MBL BEYOND EXACT EIGENSTATES AND CLOSED SYSTEMS

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November 2015

Motivation

- Much MBL theory formulated in terms of **exact eigenstates** (failure of ETH, entanglement properties etc)
- Almost all MBL theory formulated in terms of **closed systems**
- These are useful idealizations, but…
- To make contact with experiment, we must do better.

Outline of talk

- Beyond eigenstates: spectral approach to localization
  - Correlation functions of local operators
  - How to diagnose localization (and many body localization) in experiment
  - Weak v strong localization
- Beyond closed systems: MBL coupled to a bath
  - Weak coupling to a bath
  - Implications for experiment and theory
- Delocalize the system – or localize the bath?

Spectral functions

- Lets look at *correlation functions of local operators*

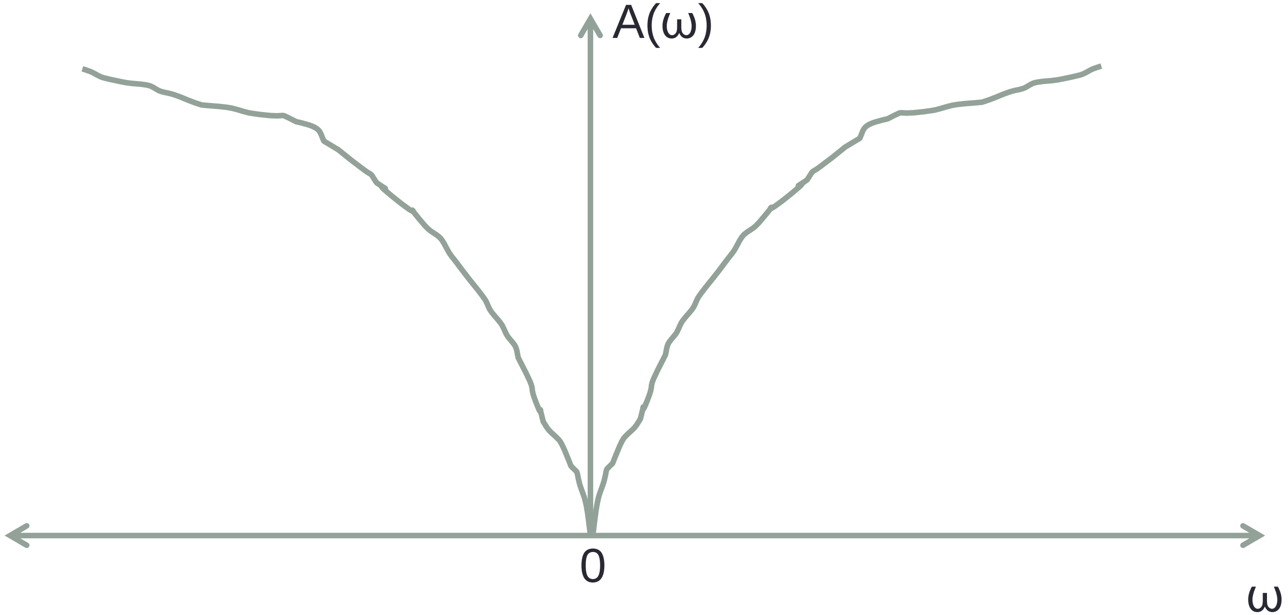
$$H = \sum_{i=1}^{L} [h_i \hat{S}_i^z + J\hat{\vec{S}}_i \cdot \hat{\vec{S}}_{i+1}],$$

$$\begin{aligned} A_j(\omega) \; &\equiv \; \mathrm{Im} \int_0^\infty dt e^{i\tilde{\omega}t} \mathrm{Tr}\, \left[\rho S_j^- e^{-iHt} S_j^+ e^{iHt}\right] \\ &+ \; \mathrm{Im} \int_0^\infty dt e^{i\tilde{\omega}t} \mathrm{Tr}\, \left[\rho S_j^+ e^{-iHt} S_j^- e^{iHt}\right], \end{aligned}$$

Imagine system prepared in arbitrary state e.g. Gibbs state
RN, S. Gopalakrishnan and D.A. Huse, PRB (2014)

How to diagnose localization

- Spatially averaged spectral functions
- Localized states look gapped at all temperatures

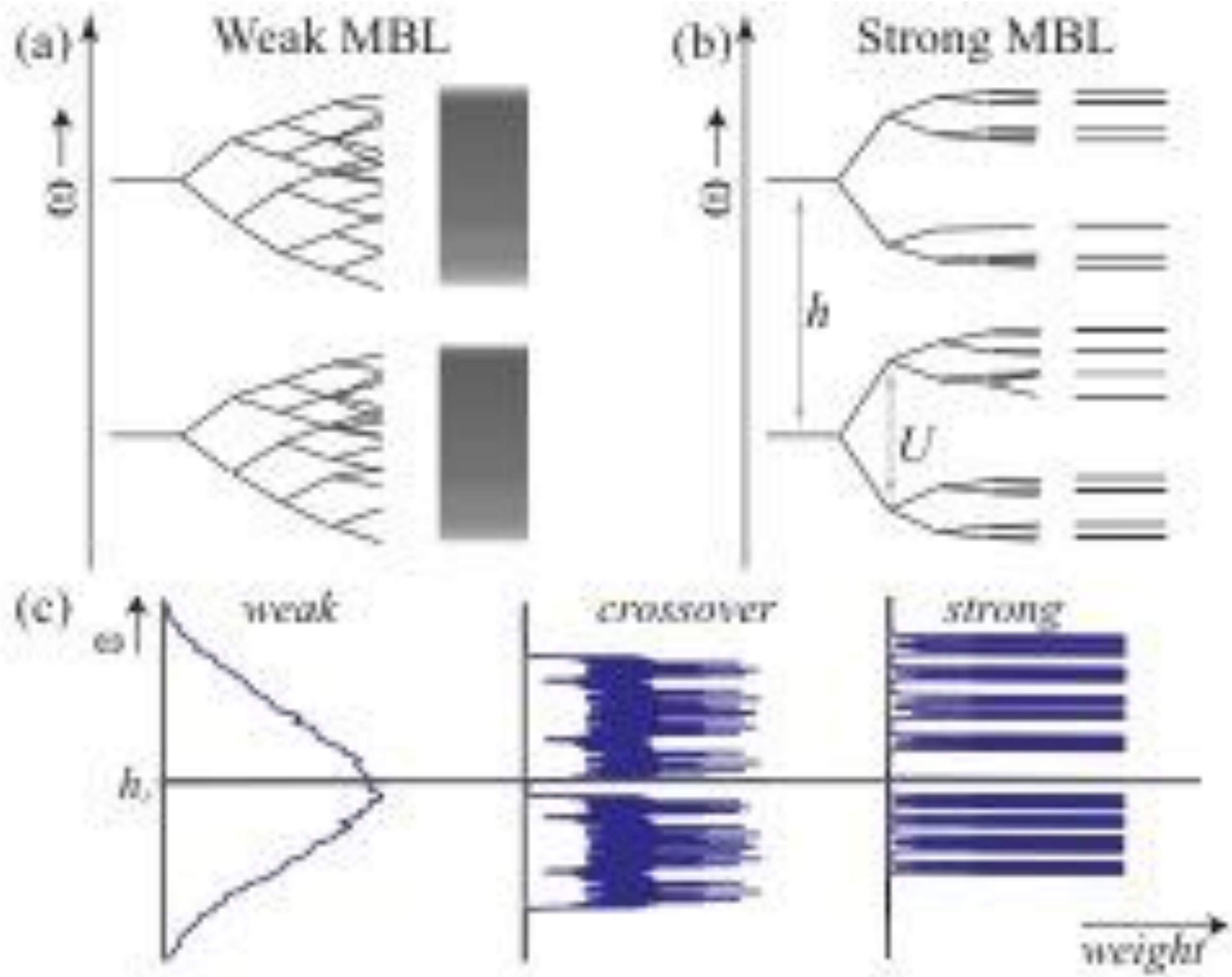


Single site spectral functions

- No spatial average (e.g. spectroscopy with good resolution, STM).
- Thermal phase: continuous bands
- Localized phase: discrete

$$H = E_0 + \sum_i \tau_i^z + \sum_{ij} J_{ij}\tau_i^z\tau_j^z + \sum_{n=1}^{\infty}\sum_{i,j,\{k\}} K_{i\{k\}j}^{(n)}\tau_i^z\tau_{k_1}^z\ldots\tau_{k_n}^z\tau_j^z \ ,$$

- Non-interacting localized phase: finite set of delta functions
- MBL: infinite set of delta functions (one from every eigenstate), with thermal weight


(a)
Weak MBL
ω
(b)
Strong MBL
ω
h
U
(c)
weak
crossover
strong
ω
weight

How to diagnose MBL

- Look for hierarchy of gaps
- See whether the number of gaps depends on the temperature (MBL only)

Beyond isolated systems

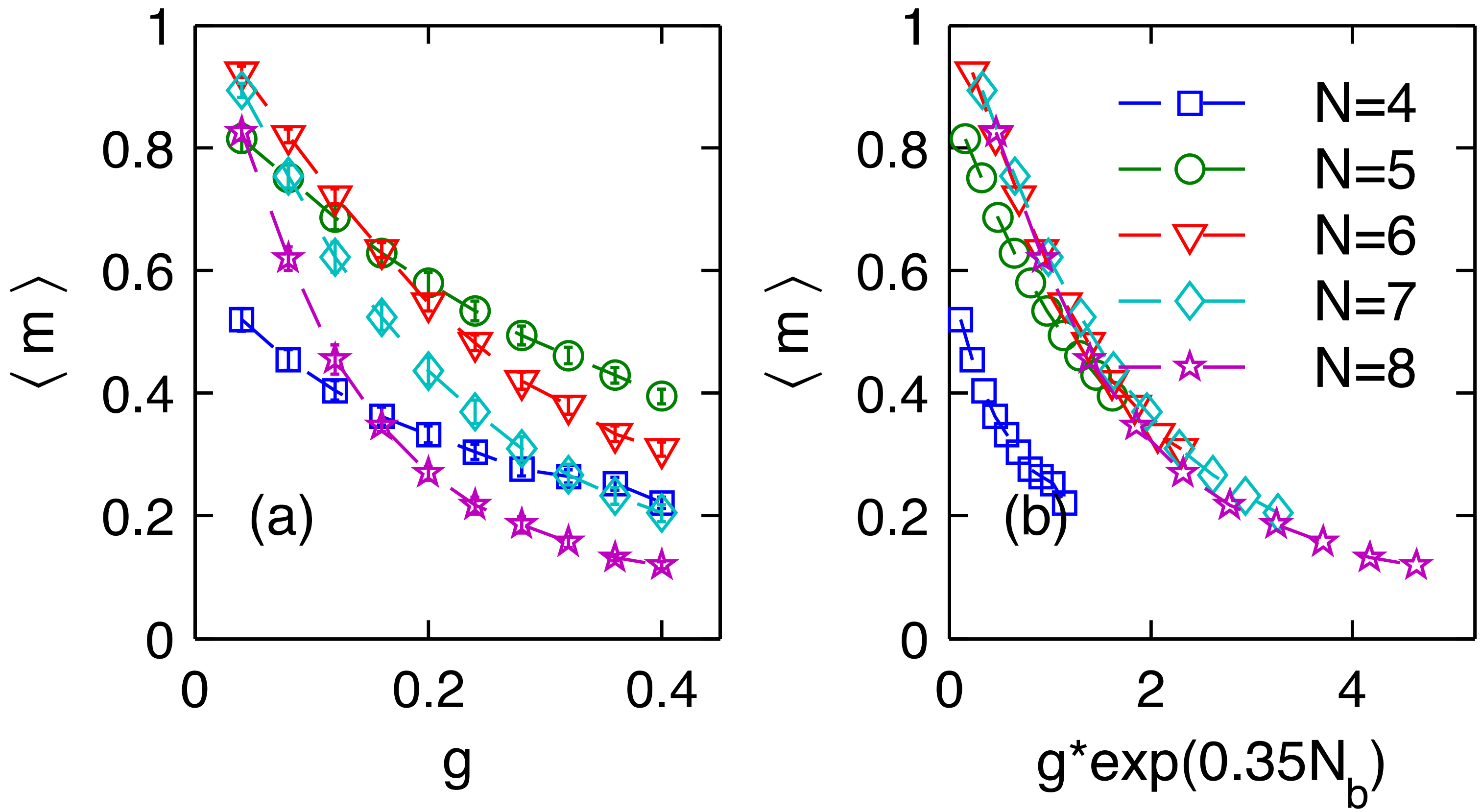
$$H_0 = \Sigma_{i=1}^{N-1} 2J\vec{\sigma}_i.\vec{\sigma}_{i+1} + \Sigma_{i=1}^{N} h_i \sigma_i^z$$

$$H_{bath} = \Sigma_{i=1}^{N_b-1} 2J_b S_i^z S_{i+1}^z + \Sigma_{i=1}^{N_b} h_b S_i^z + \Sigma_{i=1}^{N_b} g_b S_i^x$$

$$H_{int} = g\Sigma_{i=1}^{N-1} \sigma_i^+ \sigma_{i+1}^- S_{i+(N_b-N)/2}^x + h.c.$$

RN, S. Gopalakrishnan and D.A. Huse, PRB (2014)
S. Johri, RN and R.N. Bhatt, PRL (2015)

Eigenstate properties

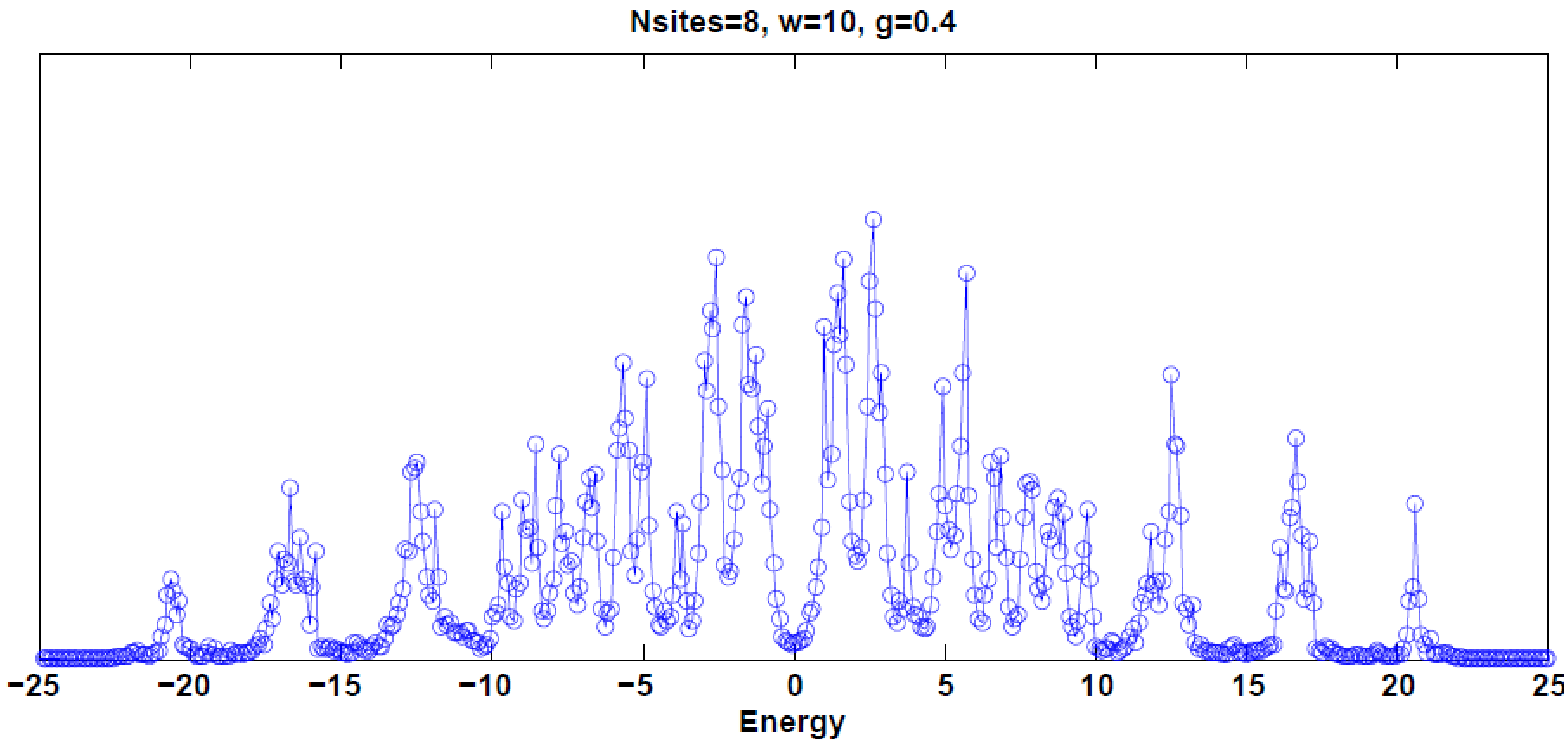


Spectral signatures are robust

- Look at spectral functions coarse grained on the scale of the many body level spacing in the bath
- Isolated system: delta functions
- Coupled to a bath: Lorentzians
- As long as Lorentzian width << size of largest gaps, system still looks MBL in some meaningful sense.

Clustering in Spectral Functions

(p-bits)



Line broadening as a function of g?

- Non-interacting: $\Gamma_1(g) \sim g^2/t.$
- N strongly interacting spins: $\Gamma_m(g) \sim Ns(T)g^2/t,$
- Large MBL system: $\Gamma_m(g) \sim \frac{g^2}{t} s(T)\xi^d \ln^d\left(\frac{tU_0}{g^2 s(T)\xi^d}\right)$
- Can diagnose MBL through g dependence (and T dependence) of linewidth.
- Can also diagnose MBL through parametric separation of t_1 and t_2 times.

Summary of spectral approach

- Diagnose MBL through correlation functions of local operators
- Experimentally measurable
- Robust to coupling to a bath
- Look for: discrete spectral functions, gapped at all temperatures, hierarchy of gaps (which depends on temperature), g dependence of linewidth, parametric separation of t_1 and t_2 times.

Applications of spectral language

- **MBL transition:** Near localization/delocalization transition, on the metallic side, the `internal heat bath' starts to die.
- Spectral approach gives self consistent mean field theory of transition (**Gopalakrishnan and RN, PRB 2014**).
- **Can MBL stabilize quantized Hall effect at non-zero temperatures?**
- Topological obstruction to full localization. Do critical states act as heat bath and destroy localization? **RN and Potter, PRB 2014**
- **MBL in 2D continuous space?** Relevant for cold atoms.
- **RN, PRB 2014.**

Outlook

- Thus far, assumed coupling to bath destroys MBL
- Alternative possibilities exist
- Possibility 1: **`Zeno localization'** – Huse, **RN,** Pietracaprina, Ros, Scardicchio, PRB 92, 014203 (2015)
- Possibility 2: coupling to MBL system introduces disorder into bath, which *localizes* the bath.
- **MBL proximity effect – RN, arXiv: 1506.05468**
- **This can be shown to happen in certain special limits (bath near atomic limit, a particular strong coupling limit)**

  See also Bela Bauer talk, W. deRoeck talk

What happens away from special limits?

- When does the bath delocalize the system, and when does the system localize the bath?
- A general theory of MBL `in open systems’ (i.e.coupled to baths)?

Thanks to collaborators…

- Sarang Gopalakrishnan (Harvard/Caltech)
- David Huse (Princeton)
- Sonika Johri (Princeton)
- Ravin Bhatt (Princeton)
- Andrew Potter (Berkeley)
- Antonello Scardicchio (ICTP Trieste)
- Valentina Ros (ICTP Trieste)
- F. Pietracaprina (ICTP Trieste)

The model

$$H_c = \sum_{\langle ij \rangle} t_c c_i^\dagger c_j + U c_i^\dagger c_i c_j^\dagger c_j + \sum_i \varepsilon_i c_i^\dagger c_i$$

$$H_d = \sum_{\langle ij \rangle} t_d d_i^\dagger d_j + \lambda d_i^\dagger d_i d_j^\dagger d_j$$

$$H_{int} = \sum_i g_i c_i^\dagger c_i d_i^\dagger d_i$$

Localization in coupled c-d system

- Strategy: start from a limit in which the coupled system is manifestly localized
- Perturb about that limit and argue that locator expansion converges
- This works in three regimes:
  - Bath close to an atomic limit (small t_d), weak coupling
  - Bath close to non-interacting limit and system strongly localized, weak coupling
  - *Strong* coupling limit G → ∞

- The moral: MBL *can* survive coupling to a bath if localization is strong and bath is weak – by destroying bath.