Early Breakdown of Area-law Entanglement in many-body delocalization
Or
MBL breaks down earlier than you might think

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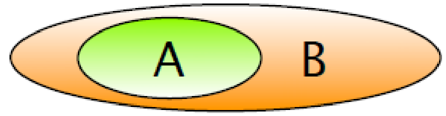
T. Devakul and RRPS PRL 115, 187201 (2015)

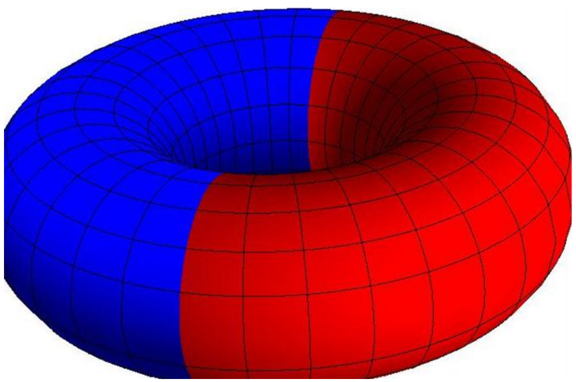
OUTLINE:

1. Entanglement Entropy of Eigenstates: ETH vs MBL
2. Perturbation theory/ NLC formally in thermodynamic limit
3. The NLC method
4. Results for the random-field Heisenberg chain
5. Some results in 2D
6. Conclusions



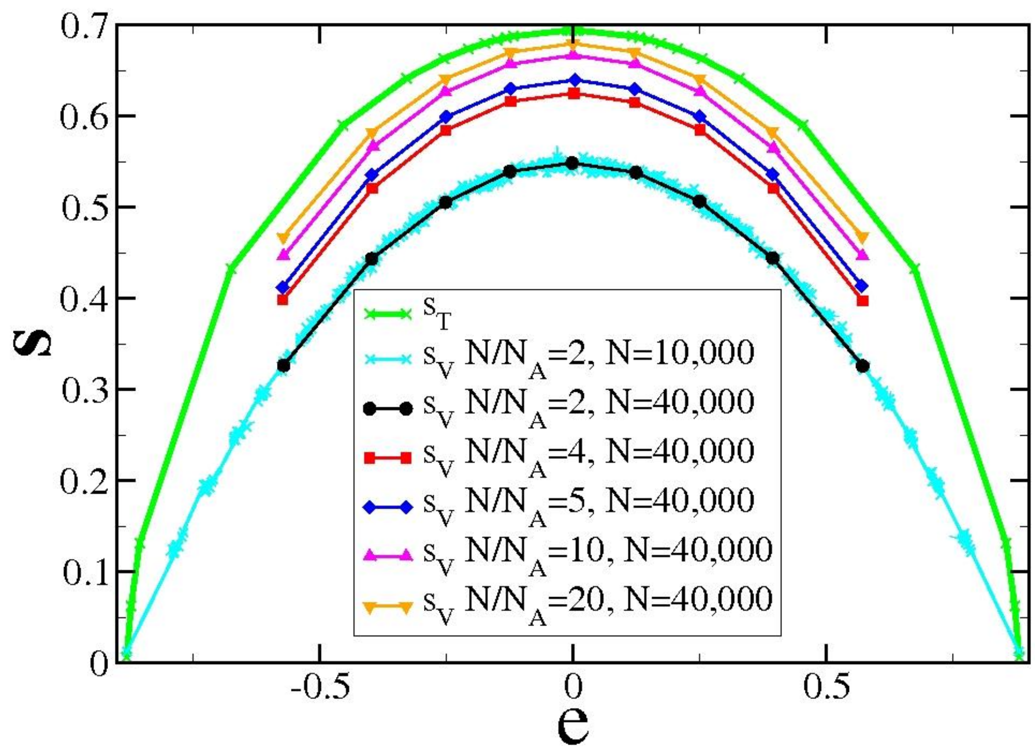
Entanglement entropy of eigenstates can distinguish ETH and MBL

ETH : Thermal entropy --- Volume Law



Free fermion model

M. Storms, RRPS PRE 2014

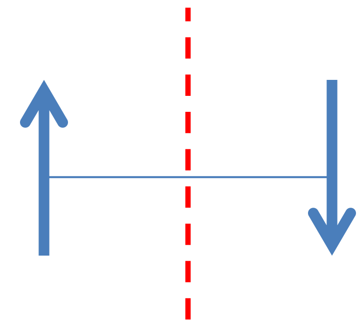


MBL: Local entanglement ---- Area Law even for finite energy-density eigenstates

Perturbation theory/ NLC in thermodynamic limit

Random Field XXZ model

$$H = J \sum S_i S_j \; + \sum h_i \, S_i{}^z$$



Can we show area-law and its breakdown?

Perturbation theory in J ?
But energy denominators cause problems.
When degenerate we should allow for superposition of unperturbed states.

Numerical Linked Cluster (NLC) expansion fixes the problem.
Keep the graphical structure of perturbation theory.
But use exact diagonalization to obtain contributions of clusters.

Numerical Linked Cluster Method

$$S(c) = \sum_{\alpha} \frac{e^{-\beta\epsilon_{\alpha}^{c}}}{\mathcal{Z}} s(|\alpha^{c}\rangle)$$

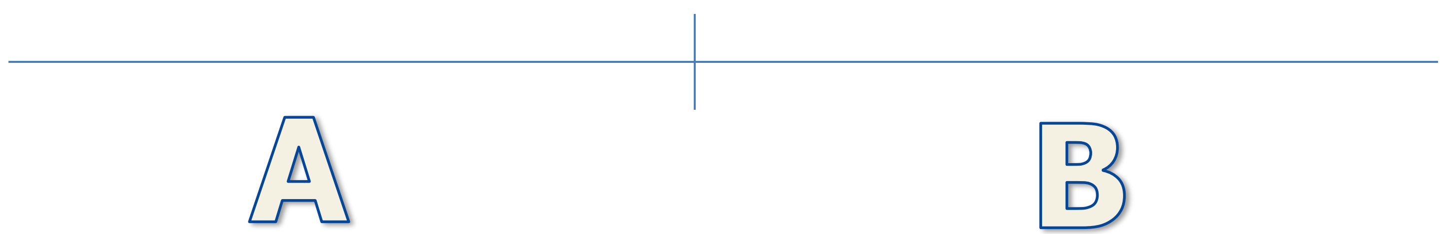
Von Neumann entropy averaged over all states (Boltzmann weighted)

NLC $S(\mathcal{L}) = \sum_c \widetilde{S}(c)$

For the infinite lattice

$$\widetilde{S}(c) = S(c) - \sum_{c' \subset c} \widetilde{S}(c')$$

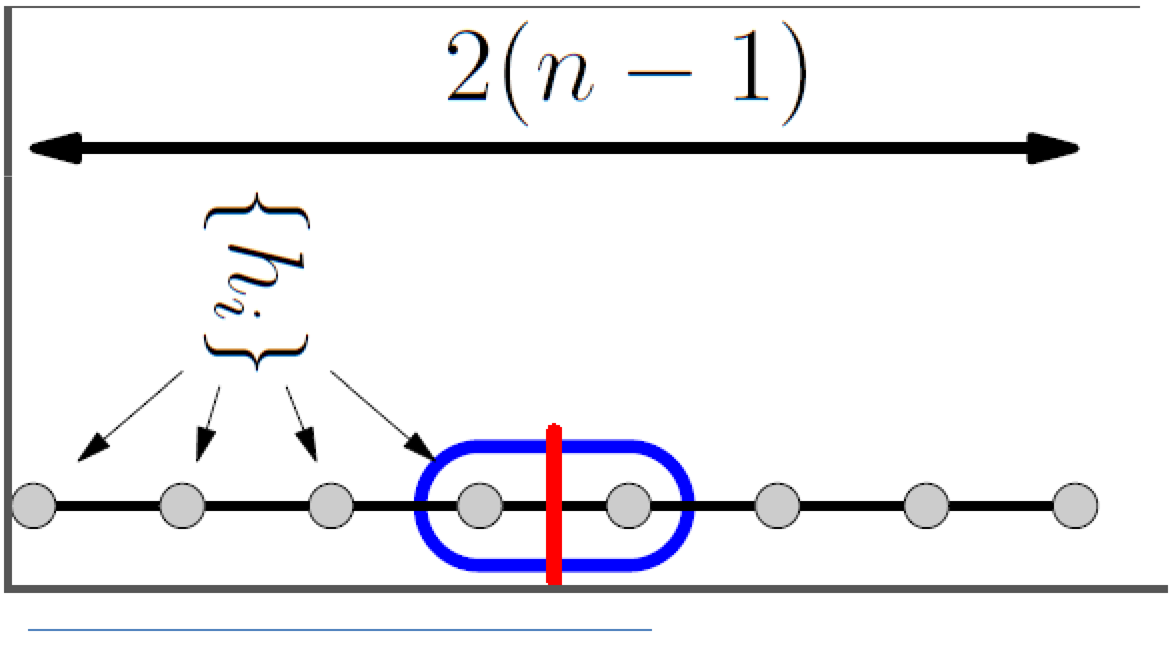
Contribution of a cluster



Only connected clusters that cross the interface contribute
Convergence of NLC implies area-law

Disorder Averaging

In one-d only a finite part of the chain matters in order n
We assign a field to every site and calculate properties
Then average over disorder at the end

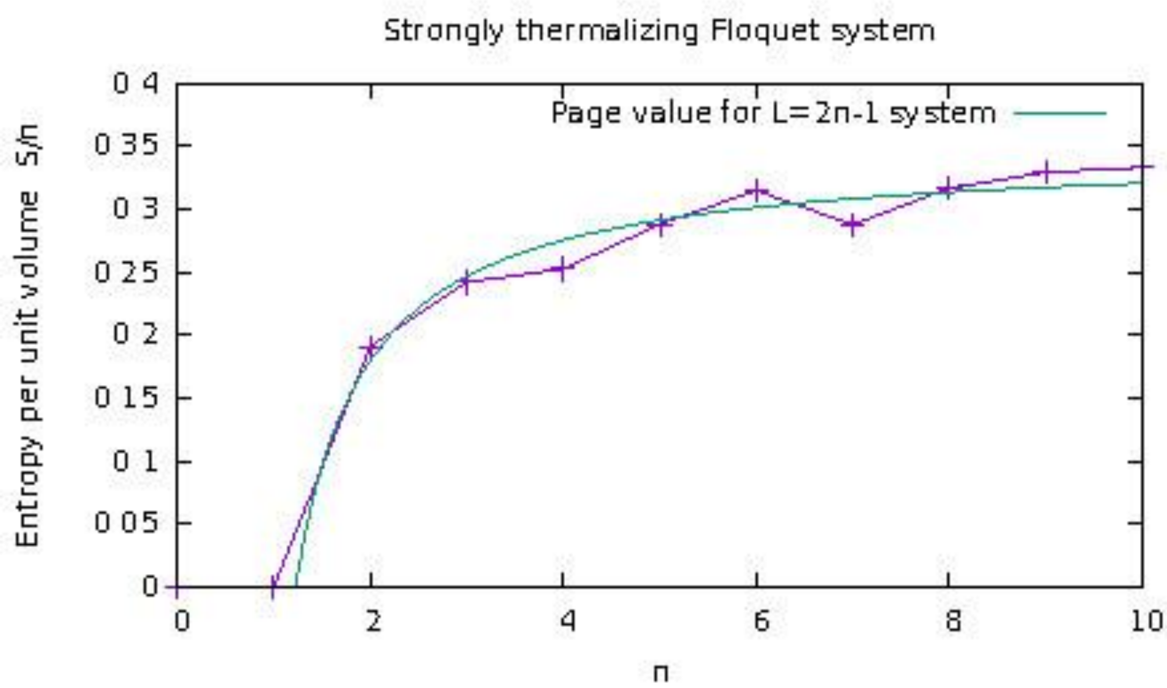


n=5

Gives us distribution over disorder
Mean, Median, Standard Deviation,
Tails of Distribution, …….

NLC is non-perturbative: Can get MBL and ETH phases

Random state (Haar measure) gives $\frac{S}{n} = \frac{\ln 2}{2} + O(1/n)$



Floquet Model has a strongly thermalizing Phase PRE (2015) Zhang, Kim and Huse

$$H = \sum_{i=1}^{L} g\sigma_i^x + \sum_{i=1}^{L} h\sigma_i^z + \sum_{i=1}^{L-1} J\sigma_i^z\sigma_{i+1}^z$$

$$U_F(\tau) = e^{-iH_x\tau}e^{-iH_z\tau}$$

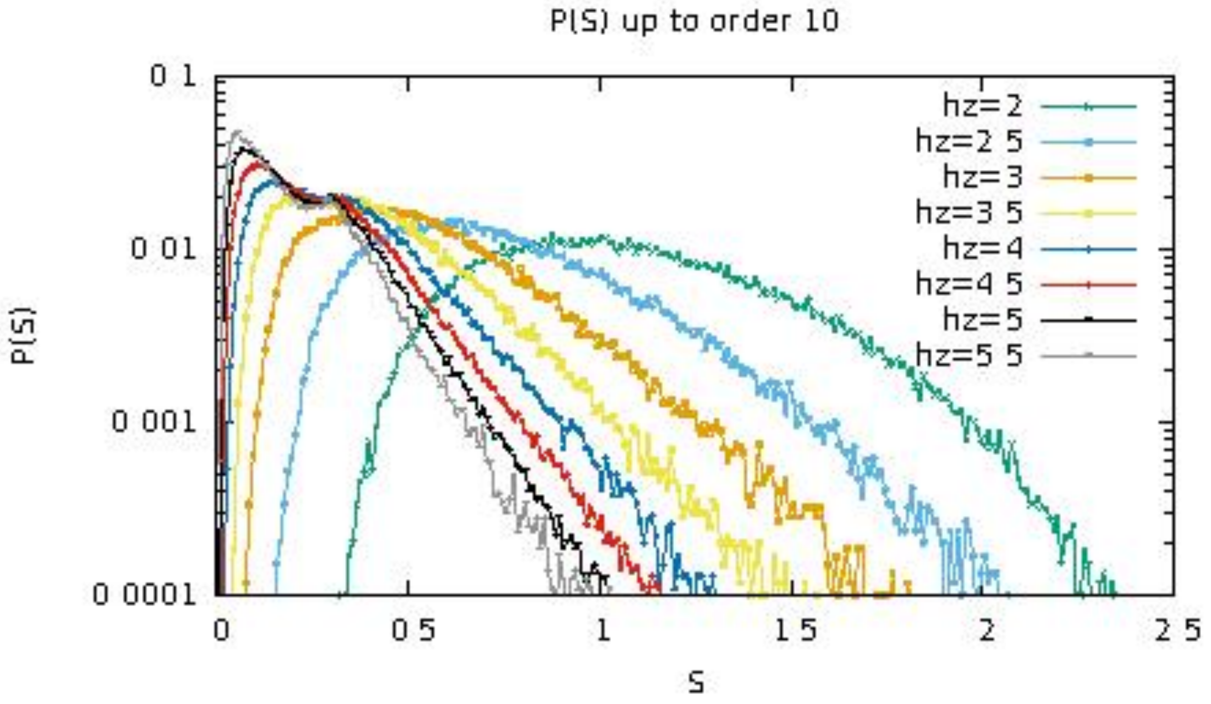
Heisenberg chain in a random field

$$\mathcal{H} = \sum_i h_i S_i^z + \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j, \qquad h_i \in [-h, h].$$

Has been studied numerically by many authors
A. Pal and D. Huse PRB 2010

For what h values is there an MBL phase?

Probability distribution of eigenstate-averaged entropies ($\beta = 0$)



Exponential decay at large S
Clear difference between large and small h at low S

Thermal Phase

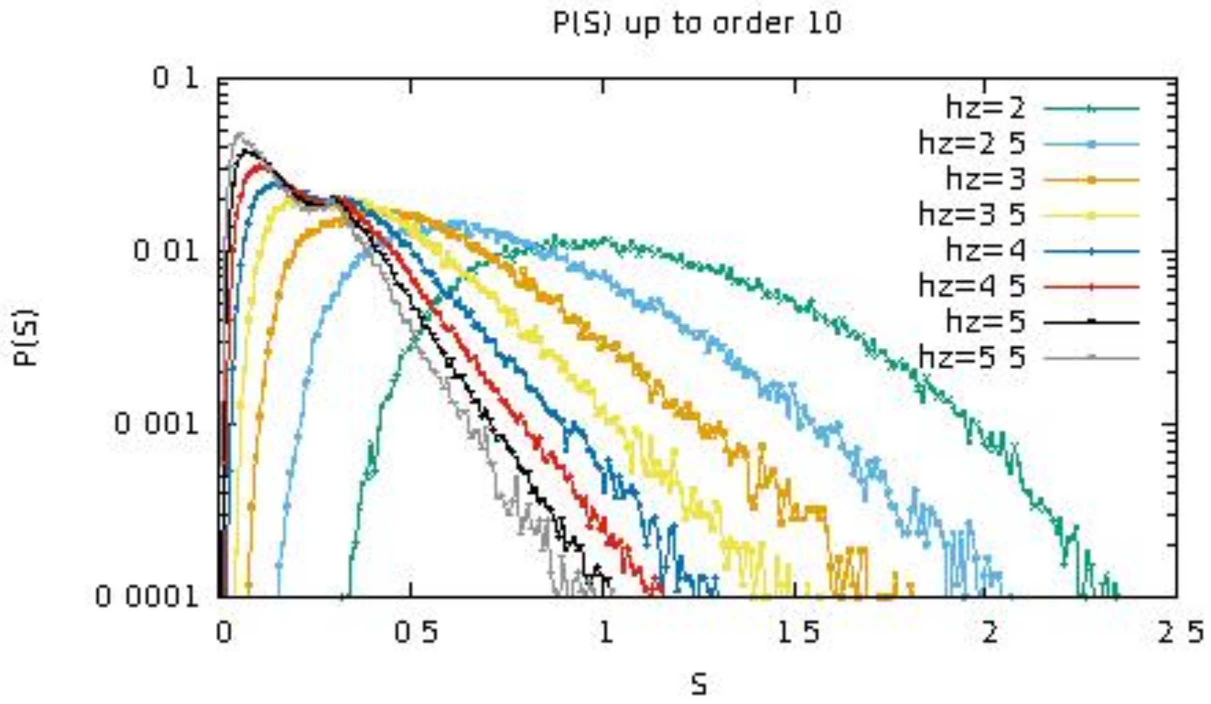
S should scale with n

Width should presumably scale with $\sqrt{n}$

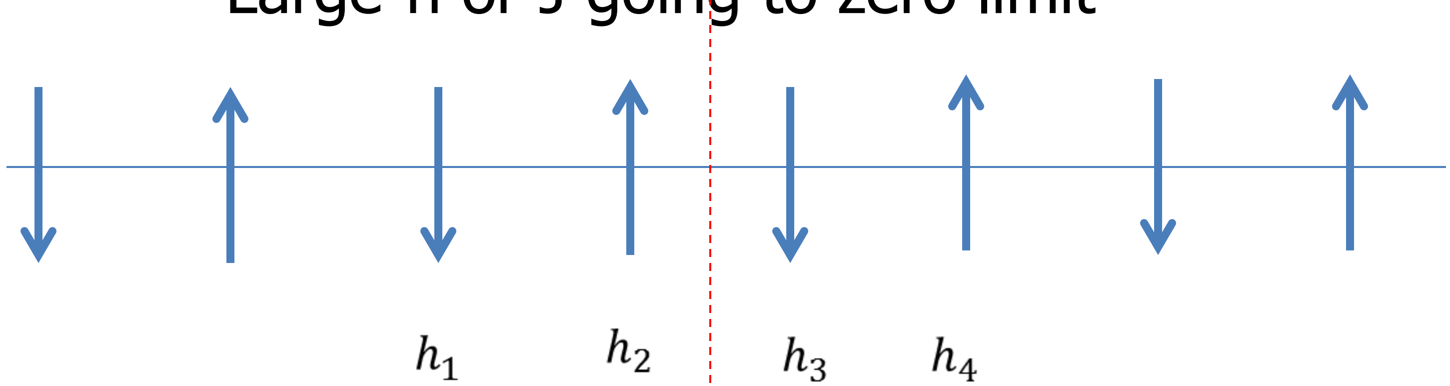
MBL phase

Peak at small S

Shoulder at $\ln 2\ /2$



Deep in MBL Phase
Large-h or J going to zero limit



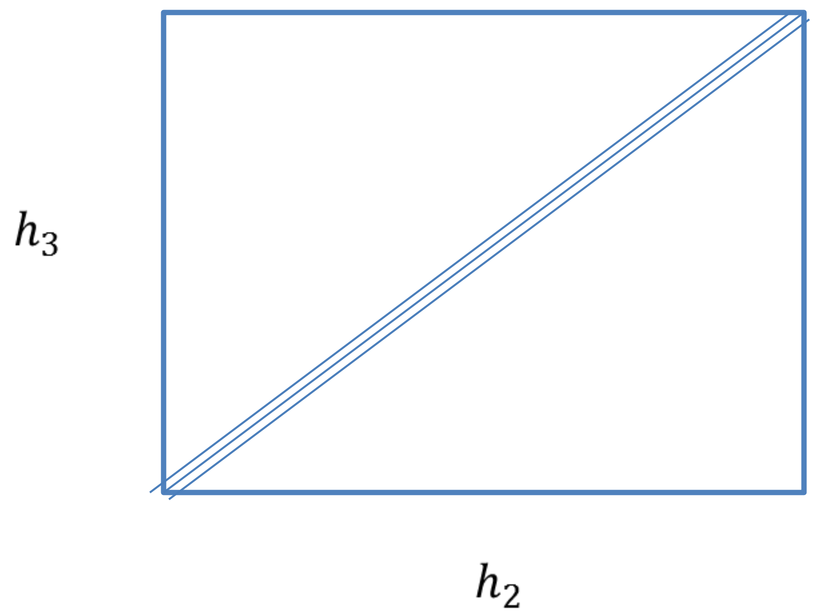
If $|h_2 - h_3| \gg J$ entanglement is small typically $\sim |\frac{J}{h_2 - h_3}|^2$

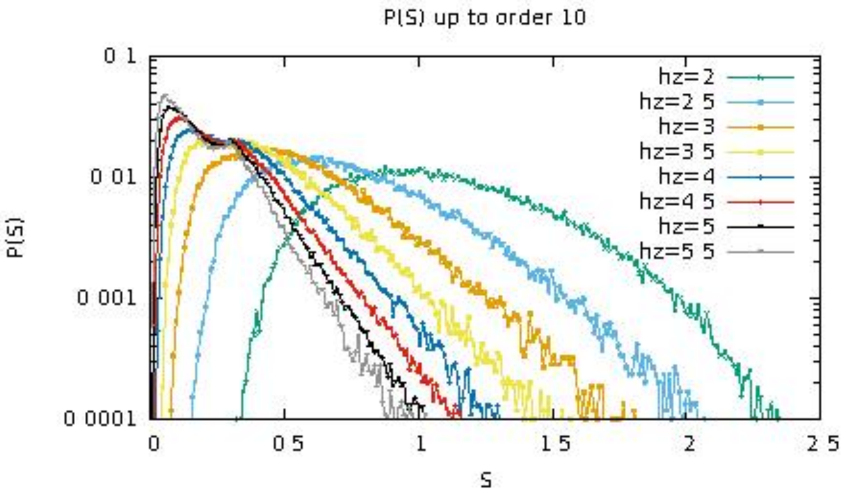
If $|h_2 - h_3| \ll J$ Resonance
States of 4 type

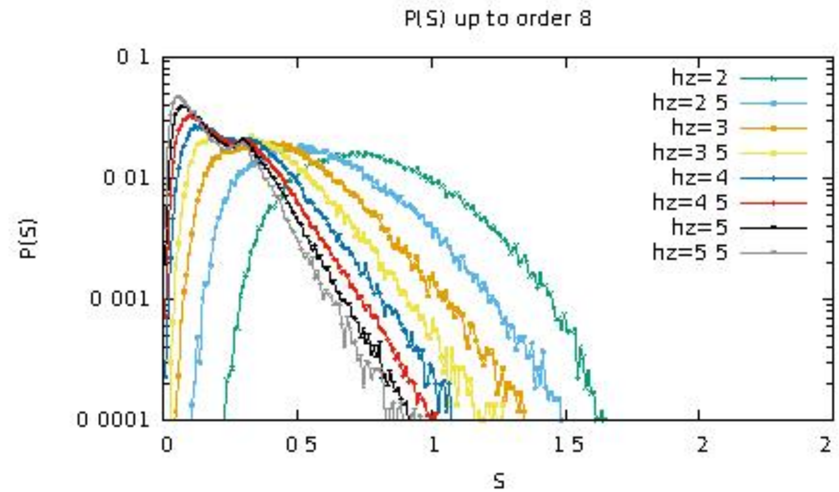
$\uparrow\uparrow$ $\quad$ $\downarrow\downarrow$ $\quad$ s=0

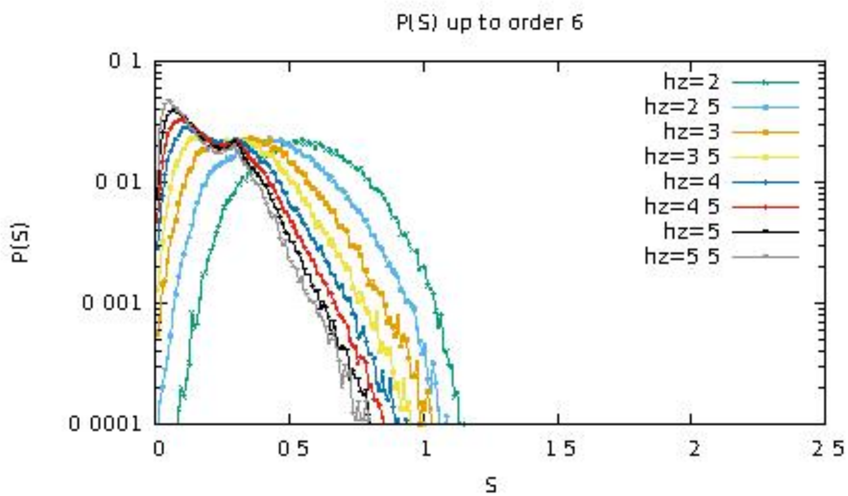
$\uparrow\downarrow - \downarrow\uparrow$

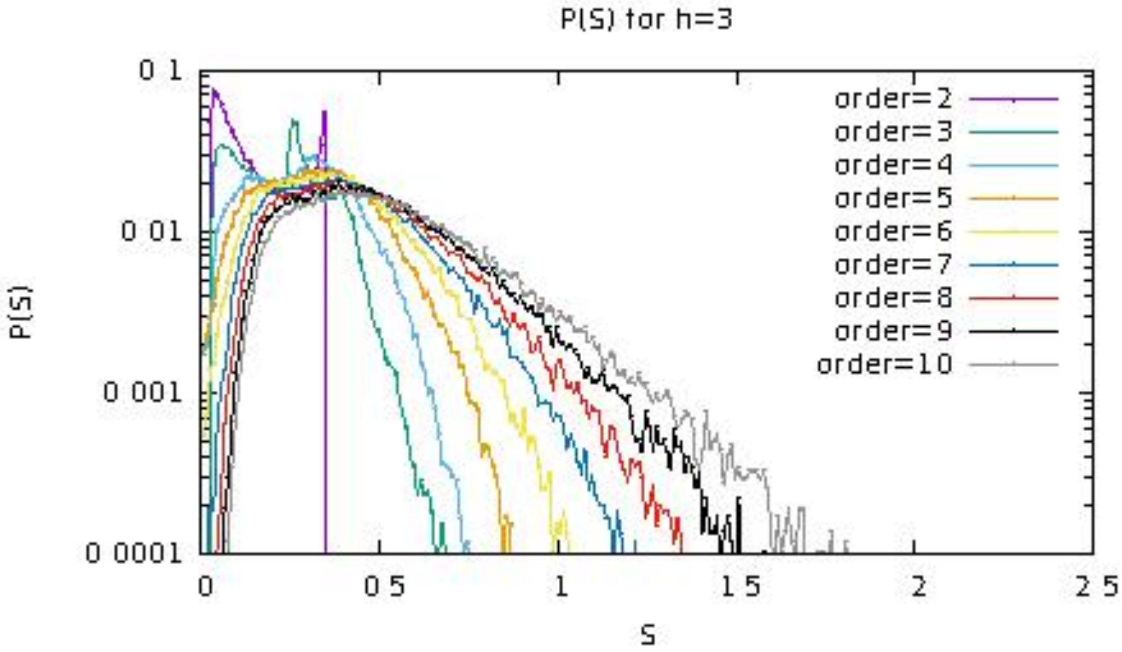
$\uparrow\downarrow + \downarrow\uparrow$ $\quad$ s=$\ln 2$



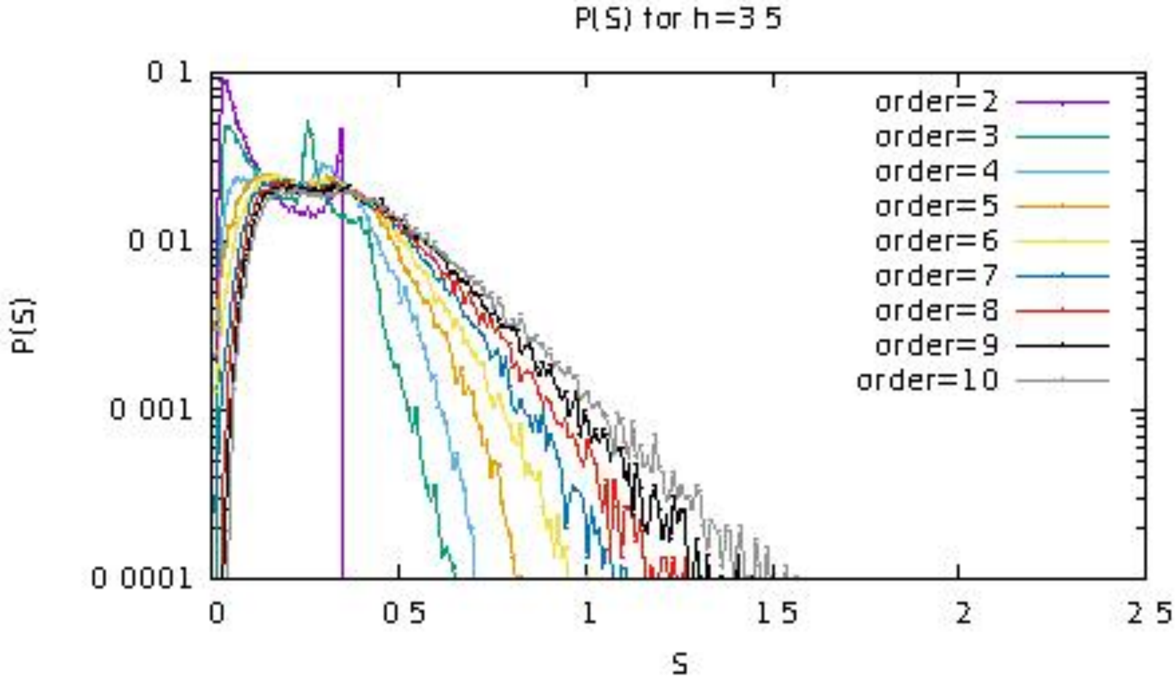

P(S) up to order 10
0 1
0 01
0 001
0 0001
P(S)
0
0 5
1
1 5
2
2 5
S
hz=2
hz=2 5
hz=3
hz=3 5
hz=4
hz=4 5
hz=5
hz=5 5


P(S) up to order 8
0 1
0 01
0 001
0 0001
P(S)
0
0 5
1
1 5
2
2 5
S
hz=2
hz=2 5
hz=3
hz=3 5
hz=4
hz=4 5
hz=5
hz=5 5

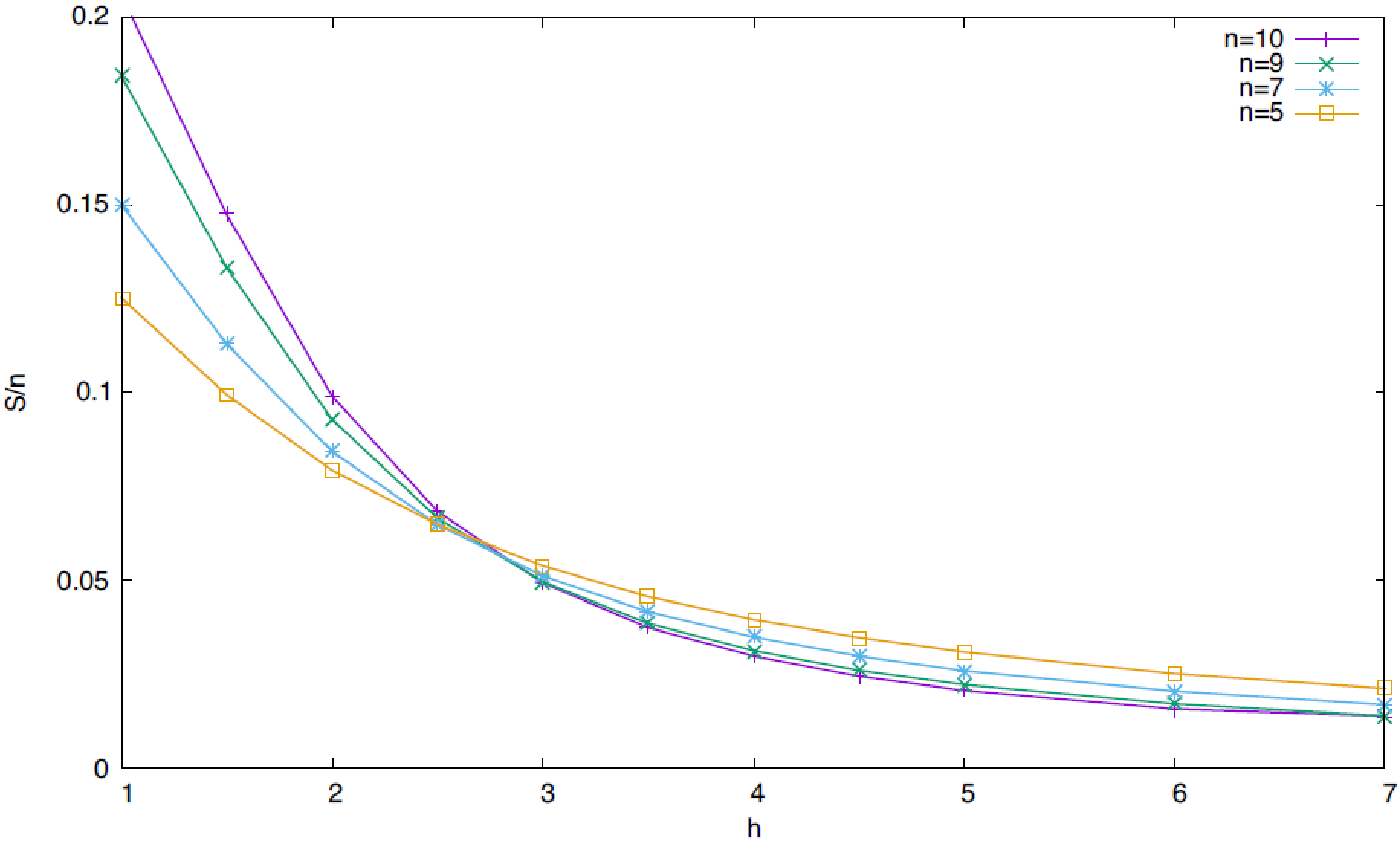

P(S) up to order 6
0 1
0 01
0 001
0 0001
P(S)
0
0 5
1
1 5
2
2 5
S
hz=2
hz=2 5
hz=3
hz=3 5
hz=4
hz=4 5
hz=5
hz=5 5



h=3.5 not in MBL?

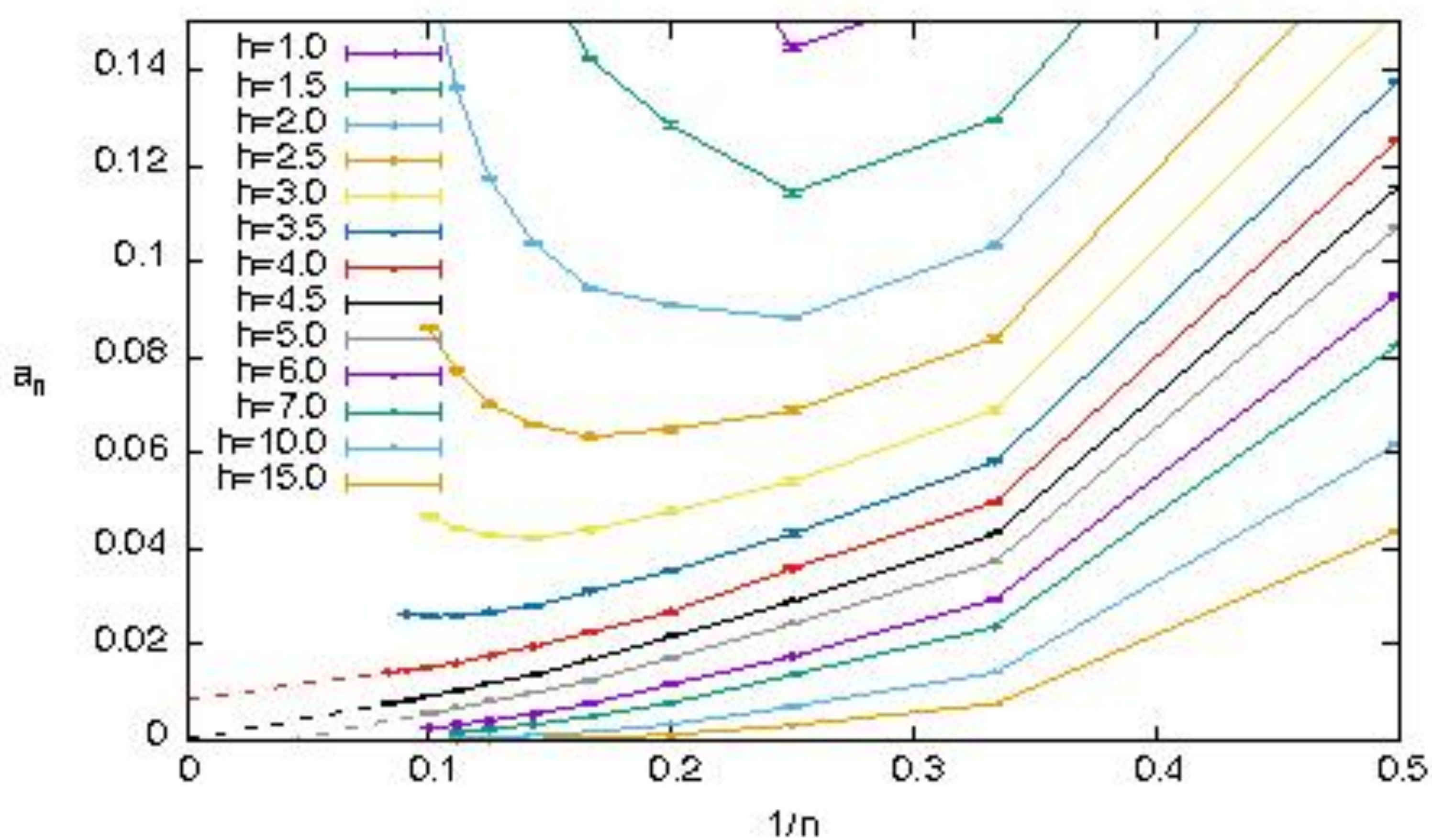


Looking for Volume Law: Partial sums at different order of NLC



Not a sensitive way to detect the transition

$S = \sum_n a_n$: a_n disorder averaged contribution from all n-site clusters
Is the sum converging?



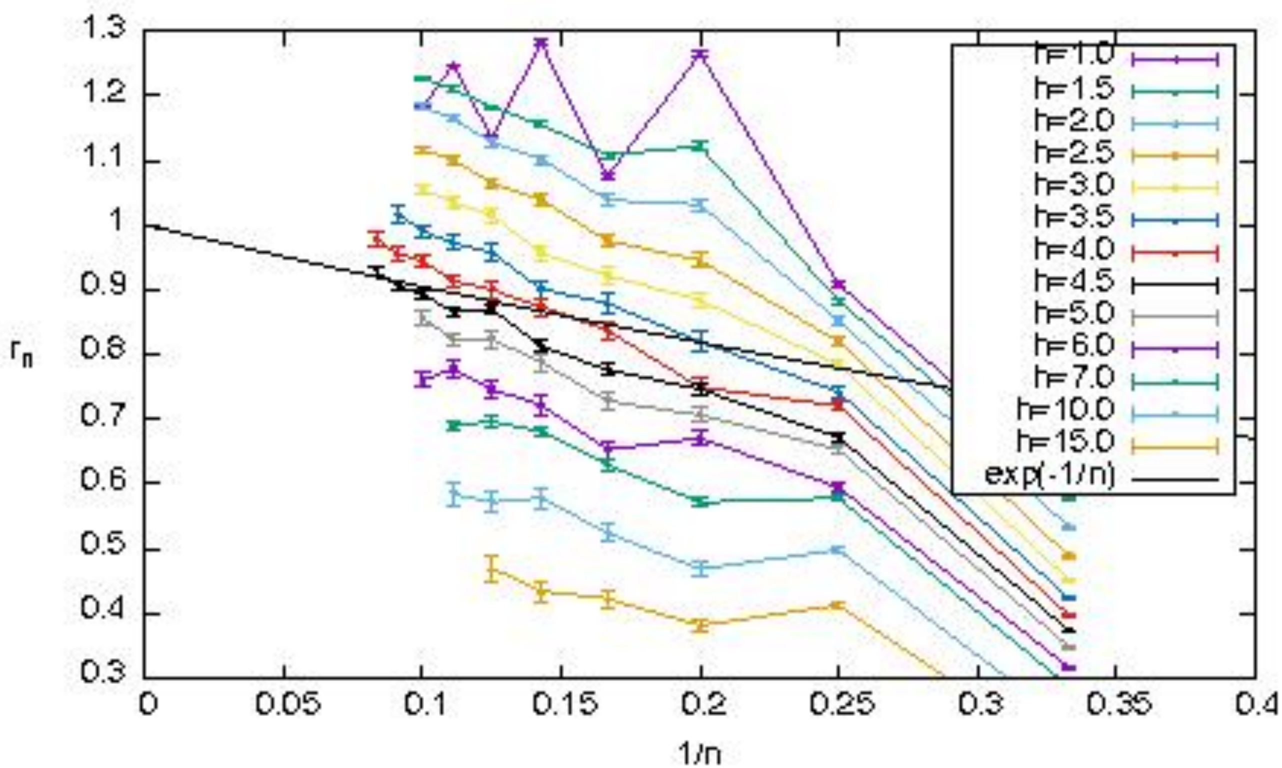
Trend: What appears convergent at small n turns around at larger n
MBL must have exponential convergence:
Must approach zero with a vanishing slope : critical h closer to 4.5

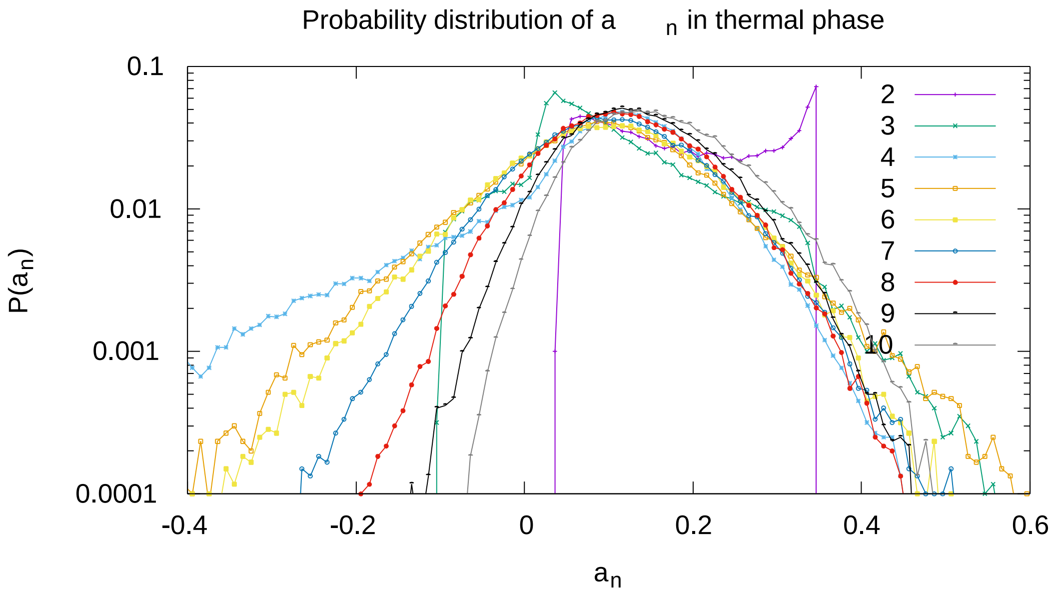
Ratios: $r_n = a_n/a_{n-1}$ are even more sensitive to detecting onset

$$a_n = C n^{-k} \exp(-n/\xi)$$

$$r_n = a_n/a_{n-1} = (1 - k/n) \exp(-1/\xi)$$

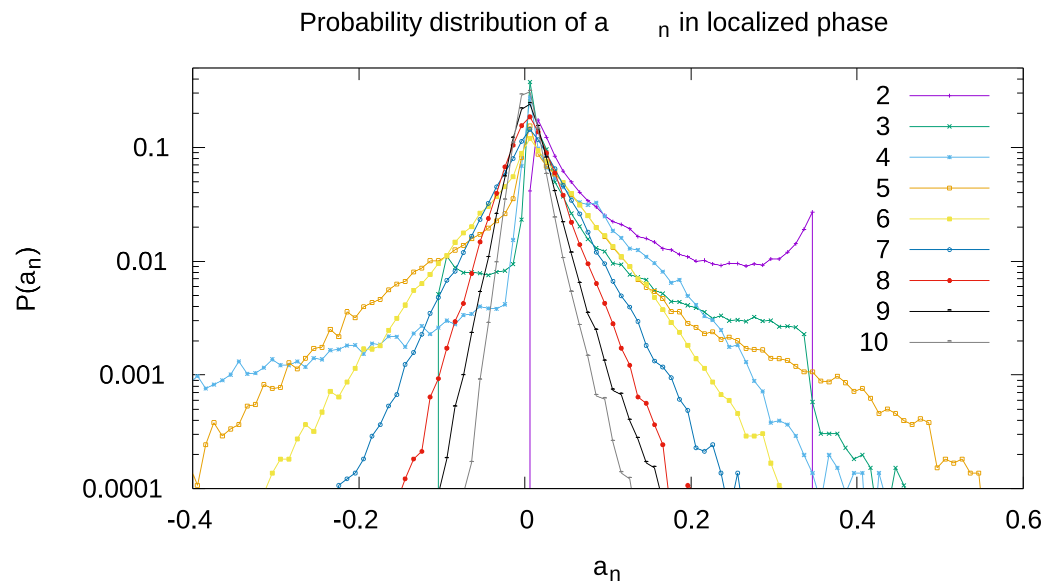
$$r_n \leq r_\infty = \exp(-1/\xi) < \exp(-1/n)$$




Probability distribution of a n in thermal phase
P(a n)
0.1
0.01
0.001
0.0001
-0.4
-0.2
0
0.2
0.4
0.6
a n
2
3
4
5
6
7
8
9
10

h=2

h=5.5


Probability distribution of a n in localized phase
P(a n)
0.1
0.01
0.001
0.0001
-0.4
-0.2
0
0.2
0.4
0.6
a n
2
3
4
5
6
7
8
9
10

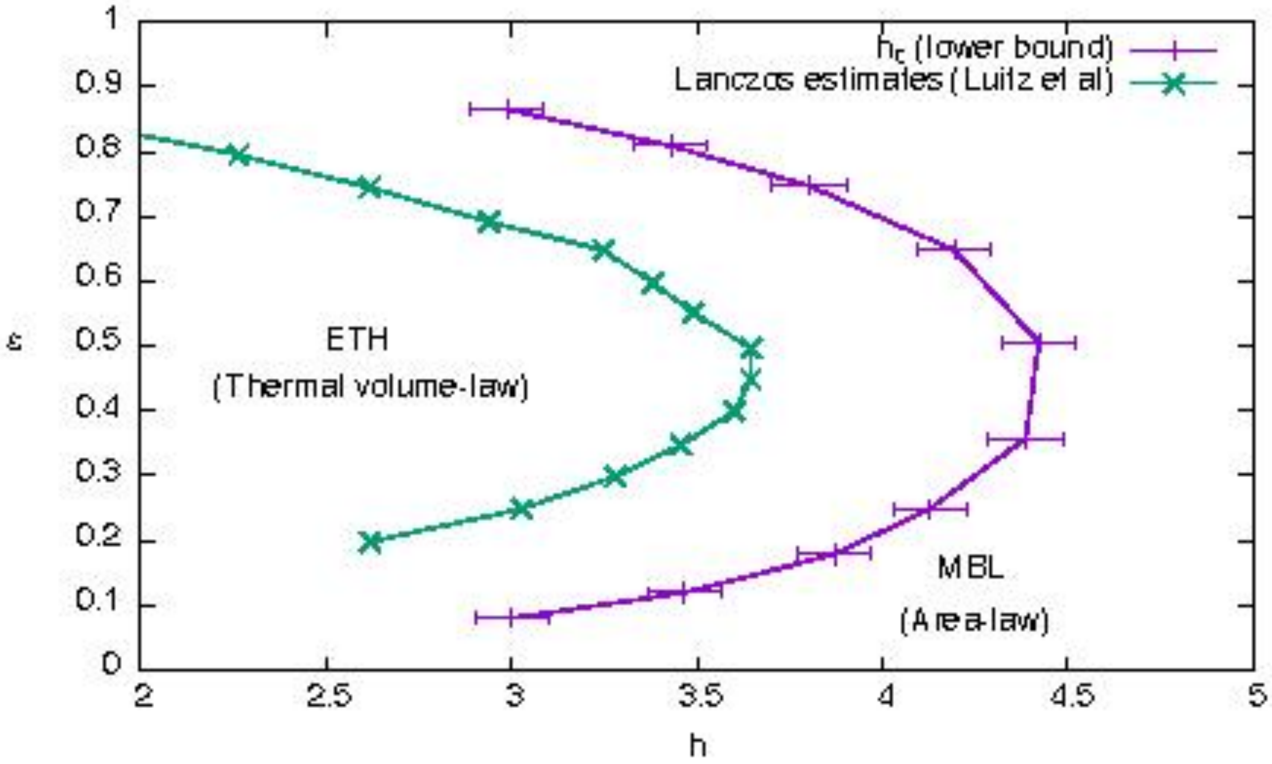
Mobility Edge? Look at states in a temperature window

$$S(c) = \sum_{\alpha} \frac{e^{-\beta \epsilon_{\alpha}^{c}}}{Z} s(|\alpha^{c}\rangle)$$

Each temperature window corresponds to a definite energy
When do states at that energy have `area-law entanglement’

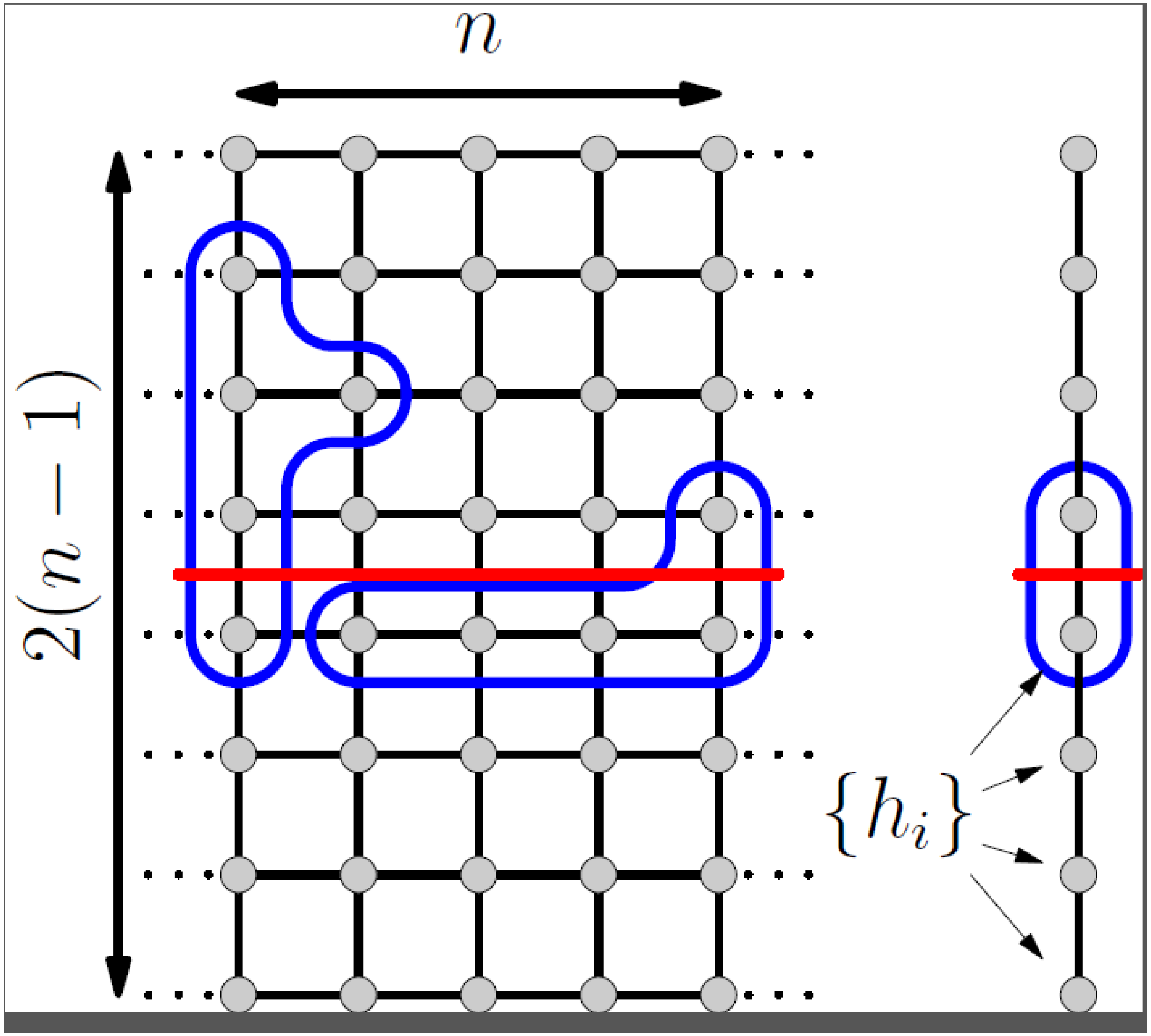
Mobility Edge? Look at states in a temperature window

Compare with ED (Lanczos) D. J. Luitz, N. Laflorencie, F. Alet, PRB 2015



Looking for the full transition systematically overestimates MBL

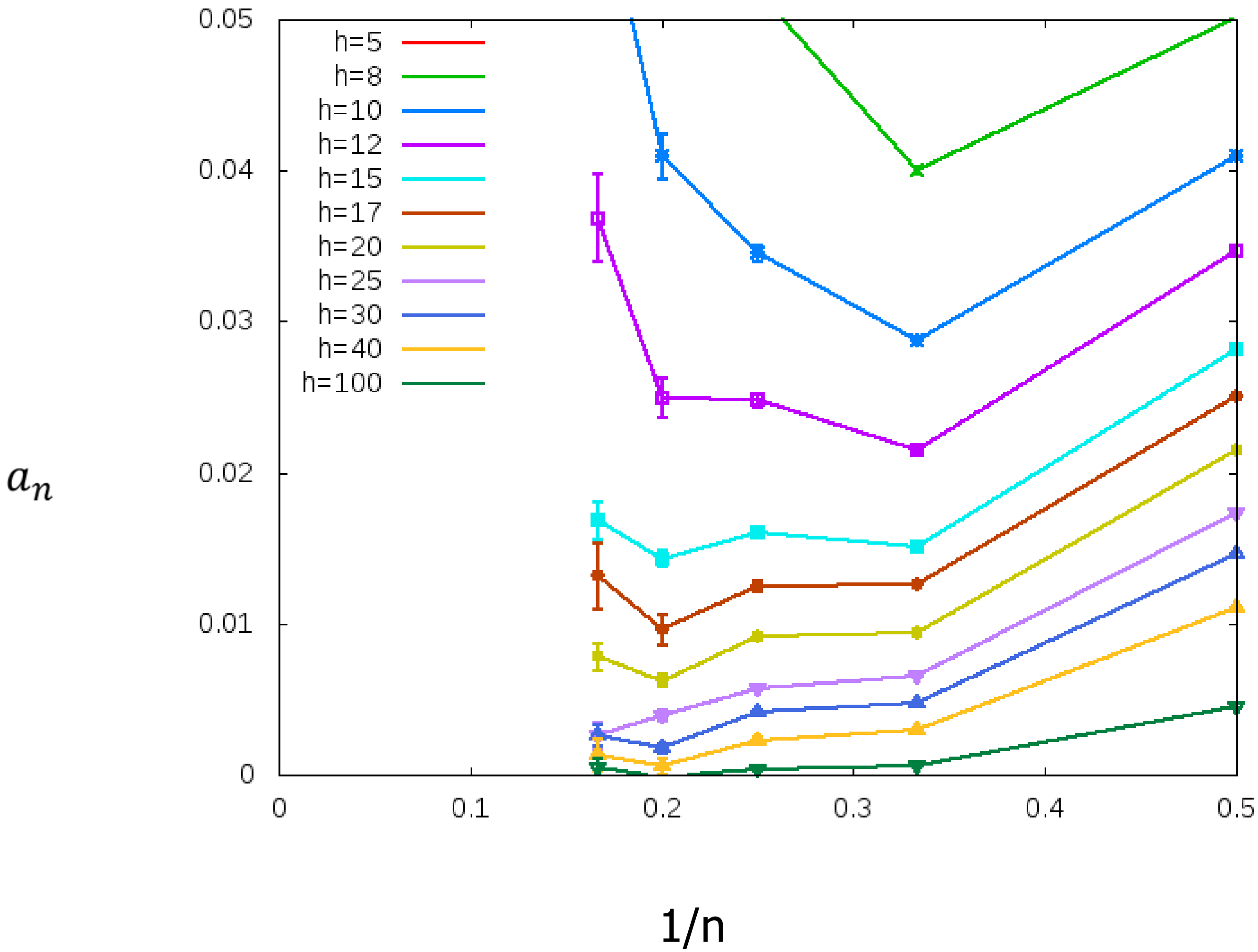
Higher Dimension



A

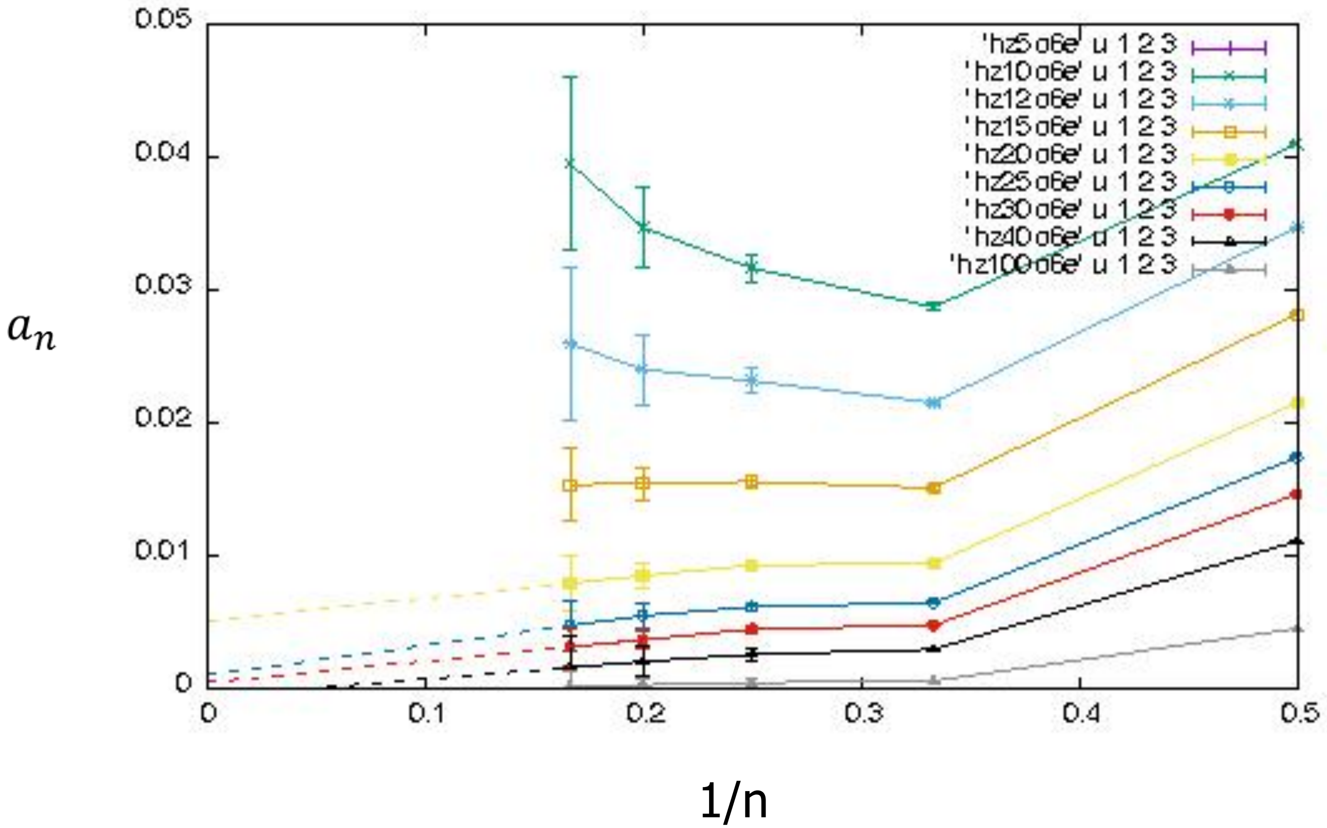
B

There should be self averaging from transverse extent. Focus on the mean



Critical h is closer to 40 (does not scale with z)

Strong alternation --- use Euler like summation/averaging



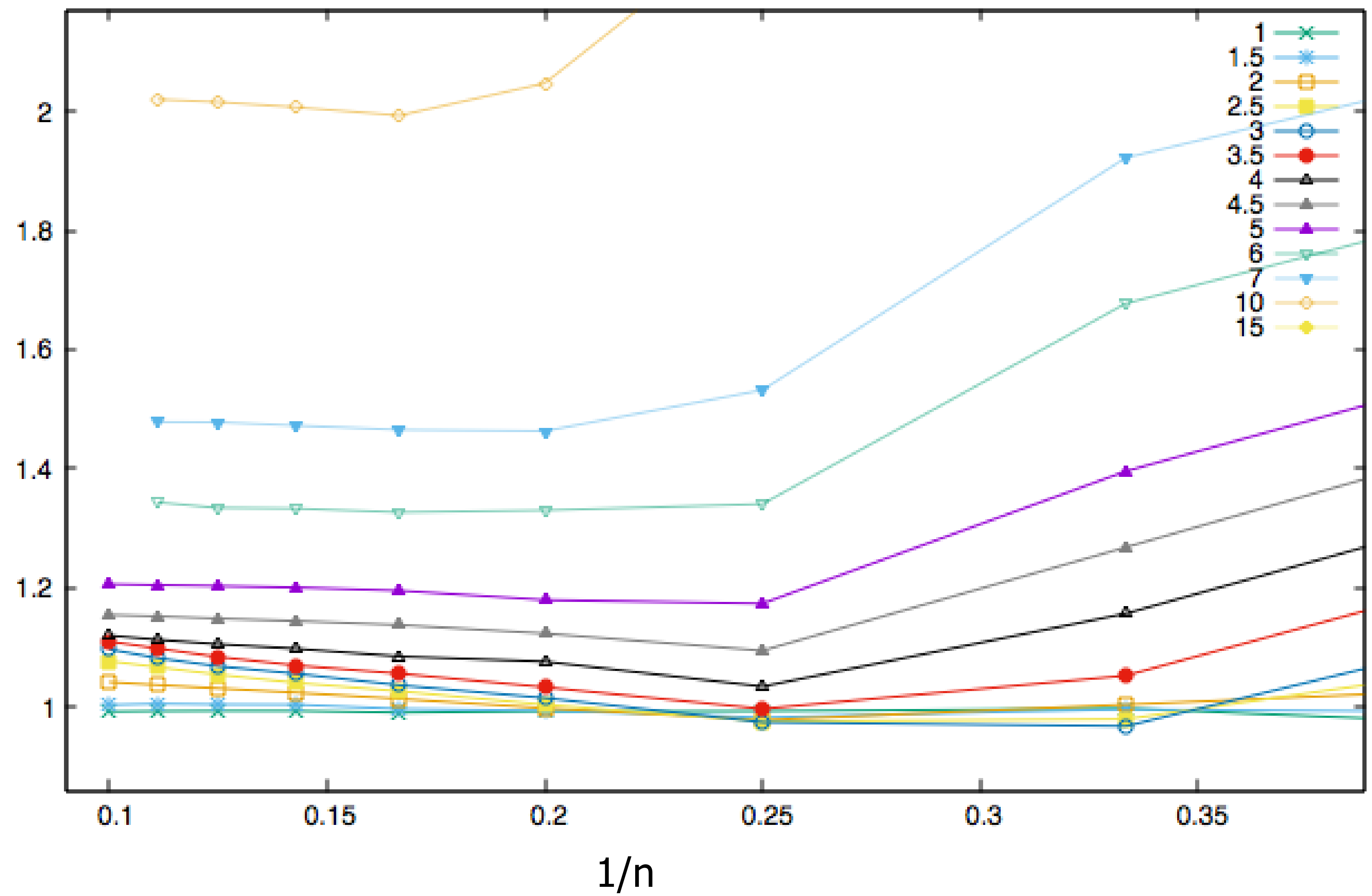
Critical h is closer to 40 (does not scale with z)

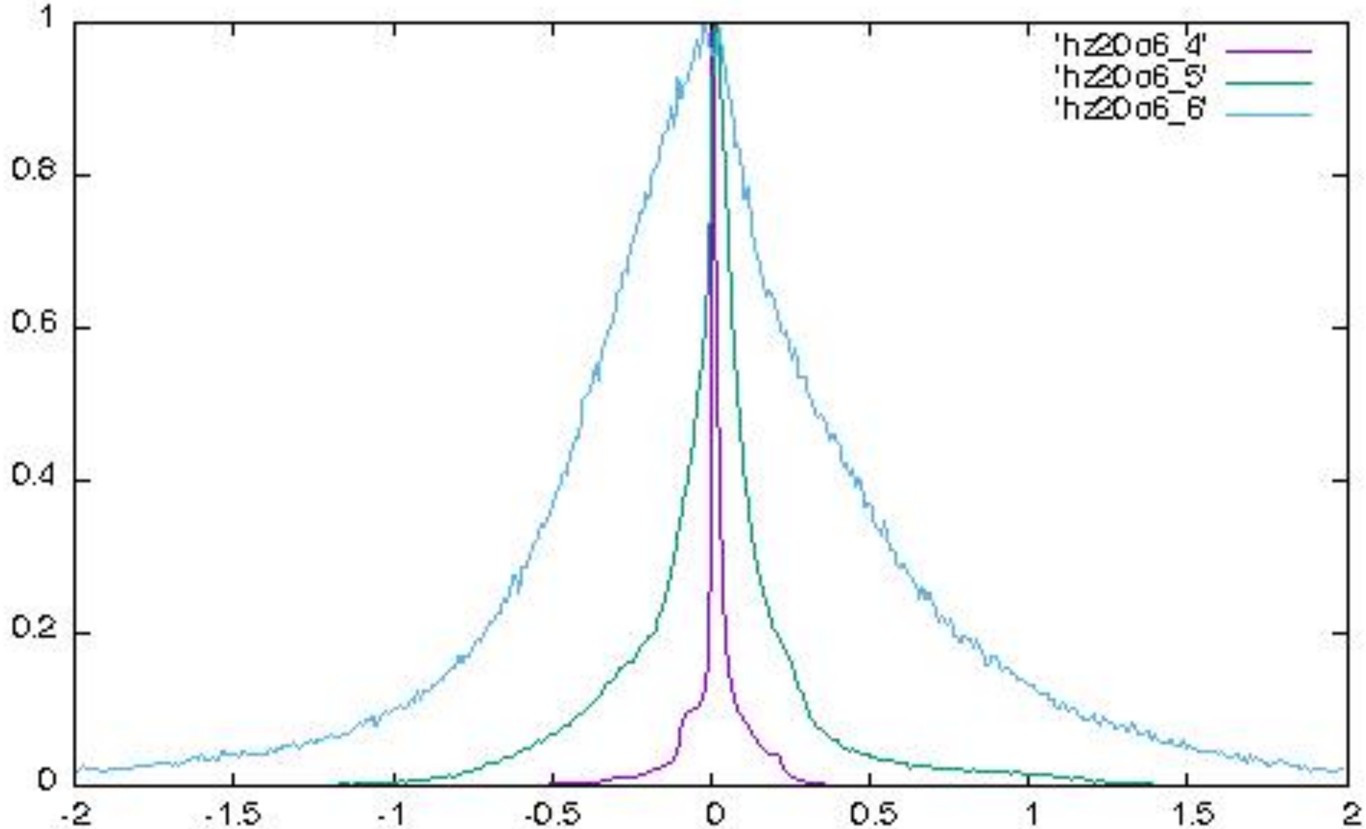
Conclusions

- Numerical Linked Cluster Expansion allows calculation of eigenstate entanglement thermally averaged over all eigenstates
- There appears a clear difference between MBL and ETH phases at finite orders
- States that appear localized in low orders can become delocalized ultimately
- Finite size studies overestimate the MBL phase
- Analysis at finite T suggests a mobility edge
- MBL phase shrinks rapidly with increasing dimensions

THE END

Ratio of median to mean





'hz20o6_4'
'hz20o6_5'
'hz20o6_6'
1
0.8
0.6
0.4
0.2
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

Looking at median instead of mean gives similar results