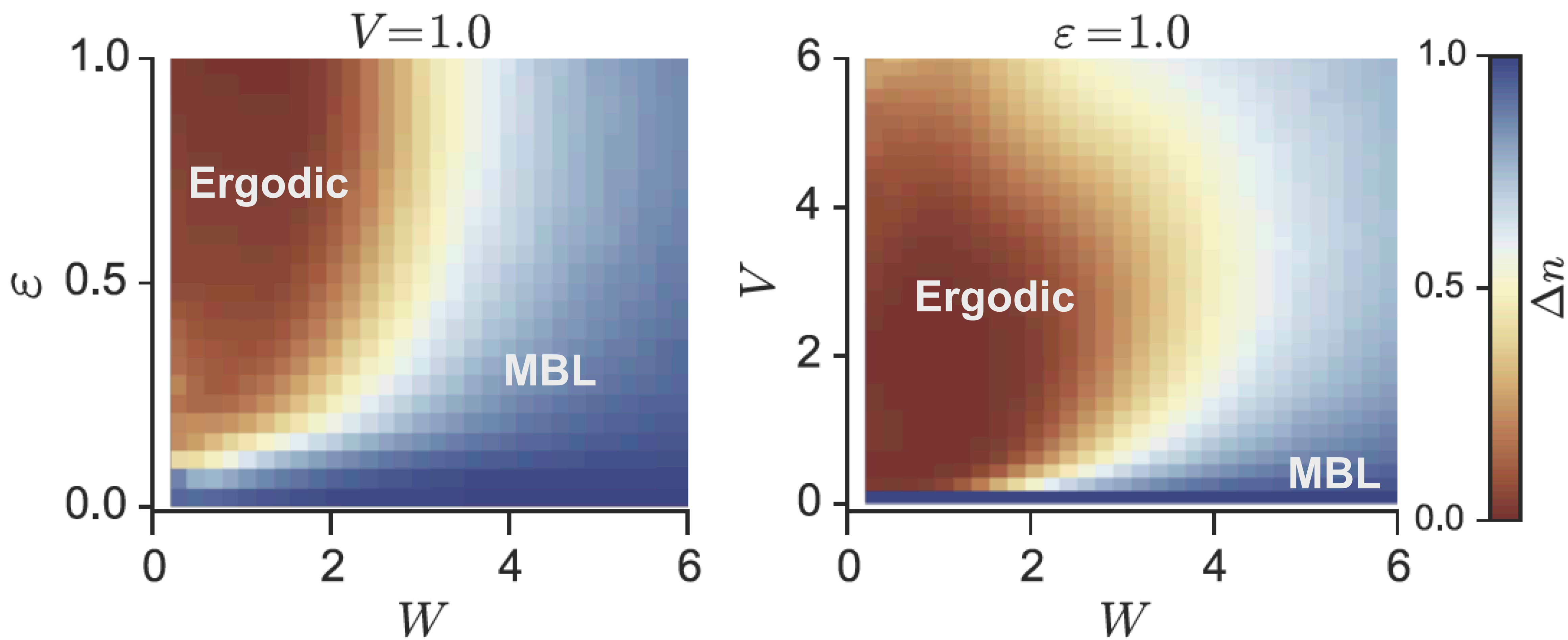


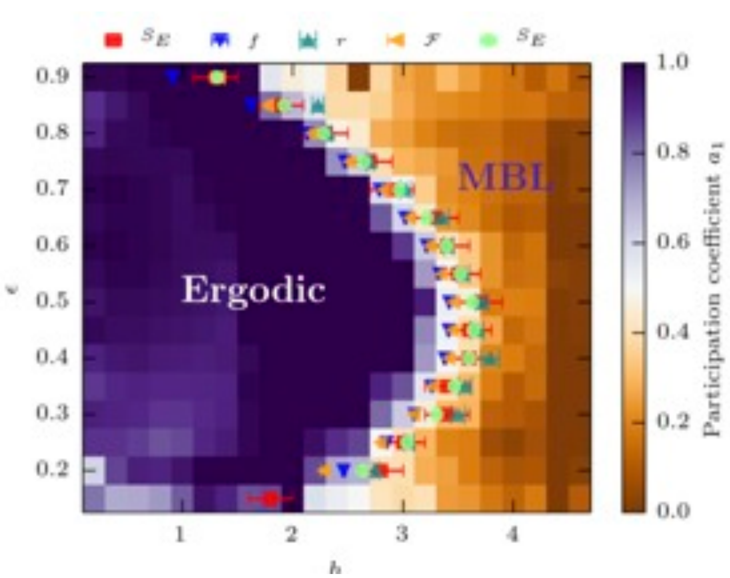
$$E \approx \langle \psi_n | H | \psi_n \rangle = \mathrm{tr}(\rho(T) H) \qquad \rho^{(1)}_{ij}(T) = \mathrm{tr}[\rho_{\mathrm{can}}(T) c_i^\dagger c_j] \to \langle n_\alpha(T) \rangle$$

Finite-size effects typical for non-integrable system, Sorg, Vidmar, Pollet, FHM PRA 2014

Discontinuity in occupations: “Phase” diagrams

$$\Delta n = n_{N+1} - n_N$$

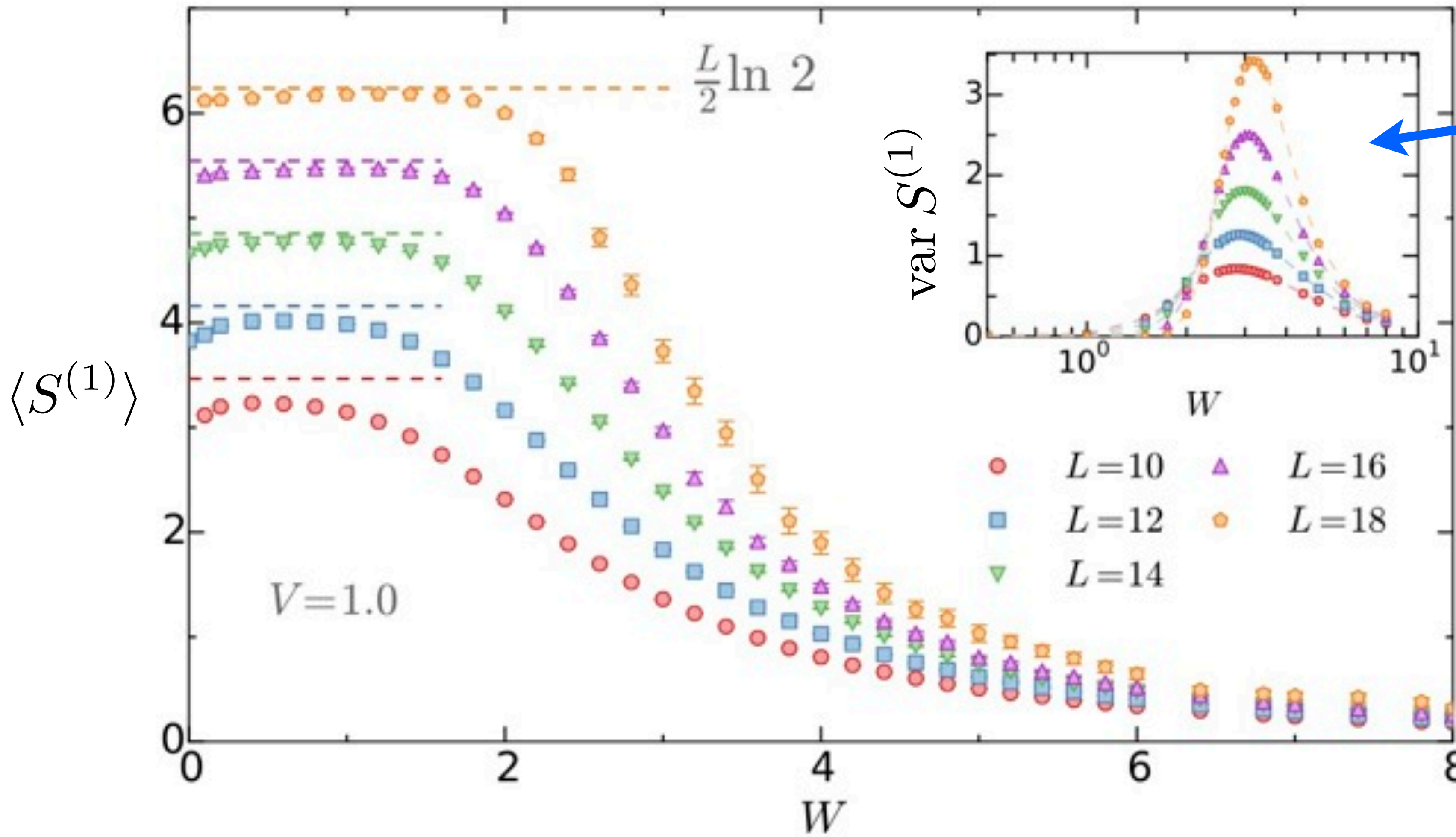




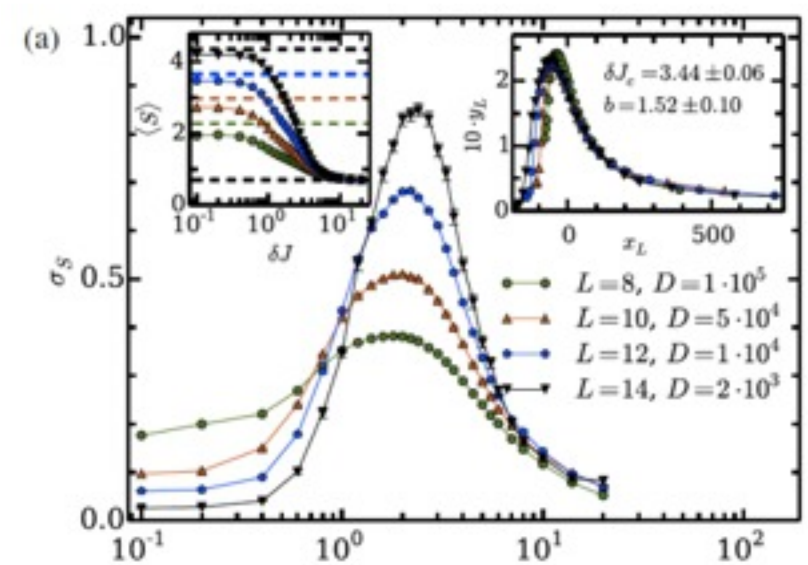
Step-like discontinuity similar to n(k) of a Fermi-liquid agrees with known phase diagram

One-particle occupation entropy

$$S^{(1)} = -\sum_{\alpha}[n_\alpha \ln(n_\alpha)]$$



Similar to entanglement entropy



Kjall, Bardarson, Pollmann
PRL 2014

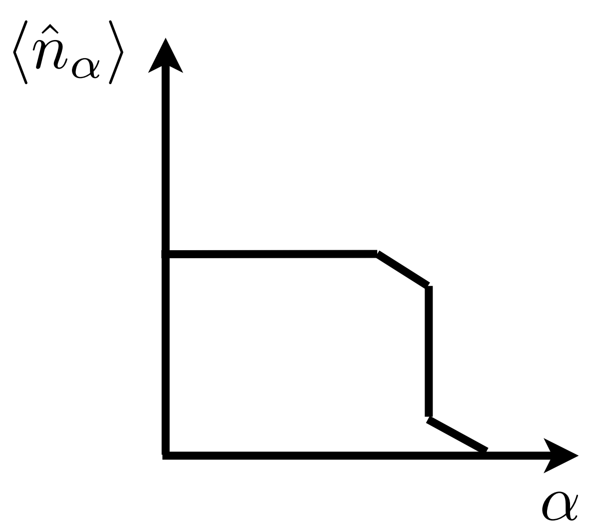
Maximum in fluctuations at transition!

MBL & Fermi-liquids

→ Connection to conserved charges & Fermi-liquid interpretation

(one realization!)

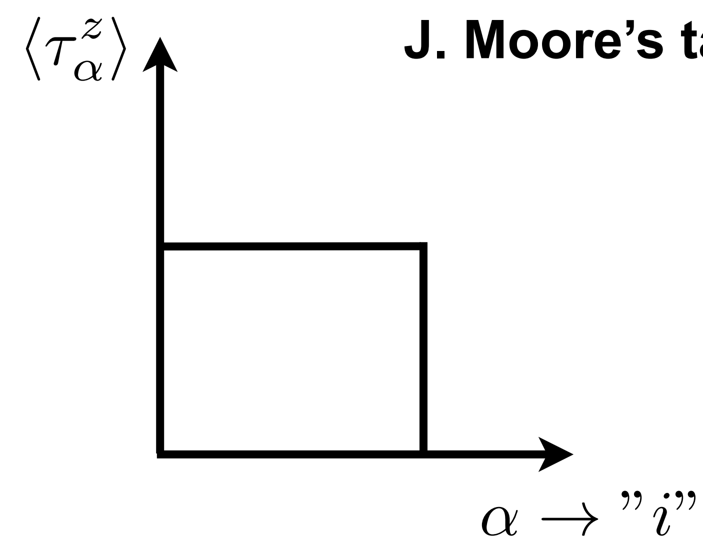
See also E. Altman's talk, J. Moore's talk



"Physical" particles in FL

OPDM eigenstates **l-bits**

$$\hat{n}_\alpha = c^\dagger_\alpha c_\alpha \leftrightarrow \tau^z_\alpha$$



Quasi-particles in FL

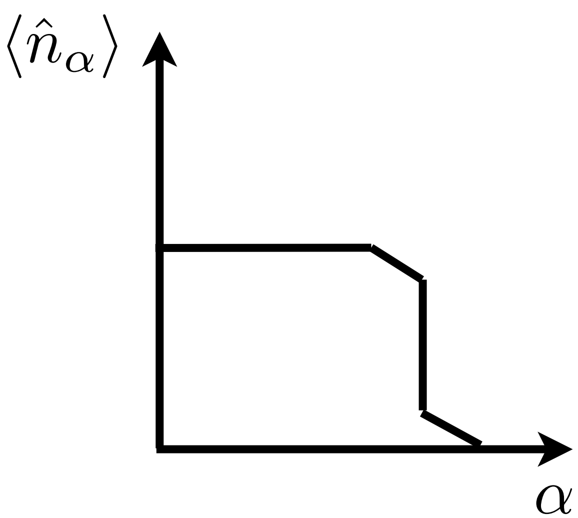
$$H = \sum_i \epsilon_i \tau^z_i + \sum_{i,j} J_{i,j} \tau^z_i \tau^z_j + \sum_{i,j,\{k\}} K^n_{i,\{k\},j} \tau^z_i \tau^z_{k_1} \cdots \tau^z_{k_n} \tau^z_j$$

$$\tau^z_i = \sum_j a^{(i)}_j n_j + \sum_{l,j,m} b^{(i)}_{l,m} c^\dagger_l n_j c_m + \ldots$$

Huse, Nandishkore, Oganesyan Phys. Rev. B 90, 174202 (2014)
Serbyn, Papic, and Abanin, Phys. Rev. Lett. 111, 127201 (2013)
Vosk, Altman, Phys. Rev. Lett. 110, 067204 (2013)

Outlook

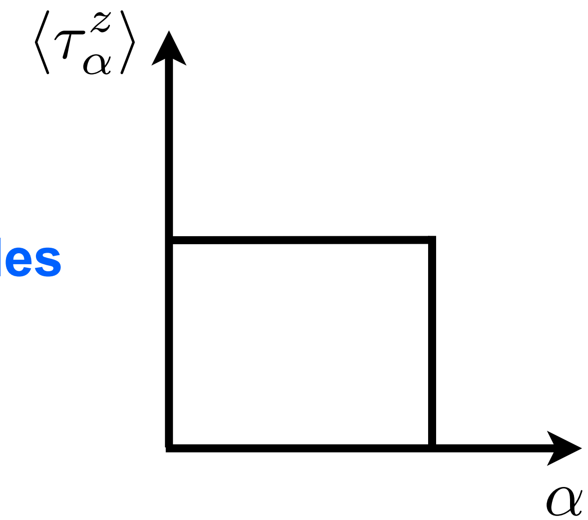
→ **Connection to conserved charges & Fermi-liquid interpretation**



$$\hat{n}_\alpha = c^\dagger_\alpha c_\alpha \leftrightarrow \tau^z_\alpha$$

“Physical” particles in FL

Quasi-particles in FL



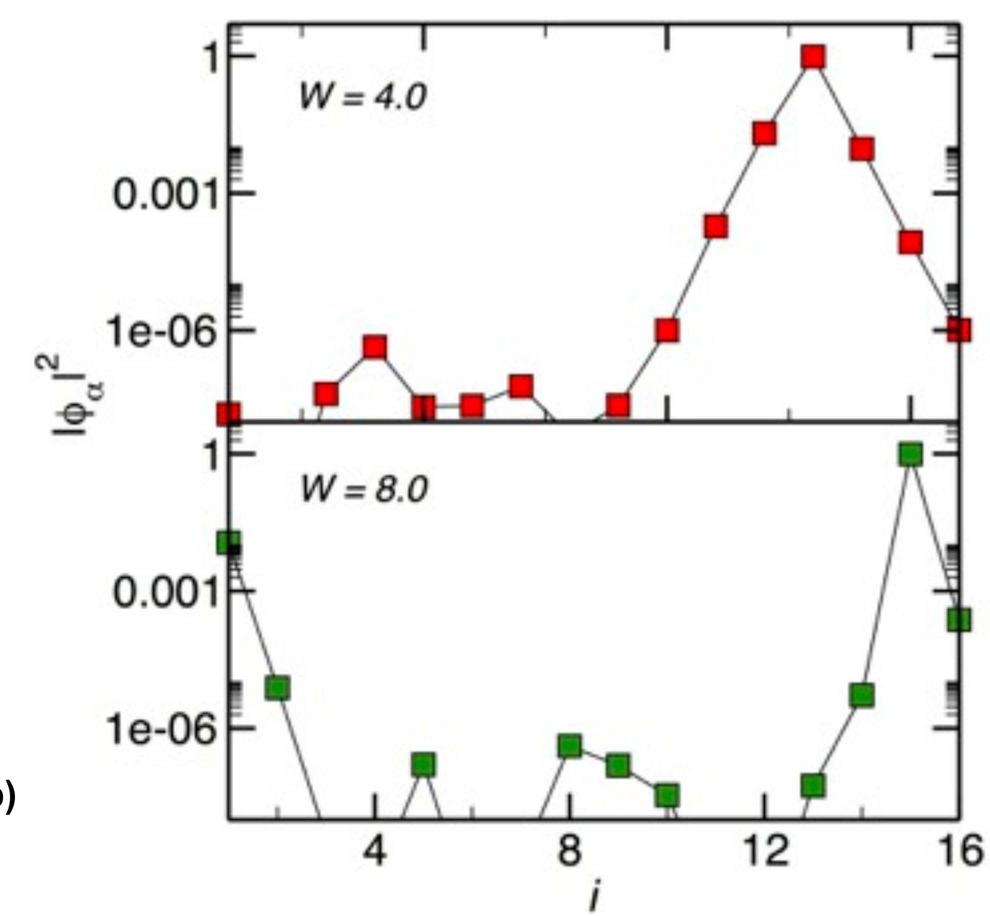
→ **Quantitative analysis - scaling**

→ **Larger systems: DMRG**
Khemani, Pollmann, Sondhi arXiv:1509.00483; Yu, Pekker, Clark arXiv:1509.01244
Lim, Sheng arXiv:1510.08145; Karrasch, Kennes arXiv:1511.02205

→ **Localization length from natural orbitals**

→ **Multifractal natural orbitals? See Kravtsov’s talk**

→ **Connect to OL experiments**
Schreiber et al. Science 349, 842 (2015); Bordia et al. arXiv:1509.00478 (Bloch group)



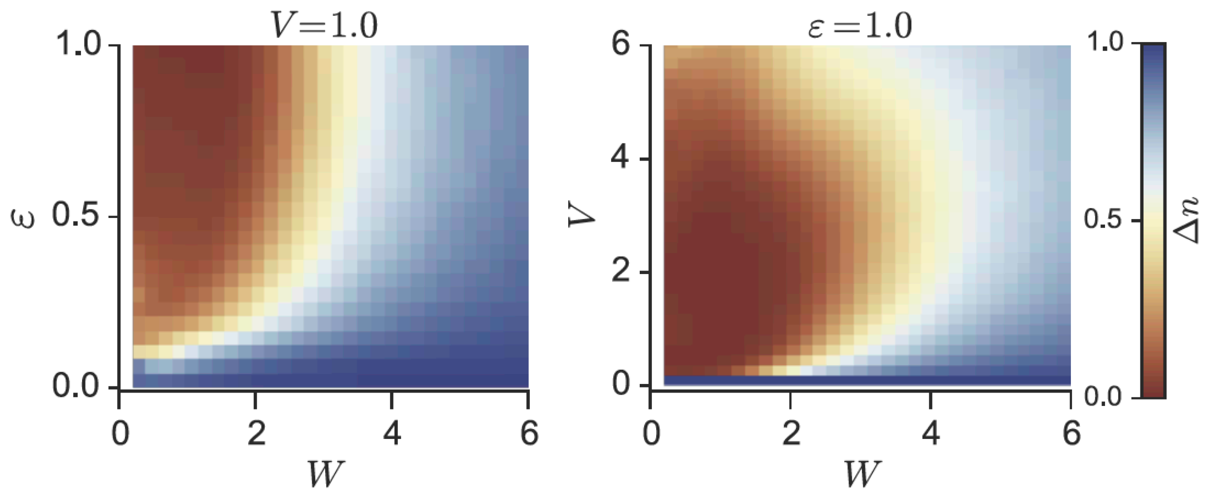
Summary

→ **Single-particle description based on one-particle density matrix**

→ **Natural orbitals (de)localized in (ergodic)MBL phase**

→ **Discontinuity in occupations!**
Reminiscent of Fermi-liquid:
distinguishes MBL from Anderson insulator

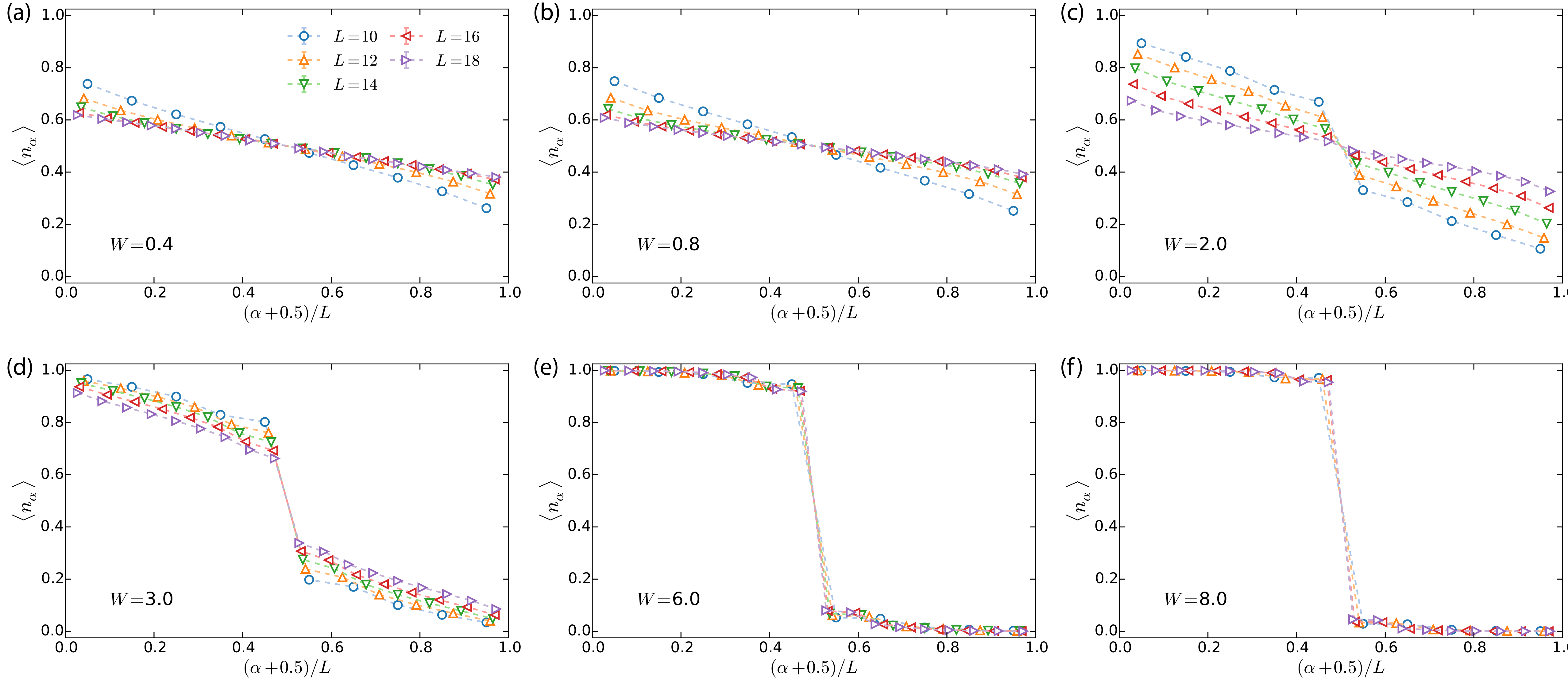
→ **Addtional tool for diagnostics of MBL phase**



Bera, Schomerus, FHM, Bardarson,
Phys. Rev. Lett. 115, 046603 (2015)

Thank you!

Finite-size dependence of occupation discontinuity



OPDM eigenvalues in individual realizations