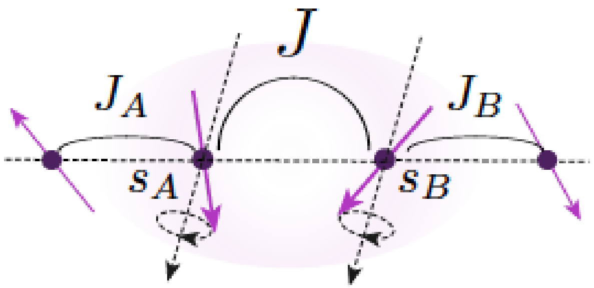
Dynamics of random Heisenberg model (mostly from real space RG)

Ivar Martin



With: Kartiek Agarwal, Eugene Demler

Phys. Rev. B 92, 184203 (2015)

Nov 2015. KITP: MBL

Outline

- Random bond Heisenberg
  - Noise
  - Connection to MBL, and its absence
- Heisenberg + random h
  - Bracketing the transition

Motivation

Flux noise in qubits

Shows MBL trans.

Model 1: Random Bond Heisenberg

$$H = \sum_i J_i \vec{S}_i . \vec{S}_{i+1}$$

$$P(|J|)$$

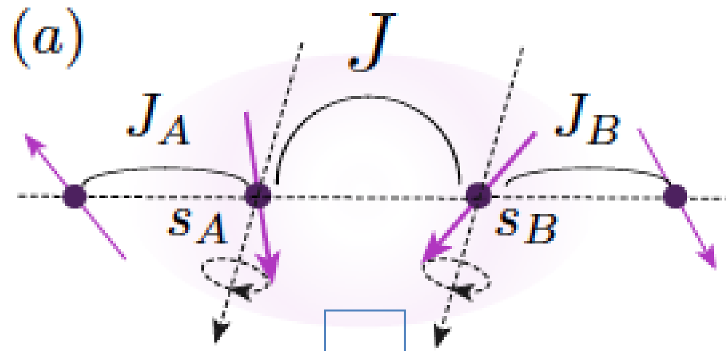
Some Ferro, some Anti-Ferro bonds : say 50/50.

$$\vec{M}(t) = \sum_i g(r_i) \vec{S}(r_i)$$

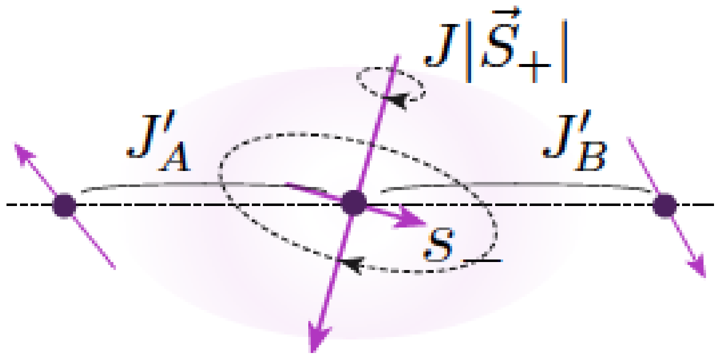
$$\text{Noise} \; : \; S(t) = \langle [M(t), M(0)]_+ \rangle$$

How do we compute the dynamics of this complicated object?

RSRG treatment of the problem



Eliminate fastest bond



+ Noise @ $\omega = J|\vec{S}_+|$

$$S_+ \in [|S_A - S_B|, S_A + S_B]$$

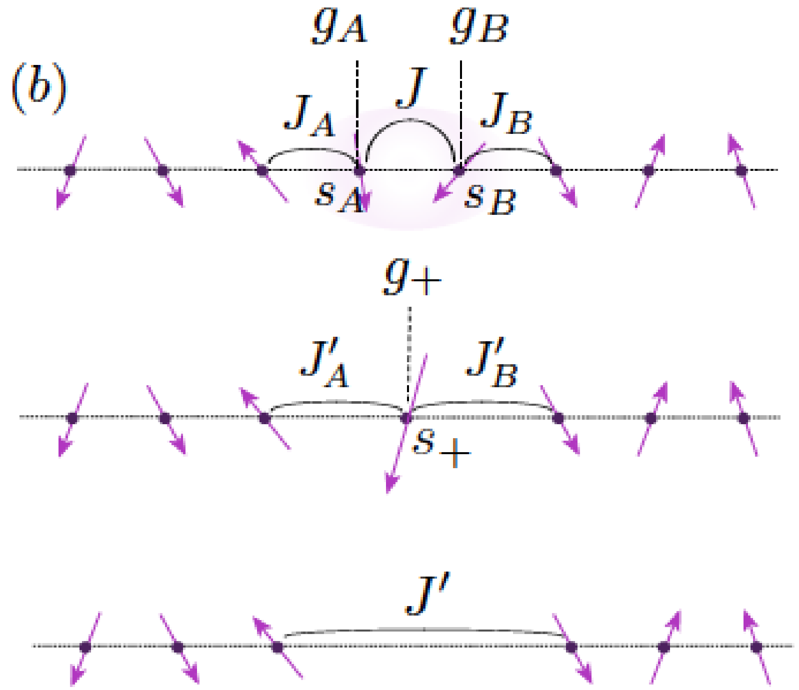
$$J'_{A(B)} = J_{A(B)}(\vec{S}_{A(B)} \cdot \vec{S}_+)/\left|\vec{S}_+\right|^2$$

Near T = 0, Thermodynamics: Dasgupta-Ma-Hu (1979), R.Bhatt and P. Lee (1982), D. Fisher (1995), Westerbrg et al (1996), Motrunich et al (2001)

Excited states: Vosk-Altman (2012), Pekker et al (2014), Potter et al (2015)

RG procedure for Noise

$$\vec{M}(t) = \sum_i g(r_i)\vec{S}(r_i)$$



Noise :

$$S(t) = \langle [M(t), M(0)]_+ \rangle$$

Examples:

$S(q,\omega)$:

$$g(r_i) = cos(qr_i)$$

Magnetic field at R:

$$g(r_i) = 1/|\vec{R} - \vec{r}_i|^3$$

$$g_+ = g_A(\vec{S}_A \cdot \vec{S}_+)/\left|\vec{S}_+\right|^2 + g_B(\vec{S}_B \cdot \vec{S}_+)/\left|\vec{S}_+\right|^2 =$$

$$(g_A + g_B)/2 + (g_A - g_B)/2\left[\left|\vec{S}_A\right|^2 - \left|\vec{S}_B\right|^2\right]/\left|\vec{S}_+\right|^2$$

Agarwal, Demler, Martin: http://arxiv.org/pdf/1506.00643.pdf, PRB (2015)

Quantum Noise calculation

$$M(t) = M_S(t) + M_F(t)$$

$$M_S(t) = \sum_{i \neq A,B} g_i \vec{S}_i + g_+ \vec{S}_+$$

$$M_F(t) = (g_A - g_B)/2 \left[\vec{S}_- - \vec{S}_+ (\vec{S}_- \cdot \vec{S}_+)/|\vec{S}_+|^2 \right]$$

Magnitude

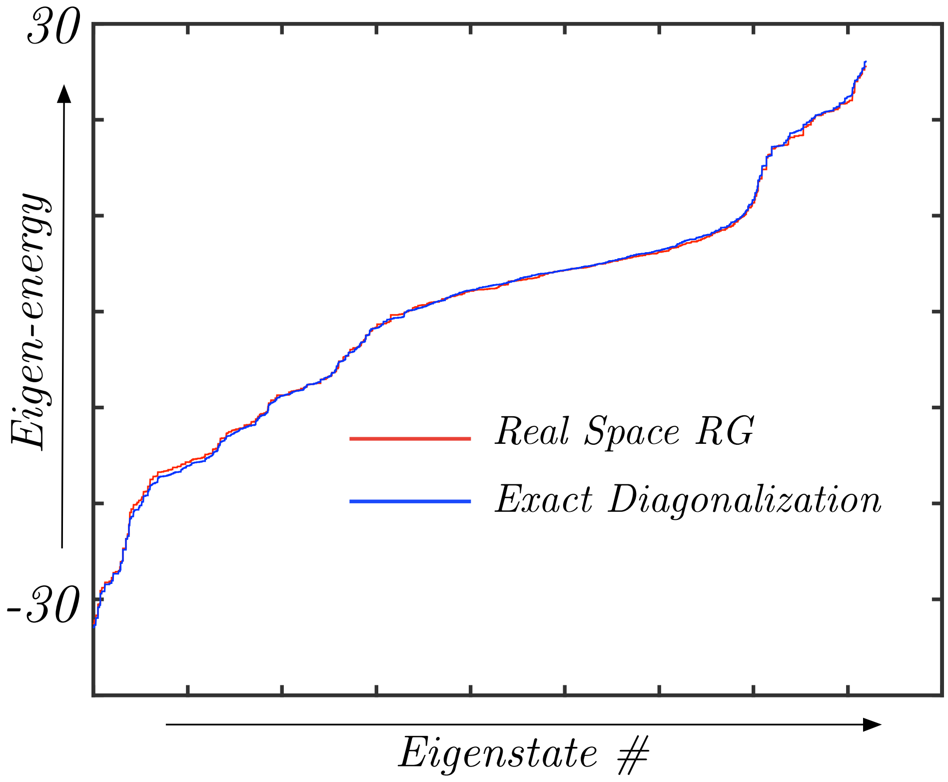
$$\left| \vec{S}_- - \vec{S}_- . \vec{S}_+ \frac{\vec{S}_+}{|\vec{S}_+|^2} \right|^2 = 2s_1(s_1+1) + 2s_2(s_2+1) - s(s+1) - \frac{(s_1(s_1+1) - s_2(s_2+1))^2}{s(s+1)}$$

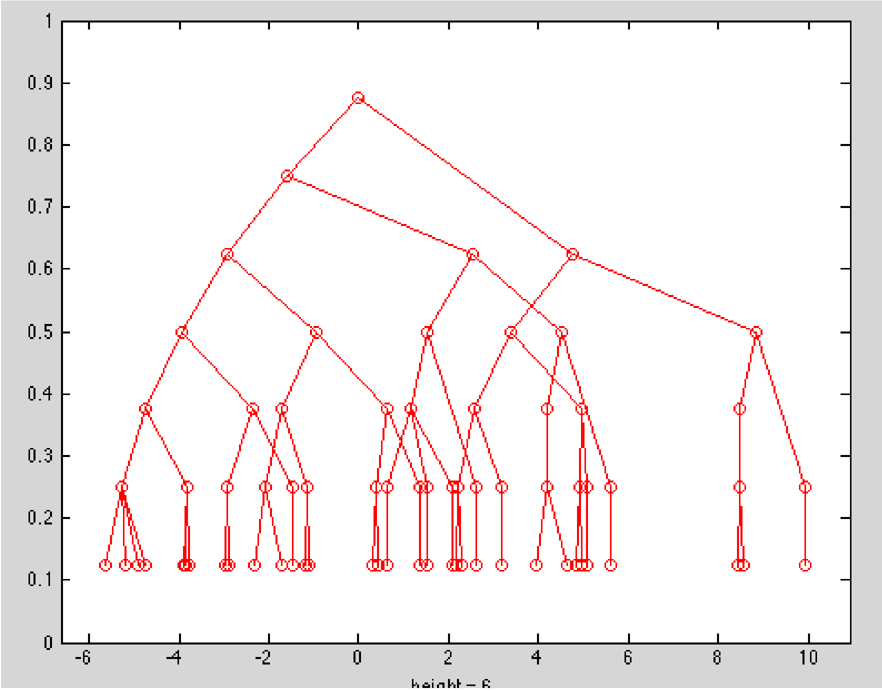
Frequency

$$\omega = J|\vec{S}_+|$$

$$\omega(s) = Js$$

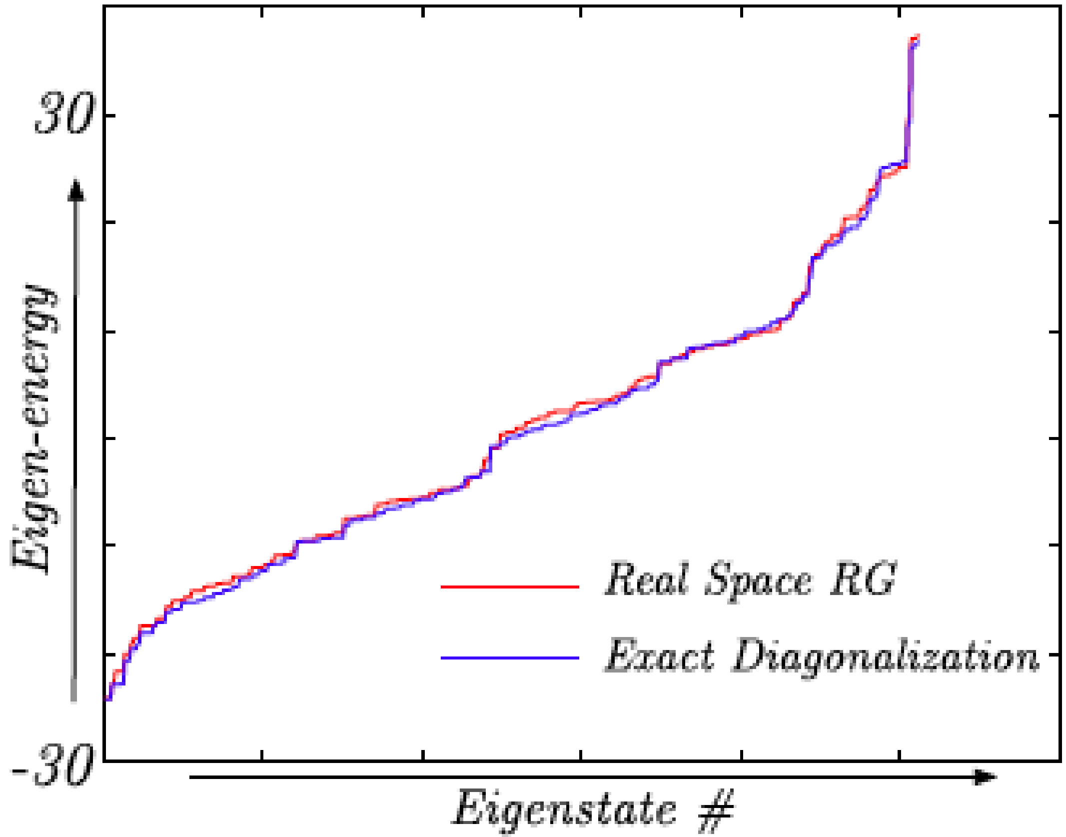
RG comparison with Exact Diag. - disordered ½ Ferro, ½ Anti-Ferro





Cf X-RSRG of Pekker at al (2014) for Ising

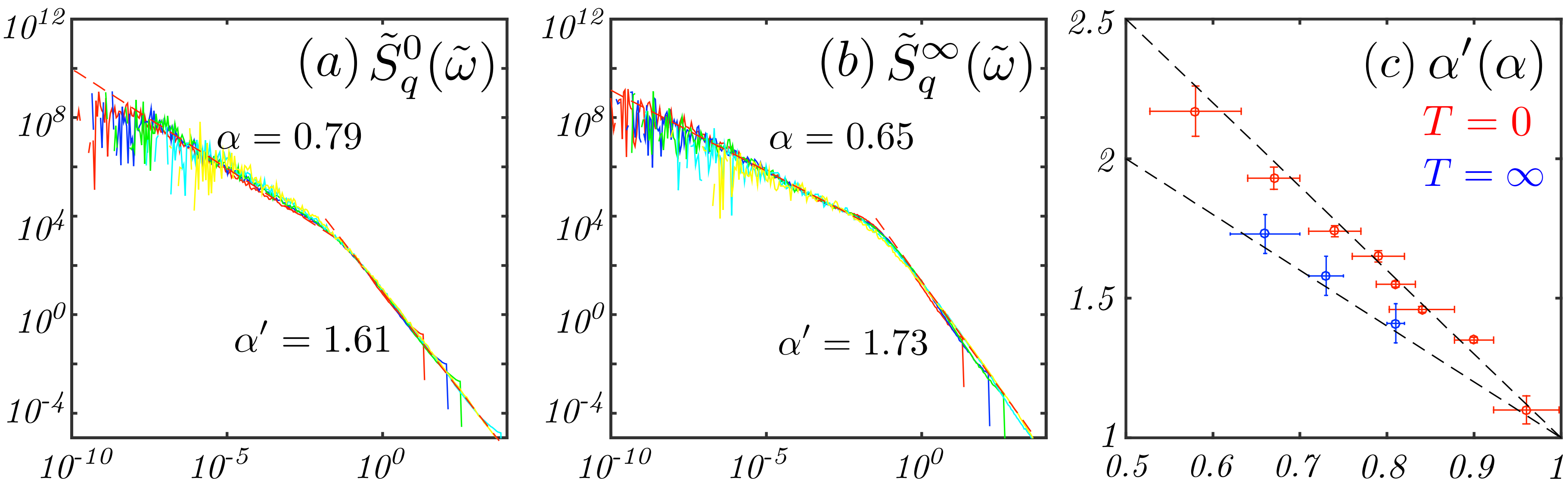
RG comparison with ED in 2D, 3x3



The structure factor (harmonic probe) $S_q(\omega)$

$$g_q = \cos(\vec{q}.\vec{r}) \qquad \text{generic probe}: \; N(\omega) = \sum_q g_q^2 S_q(\omega)$$

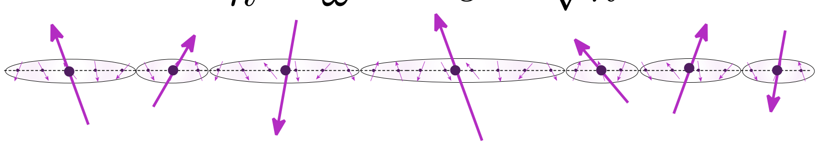
Piece-wise power-law form of the structure factor
at zero/infinite temperature.



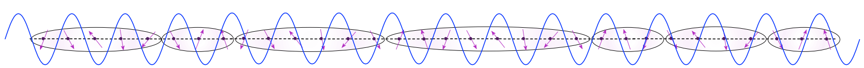
Why two power-law form?

$$T = \infty$$

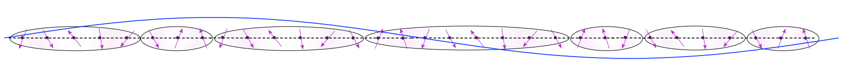
$$n \sim \omega^{-\beta} \quad s \sim \sqrt{n}$$



$$qn(\omega) \gg 1 \quad S_q(\omega) \sim 1/\omega^{\alpha} \quad \alpha < 1$$

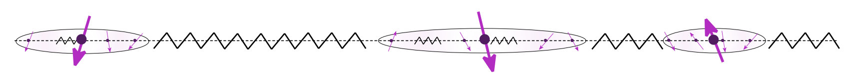


$$qn(\omega) \ll 1 \quad S_q(\omega) \sim 1/\omega^{\alpha'} \quad \alpha' > 1$$



$$T = 0$$

$$n \sim \omega^{-\beta} \quad s \sim \sqrt{n}$$



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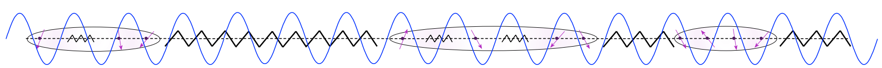

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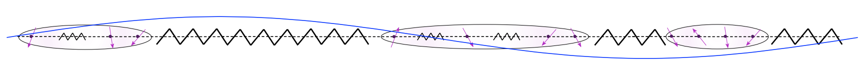
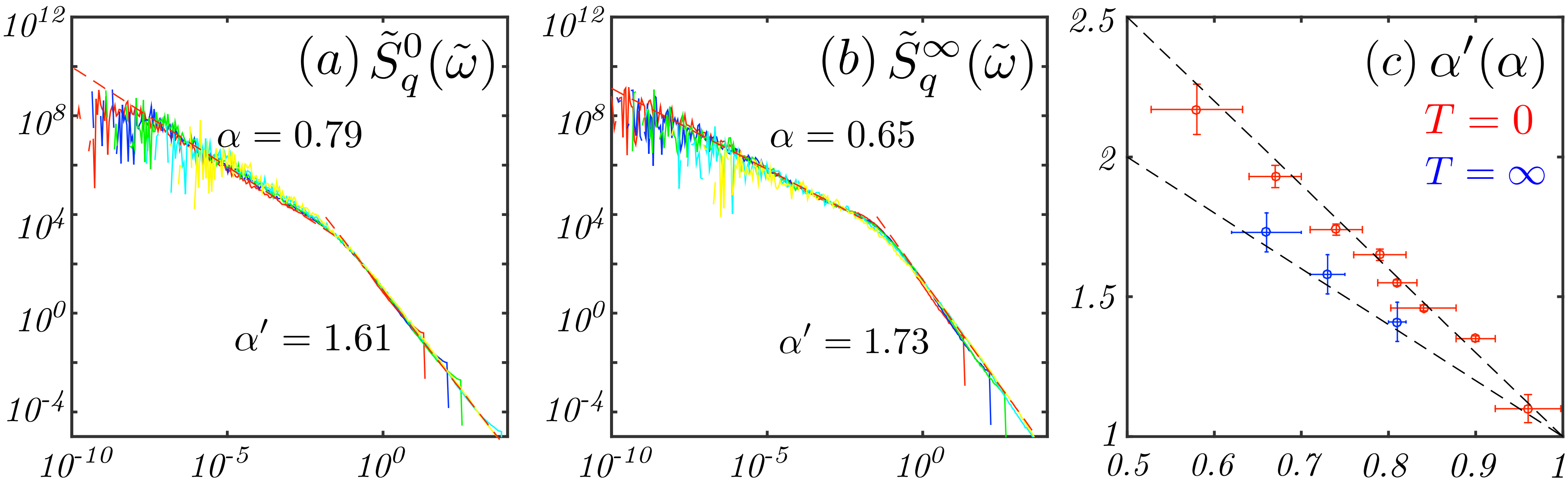


The Structure factor (harmonic probe): $S_q(\omega)$



$$S_q(\omega) \underset{T=0}{=} q^{1/2-1/\beta} g_0\left(\frac{\omega}{q^{1/\beta}}\right) = \begin{cases} 1/\omega^{1-\beta/2} & \omega \ll q^{1/\beta} \\ q^2/\omega^{1+3\beta/2} & \omega \gg q^{1/\beta} \end{cases}$$

$$S_q(\omega) \underset{T=\infty}{=} q^{-1/\beta} g_\infty\left(\frac{\omega}{q^{1/\beta}}\right) = \begin{cases} 1/q\omega^{1-\beta} & \omega \ll q^{1/\beta} \\ q^2/\omega^{1+2\beta} & \omega \gg q^{1/\beta} \end{cases}$$

Understanding the form of $S_q(\omega)$

- Numerical : True scaling collapse over 10 orders of magnitude in frequency and noise magnitude.
- Analytical (Specific) at T=0, T=∞ :

$$P_{F(AF)}(\Delta, S_L, S_R) \sim \frac{1}{\Delta_0^{1-\beta}} Q_{F(AF)}\left(\Delta\Delta_0^{-1}, S_L\Delta^{-\beta/2}, S_R\Delta^{-\beta/2}\right)$$

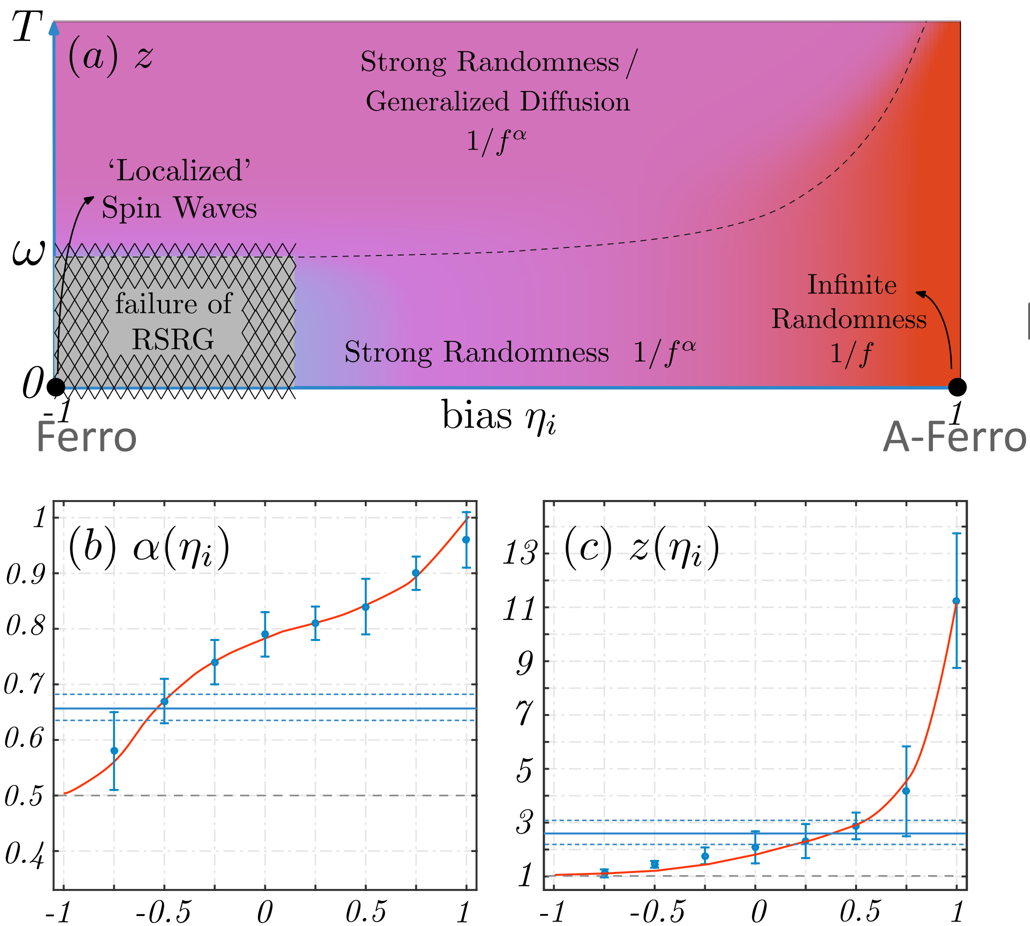
Westerberg et al, 1997

- Analytical (Generalized Diffusion) at T=∞ :

$$G_q(\omega) = 1/[-i\omega + Dq^2] \longrightarrow G_q(\omega) = 1/[-i\omega + q^{1/\beta} f(\omega/q^{1/\beta})]$$

For details:
Agarwal, Demler, Martin: http://arxiv.org/pdf/1506.00643.pdf, PRB (2015)

1/f noise and MBL?

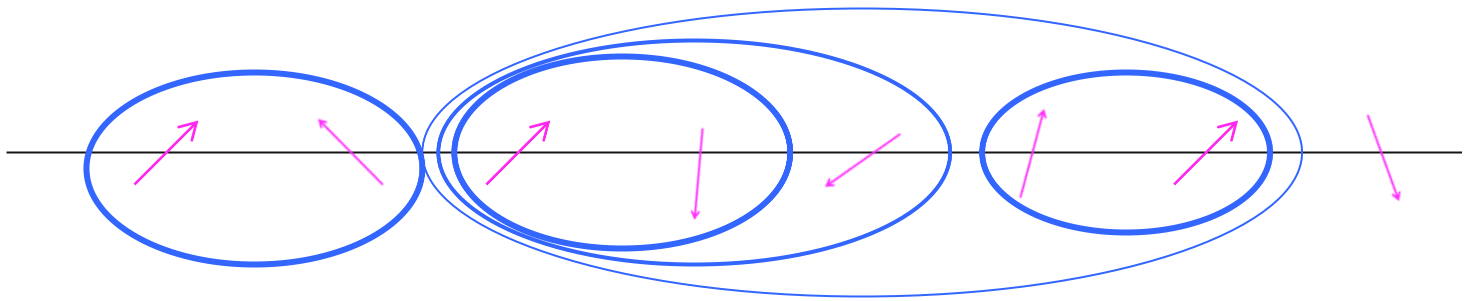


Decay of correlations =
Fourier transform of Noise Spectrum

$$S_q(f) = 1/f^{\alpha} \Rightarrow 1/t^{1-\alpha}$$

$$1/f \rightarrow \text{MBL}$$

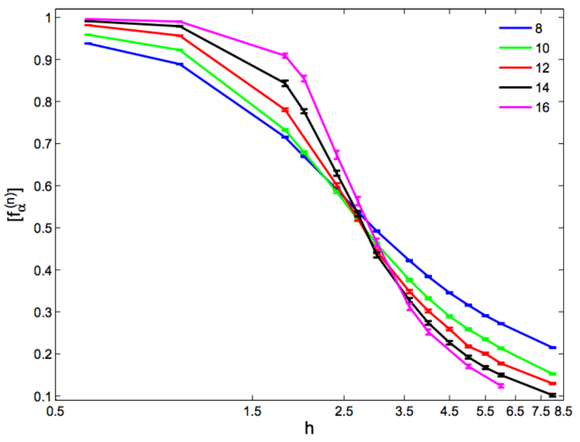
No MBL in isotropic Heisenberg.



Nested *Nonlocal* Integrals of Motion $|\mathbf{S}_+|$. Cf τ_z of Huse, Vadim, Rahul

Ways out

- Ising anisotropy?
- dipole-dipole interactions?
- random field? – Pal and Huse 2010



Model 2: Heisenberg with random field

$$H=\sum_{i=1}^{L}[h_i\hat{S}_i^z + J\hat{\vec{S}}_i\cdot\hat{\vec{S}}_{i+1}]$$

$$h_i \in [-h,h]$$

- RG:
  - Eliminate fastest frequency (strongest bond, largest h, …)
- Under RG generate new terms

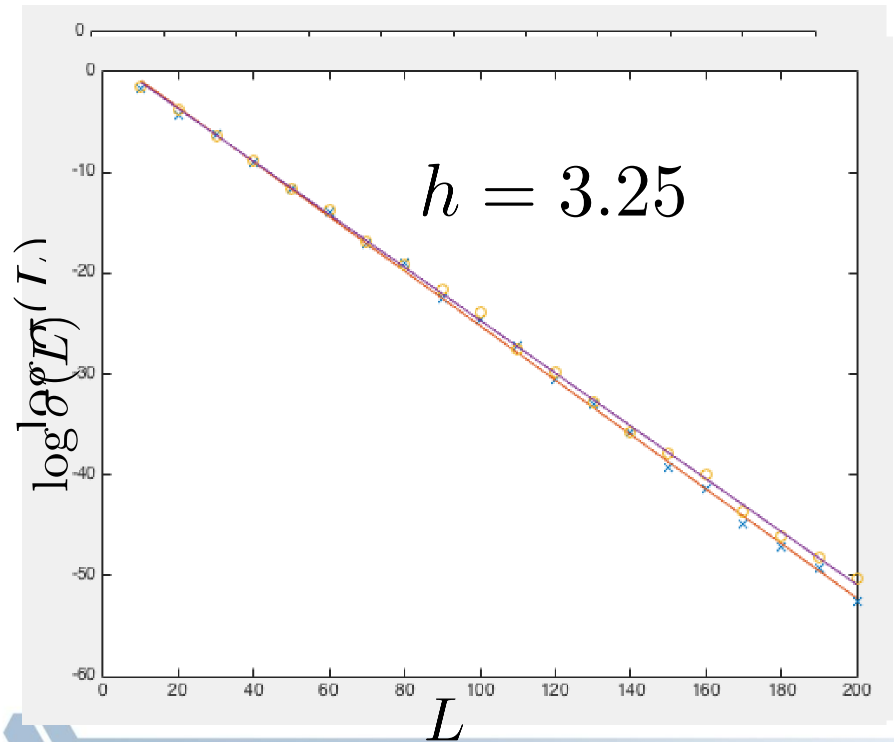
$$H=\sum_i h_i S_i^z + b_i(S_i^z)^2 + \sum_{\langle ij\rangle} J_{ij}\vec{S}_i.\vec{S}_j + J_{ij}^{\perp}\left(S_i^x S_j^x + S_i^y S_j^y\right)$$

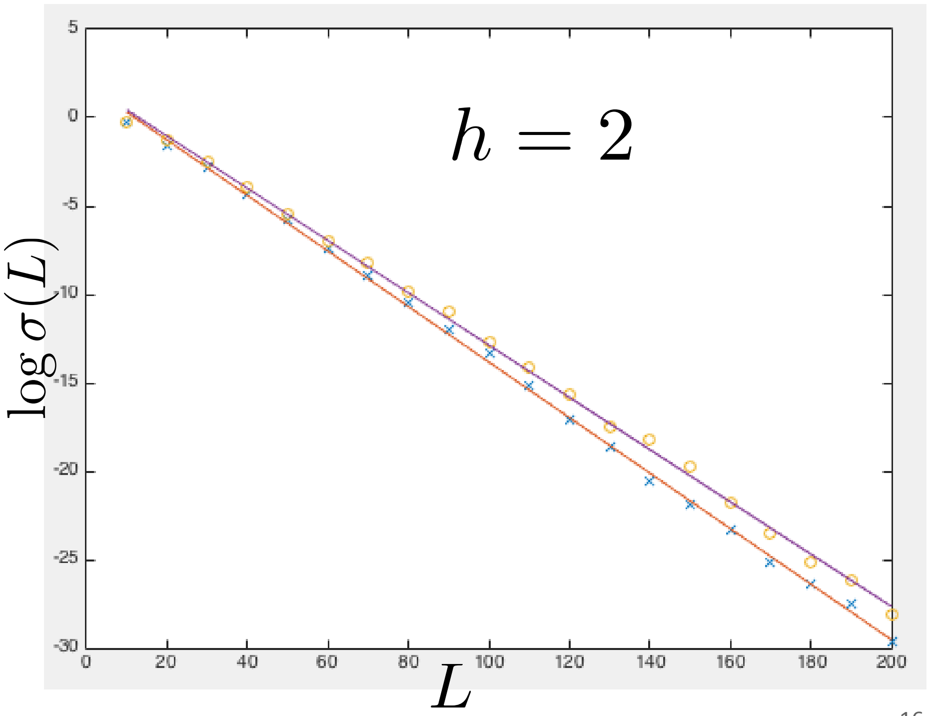
Results - spin conductivity (preliminary)

- Run RG till only two spins remain
- Calculate spin conductivity, using Larkin-Matveev 1987 (measure of residual spin coupling)

$$\sigma = \frac{4e^2}{\pi\hbar} \frac{\Gamma_l \Gamma_r |t|^2}{|(\epsilon - \epsilon_l + i\Gamma_l)(\epsilon - \epsilon_r + i\Gamma_r) - |t|^2|^2} \quad \Gamma_l, \Gamma_r \to 0$$

XY(Aderson) vs Heisnberg





RSRG fails on the conducting side (small h). Alternatives?

- Problem with RG: At T = ∞ and small h, individual spins are busy interacting with each other and don't "see" h. → Cannot apply RSRG

- What if we replace them with classical spins?

Inspiration: Quantum vs Classical RG at T = ∞

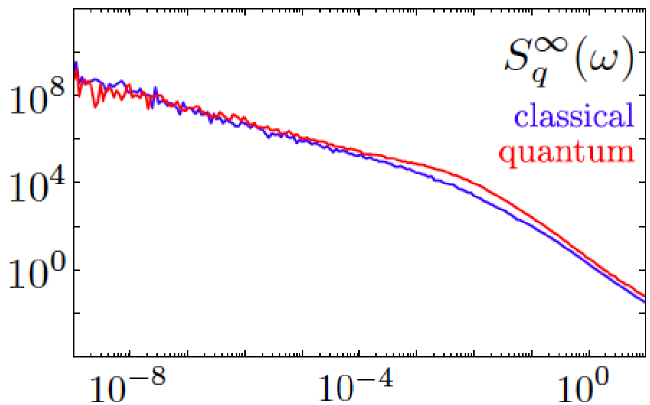
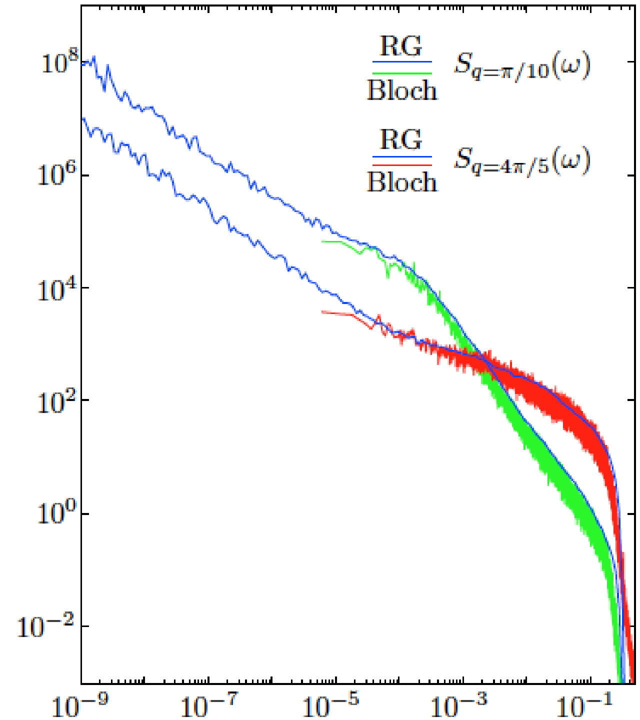
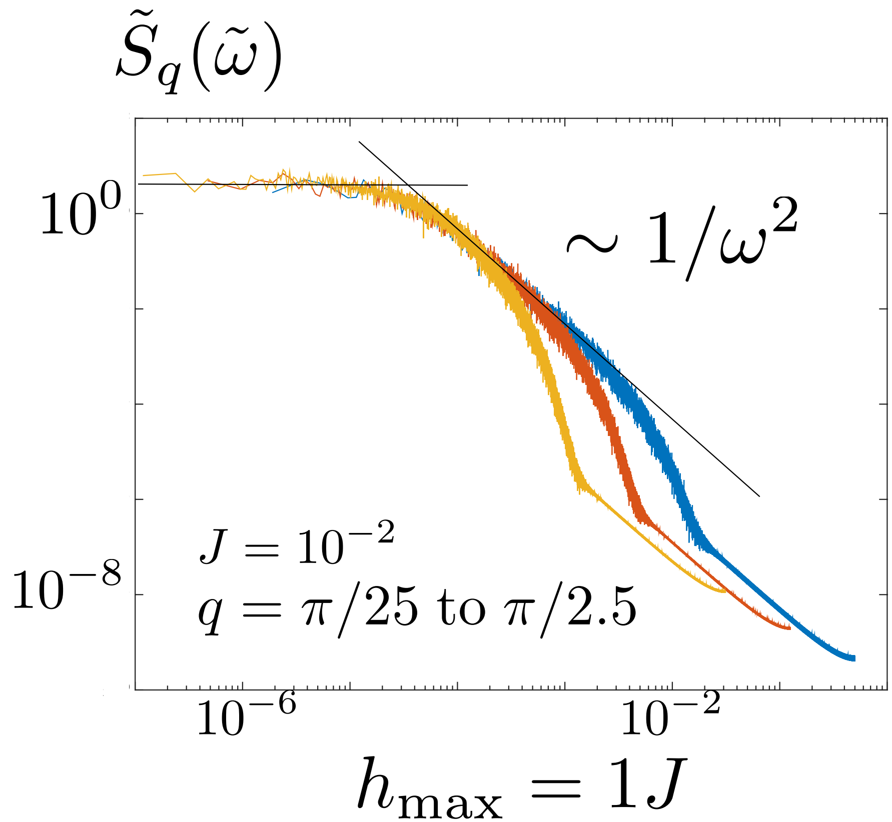
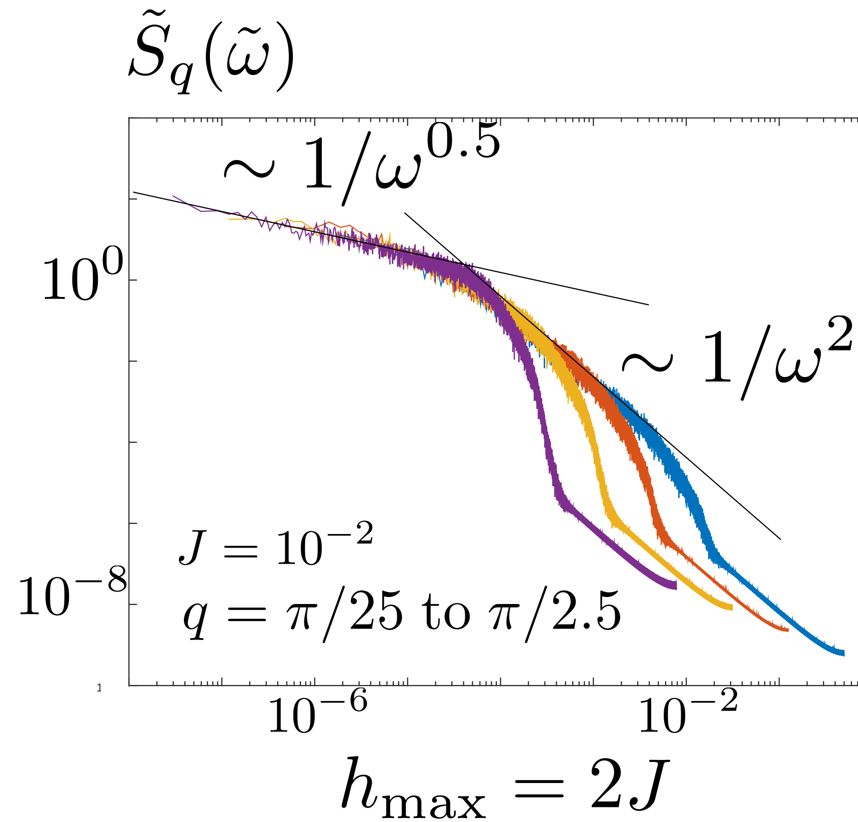


FIG. 13. Infinite-temperature $S_q^\infty(\omega)$ for classical spins of magnitude $|\vec{S}| = 1/2$ is plotted (blue) and compared with the quantum result (red) at infinite temperature. $L = 15000$, $D = 3$, $z_i = 0$.



Classical spin chain: diffusion to anomalous diffusion to spin-glass?

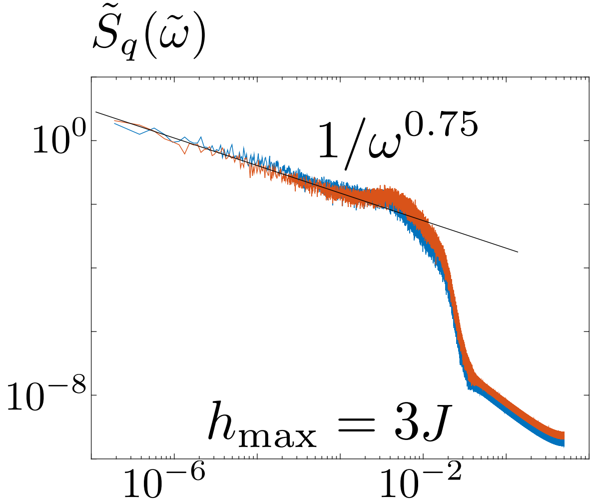


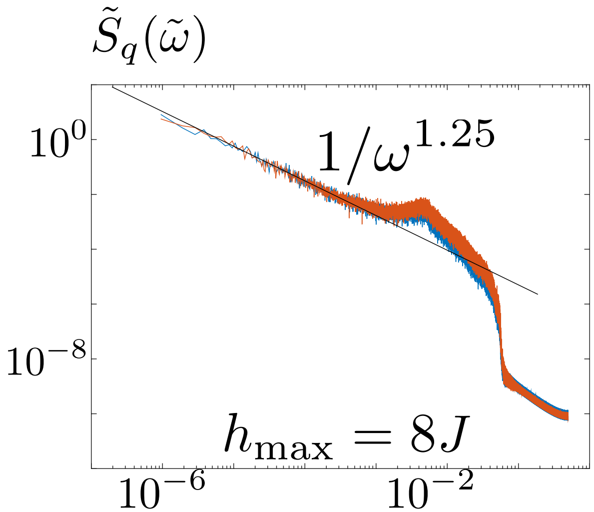


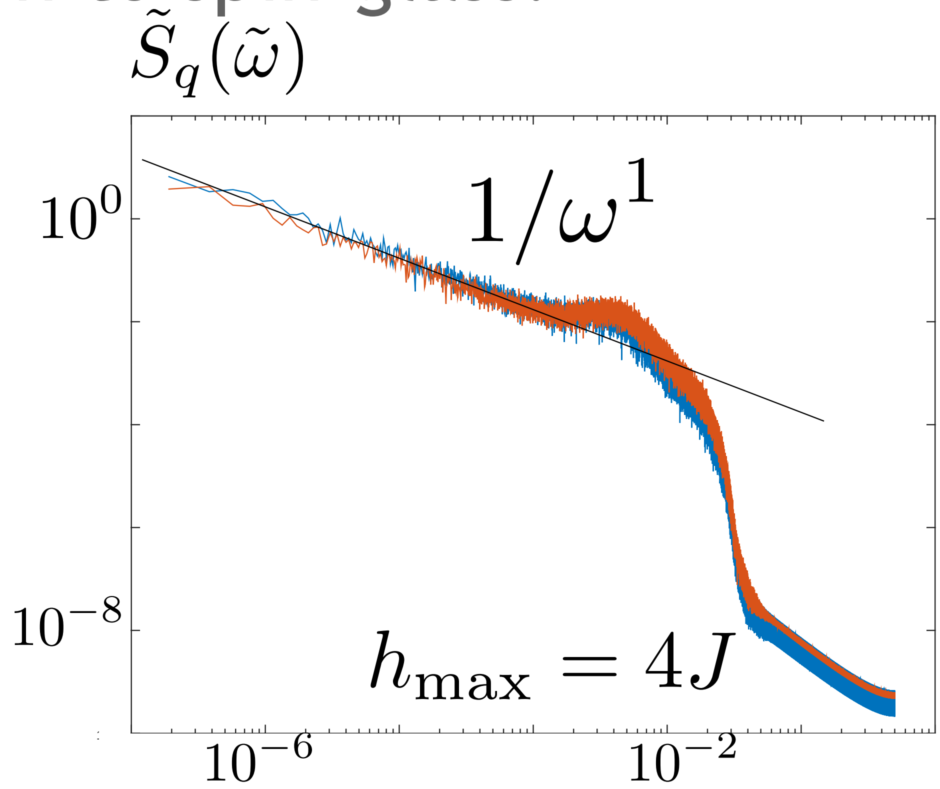
Attempted scaling collapse at hmax = 1
At large frequencies, large q, microscopic scales destroy the collapse a little.

Scaling collapse according to the anomalous diffusion form. The power-laws agree well

Classical spin chain: diffusion to anomalous diffusion to spin-glass?



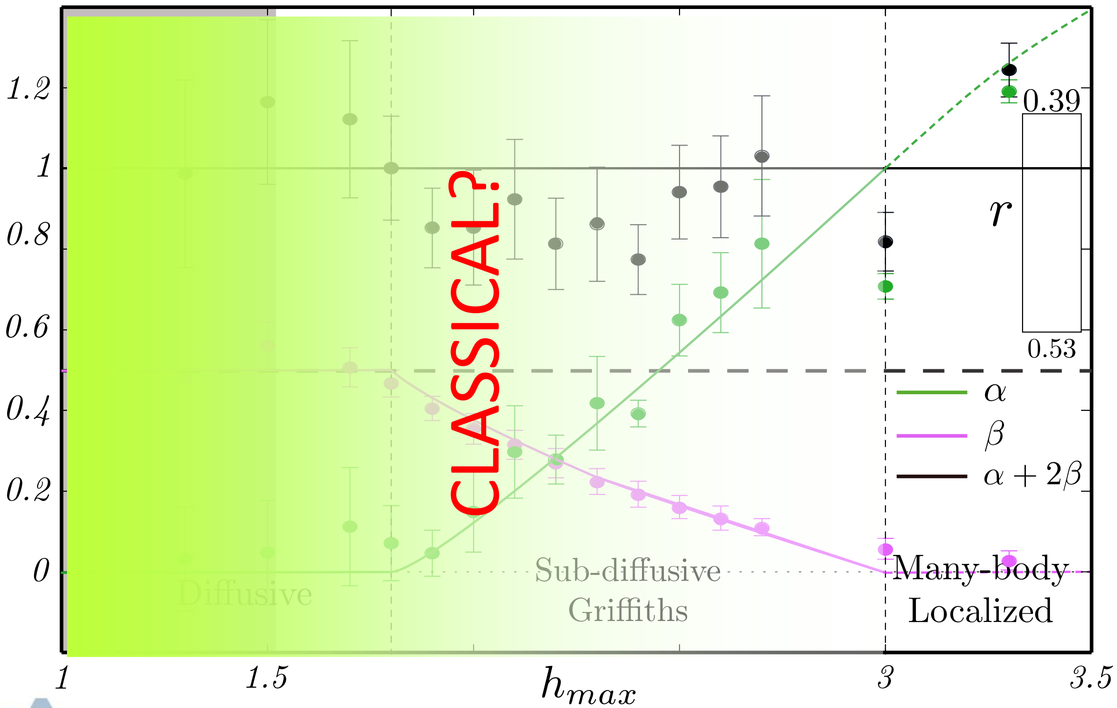




High q ($\pi/2.5, \pi/1.75$) were used to see low frequency power-law. It goes as 1/f very close to the agreed MBL transition point for quantum spin-1/2s. $J = 10^{-2}$ was used.

XXZ model with Random Magnetic Fields

Diffusive?	Sub-diffusive Griffiths	Many-Body Localized	h_{max}
DC transport Memory loss	No DC transport Memory loss	No DC transport Memory Retention	



K. Agarwal et al.
Phys. Rev. Lett. **114**, 160401

Summary

- RG to Random Heisenberg dynamics
  - 1/f noise spectra
    - From Numerics
    - From Generalized diffusion ansatz (T = inf)
    - From master equation
  - Violation of linear response at T = 0
  - Quasi 1D systems, connection to flux noise in qubits
  - Connection to MBL
- Heisenberg with random fields
  - MBL side – RSRG, quantum spins
  - (Anomalous) Diffusion – Equations of motion, classical spins
- *MBL transition = Quantum to Classical transition?*