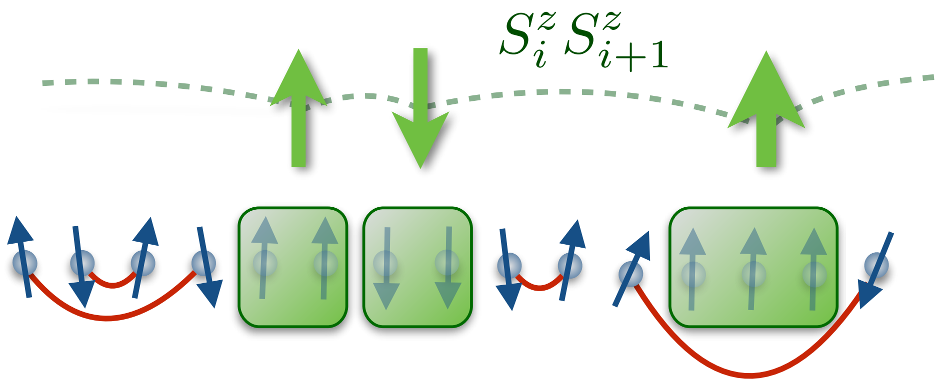
Particle-Hole Symmetry and Many-Body Localization

Romain Vasseur



MBL Workshop, KITP

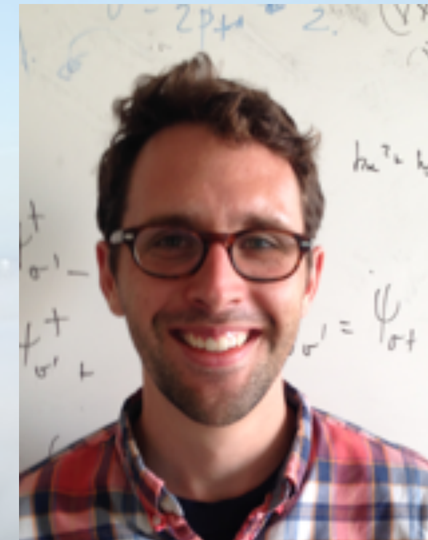
RV, A.J. Friedman, S.A. Parameswaran & A.C. Potter, arXiv:1510.04282

RV, A.C. Potter and S.A. Parameswaran, PRL 2015, arXiv:1410.6165

Collaborators

Aaron Friedman

Sid Parameswaran



Andrew Potter

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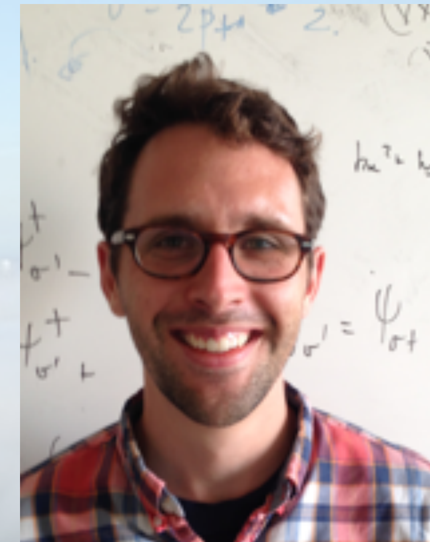

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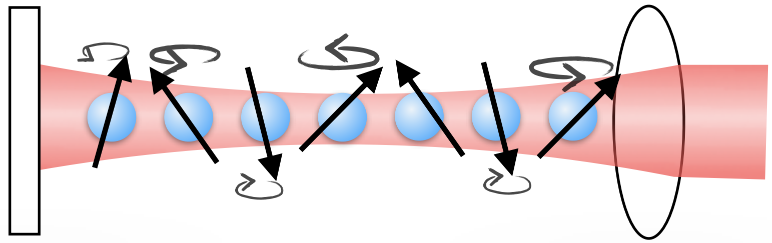
Thermalization in isolated quantum systems?

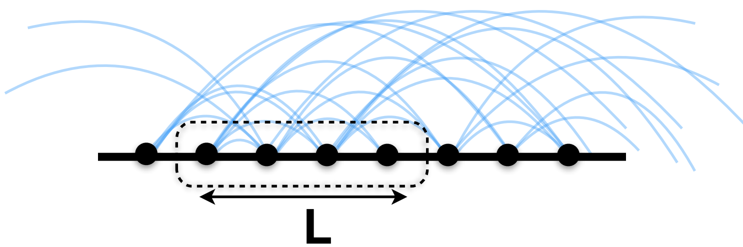


Yes

No

CLASSICAL EQUILIBRIUM WORLD

Thermalization (ETH)

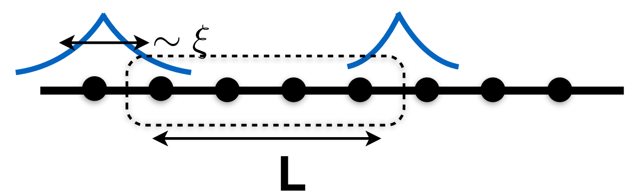


- No local memory of initial conditions
- Usual intuition OK: Stat mech
- Excited eigenstates are highly entangled
- Rapid spreading of entanglement or energy

Deutsch '91, Srednicki '94, …

QUANTUM WORLD

Many-body localization



- Local quantum information persists to infinite time: generic "integrability"
- Excited eigenstates have low entanglement ~ gapped groundstates
- No energy transport

Anderson '58, Fleishman & Anderson '80, Gornyi, Mirlin & Polyakov '05, Basko, Aleiner & Altshuler '05, Oganesyan & Huse '07, Pal & Huse '10, Bauer & Nayak '13, …

Thermalization in isolated quantum systems?

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Some questions…

Highly excited states with area law entanglement: 'look like' g.s.!

Bauer & Nayak '13, Serbyn, Papic & Abanin '13

⇨ Qualitatively new phenomena: e.g. ordering below lower critical dimension, quantum coherence at high energy density

Huse et al '13, Bauer & Nayak '13, Bahri et al '13, Chandran et al '14, Pekker et al '14, …

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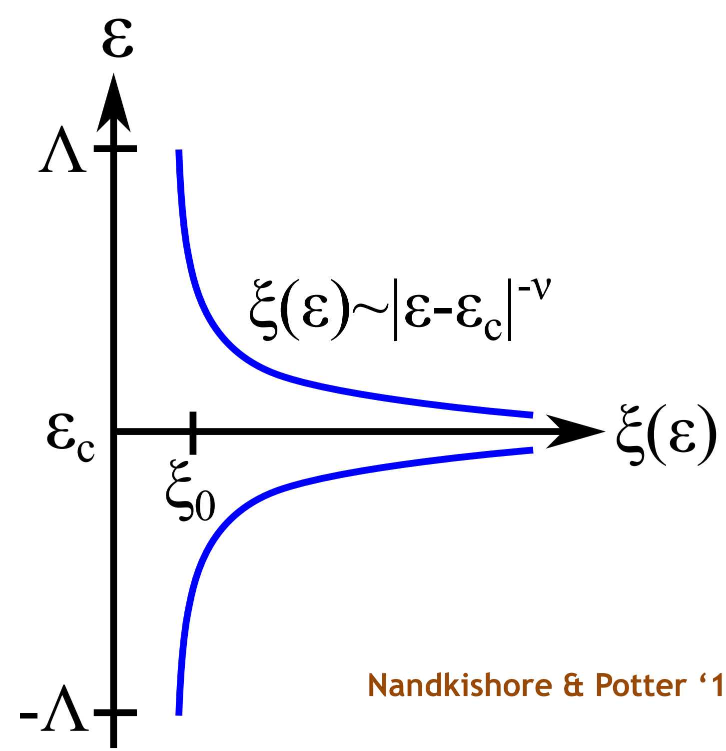
- **Can MBL protect any type of symmetry-breaking, topological or SPT order?**
- **Role of symmetries?**
- **Usually, start from completely localized single-particle orbitals. What about "marginal localization"?**



Nandkishore & Potter '14

Prototypical MBL system

Fermions

$$H = \sum_i -t(c^\dagger_{i+1} c_i + \text{h.c.}) + U n_i n_{i+1} + \mu_i n_i$$

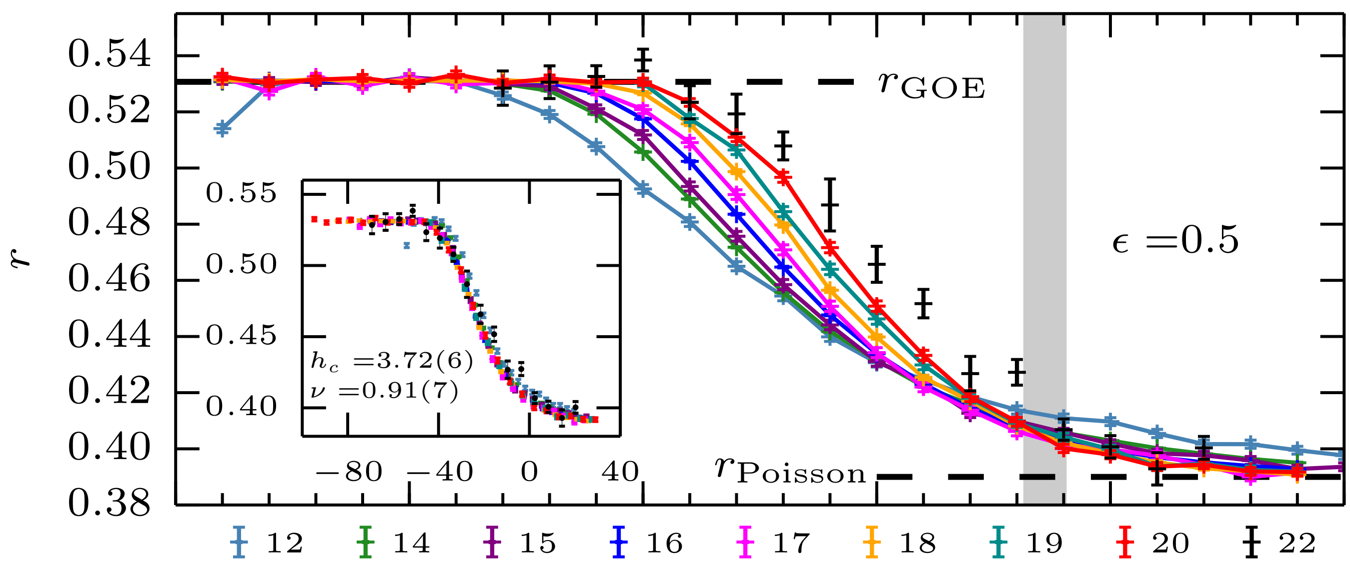
⇕ **Jordan-Wigner**

Spins

$$H = \sum_i J(S^x_i S^x_{i+1} + S^y_i S^y_{i+1} + \Delta S^z_i S^z_{i+1}) + h_i S^z_i$$

Oganesyan & Huse '07, Pal & Huse '10…

$$h_i \in [-W, W]$$



Luitz, Laflorencie, Alet '14

Model and symmetries

Fermions

$$H = \sum_i -t_i(c_{i+1}^\dagger c_i + \text{h.c.}) + U_i n_i n_{i+1}$$

⇕ Jordan-Wigner

Spins

$$H = \sum_i J_i(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta_i S_i^z S_{i+1}^z)$$

$$J_i \in (0, 1]$$

$$\Delta_i \in [-\Delta, \Delta]$$

$$P(\log J^{-1}) = \frac{1}{W} \mathrm{e}^{-\frac{\log J^{-1}}{W}}$$

Symmetries: $U(1) \rtimes \mathbb{Z}_2$

$U(1)$ → charge conservation

$\mathbb{Z}_2$ → Particle Hole (PHS)

$$\mathcal{C} = \prod_i \sigma_i^x$$

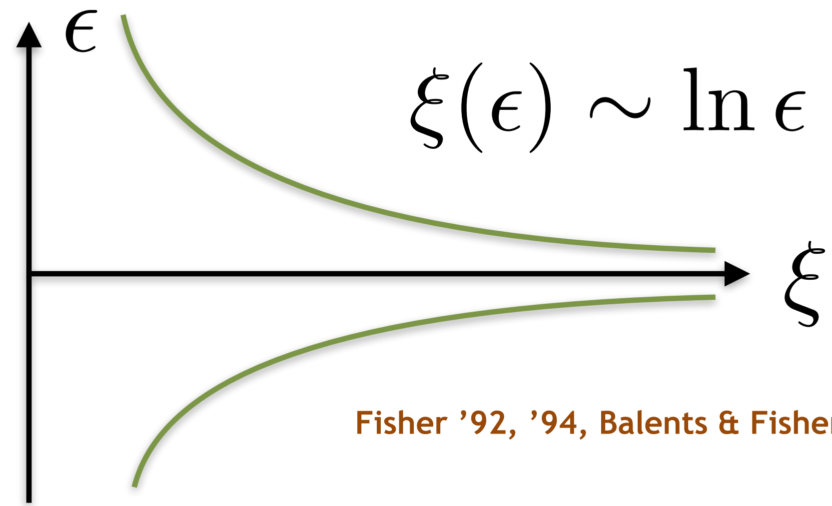
$$\mathcal{C} c_i \mathcal{C}^{-1} = (-1)^{i+1} c_i^\dagger$$

Why should you care?

- Groundstate not fully localized: "random-singlet" (quantum critical) Fisher '92

- Interactions irrelevant in the groundstate
what about highly excited-states?

- Non-interacting case $\Delta = 0$ has a diverging SP localization length + massive degeneracies of excited states



Fisher '92, '94, Balents & Fisher '97

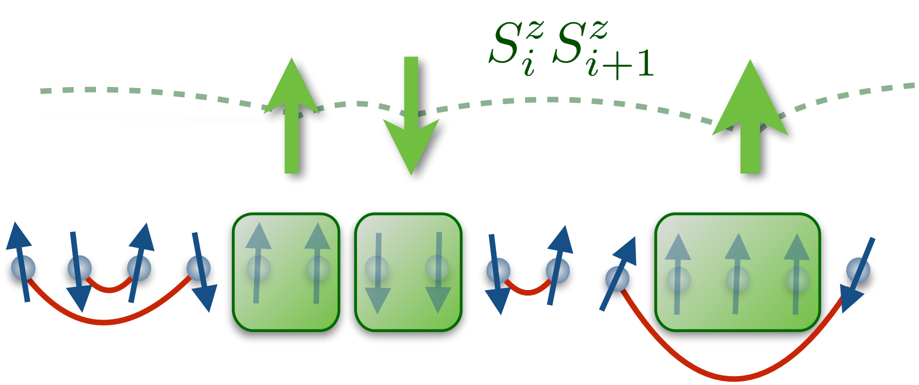
- Particle-hole symmetric MBL???

Outline

1. Strong disorder RG and spontaneous PHS breaking

2. Numerics

3. Instability of certain types of excited-state SPT order

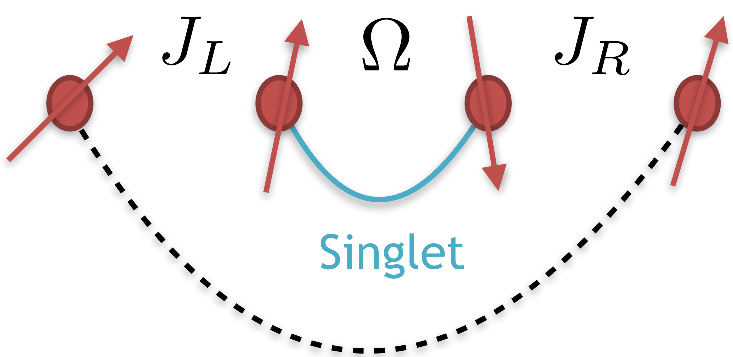


Strong Disorder Renormalization Group

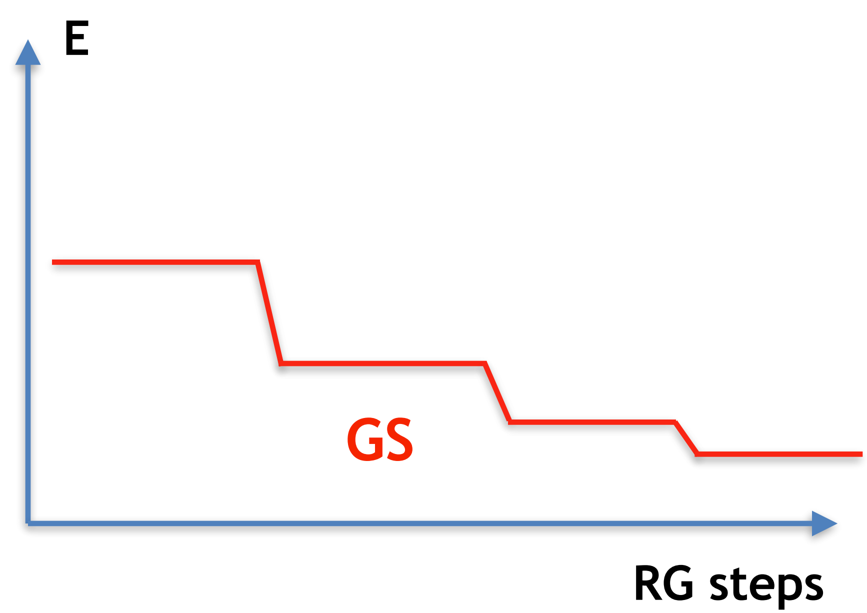
Random Singlet Physics (Groundstate)

Groundstate T=0 (AF chain): $$H = \sum_{i=1}^{L-1} J_i \left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta_i S_i^z S_{i+1}^z \right)$$

Ma-Dasgupta-Hu ‘79, Bhatt & Lee ‘79, Fisher ’92, ’94



$$J_{\text{eff}} \sim \frac{J_L J_R}{\Omega}$$



Strong Disorder Renormalization Group

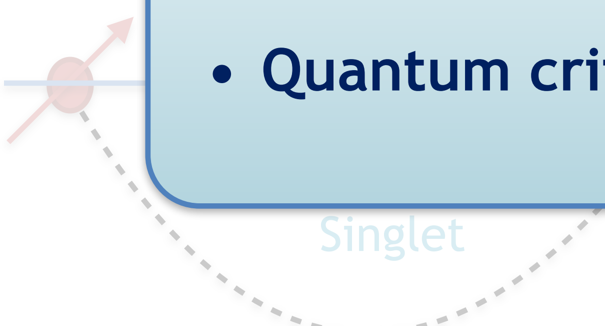
Random Singlet Physics (Groundstate)

Ground…

Ma-Dasgupt…

- Asymptotically **exact**
- Interactions **irrelevant** near $\Delta_i = 0$
- Quantum critical: S ~ log L, algebraic correlations…

T=0: Refael & Moore, '04



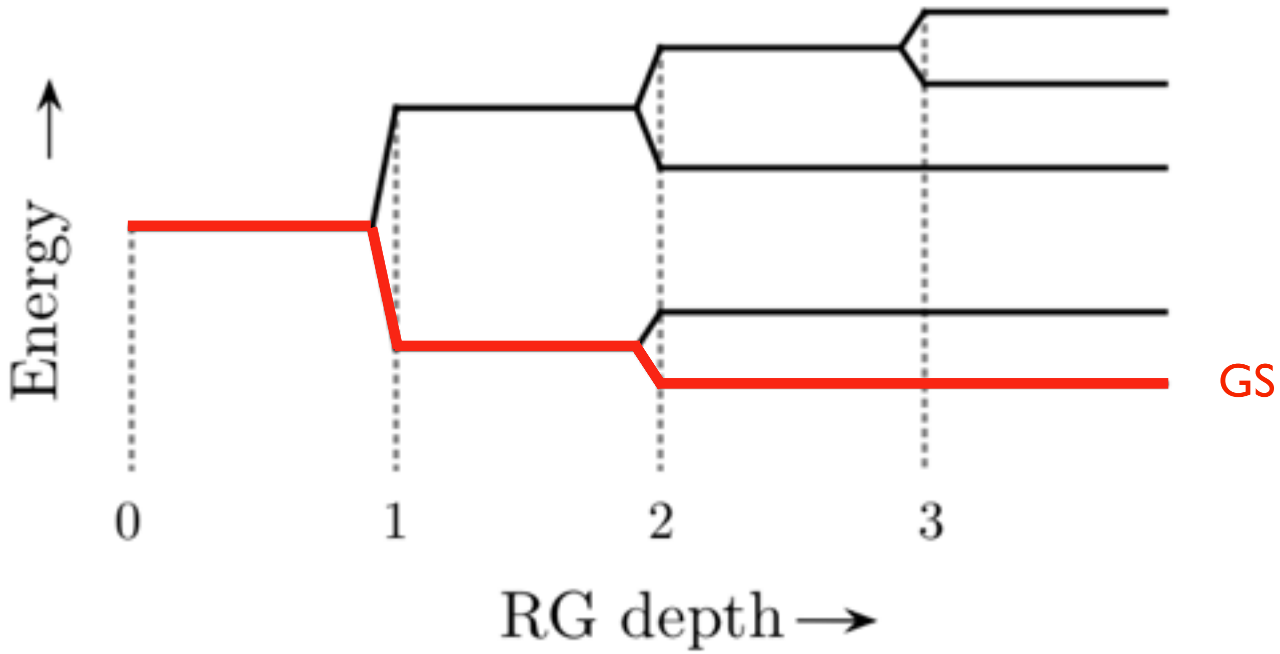
$$J_{\text{eff}} \sim \frac{J_L J_R}{\Omega}$$

RSRG

project strong bond onto groundstate manifold

T=0 : target g.s. by always picking lowest-energy outcome

"walking down the RG tree"



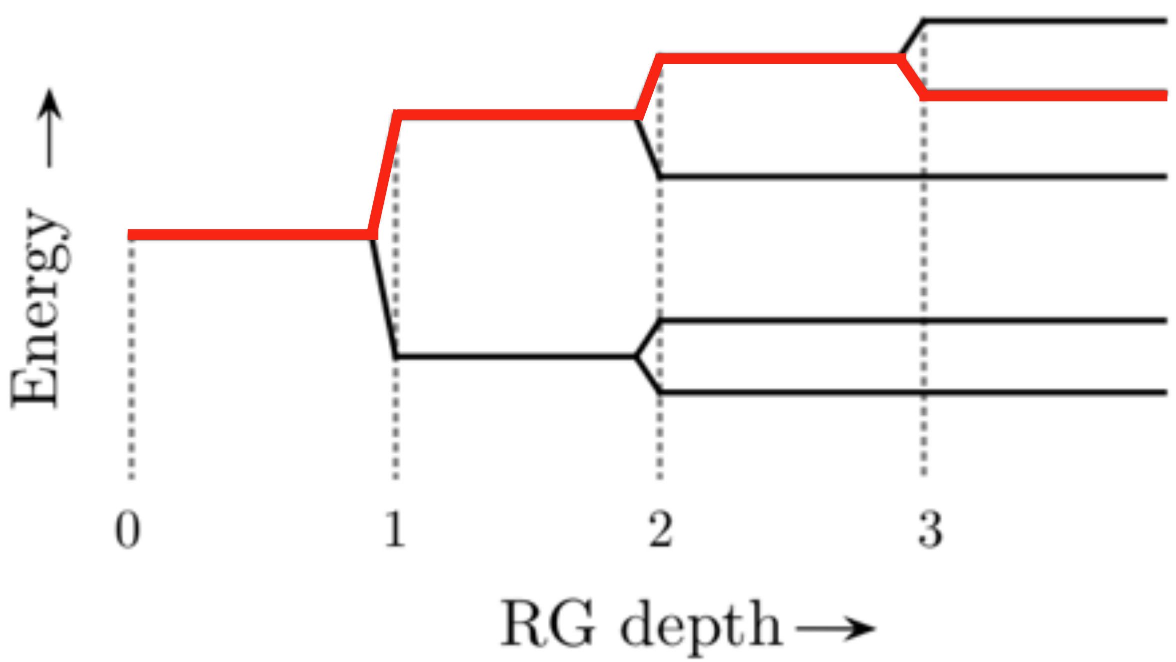
Ma-Dasgupta-Hu '79, Bhatt & Lee '79, Fisher '92, '94

RSRG-X

project strong bond onto groundstate manifold

excited

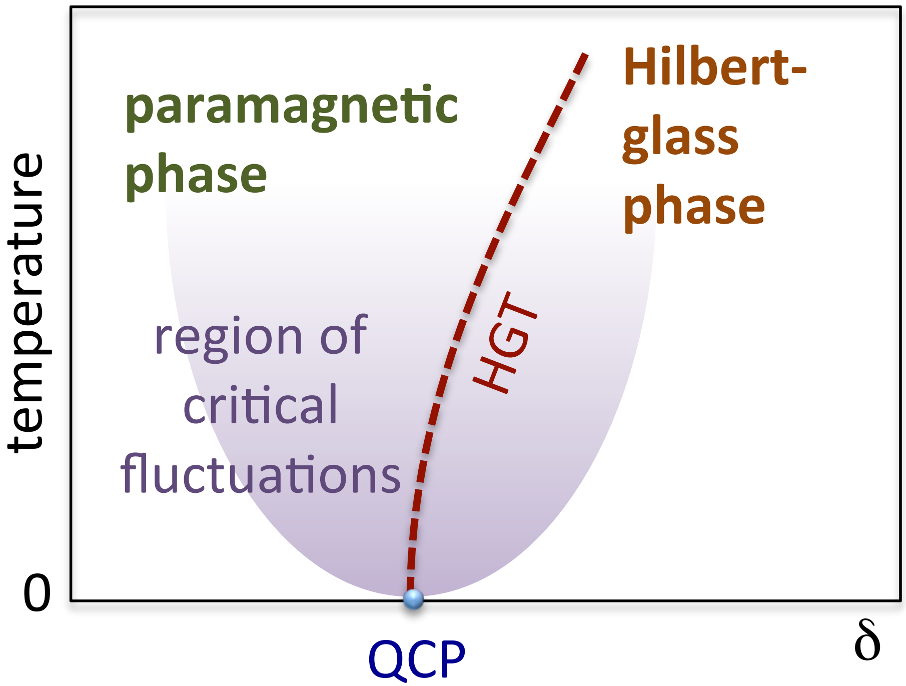
iteratively resolves smaller and smaller energy gaps

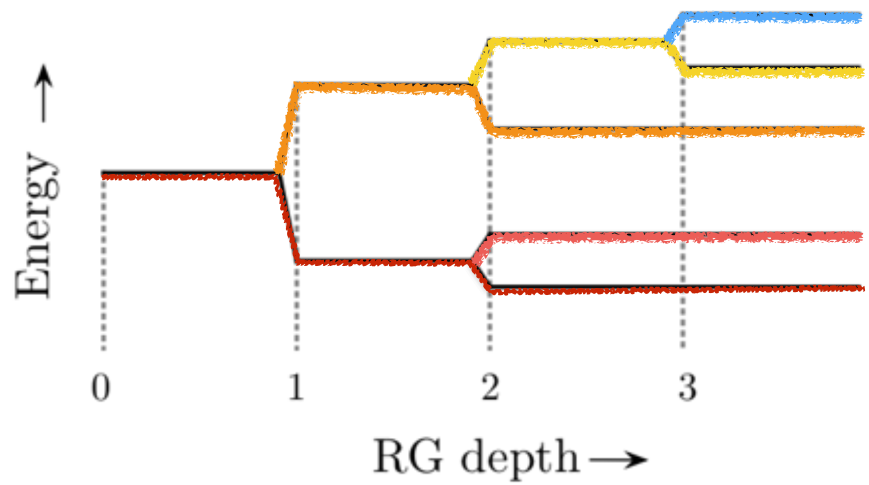


Pekker et al '13 (Ising, MC Sampling)

Related Dynamical RG: Vosk & Altman '13

RSRG-X





$$\mathcal{H}_{\mathrm{hJJ'}} = -\sum_i \left(J_i \sigma_i^x \sigma_{i+1}^x + h_i \sigma_i^z + J_i' \sigma_i^z \sigma_{i+1}^z\right)$$

Pekker et al '13 (Ising, MC Sampling)

Related Dynamical RG: Vosk & Altman '13

RV, Potter, Parameswaran PRL '15

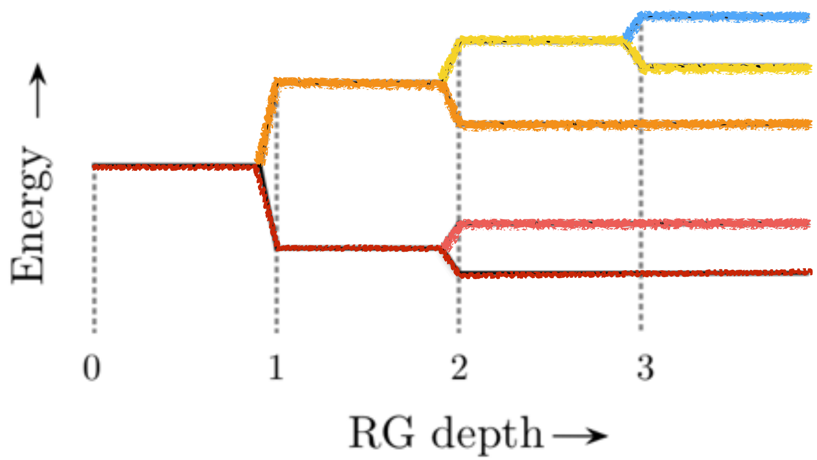
Analytic flow equations at infinite T

Monte Carlo sampling of RG tree

infinite family of models

Quantum critical glasses

RV, Potter, Parameswaran PRL '15



"Critical MBL"

Properties:

1. Log-entanglement scaling
   - Like 1D T=0 QCP, (but in highly excited states!)
   - Not thermal
2. Power-law correlations
3. Slow dynamics (z=∞)

Quantum Critical Excited States

New nonequilibrium universality classes, exact exponents

$$S(L) \sim \log L$$

T=0: Refael & Moore, '04

$$\overline{\langle \mathcal{O}(r)\mathcal{O}(0)\rangle} \sim \frac{1}{r^{2\Delta_{\mathcal{O}}}}$$

Quantum critical glasses

RV, Potter, Parameswaran PRL '15

"Critical MBL"

Properties:

Is that what happens for random-bond XXZ?

"random-singlet" physics in excited states?

Vosk & Altman '13: YES (BUT special initial state)

This talk: NO (generic initial state)

Excited States

$$S(L) \sim \log L$$

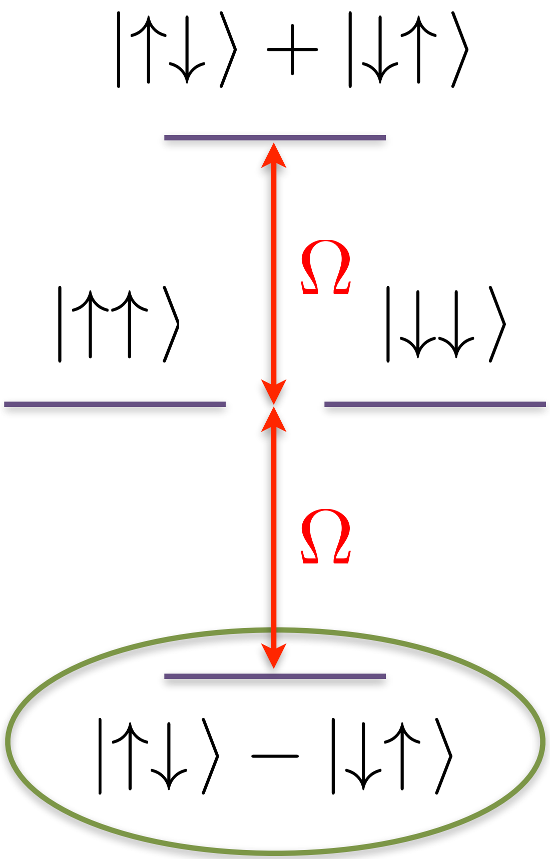
T=0: Refael & Moore, '04

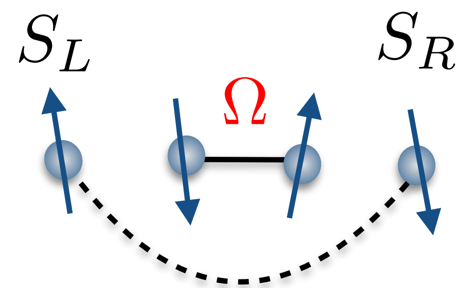
$$\overline{\langle \mathcal{O}(r)\mathcal{O}(0)\rangle} \sim \frac{1}{r^{2\Delta_{\mathcal{O}}}}$$

New nonequilibrium universality classes, exact exponents

RSRG-X: random XX chain

Huang & Moore, '04

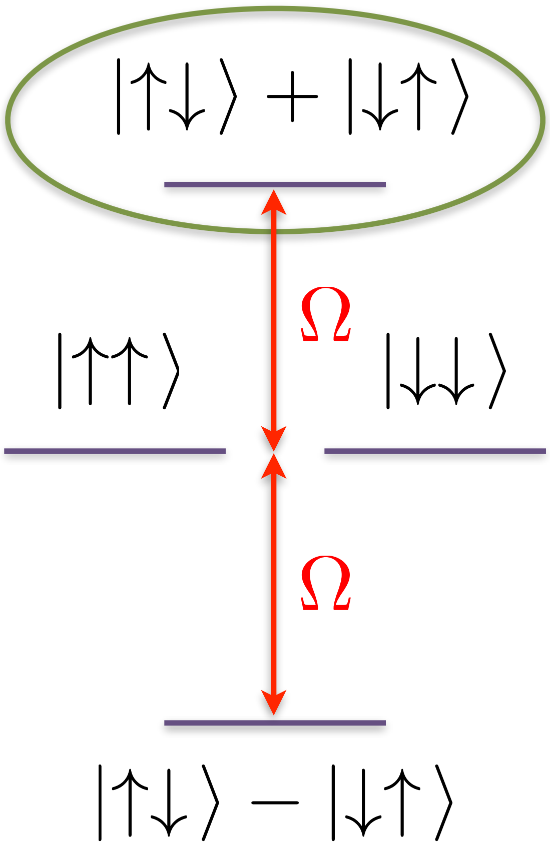


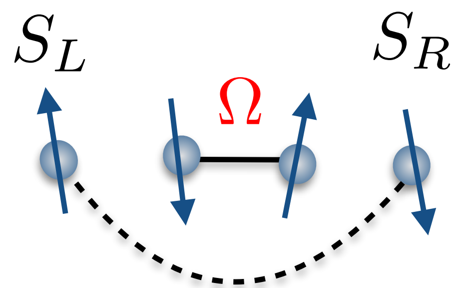


$$J_{\text{eff}} = +\frac{J_L J_R}{\Omega}$$

RSRG-X: random XX chain

Huang & Moore, '04





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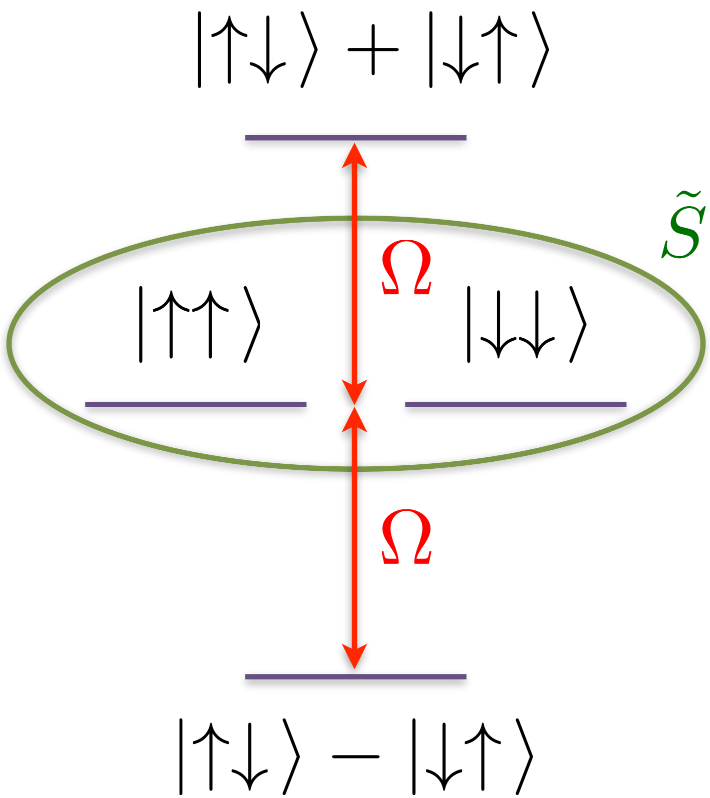
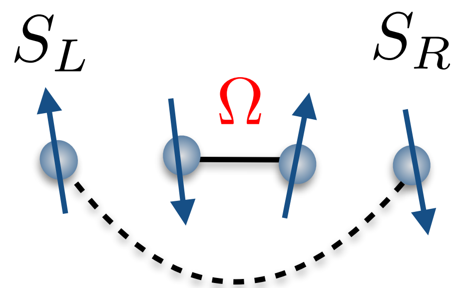


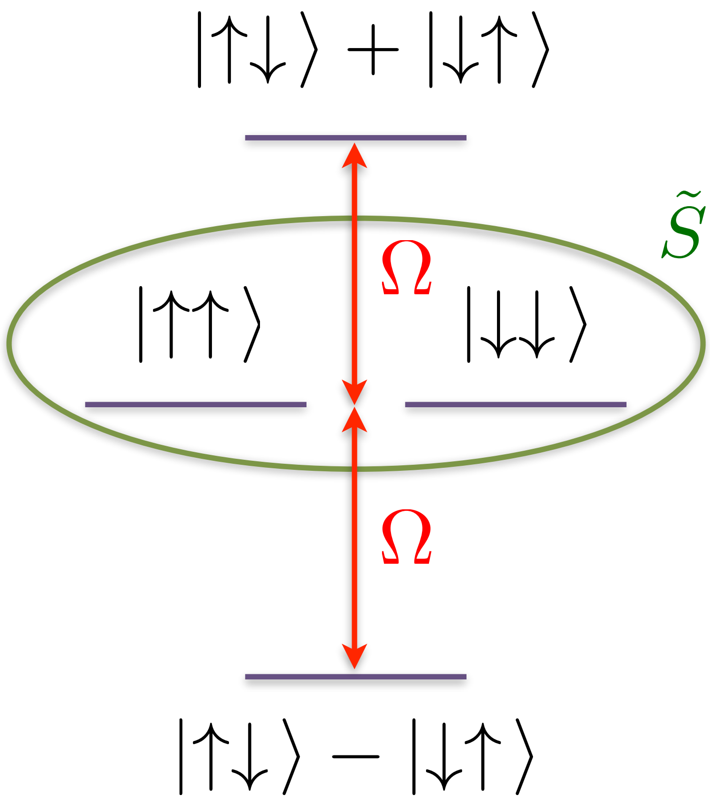


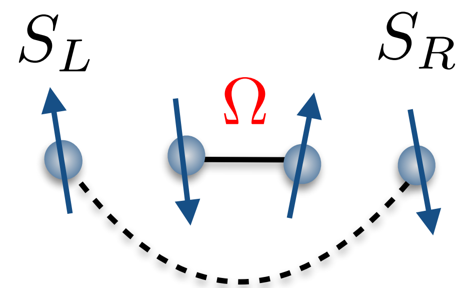
$$J_{\text{eff}} = -\frac{J_L J_R}{\Omega}$$

Effective spin $\tilde{S}$ decouples!

RSRG-X: random XX chain

Huang & Moore, '04





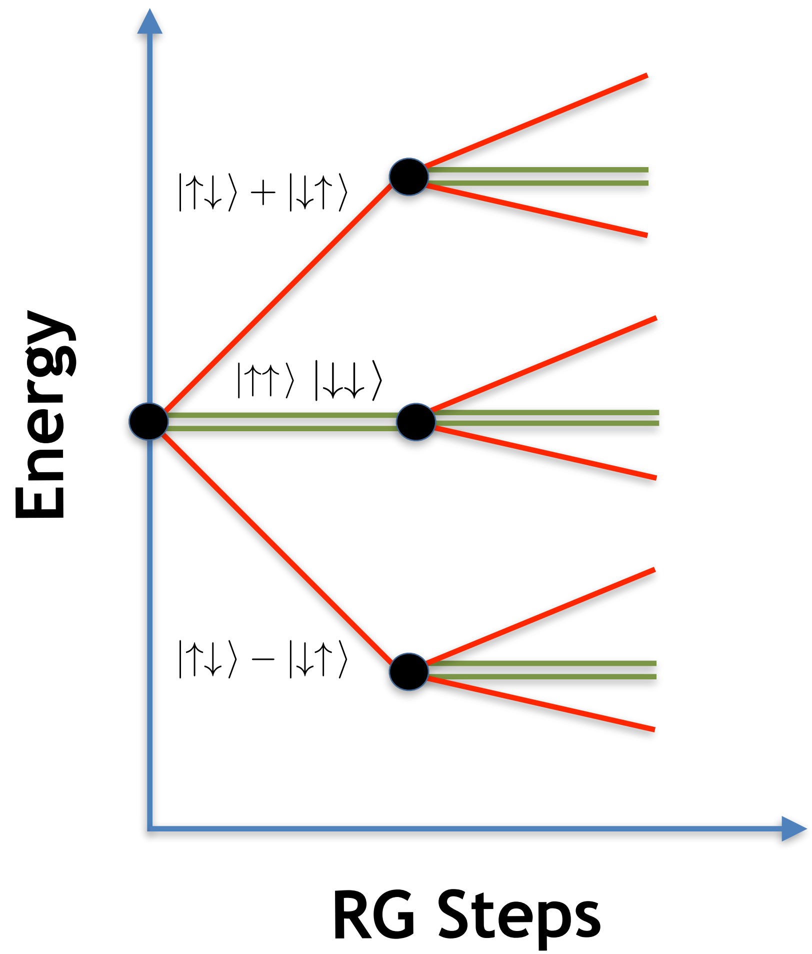
$$J_{\text{eff}} = -\frac{J_L J_R}{\Omega}$$

Effective spin $\tilde{S}$ decouples!

$$H_{\text{eff}}^{XX} = \pm \frac{J_L J_R}{2\Omega} \left(S_L^+ S_R^- + \text{h.c.} \right)$$

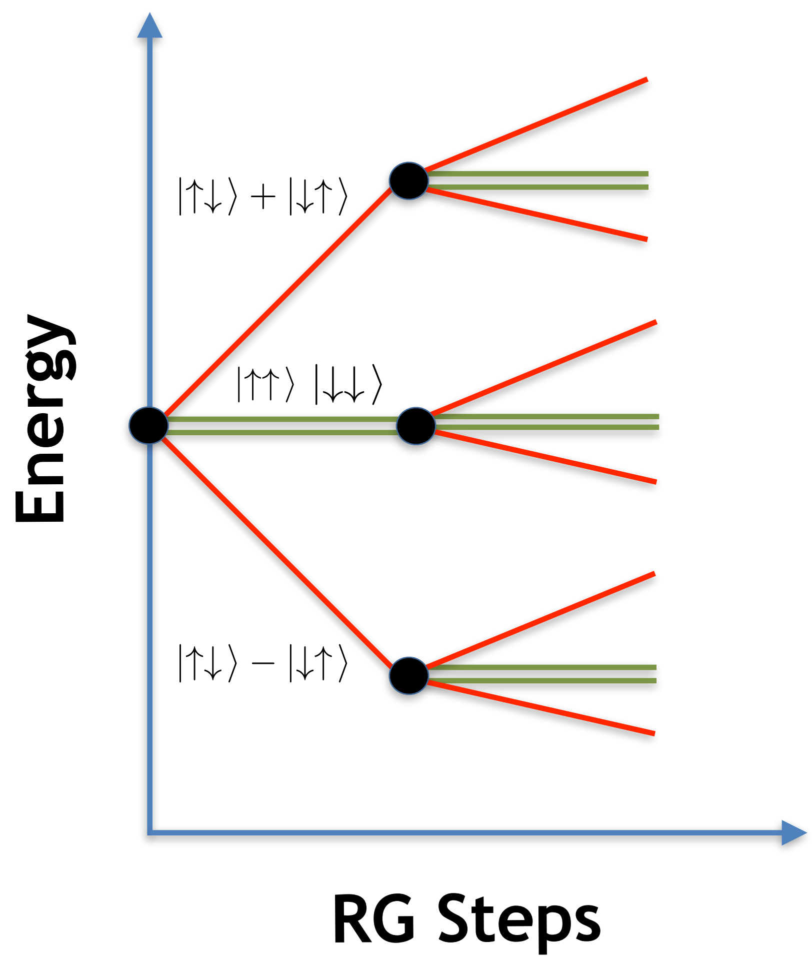
RG flow equations same as T=0 case
"Quantum critical glass"

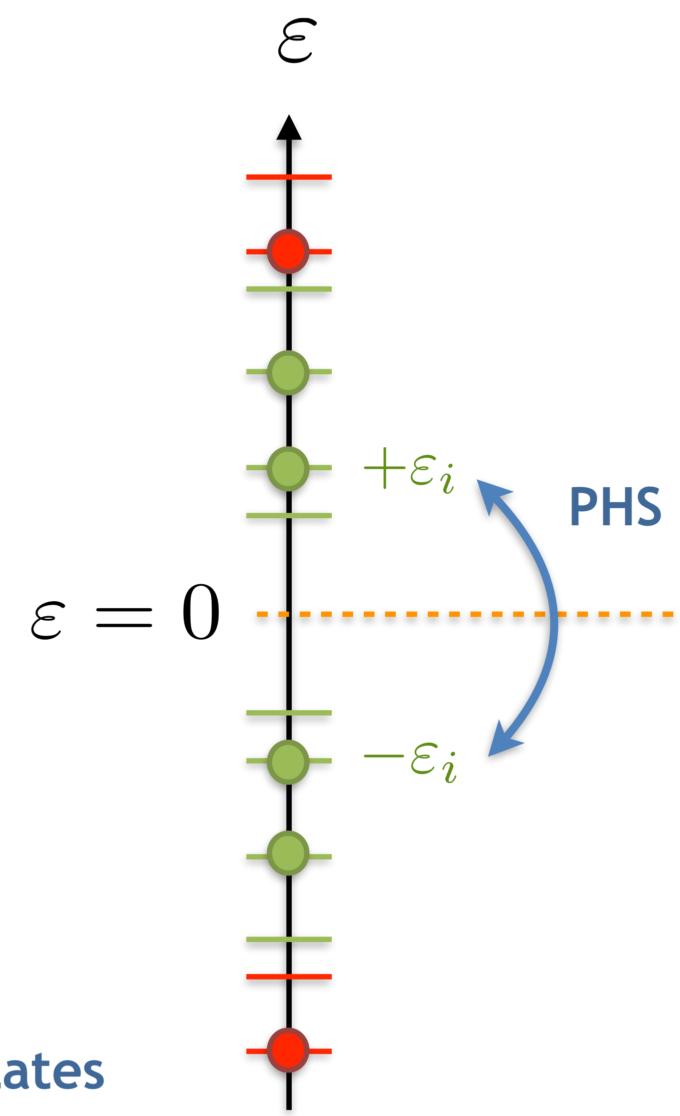
Massive degeneracies



Exponential degeneracy of highly excited states

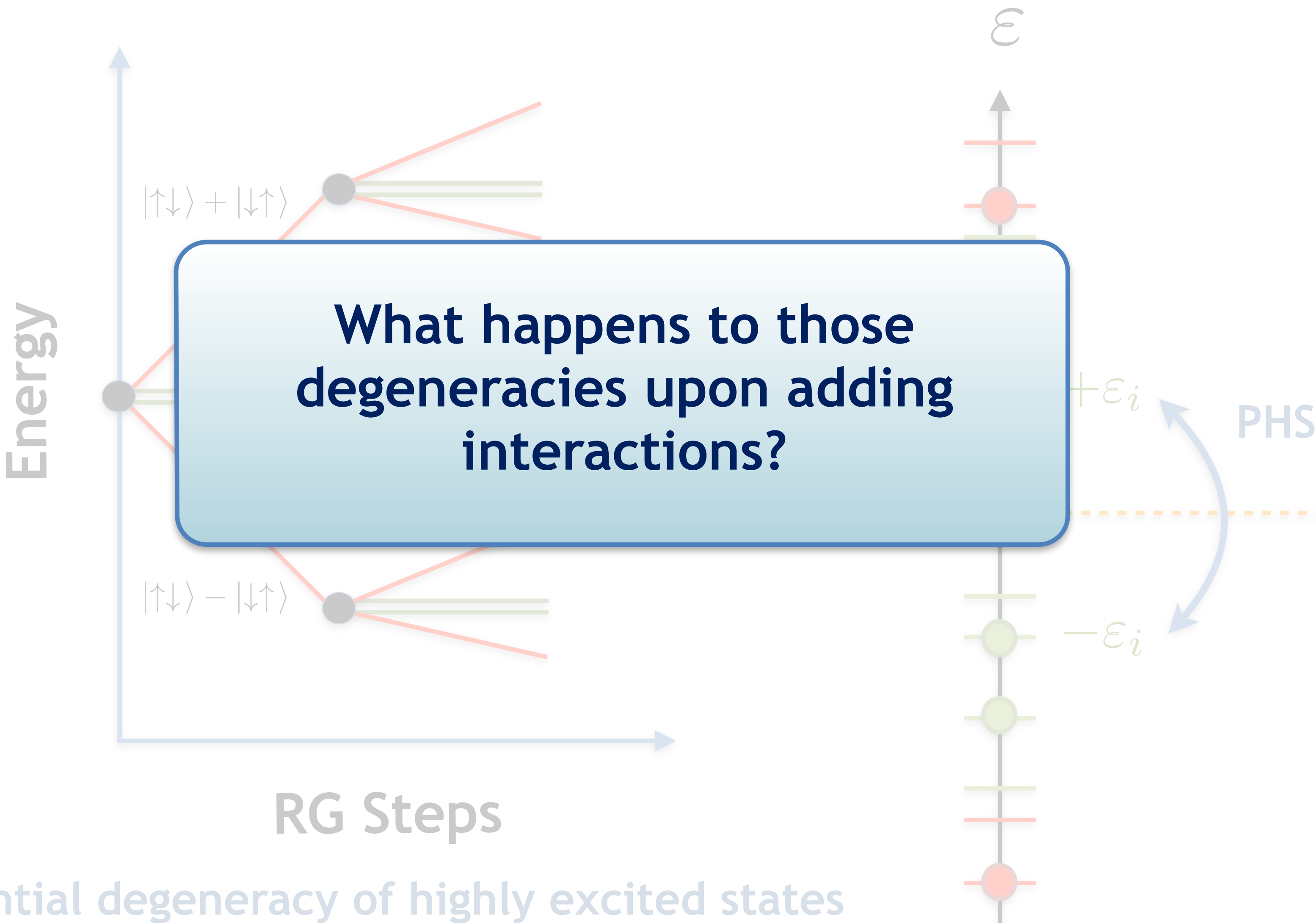
Massive degeneracies





Exponential degeneracy of highly excited states

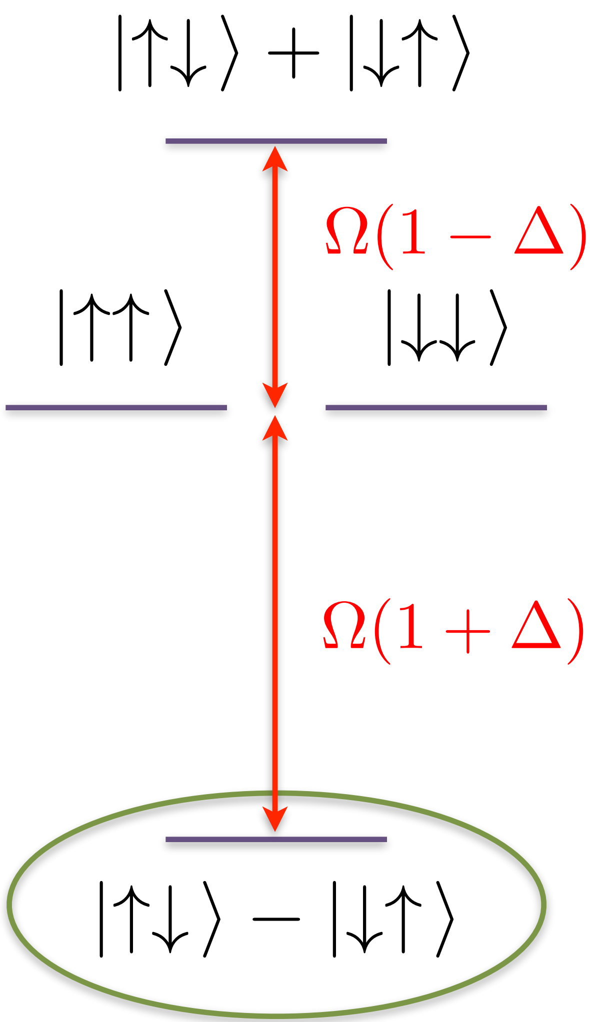
Massive degeneracies

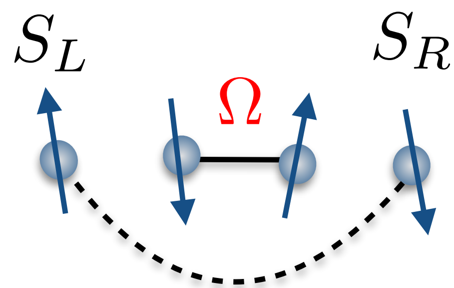


What happens to those degeneracies upon adding interactions?

Exponential degeneracy of highly excited states

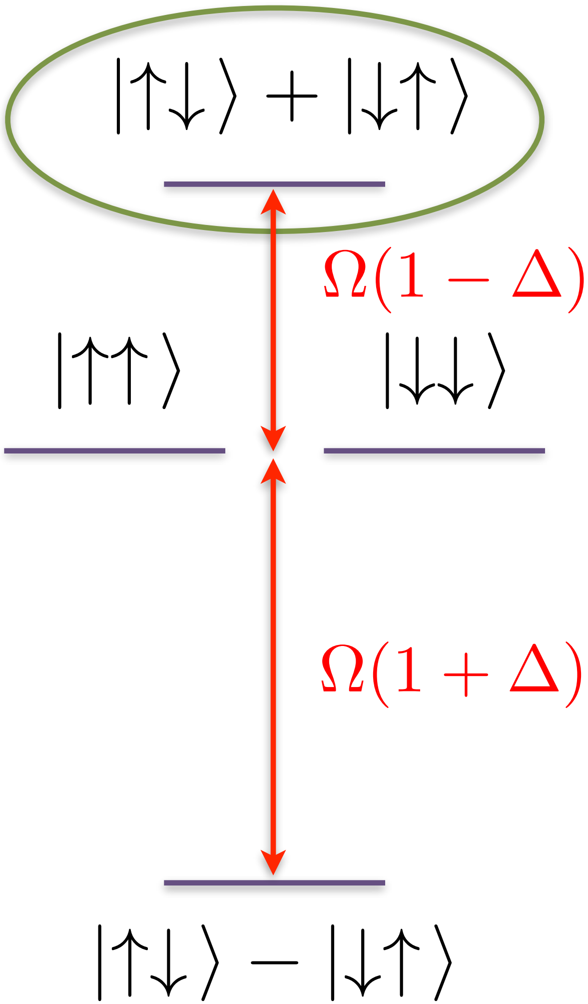
RSRG-X: random XXZ chain

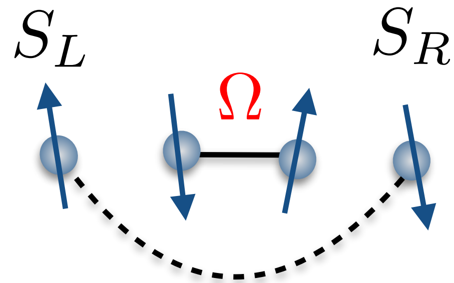




$$J_{\text{eff}} = \frac{J_L J_R}{(1+\Delta)\Omega}$$

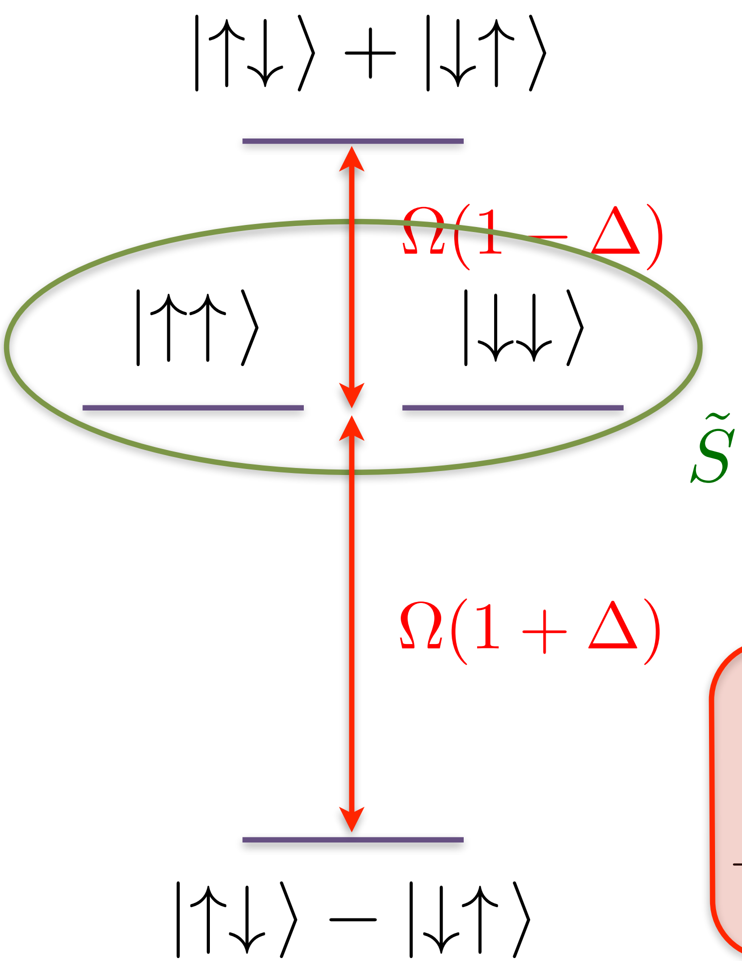
RSRG-X: random XXZ chain

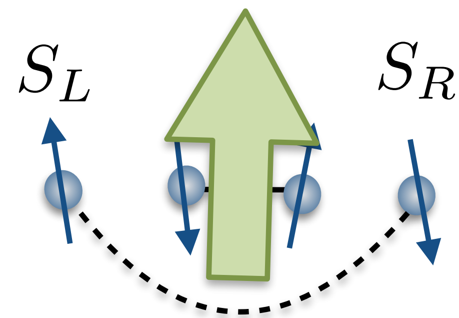




$$J_{\text{eff}} = \frac{J_L J_R}{(1-\Delta)\Omega}$$

RSRG-X: random XXZ chain

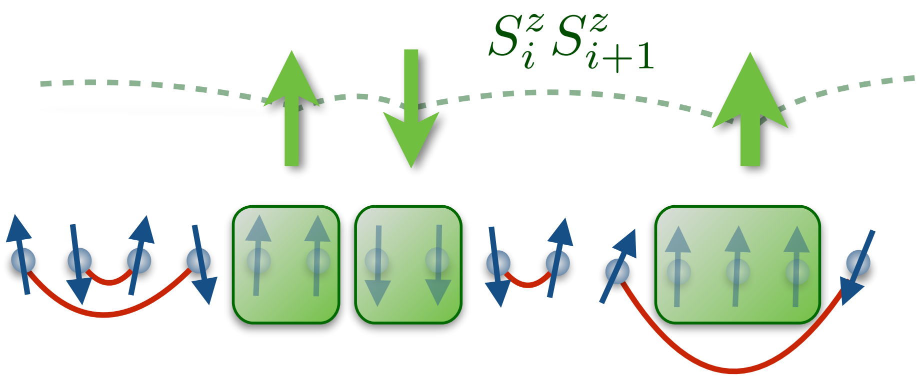




No direct flip-flop term due to charge/spin conservation!

$$H_{\text{eff}} = J_L \Delta_L S_L^z \tilde{S}^z_{\text{eff}} + J_R \Delta_R S_R^z \tilde{S}^z_{\text{eff}} + \frac{J_L J_R}{\Omega(\Delta^2 - 1)} \left[\frac{S_L^+ S_R^-}{2} + \Delta S_L^+ S_R^+ \tilde{S}^-_{\text{eff}} + \text{h.c.} \right] + \ldots$$

Spin Glass MBL phase



$$H_{\text{eff}} = J_L \Delta_L S_L^z \tilde{S}_{\text{eff}}^z + J_R \Delta_R S_R^z \tilde{S}_{\text{eff}}^z$$

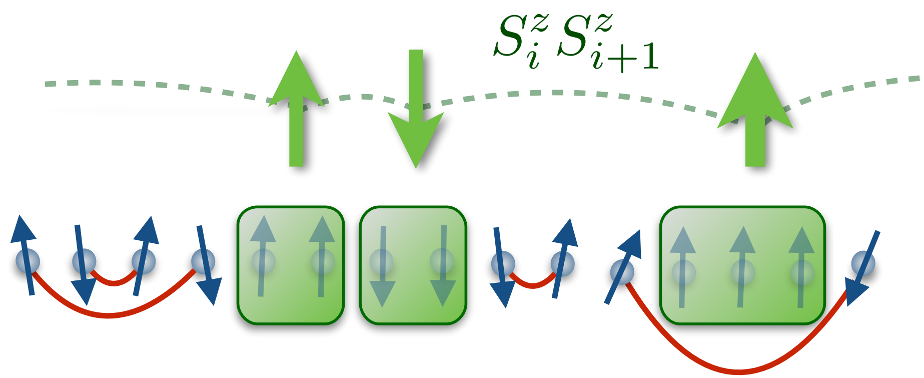
- Form larger “superspins”, increasingly harder to flip
- Interact via strong Ising interactions, and very weak flip-flop terms
- Eigenstates = superspins with random pattern of magnetization

Spin Glass MBL phase

Many-body localized phase with spontaneously broken particle hole symmetry (Spin Glass)

- Form larger "superspins", increasingly harder to flip
- Interact via strong Ising interactions, and very weak flip-flop terms
- Eigenstates = superspins with random pattern of magnetization

Remarks



- Genuine **MBL phase**, quantum criticality destroyed by interactions

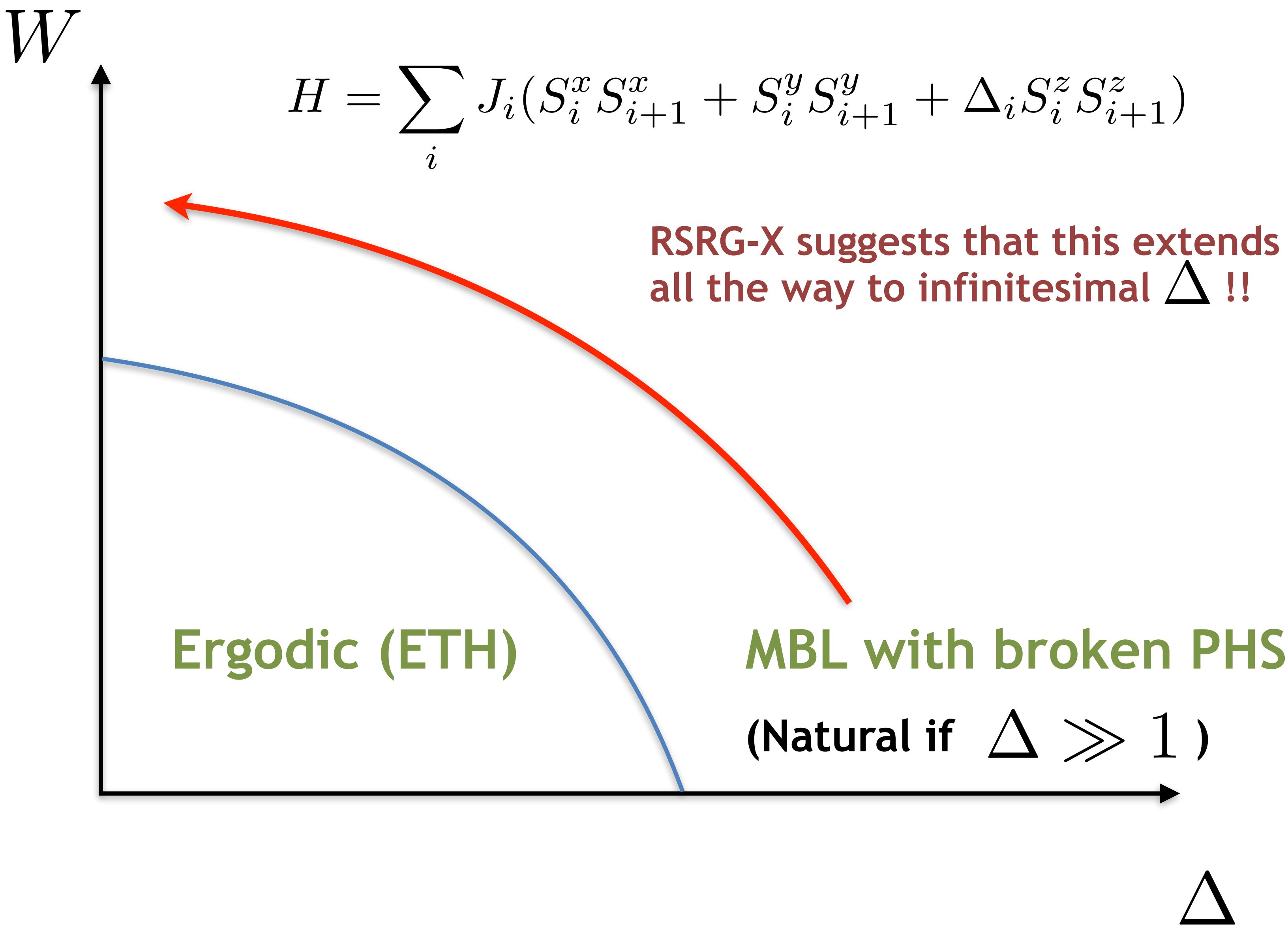
- No particle-hole symmetric ("paramagnetic") MBL phase:

PHS ~~$h_i \tilde{S}_i^z$~~ ~~$h_i \tilde{S}_i^{x,y}$~~ U(1)

- Middle of many-body spectrum **more localized** than edges!

Expected $T = \infty$ phase diagram

$$H = \sum_i J_i (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta_i S_i^z S_{i+1}^z)$$

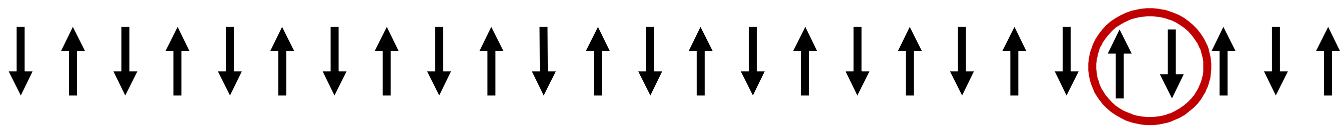


vs. Dynamical RG

Vosk & Altman ’13

Initial Neel State

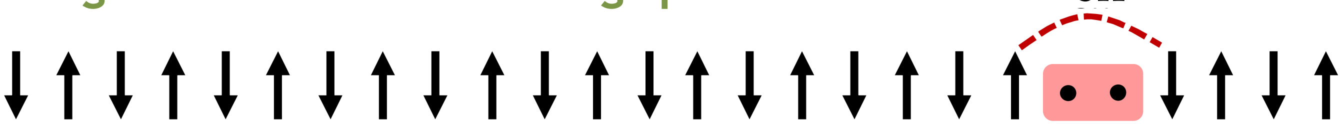
$H(\Omega)$



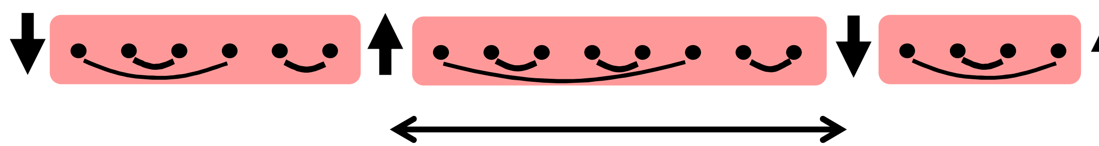
Integrate out fast oscillating spins

J_{eff}

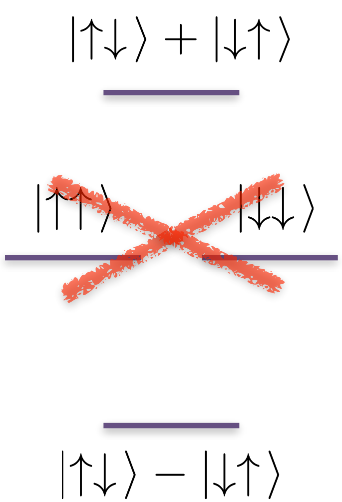
$H(\Omega - \delta\Omega)$



Equivalent to RSRG-X
but superspins projected
out



$L(t) \sim [\,\log(t)\,]^2$



Non-generic random-singlet like physics

Pictures from Ehud Altman

Numerics

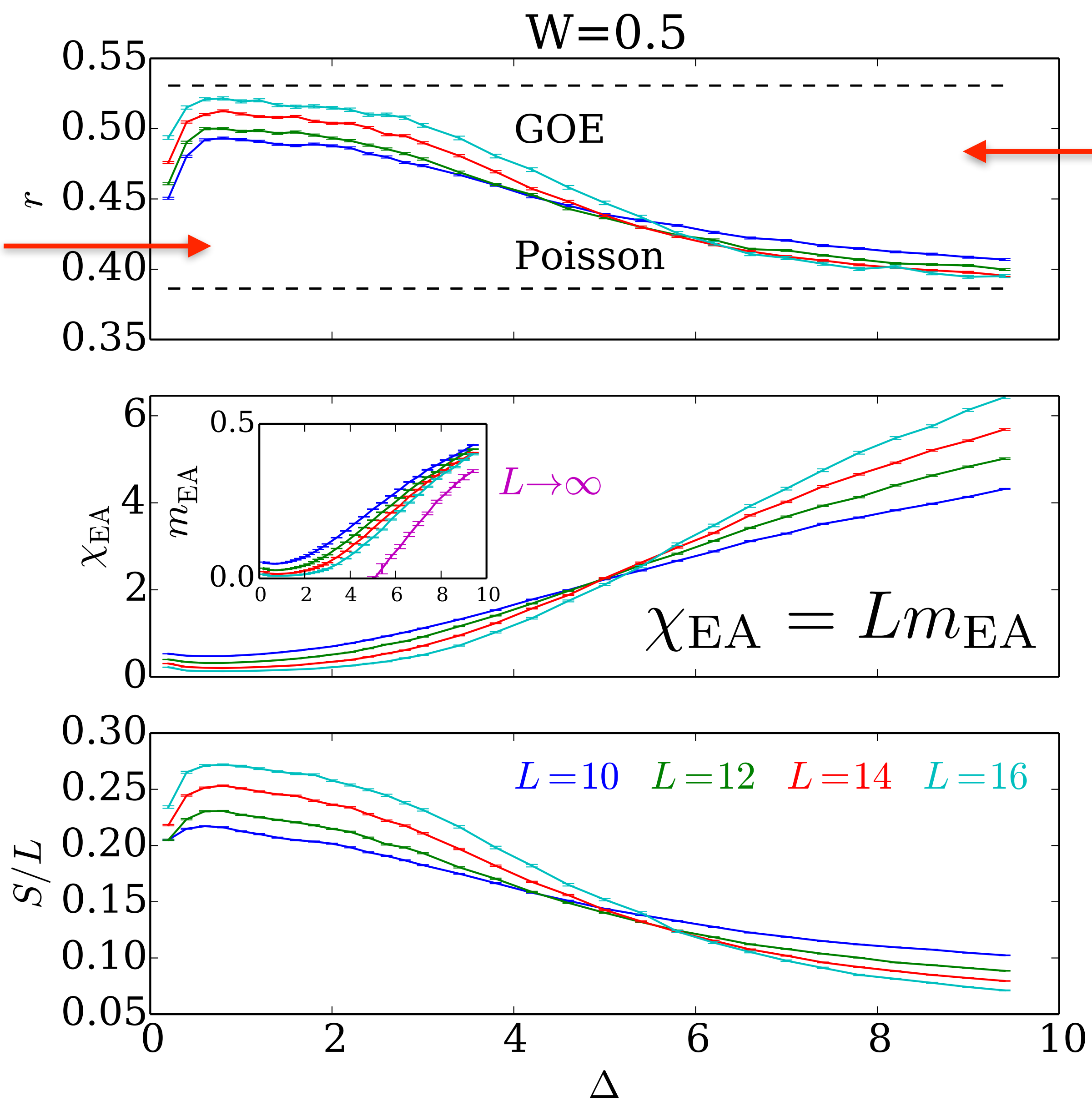
Could higher-order terms flip the superspins? Quantum criticality?

	Ergodic	MBL (SG)	Quantum critical glass
Level Statistics	GOE	Poisson	Poisson
Entanglement entropy	$S \sim L$	$S \sim 1$	$S \sim \ln L$
Spin Glass order	$m_{\text{EA}} = 0$	$m_{\text{EA}} \neq 0$	$m_{\text{EA}} = 0$ (but algebraic correlations!)

$$m_{\text{EA}} = \frac{1}{L^2} \sum_n \sum_{i \neq j} \langle n | \sigma_i^z \sigma_j^z | n \rangle^2$$

Kjall, Bardarson & Pollmann '14

Weak disorder

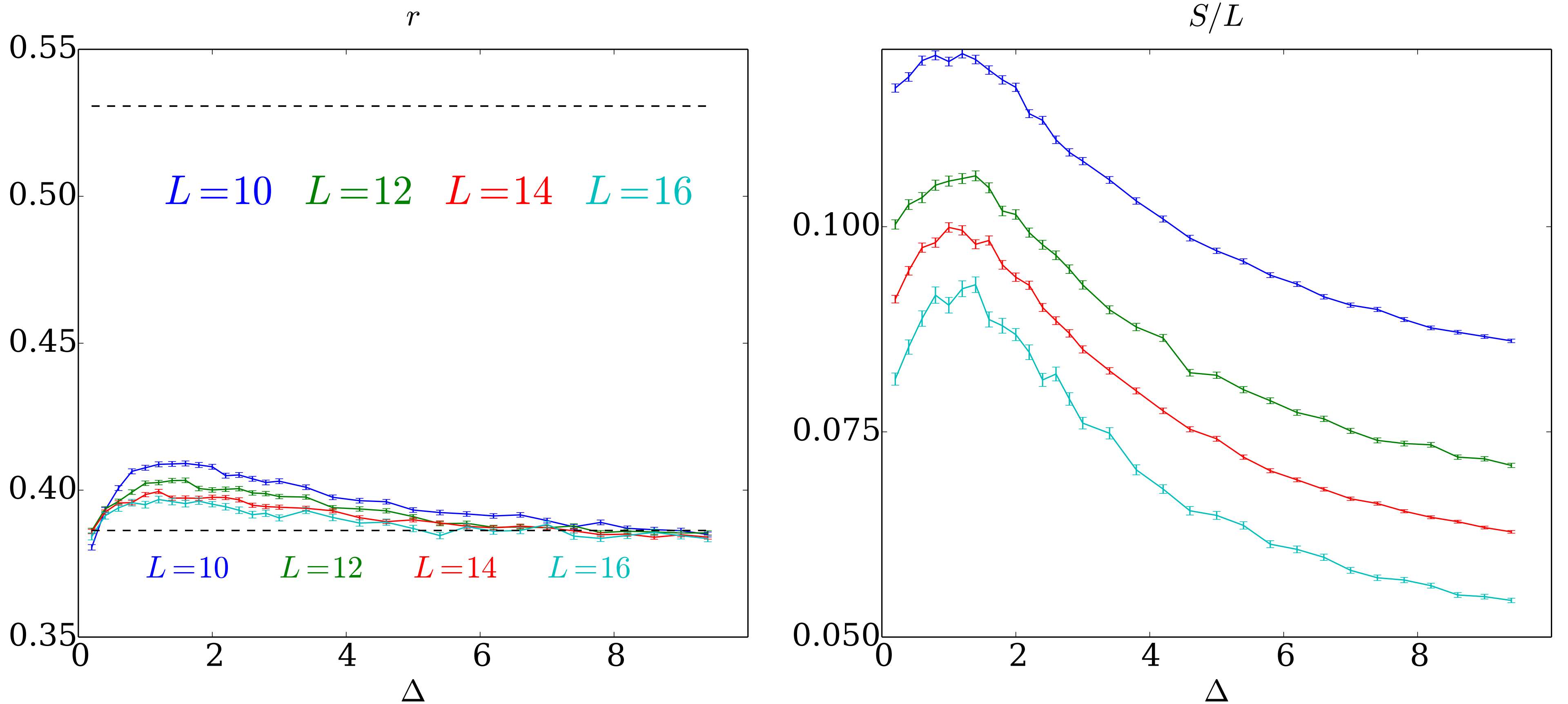


Ergodic (ETH) phase

MBL phase with SG order

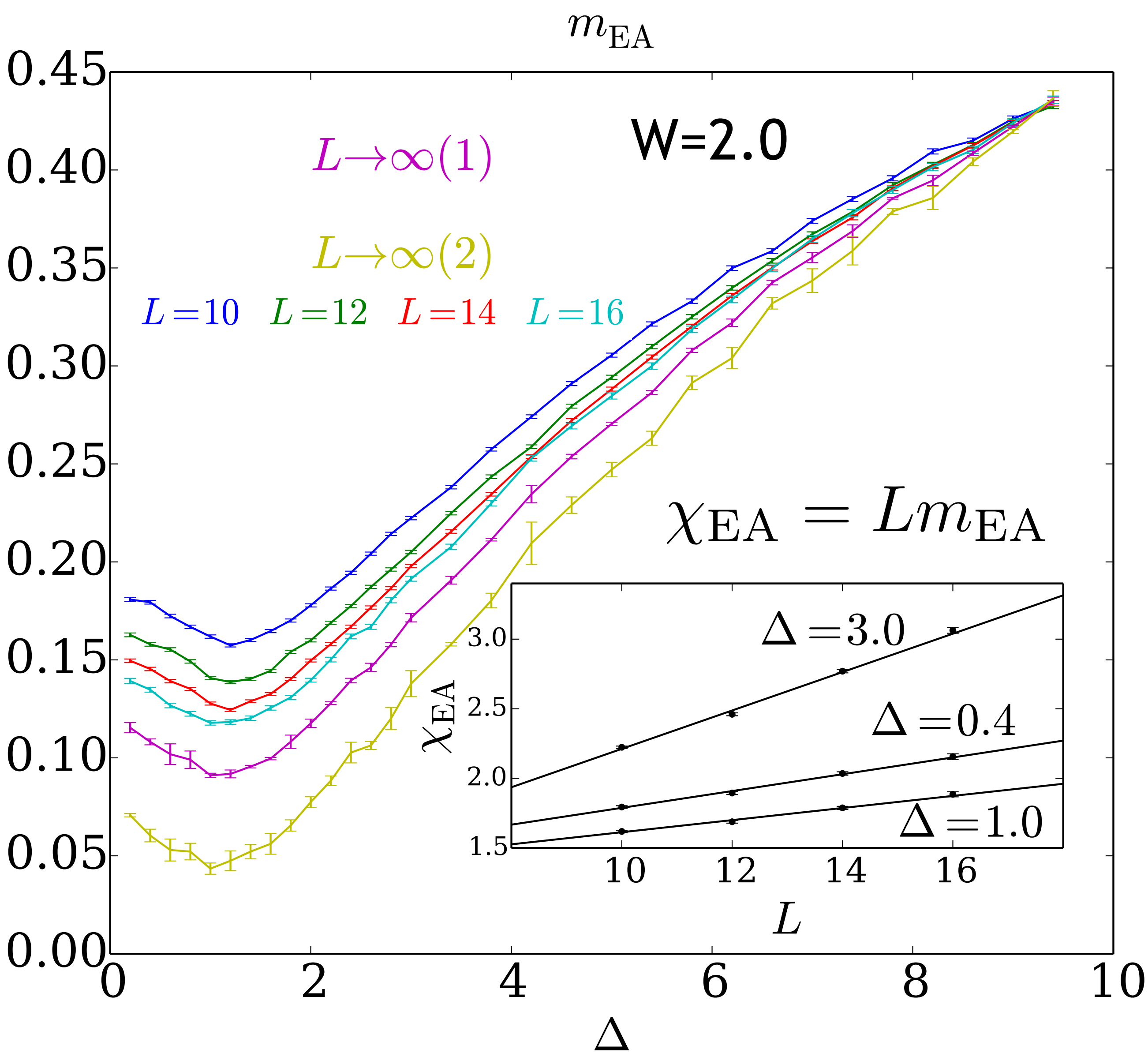
Strong disorder: spin glass

W=2.0



Many-body localized phase… but could be critical?

Strong disorder: spin glass

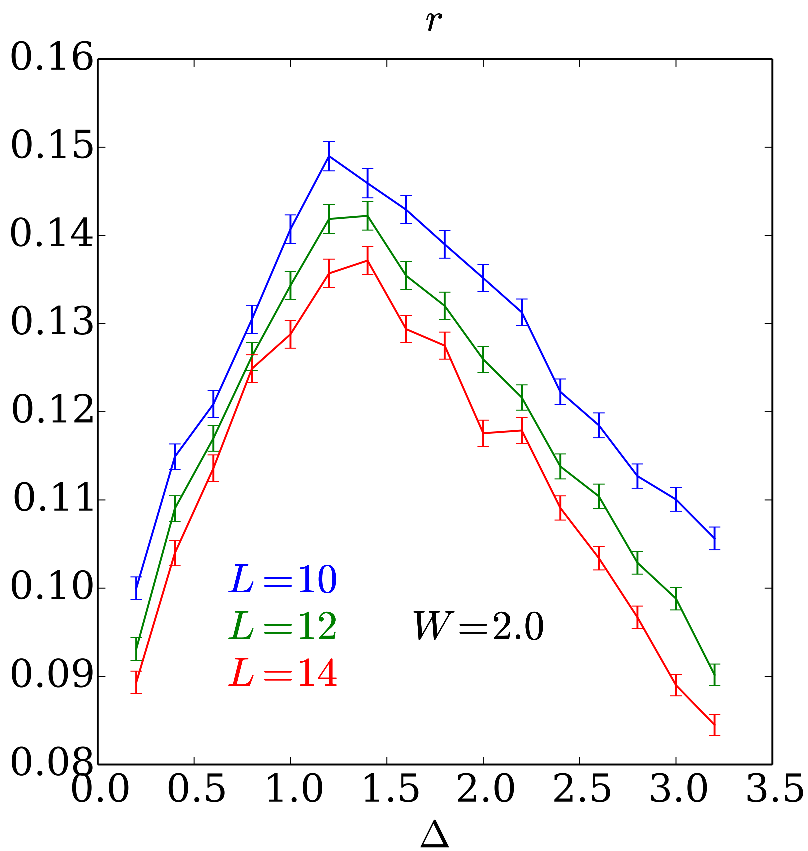


Data consistent with

$$\chi_{\mathrm{EA}} \sim L$$

$$\left(\lim_{L\to\infty} m_{\mathrm{EA}} \neq 0\right)$$

Pairing of eigenstates



Do not restrict to given "Ising" sector:

$$|n\rangle_{\pm} = \frac{|n\rangle \pm \mathcal{C}\,|n\rangle}{\sqrt{2}}$$

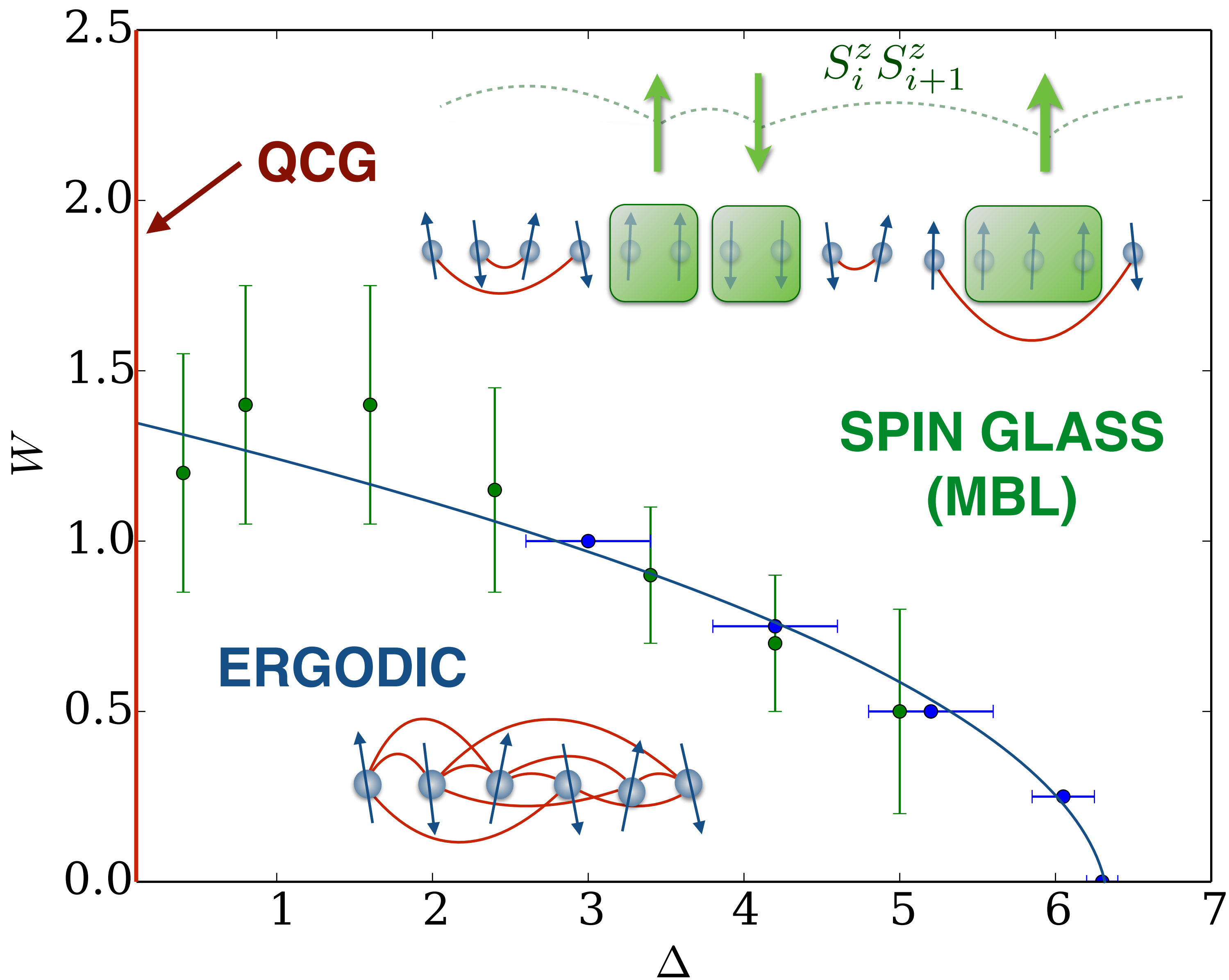
$$e^{-L/\xi} \;\; \text{VS} \;\; \delta \sim e^{-(\ln 2)L}$$

Strong disorder: pairing, r ratio ↘

Huse, Nandkishore, Oganesyan, Pal & Sondhi '13

Again consistent with spontaneously broken PHS

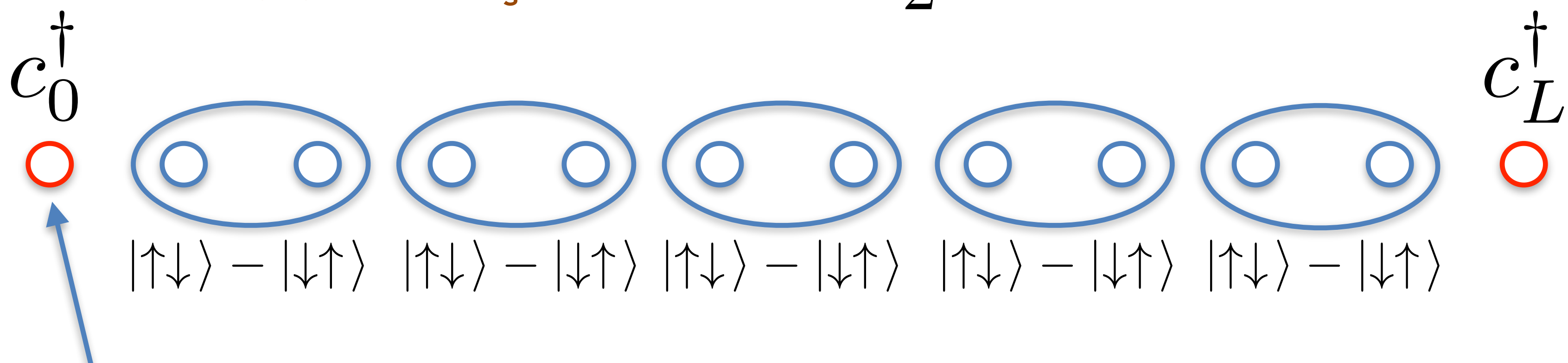
Phase diagram



Consequence for SPT order

T=0 Dimerized XXZ chain: $J_i = \frac{J}{2}(1 - \delta(-1)^i)$

Su-Schrieffer-Heeger



Topological fermionic zero mode

Protected by symmetry

$$U(1) \times \mathbb{Z}_2^{\mathcal{S}}$$

CLASS AIII

Chiral sublattice symmetry: $\mathcal{S} = \mathcal{C}K$

$$\mathbb{Z} \xrightarrow[\text{interactions}]{} \mathbb{Z}_4$$ **classification**

Fidkowski & Kitaev '10
Turner, Pollmann & Berg '11

Instability of excited-state SPT order

MBL can protect SPT order at finite energy density!

Chandran, Khemani, Laumann & Sondhi ’14
Bahri, Vosk, Altman & Vishwanath ’15, …

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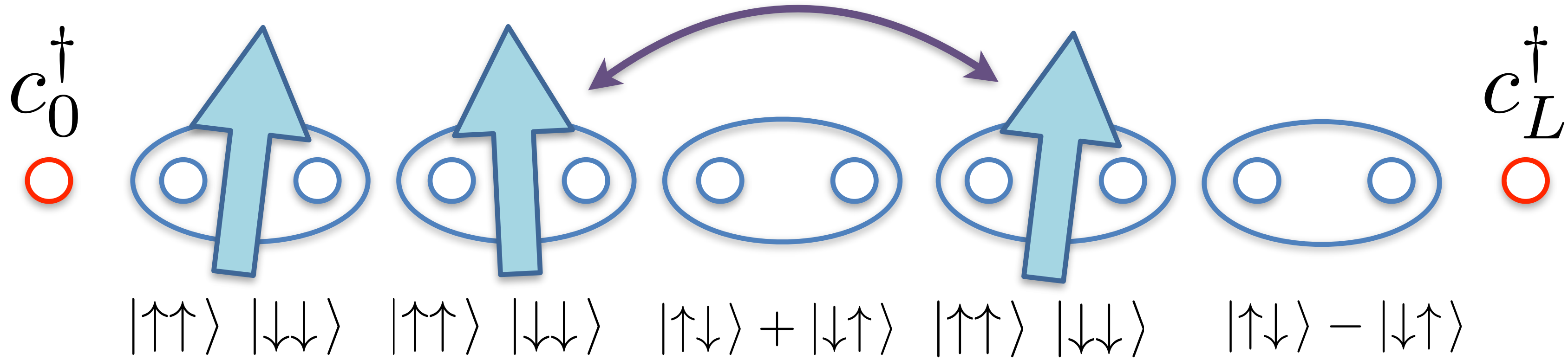
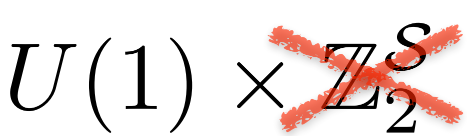
BUT

XXZ-type effective interactions





Spontaneously broken

NO SPT Order!

Previous constraints on MBL+SPT: Slagle, Bi, You & Xu '15
Potter & Vishwanath '15

Conclusion

- Quantum critical glasses: quantum-critical excited eigenstates
- Genuine MBL phase with broken PHS in random-bond XXZ chain
- Instability of certain types of excited state SPT order

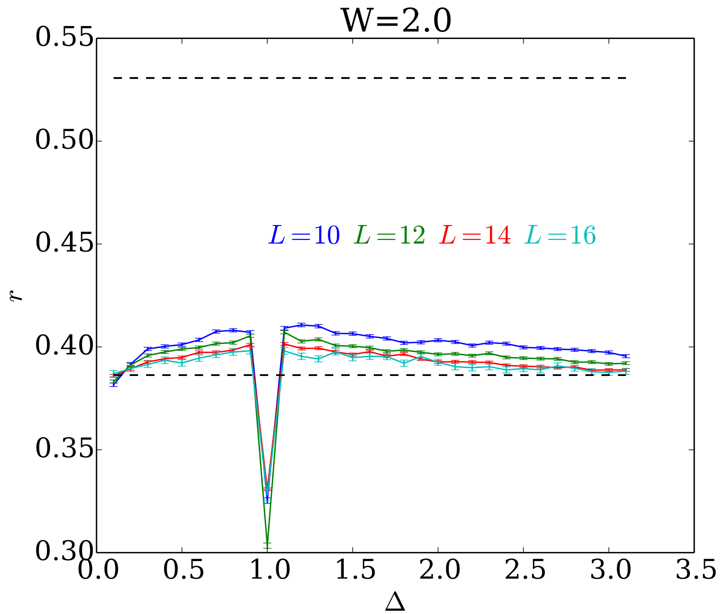
- Future directions/In progress:

1. What about PHS systems in d=2?
2. General rules for MBL + quantum order?
3. General proof of absence of PHS + MBL?

RV, A.J. Friedman, S.A. Parameswaran & A.C. Potter, arXiv:1510.04282

RV, A.C. Potter and S.A. Parameswaran, PRL 2015, arXiv:1410.6165

Uniform Δ



Qualitatively similar except at SU(2) symmetric point

(there expect thermalization)

RV, Potter, Parameswaran '14