Many-body physics with ultra-cold atoms in disorder

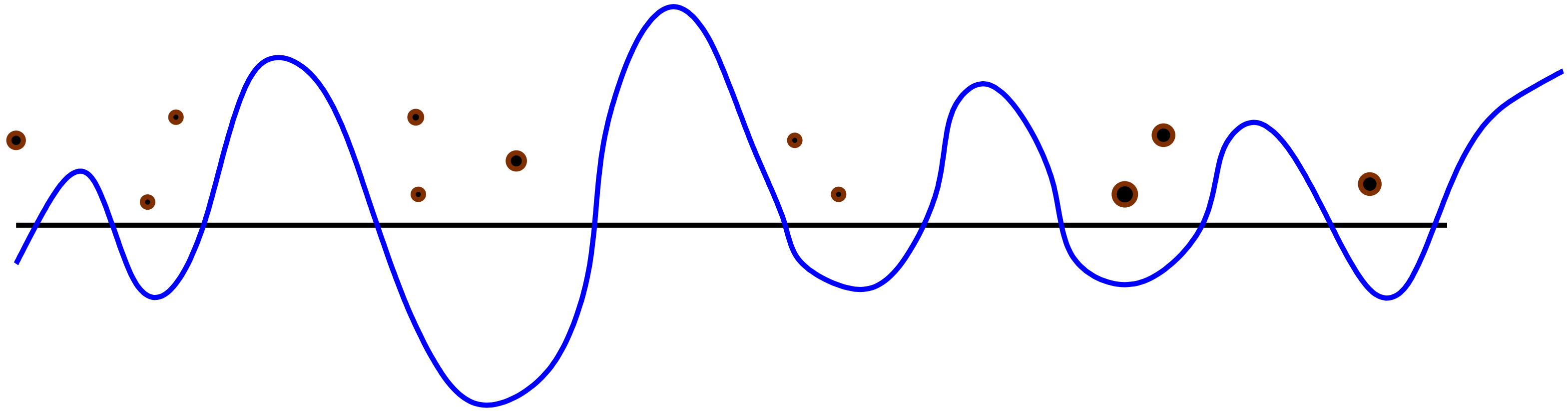
Gora Shlyapnikov
LPTMS, Orsay, France
University of Amsterdam, The Netherlands

- Introduction.
- Many-body localization-delocalization transition
- MBLDT for 1D disordered bosons
- MBLDT in the AAH model
- Phase diagram
- Conclusions

Collaborations B.L. Altshuler/I.L. Aleiner (Columbia Univ.), V. Michal (LPTMS, Orsay)

Santa Barbara, USA, October 15, 2015

Many-body system in disorder



Many-particle system in disorder $\Rightarrow$ Transport and localization properties

Anderson localization (P.W. Anderson, 1958)

Destructive interference in the scattering of a particle from random defects

Old question. How does the interparticle interaction influence localization?

Long standing problem. Crucial for charge transport in electronic systems

Appears in a new light for disordered ultracold bosons

Palaiseau, LENS, Rice, Urbana experiments. More underway

What was known and expected?

What was done?

Anderson localization of

Light

Microwaves

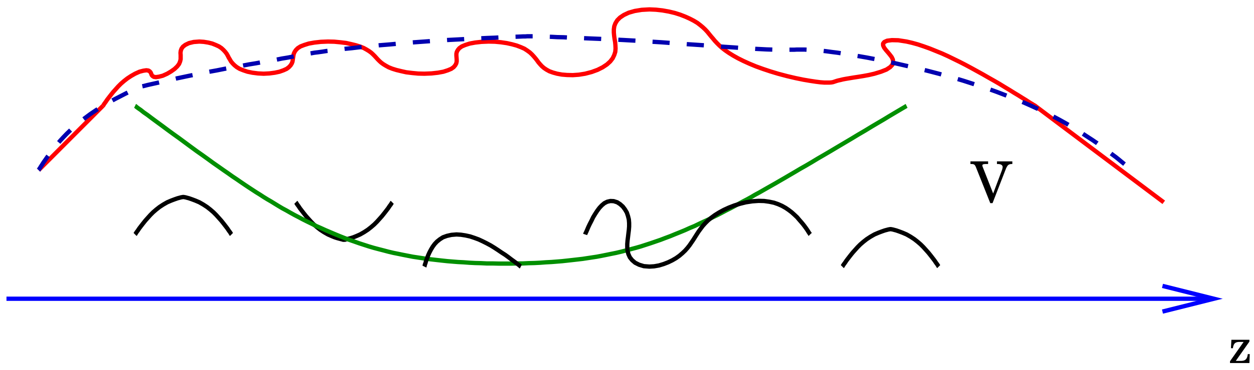
Sound waves

Electrons in solids

What is expected?

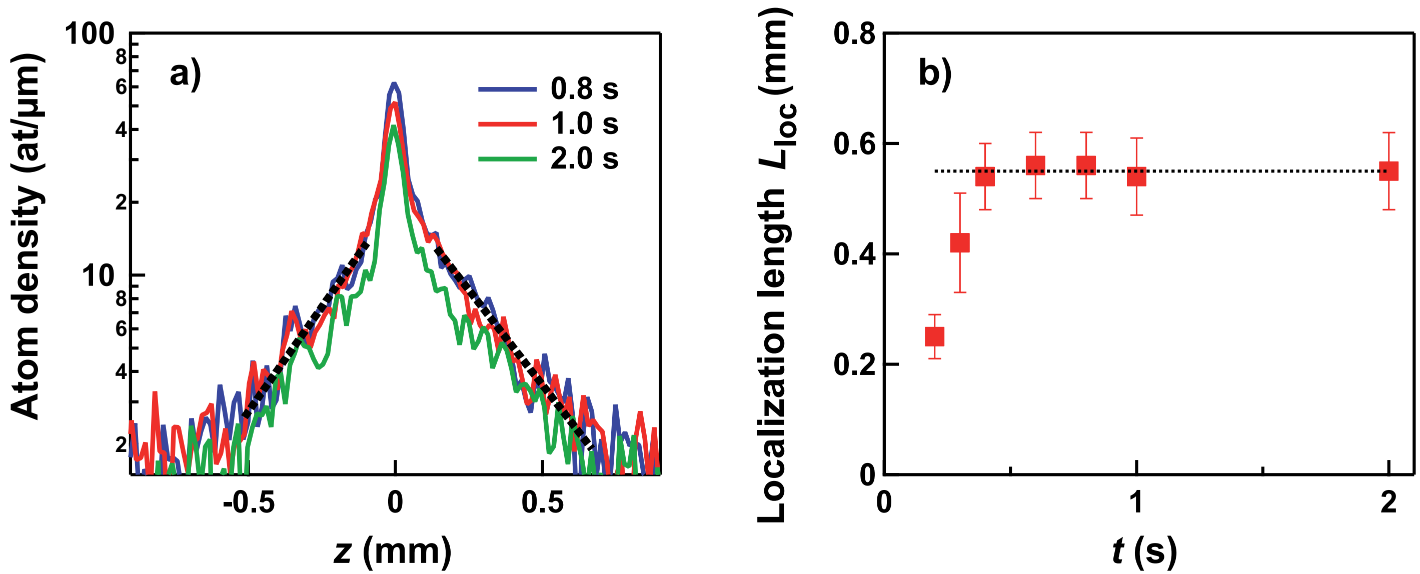
Anderson localization of neutral atoms

Experiments with cold atoms



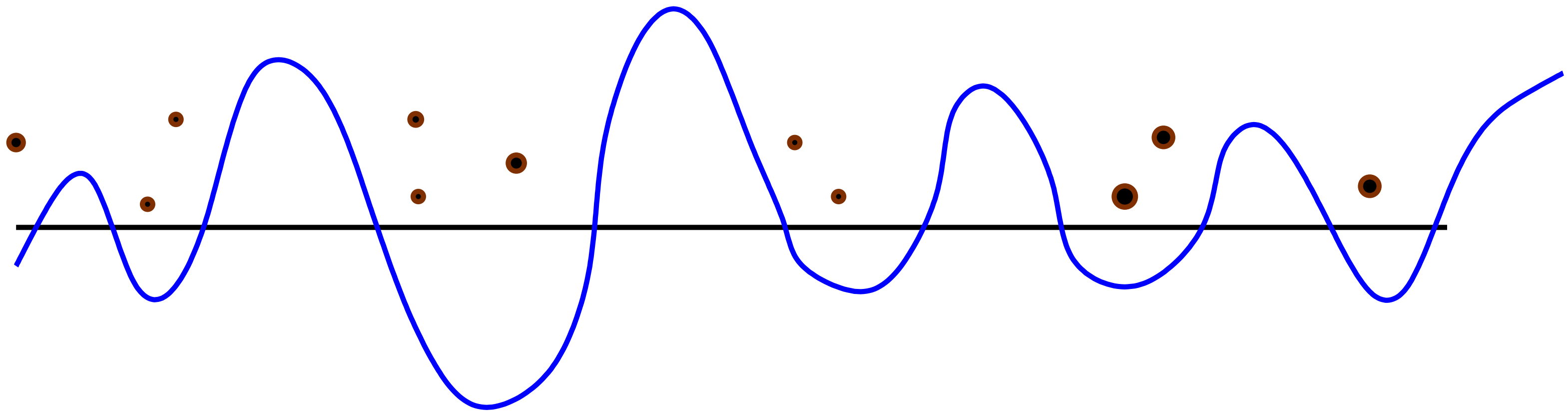
BEC in a harmonic + weak random potential $|V(z)| \ll ng \Rightarrow$ small density modulations of the static BEC. Switch off the harmonic trap, but keep the disorder $\Rightarrow$ What happens? (Orsay, LENS, Rice)

Orsay experiment



Quantum gases in disorder. What was not expected?

One-dimensional disordered bosons at finite temperature



DOGMA $\rightarrow$ No finite temperature phase transitions in 1D
as all spatial correlations decay exponentially

There is a non-conventional phase transition between two distinct states

Fluid and Insulator

Interaction-induced transition

I.L. Aleiner, B.L. Altshuler, GS, (2010)

Many-body localization-delocalization transition

(Aleiner, Altshuler, Basko 2006-2007)

How different states of two particles $|\alpha, \beta\rangle$ hybridize due to the interaction?

The probability $P(\varepsilon_\alpha)$ that for a given state $|\alpha\rangle$ there exist $|\beta\rangle, |\alpha'\rangle, |\beta'\rangle$

such that $|\alpha, \beta\rangle$ and $|\alpha', \beta'\rangle$ are in resonance:

$\langle \alpha, \beta | H_{int} | \alpha', \beta' \rangle$ exceeds $\Delta_{\alpha\beta}^{\alpha'\beta'} \equiv |\varepsilon_\alpha + \varepsilon_\beta - \varepsilon_{\alpha'} - \varepsilon_{\beta'}|$

MBLDT criterion $\quad P(\varepsilon_\alpha) \sim 1$

$\varepsilon_\alpha \approx \varepsilon_{\alpha'}; \;\; \varepsilon_\beta \approx \varepsilon_{\beta'} \Rightarrow$ Matrix element $\langle \alpha, \beta | H_{int} | \alpha', \beta' \rangle = U N_\beta \dfrac{a}{\zeta_{max}}$

Mismatch $\Delta_{\alpha\beta}^{\alpha'\beta'} = |\varepsilon_\alpha + \varepsilon_\beta - \varepsilon_{\alpha'} - \varepsilon_{\beta'}| \approx \left| \dfrac{1}{\zeta_\alpha \rho(\varepsilon_\alpha)} + \dfrac{1}{\zeta_\beta \rho(\varepsilon_\beta)} \right| \approx \dfrac{1}{(\zeta\rho)_{min}}$

MBLDT criterion

The probability that $\langle \alpha, \beta | H_{int} | \alpha', \beta' \rangle$ exceeds $\Delta_{\alpha\beta}^{\alpha'\beta'}$

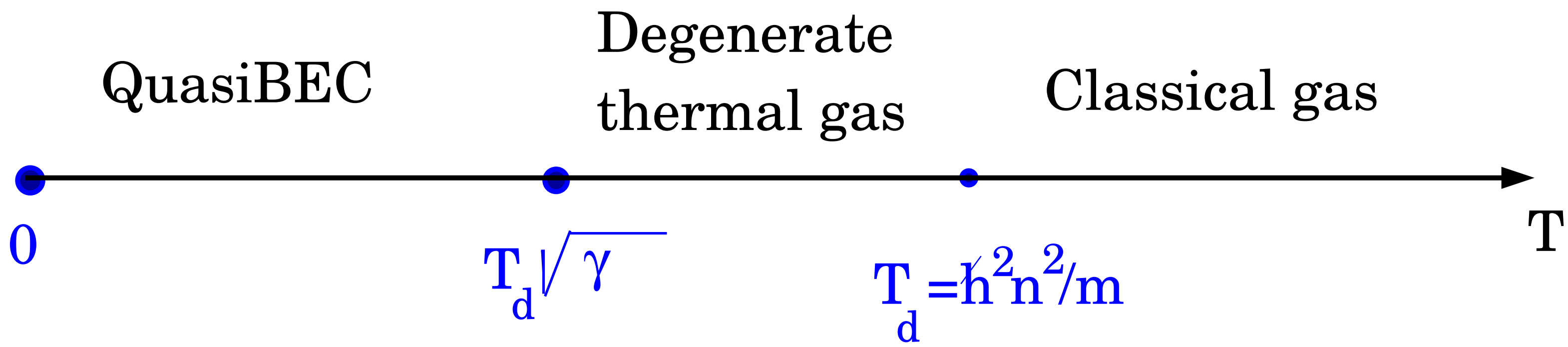
$$P_{\alpha\beta}^{\alpha'\beta'} \approx U N_\beta \frac{a(\zeta\rho)_{min}}{\zeta_{max}}$$

$$P(\varepsilon_\alpha) = \sum_{\beta,\alpha',\beta'} P_{\alpha\beta}^{\alpha'\beta'} = U \int d\varepsilon_\beta \rho(\varepsilon_\beta) \zeta_\beta N_\beta \frac{a(\zeta\rho)_{min}}{\zeta_{max}}$$

Critical coupling strength $U_c \approx \left[\int d\varepsilon_\beta \rho(\varepsilon_\beta) \zeta_\beta N_\beta \frac{a(\zeta\rho)_{min}}{\zeta_{max}} \right]^{-1}$

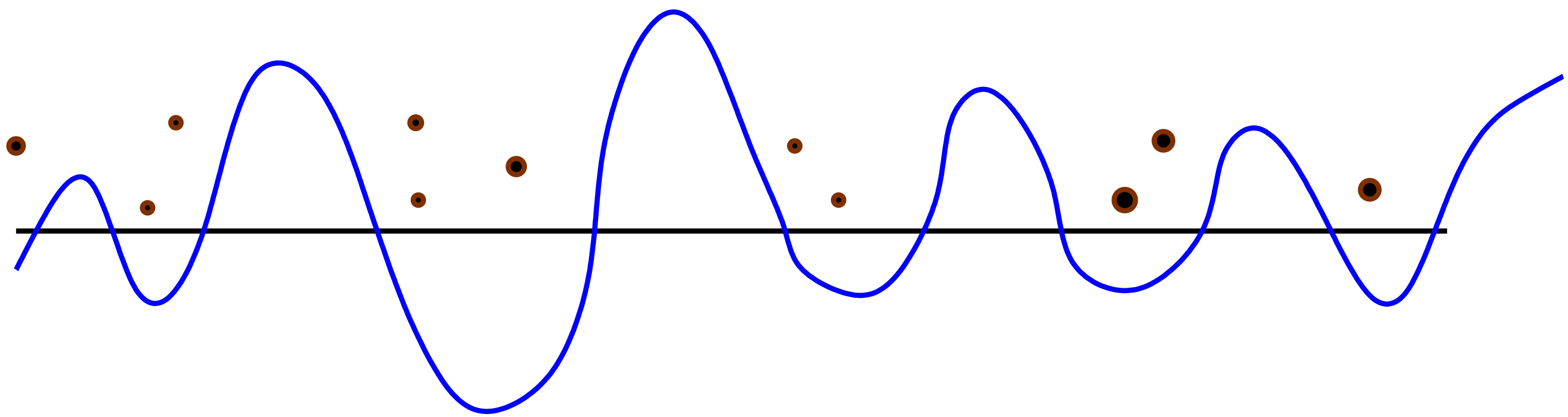
1D bosons

Interacting 1D Bose gas. No disorder $\Rightarrow$ Fluid phase



$$\gamma = \frac{mg}{\hbar^2 n} = \frac{ng}{T_d} \ll 1 \;\rightarrow\;$$ weakly interacting regime

Disordered non-interacting 1D bosons

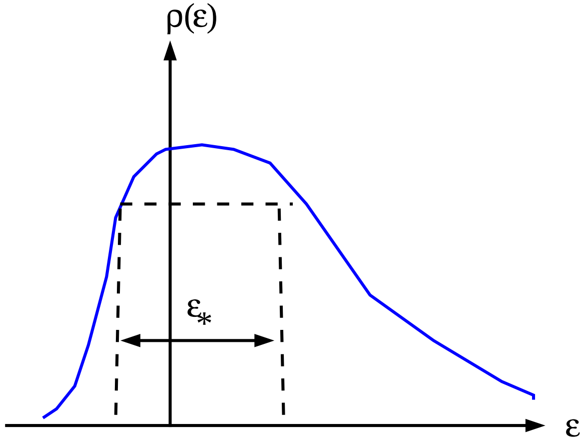


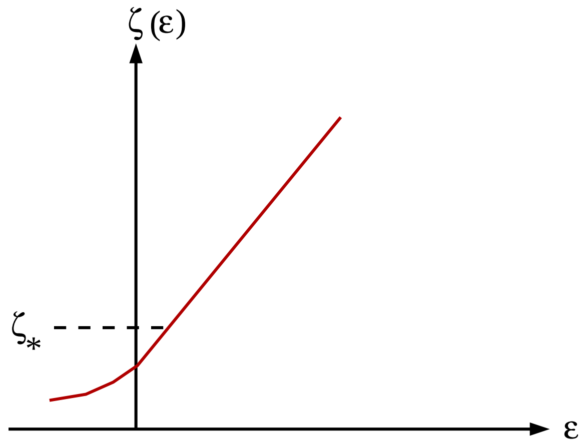
All single-particle states are localized at any energy $\rightarrow$ Anderson insulator

1D Bose gas in disorder

I.L. Aleiner, B.L. Altshuler, G.S., 2010

$$\rho(\varepsilon) \simeq \sqrt{\frac{m}{2\pi\hbar^2\varepsilon}}; \quad \zeta(\varepsilon) \simeq \frac{\hbar\varepsilon}{m^{1/2}\varepsilon_*^{3/2}} \quad \varepsilon > \varepsilon_* = U_0 \left(\frac{U_0\sigma^2 m}{\hbar^2}\right)^{1/3}$$





Classical gas $\Rightarrow T > T_d \sim \hbar^2 n^2/m; \quad \mu = T \ln n\Lambda_T$

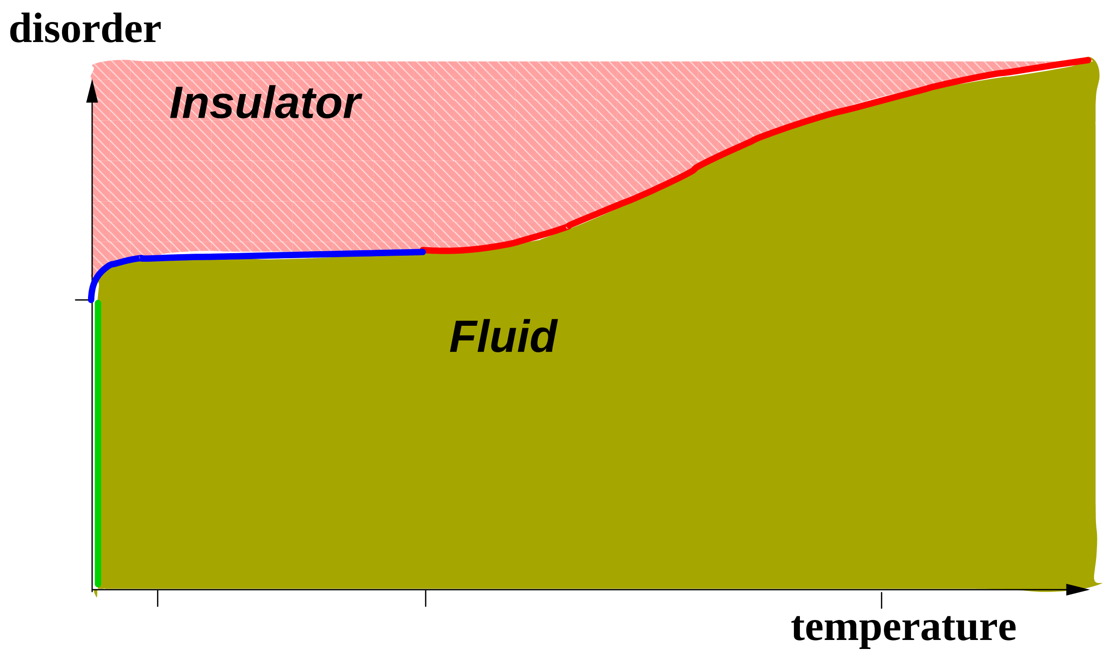
$$ng_c \sim \varepsilon_* \left(\frac{\varepsilon_*}{T}\right)^{1/2} \ll \varepsilon_*$$

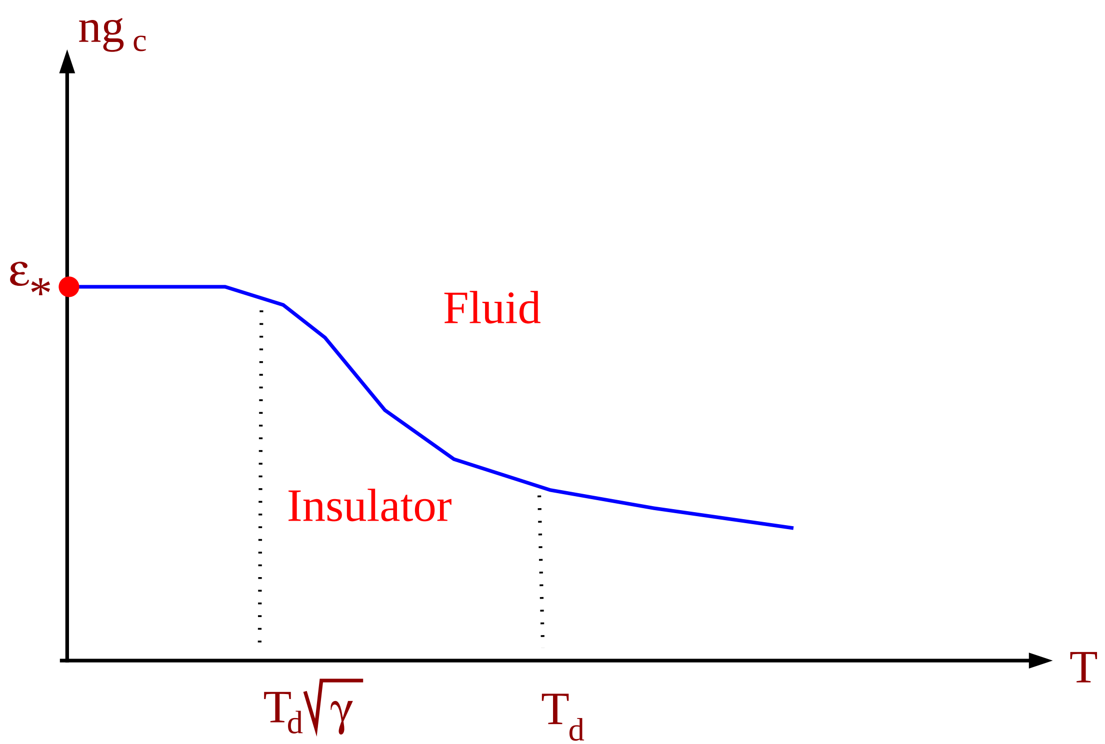
Quantum decoherent gas $\Rightarrow T_d\sqrt{\gamma} < T < T_d; \quad \mu \sim T^2/T_d$

$$ng_c \sim \varepsilon_* \left(\frac{\varepsilon_* T_d}{T^2}\right)^{1/2} \sim \frac{1}{T} \ll \varepsilon_*$$

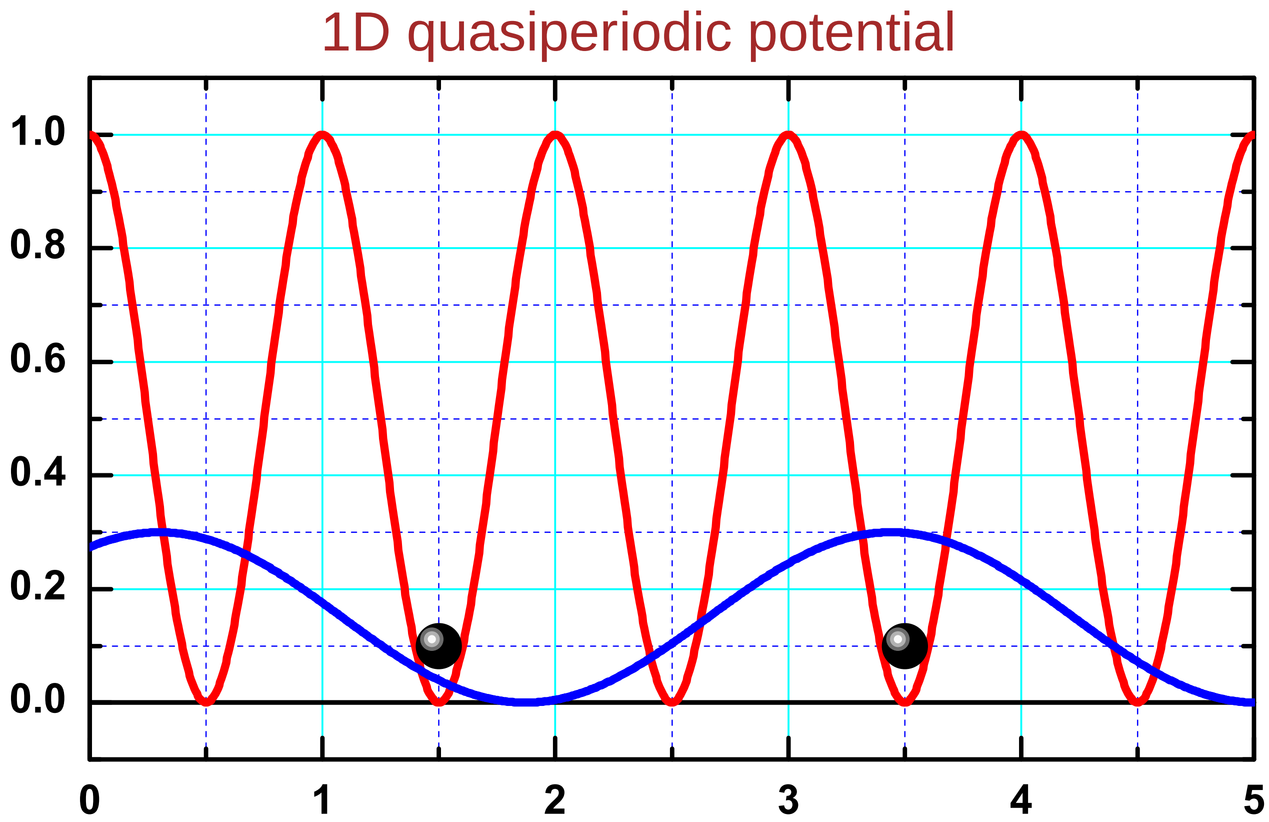
QuasiBEC $\Rightarrow T < T_d\sqrt{\gamma}; \quad ng_c \sim \varepsilon_*$

1D Bose gas in disorder





LENS experiment. What is expected?



Single-particle state

$$J(\psi_{n+1} + \psi_{n-1}) + V\cos(2\pi\kappa n)\psi_n = \varepsilon\psi_n$$

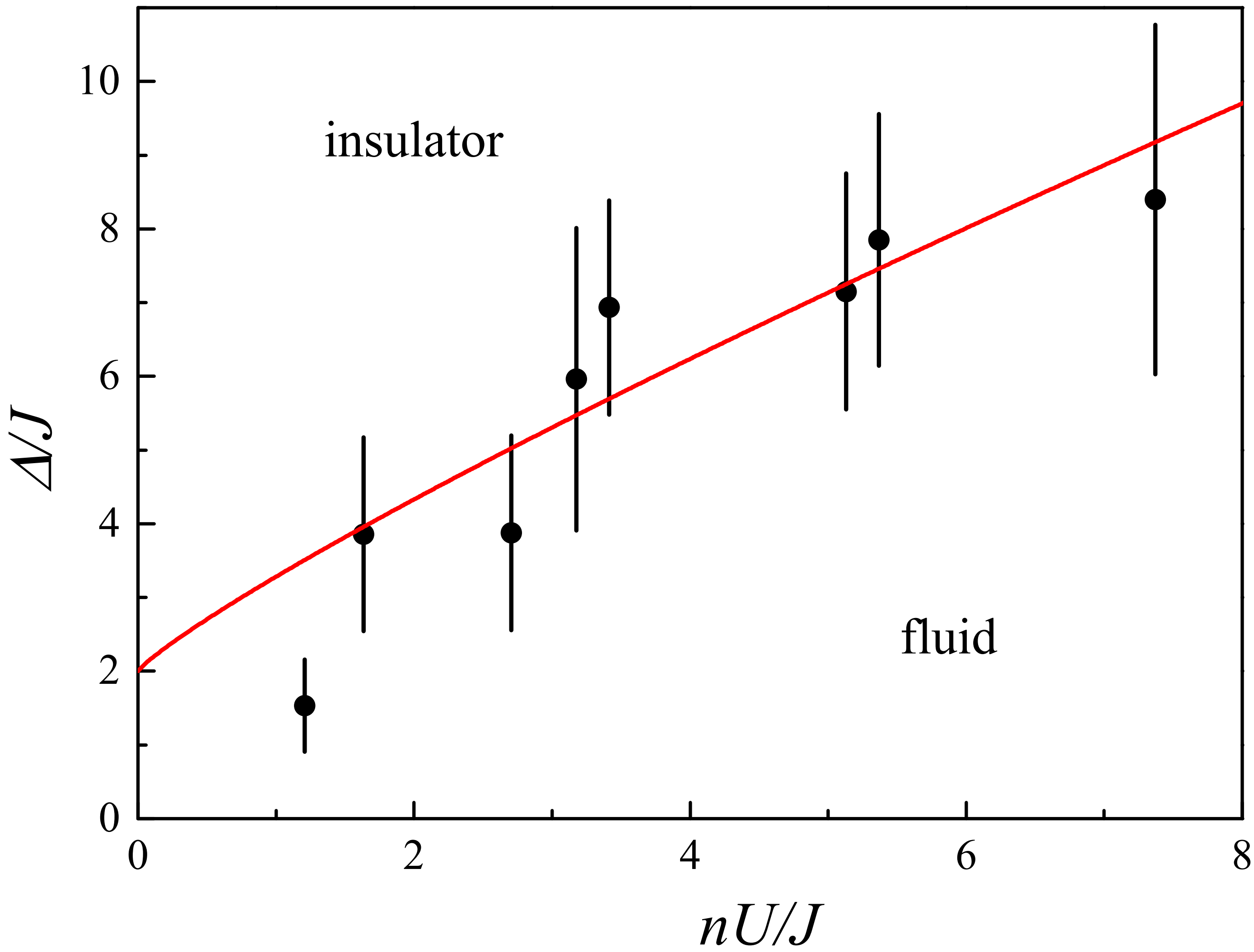
$V > 2J \rightarrow$ all single-particle states are localized

Aubry/Andre (1980)

LENS experiment

Feshbach modification of the interaction for ^{39}K

Observation of the fluid-insulator transition

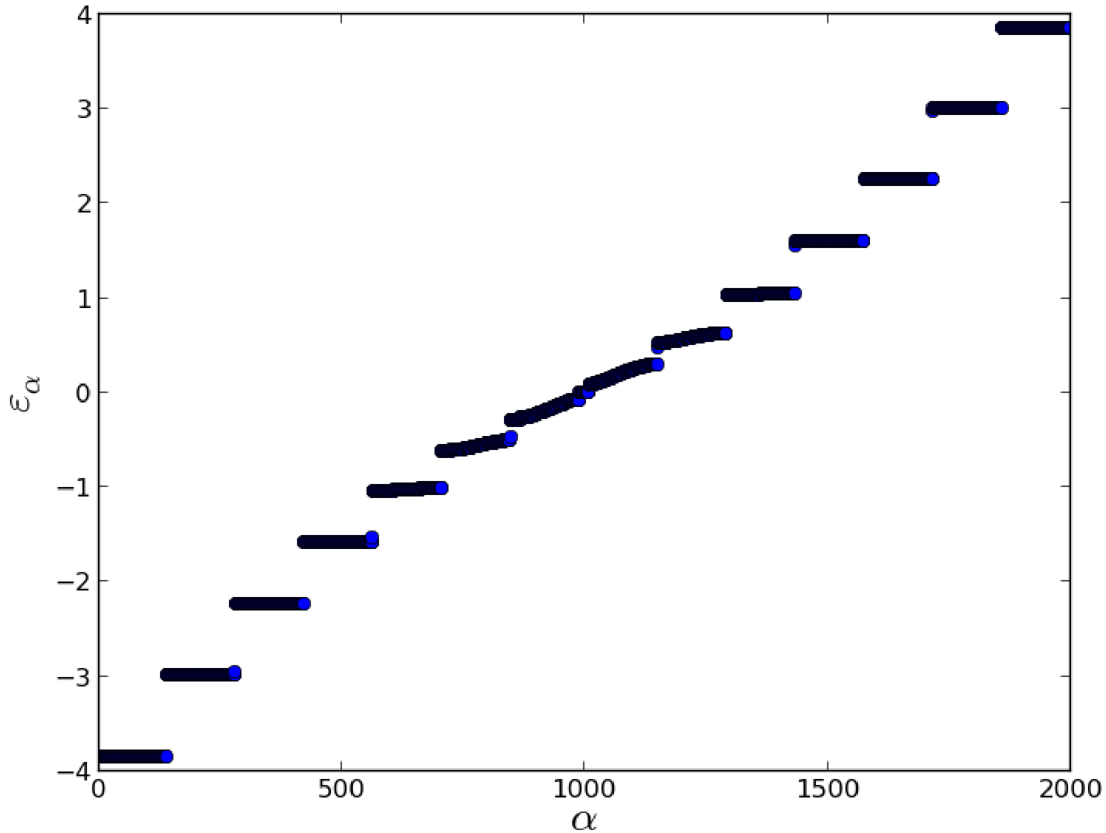


AAH model

Localization length for all eigenstates is $\zeta = a\ln^{-1}[V/2J]$

(Aubry/Andre, 1980); $\zeta \simeq Va/(V-2J) \gg a$ for V close to $2J$

Single-particle energy states for $\kappa \ll 1$ ($\kappa = \sqrt{2}/20$ and $V = 2.05J$)



Interacting bosons $H_{int} = U\sum_j n_j(n_j-1)/2$

MBLDT in the AAH model

The number of clusters $N_1 \simeq 1/\kappa$ for $\kappa \ll 1$

The width of a cluster Γ grows exp[onentially with energy

For $N_1 < \zeta$

$\zeta/N_1 \Rightarrow$ number of states of a given cluster participating in MBLDT

$T \ll 8J \rightarrow$ lowest energy cluster

MBLDT criterion $\int_0^{\Gamma_0} d\varepsilon\, \rho^2(\varepsilon)\zeta n_\varepsilon U_c = 1$

Occupation number of particle states

$$n_\varepsilon = [\exp(\varepsilon + U n_\varepsilon/\zeta - \mu)/T - 1]^{-1}$$

Chemical potential $\rightarrow \int \rho(\varepsilon) n_\varepsilon d\varepsilon = \nu$

Critical coupling at $T = 0$

$$T = 0 \Rightarrow \quad \varepsilon + U n_\varepsilon / \zeta(\varepsilon) = \mu$$

$$n_\varepsilon = \zeta(\mu_0 - \varepsilon)/U; \quad \varepsilon < \mu_0$$

$$n_\varepsilon = 0; \quad \varepsilon > \mu_0$$

$$U_c \nu \simeq \frac{2\Gamma_0}{\kappa \zeta}$$

Valid also at $T \ll \omega$

Critical coupling at finite temperatures

$$n_\varepsilon = \frac{\zeta}{2U}\left\{(\mu-\varepsilon)+\sqrt{(\mu-\varepsilon)^2+4TU/\zeta}\right\} \text{ if } n_\varepsilon \gg 1$$

$$n_\varepsilon = \exp-(\varepsilon-\mu)/T \text{ if } n_\varepsilon \lesssim 1\ (\varepsilon > \mu)$$

$$\frac{U_c(T)}{U_c(0)} \simeq \left[1+\frac{T}{8\nu J}\ln\left(\frac{T}{\omega}\right)\right];\quad \omega \ll T \ll 8J$$

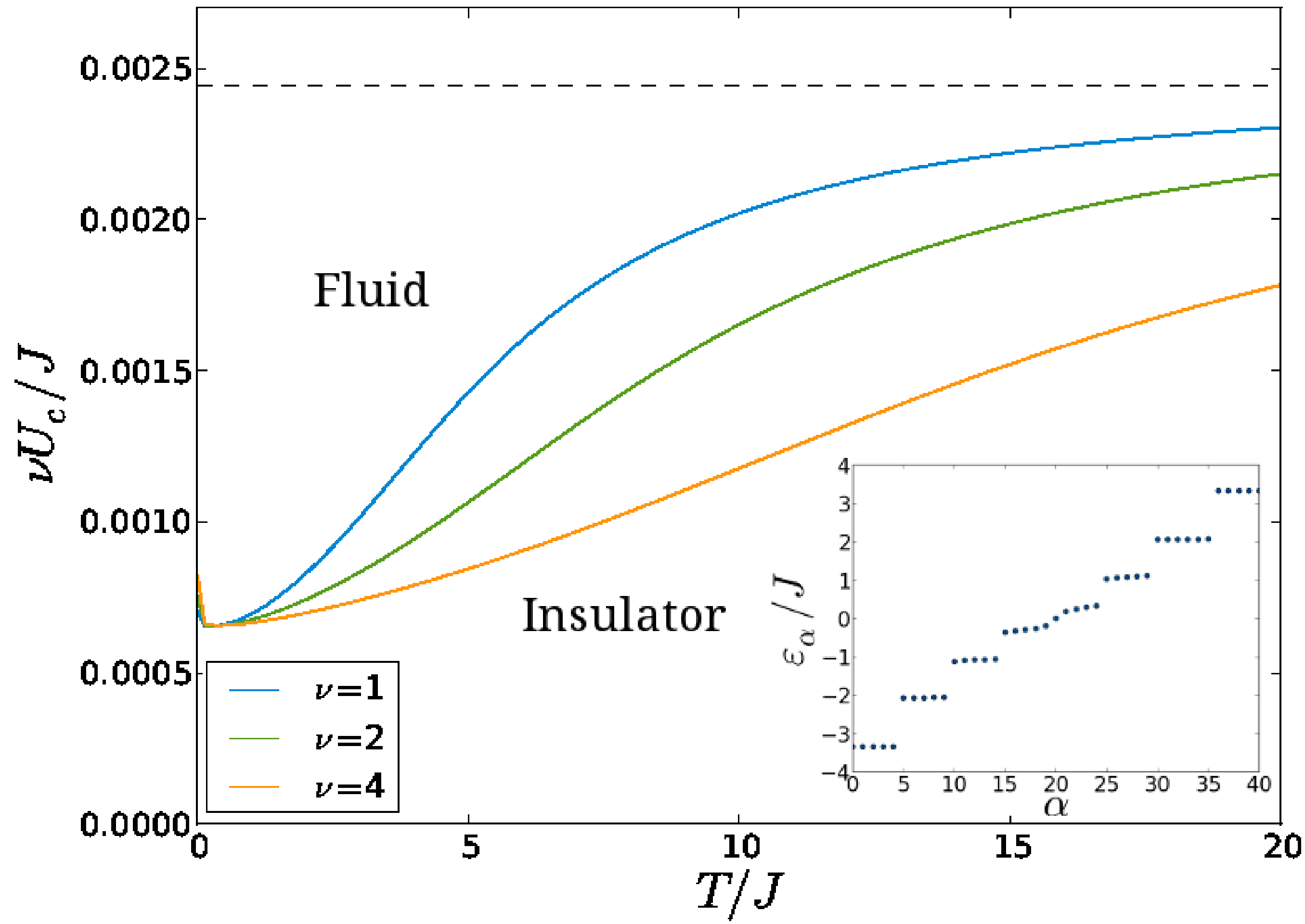
Ab initio not expected. Anomalous temperature dependence!

$$T\to\infty \quad \Rightarrow n_\varepsilon \simeq \nu;\ \mu \simeq -T/\nu$$

$$U_c\nu \simeq \frac{\Gamma_0}{\kappa^2\zeta};\qquad \frac{U_c(\infty)}{U_c(0)} = \frac{1}{\kappa}$$

Critical coupling

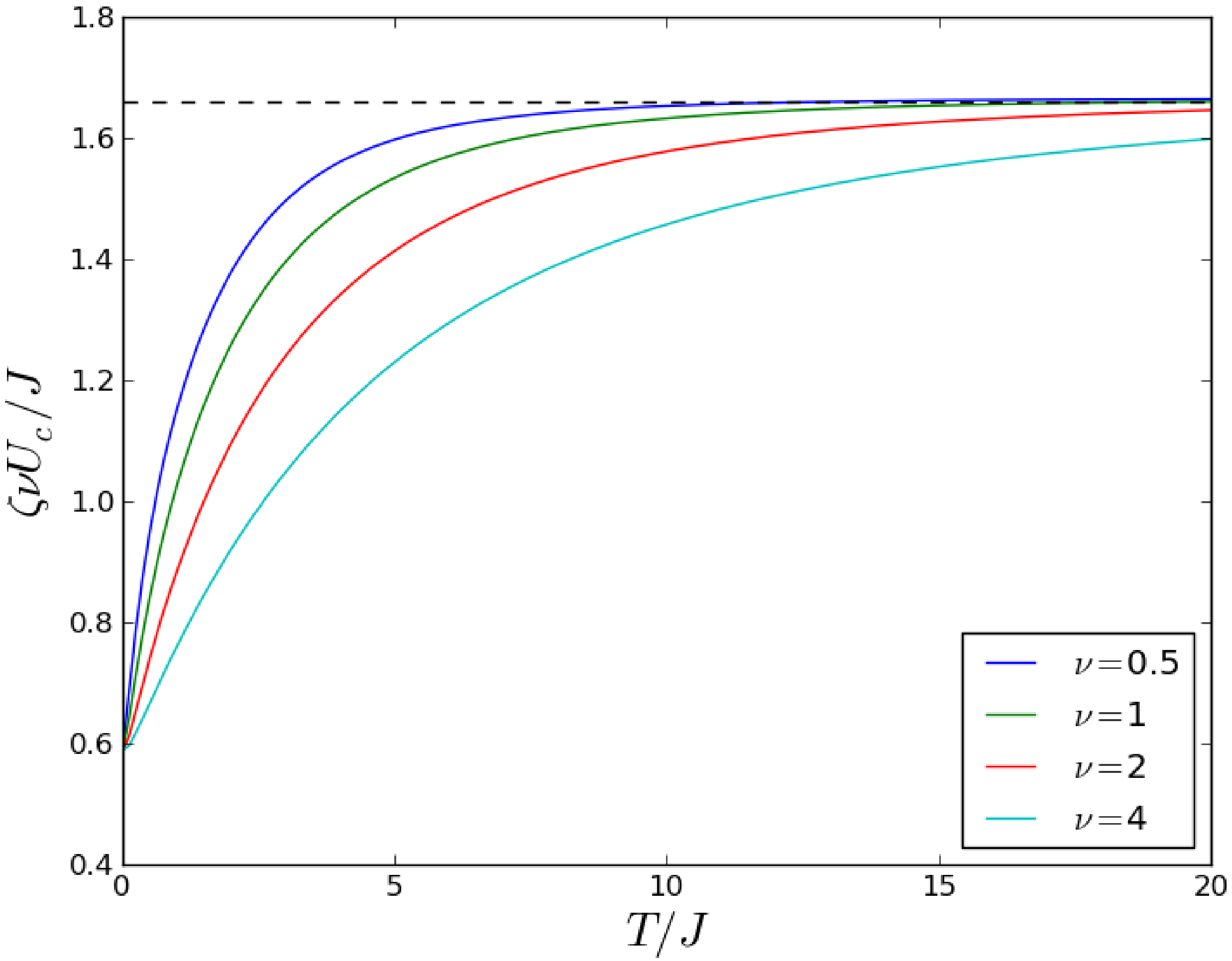
κ close to 1/8 and $V = 2.05J$



Increase in temperature favors the insulator state. ”Freezing with heating”

Critical coupling

Golden ratio $\kappa = (\sqrt{5} - 1)/2$ and $V = 2.1J$



Conclusions

1D bosons is a promising system to study the many-body localization-delocalization transition

Atoms in quasiperiodic potentials $\Rightarrow$ Increasing temperature may favor localization

Thank you for attention!