Training, Memory and Universal Scaling in Frictional Granular Matter

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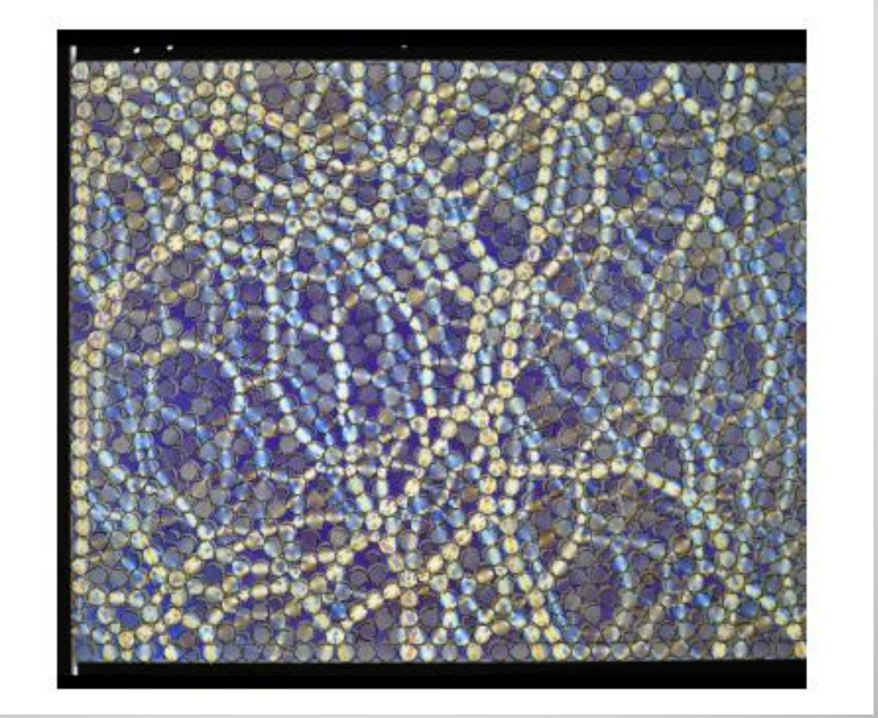
Motivation

Frictional Dynamics play an important role in seismic faulting

- The forces supported by the particles are not homogeneous in space but are concentrated into ‘‘stress chains’’ that are quite important in sustaining frictional forces



Technique pioneered and highly developed by R. Bheringer

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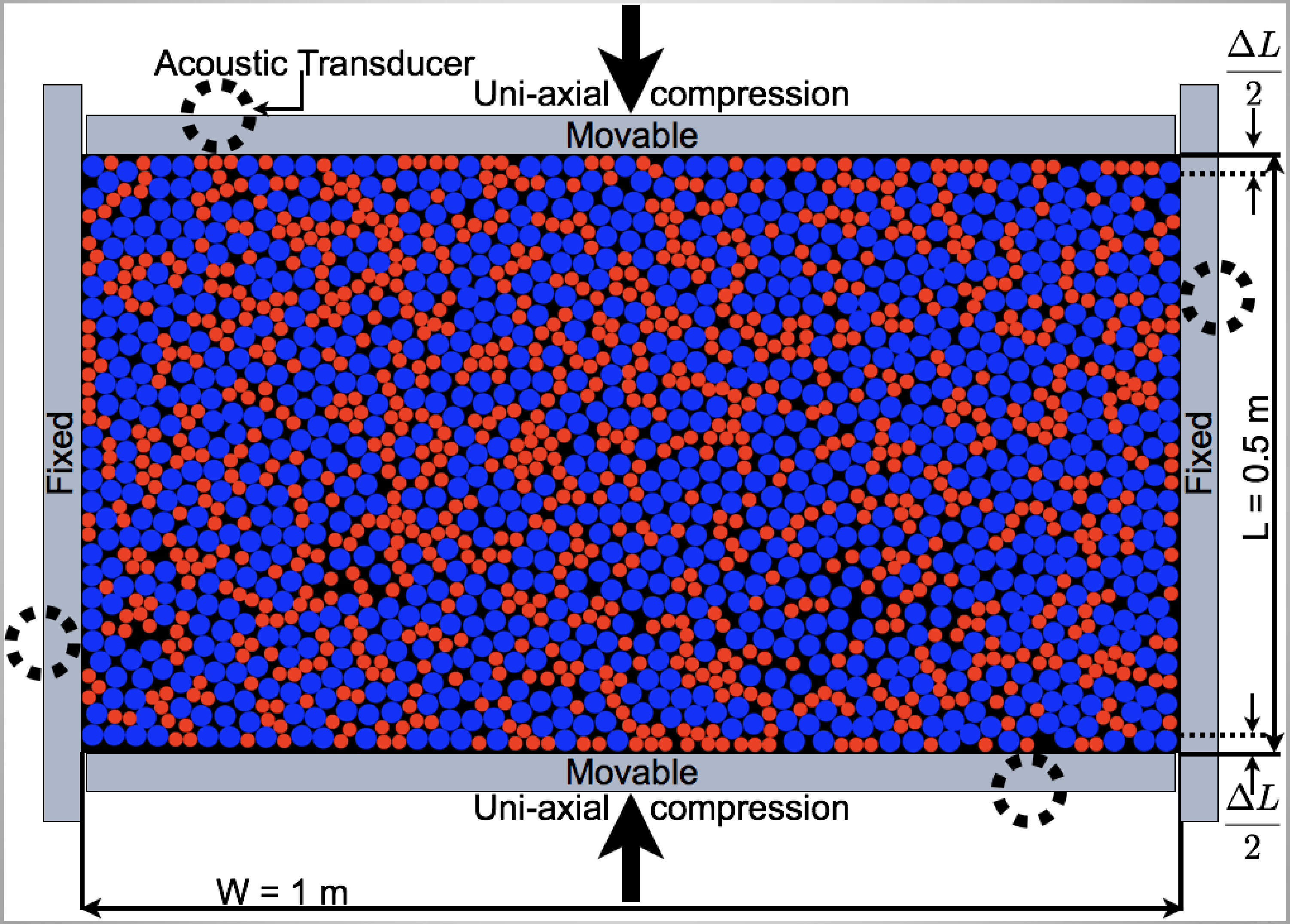
Granular Flow: Friction and the Dilatancy Transition

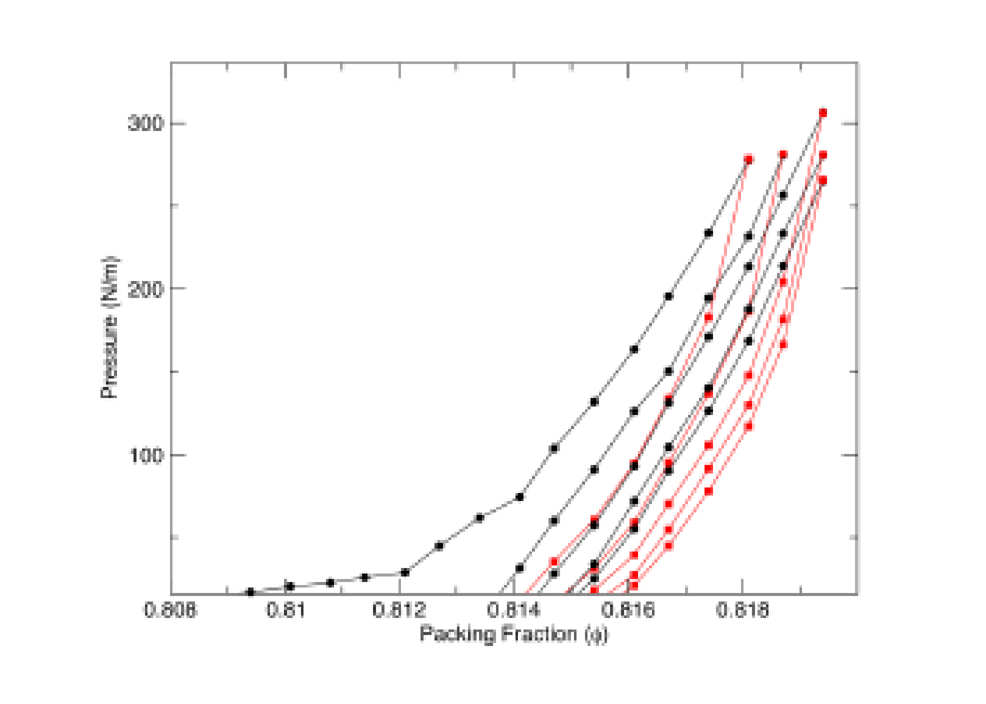
Peter A. Thompson and Gary S. Grest

The fundamental difficulties impeding our understanding lay with the very properties of granular media that make it so interesting to study in the first place: dilatancy, bistability, arching, segregation, and thixotropy. These properties conspire to create substantial hysteresis and instability, limiting experimental control and reproducibility [3,6–9]. Analytical treatments are also difficult because the boundary conditions and velocity distribution functions are poorly understood, and microstructure induces complex correlations among grains [4,6,7].

The experiment



Periodic compression and de-compression cycles



See also later lecture by S. Sastry

(i) In each cycle the system compactifies and the dissipation reduces

(ii) It appears that the areas reduce slowly, maybe with a power law

Questions

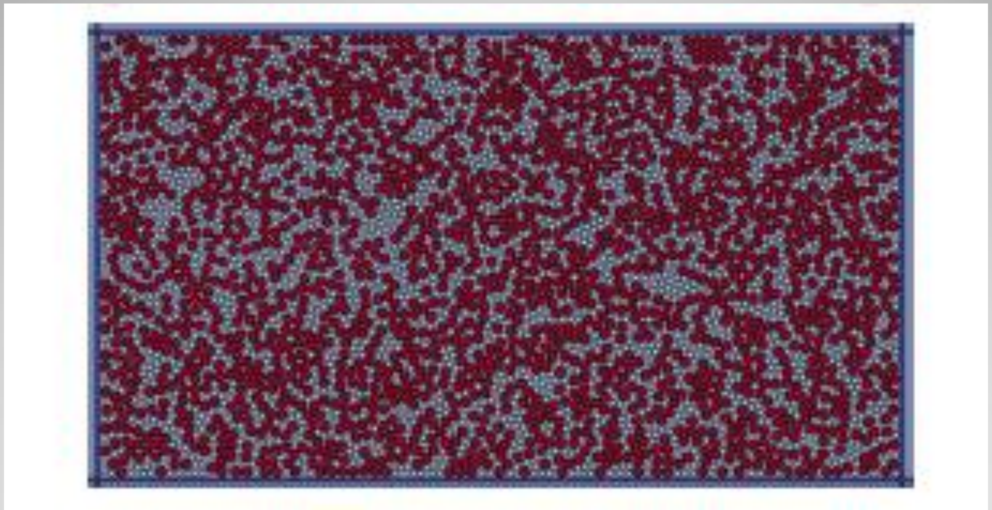
- Is the asymptotic area zero or finite?
- Is it indeed a power law, what power?
- Is the power law universal?

Answers

$$A_n = A_\infty + Bn^{-\theta}\,, \quad \theta \approx 1\,.$$

- Asymptotic area: finite due to frictional losses
- Inverse power law up to log corrections
- The power law universal

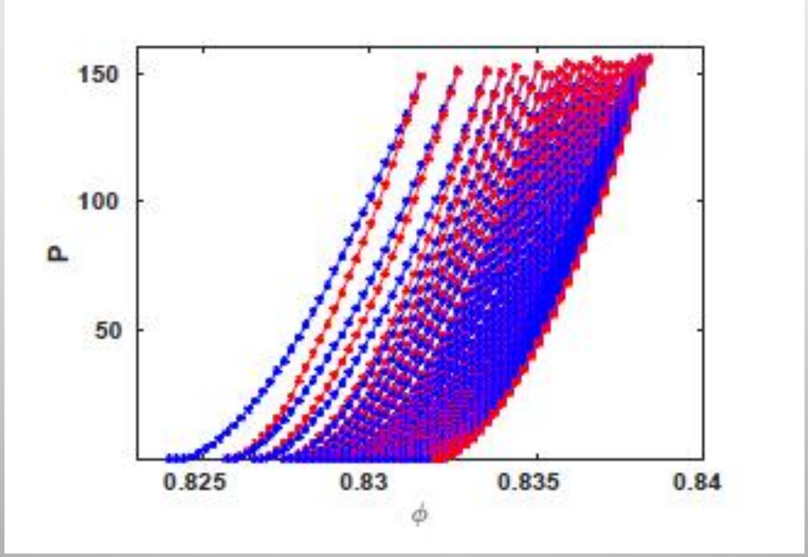
Simulations



[5] L. E. Silbert, D. Erta, G. S. Grest, T. C. Halsey, D. Levine and S. J. Plimpton, Phys. Rev. E. **64**, 051302 (2001).

Hertzian normal forces, Mindlin tangential forces, coulomb law imposed

$$F_{ij}^{(t)} \leq \mu F_{ij}^{(n)}$$



Theory

$$X_n \equiv \Phi_{n+1}(P_{\max}) - \Phi_n(P_{\max}) \; .$$

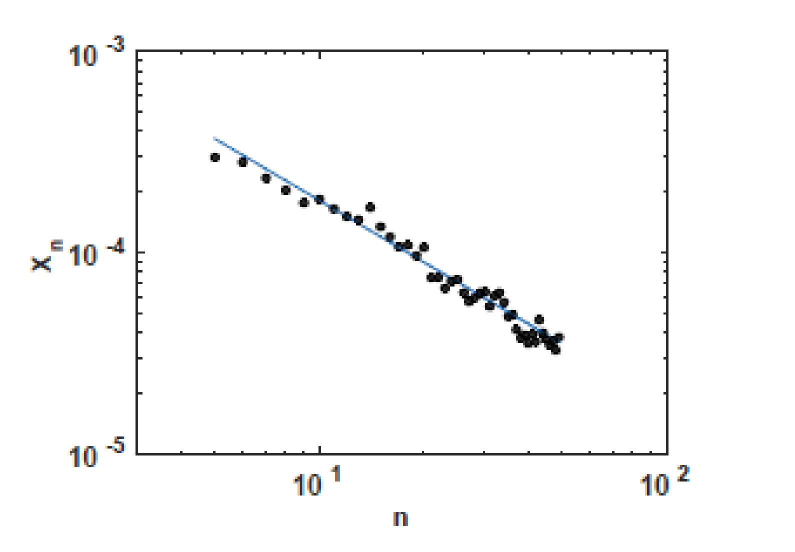
$$X_{n+1} = g(X_n) = X_n - CX_n^2 + \cdots \; .$$

The fixed point must be zero

$\sum_n X_n$ must converge;

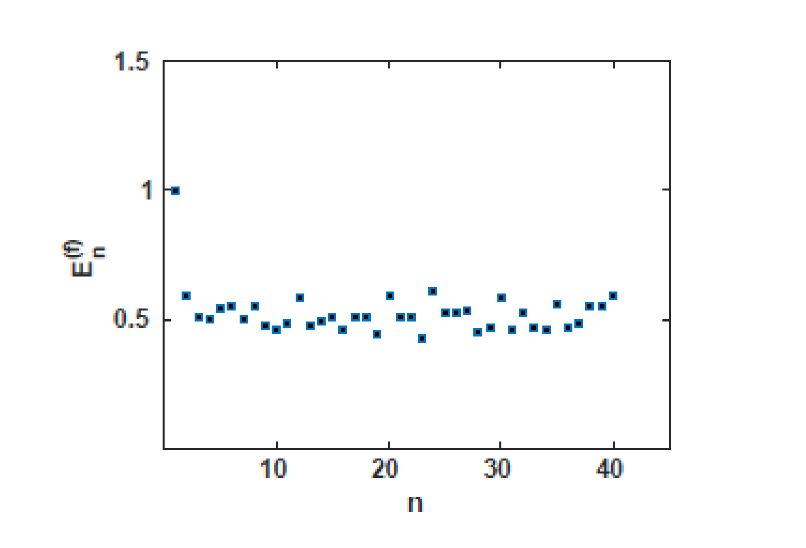
The solution of this equation for n large is

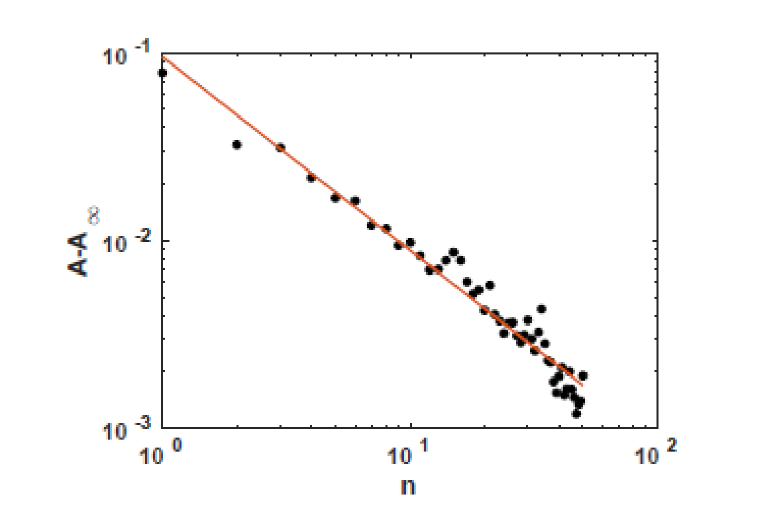
$$X_n = \frac{C^{-1}}{n} \; .$$



$$A_n \sim P_{\max} \Delta V_n \sim \frac{P_{\max} N a^2}{\phi_\infty^2} \Delta \Phi_n \; ,$$

The asymptotic area





$$A_n = A_\infty + Bn^{-\theta}\ , \quad \theta \approx 1\ .$$

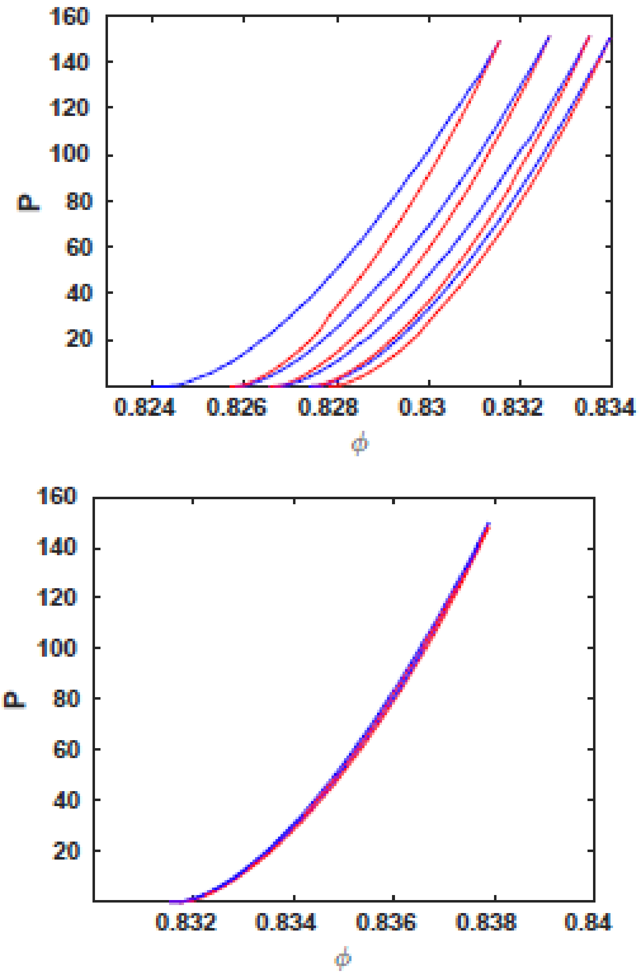


FIG. 8. Upper panel: examples of low order hysteresis loops in the $P-\Phi$ plane. The compression legs are blue and the decompression legs red. Lower panel: examples of higher order hysteresis loops in the $P-\Phi$ plane with the same color convention. The high order loops are no longer able to compactify the system further, and the compression leg begins at the same volume fraction where the decompression leg ends.

Different friction coefficient

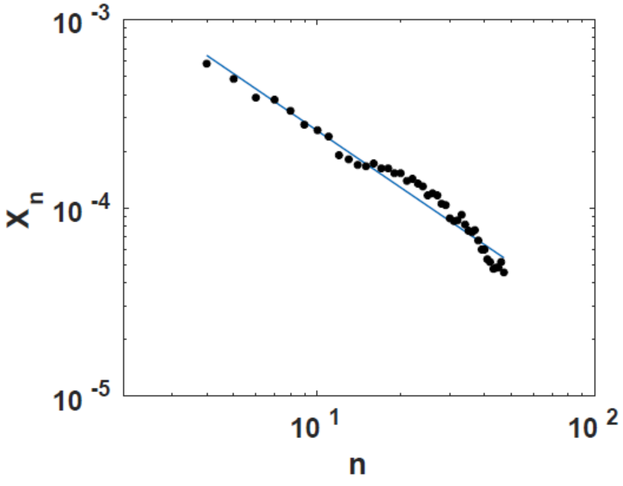


FIG. 5: Log-log plot of X_n vs. n. Here $\mu = 0.3$, the black dots are the data, the blue line is the best fitting scaling law $y = 0.003x^{-1.005}$.

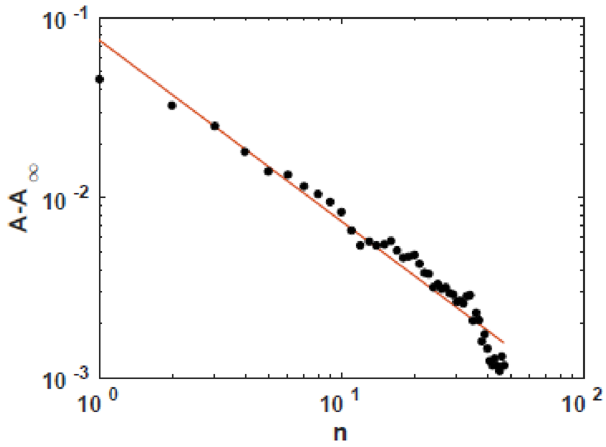
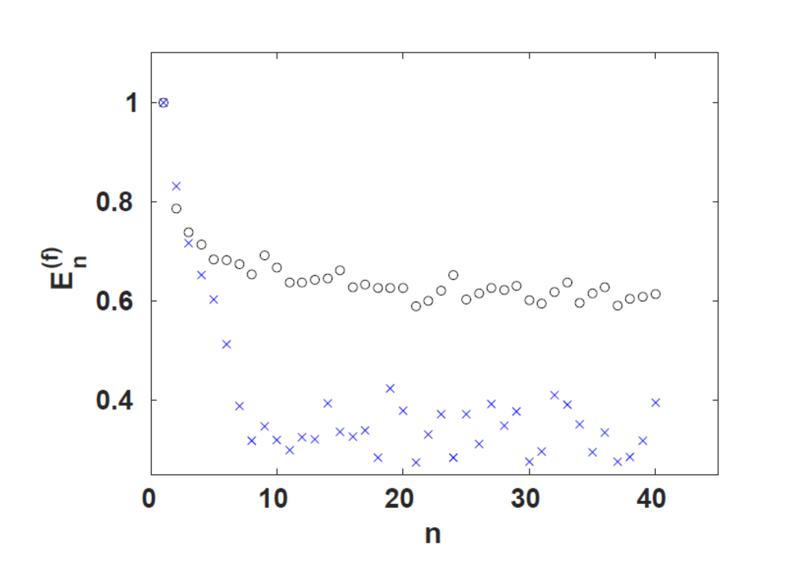
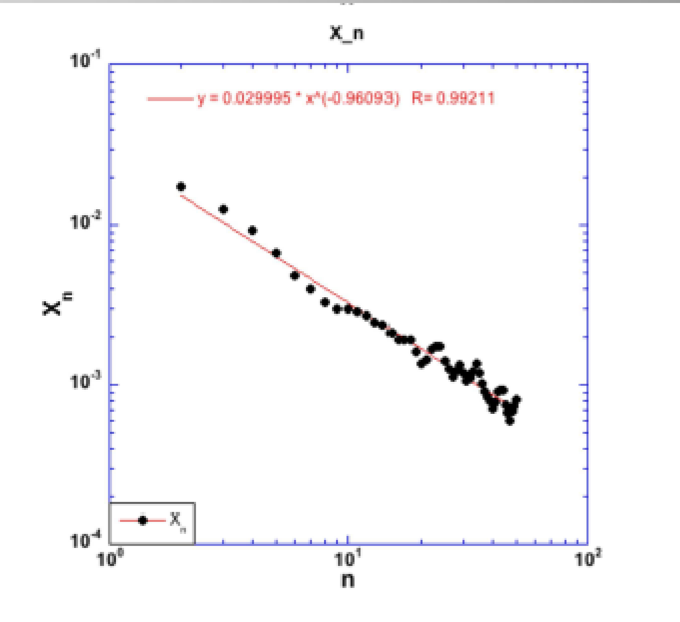
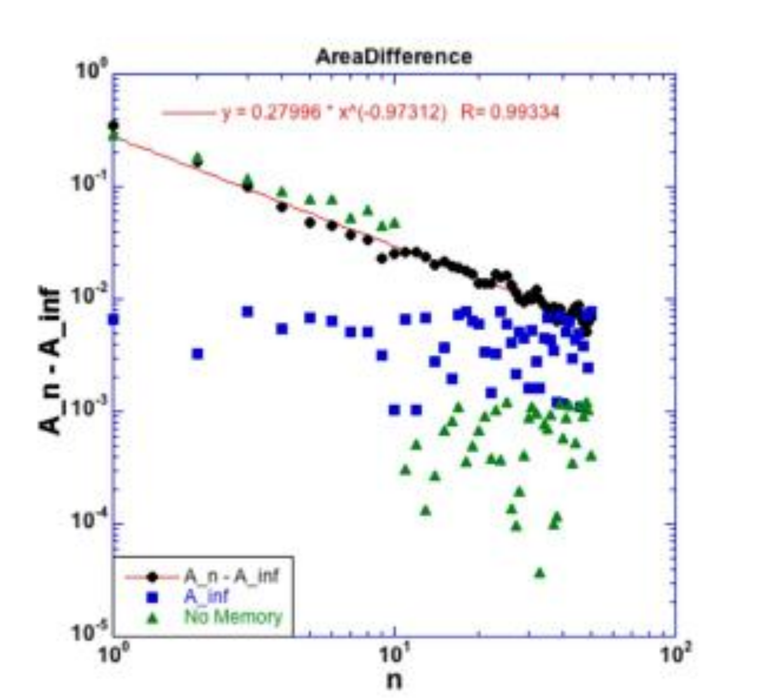


FIG. 4: The power law for the decaying areas under the hysteresis loops as measured in the numerical simulation. Here $\mu = 0.3$, Black dots are data and the red line is the best fitting power law $y = 0.075X^{-1.005}$.

Back to experiment







Conclusions and road ahead

- We have a tiny but apparently robust universal law for training and memory in frictional granular media.
- Next we focus on an interesting giant slip event.
- This requires a detailed understanding of the pdf of normal and tangential forces.
- Stay tuned. Do not go anywhere.

Thank you!

Simulations

$$\Delta_{ij} = |r_{ij} - D_{ij}|,$$

$$\begin{aligned}
\boldsymbol{F}_{ij}^{(n)} &= k_n \Delta_{ij} \boldsymbol{n}_{ij} - \gamma_n \boldsymbol{v}_{n_{ij}} \ , \ \boldsymbol{F}_{ij}^{(t)} = -k_t \boldsymbol{t}_{ij} - \gamma_t \boldsymbol{v}_{t_{ij}} && (1) \\
k_n &= k_n^{'} \sqrt{\Delta_{ij} R_{ij}} \ , \quad k_t = k_t^{'} \sqrt{\Delta_{ij} R_{ij}} && (2) \\
\gamma_n &= \gamma_n^{'} \sqrt{\Delta_{ij} R_{ij}} \ , \quad \gamma_t = \gamma_t^{'} \sqrt{\Delta_{ij} R_{ij}} \ . && (3)
\end{aligned}$$

$$F_{ij}^{(t)} \leq \mu F_{ij}^{(n)} \ ,$$