Memory in granular materials

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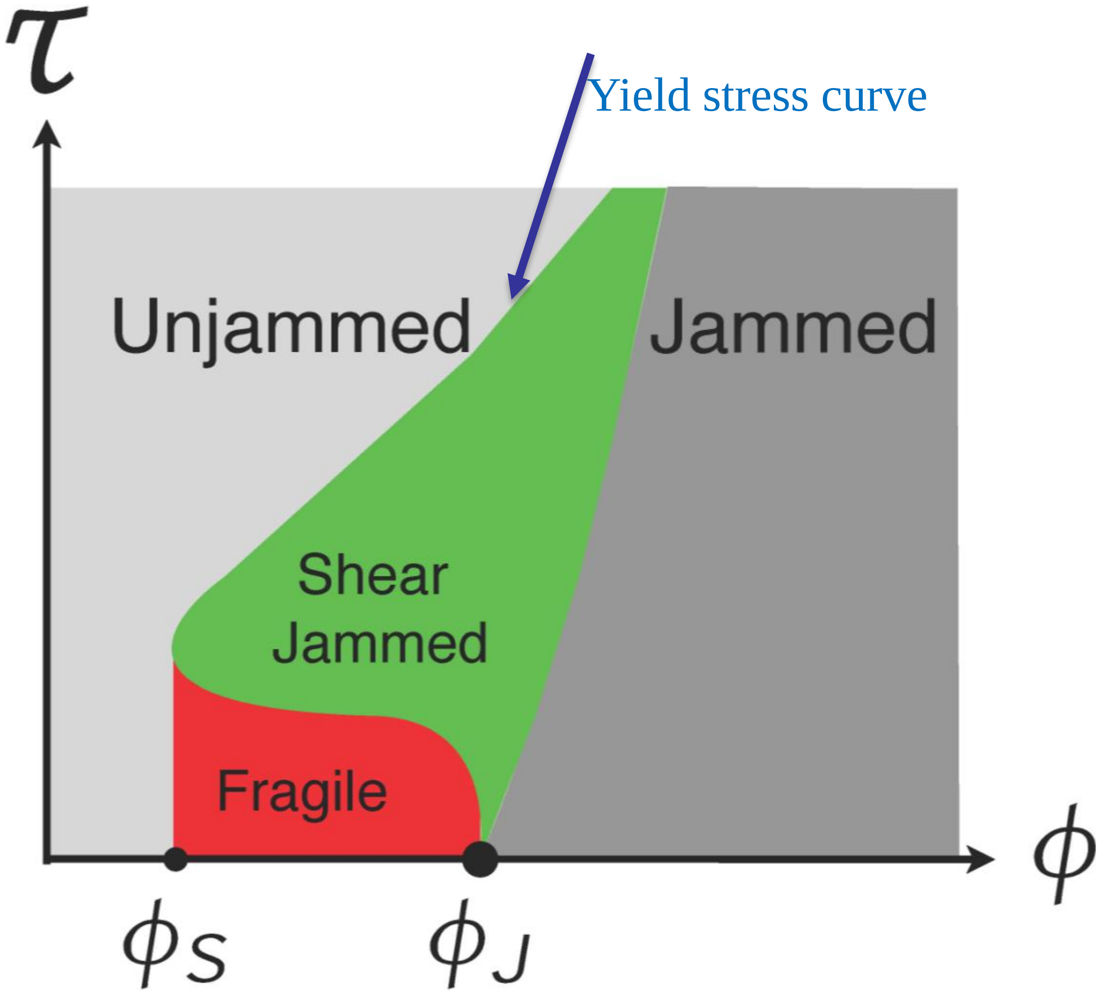
Premise: Granular materials respond to strains by forming networks—these reflect a memory of the initial state plus strain protocol

- Background
  - Shearing—can rewrite memory, but also reproduce previous states
  - Particles: elastic (soft) and frictional
  - Force networks and protocols
  - Experimental techniques
  - Shear jamming phase diagram
- Cyclic shear systematically rewrites memory—activation by shear amplitude
- What are the microscopic processes that cause shear jamming, and also rewrite the networks

Context
Shear stress τ vs. packing fraction φ

Frictional spheres; static states



Shear strain applied to granular materials can jam an initially stress-free state. Continued shear drives the system to the yield stress curve

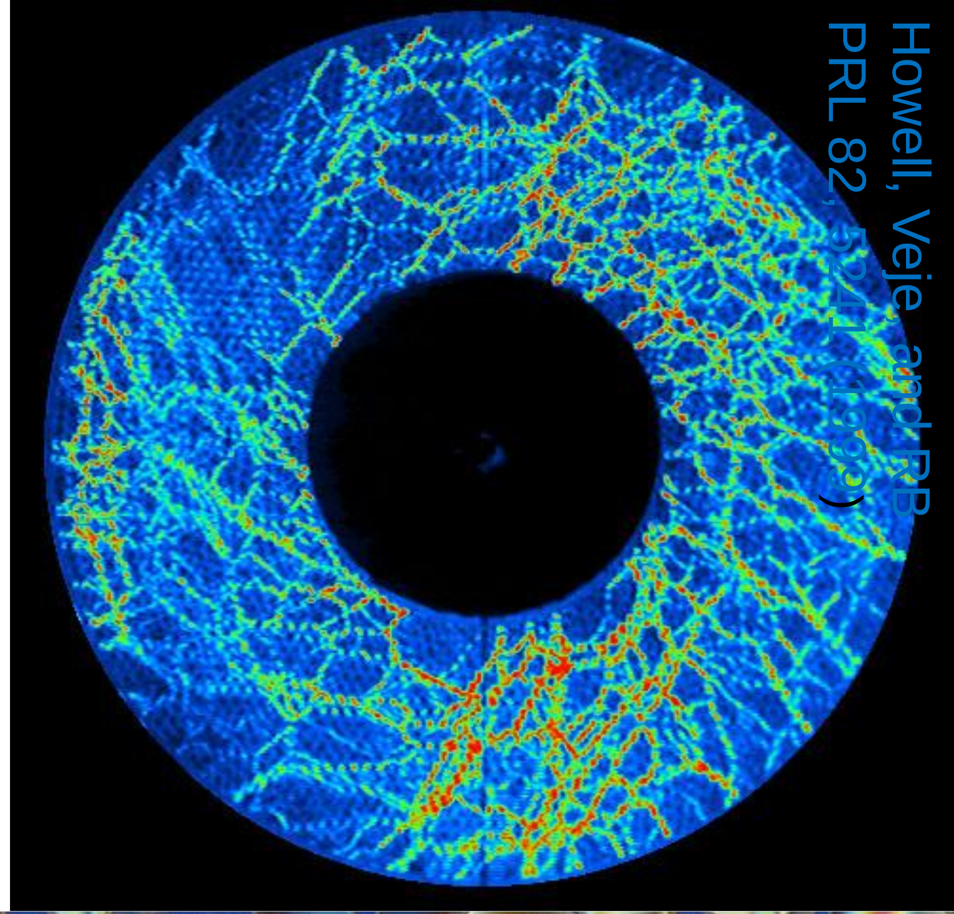
- The macroscopic state diagram includes fragile, shear jammed and dynamic states at the YSC
- The initial processes leading to shear jamming generate anisotropic networks, called force chains. How should one characterize/distinguish networks?
- At a yet smaller scale, what processes enable the formation of force chains under shear?
- Do these processes lead to memory? If so, how?

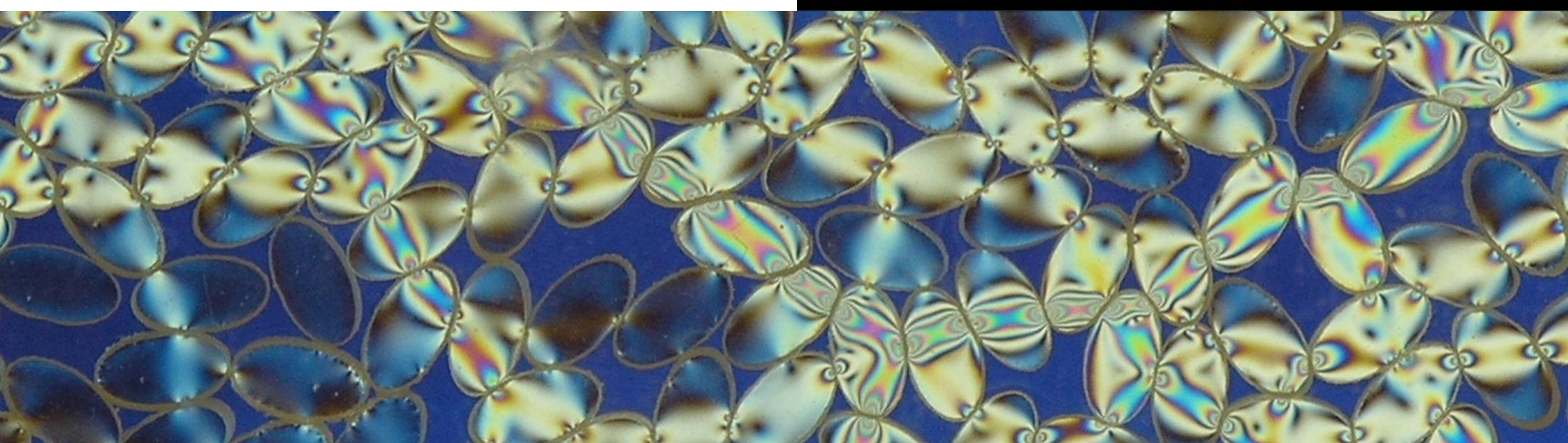
Granular Material:Dense Phases, particularly sheared, frictional

Forces are carried preferentially on force chains (**Networks**) →multiscale phenomena—grains to system

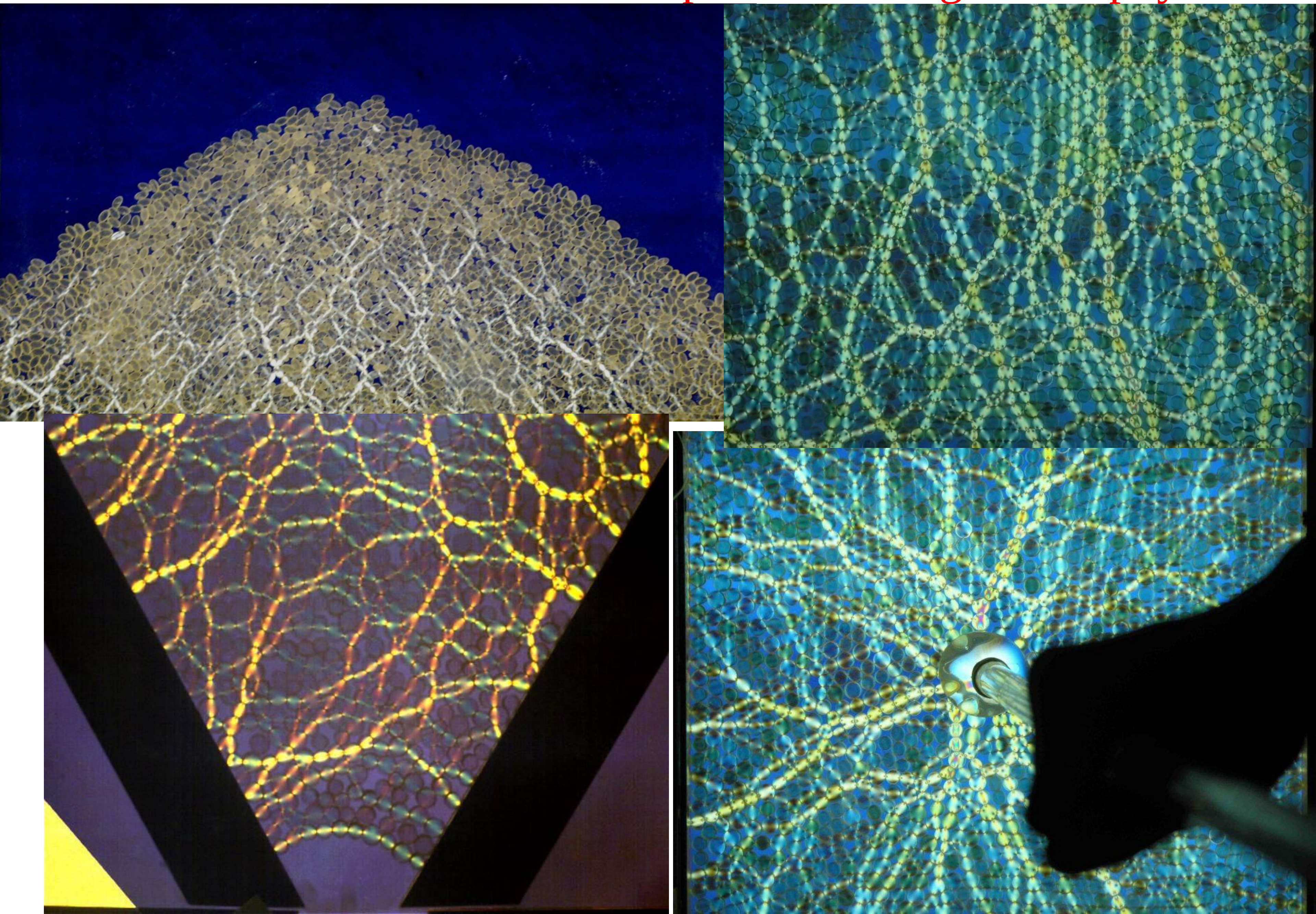
Deformation leads to large spatio-temporal fluctuations

Granular materials jam —fluid ← →solid transition (Howell, P&G1997, PRL 1999)





Force networks are an essential part of dense granular physics

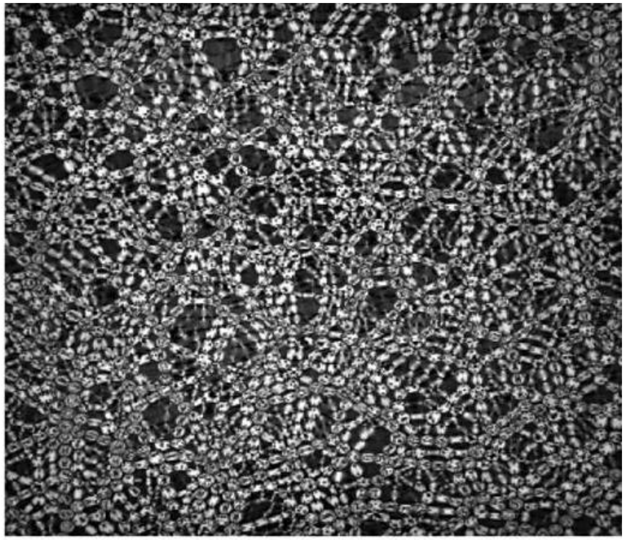


Particle properties for this discussion

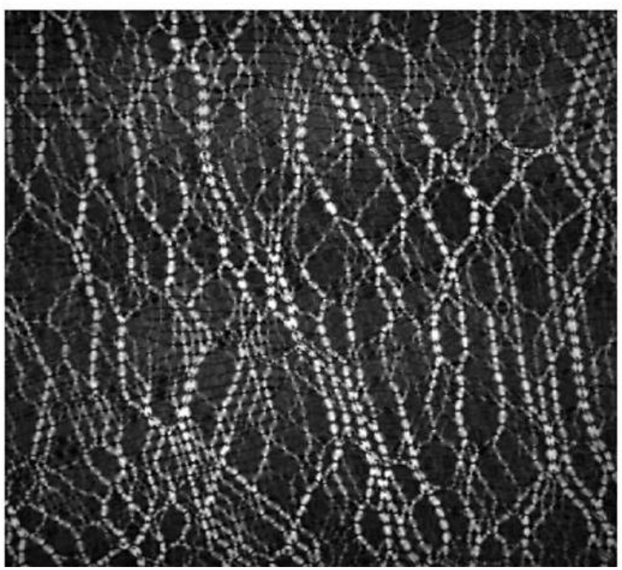
- Particles interact when they are in contact—no contact no force
- Particles interact by elastic normal forces and tangential frictional forces
- Normal force, F_n depends on the distance δ by which two particles have been pushed together (overlap)—$F_n \sim \delta^{\alpha}$…

  $\alpha = 1, 3/2$ for Hookean and Hertzian contacts resp.
- Grains typically have friction, coefficient μ…friction forces do not depend on inter-grain positions -> no potential energy—large particle size -> athermal

Relation of force networks to protocols—e.g. compression or shear



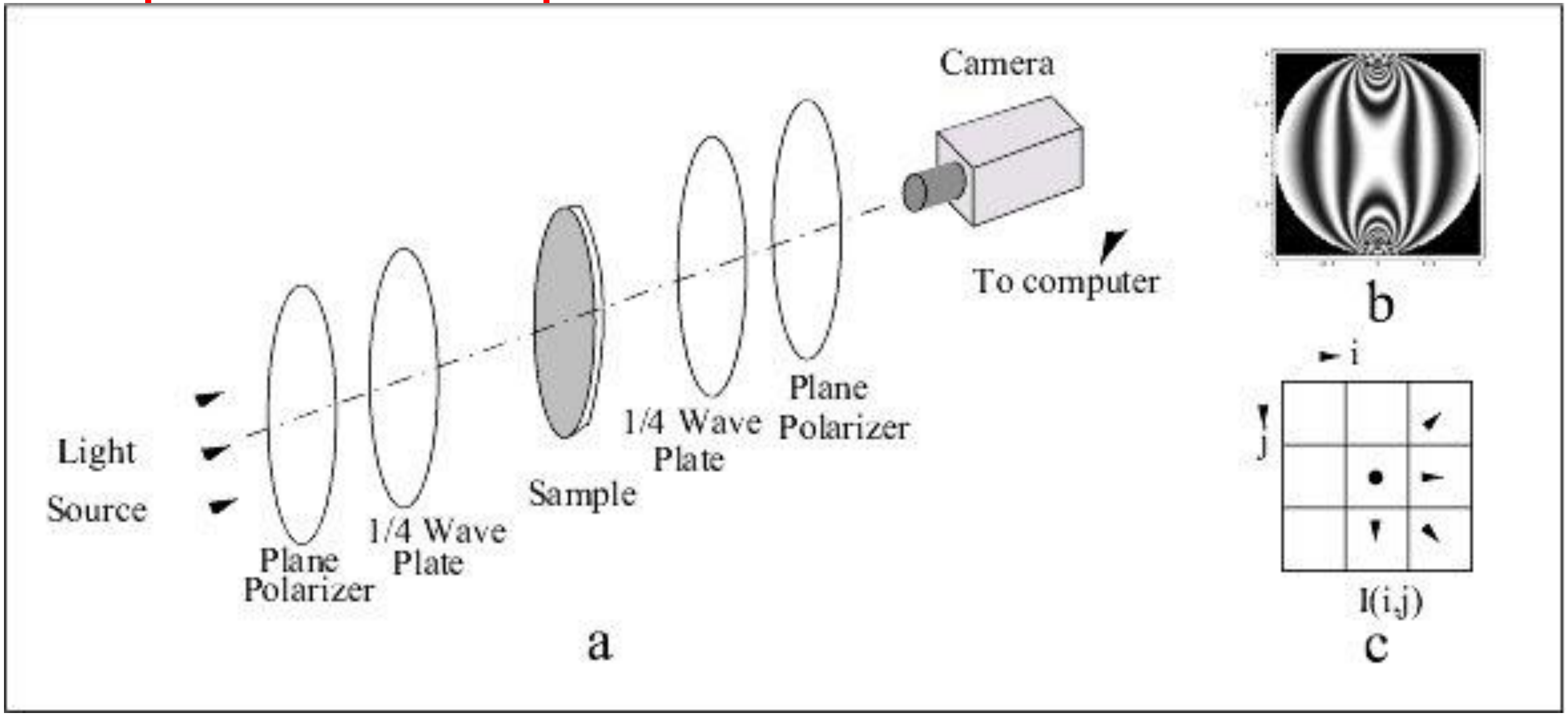
Isotropic Compression

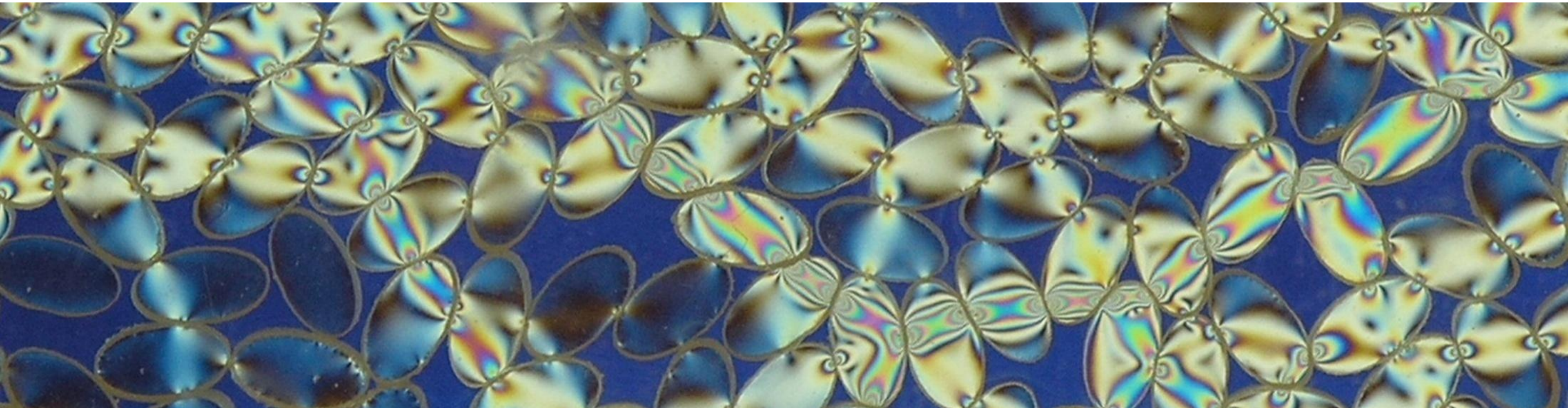


Pure Shear

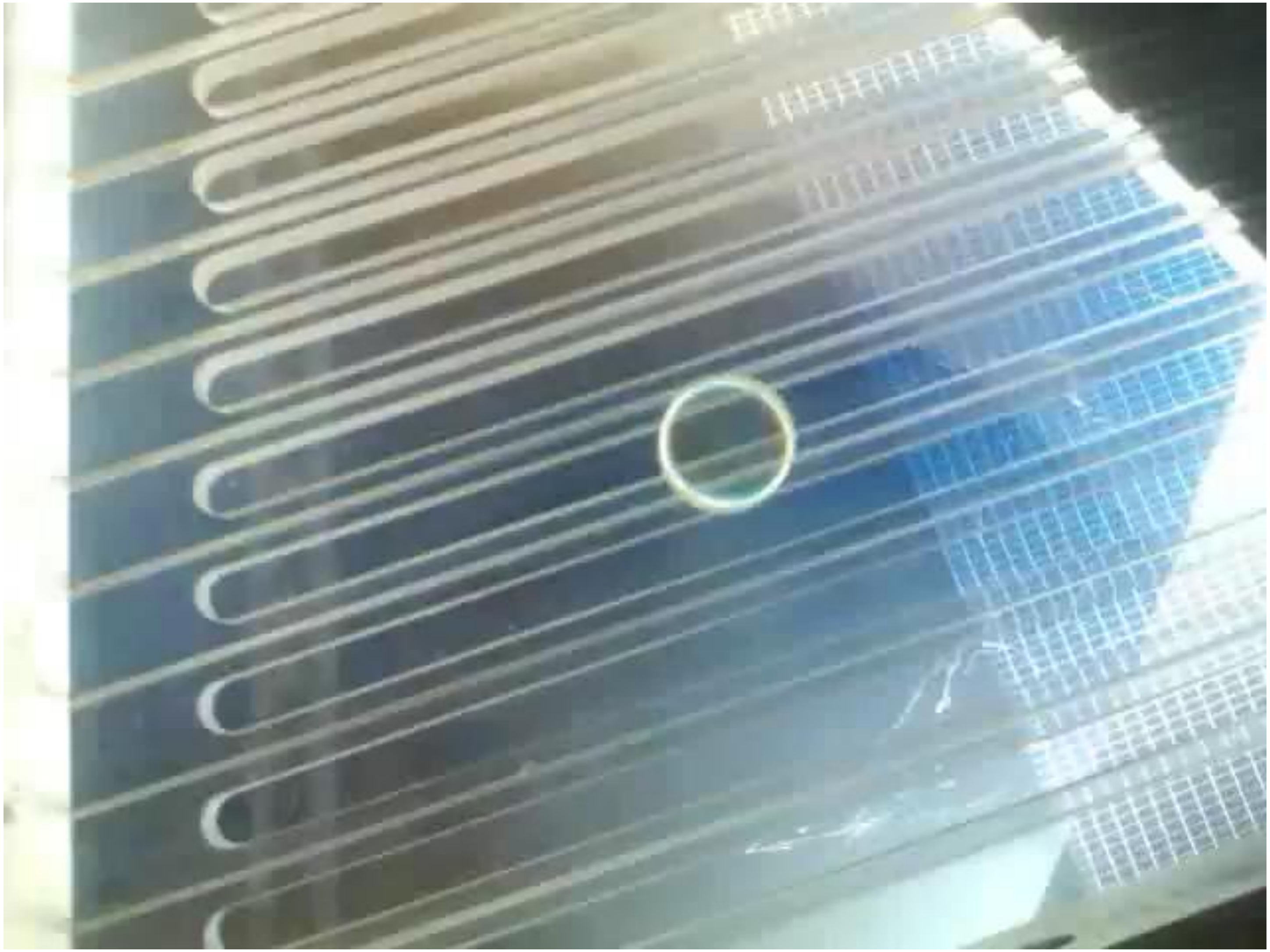
T. Majmudar and BB, Nature 2005

Measuring contact forces by photoelasticity—2D quantiative experiments from smallest scales





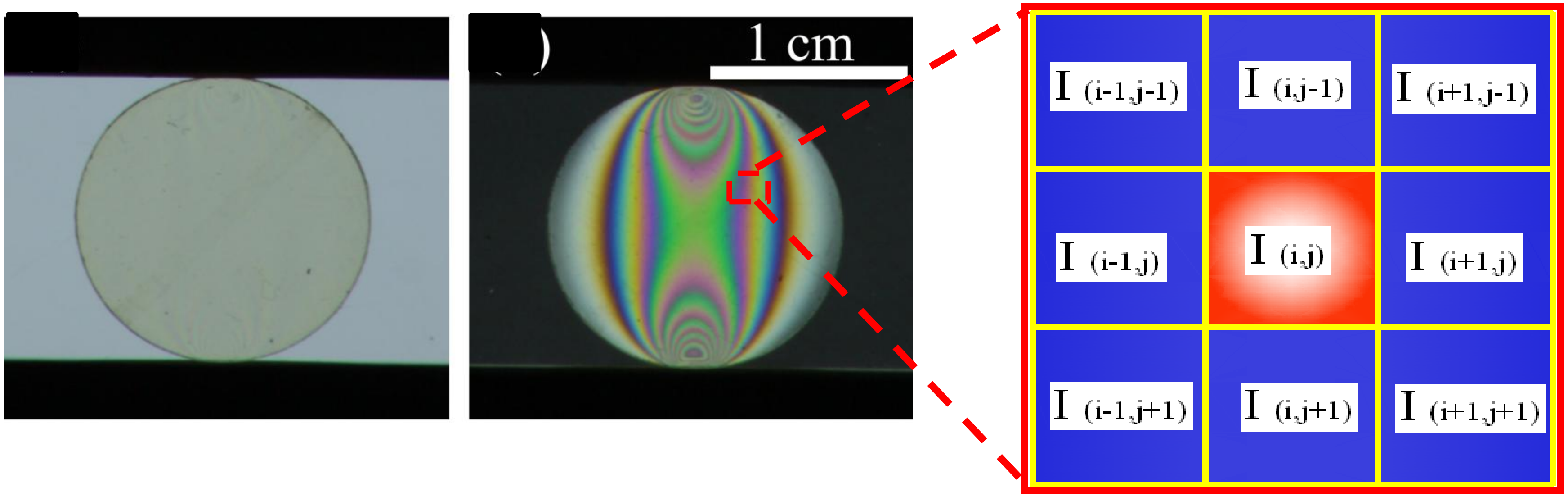
Fun with photoelasticity*



*Joshua Dijksman

Experimental advances allow grain-scale force measurements--I

D. Howell, BB, PRL 1999, PRE 1999



$$\nabla I_{i,j}^2 = \frac{1}{4}\left[\left(\frac{I_{i+1,j} - I_{i-1,j}}{2}\right)^2 + \left(\frac{I_{i,j+1} - I_{i,j-1}}{2}\right)^2 + \left(\frac{I_{i+1,j+1} - 1 - I_{i-1,j-1}}{2}\right)^2 + \left(\frac{I_{i+1,j-1} - I_{i-1,j+1}}{2}\right)^2\right]$$

$$< G^2 >= \frac{1}{N}\sum_{i,j} \nabla I_{i,j}^2$$

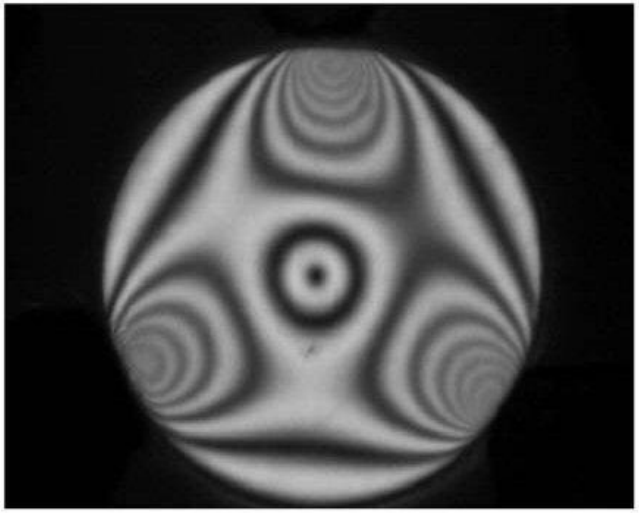
Experimental Advances allow grain-scale contact force measurements--II

T. Majmudar and BB
Nature, 2005

- Contact forces determine exact photoelastic pattern:
- Contact forces → stresses within disk (linear elasticity)
- Planar stresses give pattern:

$$I = I_o \sin^2[(\sigma_2 - \sigma_1)CT/\lambda]$$

c

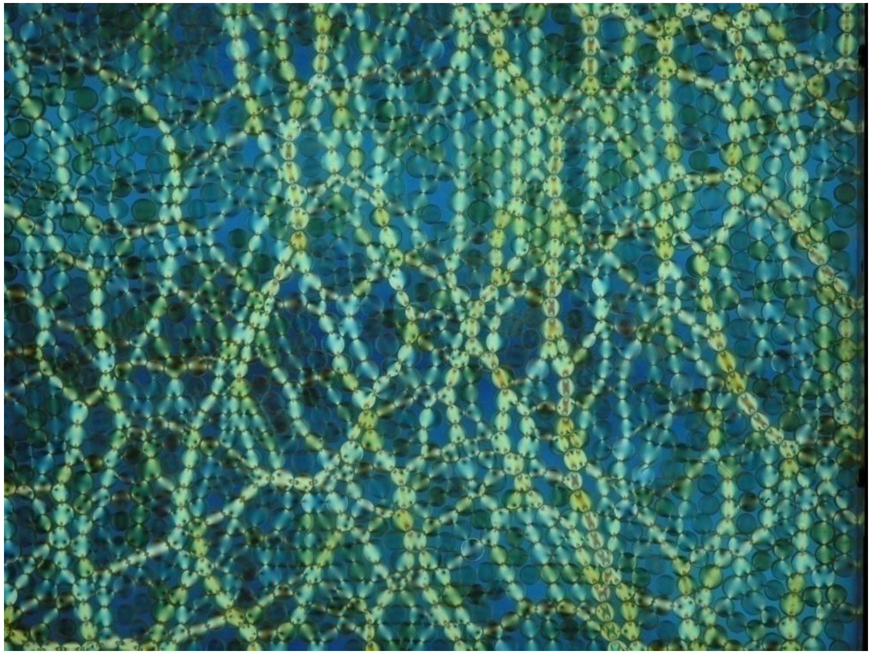


Technique for finding 2D contact forces

- Process images to obtain particle centers and contacts
- Exact solution for stresses (biharmonic equation) has contact forces as parameters
- Make a nonlinear fit to photoelastic pattern using contact forces as fit parameters
- $I = I_o \sin^2[(\sigma_2 - \sigma_1)CT/\lambda]$
- In the previous step, invoke force and torque balance to reduce unknown contact forces
- Newton's 3d law provides error checking

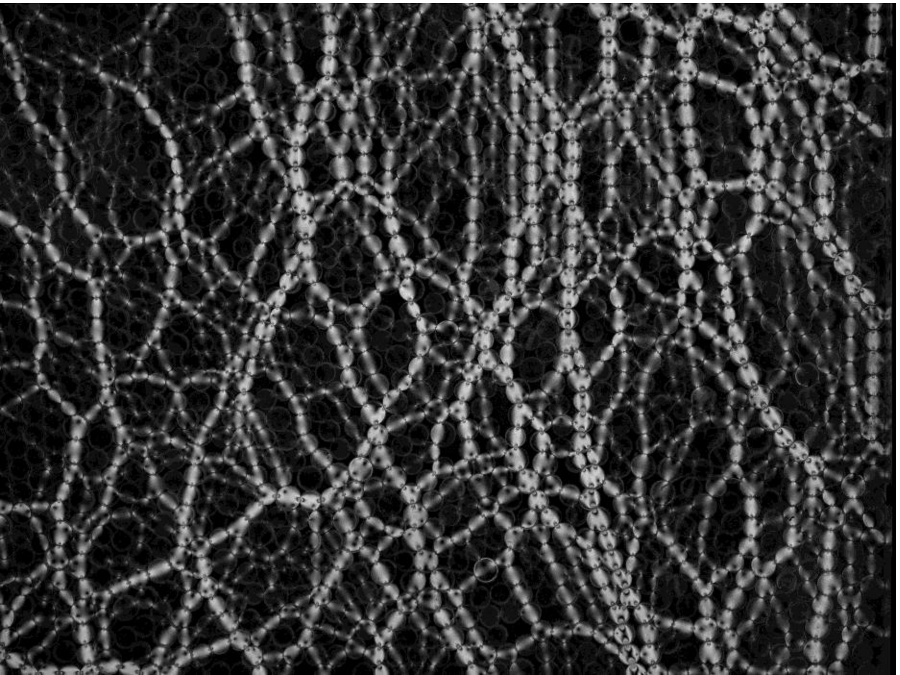
Key new approach: obtain grain contact forces

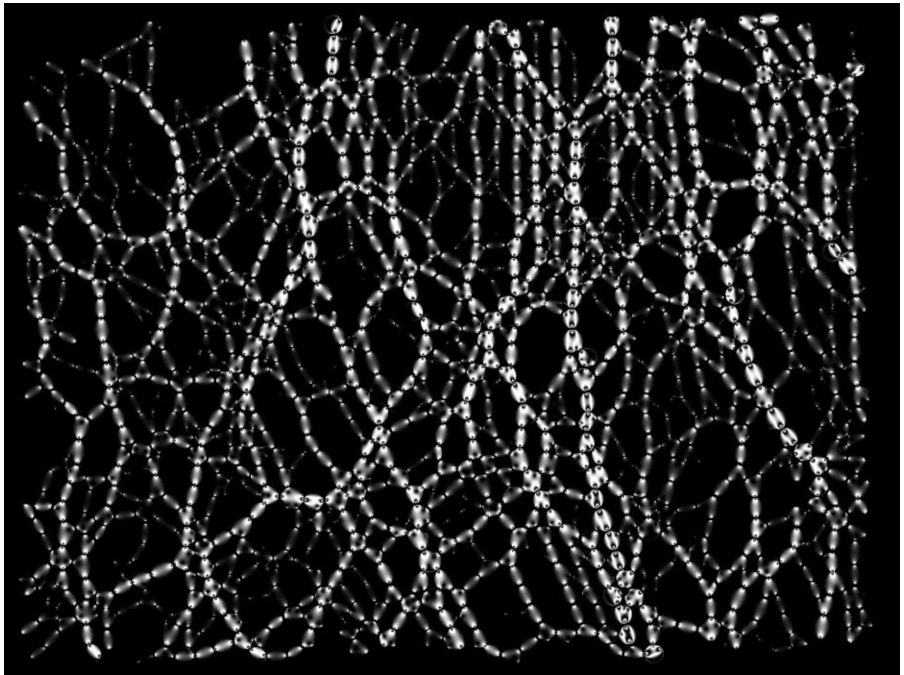
Experiment--raw



Reconstruction From force inverse algorithm

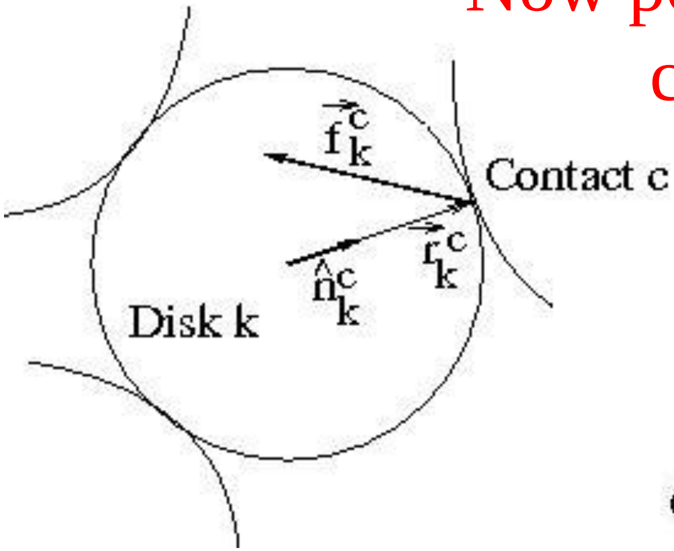
Experiment Color filtered





Obtaining stresses and fabric from experimental data

Now possible to obtain direct experimental characterizations at grain scale

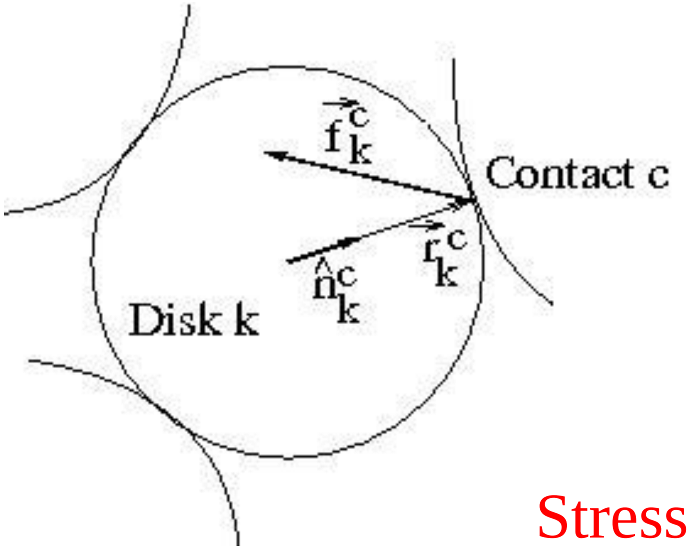


$$\hat{\sigma} = \frac{1}{V} \sum_{i \neq j} \vec{r}_{ij} \otimes \vec{f}_{ij},$$ Stress

$$\hat{R} = \frac{1}{N} \sum_{i \neq j} \frac{\vec{r}_{ij}}{\|\vec{r}_{ij}\|} \otimes \frac{\vec{r}_{ij}}{\|\vec{r}_{ij}\|},$$ Fabric

These quantities can be coarse-grained to produce continuum fields

Stresses, fabric, force moment tensor—2D
evaluate across scales: particles, networks, system



Fabric tensor

$R_{ij} = \Sigma_{k,c}\, n^c_{ik}\, n^c_{jk}$

$Z = \mathrm{trace}[R]$

Stress tensor, force moment tensor

stress: $\sigma_{ij} = (1/A)\, \Sigma_{k,c}\, r^c_{ik}\, f^c_{jk}$

Force moment $\Sigma_{ij} = \Sigma_{k,c}\, r^c_{ik}\, f^c_{jk} = A\, \sigma_{ij}$

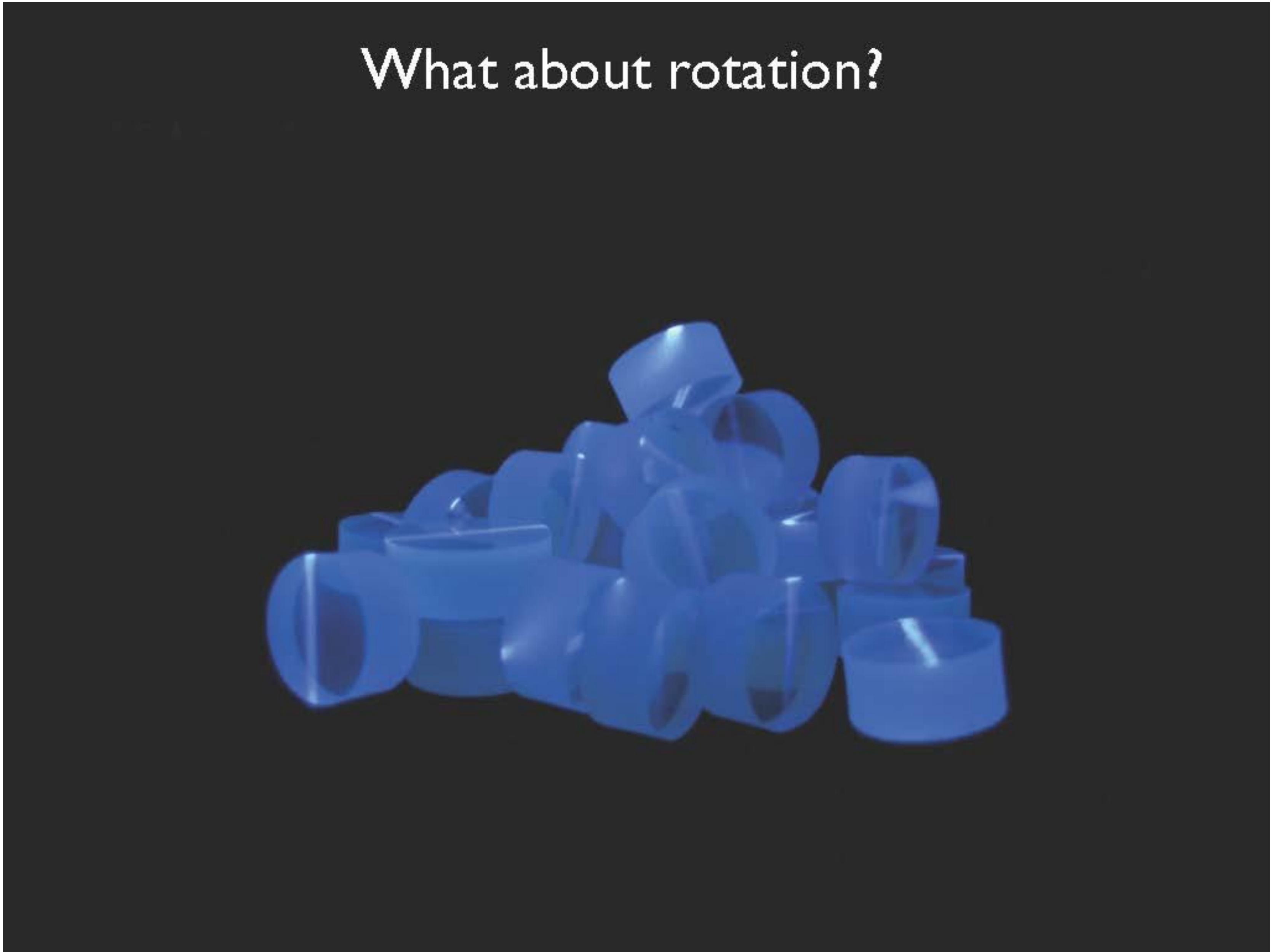
A is particle/system area

Pressure, P and shear stress

$P = \mathrm{Tr}(\sigma)/2 = (\sigma_2 + \sigma_1)/2$

$:\tau = (\sigma_2 - \sigma_1)/2$

Displacements and rotations of grains



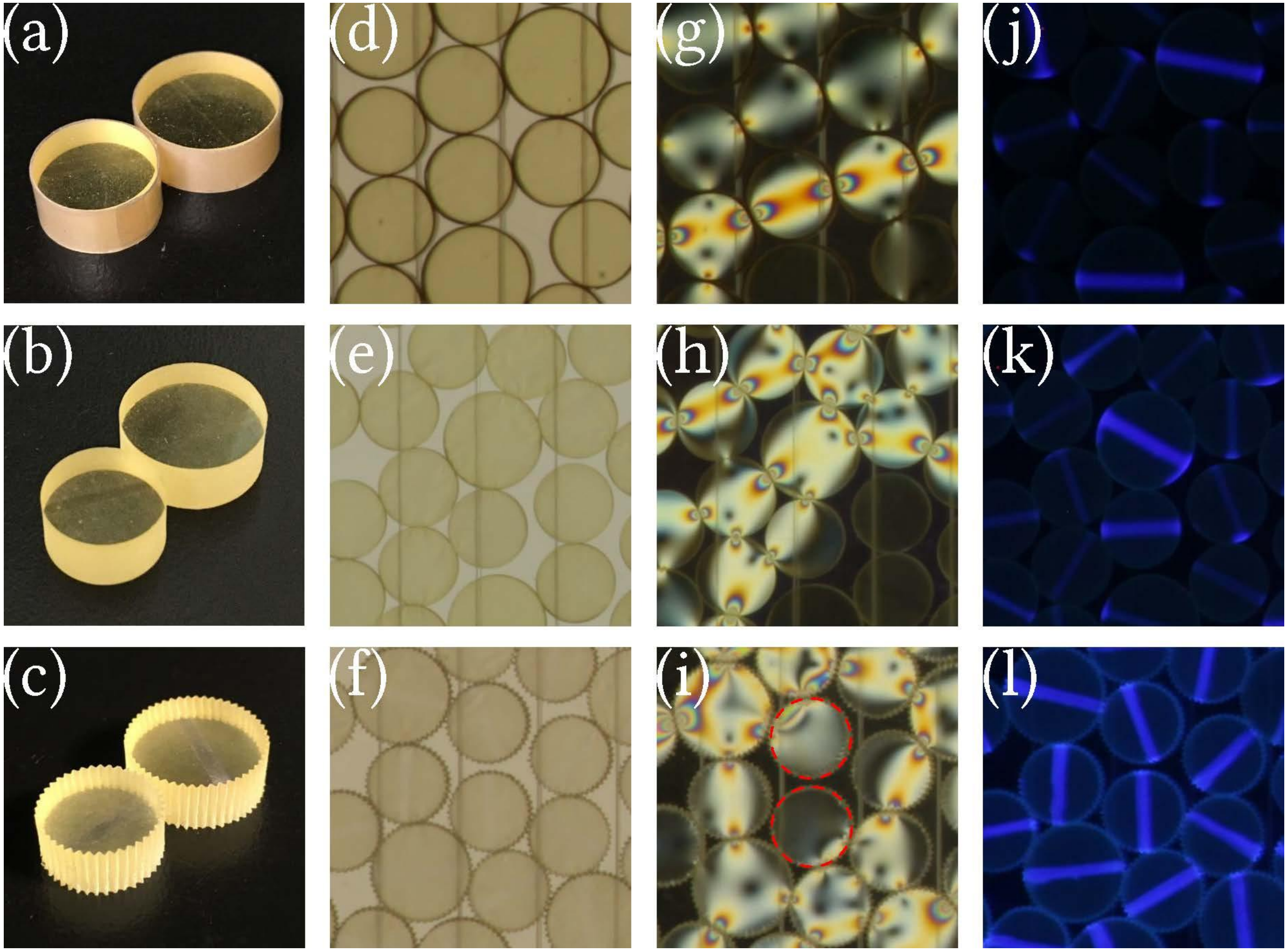
Track Particle: Forces/Displacements/Rotations

Following a small strain step we track particle displacements

$\mu = 0.15$

$\mu = 0.65$

$\mu >> 1$

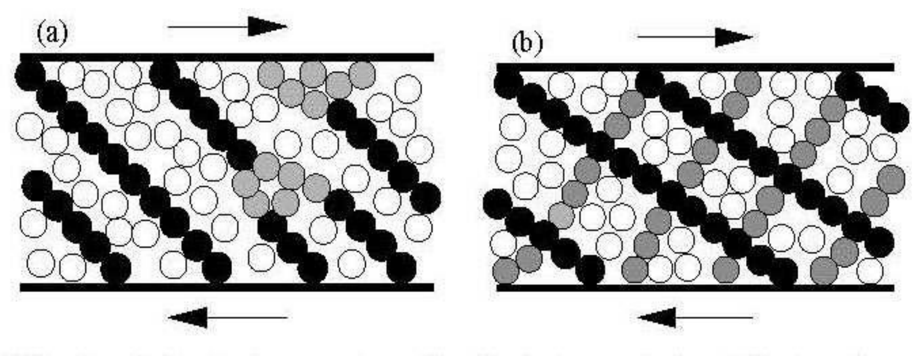


Under UV light bars Allow rotational tracking

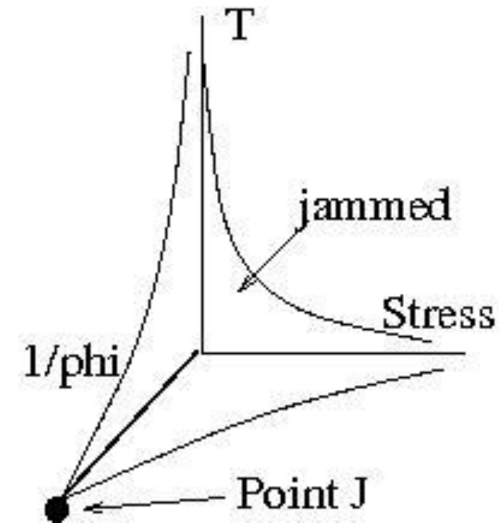
Majmudar and BB Nature, 2005; Majmudar et al. PRL 2007; Zhang et al. Gran.Matt2010; Bi, Zhang, Chacraborty, BB, Nature 2011, Ren et al. PRL 2013, Zheng et al. EPL 2014; Clark et al. PRL 2015; Cox et al. EPL 2016, Barés et al. PRE 2017, Wang et al. 2018

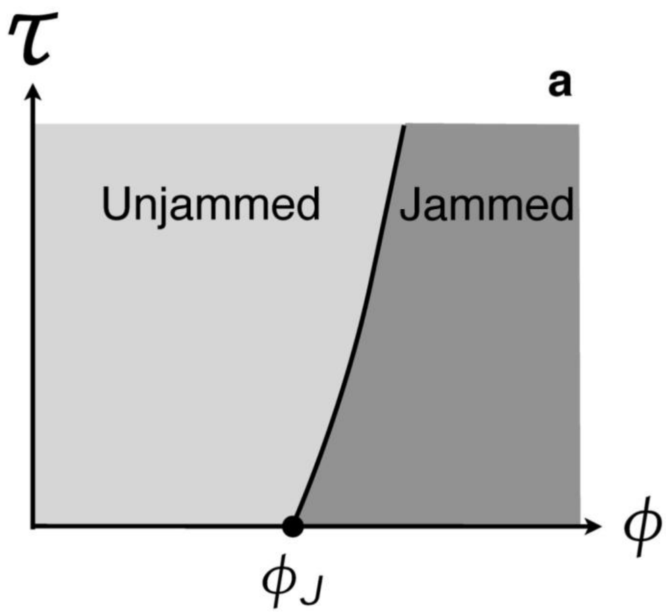
Context: Jamming and Fragility— sheared granular materials

Fragile states: ability to resist strain:
Strong in one direction but weak in reverse



Cates et al. PRL 1998

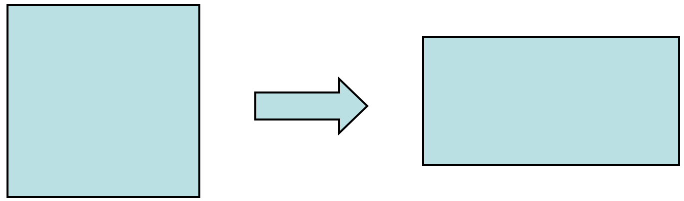




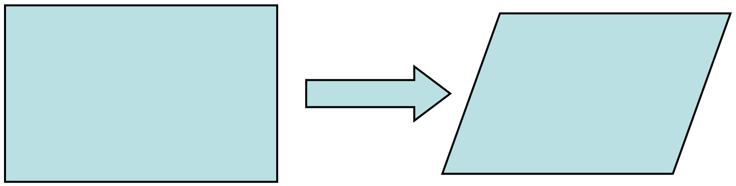
After Liu and Nagel, Nature, 1998, O'Hern et al. PRE 2003

Investigate the response to shear—creation of **stable** anisotropic states

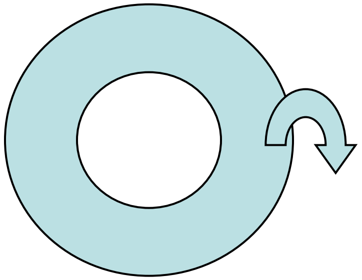
- Example 1: pure shear



- Example 2: simple shear

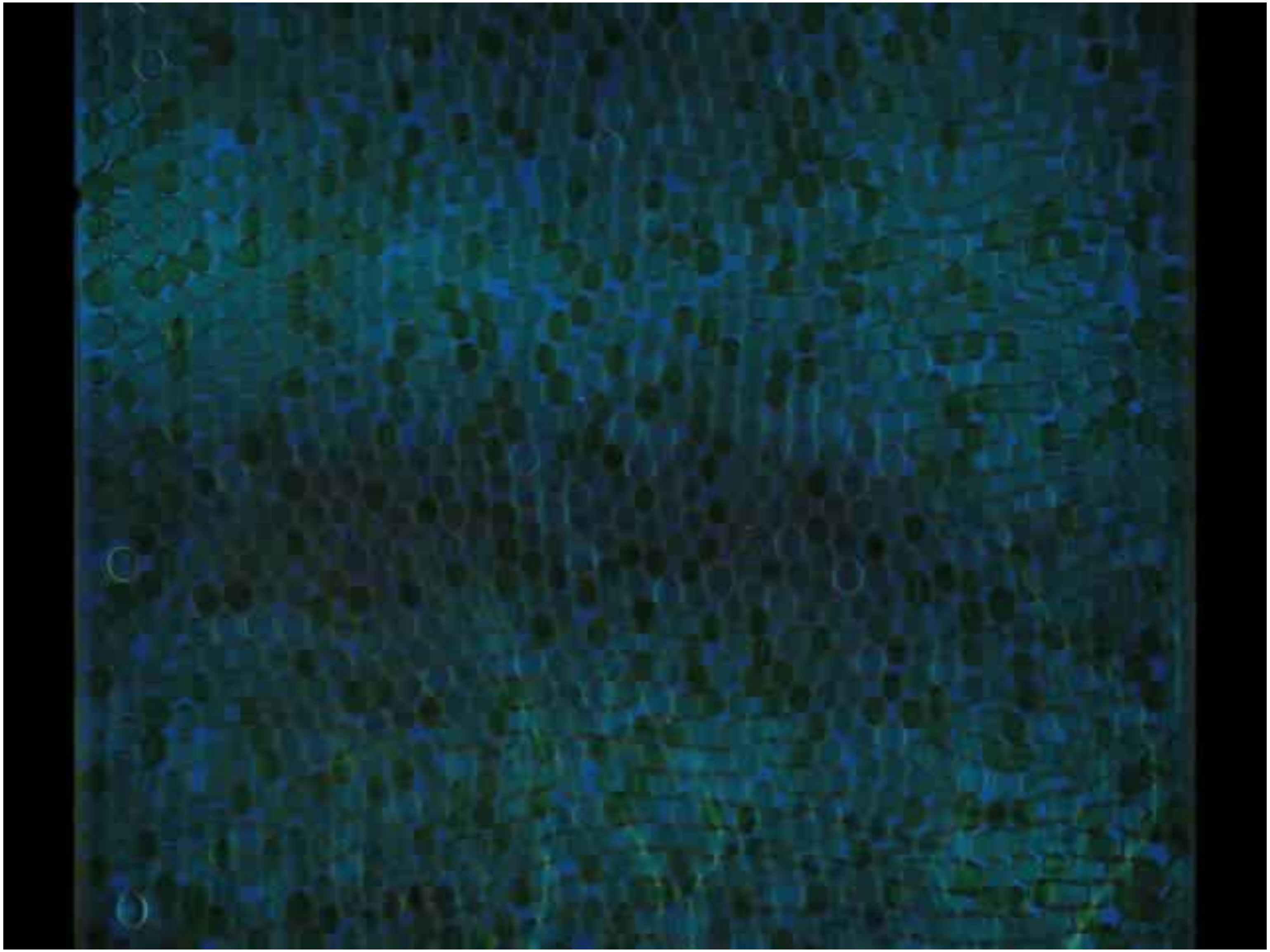


- Example 3: Couette shear

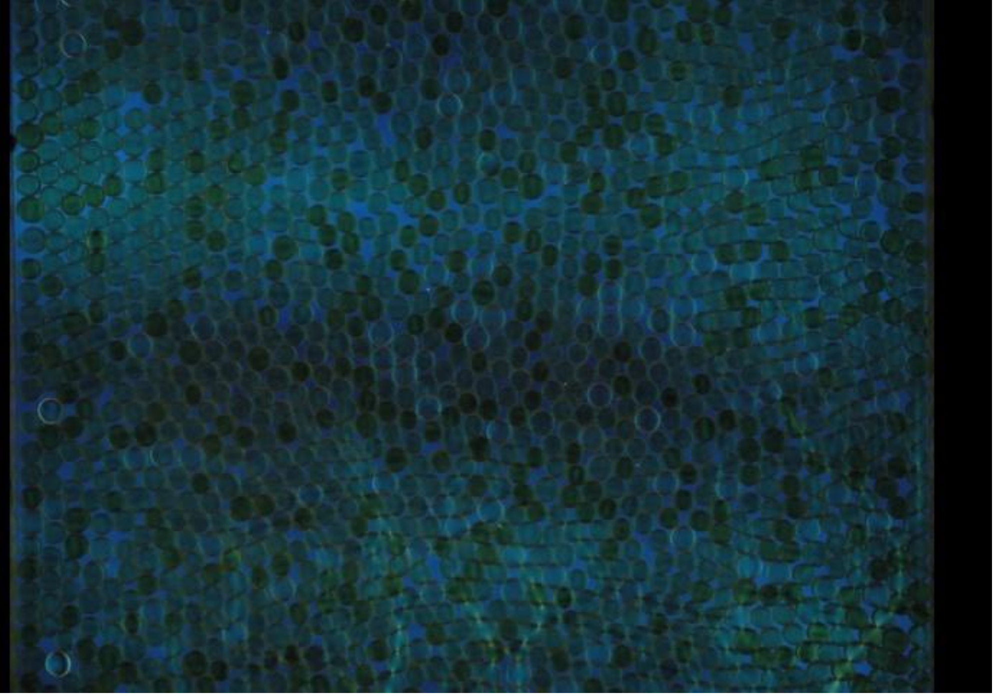


Series of experiments to map out phase diagram

Time-lapse video (one shear cycle) shows force network evolution—Frictional Shear Jamming—



Bi, Zhang, Chakraborty, RPB, Nature, 2011

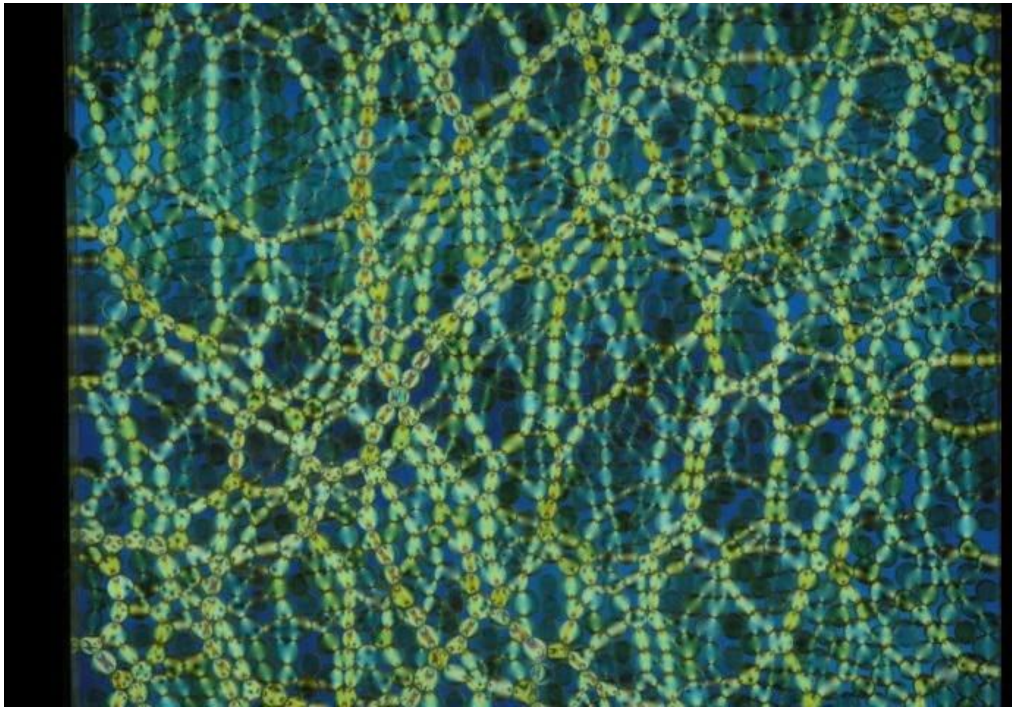


Initial and final states following a shear cycle—no change in area—Density cannot distinguish --but networks can

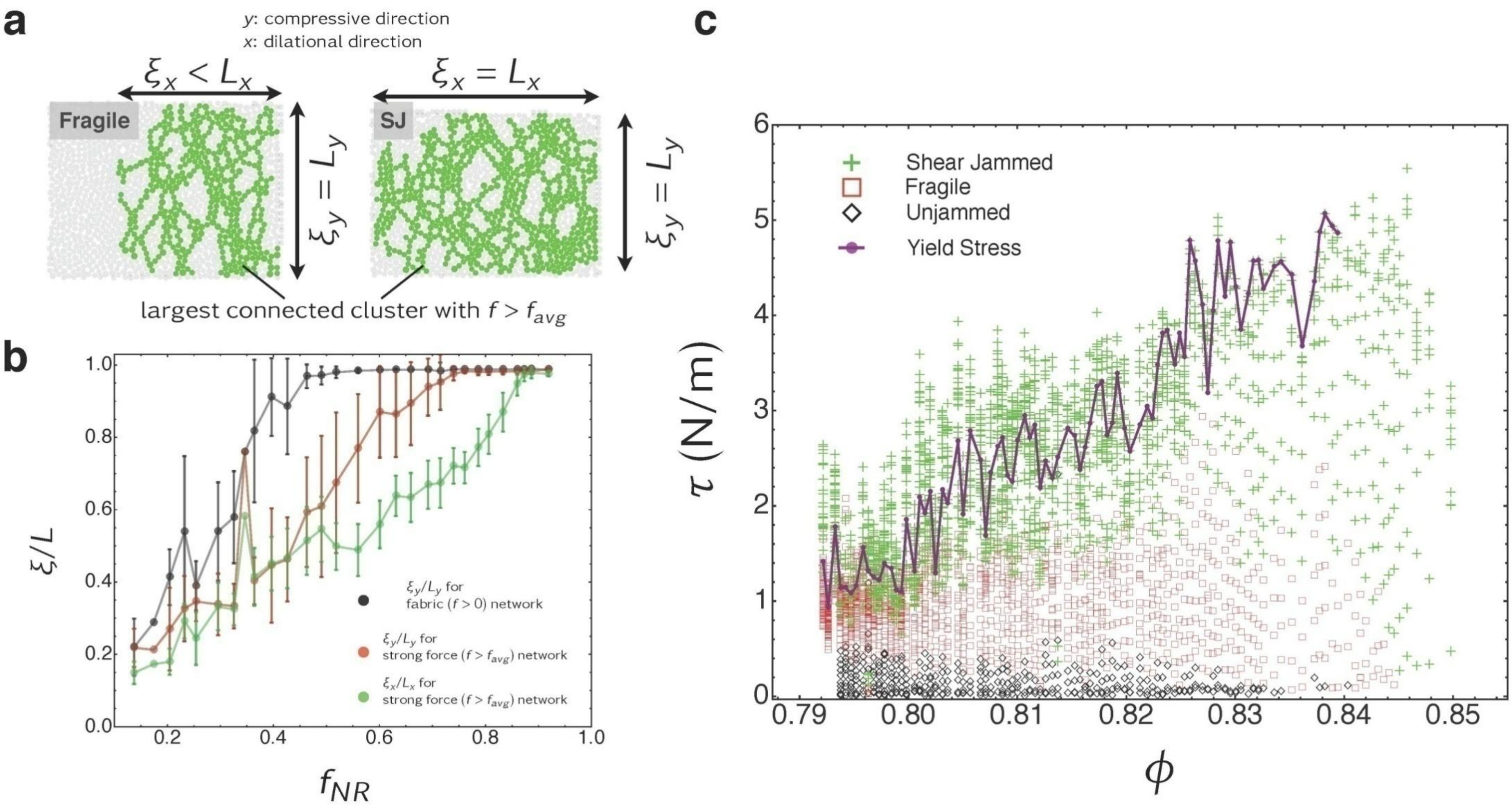
← Initial state, isotropic, no stress

Works between $\varphi_S < \varphi < \varphi_J$

Final state → large stresses jammed



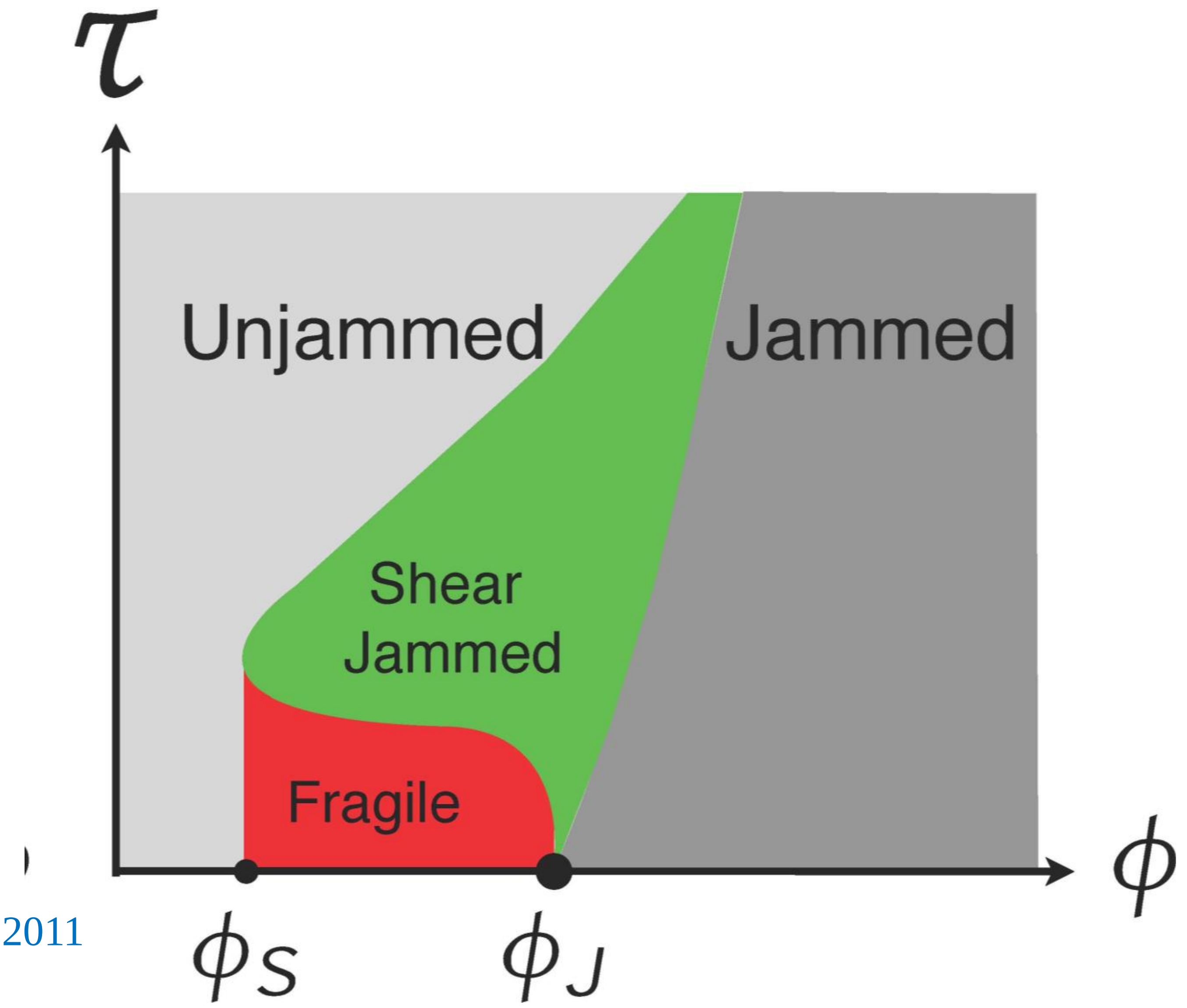
Some special properties of shear jammed states—start with Directional Percolation, Fragile and Shear-Jammed States (Bi et al. Nature, 2011)



f_{NR} = nonrattler fraction

See Otsuki and Hayakawa, Phys. Rev. E **83,** 051301 (2011)

Jamming diagram for frictional grains

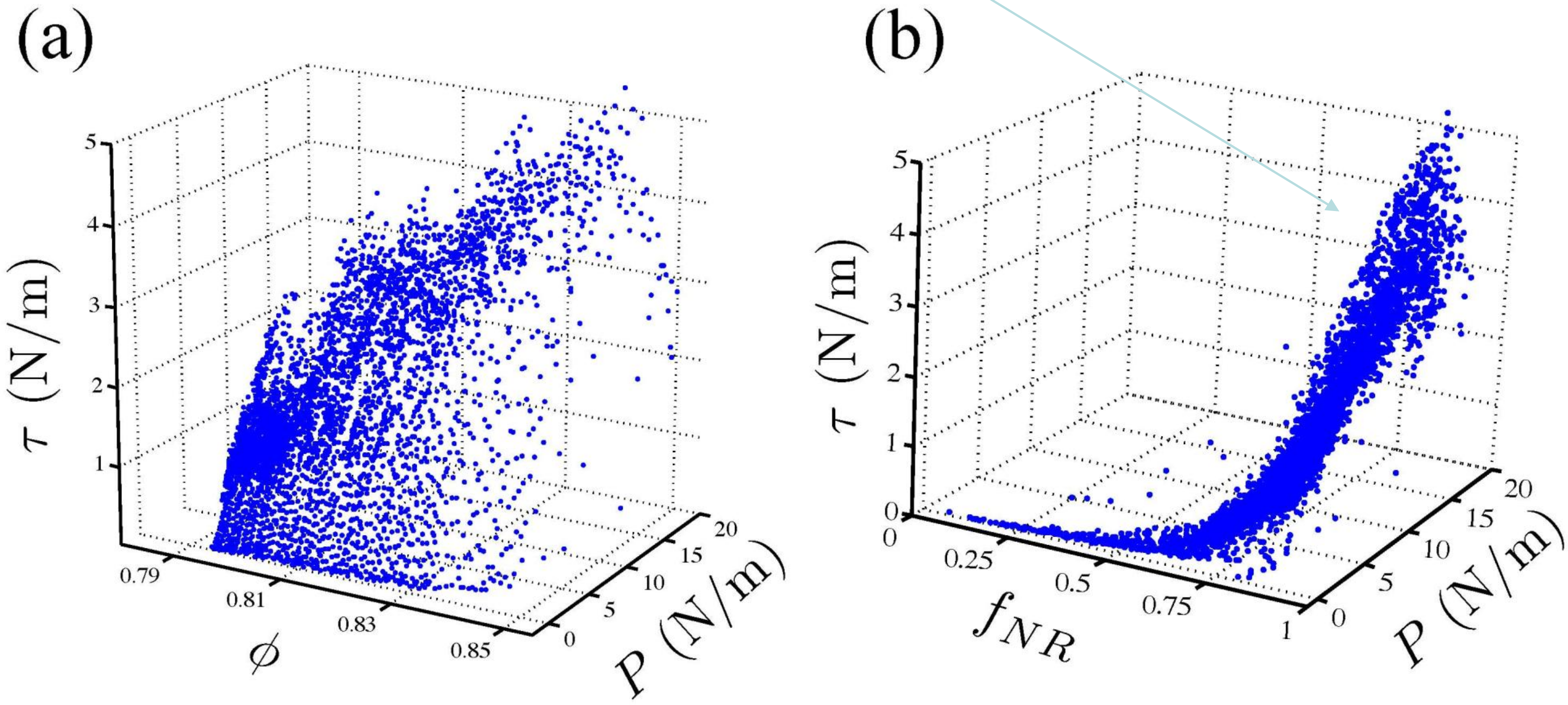


Bi et al. Nature 2011

Other features of shear jamming

Stresses vs. non-rattler fraction f_{NR}

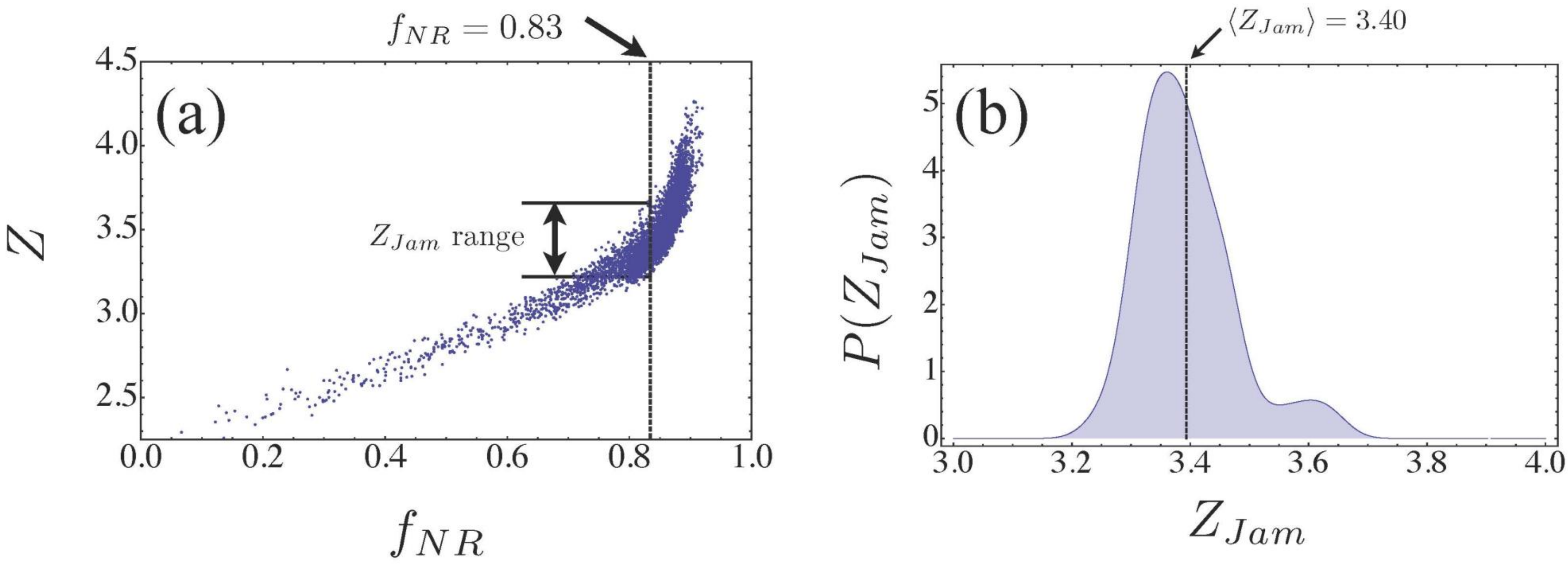
Good collapse of 'classical measures'



f_{NR} = fraction of non-rattlers—a rattler has too few contacts to be mechanically stable

Ditto for contact network properties, e.g. Z

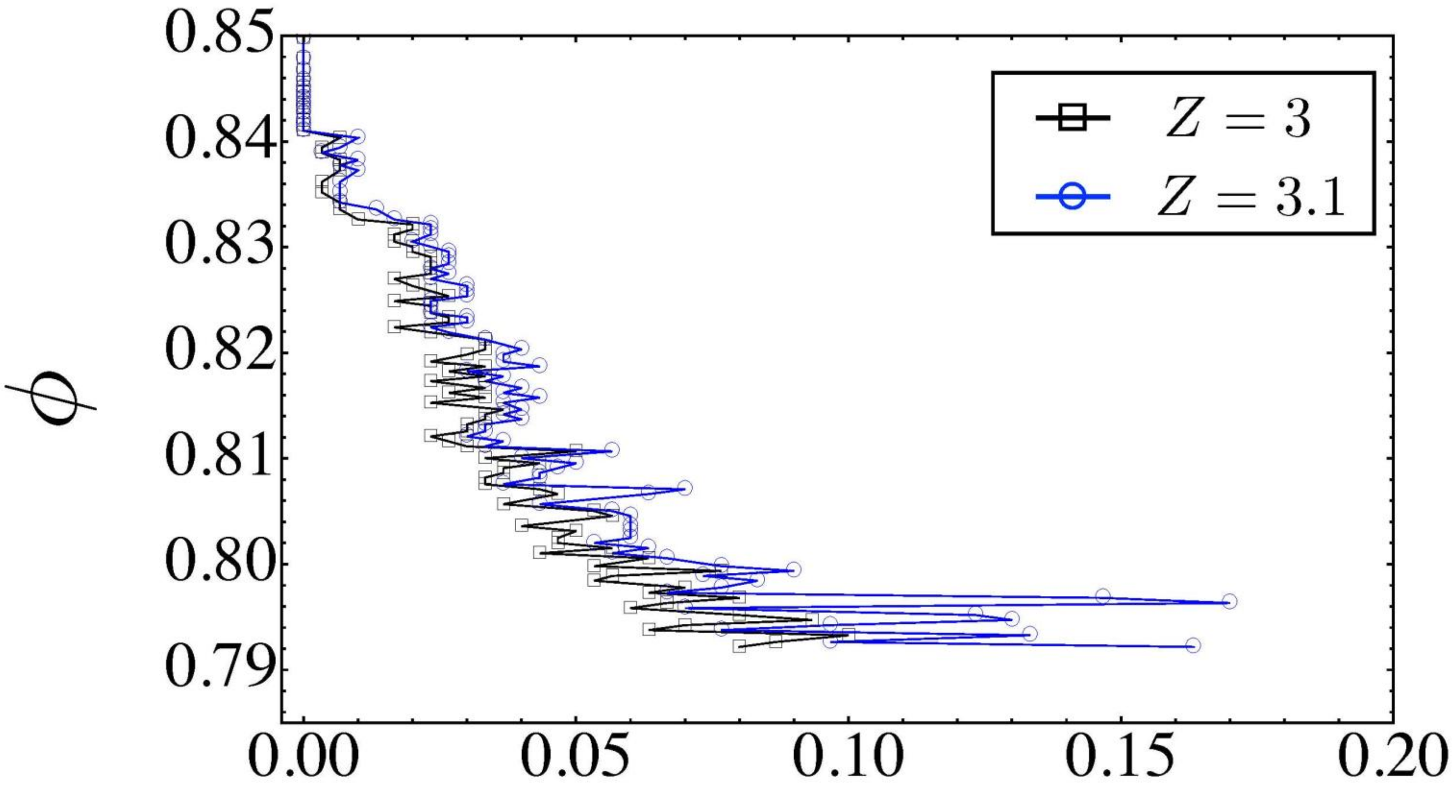
Z is average number of contacts per particle



f_{NR} = fraction of non-rattler particles
non-rattlers need at least 2 contact

Range of densities for which shear jamming can be achieved

Random dense



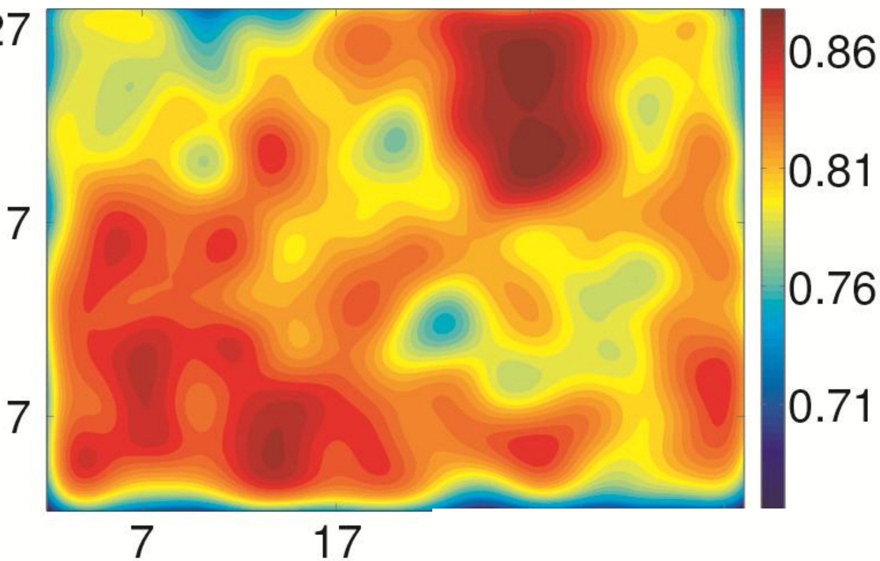
Random loose

$$\gamma_J(\phi) = min\{\gamma(Z \geq Z_{iso}^{\infty}; \phi)\}$$

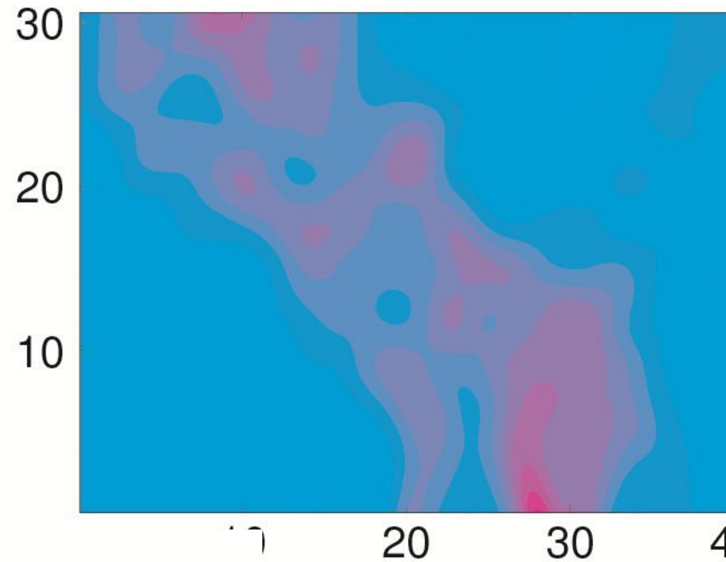
Minimum strain to shear jam

Shear band forms: result of driving soft system from wall, base friction

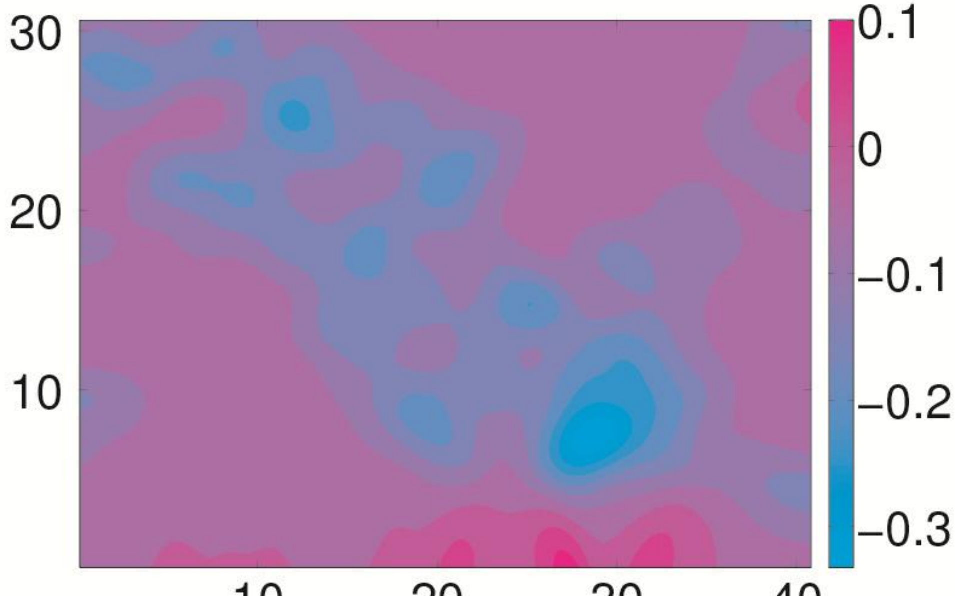
Contour plots of coarse-grained local density and strain components, at a strain of γ=9.3%}



density



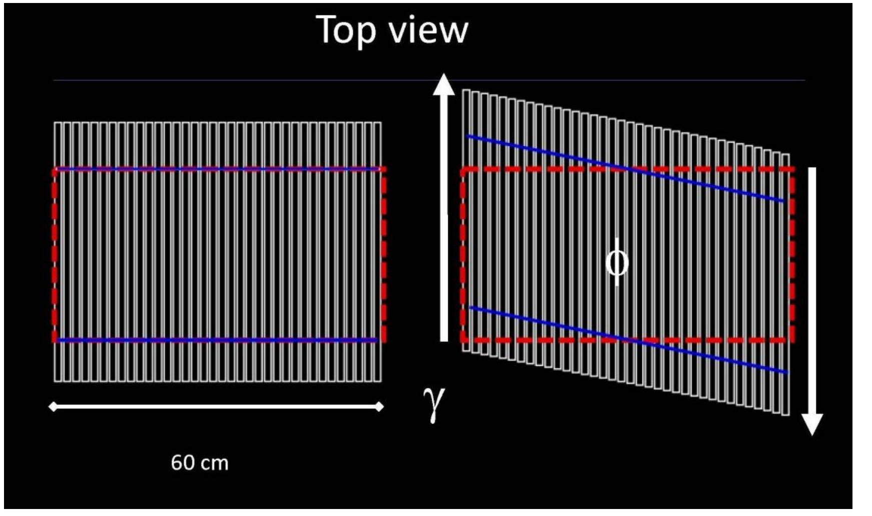
ε_{yy}



ε_{xx}

Jie Zhang,
I.Goldhirsch
BB, Supp.Prog.
Theor.Phys2010

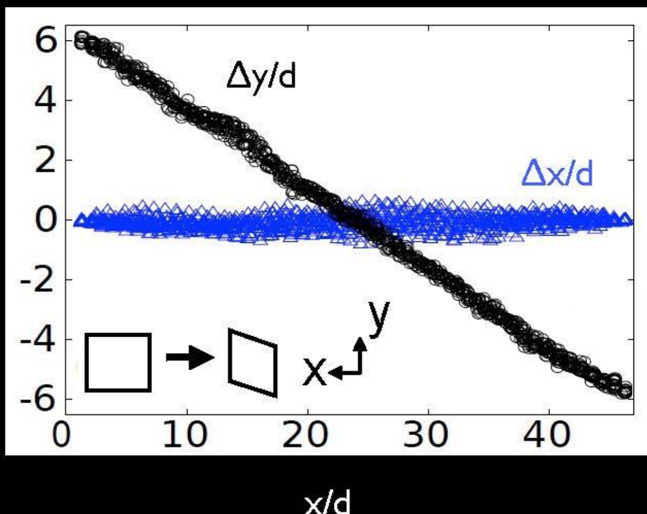
2^{nd} apparatus: uniform simple shear throughout system



Joshua Dijksman, Jie Ren, Dong Wang
BB, PRL 2013

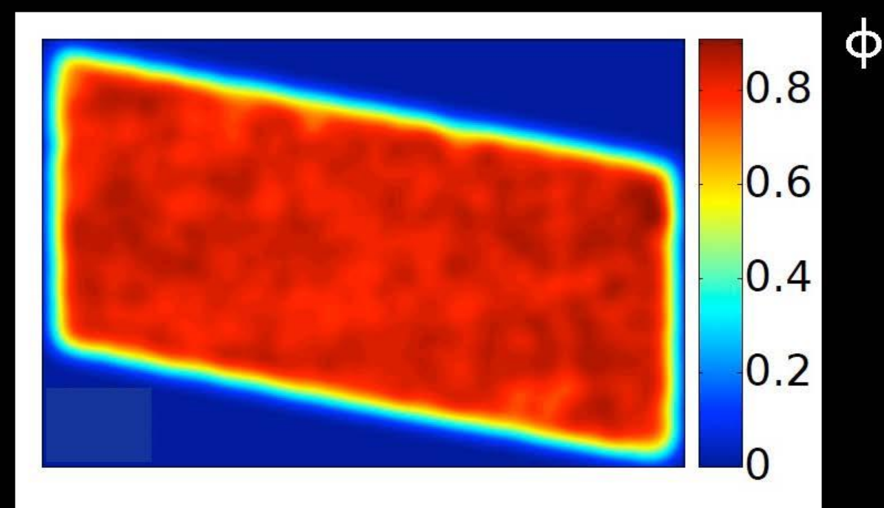
This new experimental approach supplies uniform shear—max strain ~ γ = 0.5

Particle displacements after shear

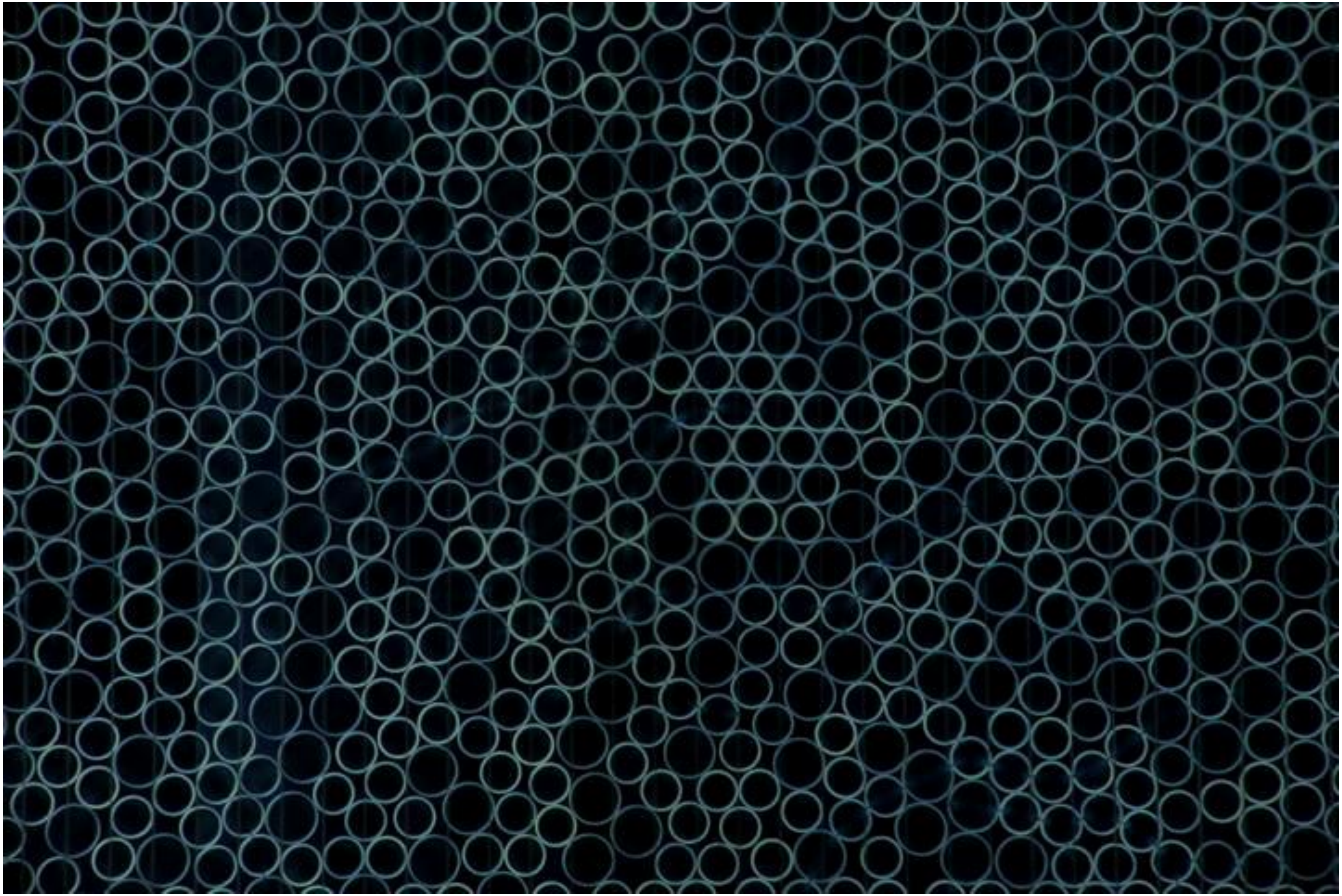


Bottom slats suppress inhomogeneities

Local packing fraction fluctuations are random



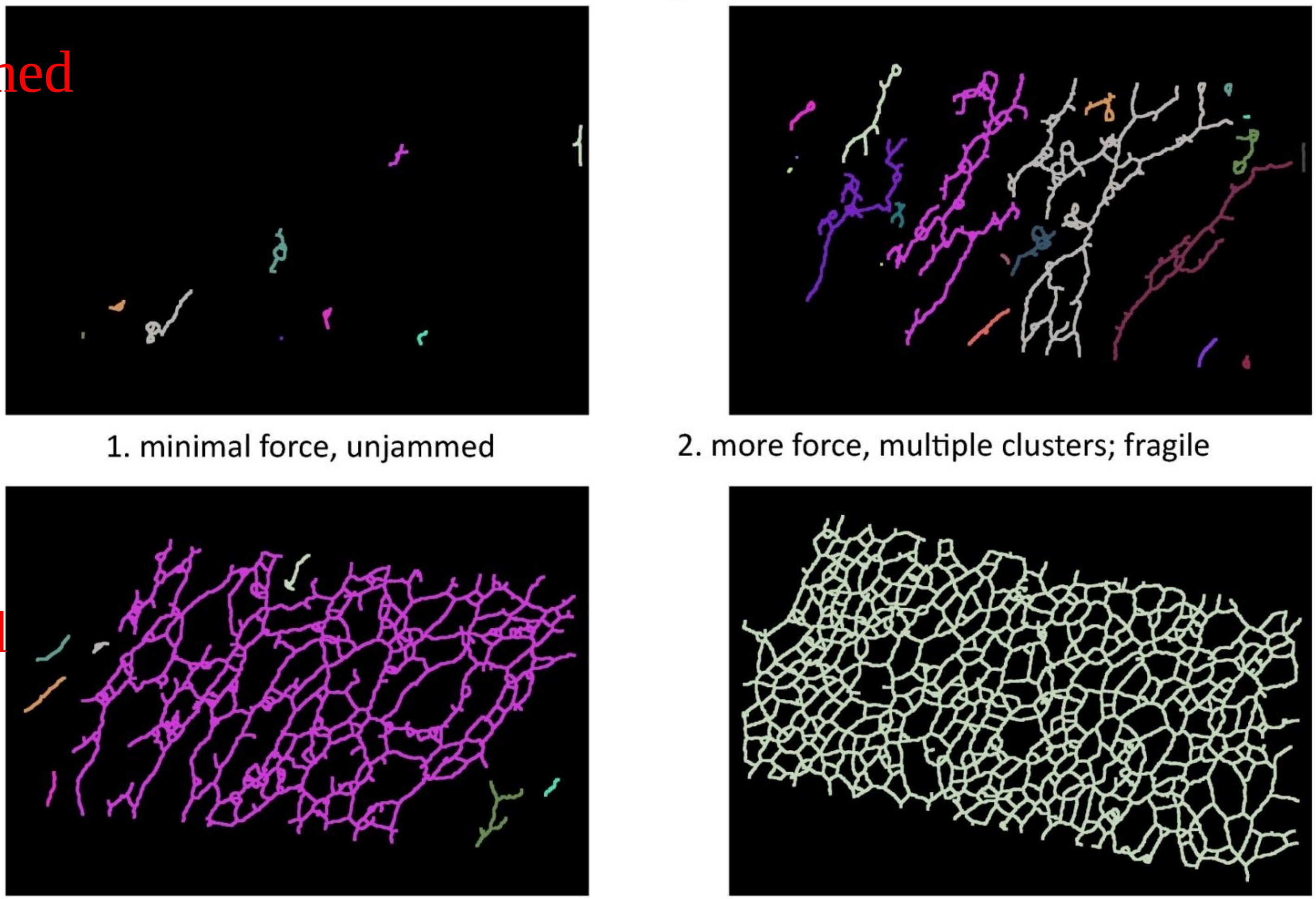
Shear-Jamming—clean experiment, constant φ—states well characterized



Networks are key to shear jamming

Increasing shear strain—first unidirectional, then all-directional percolation of strong force network (e.g. Cates et al. PRL 1998)

The force chains look differently at different stages of linear shear:



1. minimal force, unjammed

2. more force, multiple clusters; fragile

3. percolating cluster, onset of jamming

4. one large cluster, jammed

Unjammed not fragile

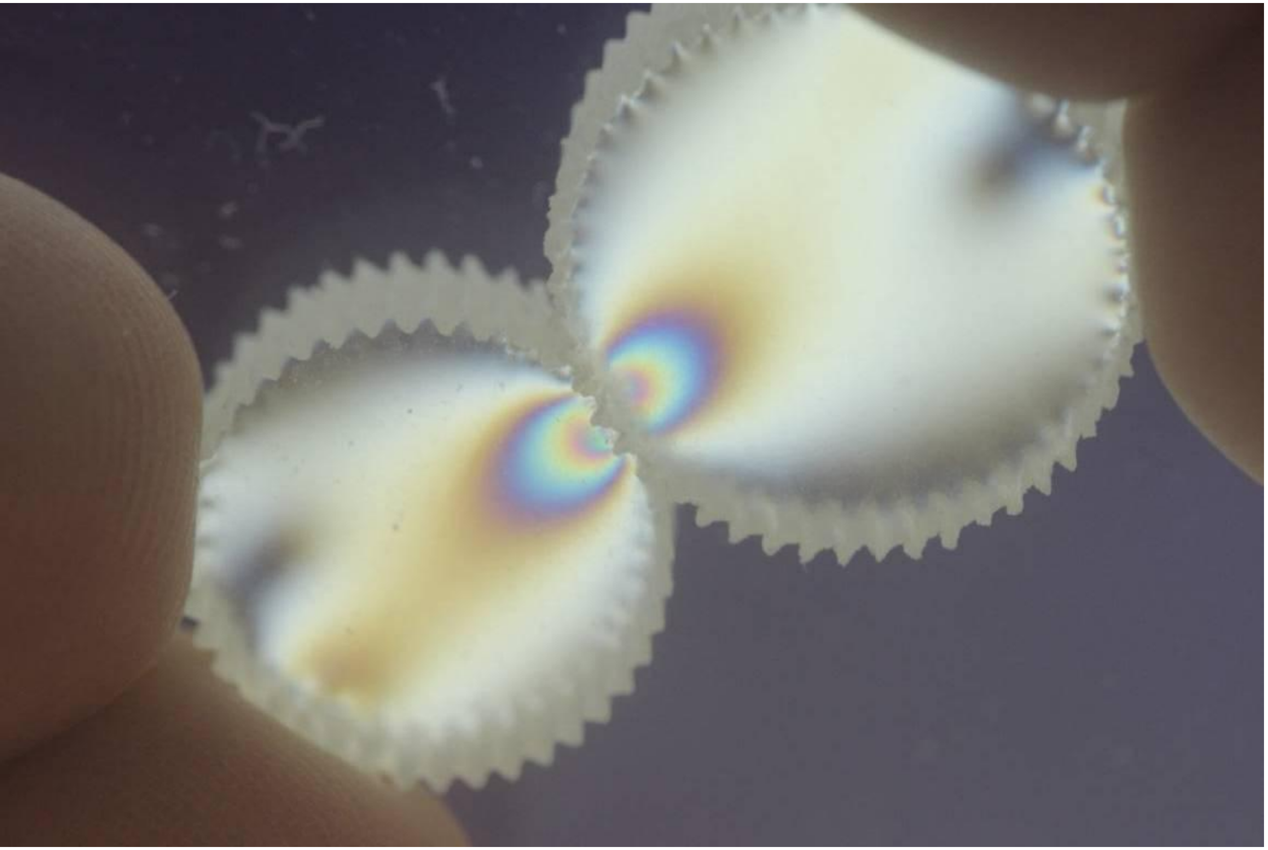
Fragile

Shear Jammed

Evolves towards more isotropic

Changing friction: higher (lower) μ gives lower (higher) φ_S

$\mu >> 1$



Make gear particles with very high μ

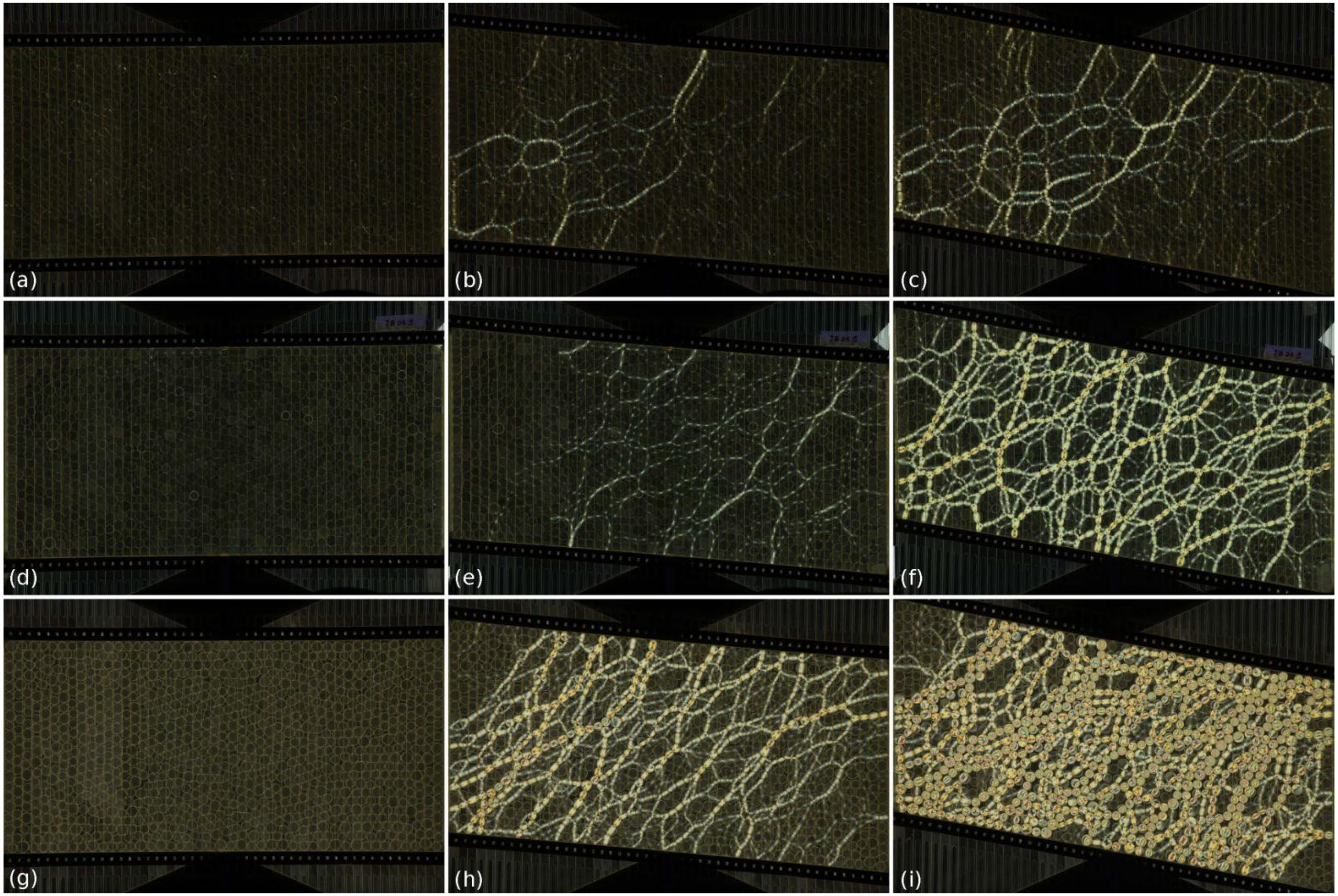
Wrap particles with Teflon for low μ

Effect of friction (Dong Wang, Jie Ren, Jonathan Barés, BB)

μ = 0.15

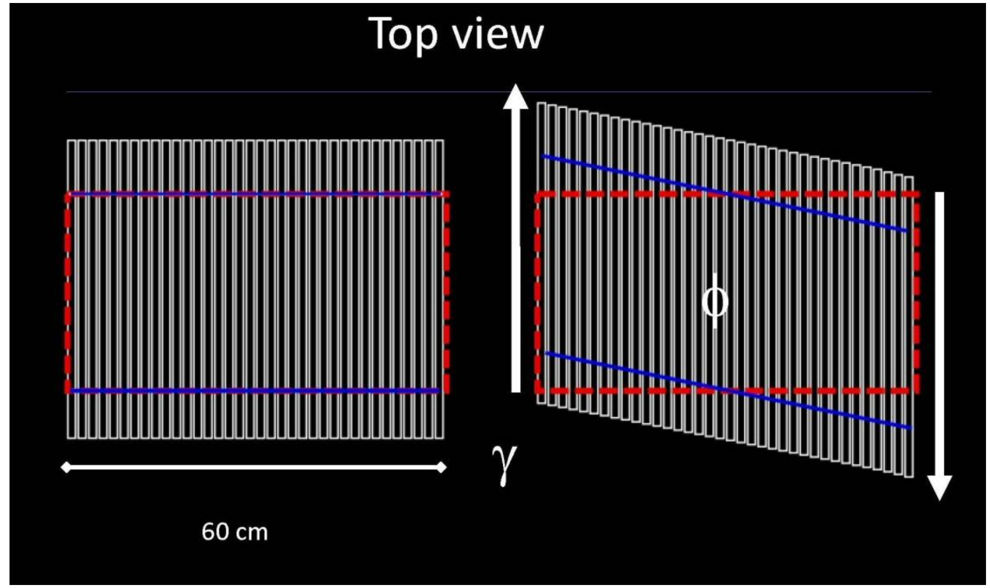
μ = 0.65

μ >> 1



Increasing strain, γ →

Does a GM have memory under cyclic shear?

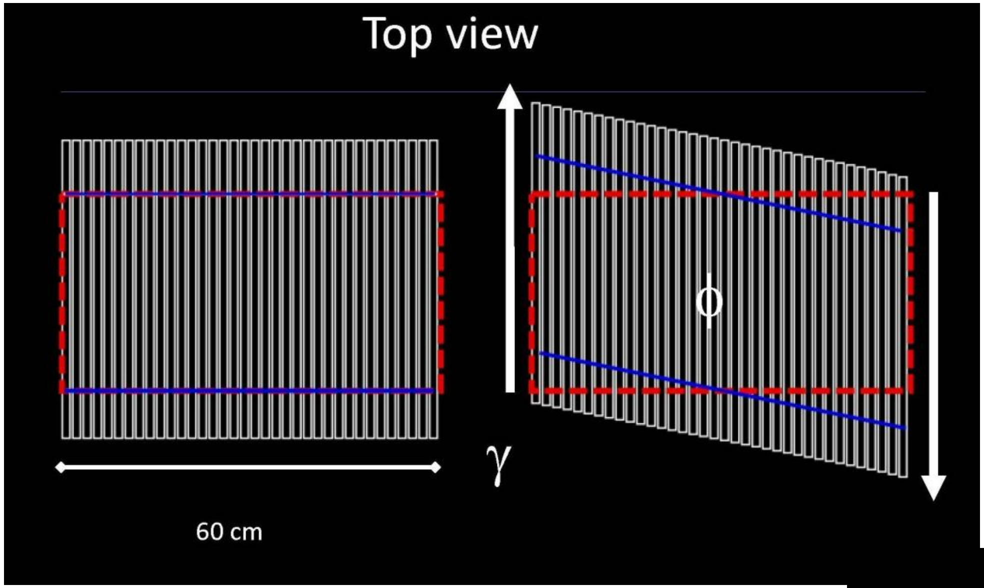
 

(Joshua Dijksman, Jie Ren, Dong Wang RPB, Phys. Rev. Lett. 2013)

Use simple shear experiment, no shear bands

Dense materials—compare to Corté et al. Nat. Phys. (2008) Fiocco et al. PRL (2014); Royer and Chaikin, PNAS (2015)

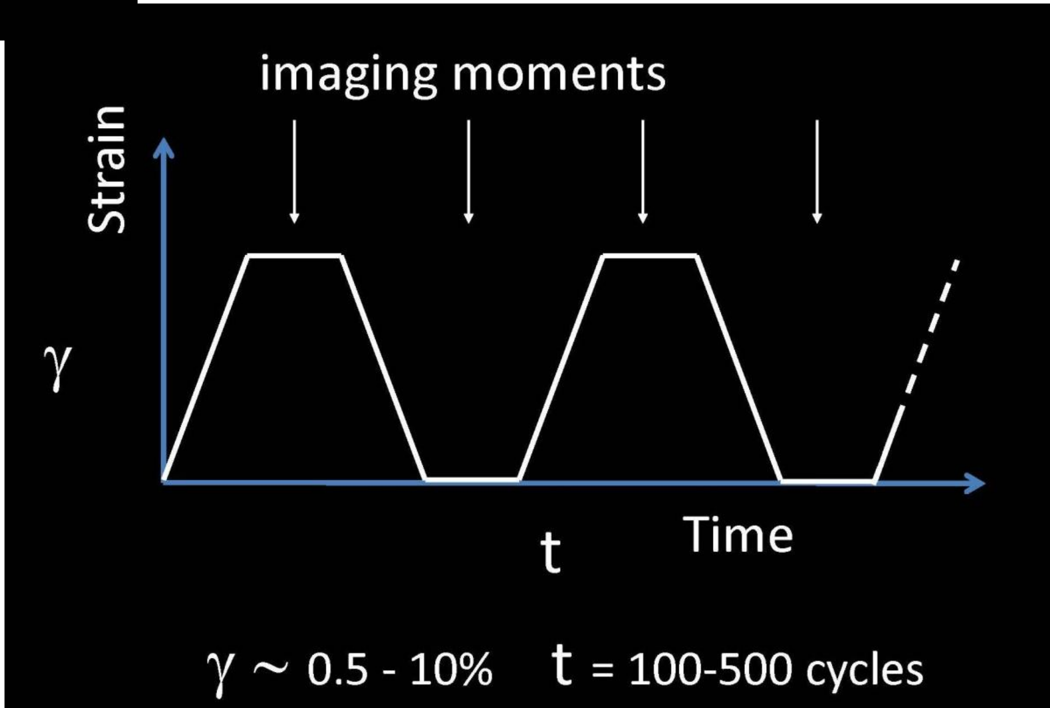
Memory forms and evolves under cyclic shear



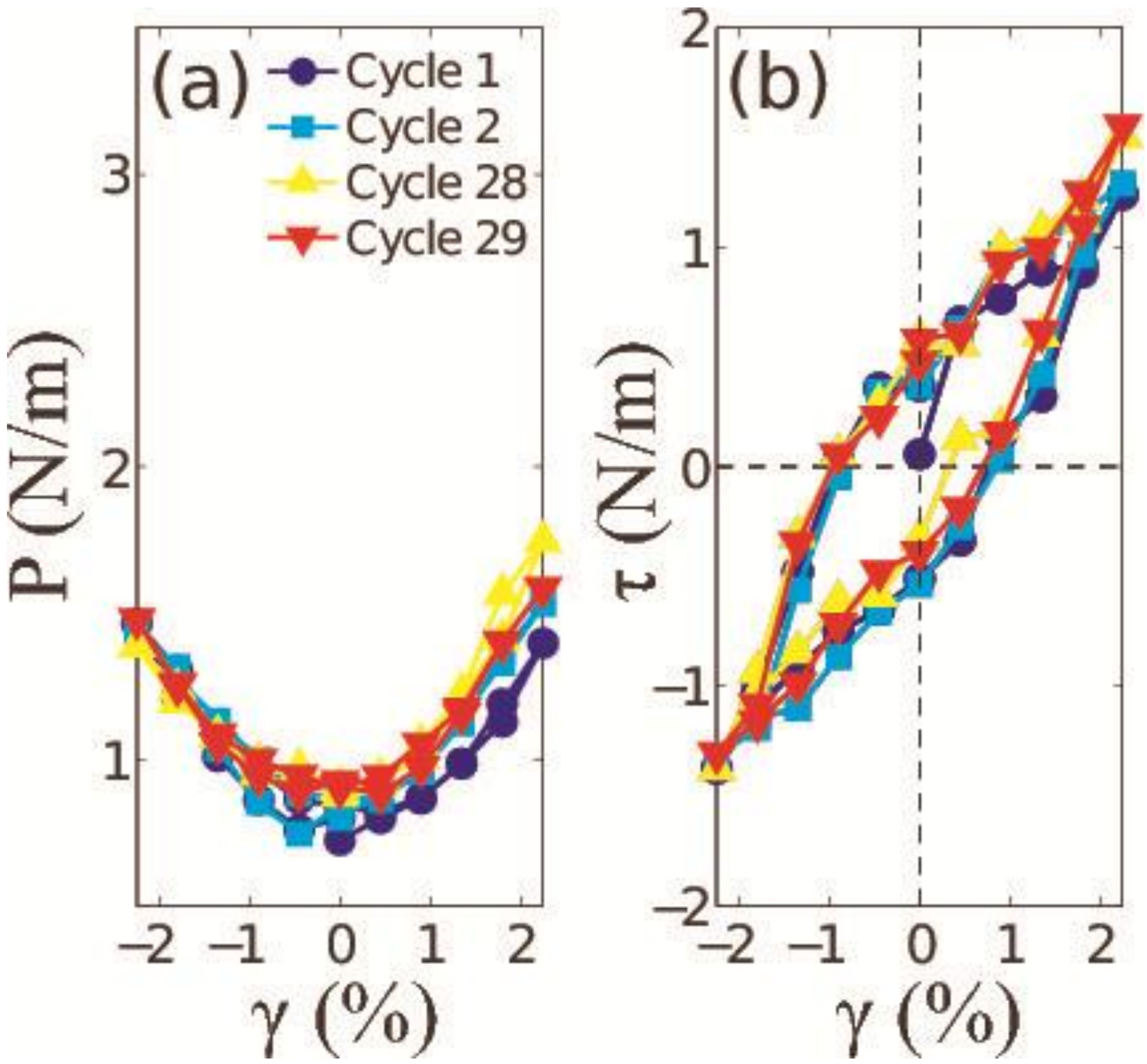
Granular analogue of dense suspension experiment

Example below is asymmetric shear

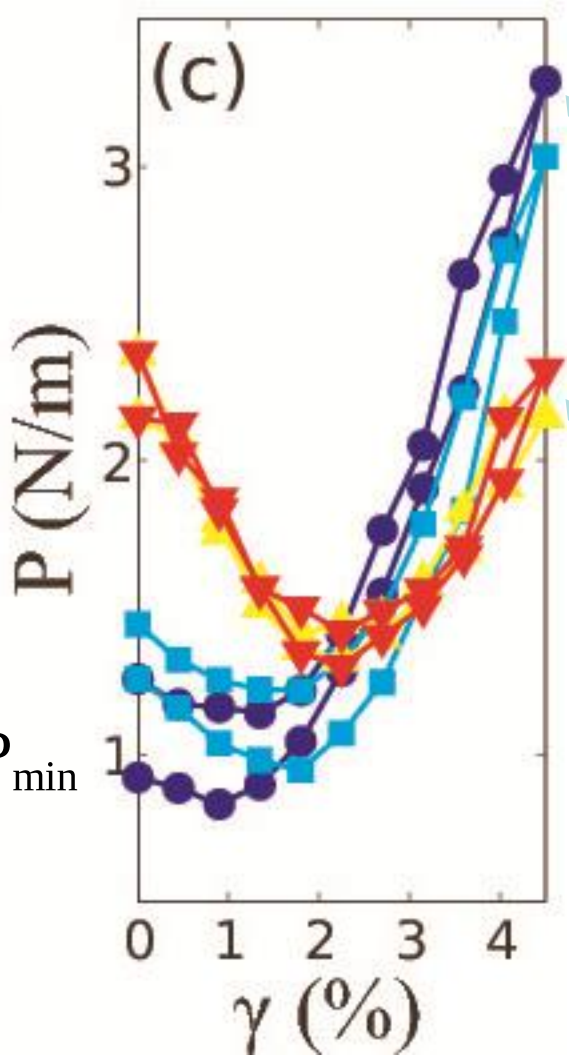
Also: symmetric cyclic shear



Apply symmetric cyclic shear—rapid relaxation to limit cycle



Apply asymmetric cyclic shear: note slow relaxation



P_{max}

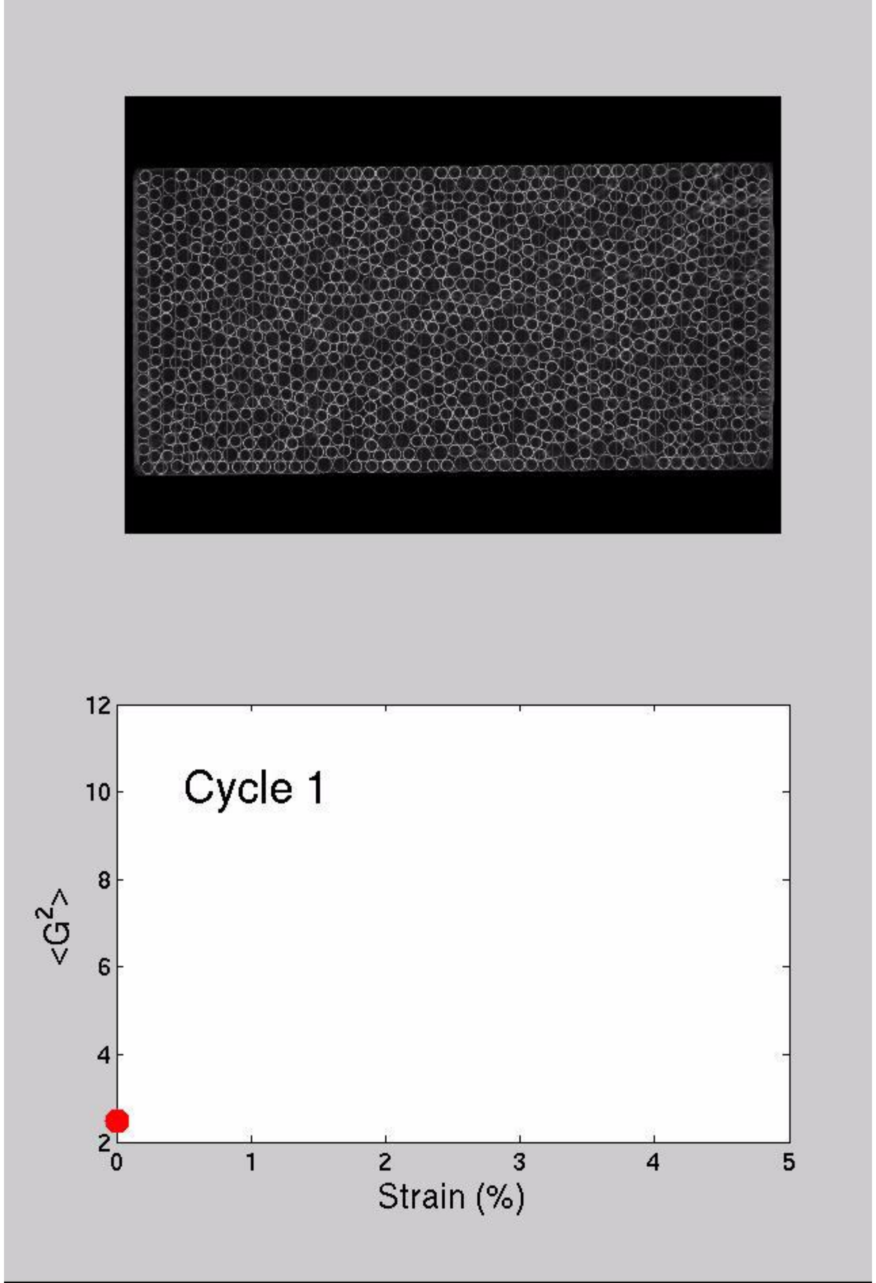
Cycle 1

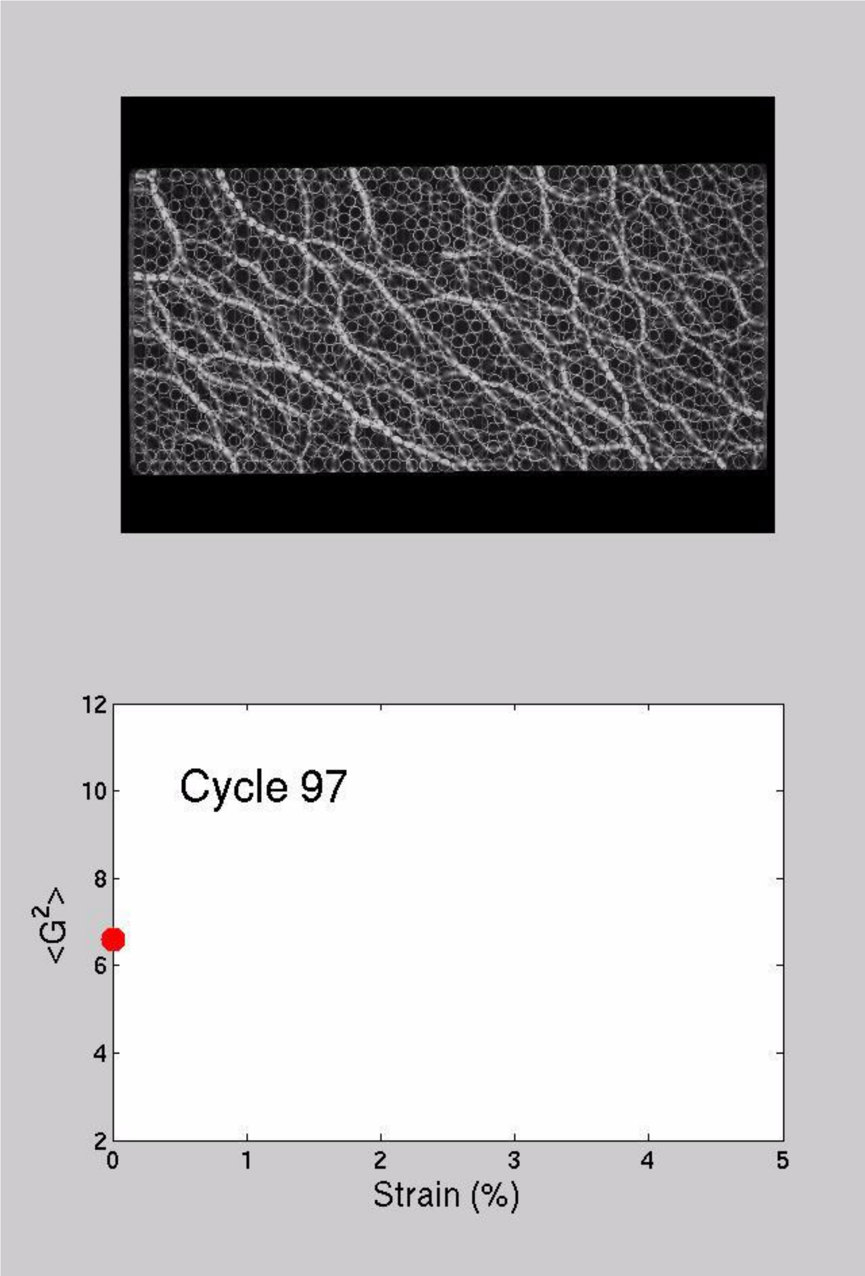
Cycle 29

P_{min}

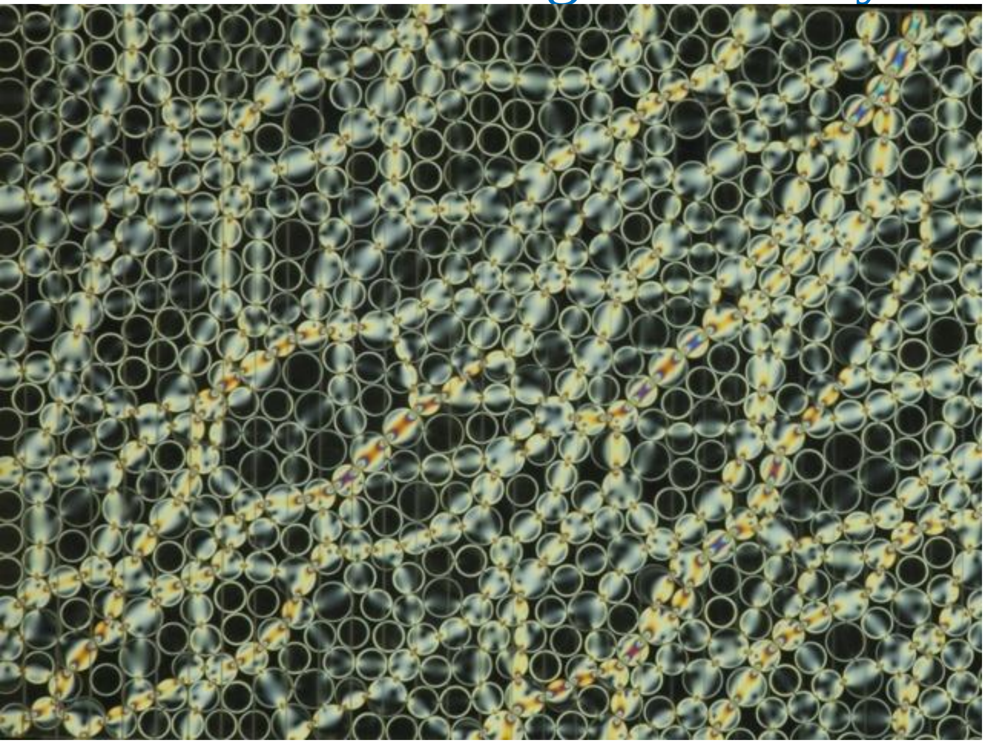
$\Delta P = P_{max} - P_{min}$

Apply asymmetric cyclic shear: note slow relaxation (time-lapse videos of quasistatic shear)

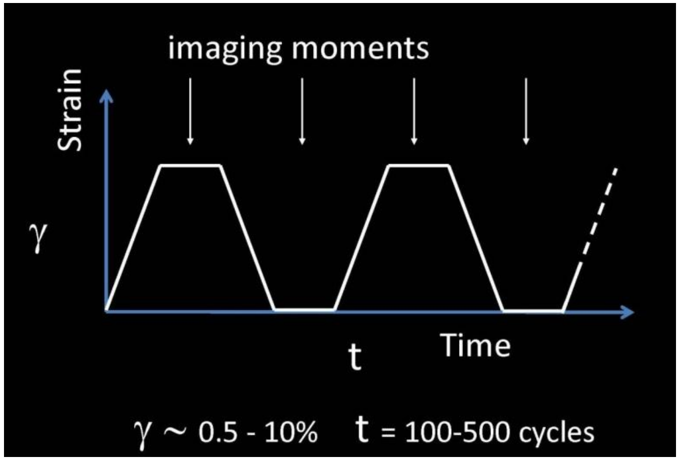




Networks are at core of evolving granular systems—e.g. Strobed images-shear cycles: stress activated process

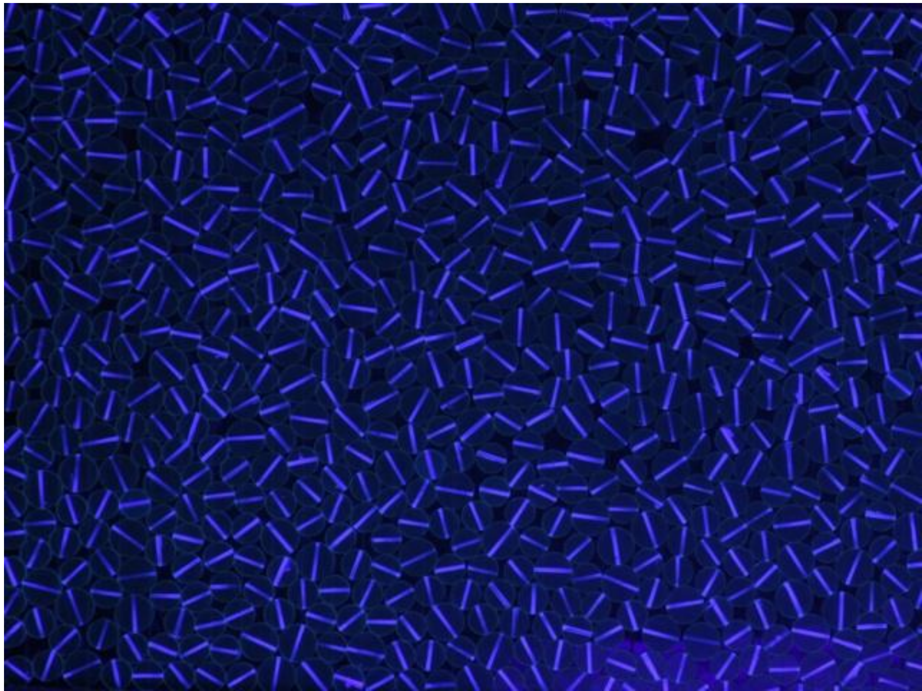


←stresses fluctuate

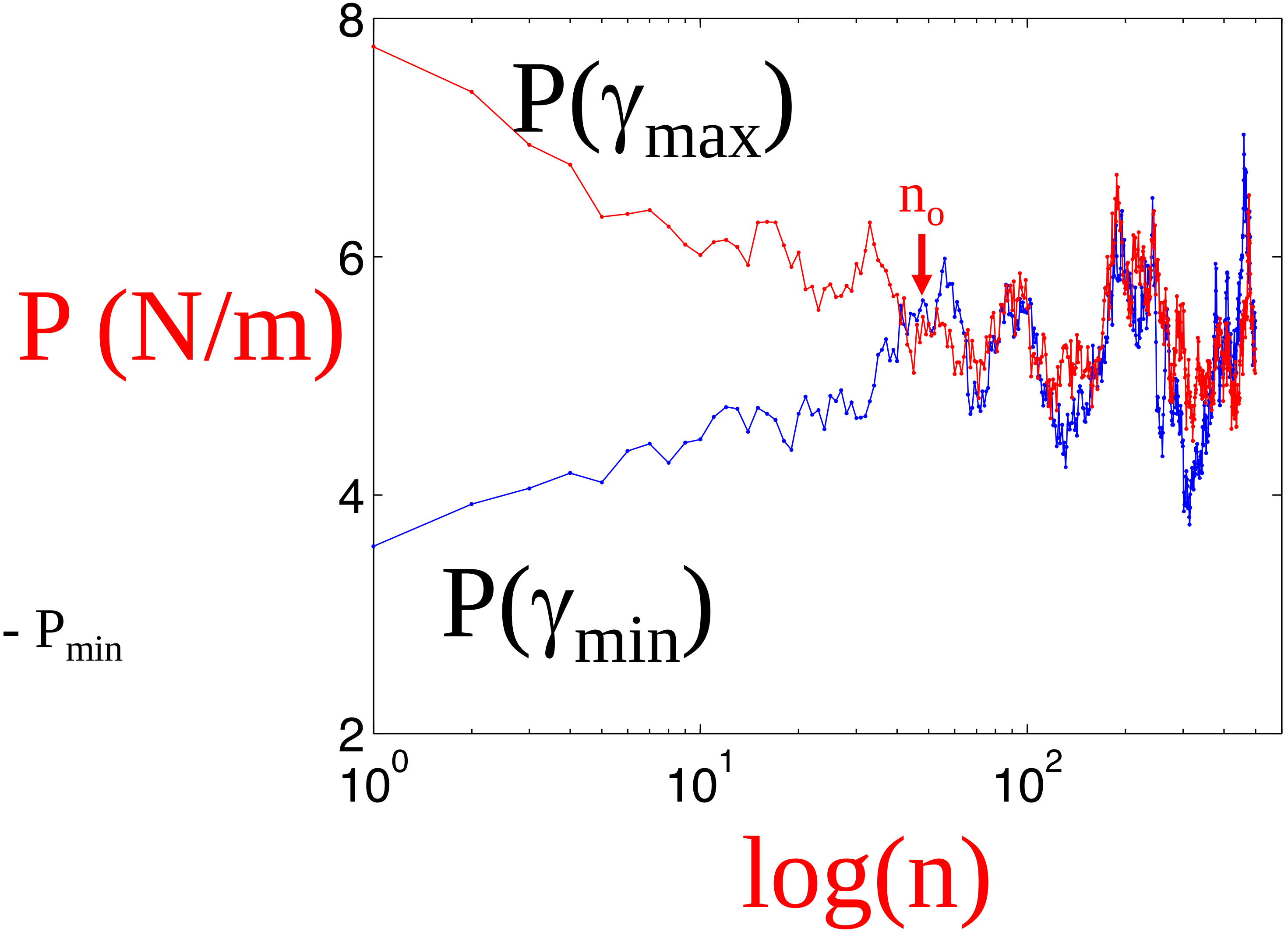


Stress, position, rotation—
All evolve over many cycles

Positions are nearly frozen →

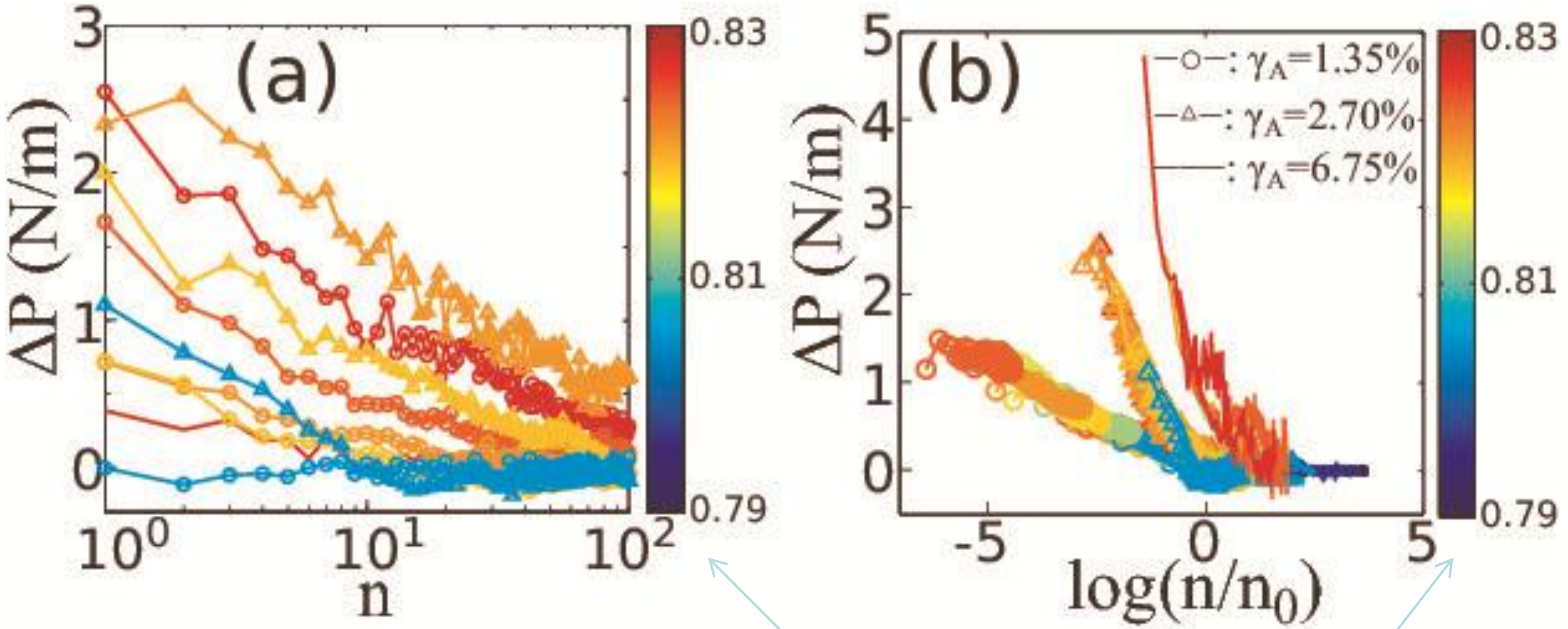


Apply asymmetric cyclic shear: note slow relaxation



$\Delta P = P_{max} - P_{min}$

Asymmetric shear:
1) log-relaxation:
2) simple φ and γ dependence

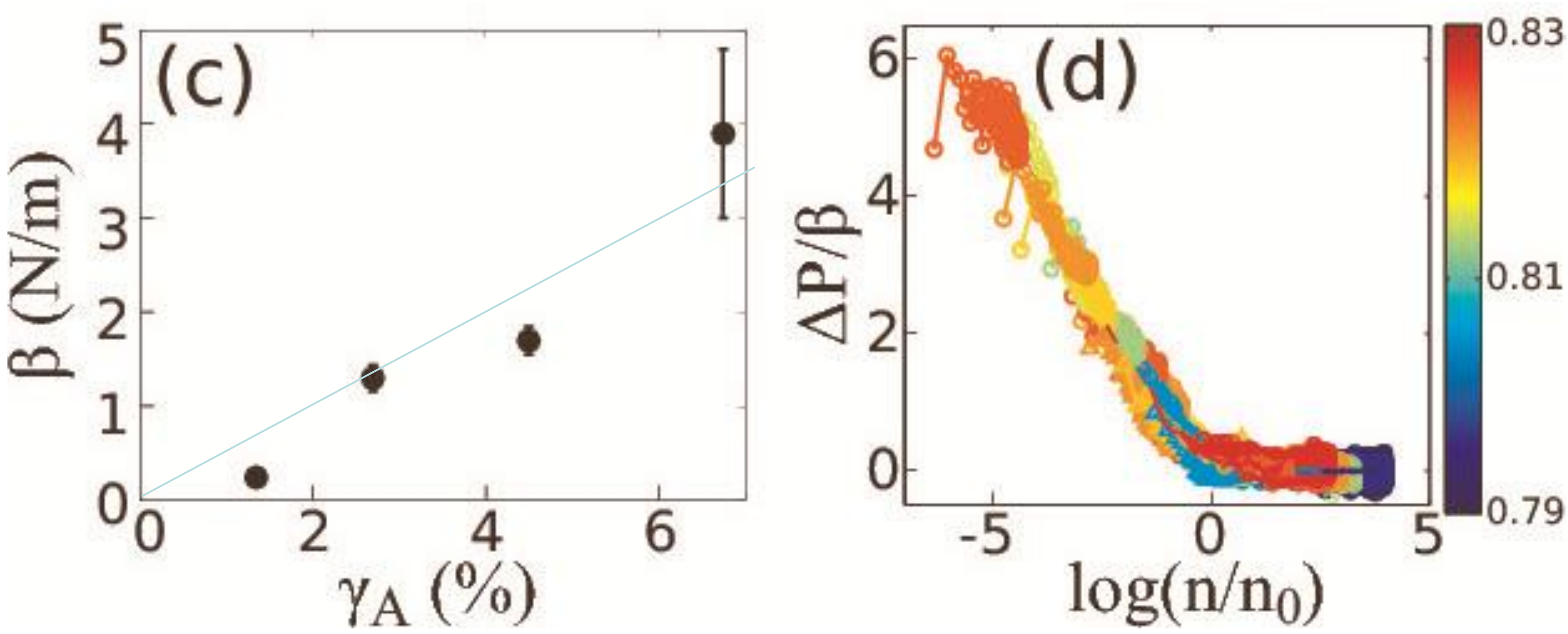


$\Delta P = -ß \ln(n/n_o)$

Colors encode density

Universal relaxation:
consistent with activated process in a stress ensemble

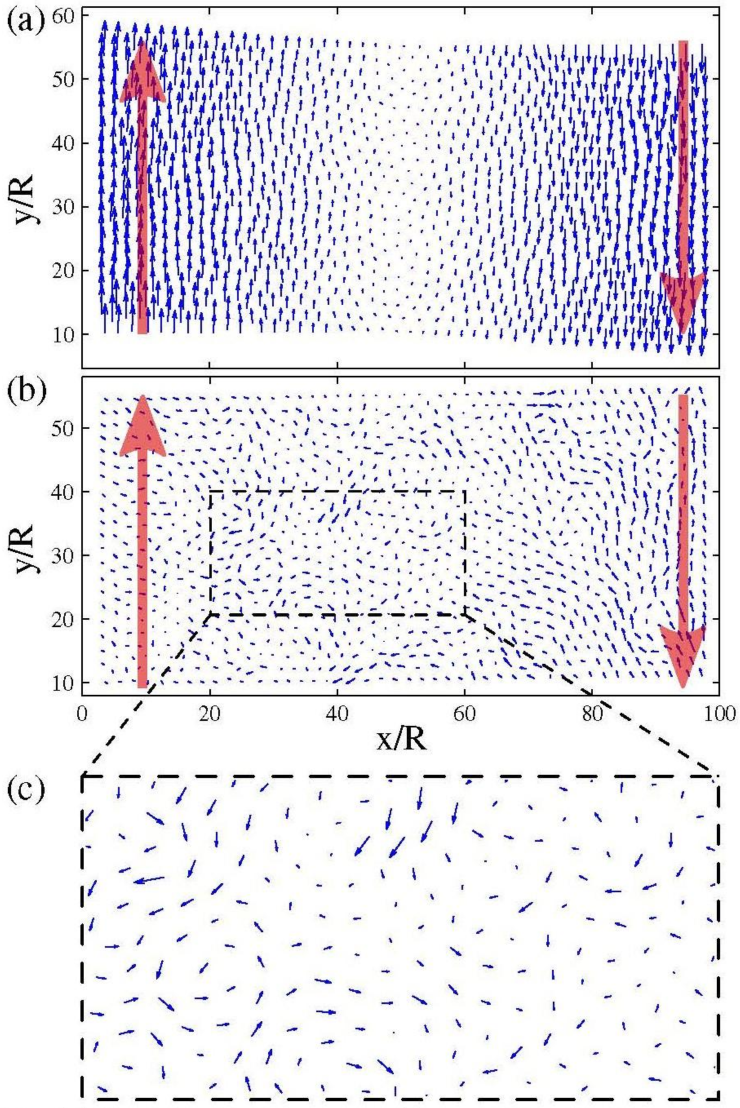
β is temperature-like—a candidate for
a granular ‘thermometer’ for shear



$\Delta P = -ß \ln(n/n_o)$ $n/n_o = \exp(-\Delta P/\beta)$

What are the microscopic processes that enable shear jamming and memory in granular materials?

Actual particle motions are very small, beyond affine strain STZ evolution is not at play here—relaxes stress hence not conducive to jamming



Affine motion of each grain, after a strain of 0.2, to scale

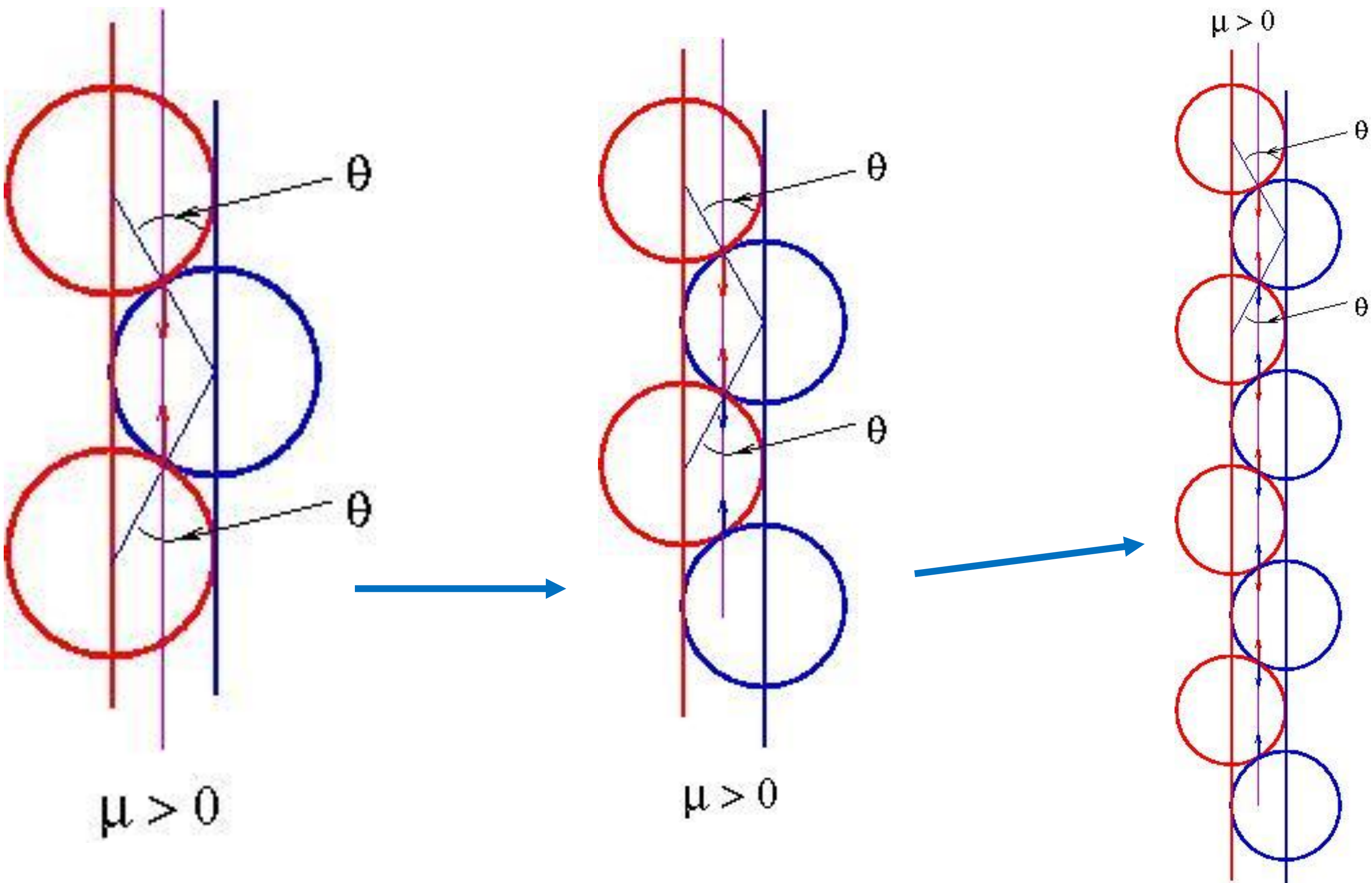
Arrows show nonaffine motion of each grain, multiplied by 3.5, after a strain of 0.2

Things that don’t happen: 1) ‘straight’ force chains

Stable if tan(θ) < μ

Continuing force chain- unique placement

No way for this

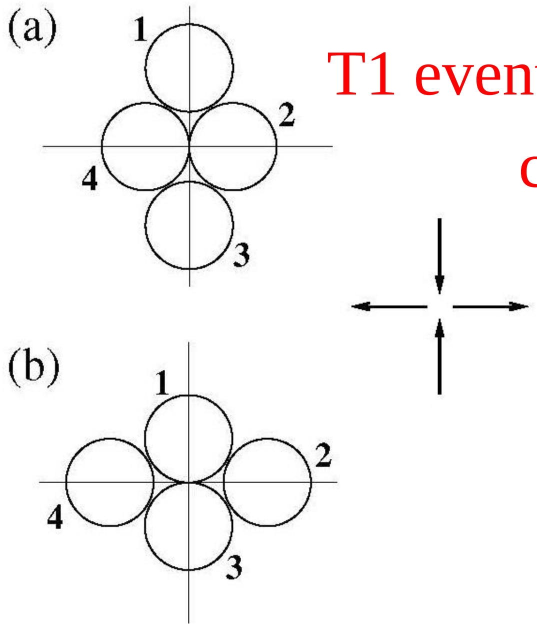


Force chains are not like this: Lines of particles possible but highly improbable

Things that don't happen 2:

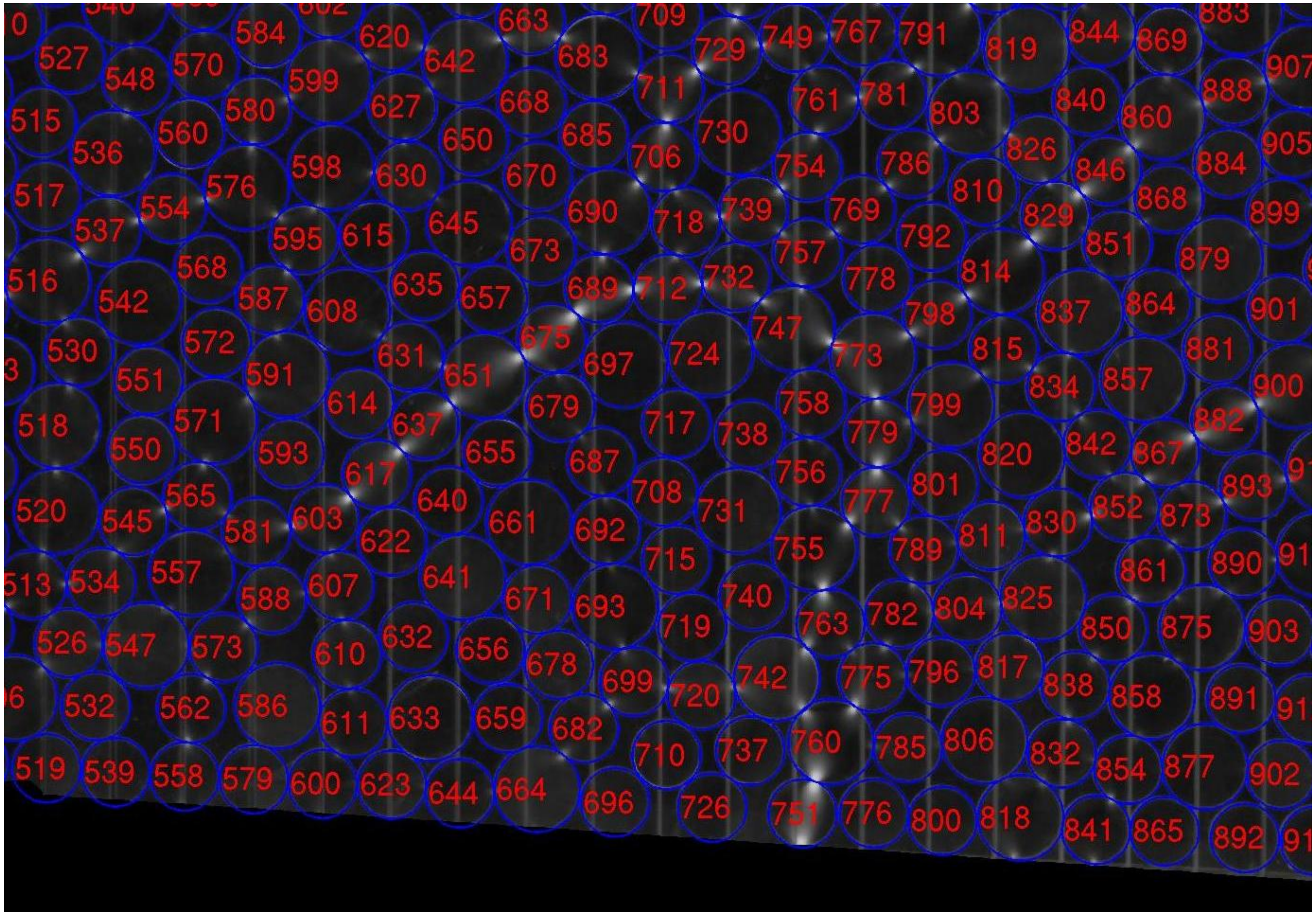
Z_{iso} = d + 1 for frictional grains—Force chains cannot stand alone

- Things that don't happen: 2) T1 events

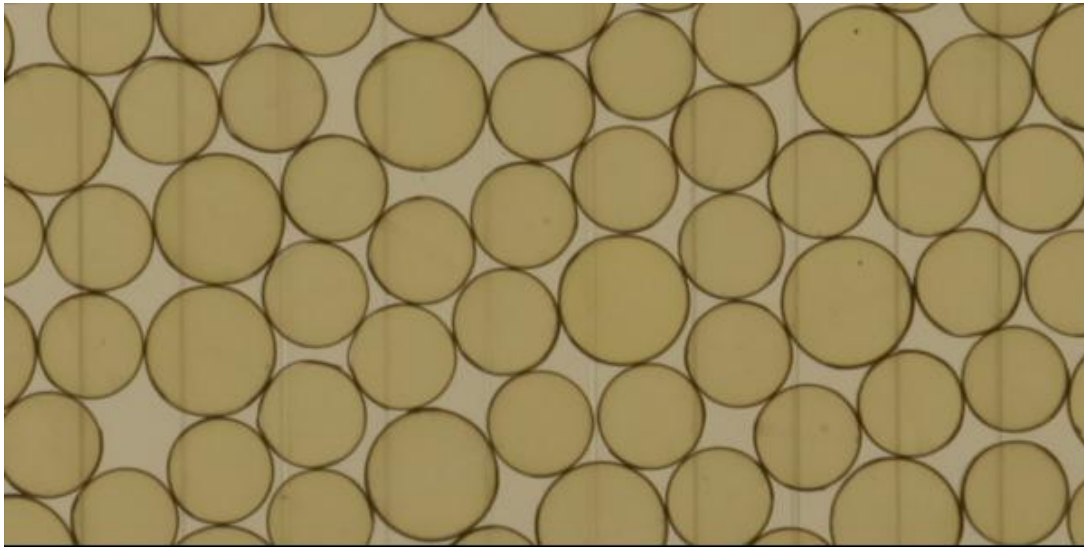


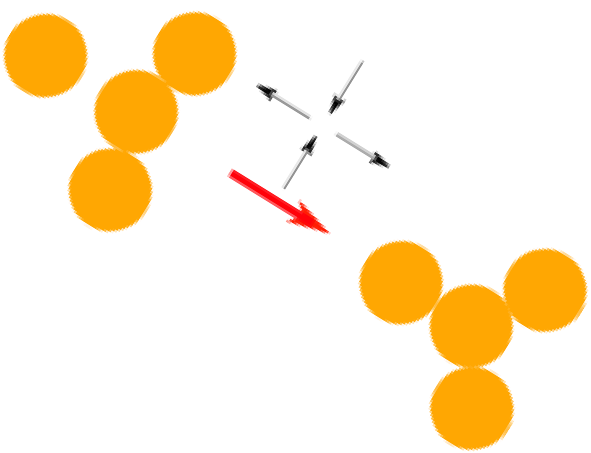
T1 events: relax stresses, geometry differs from chains

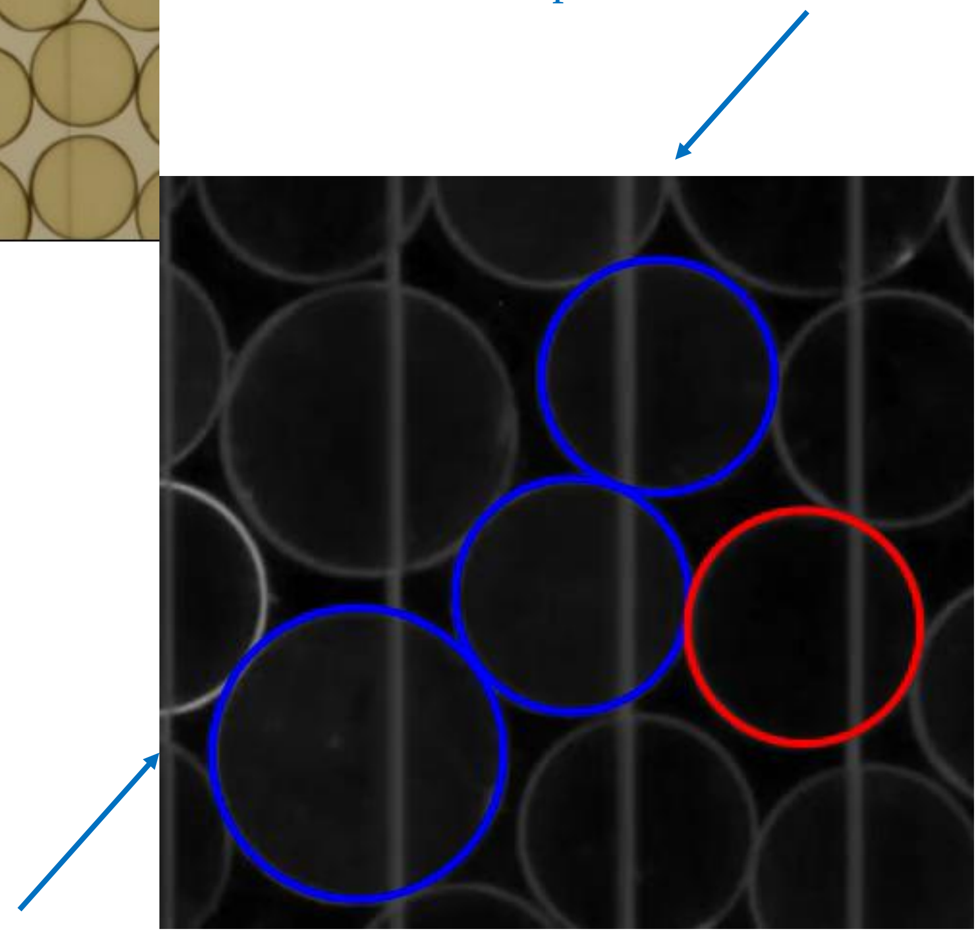
Chains bend, wiggle and intersect at branches



What does happen as chains form?
Zoom in on some local processes

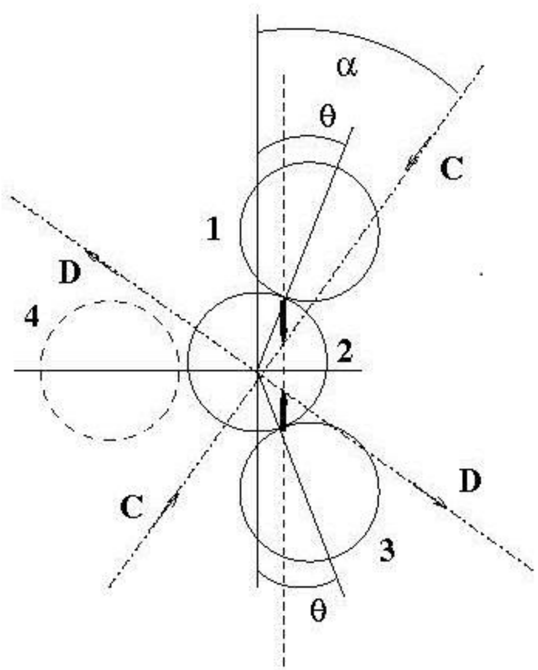






Consider small-scale configurational changes

-Trimers (captures bending) and branches-(chain mergers)



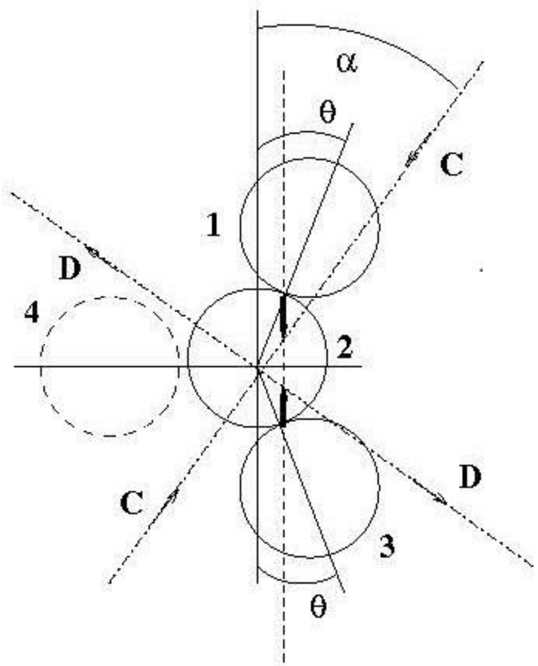
A trimer consists of three (nearly) contacting particles, e.g. 1, 2, 3.

θ measures trimer bending

α measures orientation wrt compression direction

Trimers (captures bending) and branches-(chain mergers)

α moderate: compression pushes 1, 2, and 3 together



continued compression bends trimer

pushes 2 to left faster than affine dilation for most θ

Creates new contacts, e.g. 2 and 4

Branches occur naturally due to initial packing, instability of long force chains, and bending

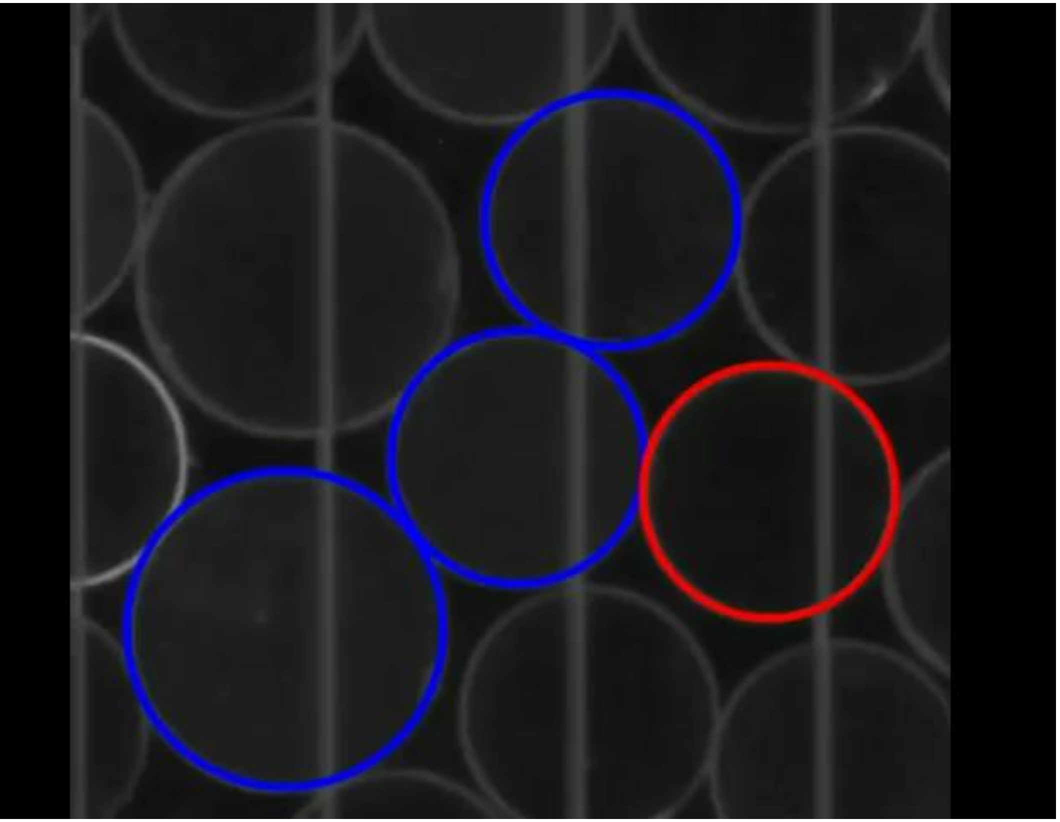
What do trimers do under shear?
trimer in compression direction

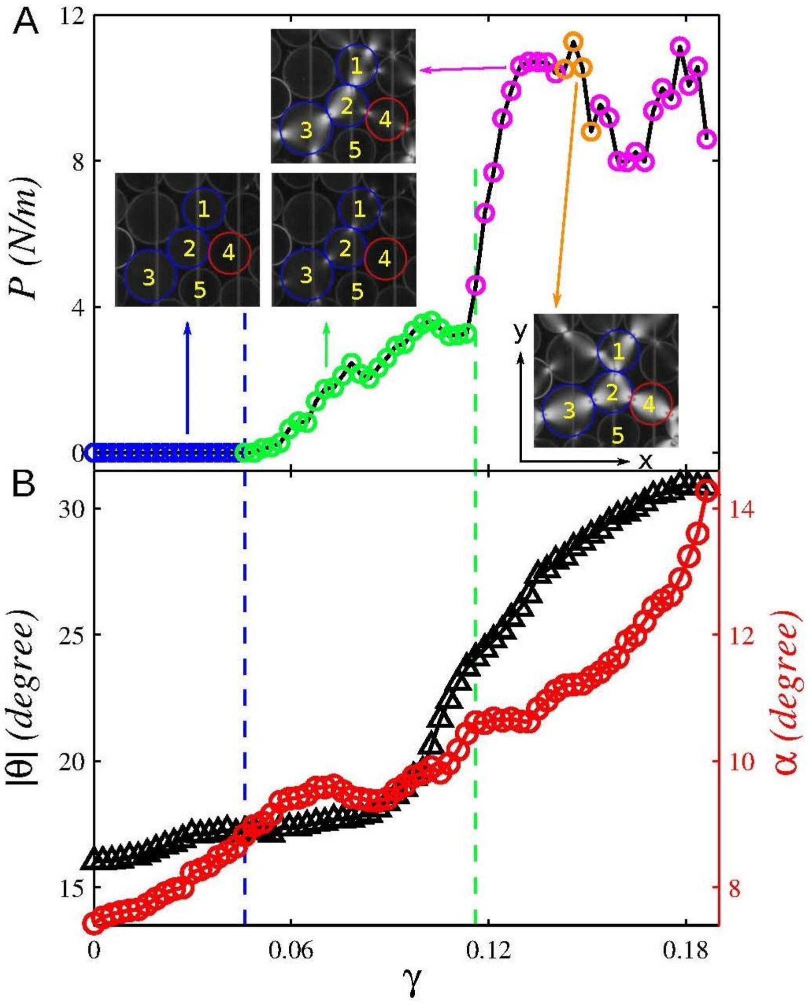
Trimer bends

Extra contact forms for middle particle

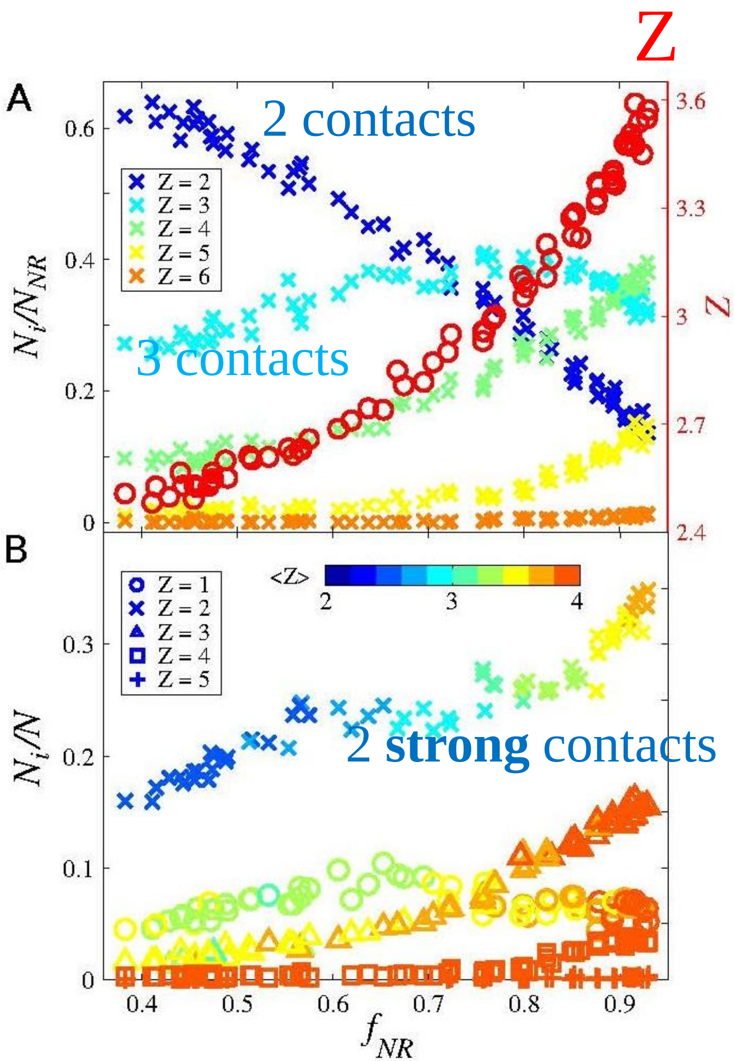
Force goes up

local processes—now slower





Contact numbers: Conversion from 2-contacts to 3 or more contacts



A: Red circles: system-wide contact number, Z.
Need Z > 3 to be jammed—all vs. non-rattler fraction

A: crosses: relative populations of particles with i contacts

B: Contact fractions for the strong network—
Symbols: number of strong Force contacts. Colors: number of total contacts

Define O to include geometric properties of a trimer

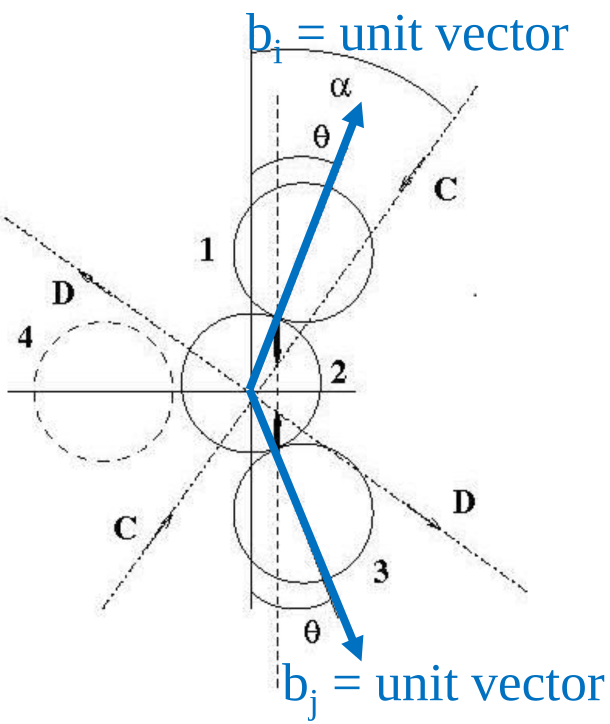
$$O = -[(\hat{b}_i \cdot \hat{b}_j - c_{ij})/[A(1 + c_{ij})] \cdot cos(2\alpha)]$$

*Normalization

$$-[(\hat{b}_i \cdot \hat{b}_j - c_{ij})/(1 + c_{ij})] \cdot cos(2\alpha),$$

Trimer ‘straightness’, 0 to 1

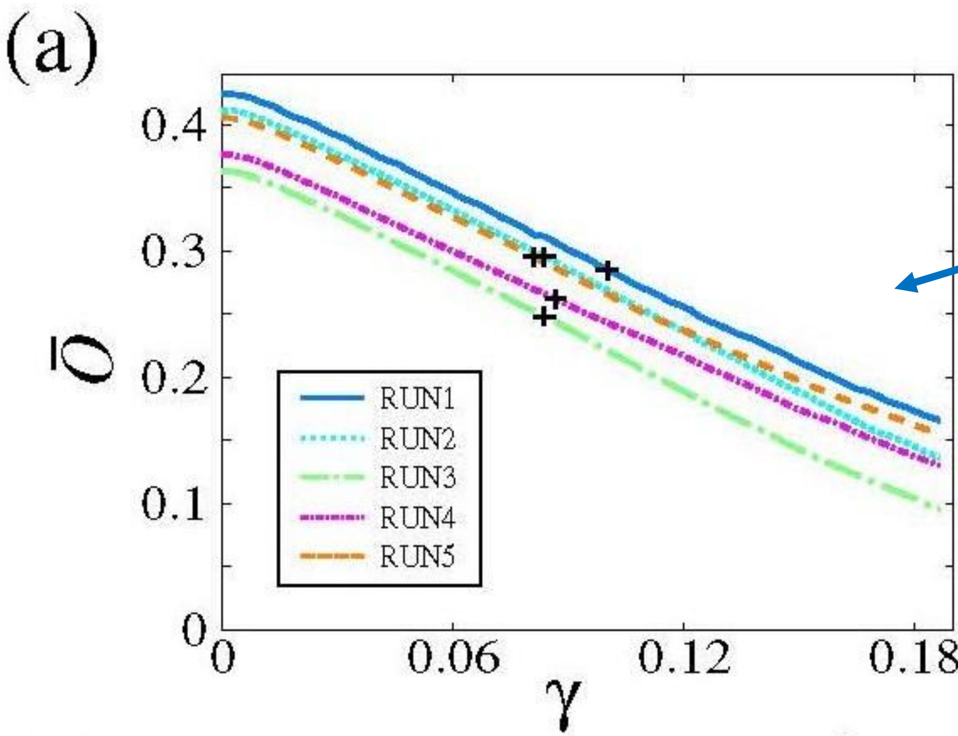
Trimer alignment with compression direction, -1 to 1



O for particles in strong network decreases with shear as trimers bend and rotate

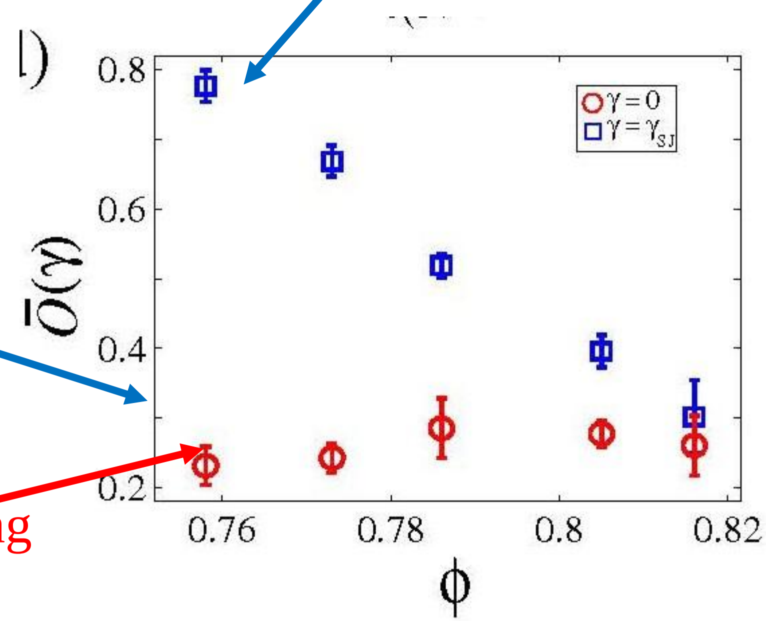
*Normalization: <O> = 1 for uniform distribution in allowed θ, and only in compression direction

<O> for trimers with O > 0



Average O vs. γ for φ = 0.805
For trimers in the network at jamming

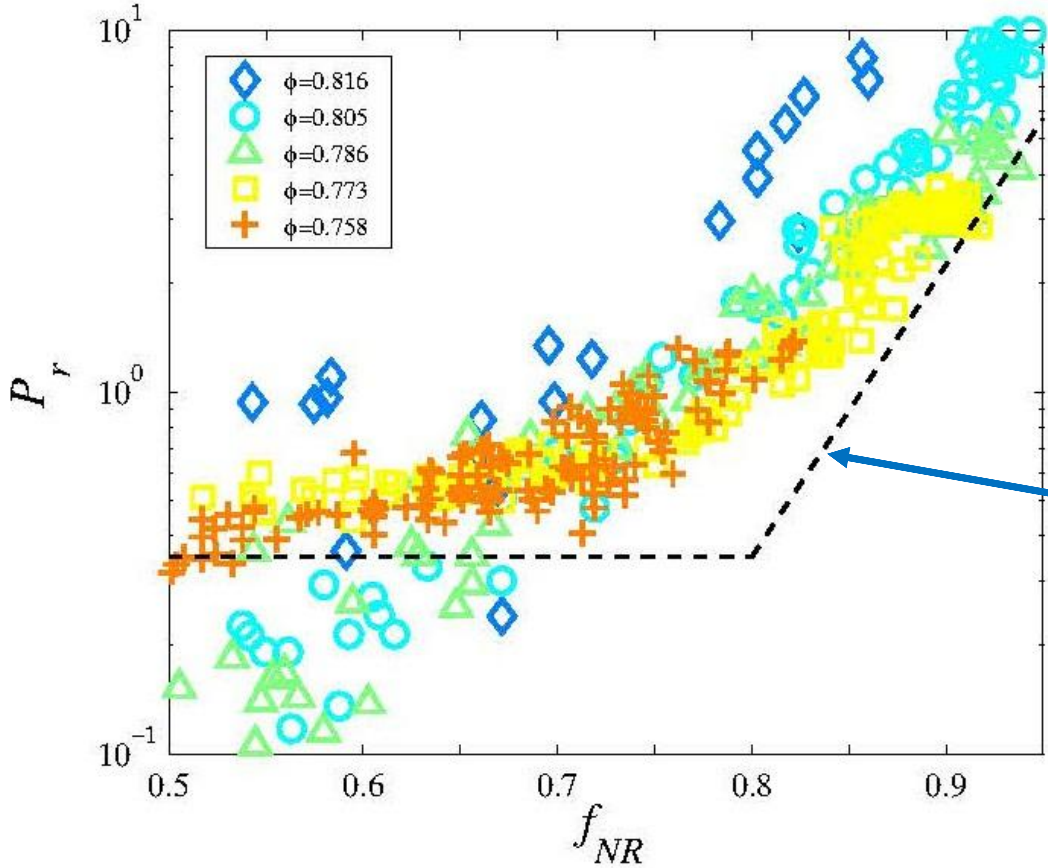
<O> before shearing



<O>, Trimers in network at shear jamming

<O> at shear jamming

Pressure from particles in trimers with O ≥ 0



Dashed line:
P from particles
in trimers
below O = 0

Shear jamming, force networks, and structural evolution

- Shear jamming **creates** strong networks at fixed volume
- Networks reflect initial conditions and protocol
- In the absence of shear banding, cyclic shearing writes and rewrites memory into force network
- **Small conformational changes enable these processes**
- Trimers, branches and contact evolution capture changes
- Provides first steps towards systematic understand of network evolution in frictional granular materials