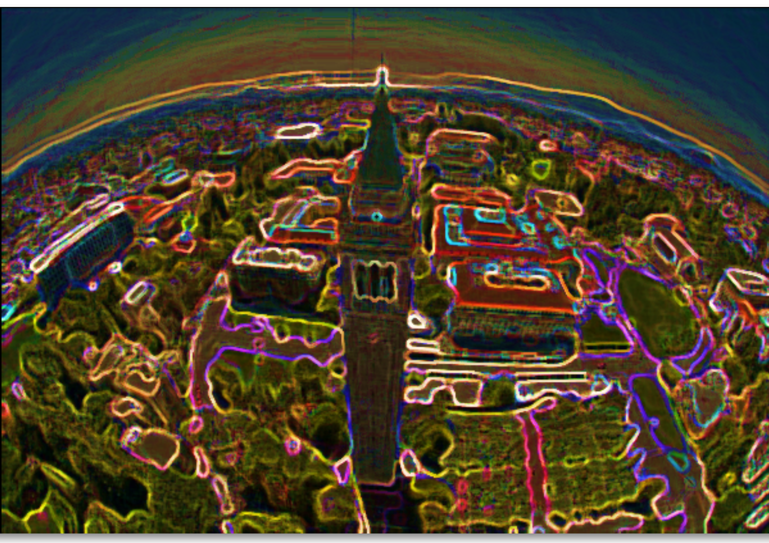
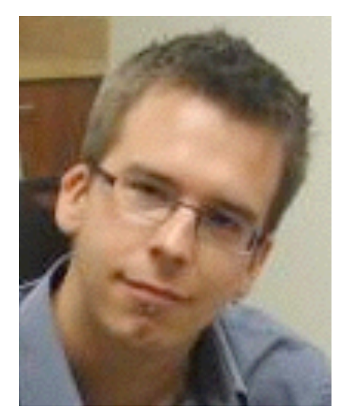


Universal scaling properties of many-body (de)-localization transitions

Andrew C. Potter
UC Berkeley

ACP, R. Vasseur & S.A. Parameswaran, PRX ‘15



Romain Vasseur
(Berkeley)

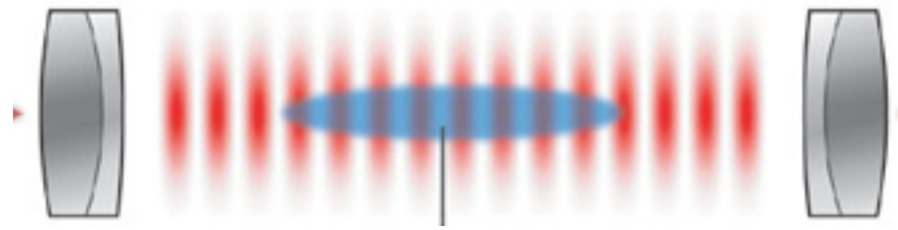
Sid Parameswaran
(Irvine)

Berkeley UNIVERSITY OF CALIFORNIA

GORDON AND BETTY MOORE FOUNDATION

Does stat. mech emerge in isolated quantum systems?

~~cold~~ hot atomic gas

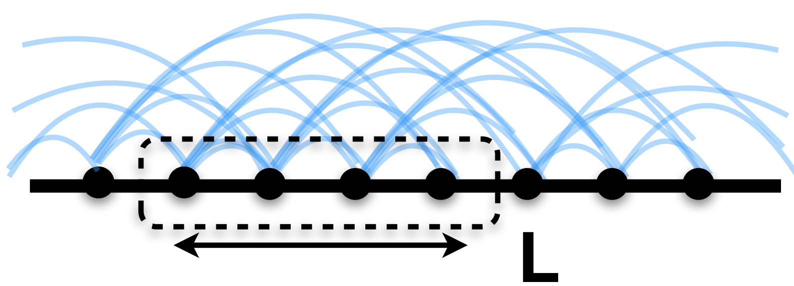


$T \sim 10^{-7}$ K

but

$T/T_F \sim 0.5$

Eigenstate thermalization

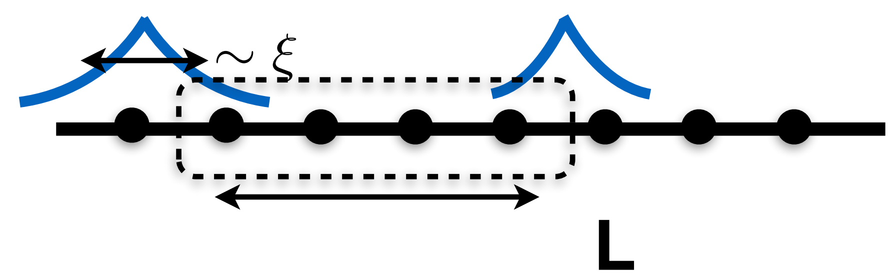


- Extensive entanglement:

  $S(L) \sim T\, L^d$

- Rapid ($\sim e^{-t/\tau}$) dephasing & decay of non-thermal correlations
- Rapid spreading of entanglement and energy

Many-body localization



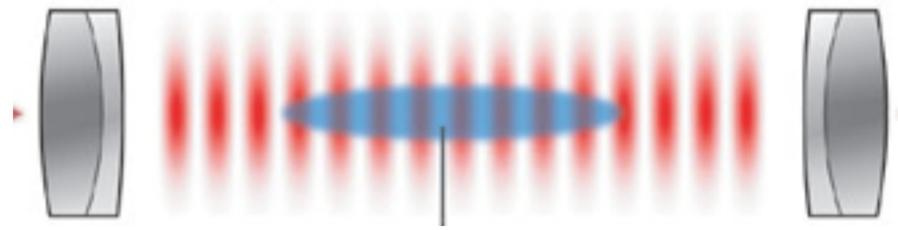
- Boundary entanglement:

  $S(L) \sim \xi\, L^{d-1}$

- Quantum coherence in “hot” matter
- No transport — initial conditions persist indefinitely

Does stat. mech emerge in isolated quantum systems?

~~cold~~ hot atomic gas



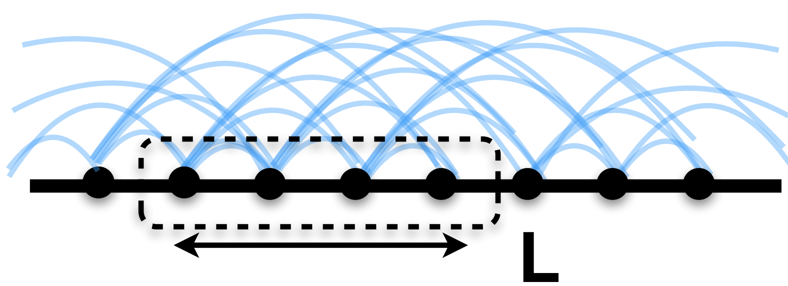
$T \sim 10^{-7}$ K

but

$T/T_F \sim 0.5$

Yes

Eigenstate thermalization

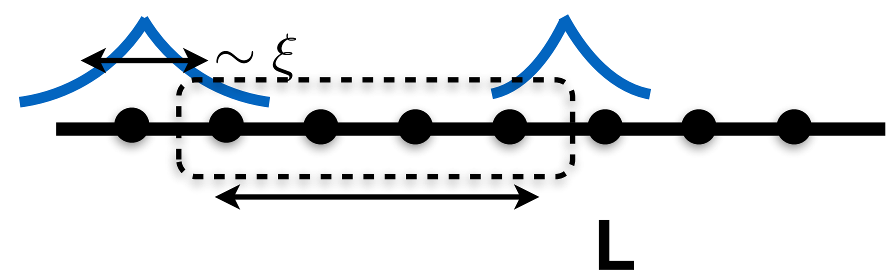


- Extensive entanglement:

  $S(L) \sim T\, L^d$

- Rapid ($\sim e^{-t/\tau}$) dephasing & decay of non-thermal correlations
- Rapid spreading of entanglement and energy

Many-body localization



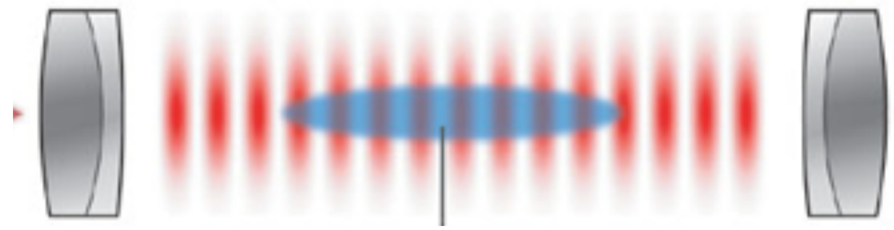
- Boundary entanglement:

  $S(L) \sim \xi\, L^{d-1}$

- Quantum coherence in "hot" matter
- No transport — initial conditions persist indefinitely

Does stat. mech emerge in isolated quantum systems?

~~cold~~ hot atomic gas



$T \sim 10^{-7}$ K

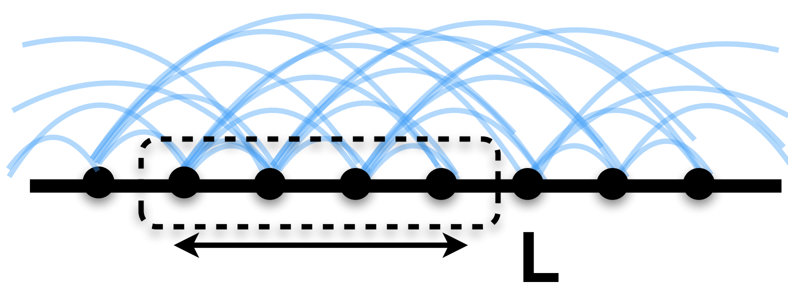
but

$T/T_F \sim 0.5$

Yes

No

Eigenstate thermalization

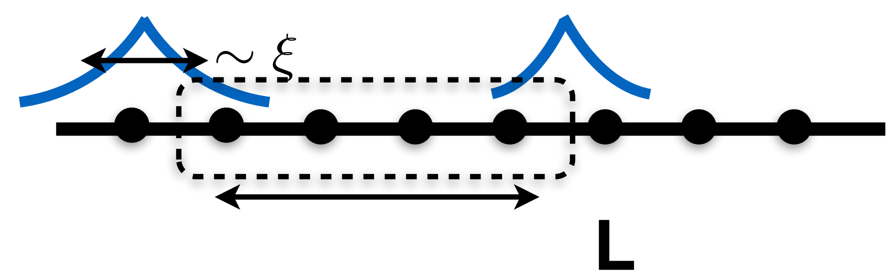


- Extensive entanglement:

  $S(L) \sim T\, L^d$

- Rapid ($\sim e^{-t/\tau}$) dephasing & decay of non-thermal correlations
- Rapid spreading of entanglement and energy

Many-body localization



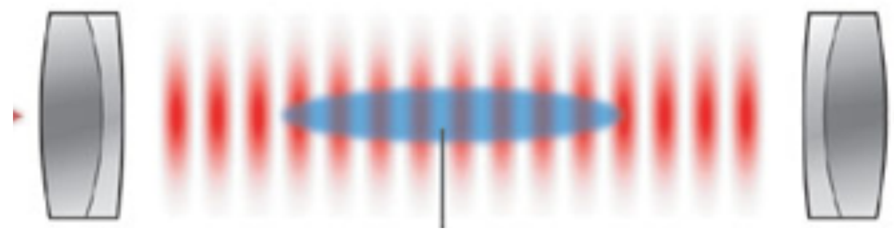
- Boundary entanglement:

  $S(L) \sim \xi\, L^{d-1}$

- Quantum coherence in “hot” matter
- No transport — initial conditions persist indefinitely

Does stat. mech emerge in isolated quantum systems?

~~cold~~ hot atomic gas



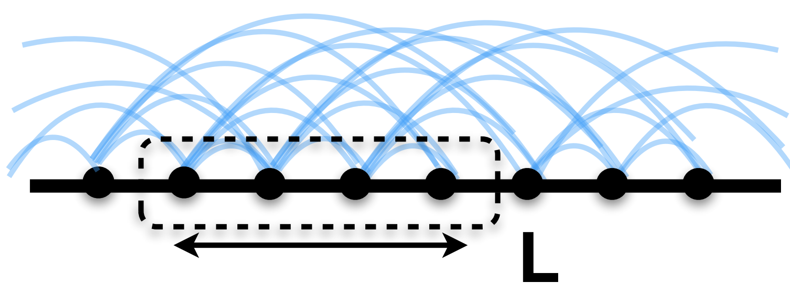
$T \sim 10^{-7}$ K

but

$T/T_F \sim 0.5$

Yes — No

Eigenstate thermalization

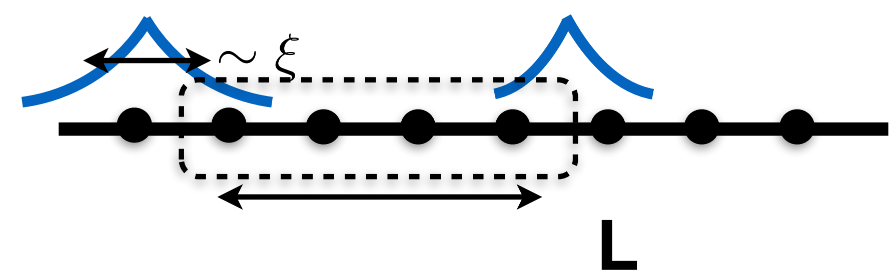


- Extensive entanglement:

  $S(L) \sim T\, L^d$

- Rapid ($\sim e^{-t/\tau}$) dephasing & decay of non-thermal correlations
- Rapid spreading of entanglement and energy

Many-body localization



- Boundary entanglement:

  $S(L) \sim \xi\, L^{d-1}$

- Quantum coherence in “hot” matter
- No transport — initial conditions persist indefinitely

Does stat. mech emerge in isolated quantum systems?

~~cold~~ hot atomic gas

T ~ 10^{-7} K
but
T/T_F ~ 0.5

Eig

- Ex

What is the nature of this transition?

- New kind of phase transition:
  - Neither classical/thermal nor quantum critical
  - Thermodynamics breaks down sharply at a critical point
- Universal scaling properties?
- How do thermal transport & dynamics slow down to stop at the critical point?

- Rapid (~e^…) dephasing & decay of non-thermal correlations
- Rapid spreading of entanglement and energy

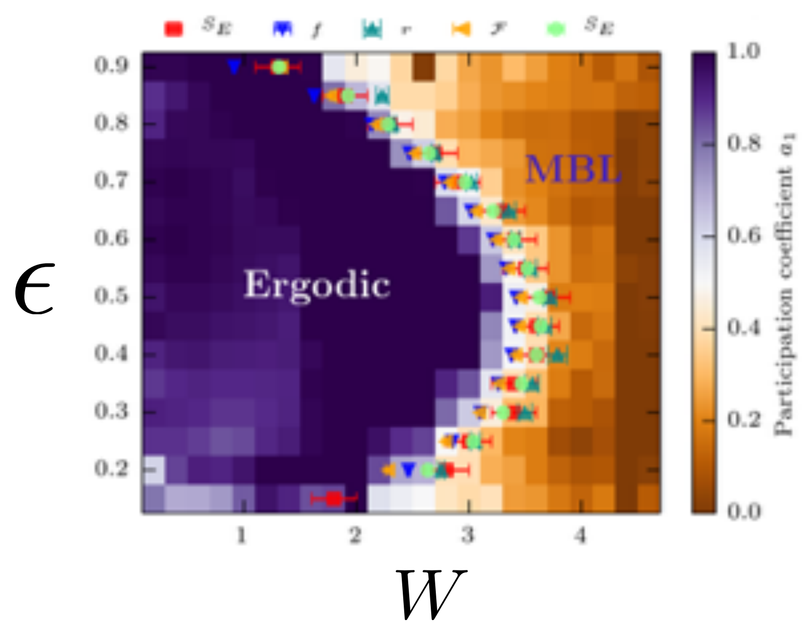
- Quantum coherence in "hot" matter
- No transport — initial conditions persist indefinitely

??

disorder

Exact Numerical Methods

ED (1D Interacting fermions + disorder)



Universal scaling: $S = L \times \mathcal{S}(L/\xi)$

Continuous (2nd order) phase transition

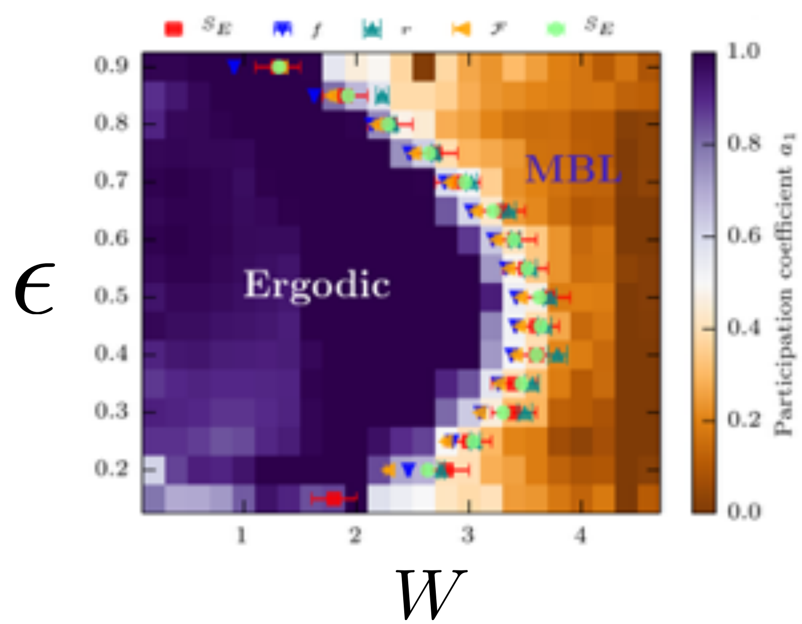
Diverging length scale:

$$\xi \sim \frac{1}{|W - W_c|^{\nu}} \qquad \nu_{\mathrm{ED}} \approx 0.8$$

Pal, Huse '10; Agarwal, et al '14; Kjall, Bardarson, Pollmann '14;
Luitz, Laflorencie, Alet '14 (Figure); Bar Lev, Cohen, Reichman '14, many others…

Exact Numerical Methods

ED (1D Interacting fermions + disorder)



Universal scaling: $S = L \times \mathcal{S}(L/\xi)$

Continuous (2nd order) phase transition

Diverging length scale:

$$\xi \sim \frac{1}{|W - W_c|^{\nu}} \qquad \nu_{\rm ED} \approx 0.8$$

But… Harris/Chayes Bound:

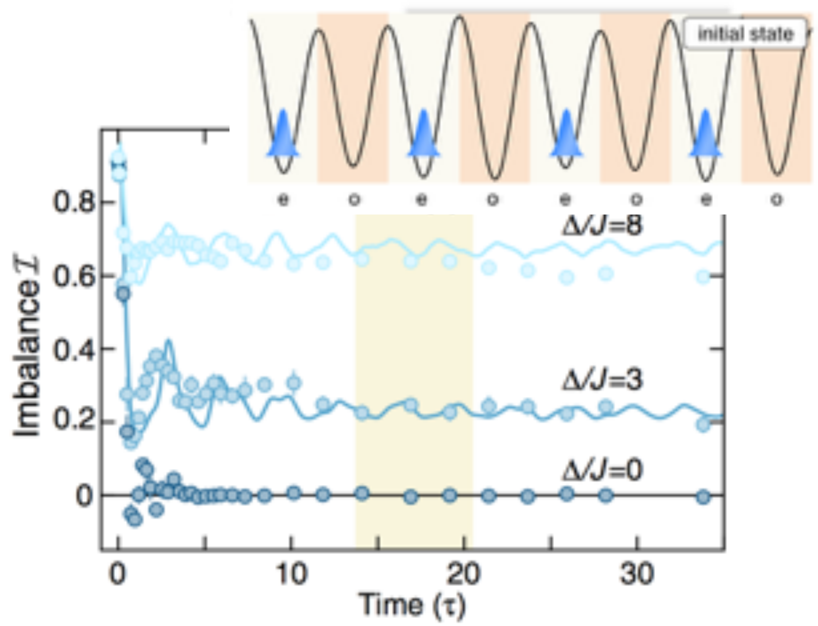
$$\nu \geq 2/d$$ (0.8 < 2 !!)

Harris (Perturbative); Chayes, Chayes, Fisher, Spencer (General)
see also: Chandran, Laumann, Oganesyan arXiv '15 (MBL, Monday talk)

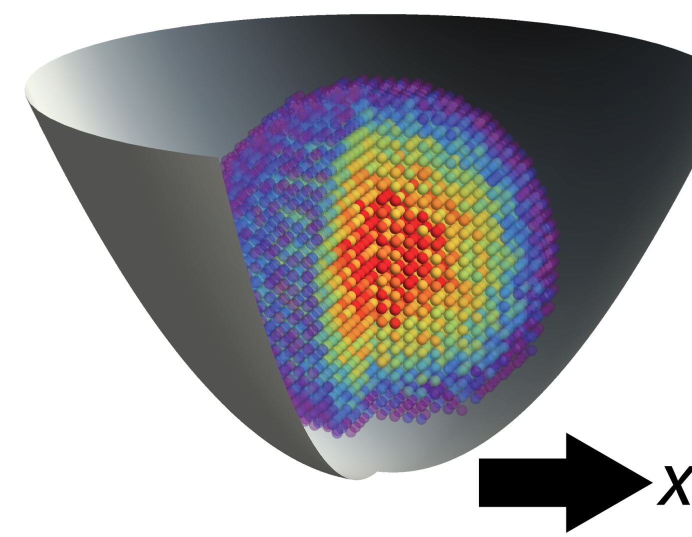
Pal, Huse '10; Agarwal, et al '14; Kjall, Bardarson, Pollmann '14;
Luitz, Laflorencie, Alet '14 (Figure); Bar Lev, Cohen, Reichman '14, many others…

Experiments

Cold atoms

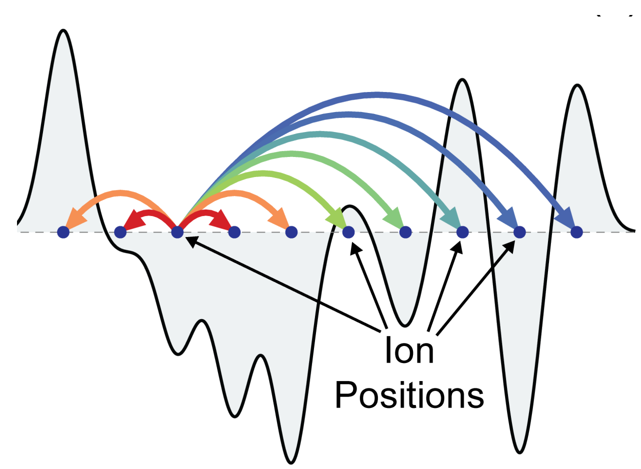


Schrieber et al. (I. Bloch Group) ’15
(First Talk of Today)



De’Marco Group (last talk)

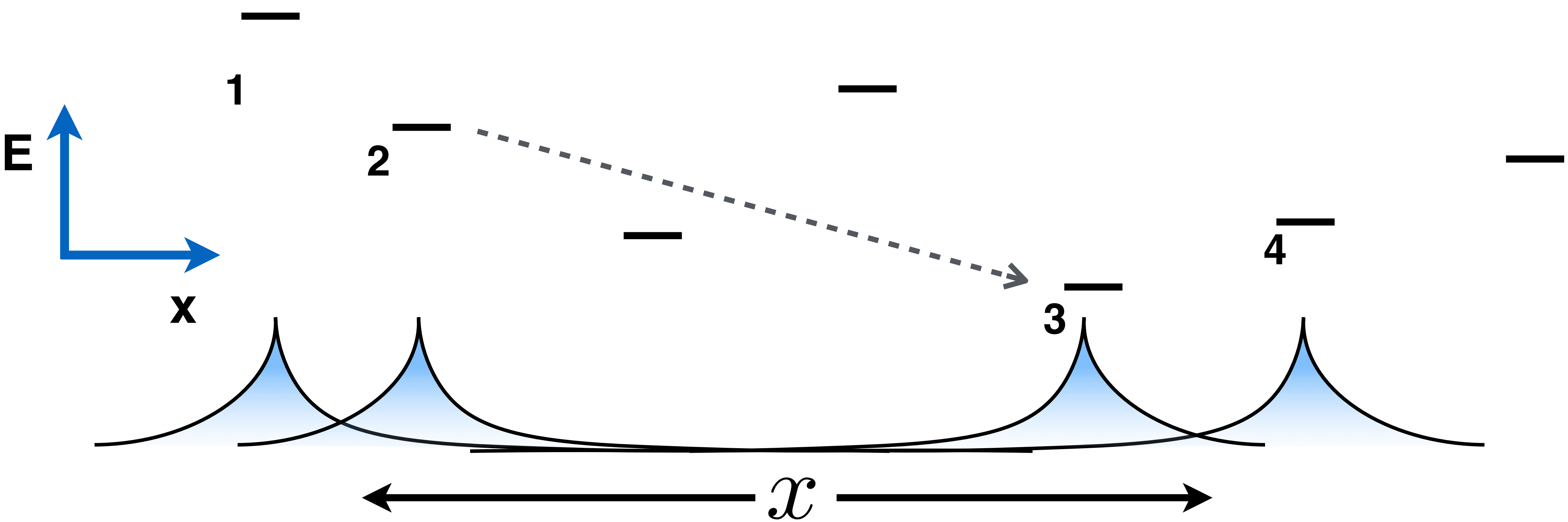
Trapped ions (Long-range interactions)



Smith et al. (C. Monroe Group) ’15 (Friday talk)

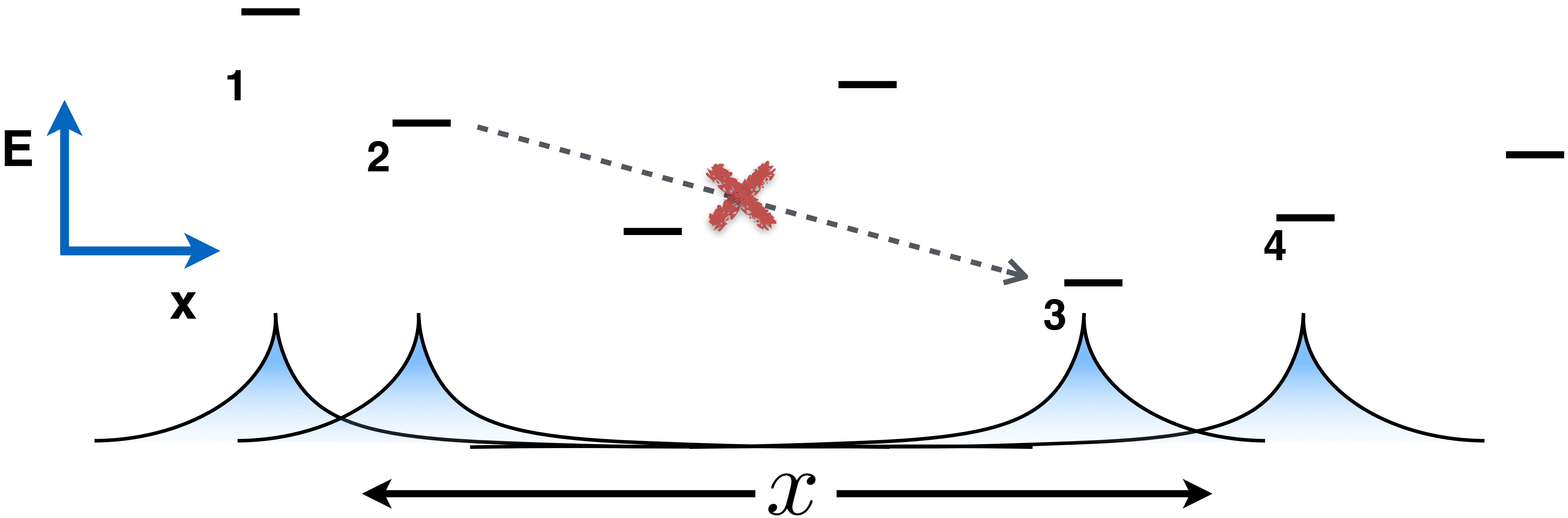
Resonances (two-particle)

$$H = \sum_{\alpha} \varepsilon_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$



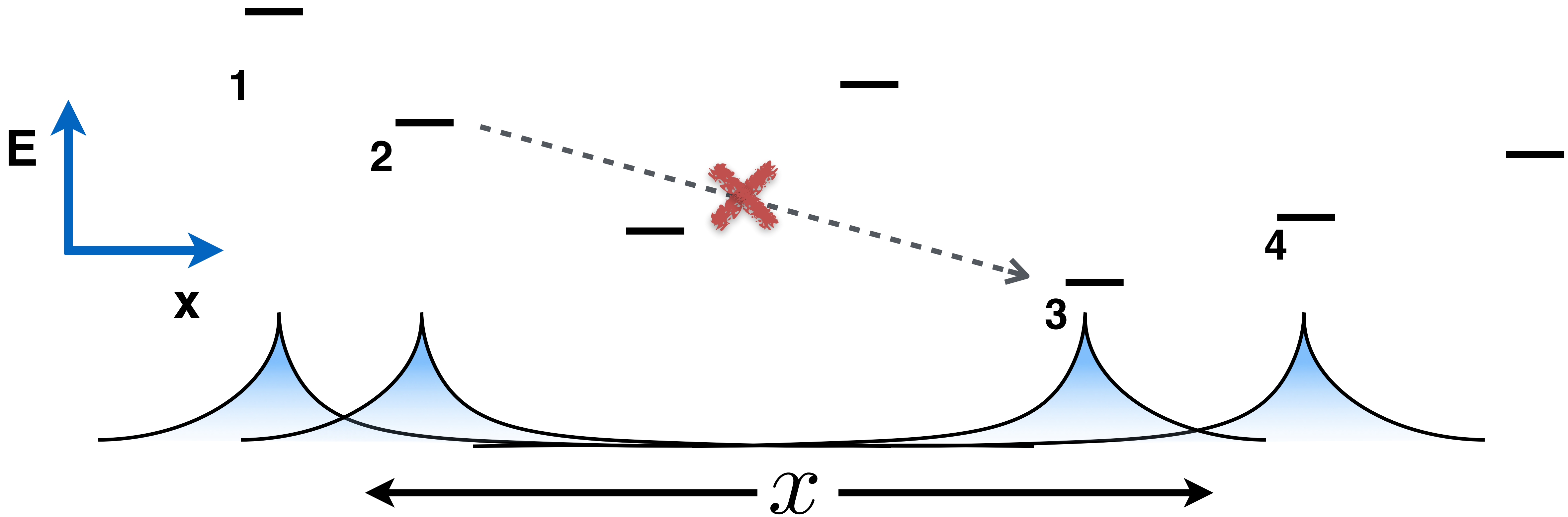
Resonances (two-particle)

$$H = \sum_{\alpha} \varepsilon_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$



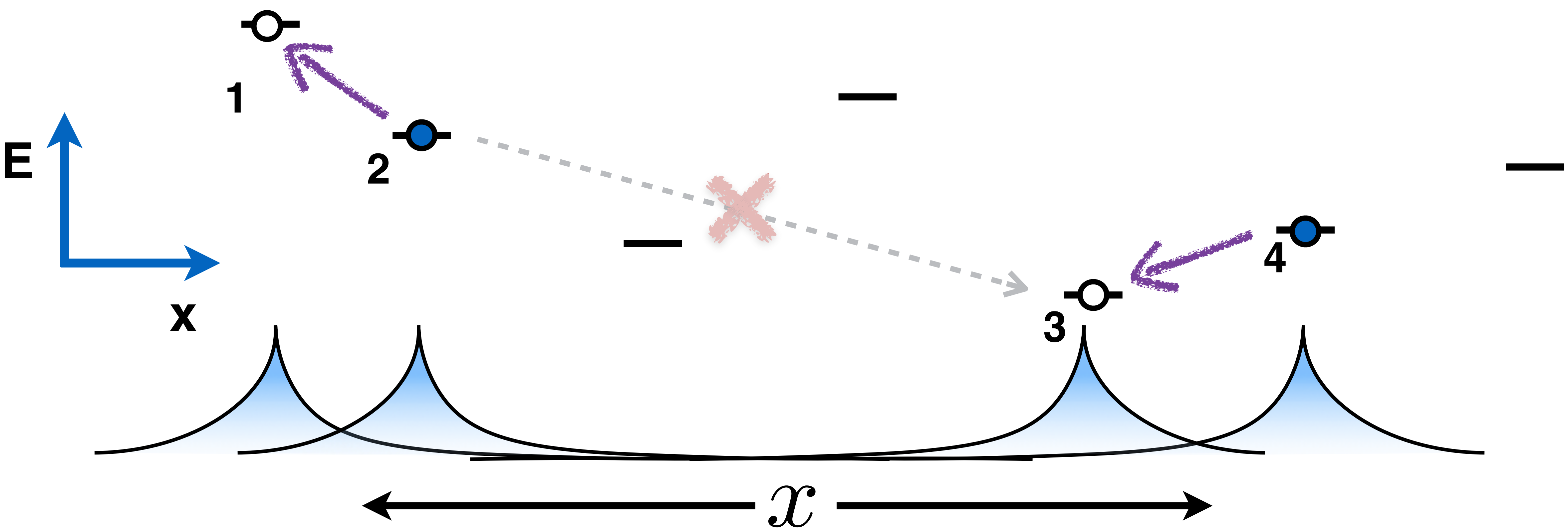
Resonances (two-particle)

$$H = \sum_{\alpha} \varepsilon_\alpha c^\dagger_\alpha c_\alpha + \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} c^\dagger_\alpha c^\dagger_\beta c_\gamma c_\delta$$



Resonances (two-particle)

$$H = \sum_{\alpha} \varepsilon_\alpha c_\alpha^\dagger c_\alpha + \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta$$

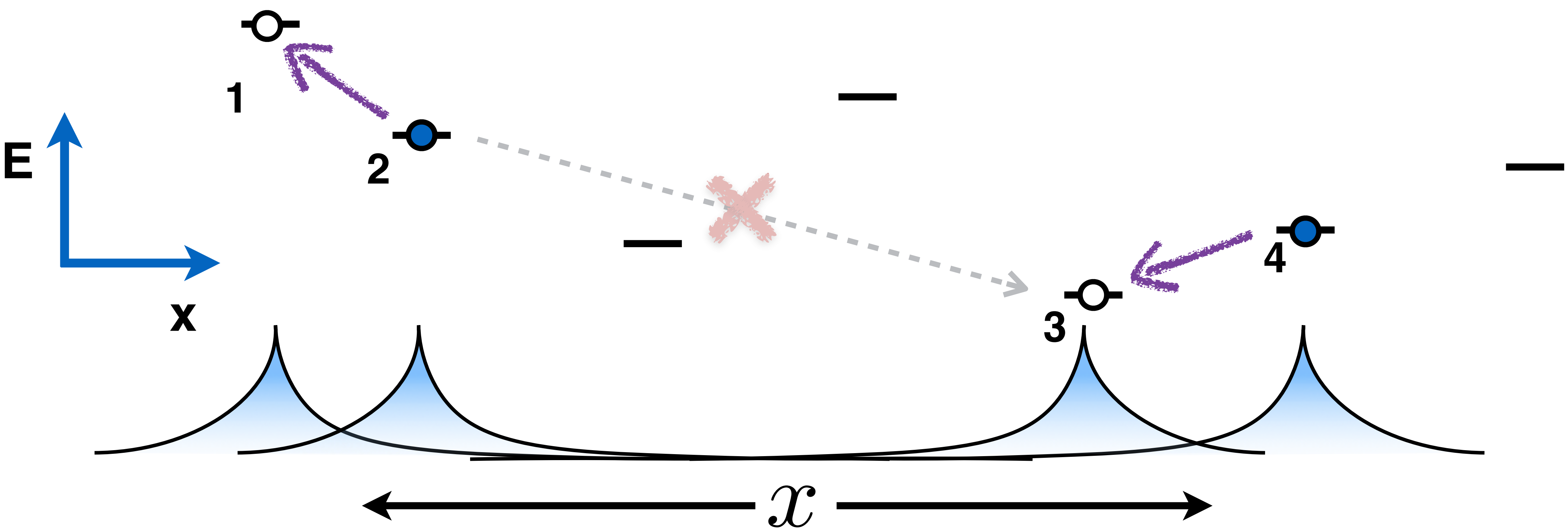


$$\Gamma(x) \sim V e^{-x/x_0}$$

$$\delta E = \delta E_{12} - \delta E_{34}$$

Resonances (two-particle)

$$H = \sum_{\alpha} \varepsilon_\alpha c_\alpha^\dagger c_\alpha + \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta$$



$$\Gamma(x) \sim V e^{-x/x_0}$$

$$\delta E = \delta E_{12} - \delta E_{34}$$

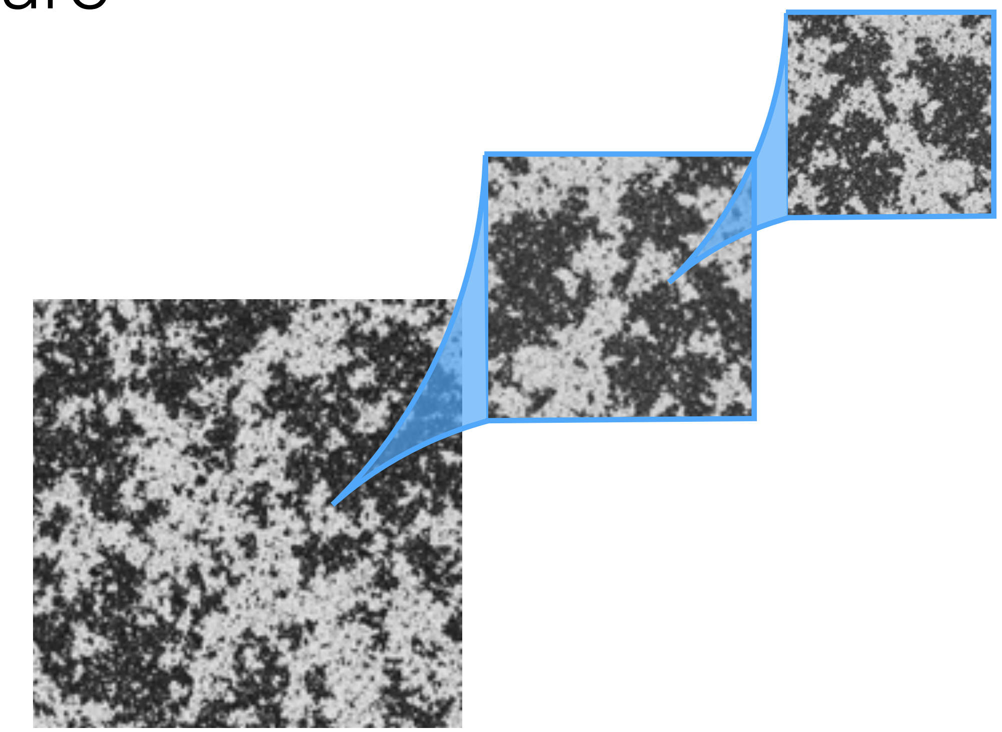
Delocalization happens through highly collective many-body resonance…

General Considerations

1. Identifying general N-body resonance is hard

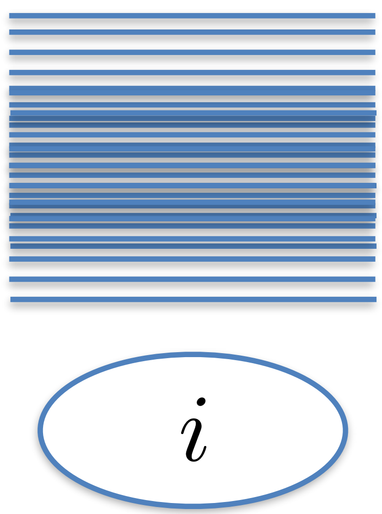
2. Critical point is thermal (entanglement monotonicity) Grover '14
   - Ignore quantum interference (strong dephasing)
   - Classical model should suffice

3. Expect (at criticality):
   - Self-similar (Fractal), Hierarchical structure
   - RG-like procedure:
     A. identify strongest resonances,
     B. form resonant clusters
     C. compute new inter-cluster couplings
     D. then see if they inter-resonate, etc…
     E. Keep only coarse grained information about clusters



Many-body resonances

Coarse grained information



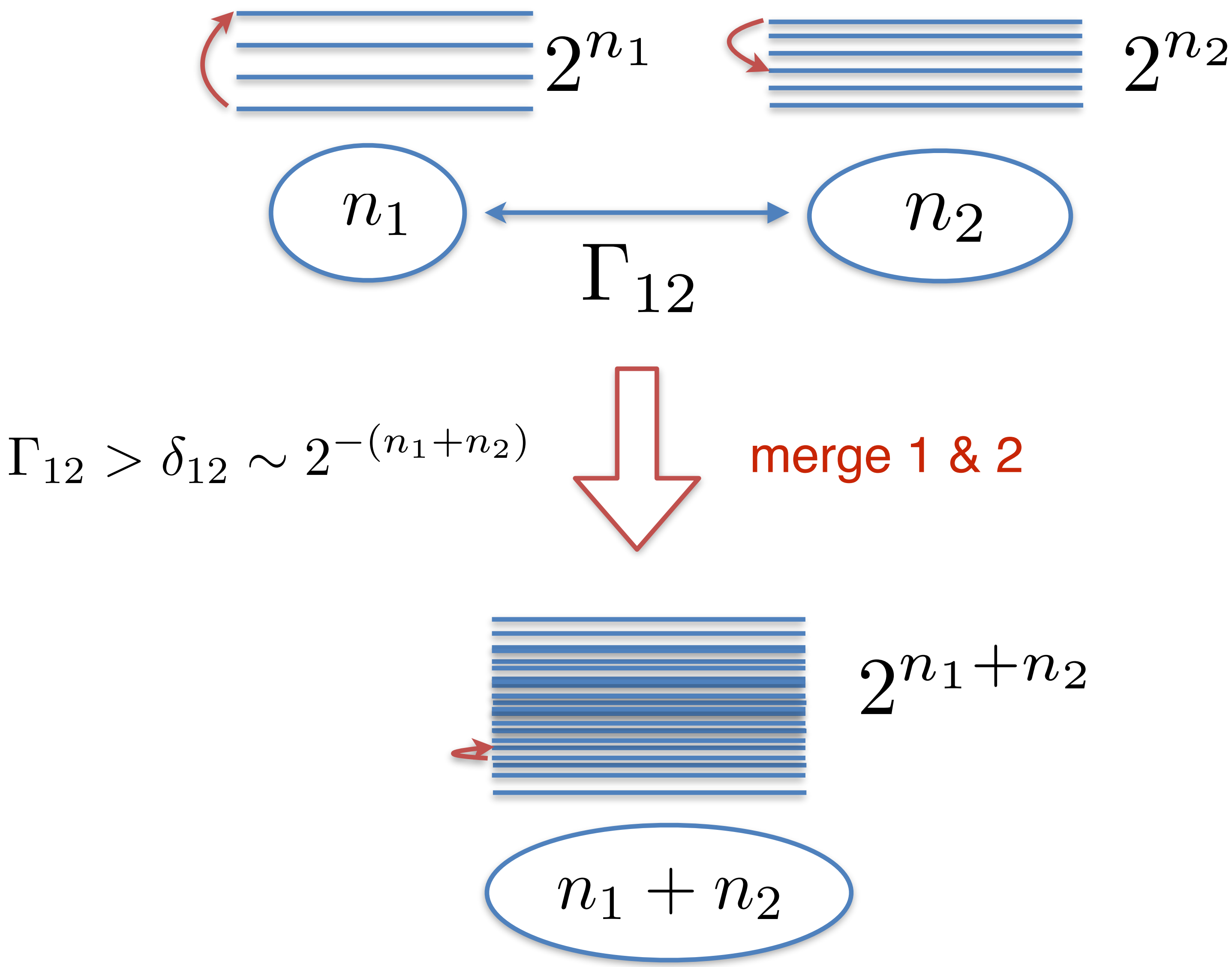
Number of DOF: $|\mathcal{H}_i| = 2^{n_i}$

Bandwidth: $\Lambda_i \approx \Gamma_{\mathrm{rms}}\sqrt{n}$

Level Spacing: $\delta_i \approx \dfrac{\Lambda_i}{|\mathcal{H}|_i} \sim 2^{-n_i}$

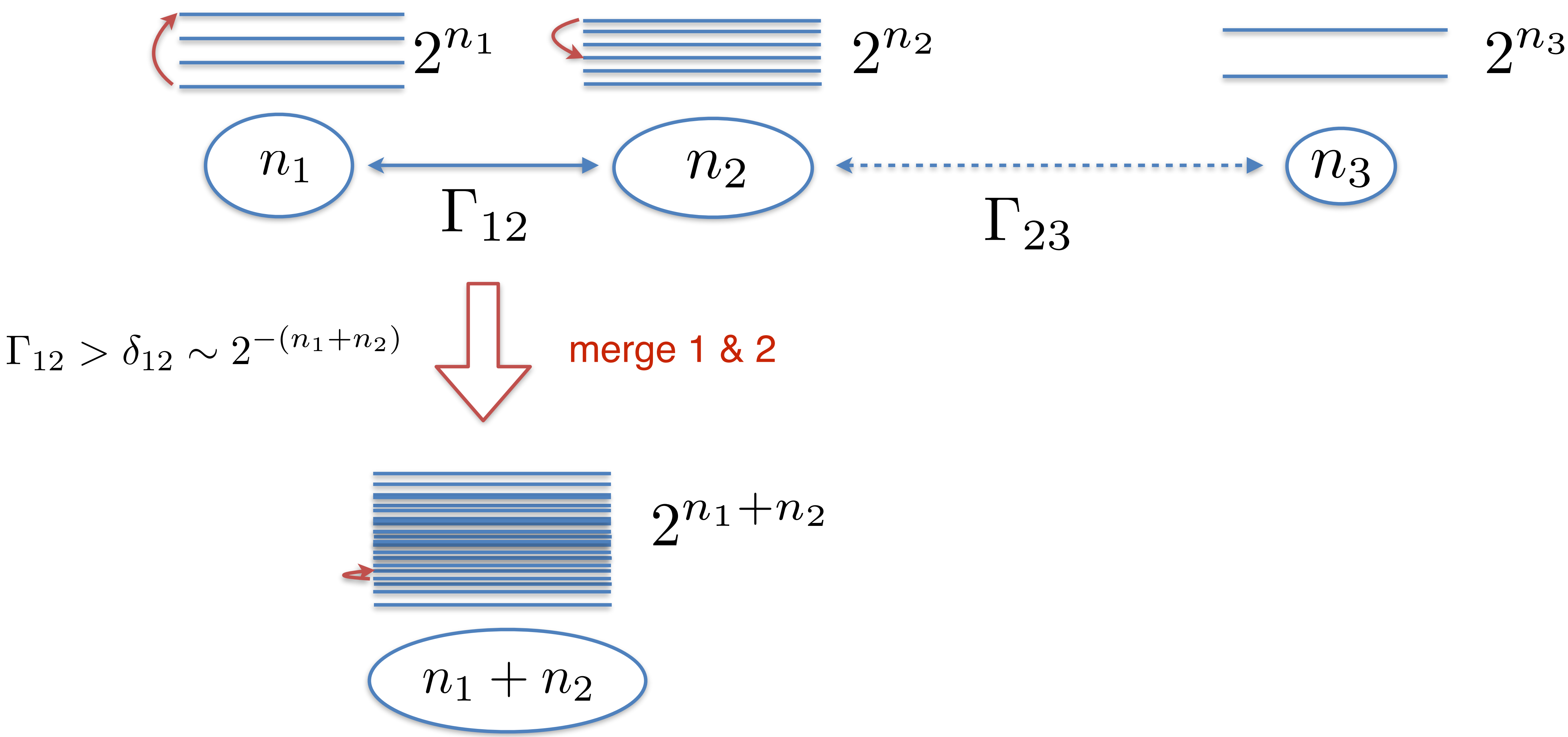
Many-body resonances

"Merging" Resonant Clusters:



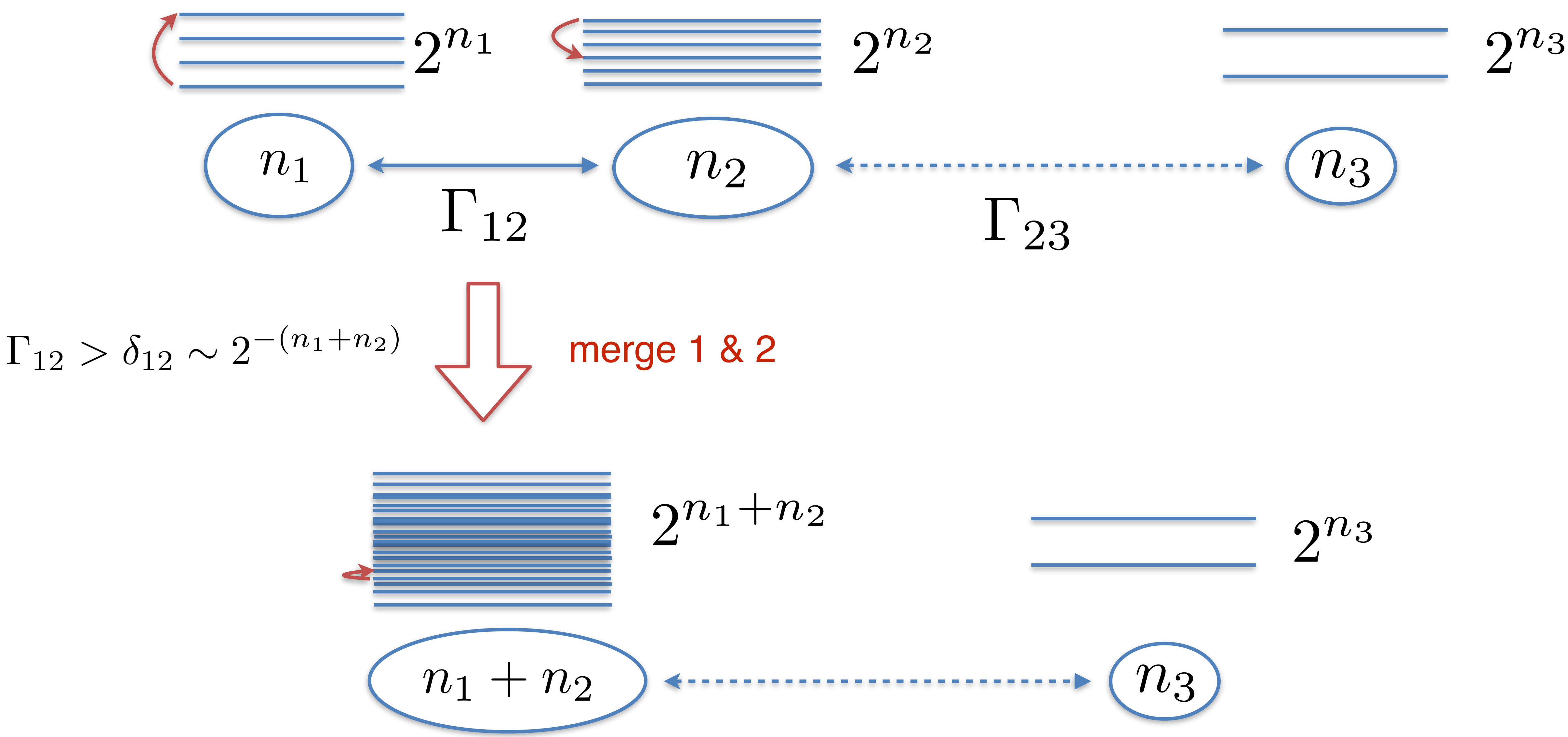
Many-body resonances

“Merging” Resonant Clusters:



Many-body resonances

"Merging" Resonant Clusters:

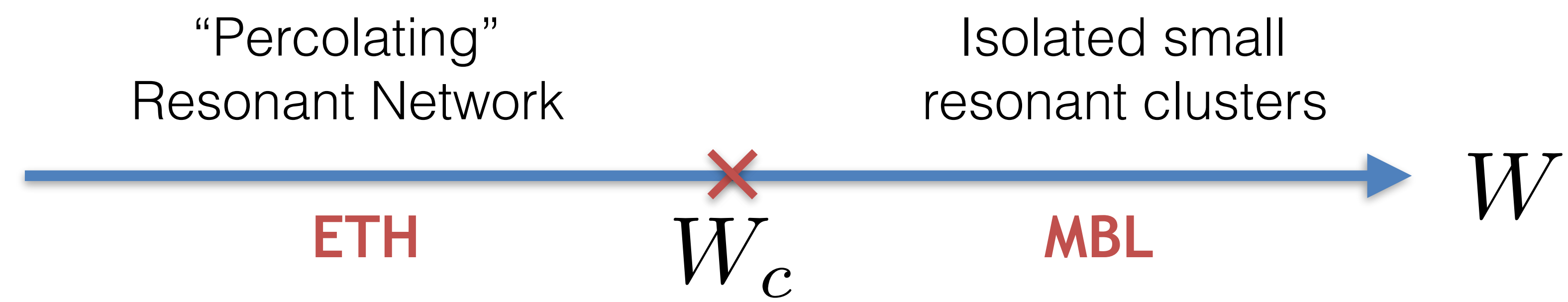


New inter-cluster couplings:

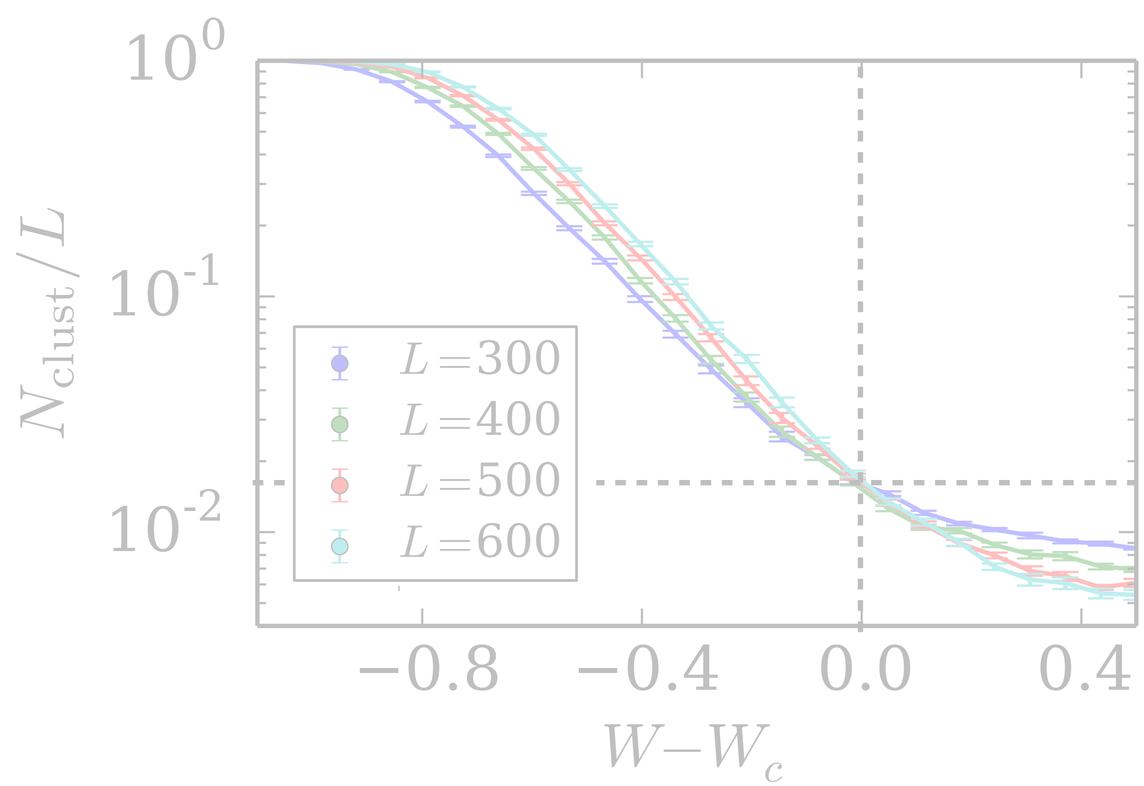
$$\Gamma_{1\cup 2;3} \approx \begin{cases} \Gamma_{23}\Gamma_{12}/\Delta_{23}, & \Gamma_{23} \ll \Delta_{23} \\ 1/(\Gamma_{23}^{-1} + \Gamma_{12}^{-1}), & \Gamma_{23} \gg \Delta_{23} \end{cases}$$

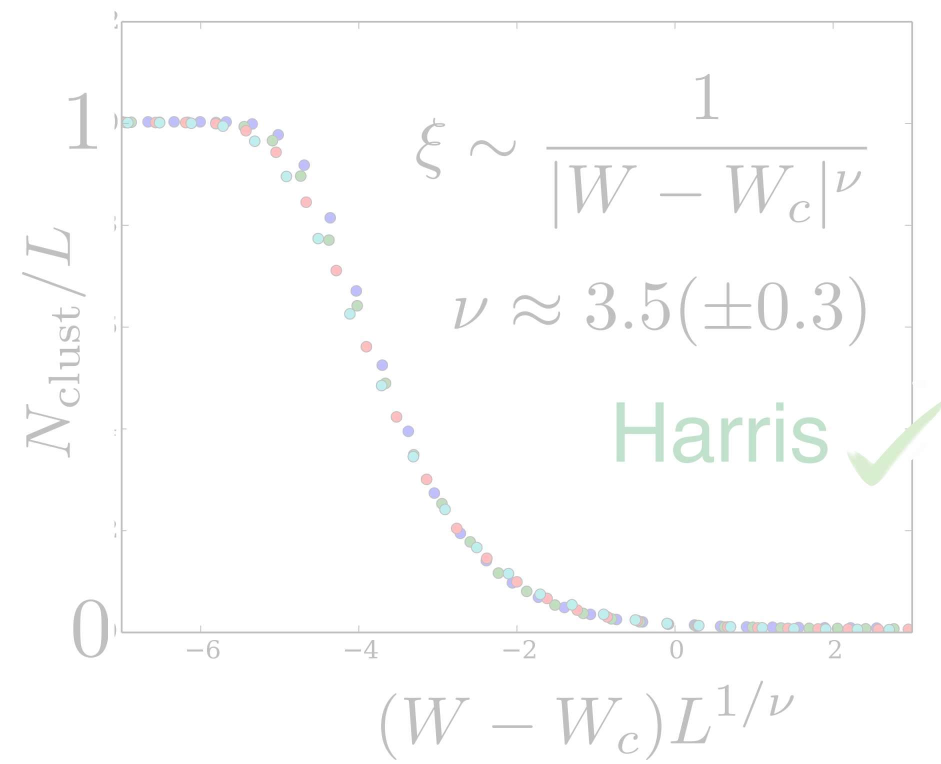
Scaling structure of the 1D MBL Transition

Possible Outcomes:



Scaling Collapse:

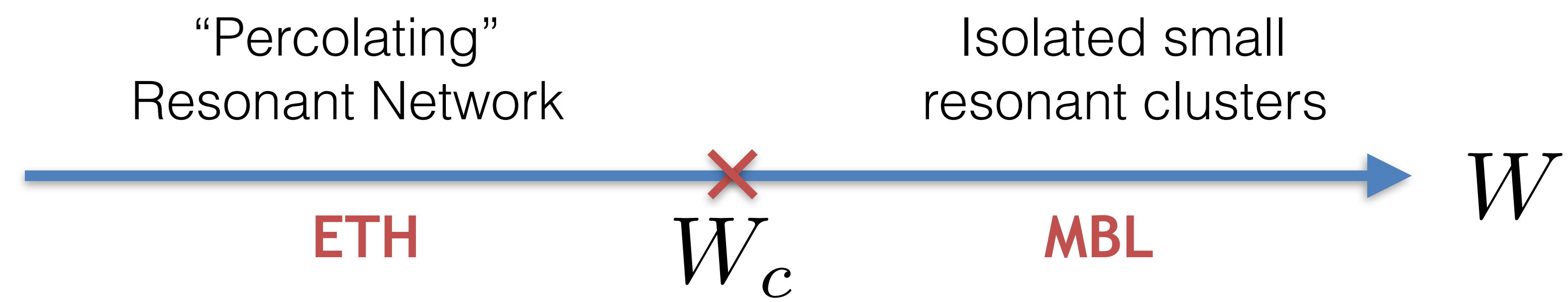




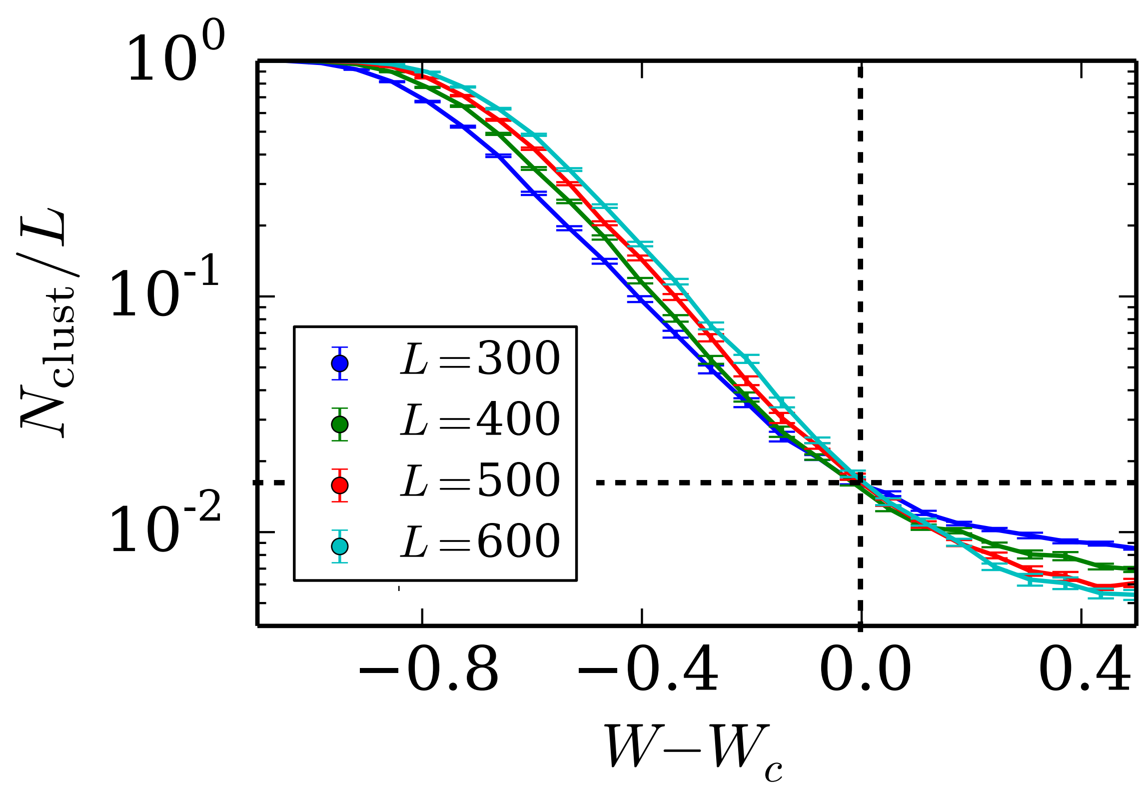
$$\frac{N_{\mathrm{clust}}}{L} = \mathcal{P}(L/\xi) \qquad \mathcal{P}(x) = \begin{cases} 1 & , x = \infty \\ p_c \approx 0.02 & , x = x_c \\ 0 & , x = 0 \end{cases}$$

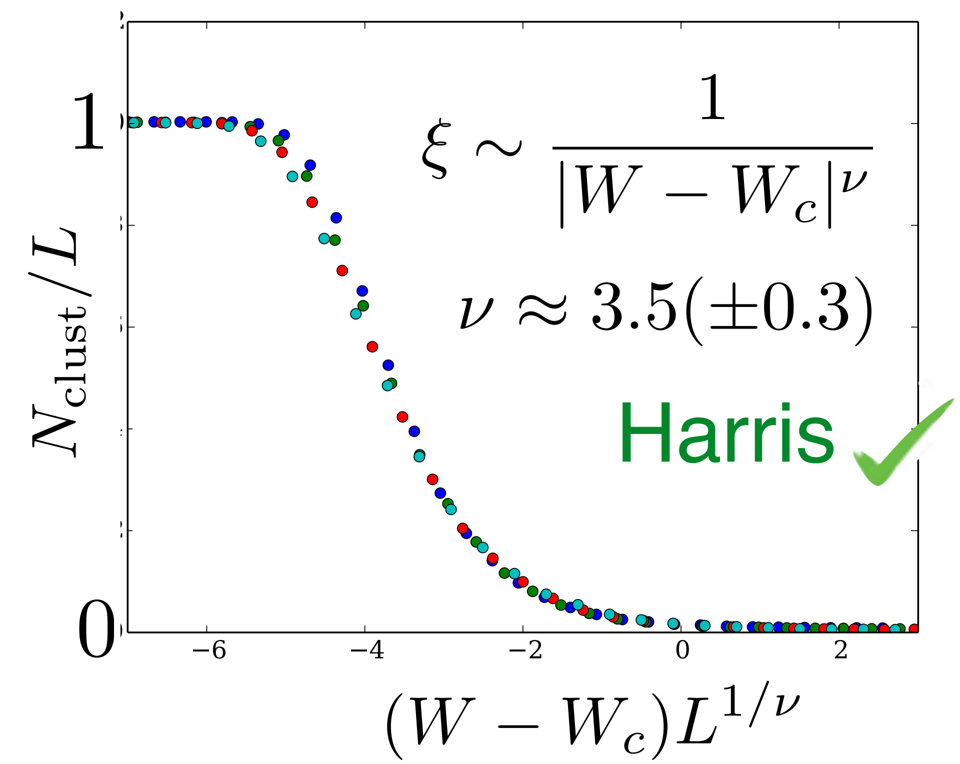
Scaling structure of the 1D MBL Transition

Possible Outcomes:



Scaling Collapse:

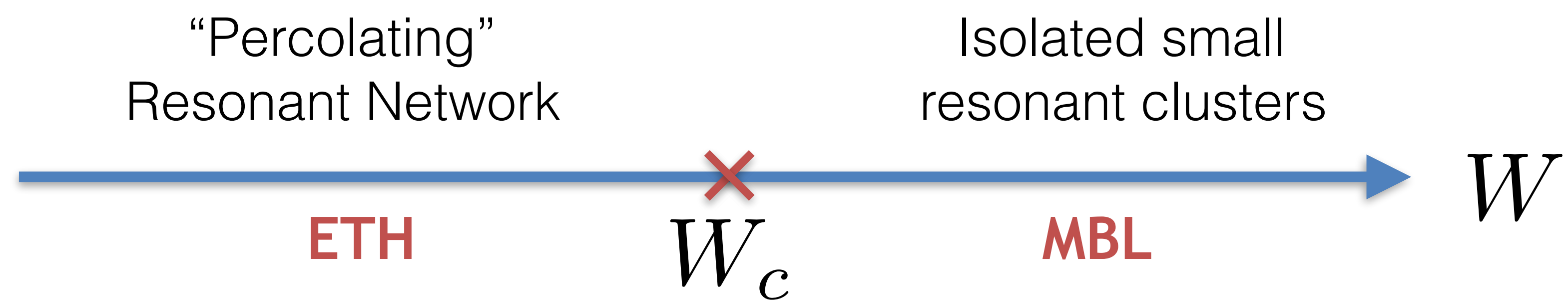




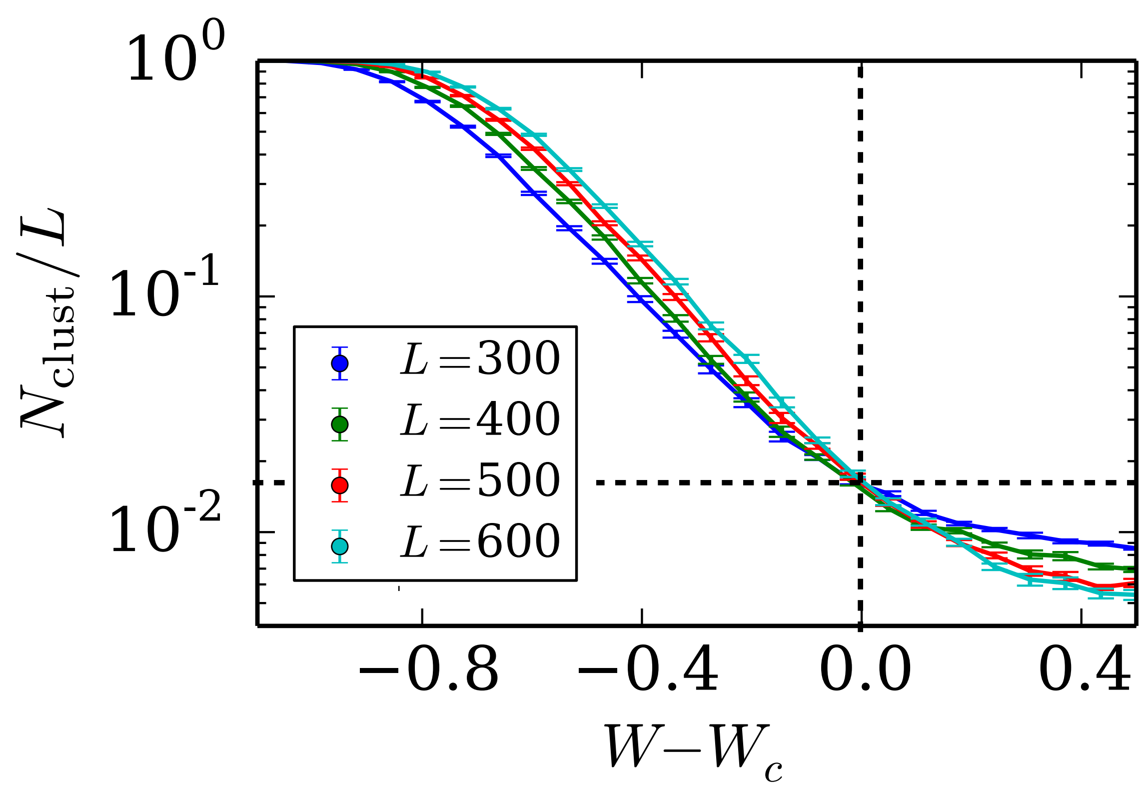
$$\frac{N_{\rm clust}}{L} = \mathcal{P}(L/\xi) \qquad \mathcal{P}(x) = \begin{cases} 1 & , x = \infty \\ p_c \approx 0.02 & , x = x_c \\ 0 & , x = 0 \end{cases}$$

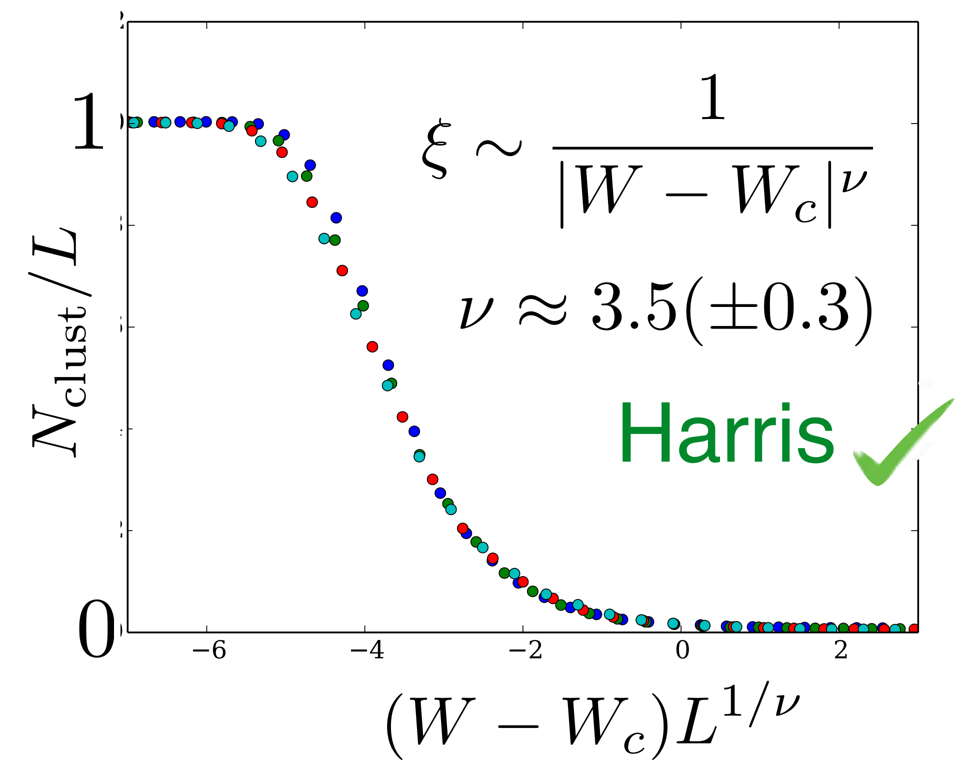
Scaling structure of the 1D MBL Transition

Possible Outcomes:



Scaling Collapse:

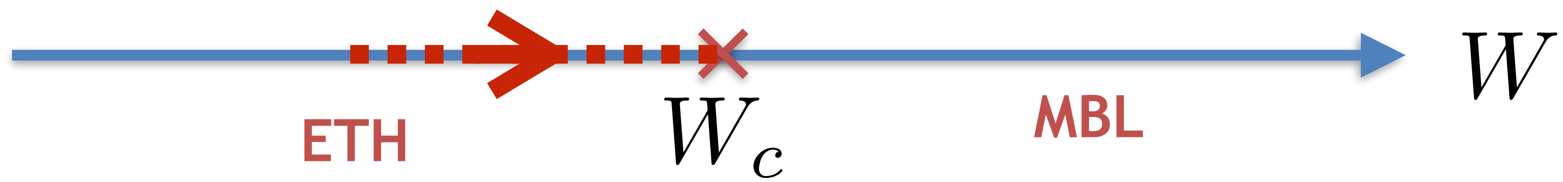


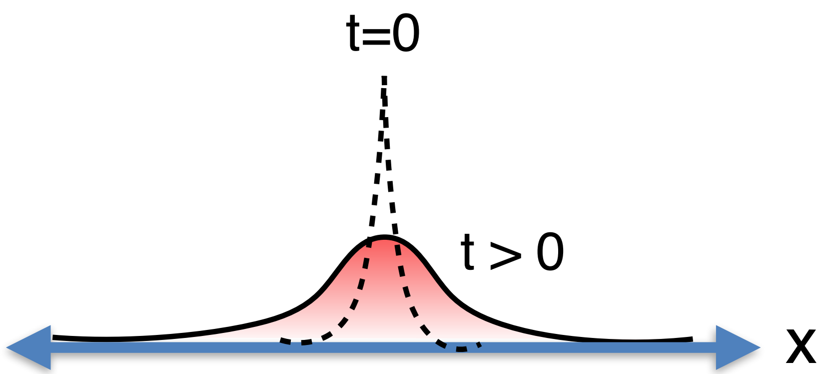


$$\frac{N_{\rm clust}}{L} = \mathcal{P}(L/\xi) \qquad \mathcal{P}(x) = \begin{cases} 1 & , x = \infty \\ p_c \approx 0.02 & , x = x_c \\ 0 & , x = 0 \end{cases}$$

Rare-resonance driven

Critical slowing down of dynamics and transport



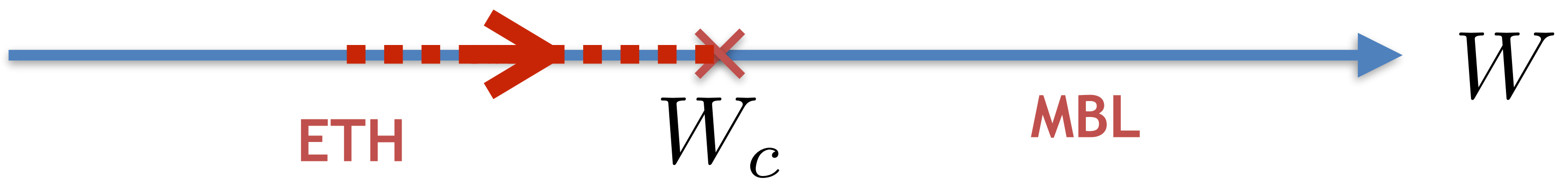


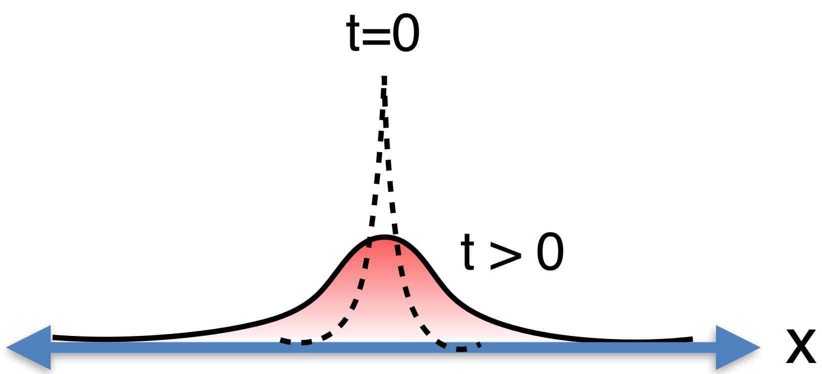
Guess: $\delta x(t) \sim \sqrt{Dt}$ $\quad D \sim \mathcal{D}(L/\xi)$

(e.g. as in single particle delocalization transition
& Mean-field treatment of MBL transition)

Gopalakrishnan, Nandkishore

Critical slowing down of dynamics and transport



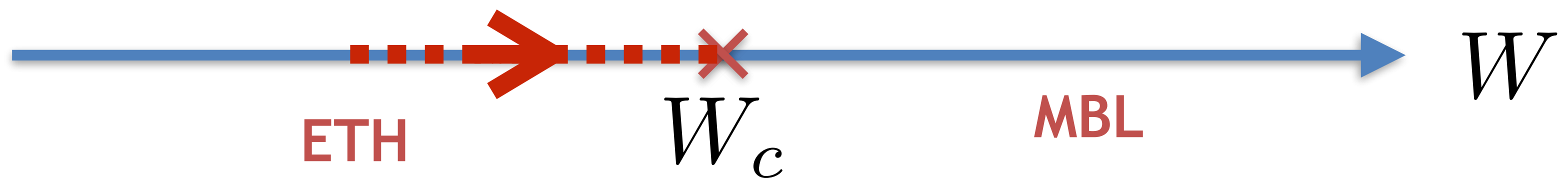


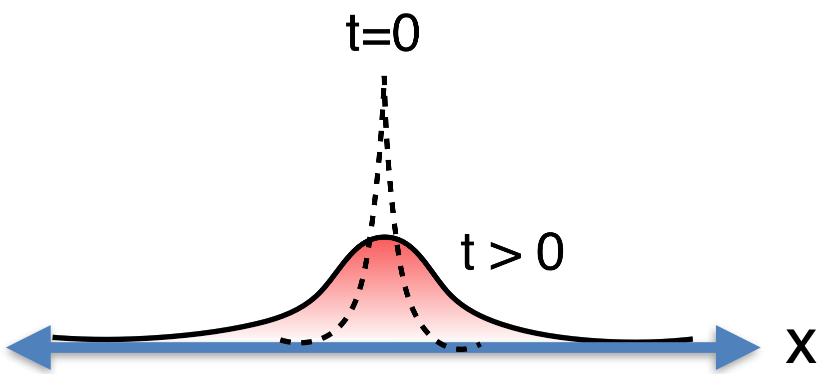
Guess: $\delta x(t) \sim \sqrt{Dt}$ $D \sim \mathcal{D}(L/\xi)$

(e.g. as in single particle delocalization transition
& Mean-field treatment of MBL transition)

Gopalakrishnan, Nandkishore

Critical slowing down of dynamics and transport





Guess: ~~$\delta x(t) \sim \sqrt{Dt}$ $\quad D \sim \mathcal{D}(L/\xi)$~~

(e.g. as in single particle delocalization transition
& Mean-field treatment of MBL transition)

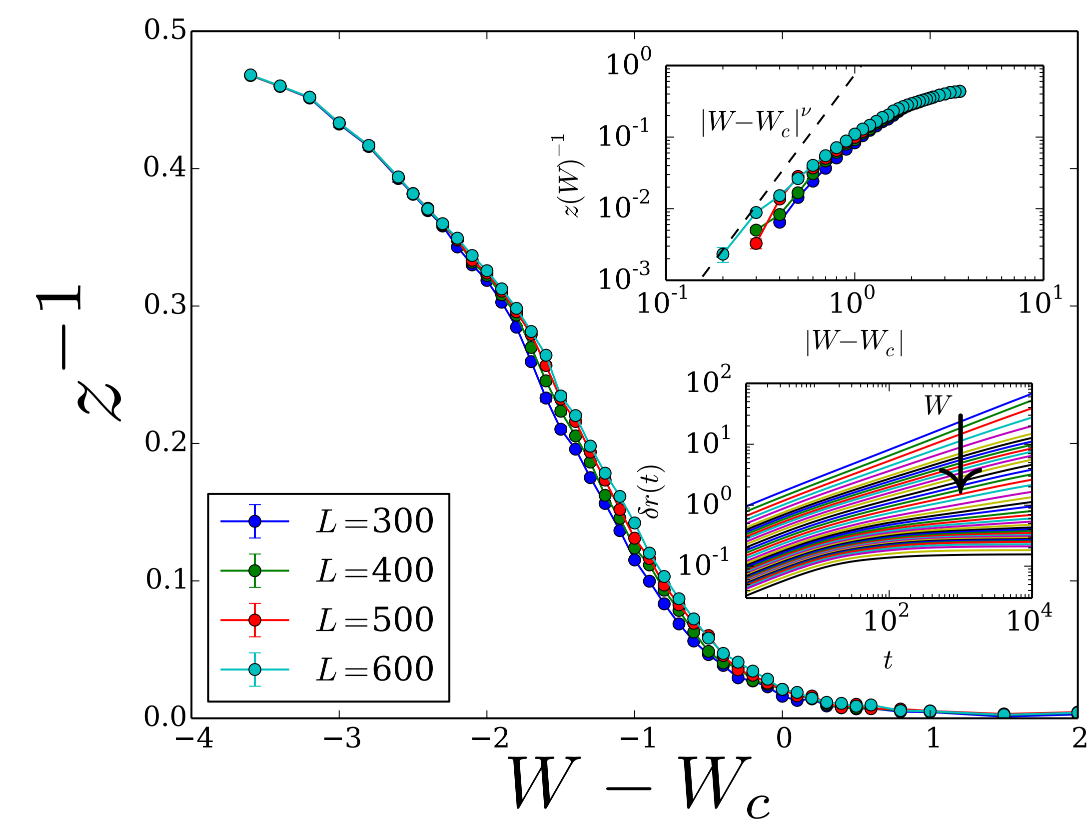
Gopalakrishnan, Nandkishore

Instead: Anomalous thermal liquid -

$$\delta x(t) \sim t^{1/z}$$

$$z \sim \frac{1}{|W - W_c|^{\zeta}} \qquad \zeta = \nu$$

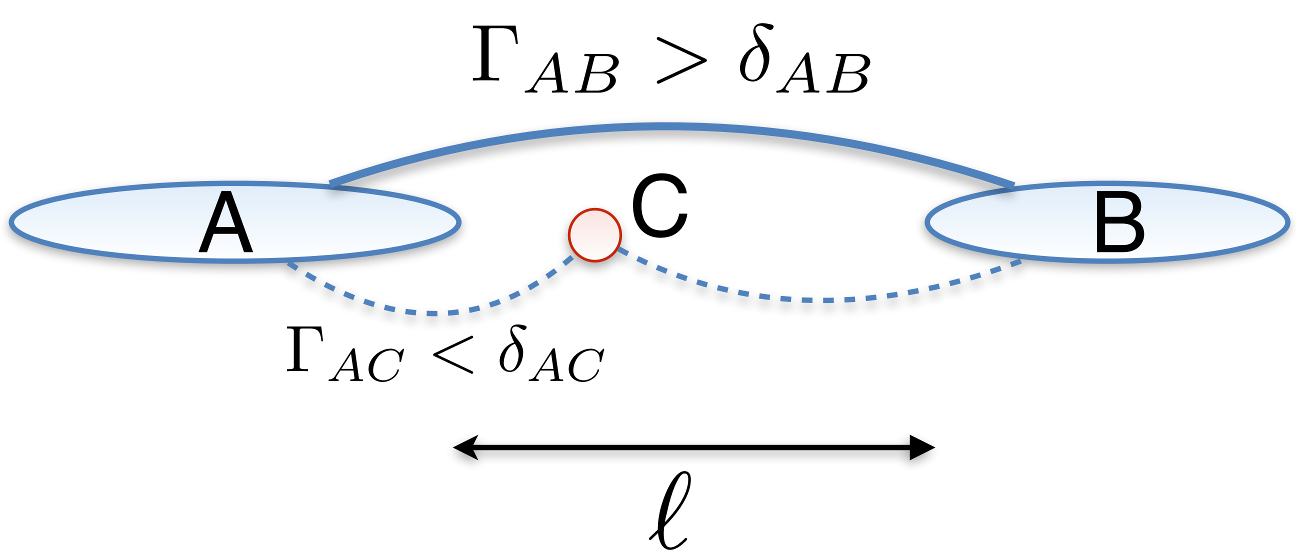
continuously evolving (scaling relation)



See also: Agarwal, et al '14 & Bar Lev, Cohen, Reichman '14; Vosk, Huse, Altman '15

Origin of subdiffusion: Gaps in Transport Path

Common Scenario:



$$\Gamma_{A\cup B;C} > \delta_{ABC}$$

$$\Gamma_{A\cup B;C} < \Gamma_{AB}$$

$$(\text{time} \sim 1/\Gamma)$$

Probability of “tunneling” gap: $P(\ell) \sim e^{-\ell/\xi}$

$$\tau(\ell) \sim e^{\ell/x_0}$$

$$P(\tau) \sim \frac{1}{\tau^{1+x_0/\xi}}$$

broad distribution of tunneling times $\mathbb{E}[\tau] = \infty$

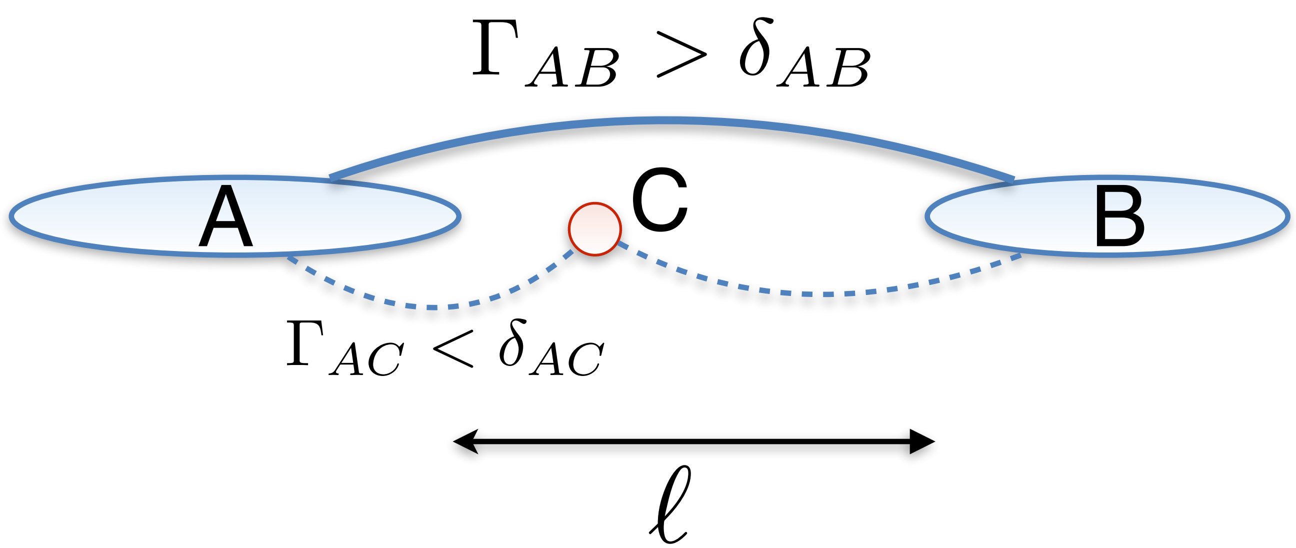
Transport through a long segment of length L dominated by rare bottlenecks:

$$\ell_* \approx \xi \log(L/\xi) \qquad \tau(L) \approx \tau(\ell_*) \approx L^{\xi}$$

See also Hulin et al ‘90, Bar Lev et al ’14; Agarwal et al ’14, Vosk, Huse, Altman ‘15

Origin of subdiffusion: Gaps in Transport Path

Common Scenario:



$$\Gamma_{A\cup B;C} > \delta_{ABC}$$

$$\Gamma_{A\cup B;C} < \Gamma_{AB}$$

$$(\text{time} \sim 1/\Gamma)$$

Probability of “tunneling” gap: $P(\ell) \sim e^{-\ell/\xi}$

$$\tau(\ell) \sim e^{\ell/x_0}$$

$$P(\tau) \sim \frac{1}{\tau^{1+x_0/\xi}}$$

broad distribution of tunneling times $\mathbb{E}[\tau] = \infty$

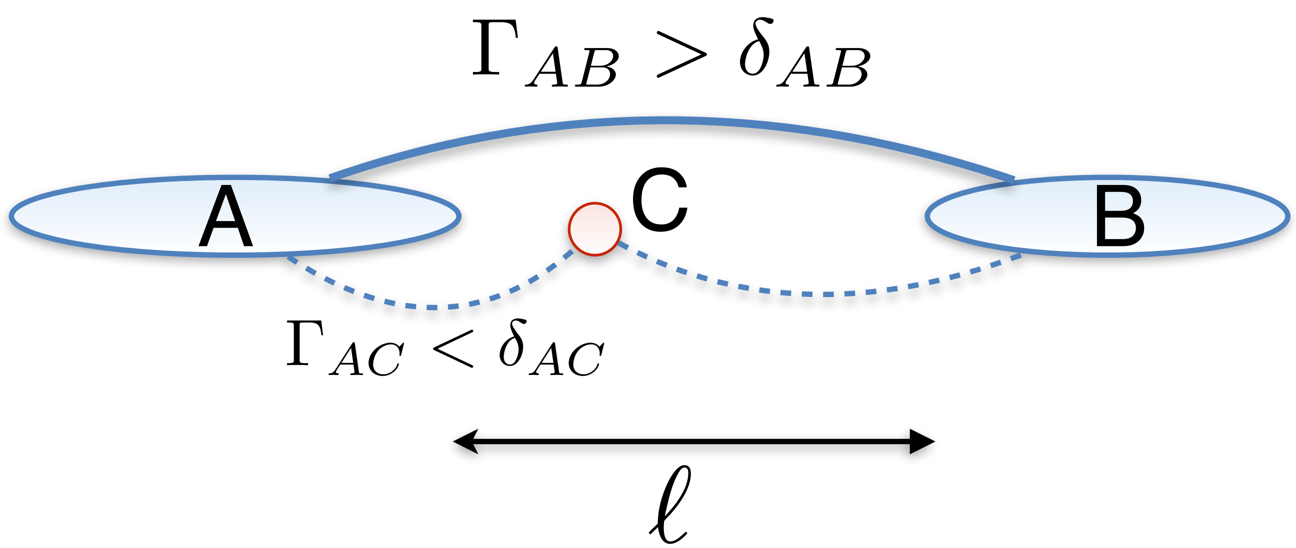
Transport through a long segment of length L dominated by rare bottlenecks:

$$\ell_* \approx \xi \log(L/\xi) \qquad \tau(L) \approx \tau(\ell_*) \approx L^{\xi}$$

See also Hulin et al ‘90, Bar Lev et al ’14; Agarwal et al ’14, Vosk, Huse, Altman ‘15

Origin of subdiffusion: Gaps in Transport Path

Common Scenario:



$$\Gamma_{A\cup B;C} > \delta_{ABC}$$

$$\Gamma_{A\cup B;C} < \Gamma_{AB}$$

$$(\text{time} \sim 1/\Gamma)$$

Probability of "tunneling" gap: $P(\ell) \sim e^{-\ell/\xi}$

$$\tau(\ell) \sim e^{\ell/x_0}$$

$$P(\tau) \sim \frac{1}{\tau^{1+x_0/\xi}}$$

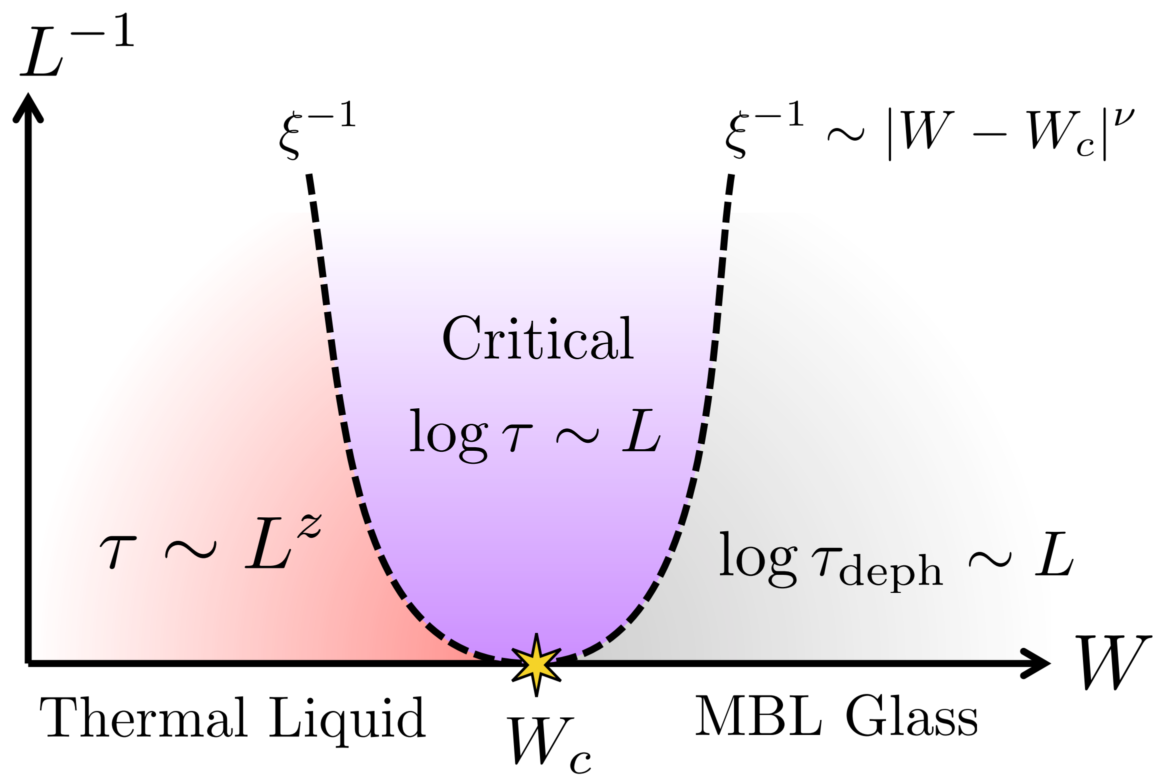
broad distribution of tunneling times $\mathbb{E}[\tau] = \infty$

Transport through a long segment of length L dominated by rare bottlenecks:

$$\ell_* \approx \xi \ \log(L/\xi) \qquad \tau(L) \approx \tau(\ell_*) \approx L^{\xi}$$

See also Hulin et al '90, Bar Lev et al '14; Agarwal et al '14, Vosk, Huse, Altman '15

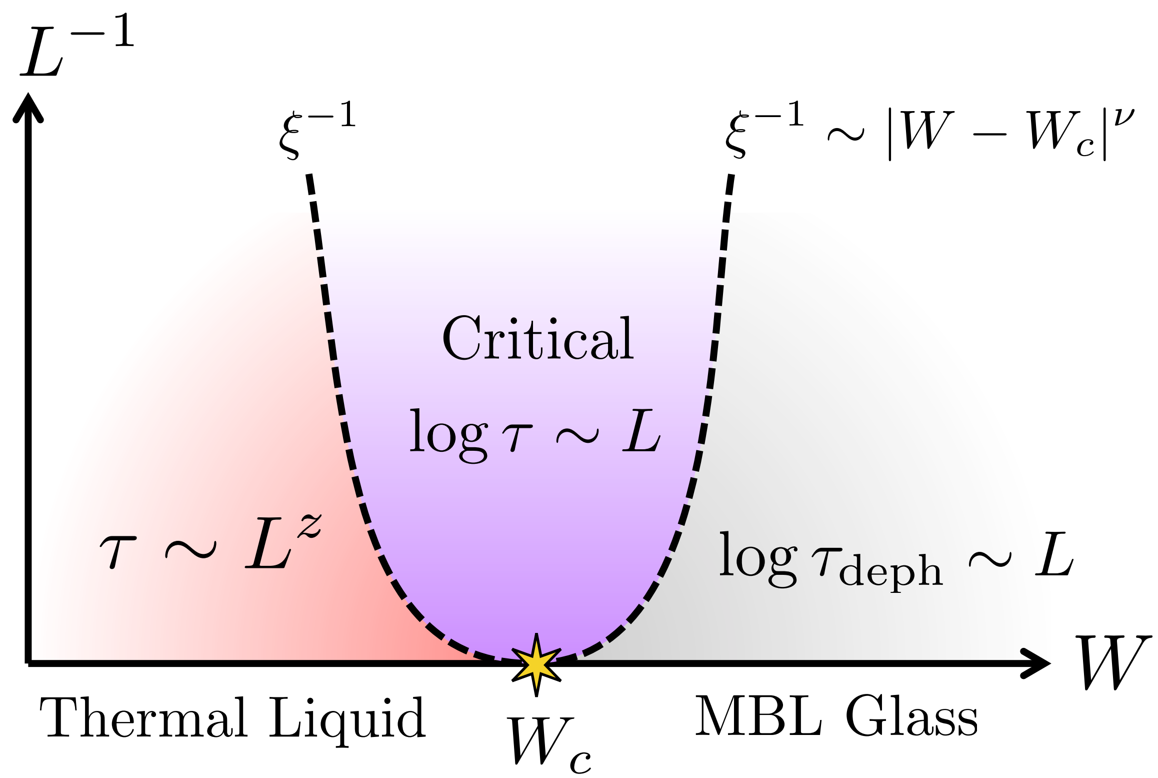
1D MBL Transition - summary



$$\nu_{1\mathrm{D}} \approx 3.5$$

(Harris/Chayes ✓)

1D MBL Transition - summary



$$\nu_{1D} \approx 3.5$$

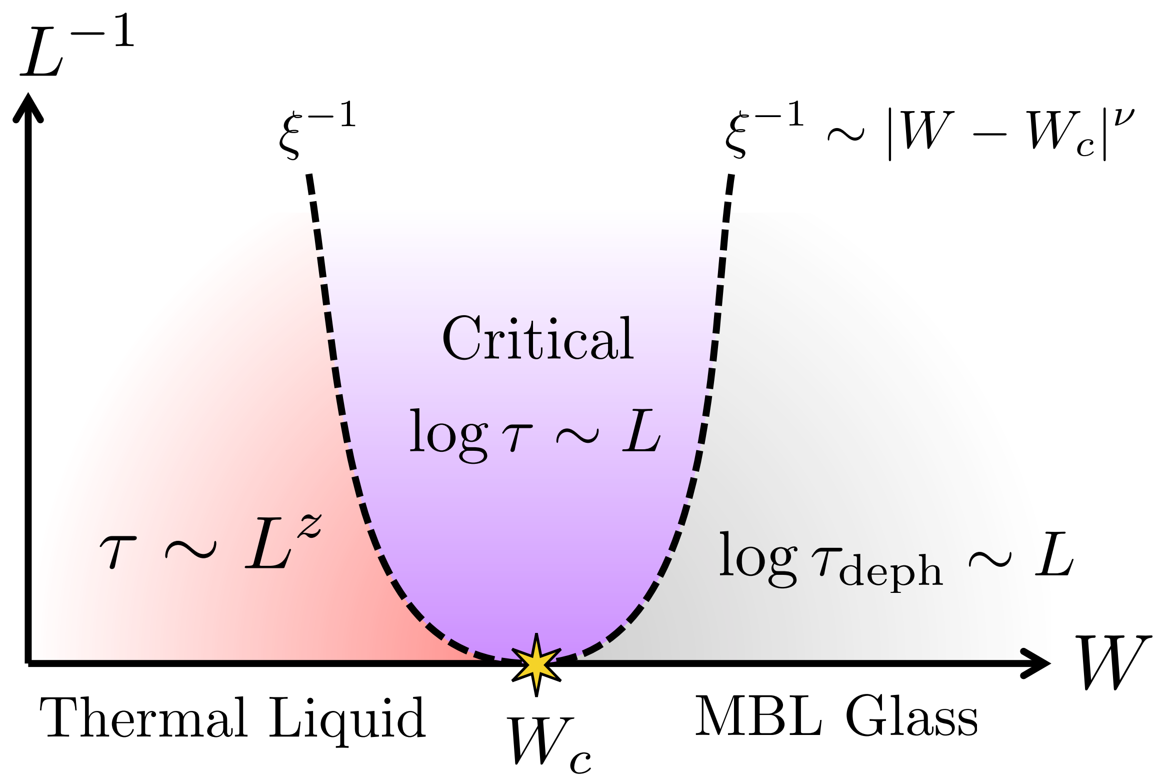
(Harris/Chayes ✓)

Nature of Transition

- single parameter scaling
- driven by rare resonances $p_c^{(1D)} \approx 2\%$

Dilute resonances typical vs average g -
Vosk Huse Altman,

1D MBL Transition - summary



$\nu_{1\mathrm{D}} \approx 3.5$

(Harris/Chayes ✓)

Nature of Transition

- single parameter scaling
- driven by rare resonances $p_c^{(1D)} \approx 2\%$

Dilute resonances typical vs average g - Vosk Huse Altman,

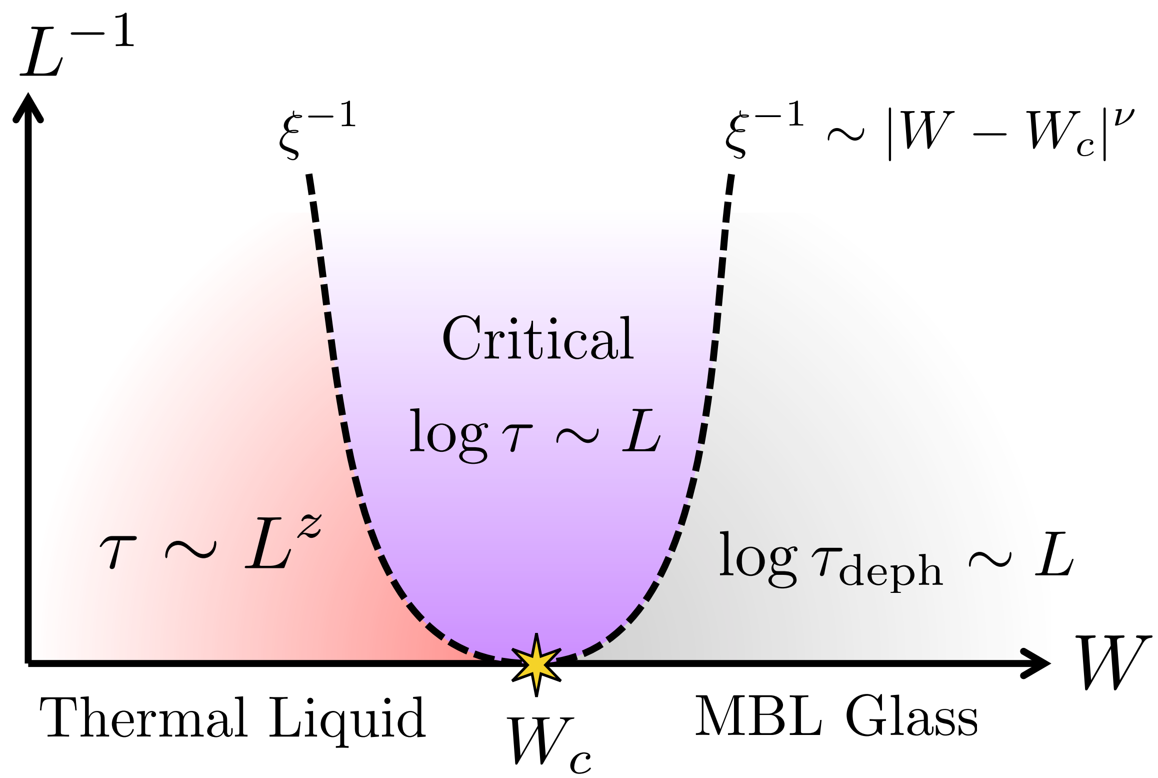
Critical region

- thermal (S ~ L)
- z=∞ (Exponentially slow transport/entanglement dynamics)
- (multi)-Fractal structure $p(\ell) \sim \frac{1}{\ell}$

Consistent w/ Entanglement monotonicity - Grover

Consistent w/ Block RG method:
Vosk Huse Altman,
& ED study of matrix elements:
Serbyn, Papic, Abanin

1D MBL Transition - summary



$$\nu_{1D} \approx 3.5$$

(Harris/Chayes ✓)

Nature of Transition

- single parameter scaling
- driven by rare resonances $p_c^{(1D)} \approx 2\%$

Dilute resonances typical vs average g - Vosk Huse Altman,

Critical region

- thermal (S ~ L)
- z=∞ (Exponentially slow transport/entanglement dynamics)
- (multi)-Fractal structure $p(\ell) \sim \frac{1}{\ell}$

Consistent w/ Entanglement monotonicity - Grover

Consistent w/ Block RG method: Vosk Huse Altman, & ED study of matrix elements: Serbyn, Papic, Abanin

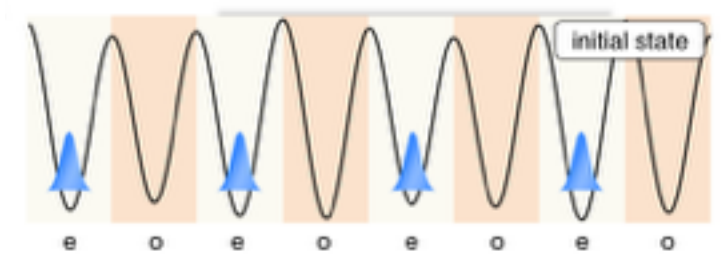
Thermal side

- Continuously evolving subdiffusion

See also: Agarwal et al '14, Bar Lev at al.'14, Vosk, Huse, Altman '15

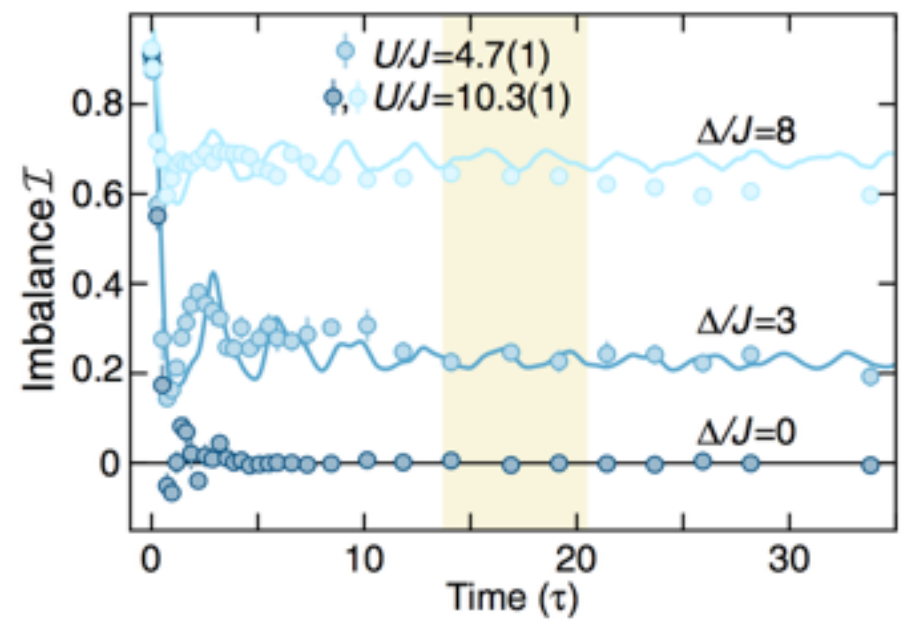
Experimental predictions

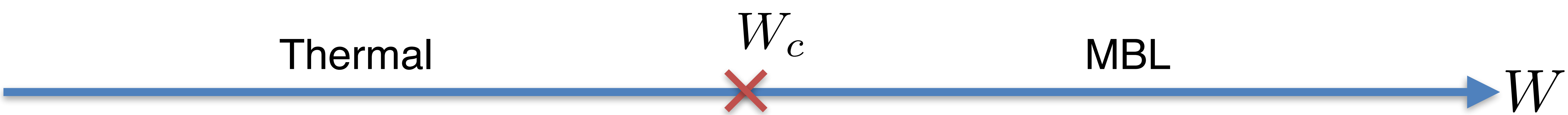
Schrieber et al. (I. Bloch Group) ‘15

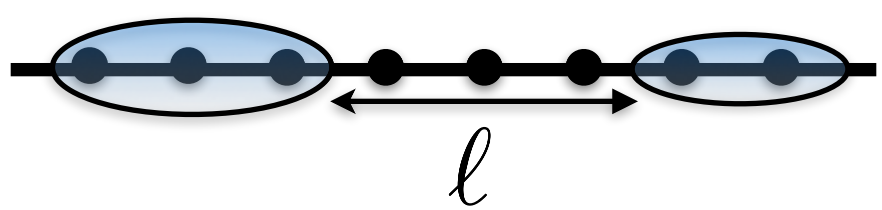


$$\mathcal{I} = n_e - n_o$$

(or any other observable with vanishing thermal average)







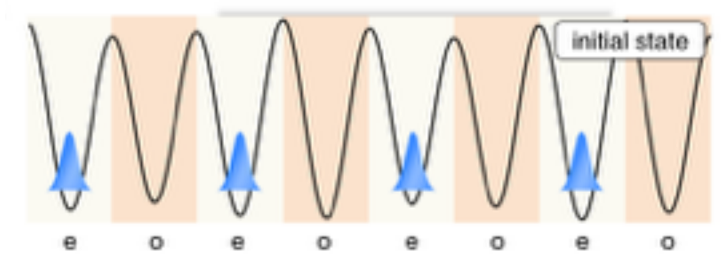
$$\tau_{\text{typ}} \sim e^{\xi/x_0}$$

$$\tau(\ell) \sim e^{\ell/x_0}$$

$$\mathcal{I}(t) \approx \int_{\log t}^{\infty} P(t) \approx t^{-1/z_{\mathcal{I}}}$$

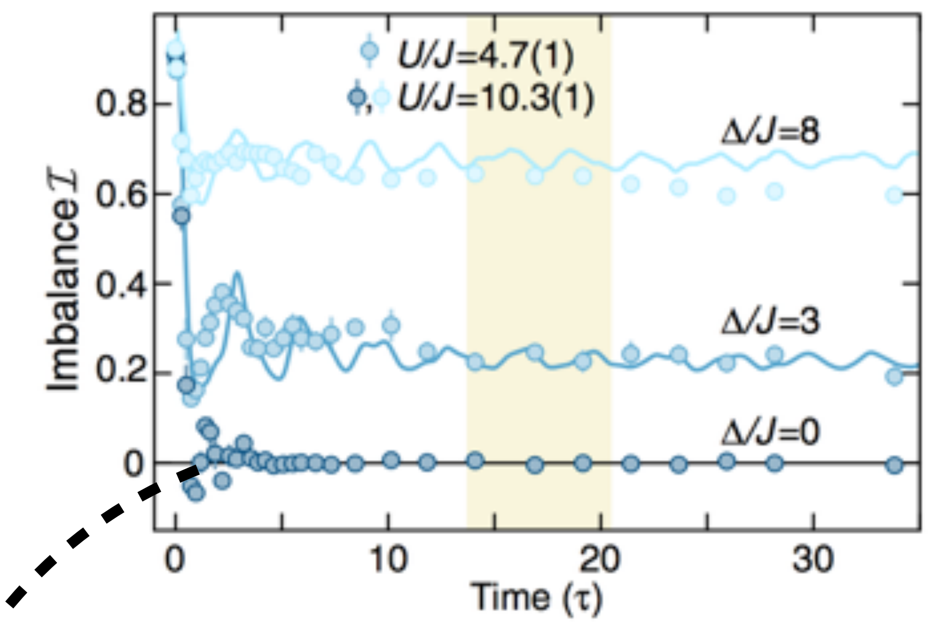
Experimental predictions

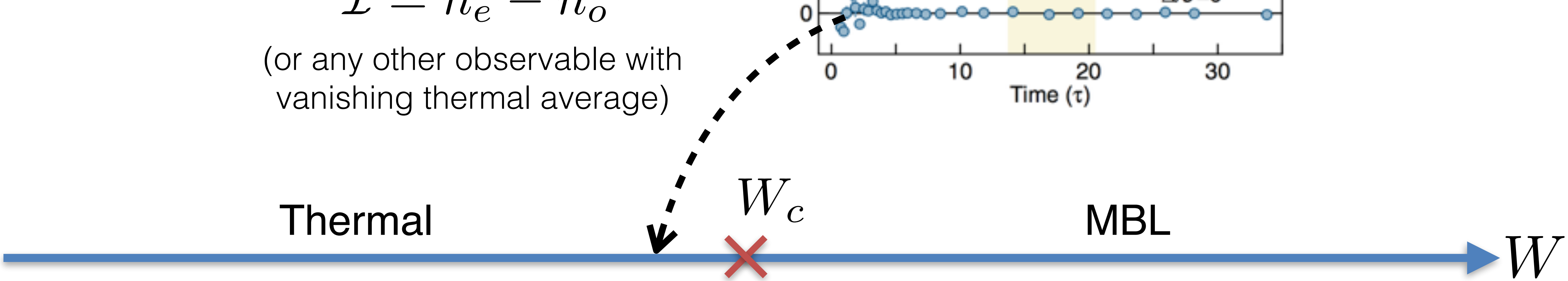
Schrieber et al. (I. Bloch Group) ‘15

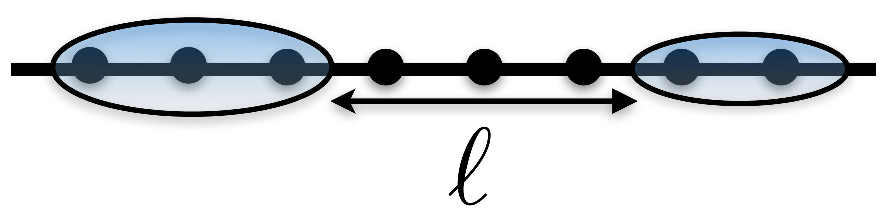


$$\mathcal{I} = n_e - n_o$$

(or any other observable with vanishing thermal average)





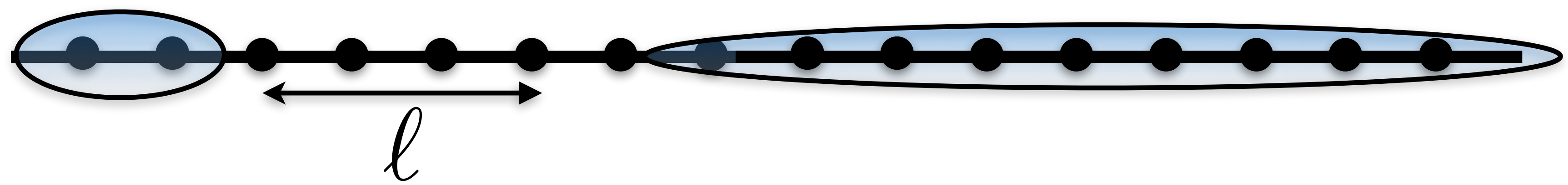


$$\tau_{\text{typ}} \sim e^{\xi/x_0}$$

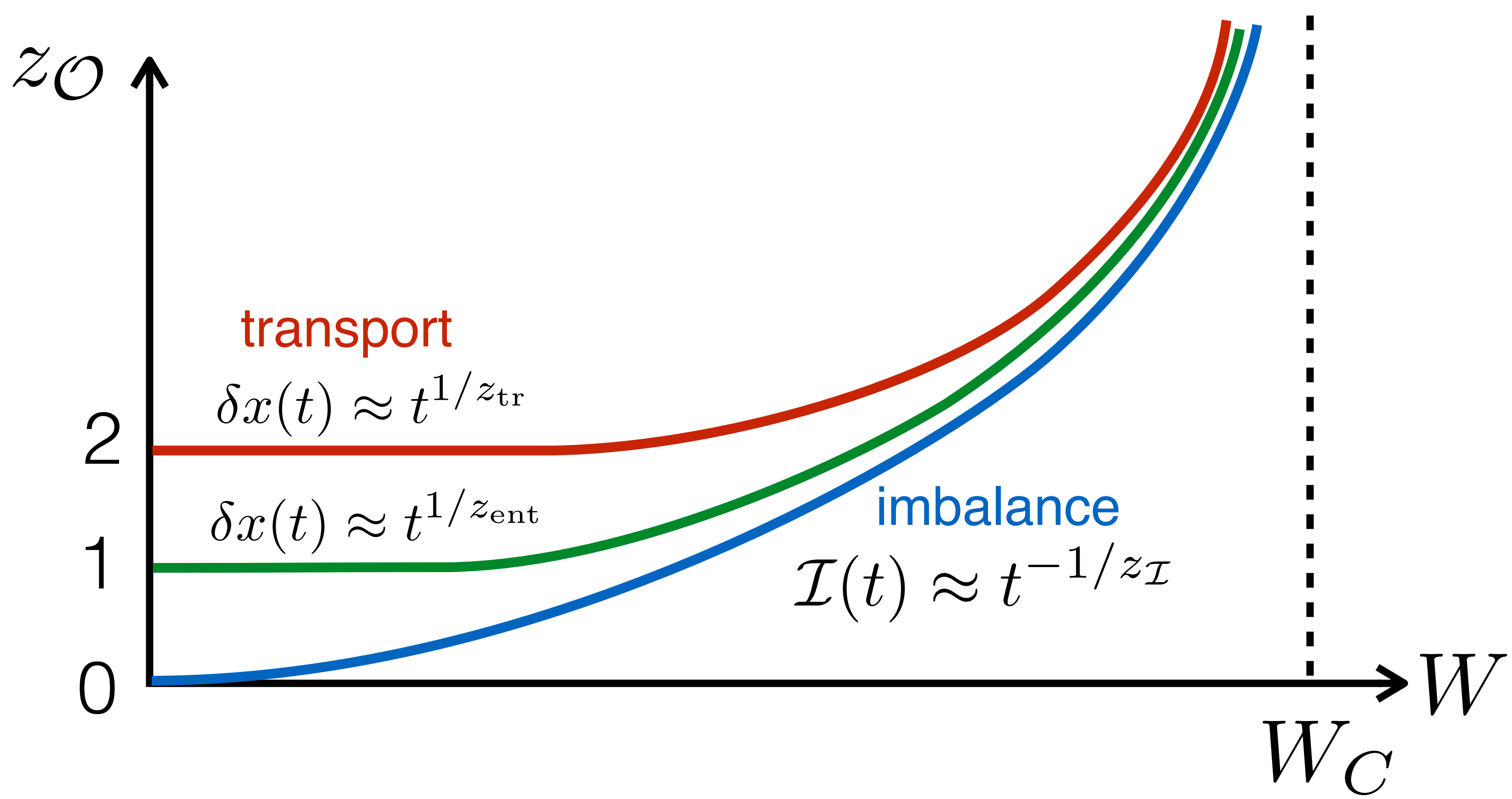
$$\tau(\ell) \sim e^{\ell/x_0}$$

$$\mathcal{I}(t) \approx \int_{\log t}^{\infty} P(t) \approx t^{-1/z_{\mathcal{I}}}$$

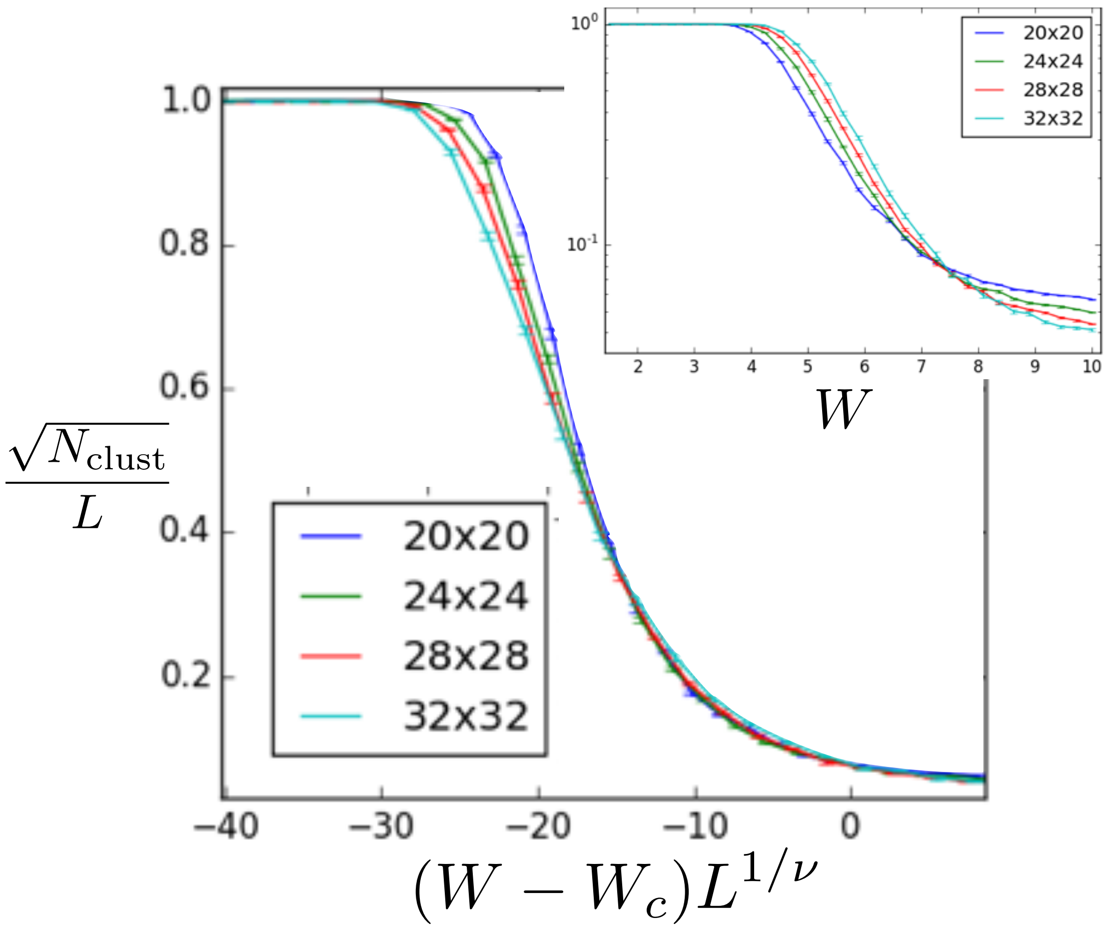
Griffiths Effects for Different Observables



$p(\ell) \approx e^{-\ell/\xi}$ $\qquad$ $\tau_{\mathcal{O}}(\ell) \approx e^{\ell/\ell_{\mathcal{O}}}$



1D vs 2D MBL Transitions



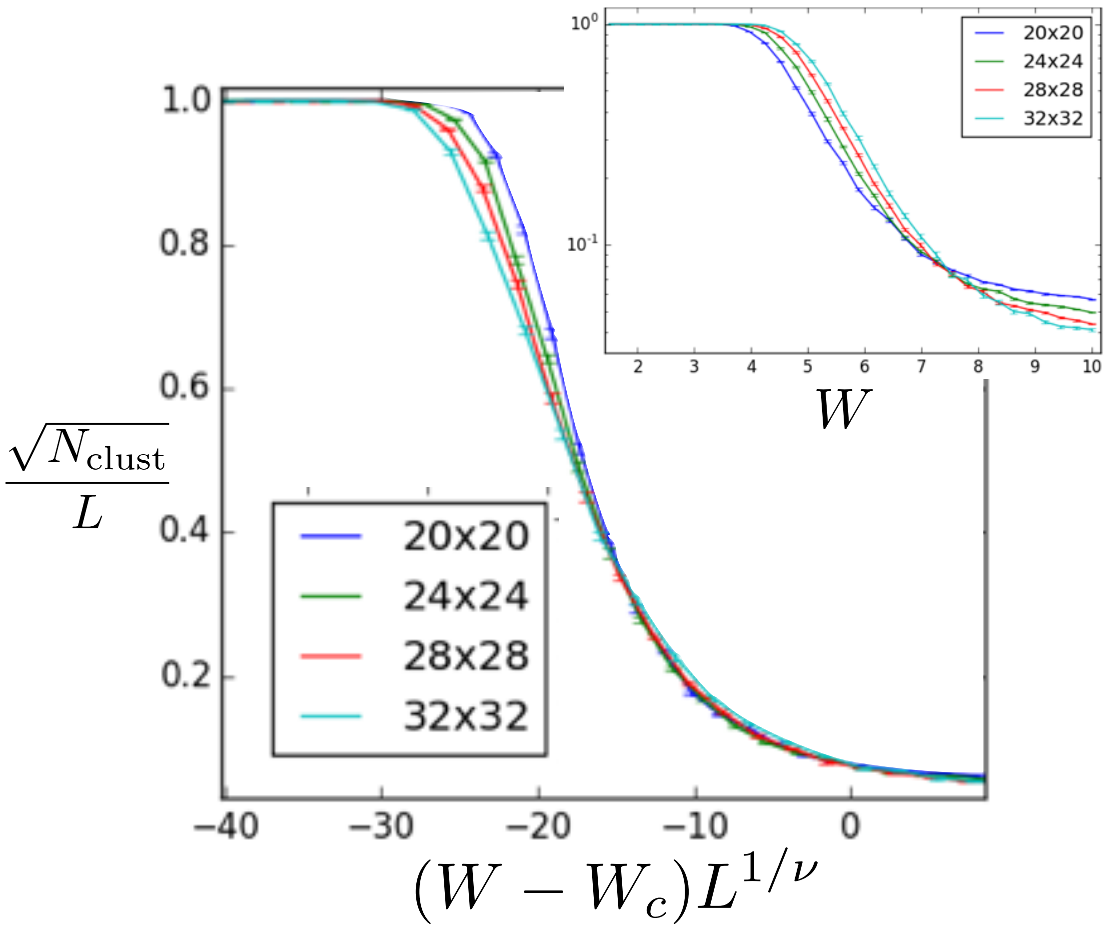
	1D	2D
Correlation Length:	$\nu_{1\text{D}} \approx 3.5$ $p_c^{(1D)} \approx 2\%$	$\nu_{2\text{D}} \approx 1.5$ $p_c^{(2D)} \approx 7\%$
Dynamics:	$\delta x(t) \sim t^{1/z}$ $z \sim \lvert\delta W\rvert^{-\nu}$ $\sigma(\omega) \sim \omega^{1-2/z}$ (Subdiffusion)	$\delta x(t) \sim \sqrt{Dt}$ $D \sim e^{-\xi}$ $\sigma(\omega) \sim \text{const}$ (Diffusion)

Similarity:
- Rare resonance driven
- Not classical percolation (p_c=50%)

Difference:
- No subdiffusion in d>1

ACP & R. Vasseur (Unpublished)

1D vs 2D MBL Transitions



	1D	2D
Correlation Length:	$\nu_{1\rm D} \approx 3.5$ $p_c^{(1D)} \approx 2\%$	$\nu_{2\rm D} \approx 1.5$ $p_c^{(2D)} \approx 7\%$
Dynamics:	$\delta x(t) \sim t^{1/z}$ $z \sim \lvert\delta W\rvert^{-\nu}$ $\sigma(\omega) \sim \omega^{1-2/z}$ (Subdiffusion)	$\delta x(t) \sim \sqrt{Dt}$ $D \sim e^{-\xi}$ $\sigma(\omega) \sim \text{const}$ (Diffusion)

Similarity:

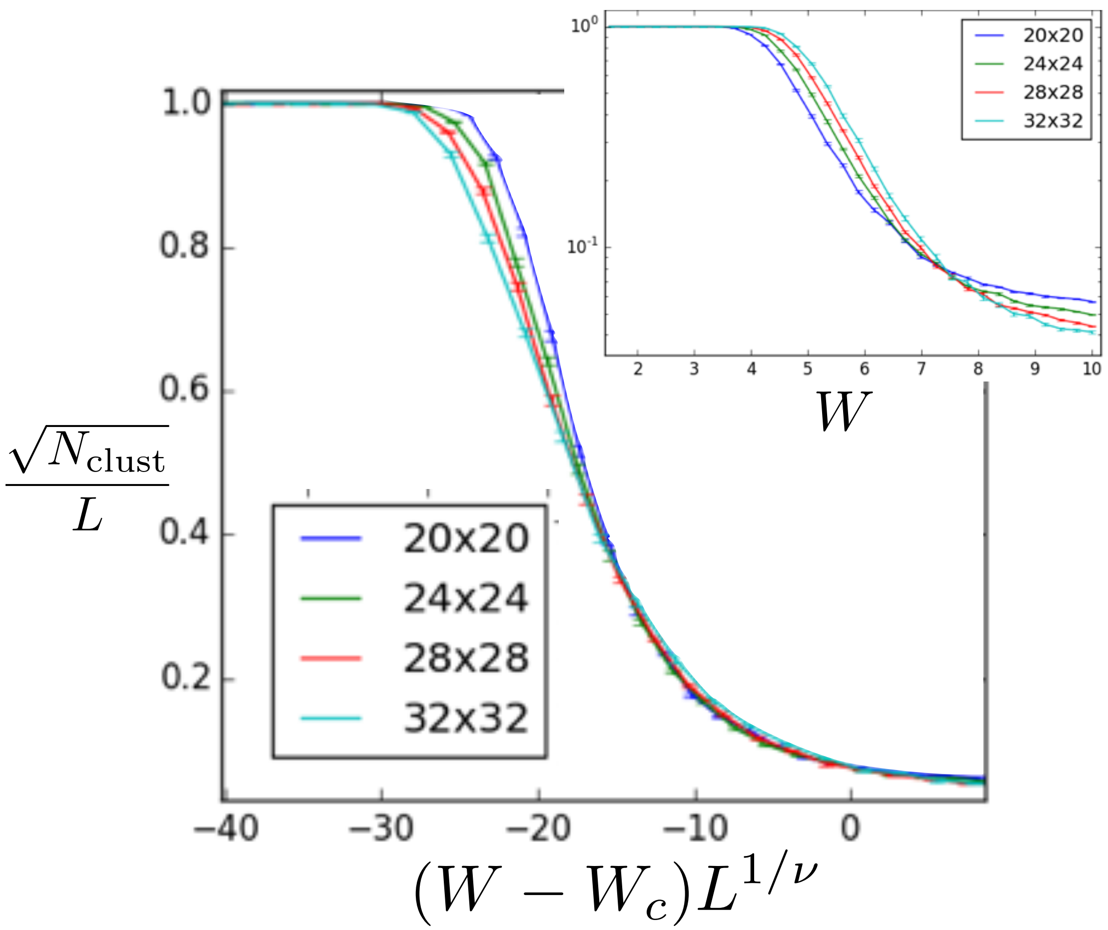
- Rare resonance driven
- Not classical percolation (p_c=50%)

Difference:

- No subdiffusion in d>1

ACP & R. Vasseur (Unpublished)

1D vs 2D MBL Transitions



	1D	2D
Correlation Length:	$\nu_{\rm 1D} \approx 3.5$ $p_c^{(1D)} \approx 2\%$	$\nu_{\rm 2D} \approx 1.5$ $p_c^{(2D)} \approx 7\%$
Dynamics:	$\delta x(t) \sim t^{1/z}$ $z \sim \lvert\delta W\rvert^{-\nu}$ $\sigma(\omega) \sim \omega^{1-2/z}$ (Subdiffusion)	$\delta x(t) \sim \sqrt{Dt}$ $D \sim e^{-\xi}$ $\sigma(\omega) \sim \text{const}$ (Diffusion)

Similarity:

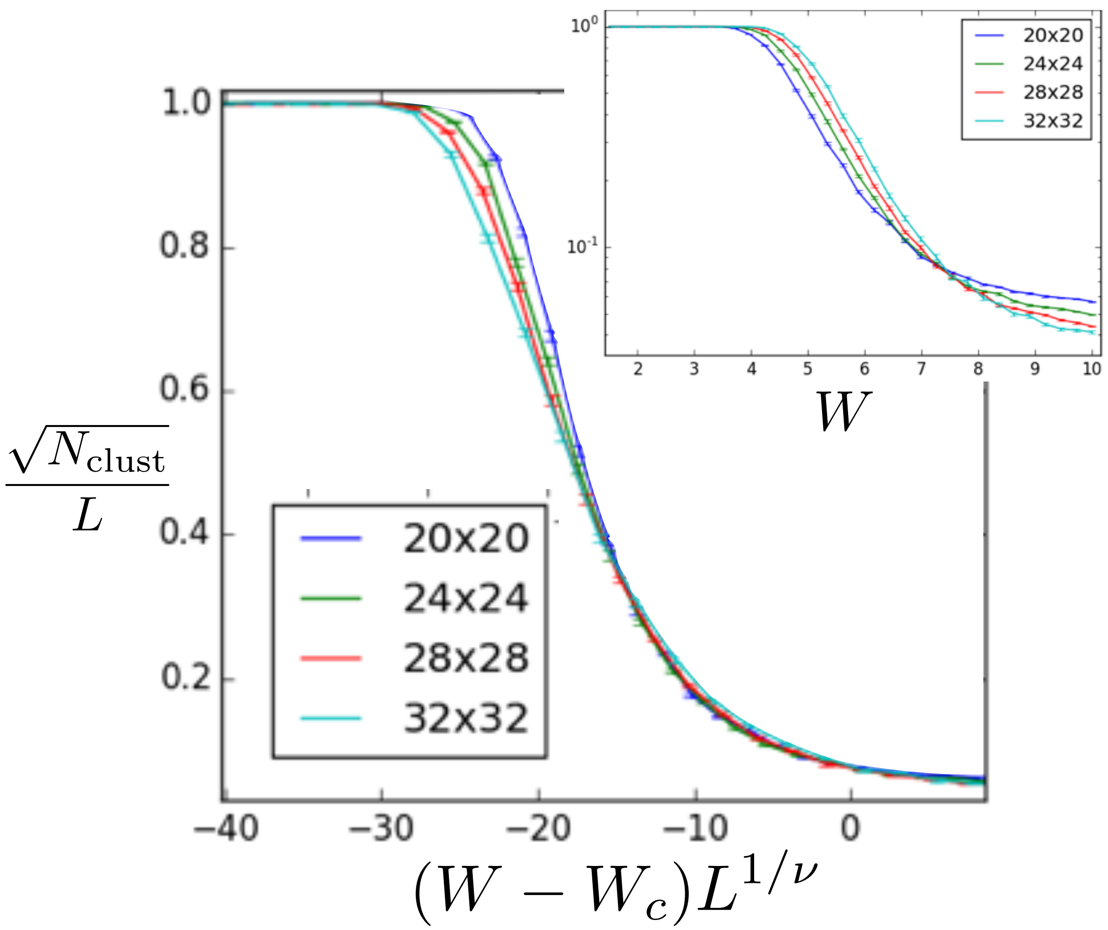
- Rare resonance driven
- Not classical percolation (p_c=50%)

Difference:

- No subdiffusion in d>1

ACP & R. Vasseur (Unpublished)

1D vs 2D MBL Transitions



	1D	2D
Correlation Length:	$\nu_{1D} \approx 3.5$ $p_c^{(1D)} \approx 2\%$	$\nu_{2D} \approx 1.5$ $p_c^{(2D)} \approx 7\%$
Dynamics:	$\delta x(t) \sim t^{1/z}$ $z \sim \lvert\delta W\rvert^{-\nu}$ $\sigma(\omega) \sim \omega^{1-2/z}$ (Subdiffusion)	$\delta x(t) \sim \sqrt{Dt}$ $D \sim e^{-\xi}$ $\sigma(\omega) \sim \text{const}$ (Diffusion)

Similarity:

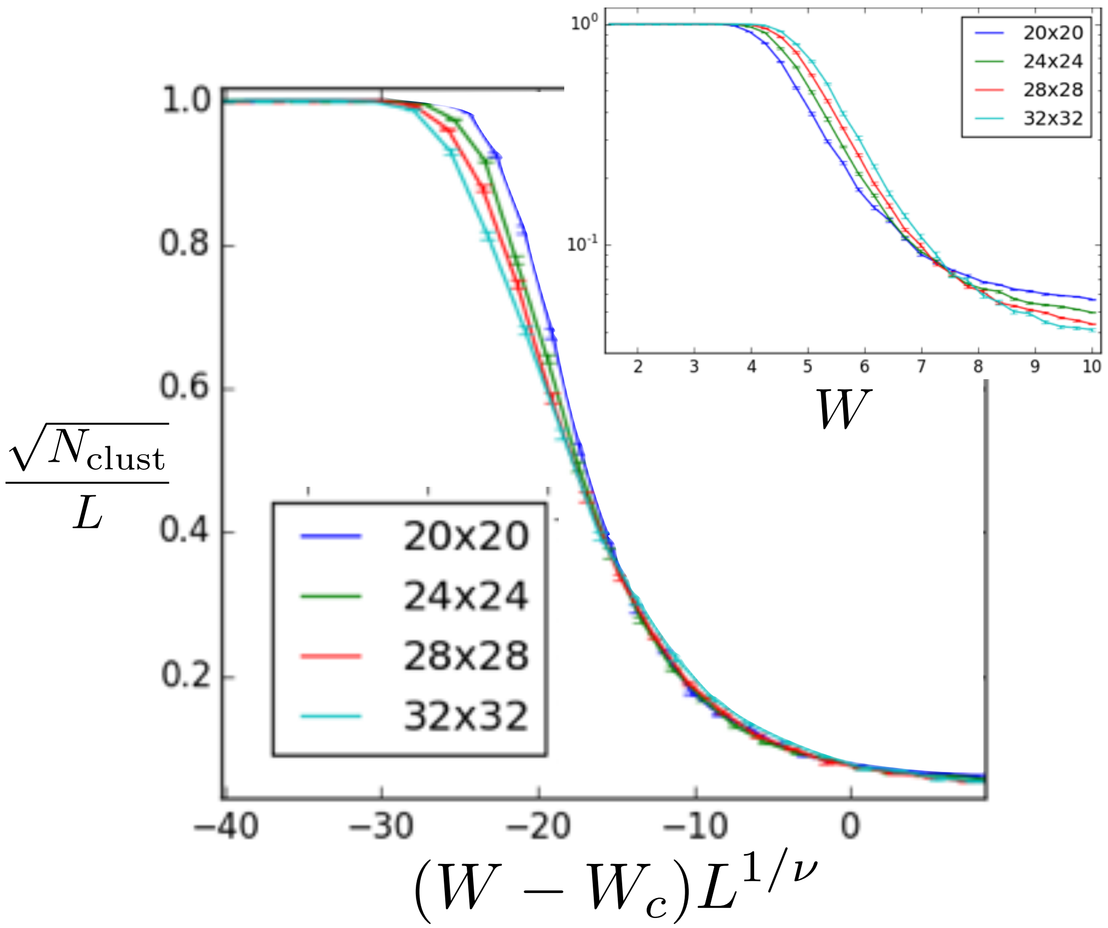
- Rare resonance driven
- Not classical percolation (p_c=50%)

Difference:

- No subdiffusion in d>1

ACP & R. Vasseur (Unpublished)

1D vs 2D MBL Transitions



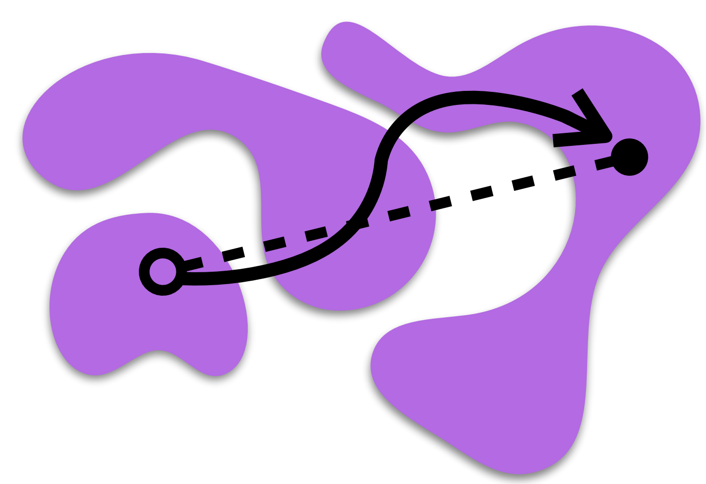
	1D	2D
Correlation Length:	$\nu_{\text{1D}} \approx 3.5$ $p_c^{(1D)} \approx 2\%$	$\nu_{\text{2D}} \approx 1.5$ $p_c^{(2D)} \approx 7\%$
Dynamics:	$\delta x(t) \sim t^{1/z}$ $z \sim \lvert\delta W\rvert^{-\nu}$ $\sigma(\omega) \sim \omega^{1-2/z}$ (Subdiffusion)	$\delta x(t) \sim \sqrt{Dt}$ $D \sim e^{-\xi}$ $\sigma(\omega) \sim \text{const}$ (Diffusion)

Similarity:

- Rare resonance driven
- Not classical percolation (p_c=50%)

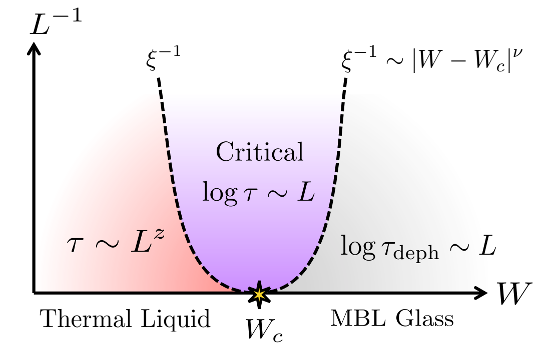
Difference:

- No subdiffusion in d>1



ACP & R. Vasseur (Unpublished)

Open questions



A. Energy density tuned transition (many-body mobility edge)

- If exists - expect same exponent (single relevant perturbation)

ED Numerics: see e.g. Luitz, Laflorencie, Alet '14;
Model proposals: Y. Huang '14;
Doubts on existence: De Roeck et al. '15

B. Time-dependent driving (Floquet MBL transition)

- Different universality class?

Abanin, De Roeck, Huveneers '15
Khemani, Nandkishore, Sondhi '15

C. Long-range interactions

- E.g. power-law interactions, critical analogs of MBL
- Inter-cluster matrix elements may renormalize strongly

Yao et al. PRL '13
Vosk Altman; Pekker, et al.;
ACP Vasseur, Parameswaran

E. Quasi-random potentials?