KITP, Santa Barbara,
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RANDOM MATRIX MODEL WITH LOCALIZATION AND ERGODIC TRANSITIONS

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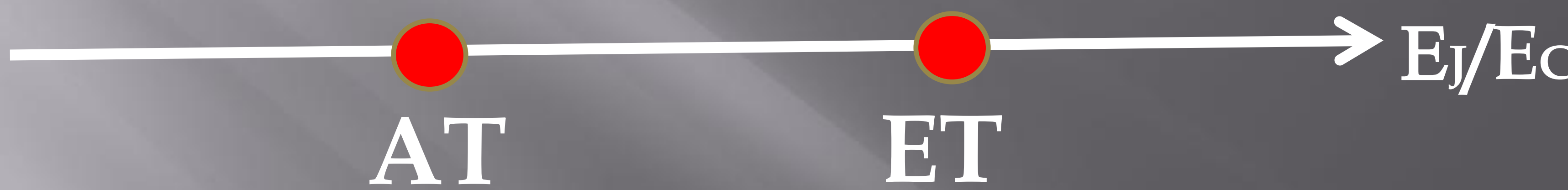
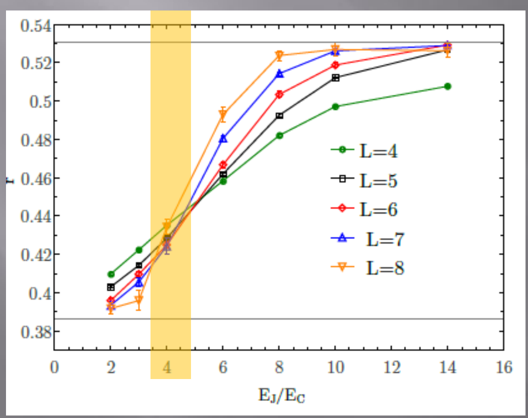
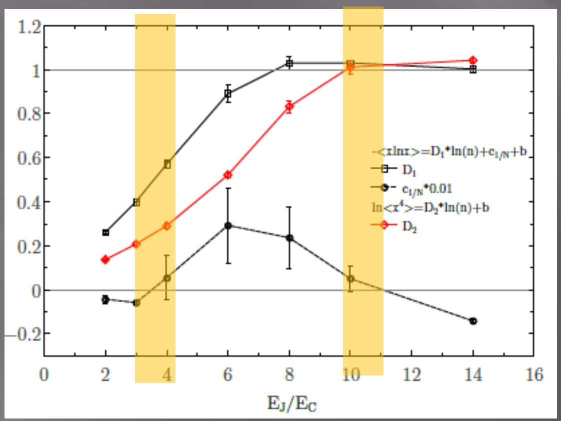
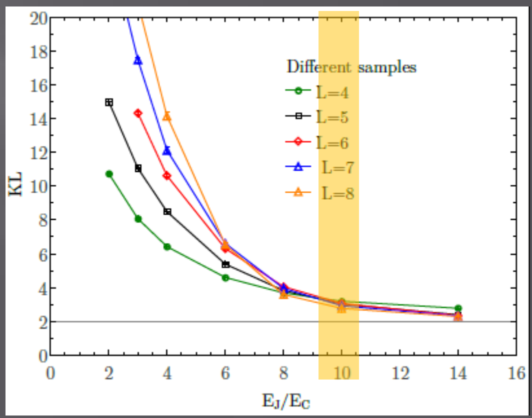
Two localization transitions in JJA?

many-body insulator **bad metal** **good metal**





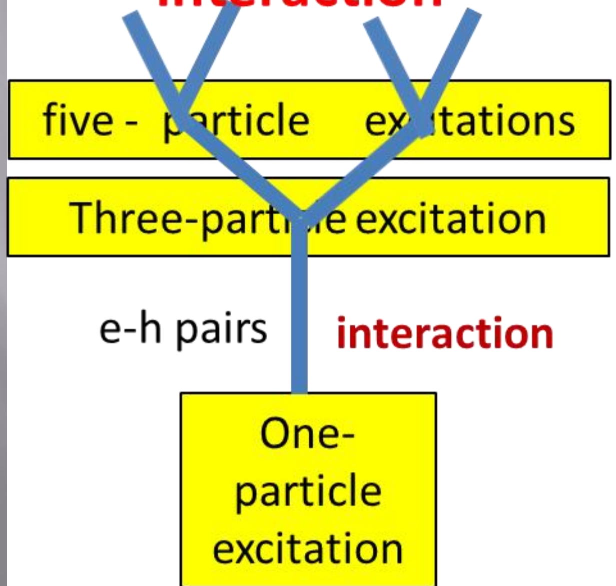




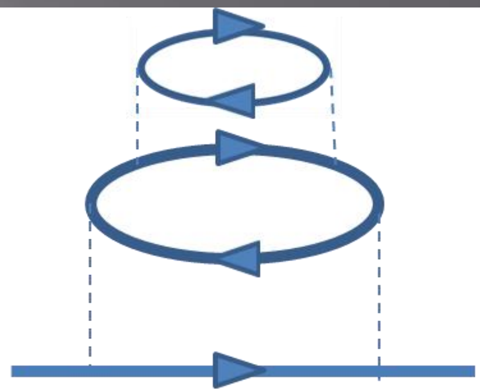
Pino Garcia, Altshuler, Ioffe, VEK

Hierarchical structure of many-body Fock space

Tree-like structure of many-body interaction



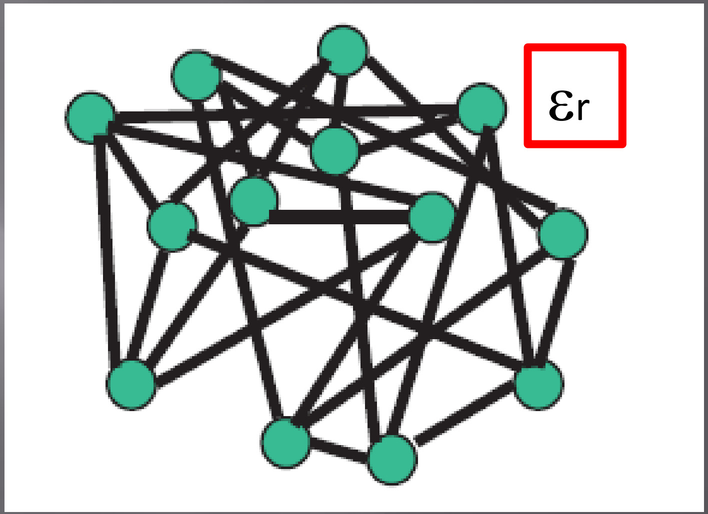
Altshuler, Gefen, Kamenev, Levitov, 1997



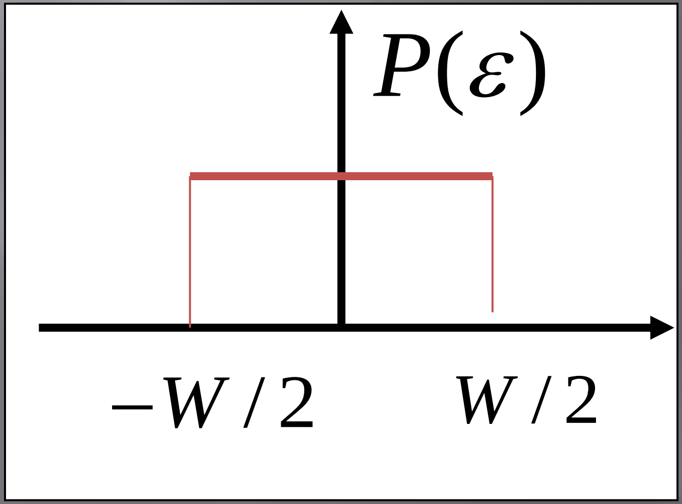
Basko, Aleiner, Altshuler, 2005

Anderson model on random regular graphs (RRG)

$$H = -I \sum_{<r,r'>} c_r^+ c_{r'} + \sum_r \varepsilon_r n_r$$



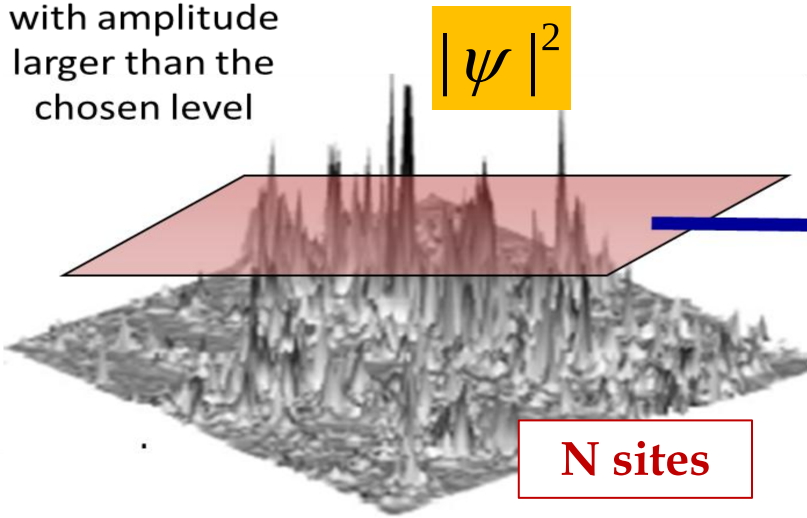
branching number K

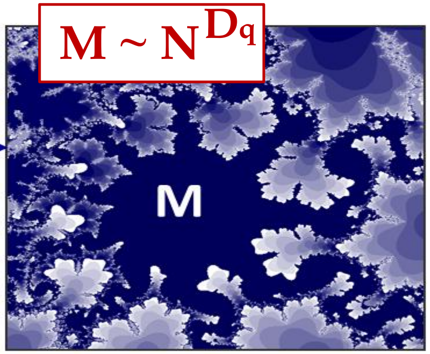


Disorder strength W

Ergodic and non-ergodic extended states

Map of the regions with amplitude larger than the chosen level



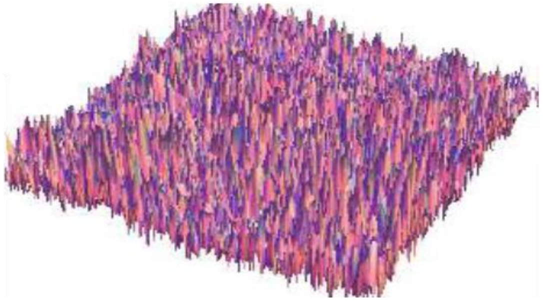


Non-ergodic, fractal states:

M/N->0

M->infty

$$I_q = \left\langle \sum_r |\Psi(r)|^{2q} \right\rangle \propto \frac{1}{N^{(q-1)D_q}}$$

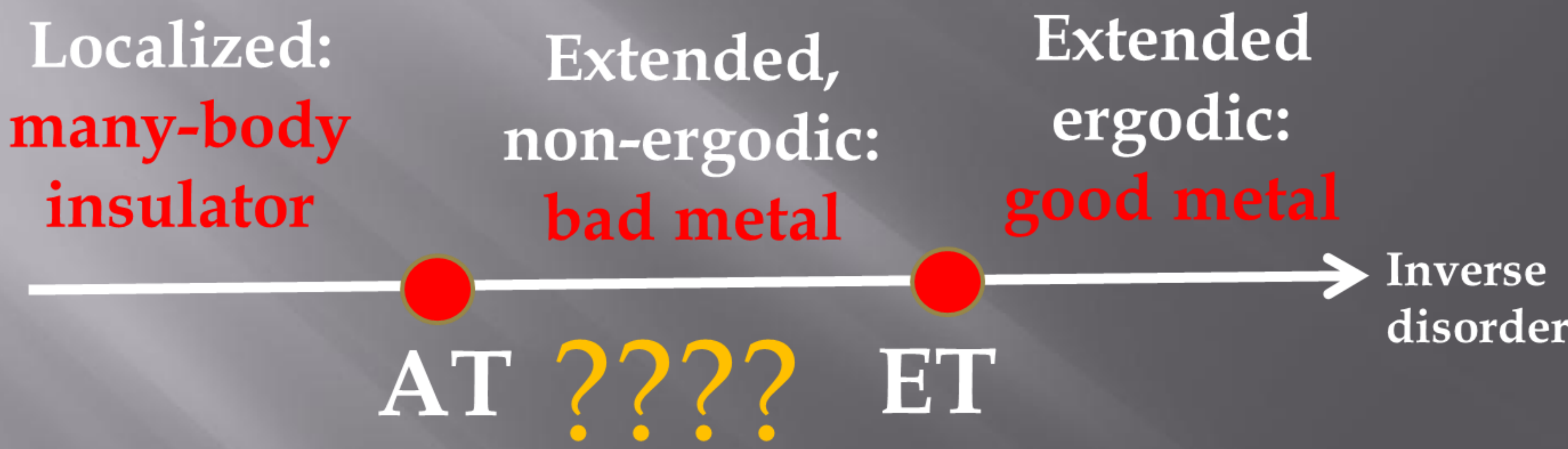


Ergodic states:

M/N->cst

M->infty

Search for a model with both localization and ergodic transitions

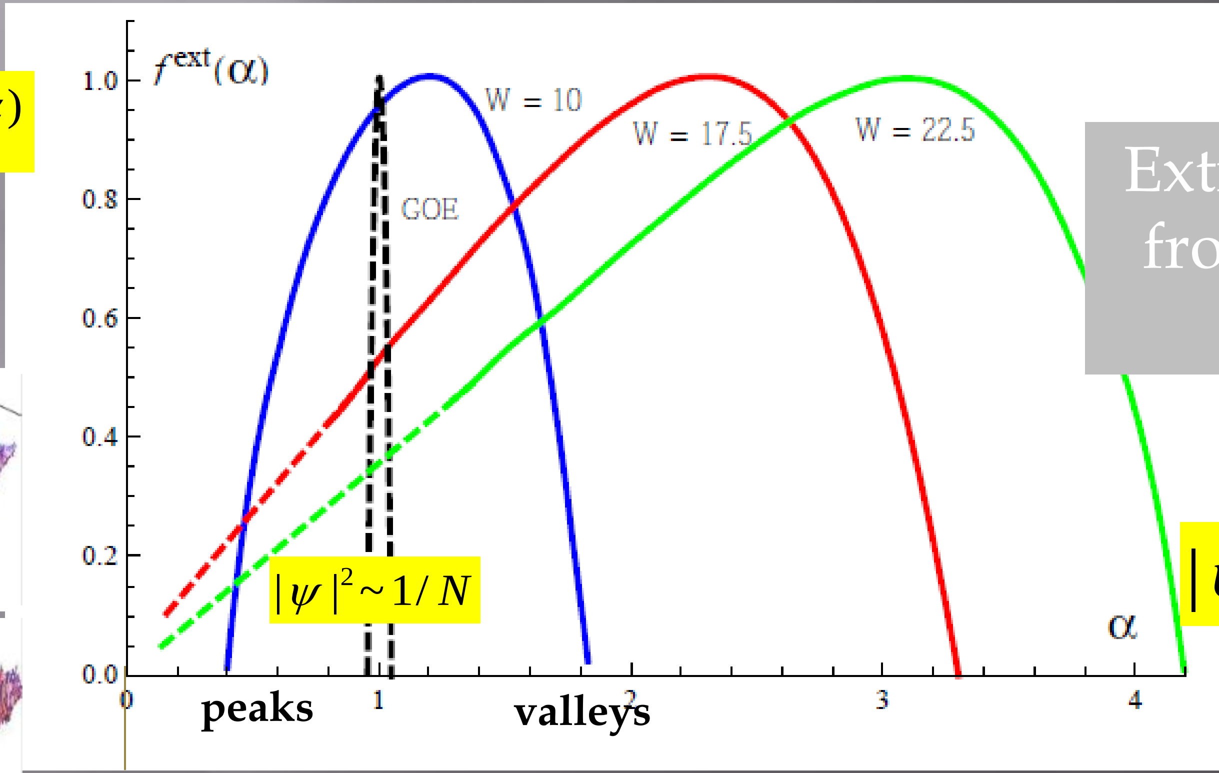


Search for ergodic transition (ET) on Bethe lattice (RRG)

Mirlin & Fyodorov: 1991-1997:

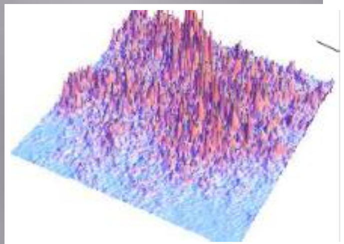
ET: No, only ergodic and localized phases

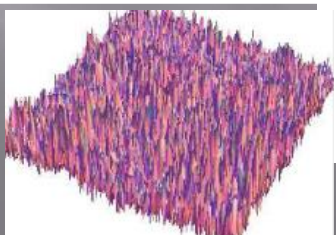
$$M = N^{f(\alpha)}$$



Extrapolation from N=2K-32K

$$|\psi|^2 \sim N^{-\alpha}$$



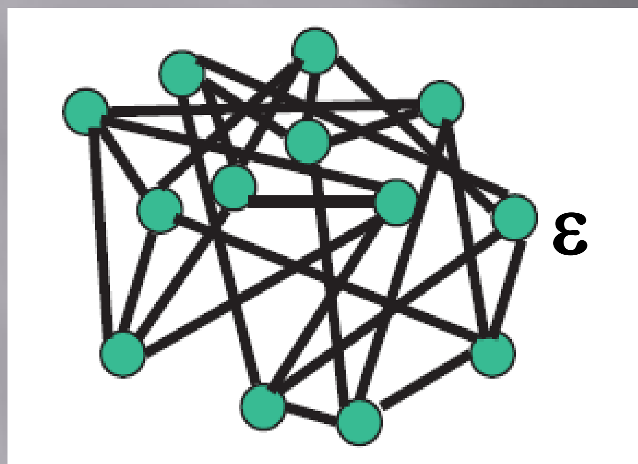


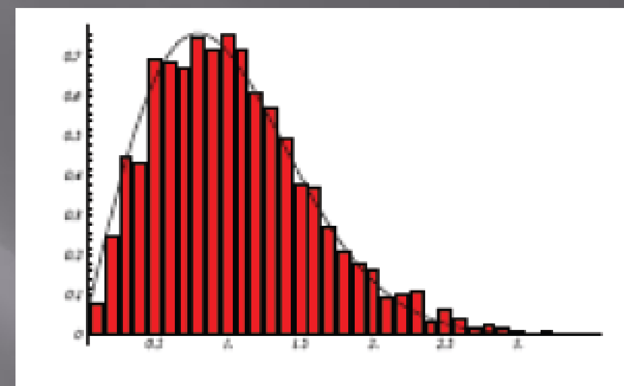
Giulio Biroli, 2011: ET – YES, but…

De Luca et al. 2014: ET - No, only non-ergodic and localized phases

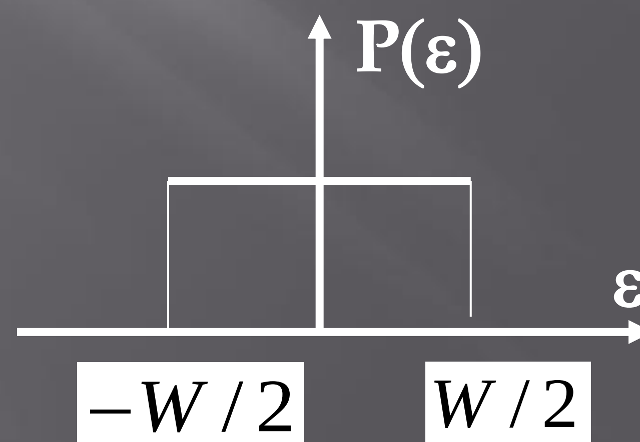
RRG: two disorder ensembles

Structural disorder: WD (Uzy Smiliansky)





On-site energy disorder:



A simpler model–relative of RRG?

WD RMT:

$$\sigma = \frac{\lambda^2}{N^\gamma} << 1$$

Mimics pristine RRG

$$\langle H_{nm}^2 \rangle = \begin{pmatrix} \sigma & \sigma & \sigma & \sigma \\ \sigma & \sigma & \sigma & \sigma \\ \sigma & \sigma & \sigma & \sigma \\ \sigma & \sigma & \sigma & \sigma \end{pmatrix}$$

Adding diagonal disorder

Special diagonal: Rosenzweig-Porter (1960) ensemble

$$\langle H_{nm}^2 \rangle = \begin{pmatrix} 1 & \sigma & \sigma & \sigma \\ \sigma & 1 & \sigma & \sigma \\ \sigma & \sigma & 1 & \sigma \\ \sigma & \sigma & \sigma & 1 \end{pmatrix}$$

Critical values of γ

$$W \sim I\ d$$

$$\langle H_{nm}^2 \rangle = \begin{pmatrix} 1 & \sigma & \sigma & \sigma \\ \sigma & 1 & \sigma & \sigma \\ \sigma & \sigma & 1 & \sigma \\ \sigma & \sigma & \sigma & 1 \end{pmatrix}$$

disorder

Hopping integral

Coordination number

$$\sigma = \frac{\lambda^2}{N^\gamma} << 1$$

$$1 \sim \frac{\lambda}{N^{\gamma/2}} N$$

→ $\gamma = 2$

I is of random sign!

$$d_{eff} \sim \sqrt{N}$$

→ $\gamma = 1$

SPECTRAL FORM FACTOR

$$K(t,t') = \langle \exp[itE_n - it'E_m] \rangle$$

Pandey 1995,
Kunz & Shapiro, 1998

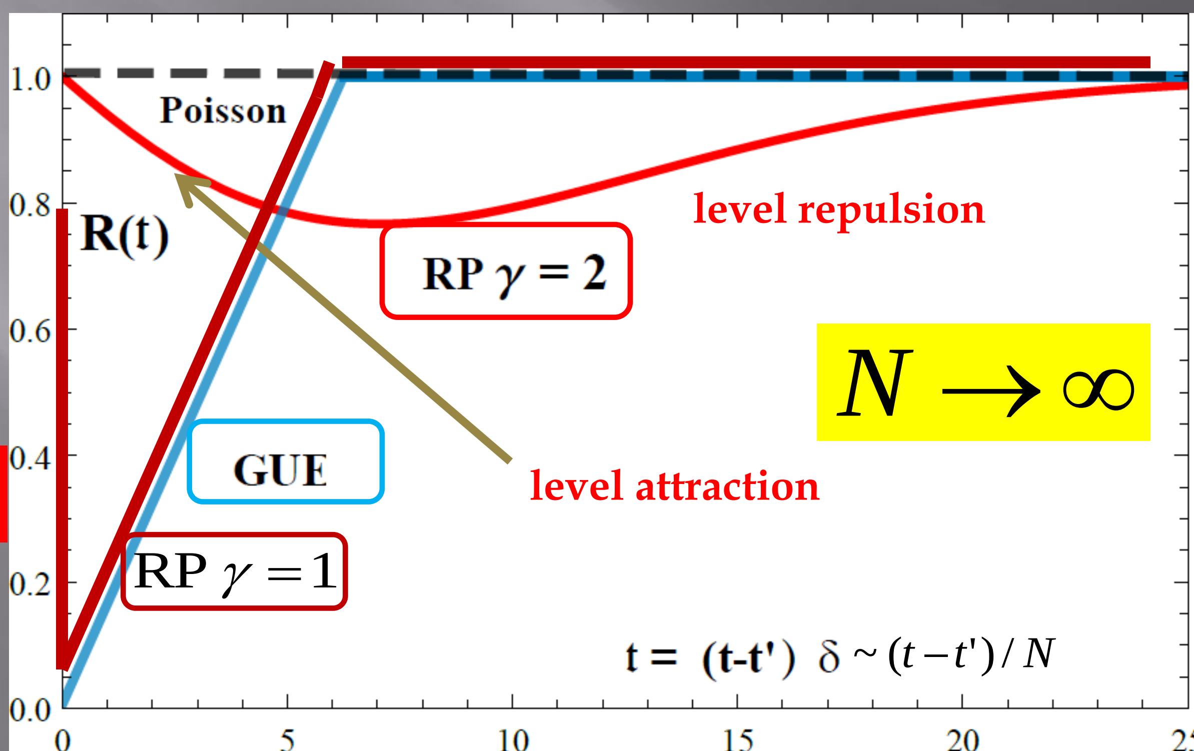
$$\sigma = \frac{\lambda^2}{N^\gamma} << 1$$

Brezin &
Hikami, 1995

γ=2

γ=1

Jump!



Generalization of Kunz & Shapiro

$$H = H_0 + A$$

GUE — Diagonal RM

$$H_0 = H - A, \; P(H_0) = \exp\left[-\frac{trH_0^2}{2\sigma}\right]$$

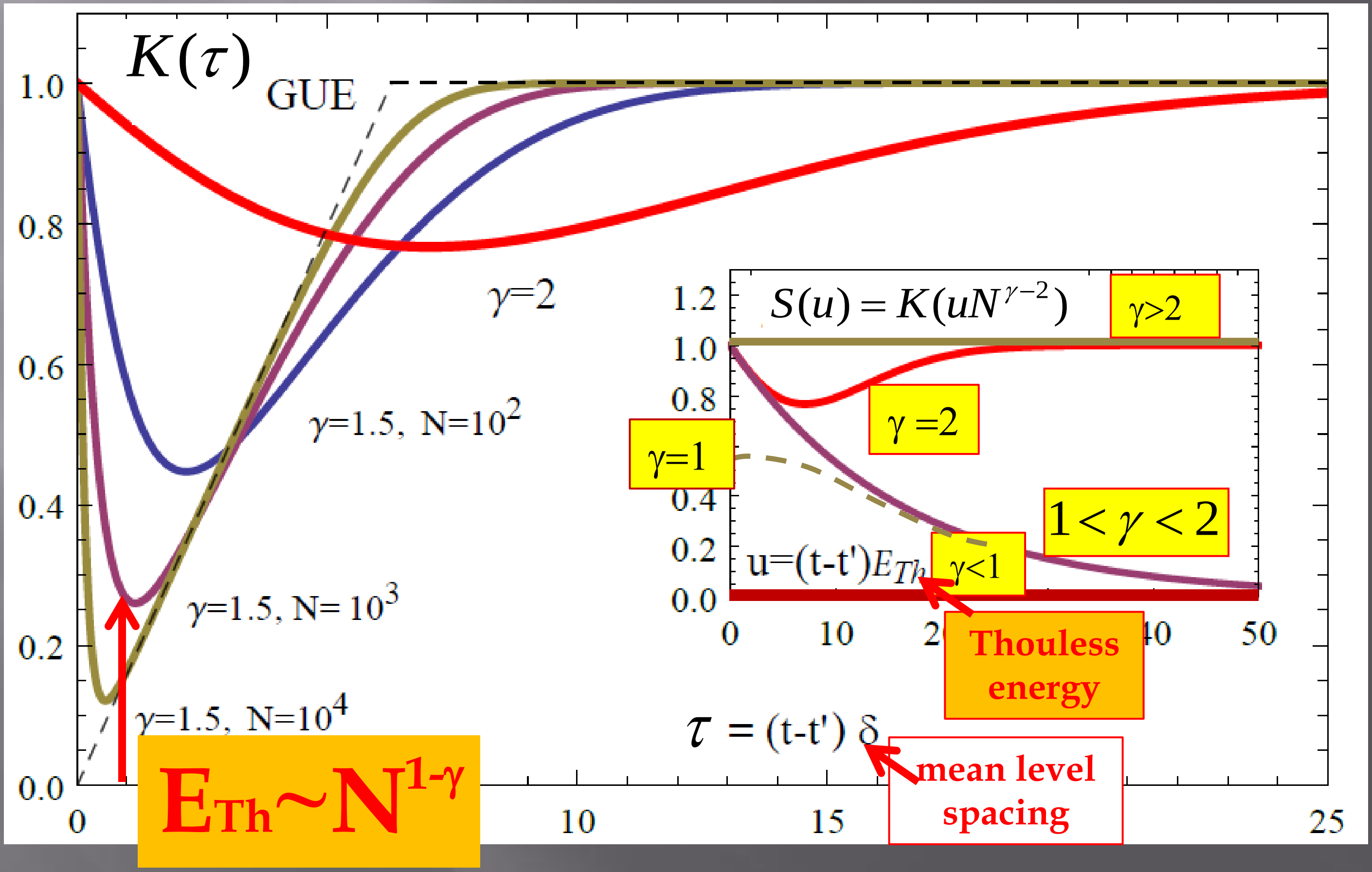
Izikson-Zuber formula of integrating over unitary matrices diagonalizing H

$$\int dU \exp\left[\frac{1}{\sigma}\,\mathrm{tr}\,A U \lambda U^{\dagger}\right] = \frac{c}{\Delta(\lambda)\Delta(a)} \det \exp\left(\frac{a_i \lambda_j}{\sigma}\right)$$

ANALYTICAL SOLUTION FOR SPECTRAL FORM-FACTOR

$$S(u) = 1 + e^{-2\pi\Lambda^2 u} e^{-\Lambda^2 u^2 N^{\gamma-2}} \left[\frac{2I_1(\kappa u^{3/2})}{\kappa u^{3/2}} - \frac{1}{4\pi}\kappa u^{5/2} N^{\gamma-2} \int_0^\infty \frac{x\,dx}{\sqrt{x+1}} I_1(\kappa u^{3/2}\sqrt{x+1})\, e^{-x u^2 \Lambda^2 N^{\gamma-2}} \right]$$

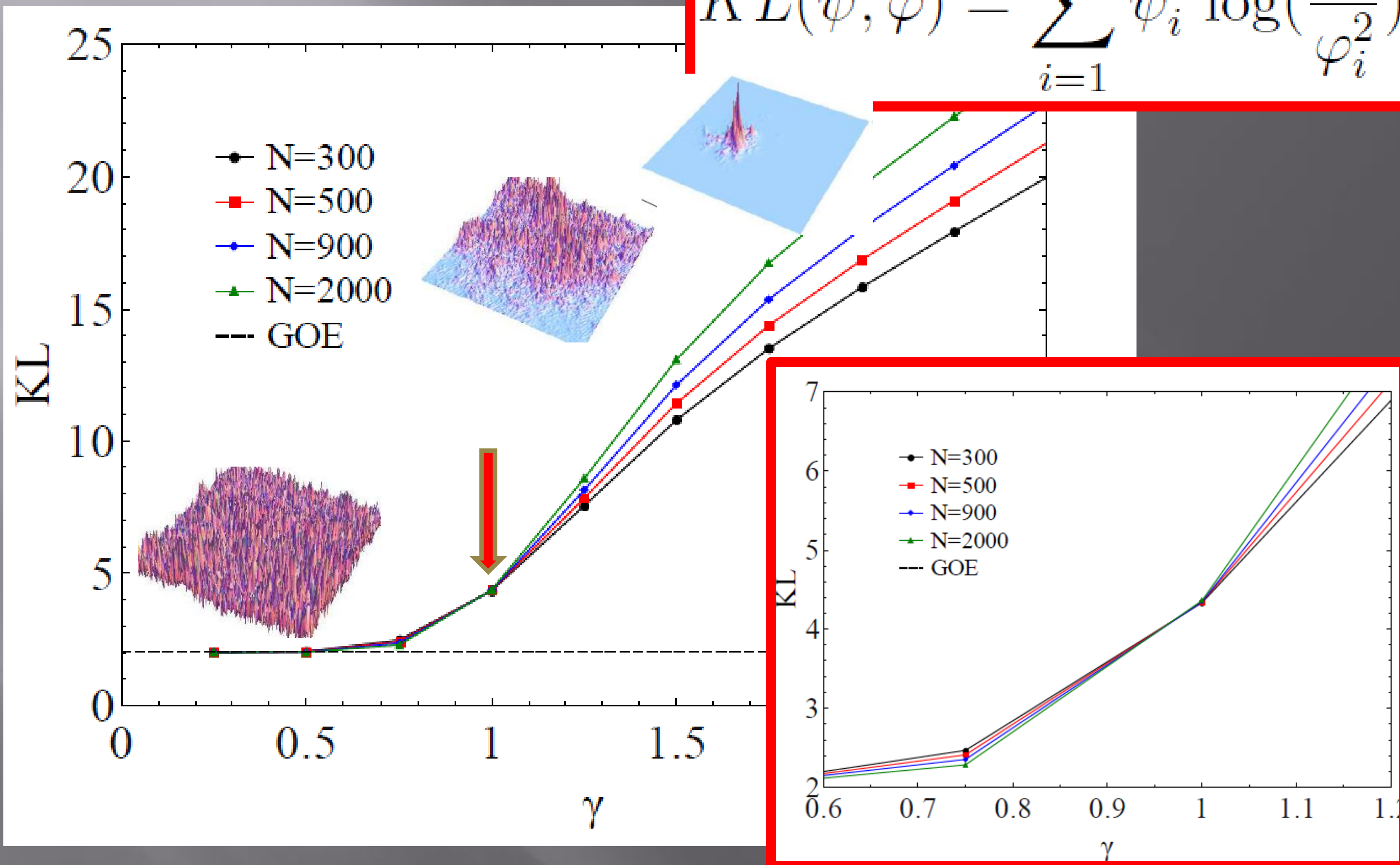
$$\kappa = \sqrt{8\pi N^{\gamma-2}}\Lambda^2 \text{ and } \Lambda = \lambda p(0)$$



KL-statistics

Kullback-Lieber

$$KL(\psi,\varphi)=\sum_{i=1}^{N}\psi_i^2\log\left(\frac{\psi_i^2}{\varphi_i^2}\right)$$

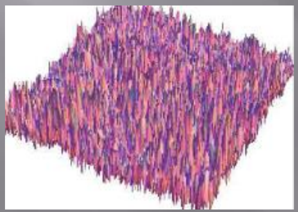


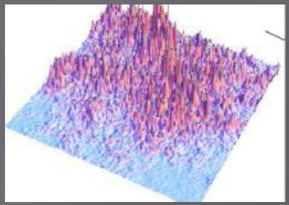
Two transition points

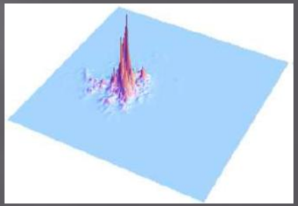
$$\sigma = \frac{\lambda^2}{N^\gamma} << 1$$

$$\langle H_{nm}^2 \rangle = \begin{pmatrix} 1 & \sigma & \sigma & \sigma \\ \sigma & 1 & \sigma & \sigma \\ \sigma & \sigma & 1 & \sigma \\ \sigma & \sigma & \sigma & 1 \end{pmatrix}$$

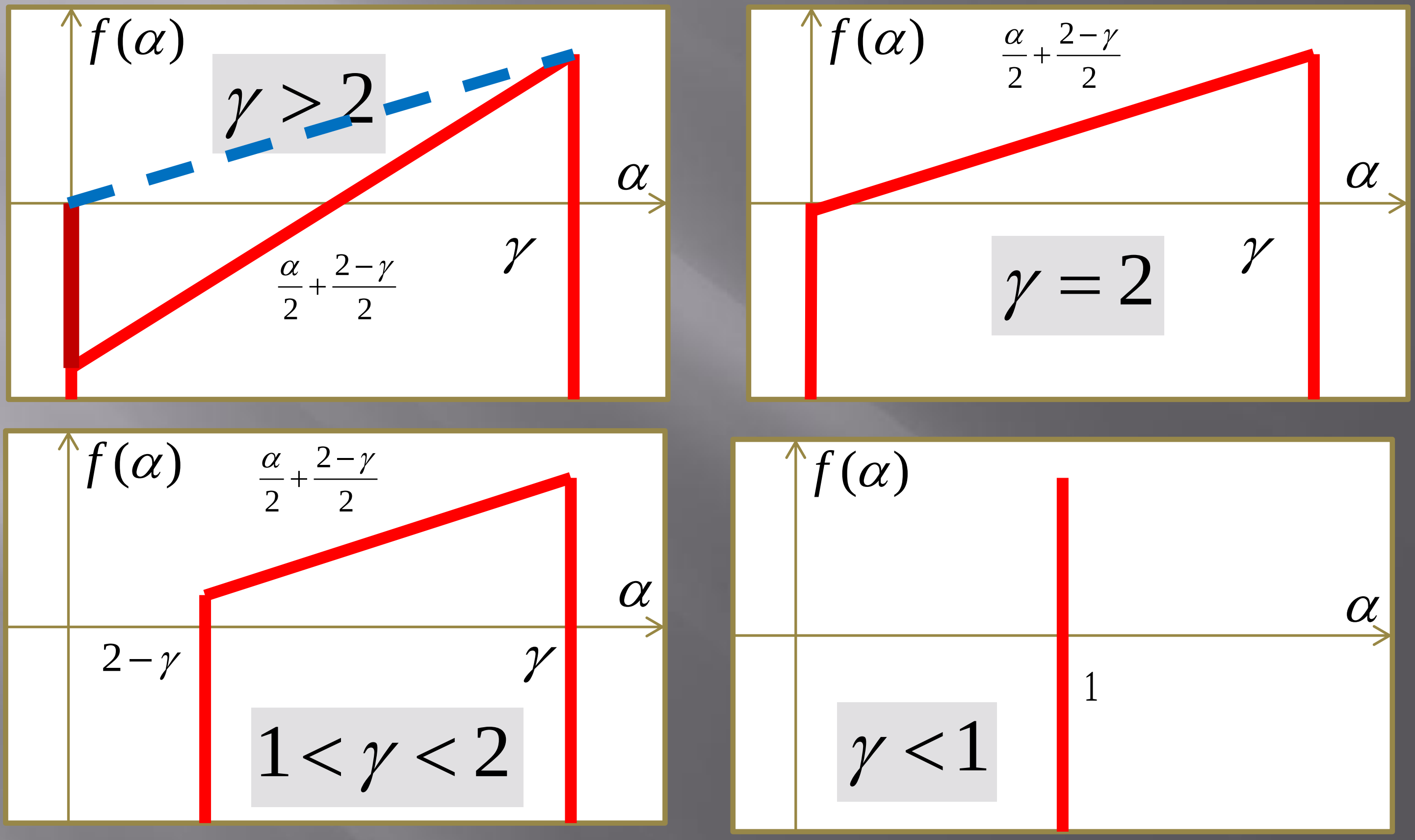
Extended ergodic — 1 — **Extended, non-ergodic** — 2 — **localized** → γ





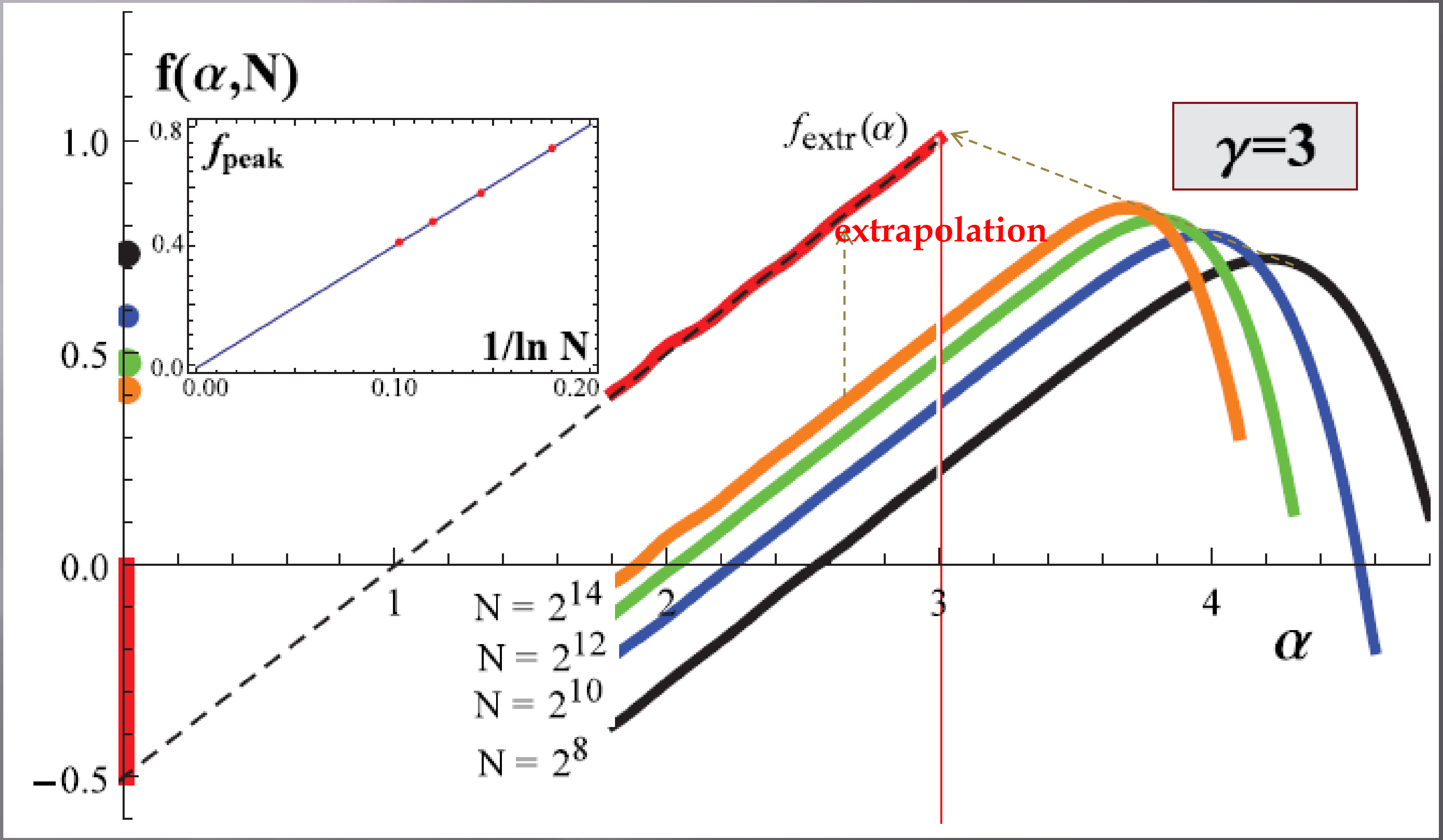


Multifractality spectrum f(α)



$$D_{q>1/2} = 2-\gamma$$

Numerics for f(α,N)

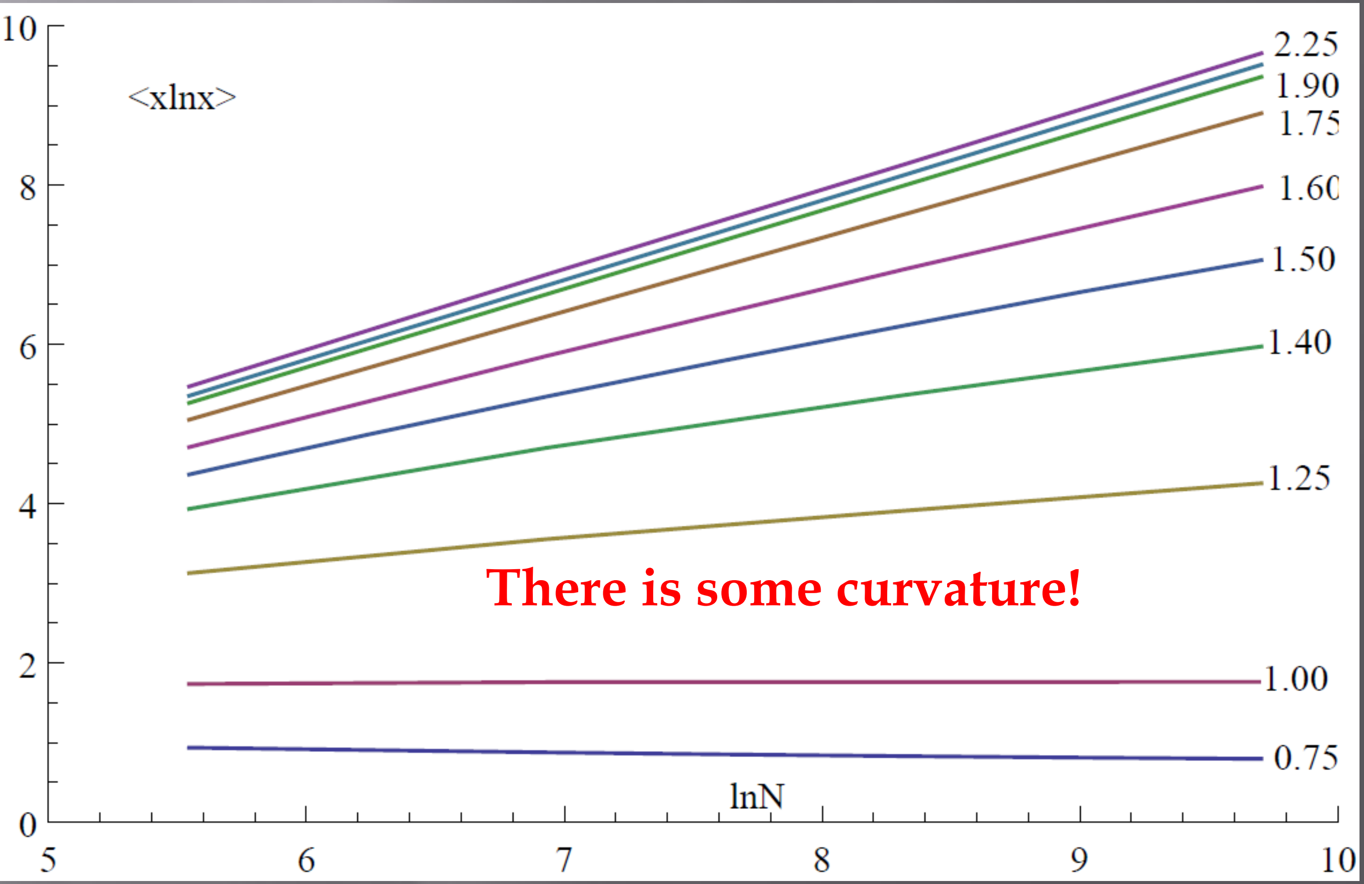


Wave function support set dimension D_1

Shannon entropy:
lnN - <x lnx>

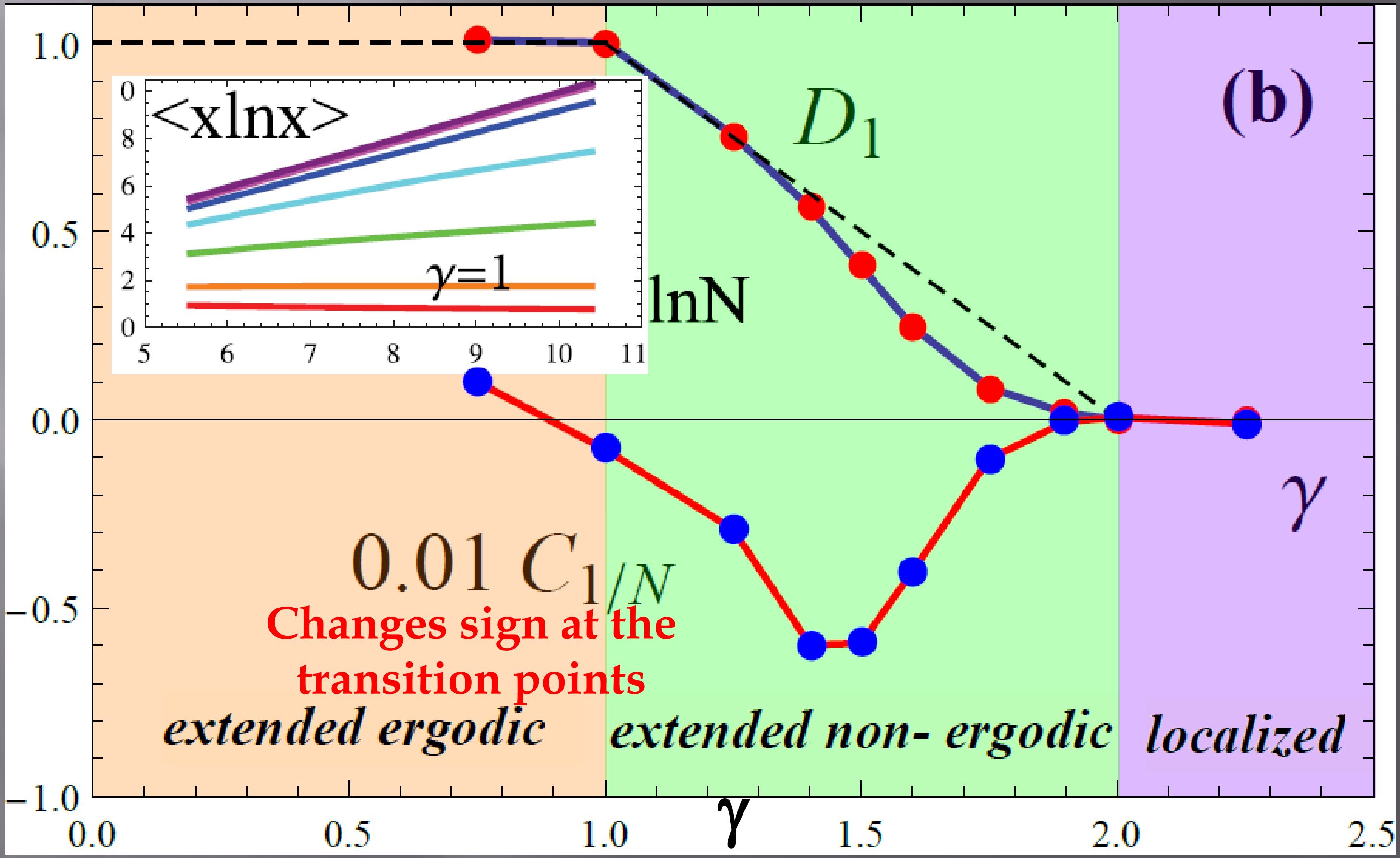
$$\langle x \ln x \rangle = (1 - D_1)\ln N + const$$

$$x = N|\Psi|^2$$

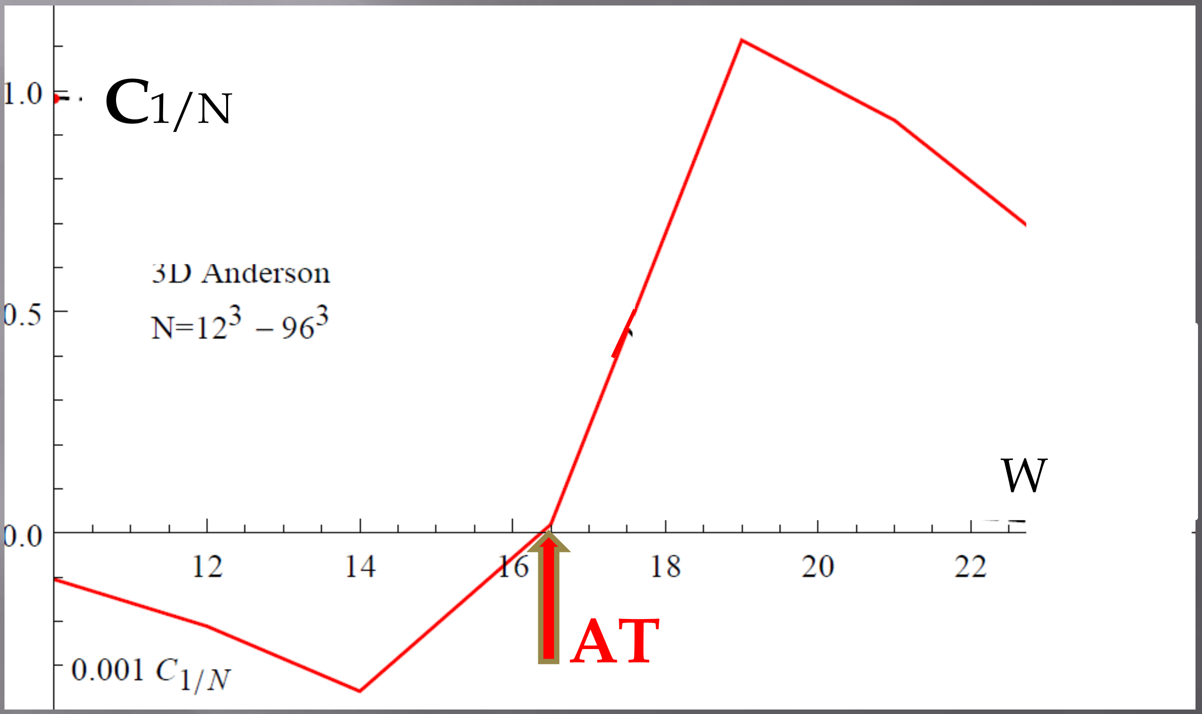


Curvature as signature of transitions

$$\langle x \ln x \rangle = (1 - D_1) \ln N + b + \frac{C_{1/N}}{N}$$



3D Anderson transition

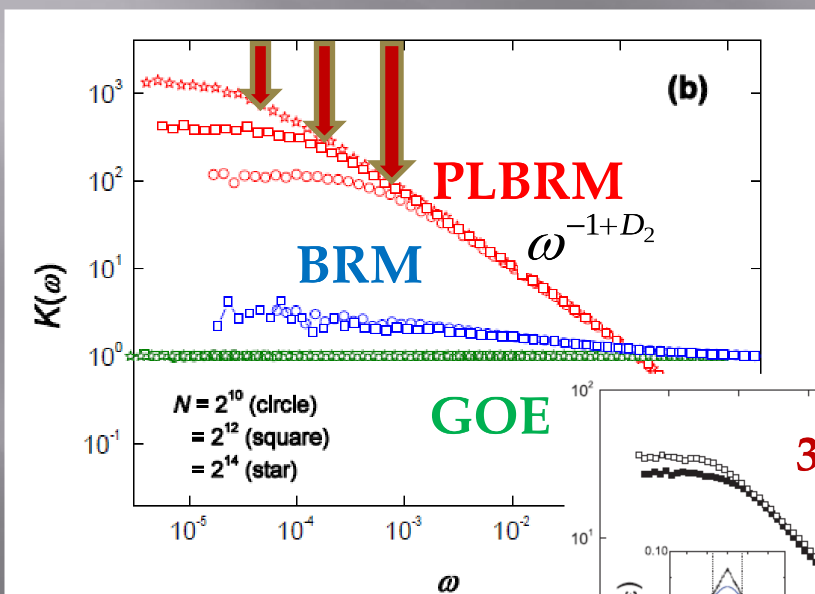


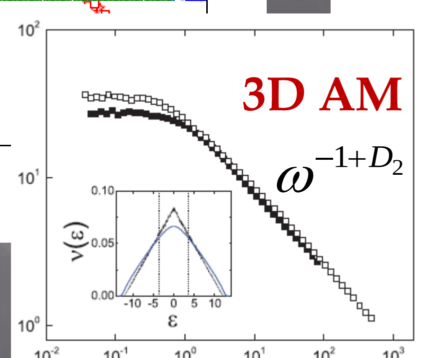
OVERLAP CORRELATION FUNCTION AND THE THOULESS ENERGY

Cuevas & VEK, PRB, 2007

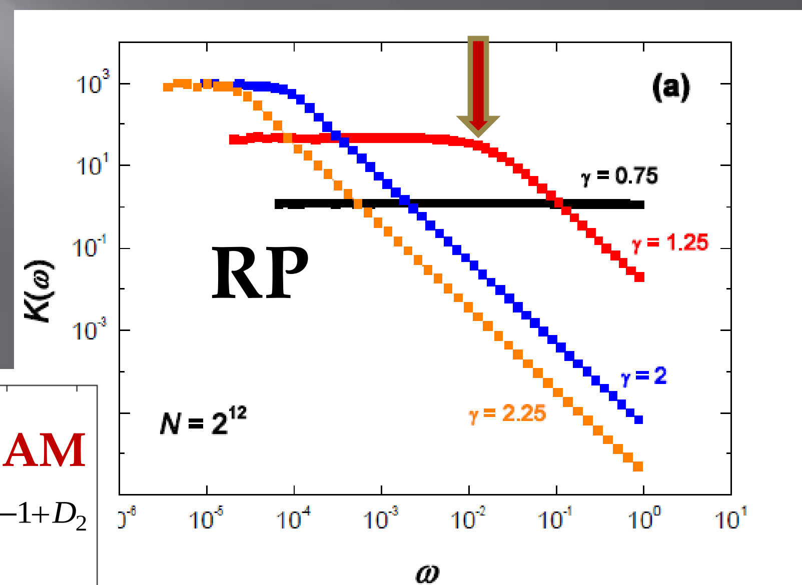
$$K(\omega) = N \sum_r |\Psi_E(r)|^2 \, |\Psi_{E+\omega}(r)|^2$$

$E_{Th} \sim \delta \sim 1/N$

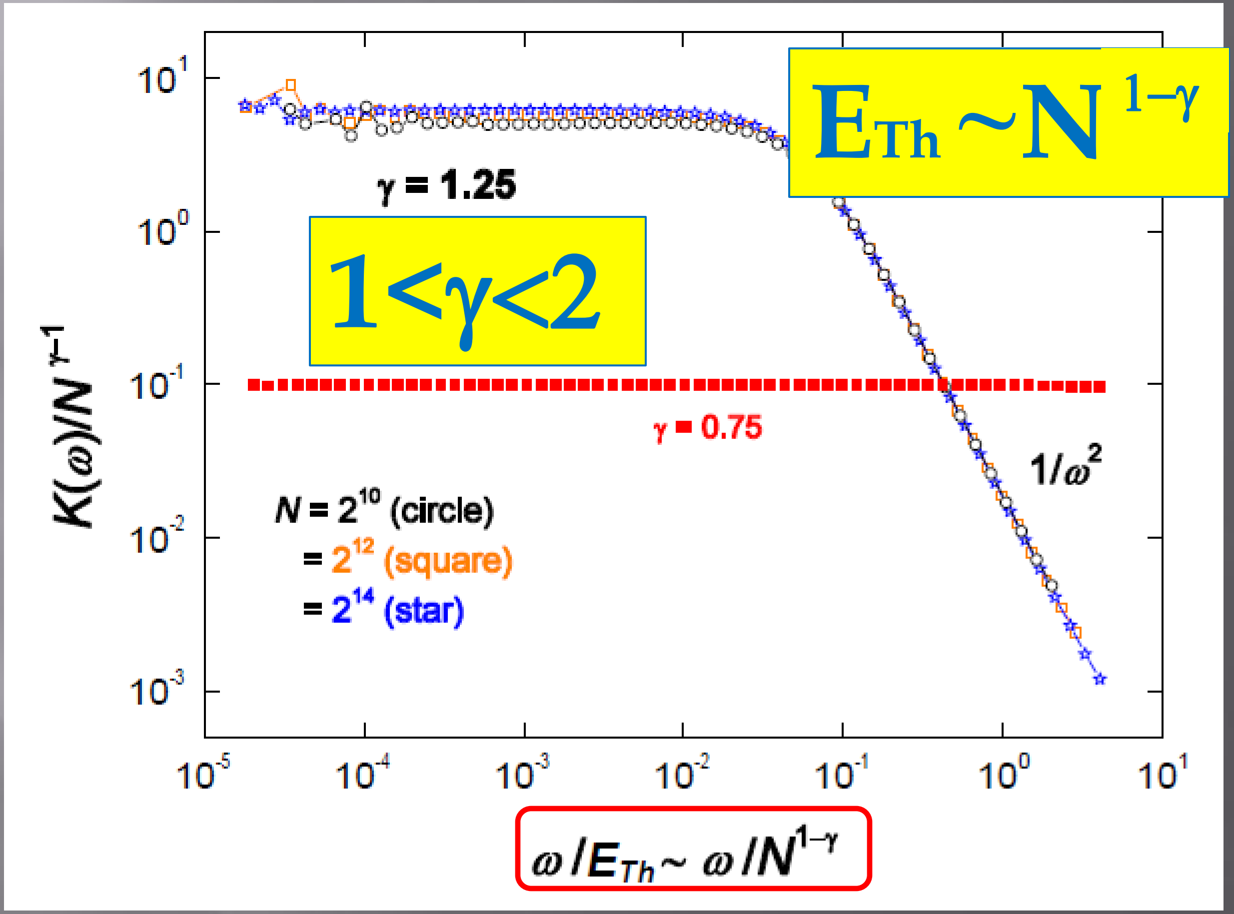




$E_{th} >> \delta$



HOW DOES E_{Th} SCALE?



The same scaling of Thouless energy as for spectral form-factor

THOULESS ENERGY AND FRACTAL DIMENSION

$D_{q>1/2} = 2 - \gamma$

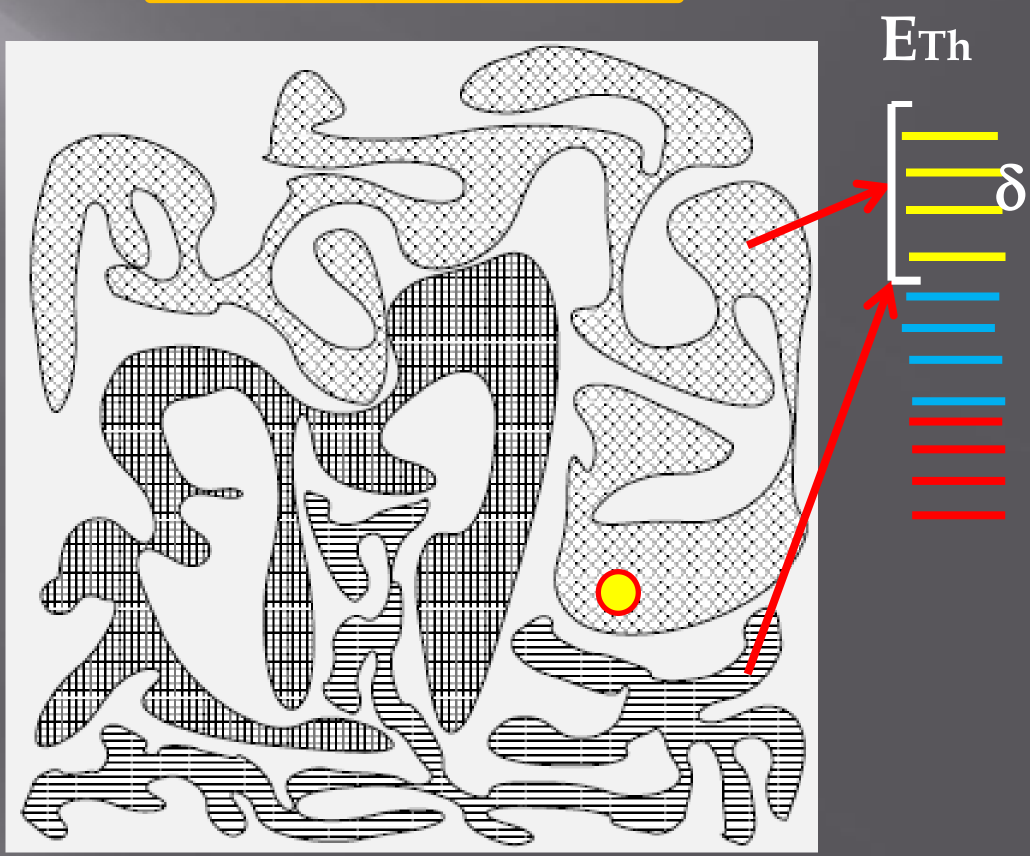
$E_{Th} = \delta N^{D}$

N sites in a sample

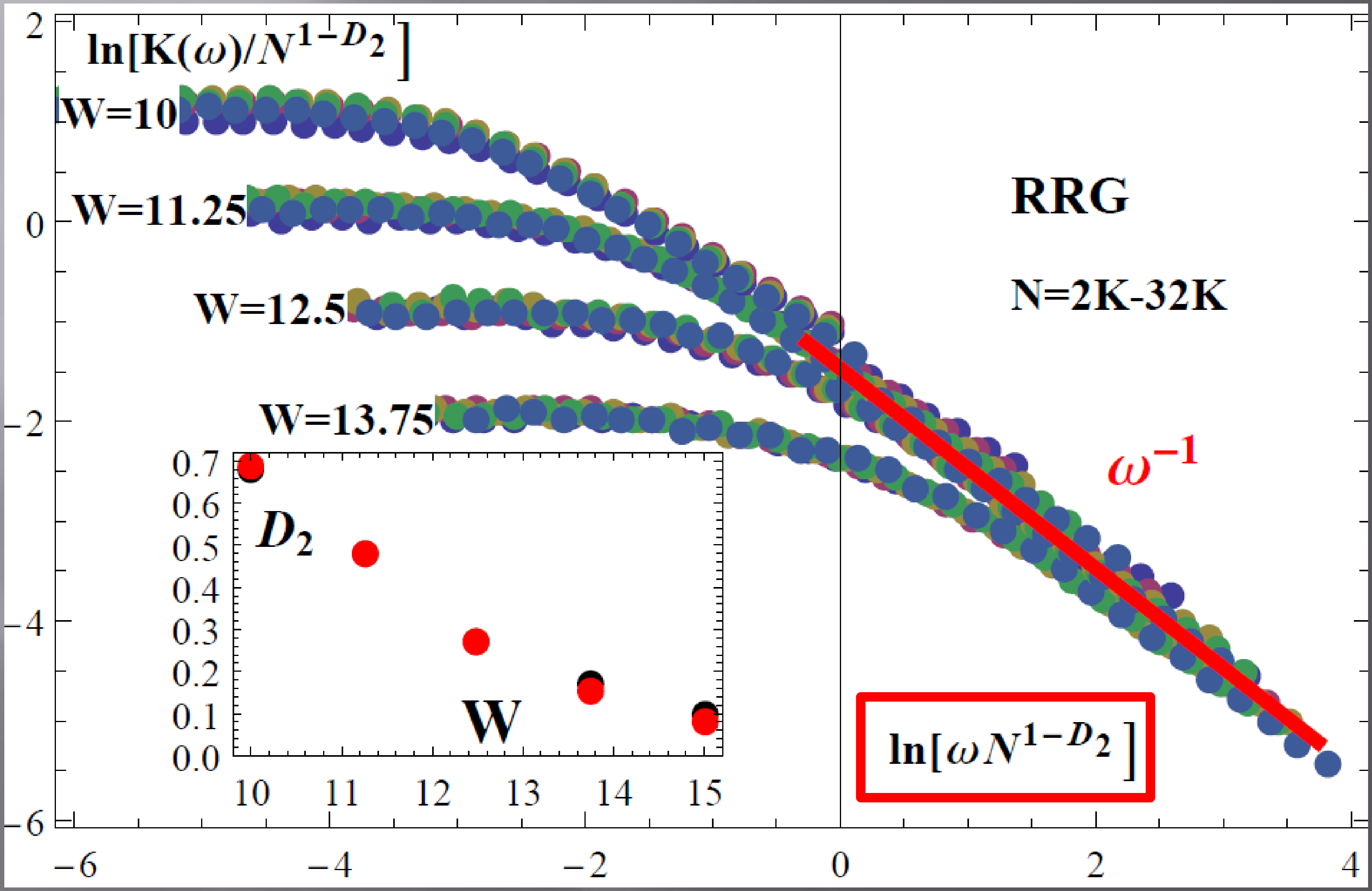
N^{1-D} fractal domains

N^{D} sites in each

Each fractal domain is a support of N^{D} wave functions leaving on it



THOULESS ENERGY ON RRG

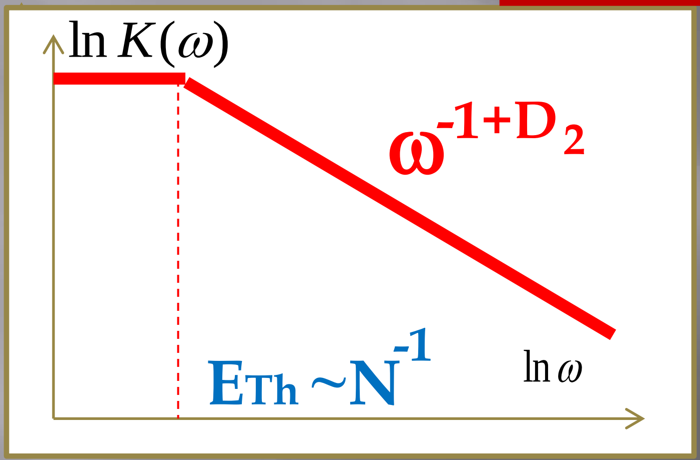


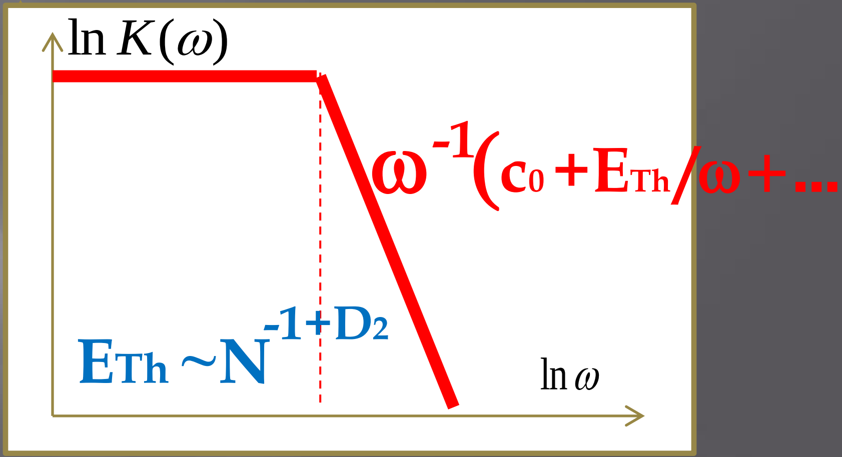
TWO TYPES OF SCALING FOR NON-ERGODIC EXTENDED STATES

Standard Chalker's scaling:
3D AT point, PRBRM

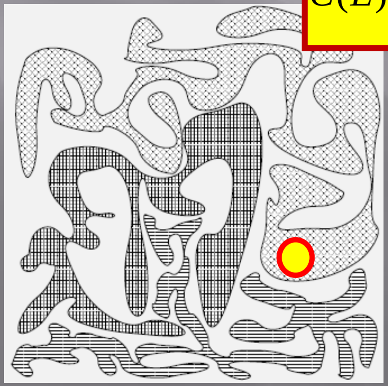
"Hierarchical " lattices: RRG,

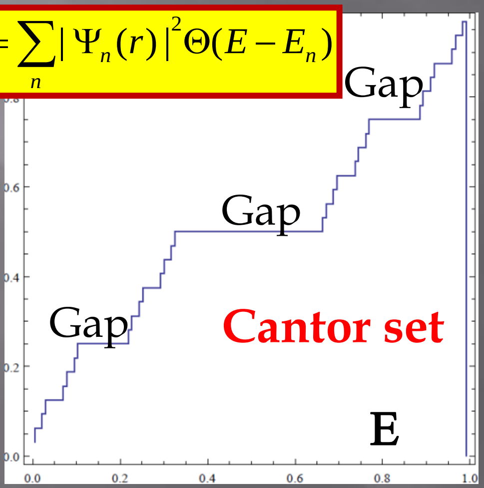
$$K(\omega) = -\partial^2_\omega \langle C(E-\omega/2)C(E+\omega/2)\rangle_E$$

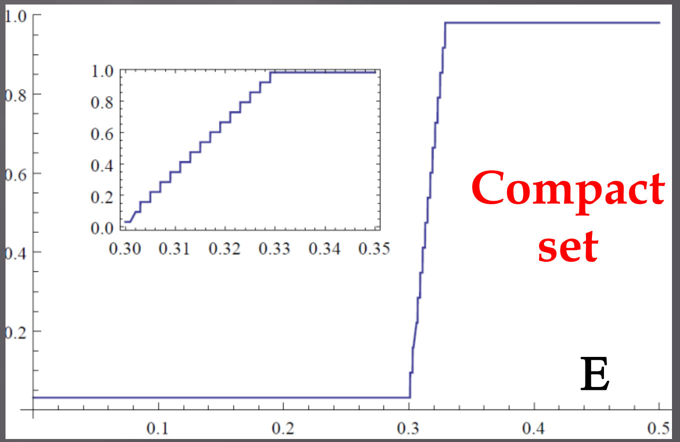




$$C(E) = \sum_n |\Psi_n(r)|^2 \Theta(E - E_n)$$







Conclusions

- Rosenzweig-Porter RM model shares many of the properties of RRG
- RP model contains both Anderson and Ergodic transitions
- They are seen in the rigorous theory of the two-level correlation function
- Perturbative treatment of the eigenfunction statistics gives the three phases: localized, non-ergodic extended, ergodic extended
- Numerics confirm existence of three phases
- <xlnx> moments (Shannon entropy) vs lnN has a curvature which changes sign at the transitions.
- Two different scalings of Thouless energy

Statistics of x=Nψ^2: perturbative arguments

$$(\psi_n(m))^2 = \frac{H_{nm}^2}{(E_n - E_m)^2}.$$

$$\int dt\, \langle \delta(x - N\psi^2)\rangle\, e^{itx} = \langle e^{itN\psi^2}\rangle = \frac{1}{N}\, e^{itN} + \langle e^{it\,NV^2/\Delta^2}\rangle.$$

$$P(V = H_{nm},\ \Delta = H_{nn} - H_{mm}) = \frac{1}{2\pi\sqrt{2\sigma}}\, e^{\frac{-\Delta^2}{4}}\, e^{\frac{-V^2}{2\sigma}}$$

$$P_{reg}(x) = \frac{1}{\sqrt{2}\pi}\frac{\sqrt{N\sigma}}{x^{3/2}}$$

$$+c\delta(x - N)$$

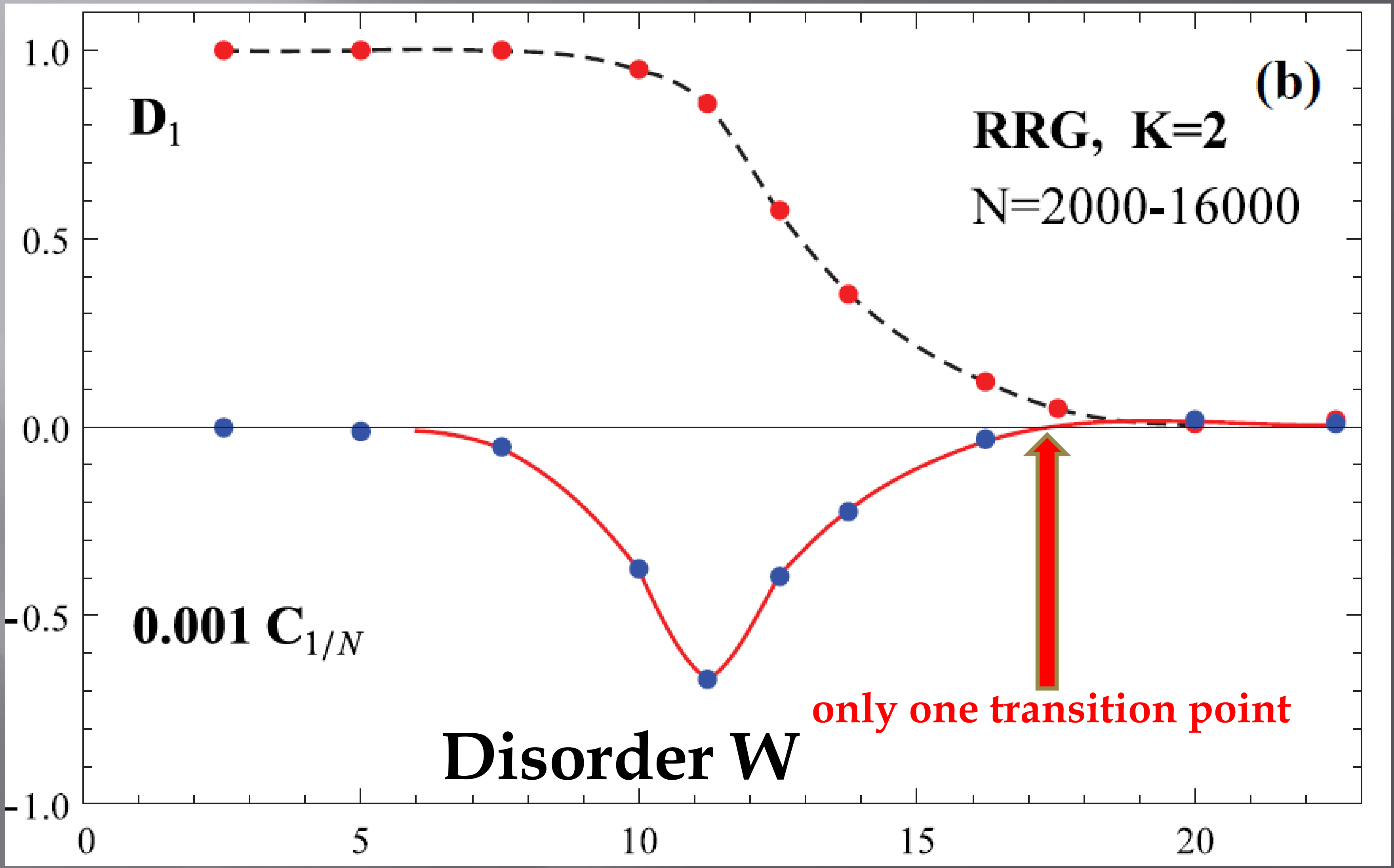
$$\int_0^\infty P(x)\, dx = 1$$

$$\int_0^\infty x\, P(x)\, dx = 1.$$

$$x_{\min} = N^{-(\gamma-1)}$$

$$x_{\max} = \begin{cases} N^{(\gamma-1)}, & \gamma < 2 \\ N, & \gamma > 2. \end{cases}$$

D1 and curvature on the RRG



Generalization of Kunz & Shapiro

$$K_1(t,t') = \frac{1}{N\tau\tau'}\oint_{\Gamma_R}\frac{dz}{2\pi i}\oint_{\Gamma_R}\frac{dz'}{2\pi i}e^{i(tz+t'z')}\left(g(z,z')^N - \rho(z/\tau)^N\rho(z'/\tau')^N\right)$$

$$K_2(t,t') = \frac{1}{N}\oint_{\Gamma_R}\frac{dz}{2\pi i}\oint_{\Gamma_R}\frac{dz'}{2\pi i}e^{i(tz+t'z')}\frac{g(z,z')^N}{(z'-z-\tau)(z'-z+\tau')}\,.$$

$$g(z,z') = 1+\tau\alpha(z)+\tau'\alpha(z')+\frac{\tau\tau'}{z-z'}[\alpha(z')-\alpha(z)]$$

$$\rho = 1+\tau\alpha(z)$$

$$\alpha(z) = \left\langle \frac{1}{z-a}\right\rangle$$

Depends on the distribution function of the diagonal matrix A=diag{a}

$$(t,t') = i\sigma(\tau,\tau') \to \frac{T}{2} \pm N^{\gamma-1}s$$

$$(z,z') \to x \pm \frac{y}{2N}$$

Rescaling and N-> infinity

Genaralization of Kunz & Shapiro

$$K_1(t,t') = \frac{1}{N\tau\tau'} \oint_{\Gamma_R} \frac{dz}{2\pi i} \oint_{\Gamma_R} \frac{dz'}{2\pi i} e^{i(tz+t'z')} \left(g(z,z')^N - \rho(z/\tau)^N \rho(z'/\tau')^N \right)$$

$$K_2(t,t') = \frac{1}{N} \oint_{\Gamma_R} \frac{dz}{2\pi i} \oint_{\Gamma_R} \frac{dz'}{2\pi i} e^{i(tz+t'z')} \frac{g(z,z')^N}{(z'-z-\tau)(z'-z+\tau')} .$$

cancels out

$\propto N^{\gamma-2}$

tend to a finite limit independent of γ

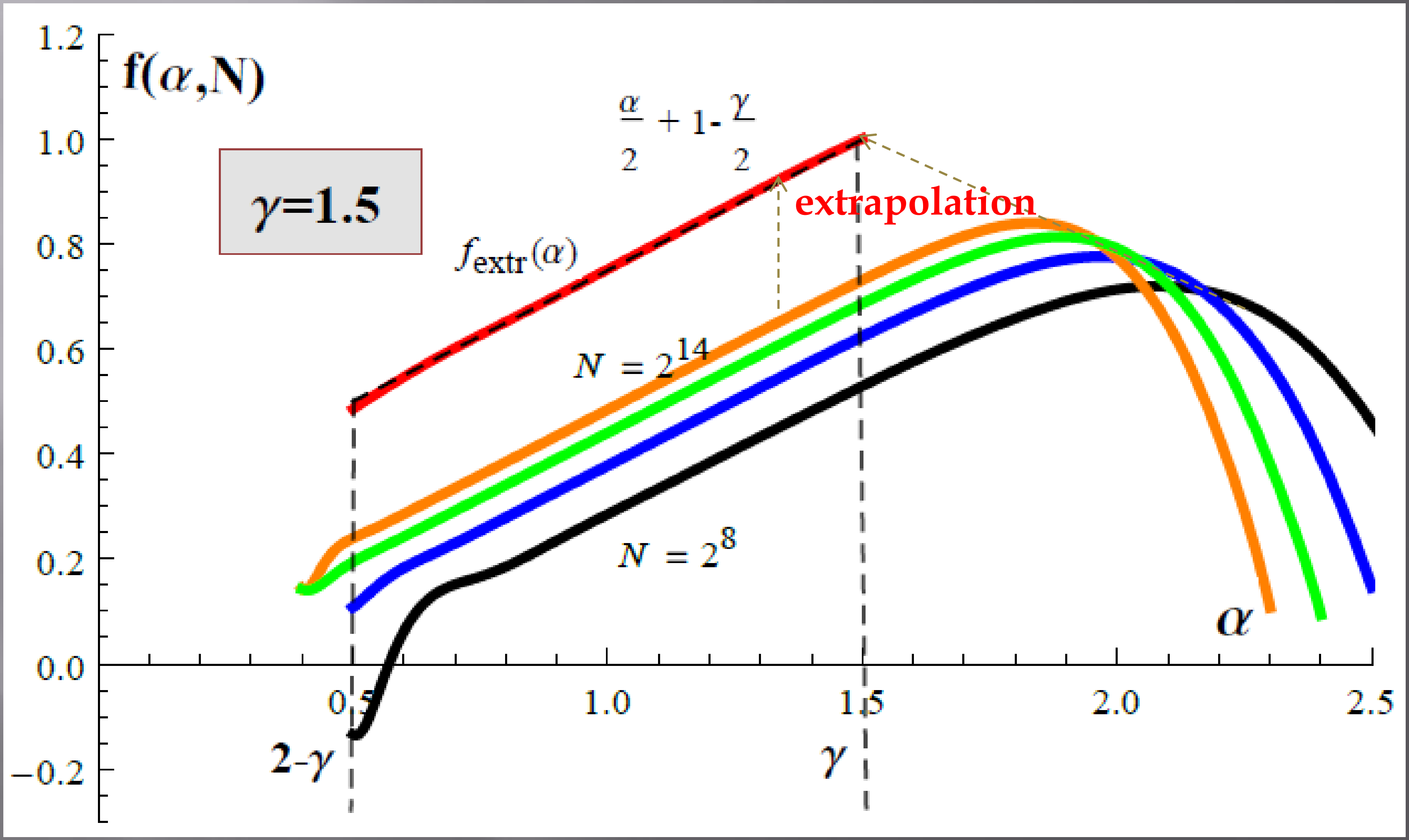
γ>2: Poisson due to infinitely fast oscillations

γ=2: regular oscillations

1< γ < 2: no oscillations

0< γ <1: rescaling does not work

Numerics for f(α,N)



Numerics for f(α,N)