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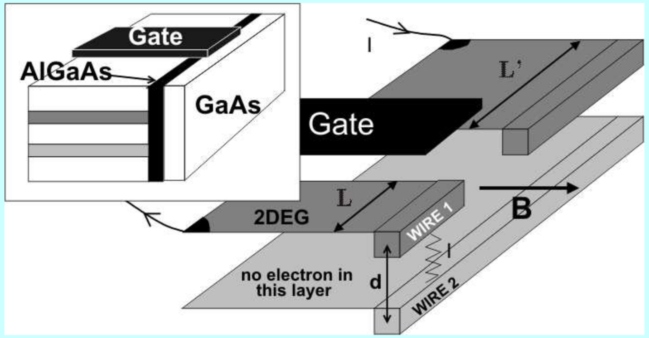
& Astronomy

The role of duality in a 1D transport of interacting particles

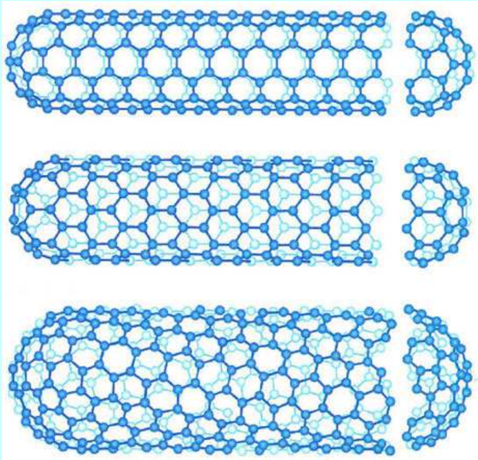
Igor Lerner

Grishin, Yurkevich, IL, PRB 69, 165108 (2004), IL, Yurkevich, Yudson, PRL 100, 256805 (2008)
Galda, Yurkevich, IL, PRB 83, 041106(R) (2011) , Galda, Yurkevich, IL, EPL, 93, 17009 (2011)
Galda, Yurkevich, Yevtushenko, IL, PRL, 110, 136405 (2013)

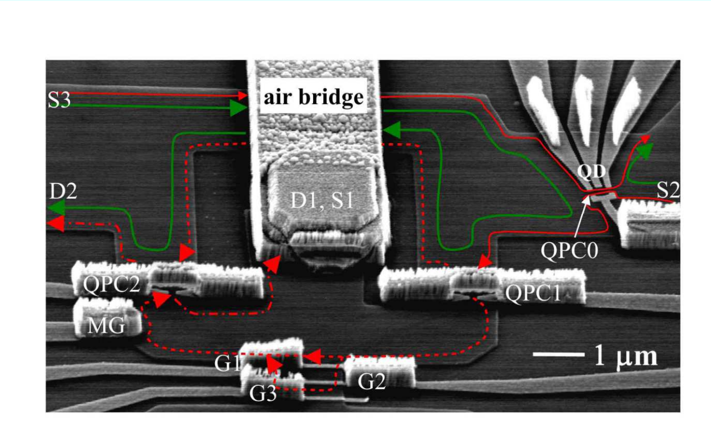
1D systems for “Luttinger” physics



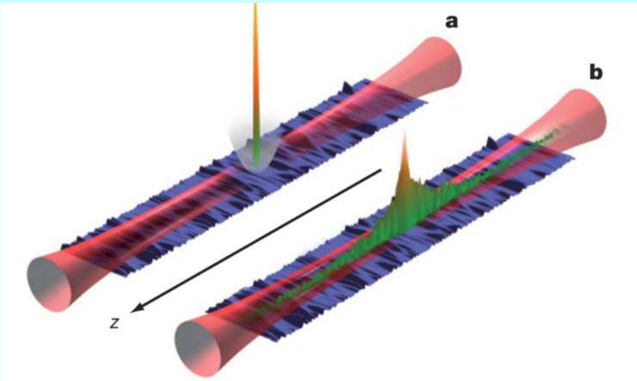
Quantum Wires



Carbon Nanotubes

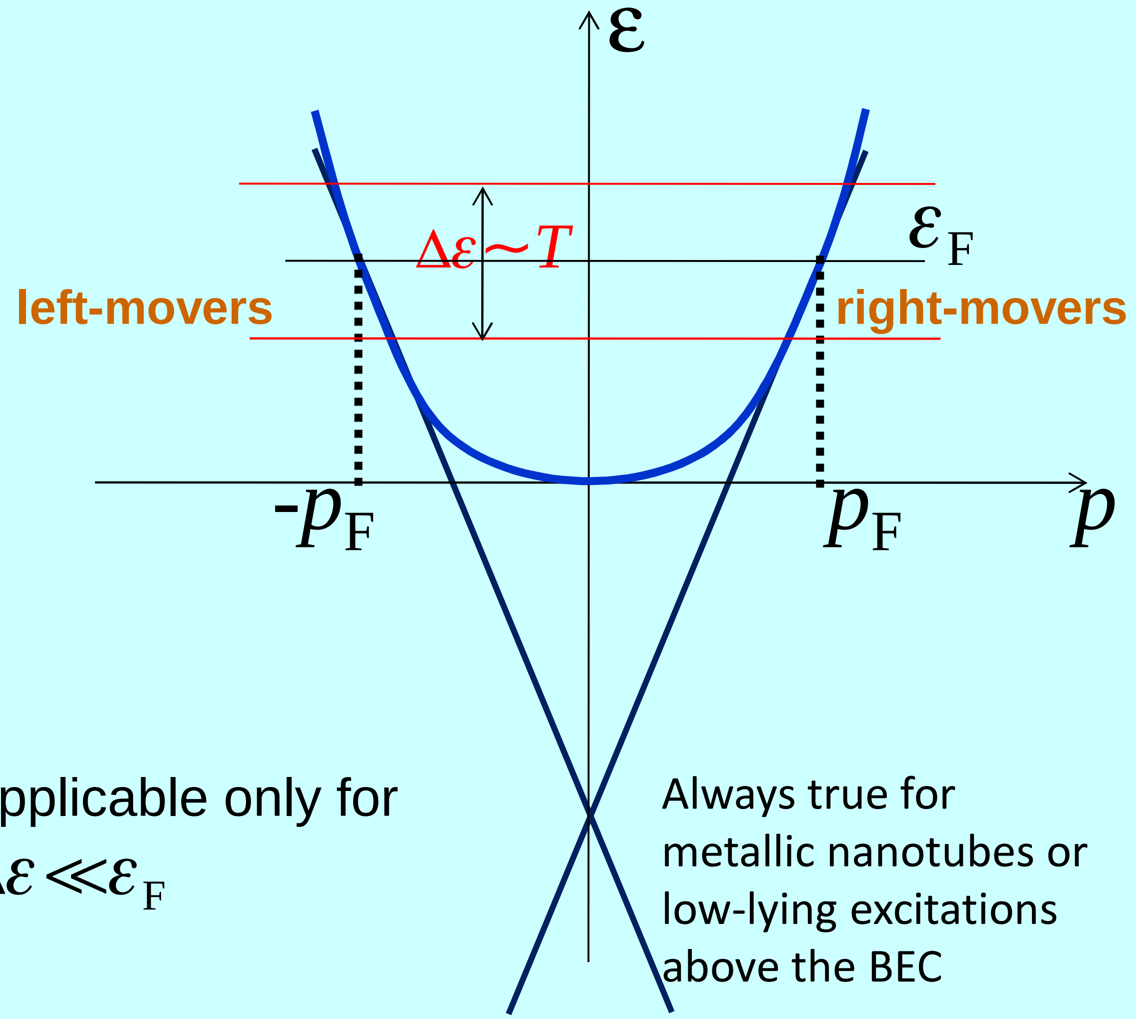


Quantum Hall Edge States



1D Optical Lattices

Tomonaga-Luttinger model



Applicable only for
$\Delta\varepsilon \ll \varepsilon_F$

Always true for metallic nanotubes or low-lying excitations above the BEC

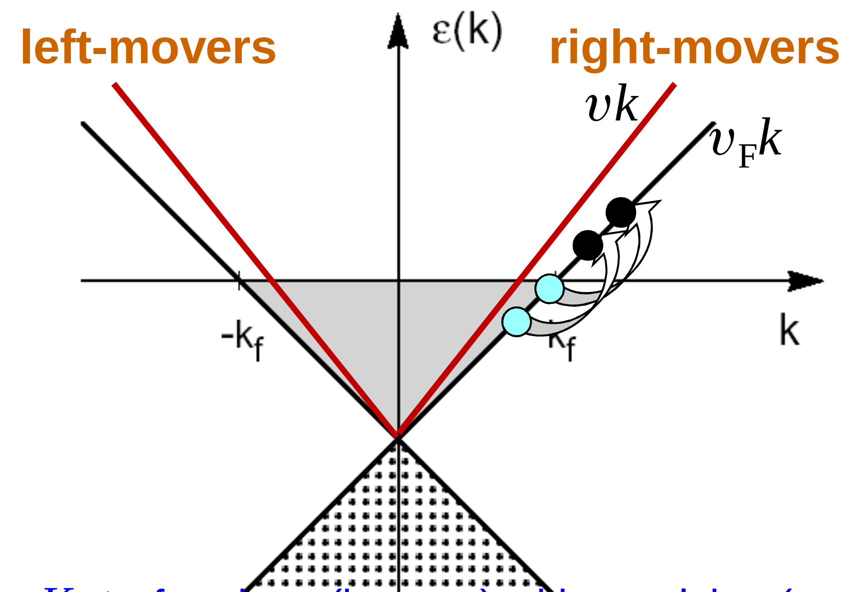
Note: e-h symmetry!
❖Linearisation is not suitable for:

- Coulomb drag
- Light absorption
- Thermoelectricity
- Polaritons

Spinless LL – bosonization

$$\mathcal{H}_0 = \sum_{\eta=\pm} \int \mathrm{d}x\, \psi^\dagger_\eta [\eta v_{\mathrm{F}} \partial_x] \psi_\eta \mapsto \frac{v_{\mathrm{F}}}{2} \sum_{\eta=\pm} \int \mathrm{d}x\, \rho^2_\eta(x)\,, \quad \rho_\eta \equiv \psi^\dagger_\eta \psi_\eta$$ is (R,L) density

Density-density interaction leads to $$H = \frac{v}{2\pi} \int \mathrm{d}x \left[\frac{1}{K} (\rho_L + \rho_R)^2 + K (\rho_L - \rho_R)^2 \right]$$



"**Bosonization**": $\psi_\eta \propto \exp[i\theta_\eta]$ results in density fluctuations $\delta\rho \propto \partial_x \theta$ and current $j \propto \partial_x \varphi$ expressed via $\theta, \varphi \equiv \theta_L \pm \theta_R$ and a ***dual*** action with the Lagrangian density

$$\mathcal{L} = \frac{1}{2\pi v K} \partial_+ \theta\, \partial_- \theta = \frac{K}{2\pi v} \partial_+ \varphi\, \partial_- \varphi$$

$$\mathcal{S} = \int \mathcal{L} \mathrm{d}x \mathrm{d}t; \quad \partial_\pm \equiv \partial_t \pm v \partial_x$$

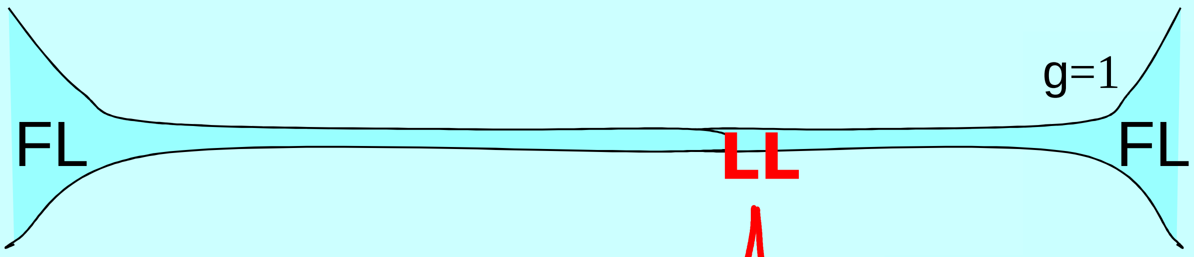
$K<1$ – fermions (bosons) with repulsion (attraction)

$K=1$ – ideal Fermi gas (hard-core Bose-gas)

$K>1$ – bosons (fermions) with attraction (repulsion)

Haldane 79,81;
von Delft & Schoeller, 98
Giamarchi 02
Gogolin, Nersesyan, Tsvelik 04

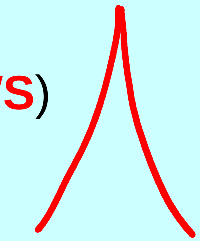
Conductance of a 1D wire: WS



– ideal dimensionless coductance

Maslov & Stone 95;
Oreg & Finkelstein 95
Ponomarenko 95; Safi & Schulz 95

but even a weak scatterer (**WS**) (e.g., a tip of STM) is “dangerous”

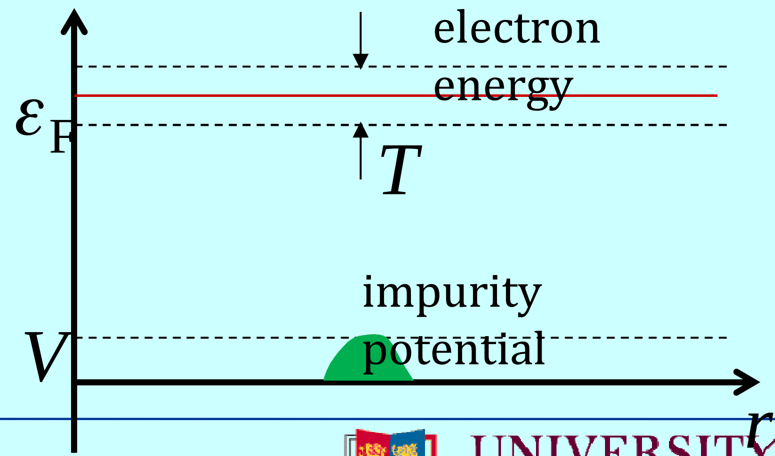


Backscattering amplitude λ changes $\mathcal{H}$ only at x=0 by adding λ cos2$\theta(0)$. It results in 0+1 dimensional (Caldeira-Leggett) action, and λ scales with energy as

$\lambda(\varepsilon) \sim \lambda\, \varepsilon^{\Delta_{\rm ws}-1}$ where $\Delta_{\rm ws}=K$ ➡ $G(T) \sim (T/T_0)^{1-\Delta_{\rm ws}}$ for fermions

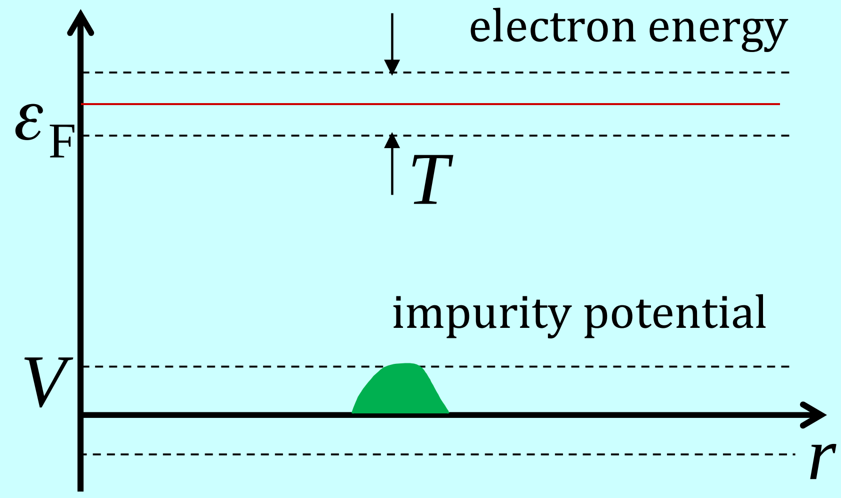
(C Kane & M Fisher 92)

An arbitrary small impurity becomes at low T impenetrable for electrons and remains irrelevant for bosons (with repulsion)

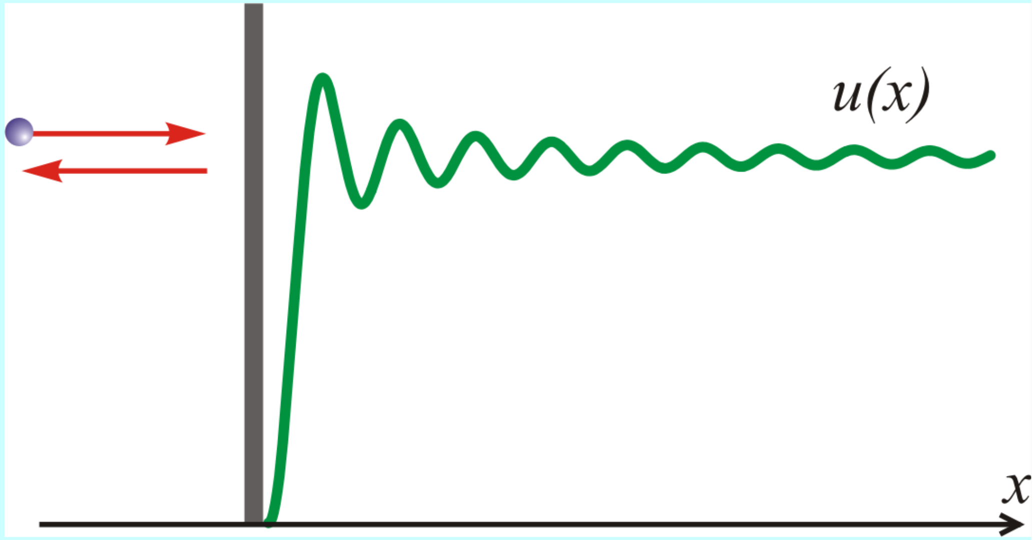


Scattering from Friedel oscillations

Without interaction a single weak impurity is irrelevant even in 1D (it will only change conductance from 1 to a finite value).



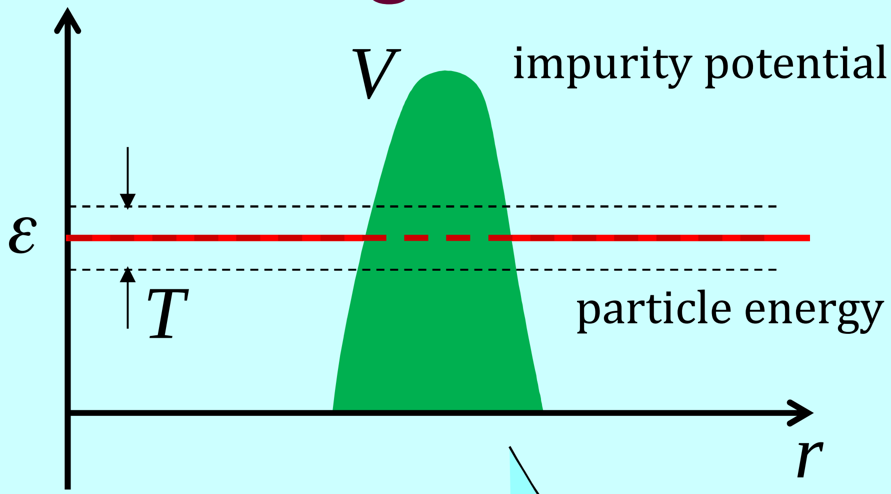
The impurity leads to Friedel oscillations of electron density



Interaction leads to backscattering of one electron on the Friedel oscillations of another.
The resulting interference kills conductance.

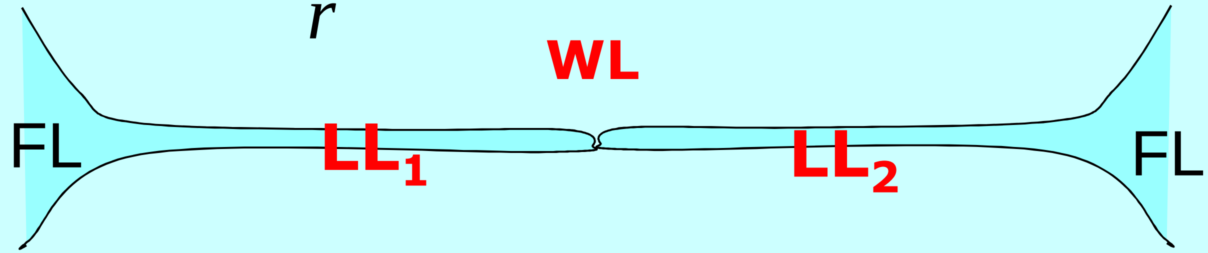
Matveev, Yue & Glazman ('92)

Strong scatterer – weak link



A single particle can only (weakly) tunnel through a high barrier.

It can be modelled as a WL between two LLs.



Tunneling amplitude t_{WL} scales with energy as

$t_{\mathrm{WL}}(\varepsilon) \sim t_0\, \varepsilon^{\Delta_{\mathrm{WL}}-1}$ where Δ_{WL}=1/K, **(C Kane & M Fisher, PRL, 92)**

$$G(T) \sim (T/T_0)^{\Delta_{\mathrm{WL}}-1}$$ for fermions

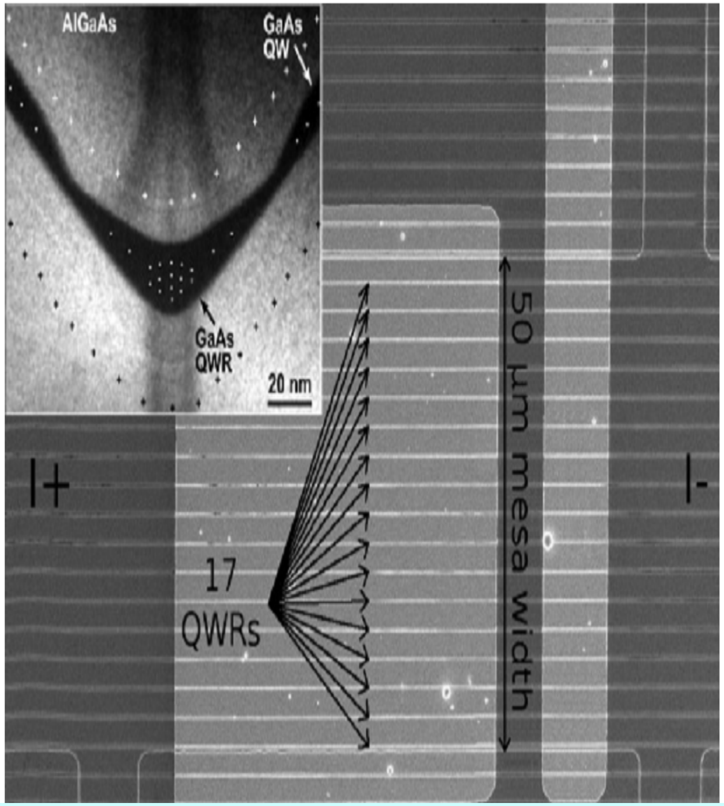
For bosons with repulsion (or fermions with attraction) Δ_{WL} <1

– an arbitrary large barrier becomes fully penetrable at low enough T

PHYSICAL REVIEW B **85**, 045315 (2012)

Experimental evidence for Luttinger liquid behavior in sufficiently long GaAs V-groove quantum wires

E. Levy,[1,*] I. Sternfeld,[1] M. Eshkol,[1] M. Karpovski,[1] B. Dwir,[2] A. Rudra,[2] E. Kapon,[2] Y. Oreg,[3] and A. Palevski[1]



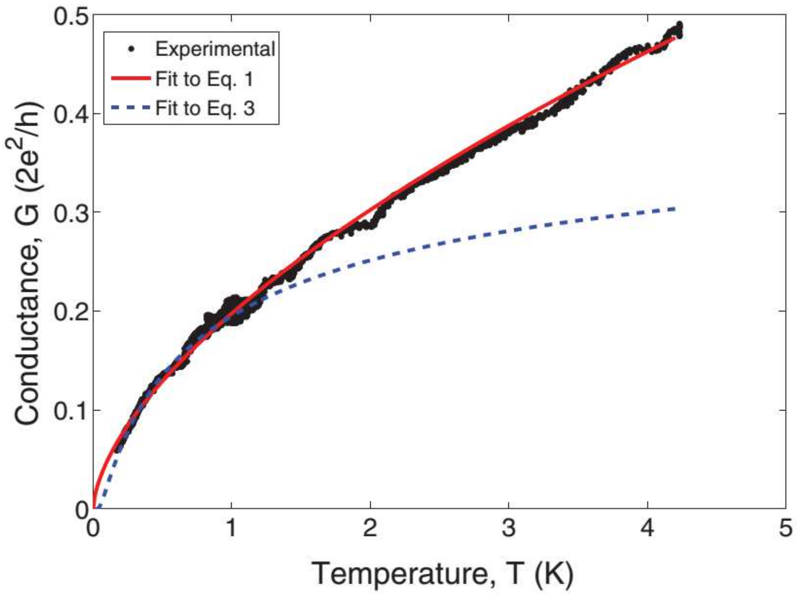


FIG. 3. (Color online) The value of the conductance at the first plateau (at $\rho_W = 17R/L = 20\ \mathrm{k}\Omega/\mu\mathrm{m}$), for the $L = 25\ \mu$m sample, as a function of temperature (dotted black points). The solid (red) curve represents the fit given by Eq. (1) with the two fitting parameters $g = 0.62 \pm 0.01$ and $T_0 = 14 \pm 0.7$ K. The dashed (blue) curve represents the fit given by Eq. (3) with $T_c = 20$ mK.

WL – WS duality

Adding weak scattering (WS), $\mathcal{L}_{\rm WS} = \lambda \cos 2\theta(0)$, and integrating over $\theta(x\neq 0)$ results in

$$\mathcal{S}_{\rm eff} = \frac{1}{K}\int \mathrm{d}\omega |\omega| \theta^2(\omega) + \lambda \int \mathrm{d}t \cos 2\theta(t)$$

The RG results in $\Delta_{\rm ws}$=K:

$\mathcal{L}$ is invariant with respect to $\theta \leftrightarrow \varphi \quad \& \quad K \leftrightarrow 1/K$

while WL enters $\mathcal{L}$ as $\mathcal{L}_{\rm WL} = t_{\rm WL} \cos 2\varphi(0)$.

Therefore WS ↔ WL ⇔ $K \leftrightarrow K^{-1}$ and $\varphi \leftrightarrow \theta$ resulting in the duality relation

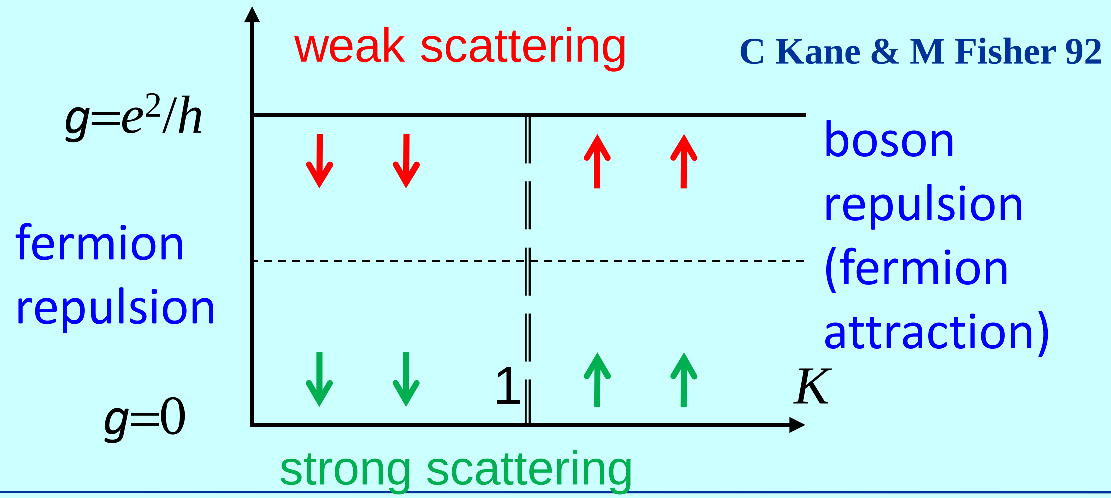
$$\Delta_{\rm WS}\,\Delta_{\rm WL} = 1$$

$\Delta<1$ ($\Delta>1$) – relevant (irrelevant) process

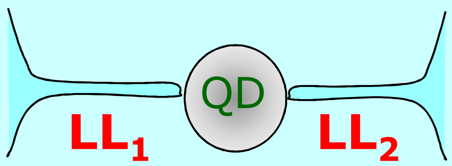
Thus duality means relevant scattering

⇓

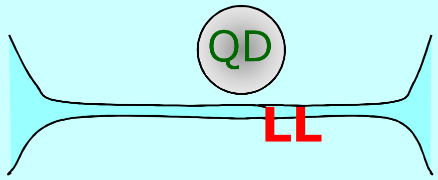
irrelevant tunnelling – and vice versa;



Duality in (anti)resonance



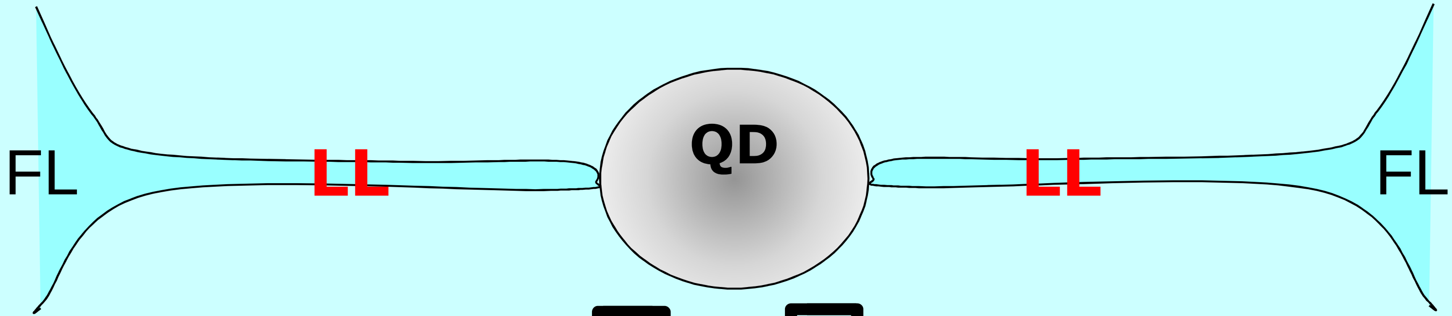
A resonant (double) WL

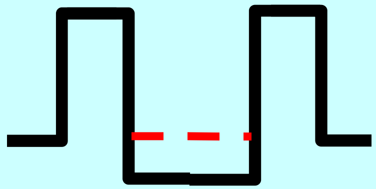


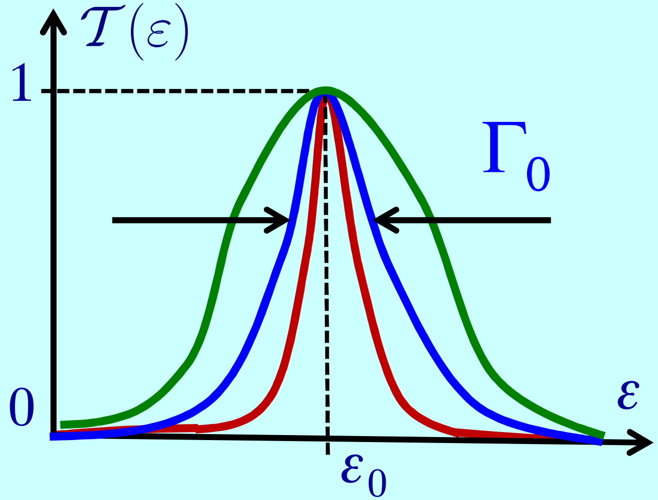
An antiresonant WS (a level in QD is hybridised with cond. el-s)

Resonant transmission through LL

The double barrier ⇔ a quantum dot with a resonant level







$$\mathcal{T}_{\rm res}(\varepsilon)\frac{\Gamma_0^2}{(\varepsilon-\varepsilon_0)^2+\Gamma_0^2}$$

Breit-Wigner resonance

$$\Gamma_0 = \pi\nu_0 |t_0|^2 \rightarrow \Gamma(\varepsilon) = \pi\nu(\varepsilon)|t_0|^2$$

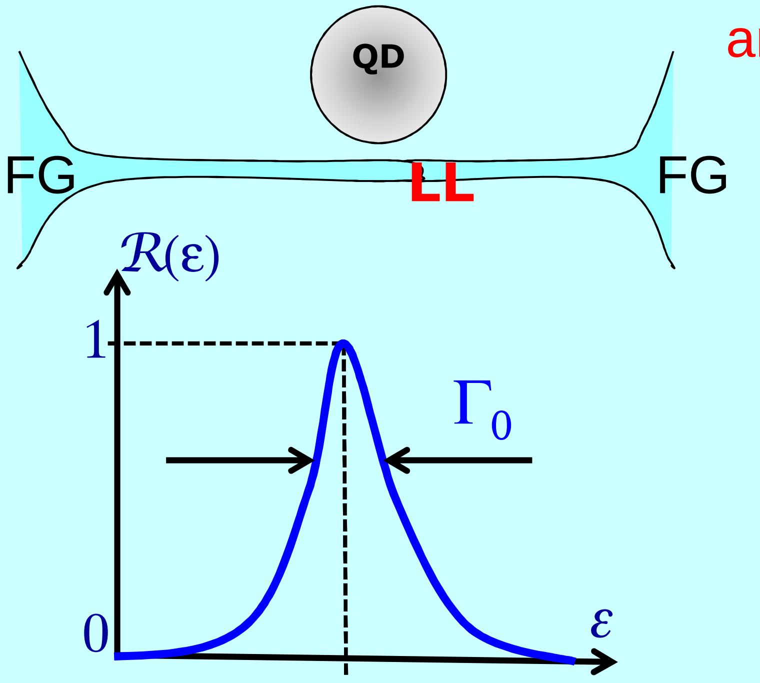
The "Luttinger liquid" effect – changing density of states $\nu_0 \rightarrow \nu(\varepsilon) \propto |\varepsilon|^\gamma$, $(\gamma = 1/K - 1)$

The LL effect when K<1

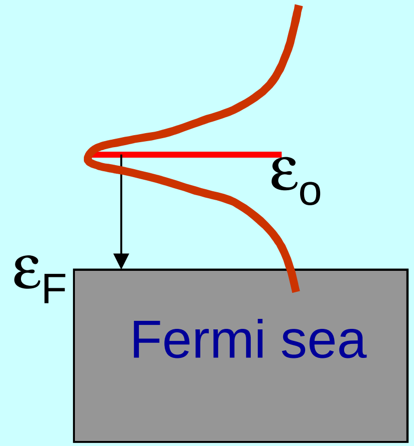
The LL effect when K>1

Kane & Fisher, PRL,96; Glazman & Nazarov, PRL,03; Polyakov & Gornyi, PRB,03

An alternative geometry



antiresonance



$$\mathcal{T}_{\text{res}}(\varepsilon) = \mathcal{R}_{\text{antires}}(\varepsilon) = \frac{\Gamma_0^2}{(\varepsilon - \varepsilon_0)^2 + \Gamma_0^2}$$

$$\mathcal{T}(\varepsilon) + \mathcal{R}(\varepsilon) = 1$$

The same LL effect? $\Gamma_0 \to \Gamma(\varepsilon) \propto \nu(\varepsilon)$ – can't be true!

Out of resonance, $\varepsilon_0 \gg \Gamma_0, \varepsilon$, $R(\varepsilon) \propto [\Gamma_0/\varepsilon_0]^2$ acts as a WS

$\Gamma(\varepsilon)$ should grow when $\varepsilon \to 0$ while $\nu(\varepsilon) \to 0$

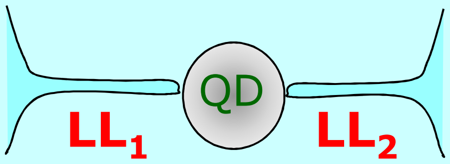
It turns out that the LL effect on Γ is opposite: $\Gamma_0 \to \Gamma(\varepsilon) \propto \nu^{-1}(\varepsilon) \propto \varepsilon^{-\gamma}$

I.L., V. Yudson, I.Yurkevich, PRL, 2008;

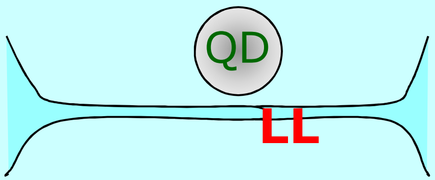
Duality in (anti)resonance

The WS – WL duality explains why $\Gamma(\varepsilon)$ are opposite for resonance/antiresonance:

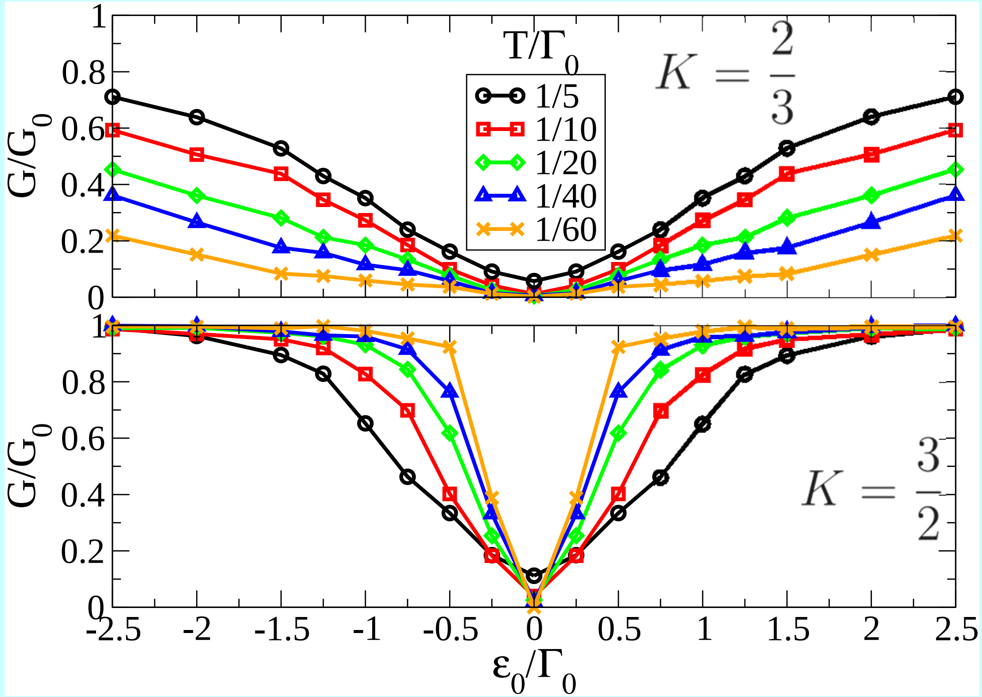
The action for the resonant WL



is dual to that for the resonant WS



by the same fields swap as for simple potential WS and WL:
$\theta \leftrightarrow \varphi, K \leftrightarrow K^{-1}$



M. Goldstein, R.Berkovits, PRL, 2010

Is duality robust?

- Does the duality survive any additional interaction, e.g. when electrons are coupled to retarded massless excitations (e.g., acoustic phonons), or when two species of ultracold atoms, e.g. fermions and bosons , are interacting?
- Can the additional coupling result in a nontrivial flow diagram for conductance when a scattering strength matters?

Adding phonons

The (longitudinal) phonon and el-ph action:

$$\mathcal{L}_{\rm ph} = -\tfrac{1}{2}\Phi\mathcal{D}_0^{-1}\Phi + g_{\text{e-ph}}\rho\Phi\,, \qquad \text{where} \qquad \mathcal{D}_0^{\rm ret}(\omega,q) = \frac{\omega_q^2}{\omega_+^2-\omega_q^2}$$

$\rho \propto \partial_x\theta$ is the total electron density; ω_q=cq

The phonon fields Φ can be integrated out resulting in the θ-only action

$$\mathcal{L}_{\rm ws} = -\frac{1}{2\pi vK}\theta(\xi)\left(\partial_t^2 - v^2\partial_x^2\right)\theta(\xi) - \frac{\alpha v}{2\pi K}\partial_x\theta(\xi)\mathcal{D}(t-t';\,x,x')\partial_{x'}\theta(\xi')\,; \qquad \xi\equiv(x,\,t)\,, \quad \alpha\equiv\frac{g^2K}{\pi v}\,.$$

Electrons are coupled via $g_{\text{e-ph}}\Phi\partial_x\theta$ to the lattice polarisation Φ made by phonons. **This irrevocably breaks the duality between the phase fields φ and θ** .

Two-component el-ph liquid

The normal modes of the full action are slow and fast "polarons" with $\omega = \pm v_\pm q$, where $v_\pm$ are the mode velocities, each with its own K

$$v_\pm^2 = \frac{1}{2}\left[v^2 + c^2 \pm \sqrt{(v^2 - c^2)^2 + 4\alpha\, v^2 c^2}\right]\ ; \quad v_- < v < v_+\,, \quad \alpha \equiv \frac{g_{\mathrm{e-ph}}^2 K}{\pi v}$$

D Loss & T Martin, 1994

We assume $\alpha<1$ ($\Leftrightarrow v_-^2>0$) to avoid the **Wentzel-Bardeen** instability

Known also for 1D fermion-boson cold atom mixtures, M Cazalilla & A Ho, 2003

However, electrons and phonons are not equal partners in the two-comp liquid: backscaterring from a defect is critical for electrons, as it is enhanced by their interaction, but not for noninteracting phonons.

Phonon scattering by defect

Extreme possibilities:

a) Translational phonons ignore the defect (el-density depletion; or, more generally, the defect which oscillates with the lattice)

b) Reflected phonons (e.g., when the defect is pinned to a substrate)

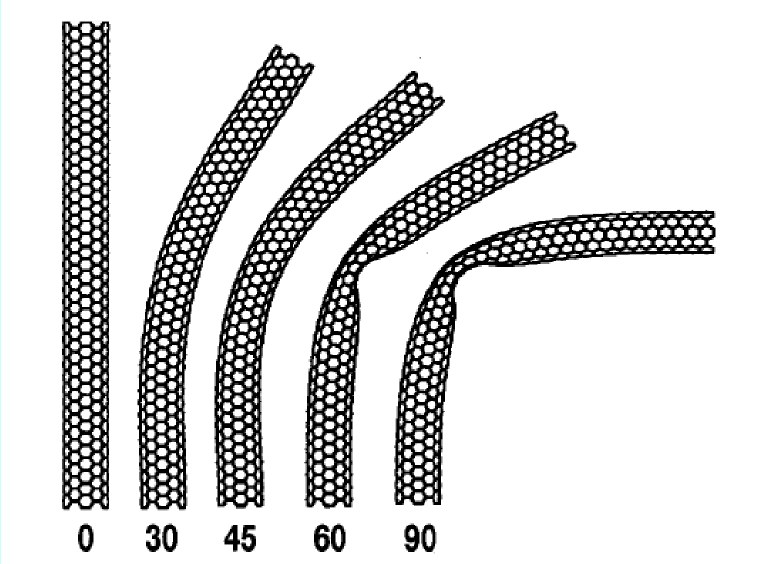
Generically, phonon scattering is described by a unitary S-matrix with a (complex) reflection coefficient r:

$r=0$ for translational phonons

$r=-1$ for reflected phonons

Crucial: phonon scattering is not critical.

The interactions (anharmonism, or el-ph) give no singular corrections to r.



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Phonons and WS

Translational phonons for are described by the phonon action with $\mathcal{D}$ =$\mathcal{D}_0$

$$\mathcal{L}_{\rm ph} = -\frac{\alpha v}{2\pi K}\partial_x\theta(\xi)\mathcal{D}(t-t';\, x,\, x')\partial_{x'}\theta(\xi')$$

For fully reflected phonons, the phonon propagator above is related to $\mathcal{D}_0$ by

$$\mathcal{D}(x,x') = \left[\mathcal{D}_0(x-x') + \mathcal{D}_0(x+x')\right]\Theta(xx')$$

In general, for phonons with the reflection amplitude *r*, it is given by

$$\mathcal{D}(x,x') = \mathcal{D}_0(x-x') - r\,\mathrm{sgn}(xx')\,\mathcal{D}_0(|x|+|x'|).$$

The full action in the presence of a single WS

$$\mathcal{S}_{\rm eff} = \tfrac{1}{2}\int\!\mathrm{d}\xi\,\mathrm{d}\xi'\,\theta(\xi)\,\mathcal{G}^{-1}(\xi,\,\xi')\,\theta(\xi') - \lambda\int\!\mathrm{d}t\,\cos(2\theta(t))$$

$$\left[\partial_+\partial_- - \mathcal{D}(\xi,\xi')\right]\mathcal{G}(\xi,\,\xi') = I$$

Solvable due to a factorability, $\mathcal{D}_0^{\rm r}(\omega;|x|+|x'|) \propto \omega_+^{-1}\mathcal{D}_0^{\rm r}(\omega;|x|)\,\mathcal{D}_0^{\rm r}(\omega;|x'|)$

WS scaling dimension

The action is quadratic at x≠0. Integrating out θ(x≠0)results in the 0d action

$$\mathcal{S}_{\rm eff} = \frac{1}{2}\int \mathrm{d}t\,\mathrm{d}t'\,\theta(t)\,\mathcal{G}^{-1}(t-t')\,\theta(t') - \lambda\int \mathrm{d}t\cos(2\theta(t))\,.$$

The Fourier transform of the retarded part of $\mathcal{G}$ (x=0,t) can be represented as

$$\mathcal{G}(\omega) = -\frac{\pi i}{2}\frac{1}{\omega+i0}\Delta(\alpha,\beta,r)\,, \qquad \beta \equiv \frac{v}{c}\,, \quad \alpha \equiv \frac{K g_{\rm ph}^2}{\pi v}\,.$$

$\Delta(\alpha,\beta,r)$ is ω-independent (equal to K at α=0). Thus the el-ph coupling doesn't change the RG scheme for WS. Calculating Δ gives a new scaling dim. of λ, $\Delta_{\rm WS}$

$$\Delta_{\rm ws} = K\,\frac{(1+r)(1+\beta\kappa)-rW}{(1+r)W\kappa - r(\kappa+\beta)}$$

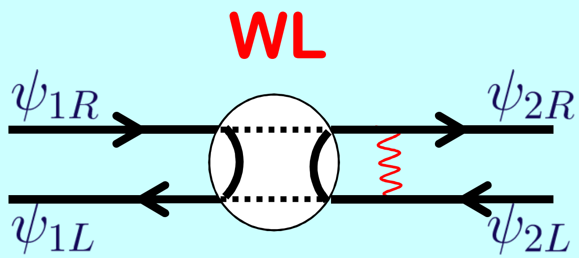
$$\kappa \equiv \sqrt{1-\alpha}\,, \qquad W \equiv \sqrt{1+2\beta\kappa+\beta^2}\,.$$

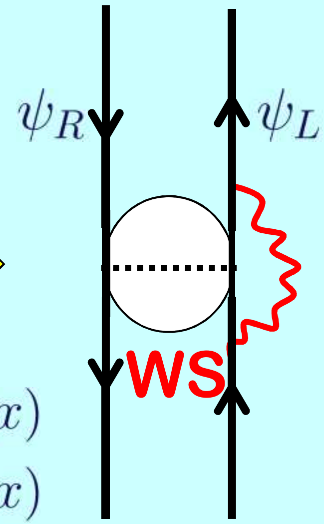
At r=0 this reproduces the result of P.San-Jose, F.Guinea, &T.Martin, 2005

Chiral unfolding for WL

As phonons coupled to field θ, the duality $\theta \to \varphi$ used for WL at α=0 wouldn't help.

Instead – the unfolding:
Eggert &Affleck, '92;
Fabrizio &Gogolin, '95





$$\psi_R(x) = \psi_{1R}(x)\Theta(-x) + \psi_{1L}(-x)\Theta(x)$$
$$\psi_L(x) = \psi_{2R}(x)\Theta(x) + \psi_{2L}(-x)\Theta(-x)$$

A problem of the interaction becoming nonlocal

was solved by rescaling $\theta \to \theta\sqrt{K}$ that reduce S_{eff} to that of free fermions, $K=1$.

Although this can't remove the el-ph interaction, we still do the unfolding and rescaling – it simplifies the phonon action which was nonlocal anyway.

Phonons and WL

As a result, the action density becomes

$$\mathcal{L}_{\rm WL} = \frac{K}{2\pi v}\left[\partial_t\theta(\xi)\mathcal{Q}^{-1}\partial_{t'}\theta(\xi') - v^2\partial_x\theta(\xi)\widetilde{\mathcal{D}}\partial_{x'}\theta(\xi')\right],$$

$$\widetilde{\mathcal{D}}^{\rm r} = \delta(x-x') + \tfrac{1}{2}\alpha\left[\mathcal{D}_0^{\rm r}(\omega;x-x') + \mathcal{D}_0^{\rm r}(\omega;x+x')\right],$$

$$\mathcal{Q}^{\rm r} = \widetilde{\mathcal{D}}^{\rm r} - \alpha(1+r)\mathcal{D}_0^{\rm r}(\omega;|x|+|x'|),$$

while the tunnelling term unfolds to $t_{\rm WL}\cos[2\theta(t)]$.

It results in the scaling dimension

$$\Delta_{\rm WL} = \frac{1}{KW}\left[\kappa + \beta - \frac{(1+r)\alpha\beta}{W + (1+r)(1+\beta\kappa - W)}\right].$$

On the face of it, $\Delta_{\rm WS}$ looks different

$$\Delta_{\rm WS} = K\,\frac{(1+r)(1+\beta\kappa) - rW}{(1+r)W\kappa - r(\kappa+\beta)}$$

$$\kappa \equiv \sqrt{1-\alpha}\,, \qquad W \equiv \sqrt{1+2\beta\kappa+\beta^2}\,.$$

Nevertheless, the duality holds $\Delta_{\rm WS}\,\Delta_{\rm WL} = 1$

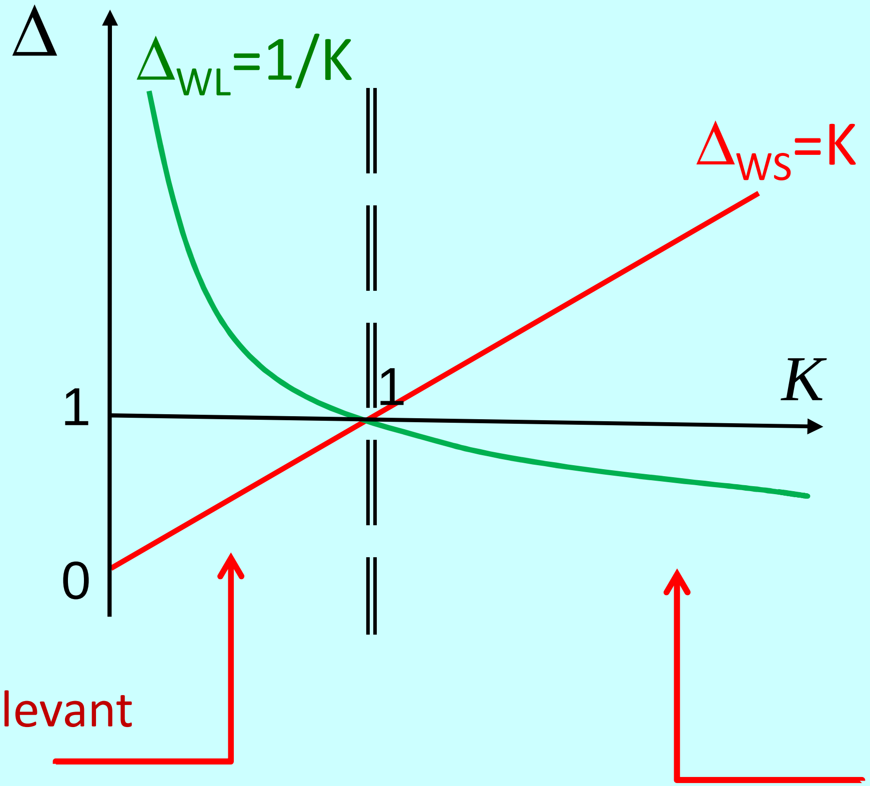
I Yurkevich, A Galda, O Yevtushenko, IL, PRL, 2013
At r=0 this reproduces the result of P.San-Jose, F.Guinea, &T.Martin, PRL, 2005

Two scenarios for backscattering

1. Electrons and phonons backscattering amplitudes change independently: one goes from weak backscattering to strong (ie weak tunnelling t_{WL}) keeping the phonon reflection coefficient r fixed

2. Backscattering amplitudes λ and r change in parallel (e.g., when bending a nanotube, or making a strong lattice coupling to an AFM tip).

Usual LL



$$\lambda_{WS} \propto \varepsilon^{\Delta_{WS}-1}$$
$$t_{WL} \propto \varepsilon^{\Delta_{WL}-1}$$

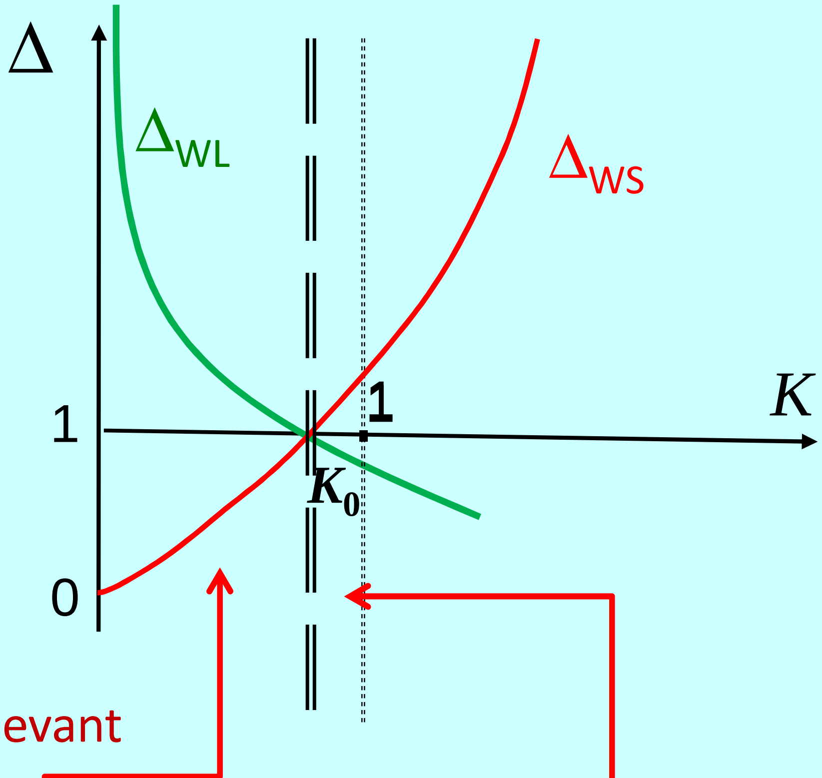
impurity relevant
(insulator)

impurity irrelevant
(conductor)

LL coupled to phonons: 1st scenario

Duality holds, $\Delta_{\rm WS}\,\Delta_{\rm WL}=1$, when el and ph scatterings are uncorrelated

$$\lambda_{\rm WS}\propto\varepsilon^{\Delta_{\rm WS}-1}$$
$$t_{\rm WL}\propto\varepsilon^{\Delta_{\rm WL}-1}$$

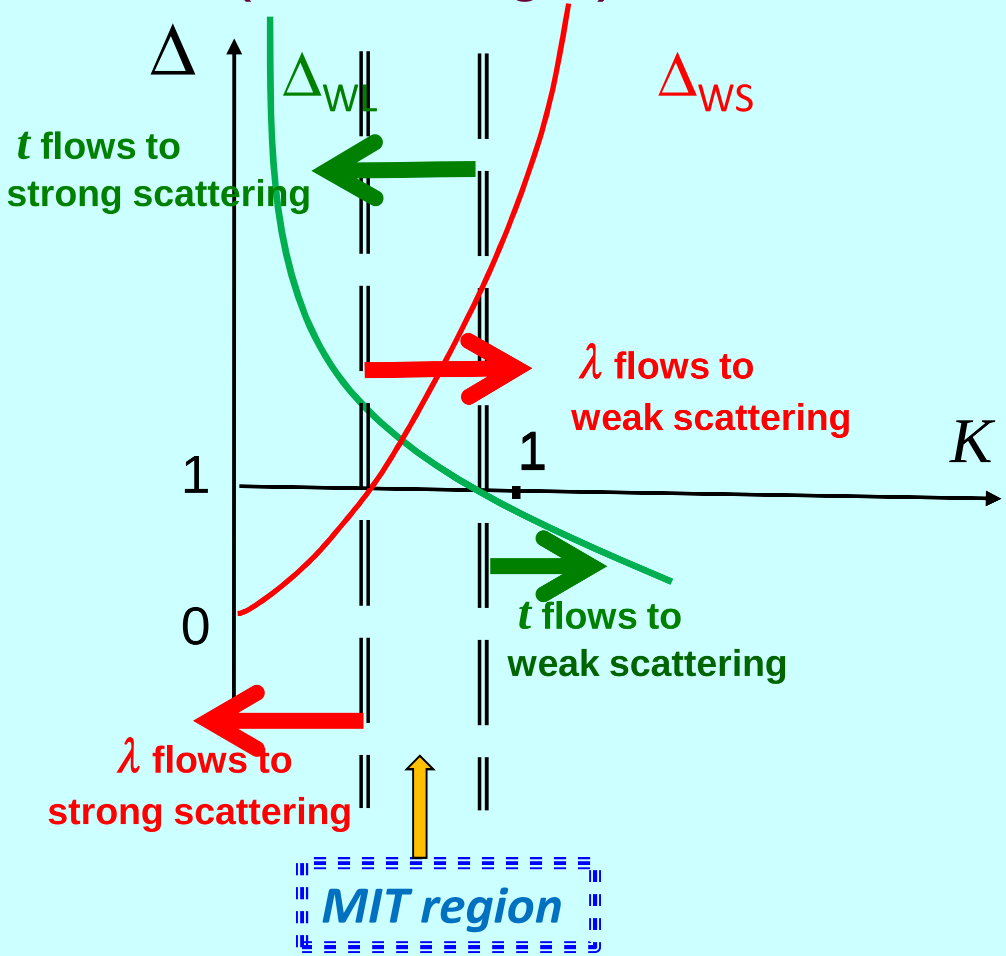


impurity relevant
(insulator)

impurity irrelevant
(conductor)

LL coupled to phonons: 2nd scenario

Duality breaks down, $\Delta_{\mathrm{WS}}\,\Delta_{\mathrm{WL}} \neq 1$, when going from WS to WL (increasing λ) also increases r.

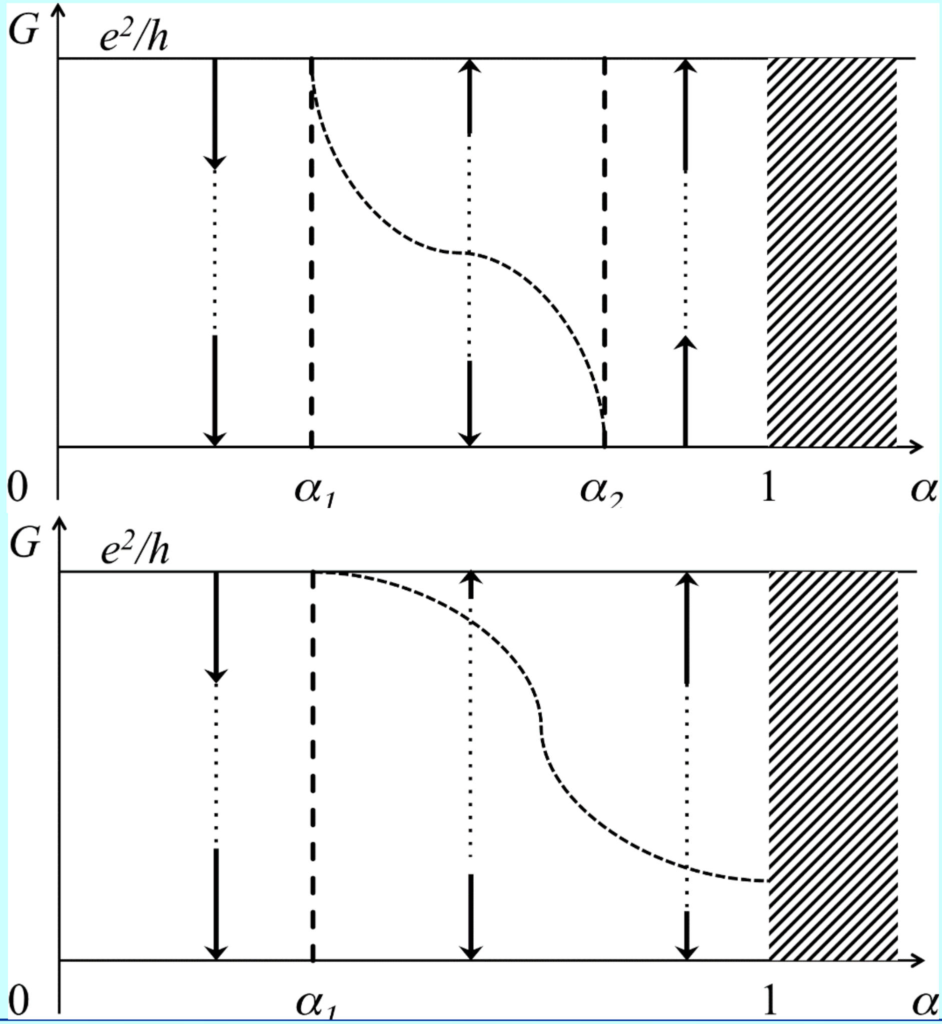


$$\lambda_{\mathrm{WS}} \propto \varepsilon^{\Delta_{\mathrm{WS}}-1}$$

$$t_{\mathrm{WL}} \propto \varepsilon^{\Delta_{\mathrm{WL}}-1}$$

Correlated scattering

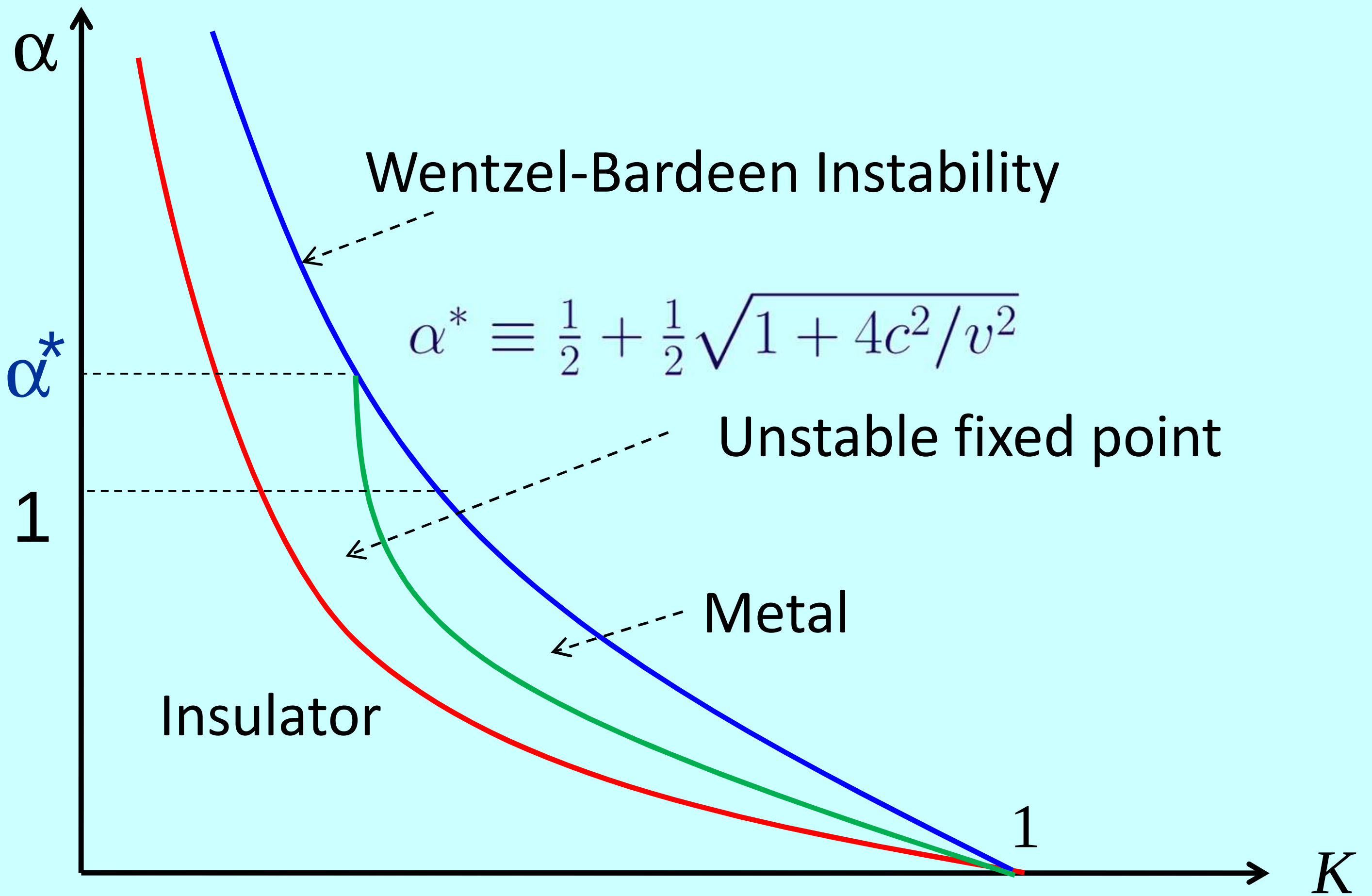
Duality is irrelevant: Δ_{WS} and Δ_{WL} are taken at different r



α_1 (or α_2) is the point where Δ_{WS} (or Δ_{WL}) equals 1

Depending on β, α_2 can exceed 1.

Phase Diagram



Galda, Yurkevich, IL, PRB 83, 041106(R) (2011))

Galda, Yurkevich, Yevtushenko, IL, PRL, 110, 136405 (2013)

Fermion-boson superflow

A two-component liquid of centaurs, which are neither fermions nor bosons, with $v_- < v, c < v_+$ fractionalises into phase-separated flows (f and b) when the f-b interaction is relatively weak: $g_{fb} < vc/K_f K_b$ (from the point where $v_-^2 < 0$).

Can flowing through a constriction cause the fractionalisation of the f-b mixture?

For the contact interaction, $K_f = 1$, weak scattering becomes irrelevant for fermions, $\Delta^f_{WS} > 1$, and even more irrelevant for bosons, $\Delta^b_{WS} > K_b > 1$,

$$\Delta^{\rm f}_{\rm ws} = (v + c/\kappa)\, W^{-1} > 1\,, \qquad \Delta^{\rm b}_{\rm ws} = K_{\rm b}\,(v/\kappa + c)\, W^{-1} > K_{\rm b} > 1\,,$$

while weak link is always relevant for both: $\Delta^b_{WL} < K_b^{-1} < 1$ and $\Delta^f_{WL} < 1$

$$\Delta^{\rm f}_{\rm WL} = \frac{v + c\kappa}{W} < 1\,, \qquad \Delta^{\rm b}_{\rm WL} = \frac{v\kappa + c}{K_{\rm b} W} < \frac{1}{K_{\rm b}} < 1\,,$$

Fermions follow bosons in “sympathetic” superflow for any impurity strength.

A full phase diagram might re-emerge, though, for fermions with a (long-range) f-f interaction – e.g. for dipole molecules.

Summary

- Duality between the weak scattering and weak tunneling governs 1D flow of interacting particles through an impurity (defect, constriction, etc)
- A coupling to massless bosons does not destroy the duality. Does it mean that there is a hidden integrability or is duality “stronger” than integrability?
- The e-ph coupling may result in a rich phase diagram: MIT with changing the impurity strength is possible
- Similarly, the fermion-boson flow through a constriction could change from free to none depending on the strength (width) of the constriction