

# Patchy memories in low dimensions: Disorder, noise, and genesis; recovering after erasure

Jie Yang, AAM, Syracuse University  
PRB 96, 214208 (2017) & to be submitted

Creighton Thomas, Olivia White; support from the NSF

KITP - Memory Formation in Matter - February 20, 2018

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Disorder, noise, and genesis;  
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*Theory & computation:  
explore possibilities for particular class of memory*

Sid Nagel, Feb. 15

## Many examples of memories in matter

Stone and chisel / Paper and pencil

Photograph / Phonograph

Computer: e.g., magnetic domains

Associative memory in neural nets (Hopfield model)

Kaiser effect: remembers largest strain

Kovacs effect: remembers waiting time

Return-point memory in magnets: Nested hysteresis curves

Pulse duration memory

Multiple transient memories (charge density waves; non-Brownian suspensions)

Multiple memories in jammed solids

Echoes: spin; (anharmonic) phonon

Aging, rejuvenation and memory in glasses

Dynamical systems - remembering initial conditions

Shape-memory alloys

Designing in function: memory

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^  
(spin)

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# Spin glass - memory features



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- Expt: bulk susceptibility

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# 2D, 1D Ising spin glass, $T=0$

$$\mathcal{H}(\vec{s}) = - \sum_{\langle ij \rangle} J_{ij} s_i s_j$$

Square/linear/ladder lattice, index  $i$  for spins,

Ising spins  $s_i = \pm 1$ ,

Gaussian distributed  $J_{ij}$ , mean 0

Two global ground states

Two configurations denoted by  $A$ ,  $\bar{A}$

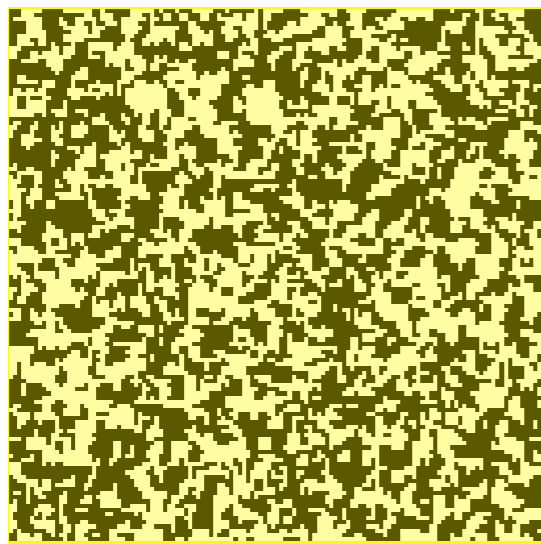
Pictures: *not*  $s_i$ , instead whether

$s_i$  aligned with  $A$  or  $\bar{A}$

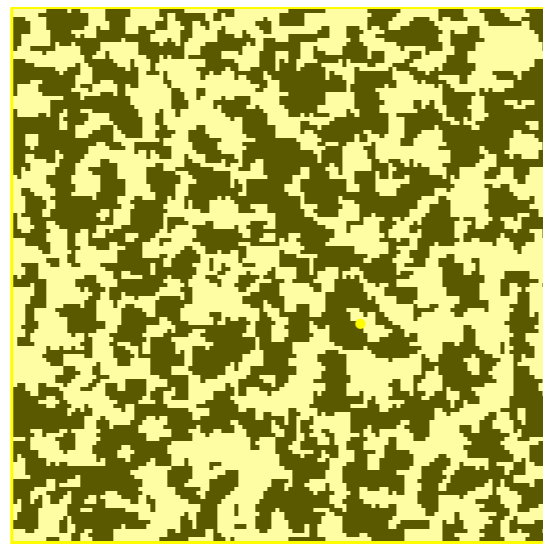
# Coarsening via patches



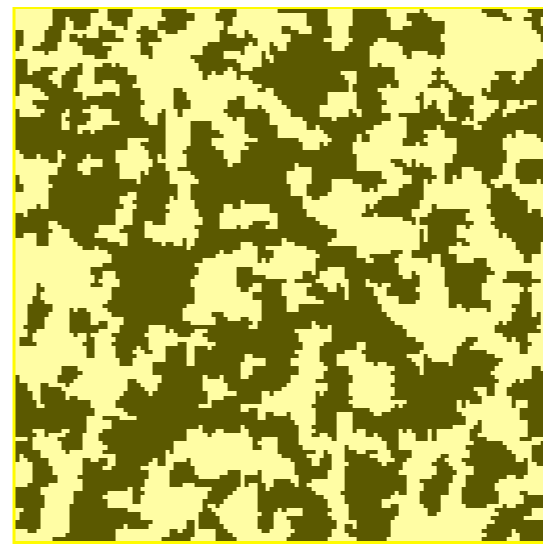
# Coarsening via patches



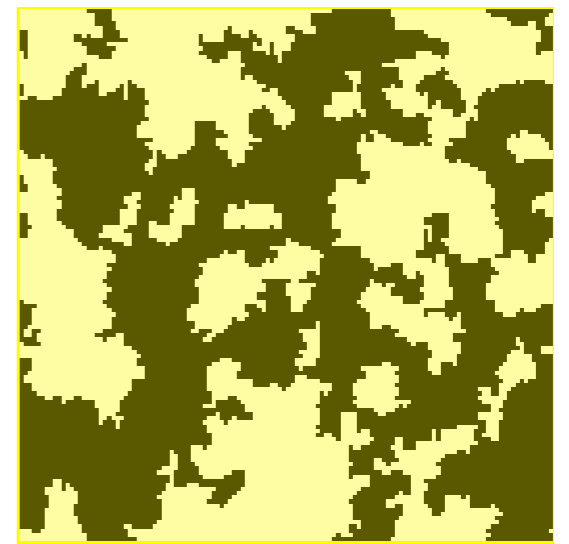
$l = 1$



$l = 2$



$l = 4$



$l = 8$

Start with random spins,  $l$  is patch size

Dark =  $A$  phase, light =  $\bar{A}$  phase.

# Inspired by, compare with, other spin glass work:

Ye, Gheissari, Machta, Newman, Stein (2016):

Detailed study with of single spin flips

Dependence of local  $T = 0$  aging on dimensionality

Chanal and Krauth (2010):

Multi-scale coupling from the past in 2DISG  $T \neq 0$

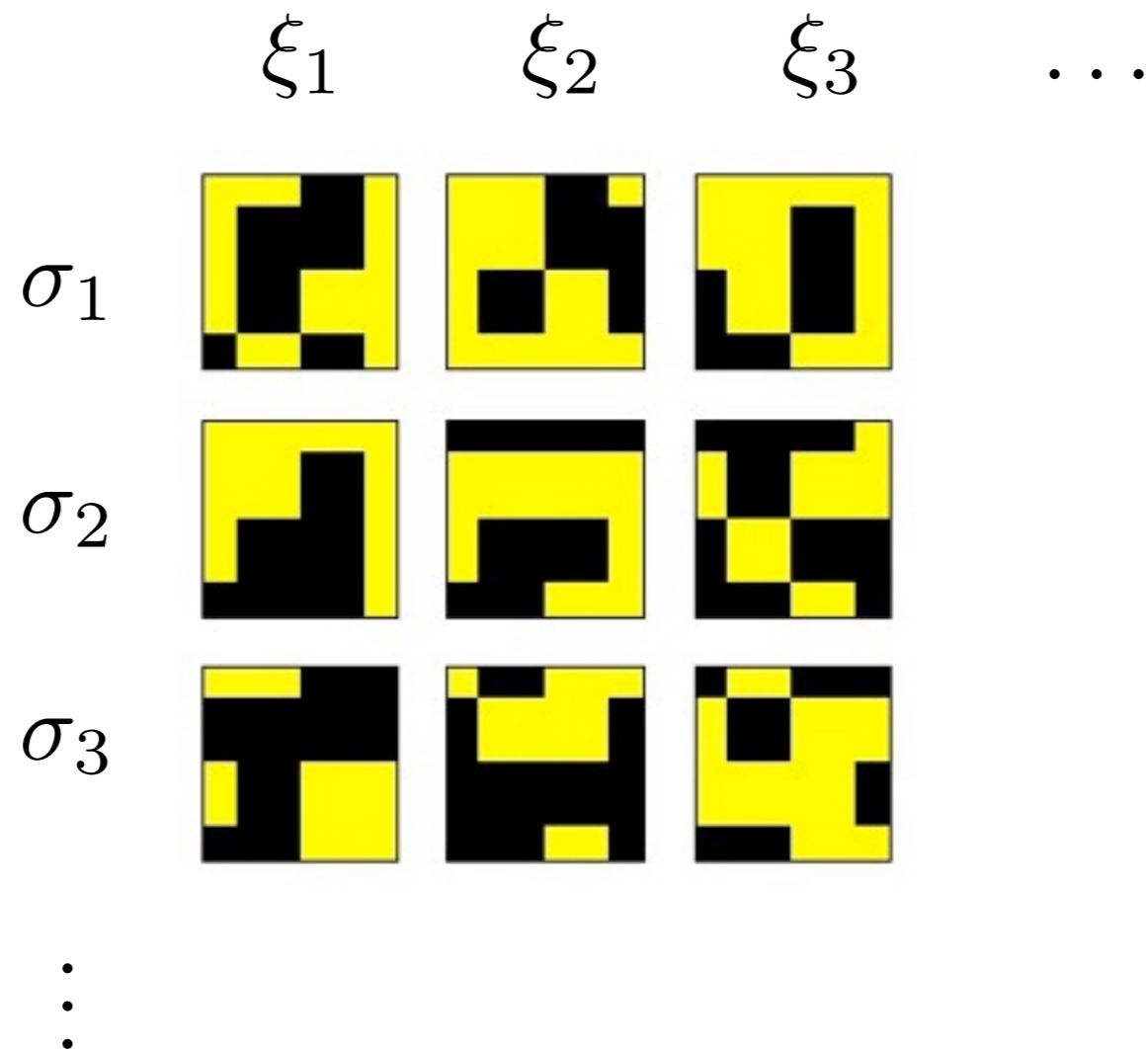
Found: final configuration not dependent on initial configuration at high  $T$

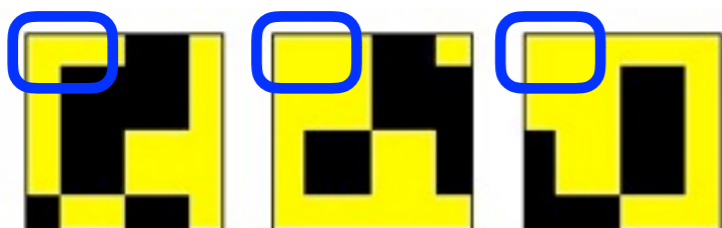
Nature/environment,  
Genesis/initial conditions,  
Nurture/history of noise

Given

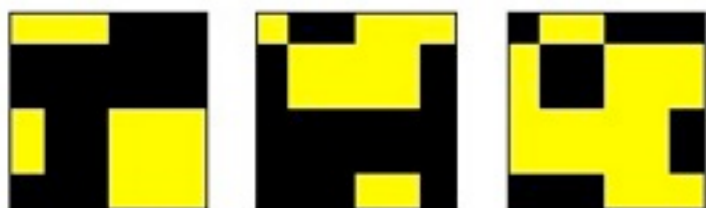
- environment =  $J_{ij}$
- genesis = initial spins  $\sigma_i$
- nurture = history of patch placements = random noise  $\xi$

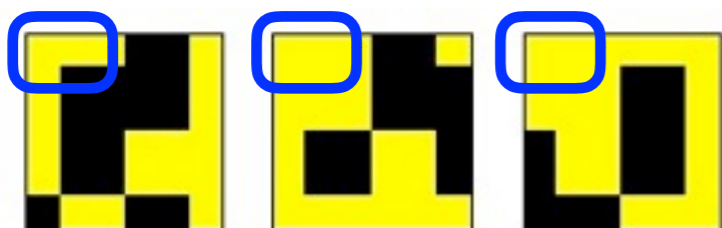
Find configurations  $s_i(J_{ij}, \sigma, \xi)$



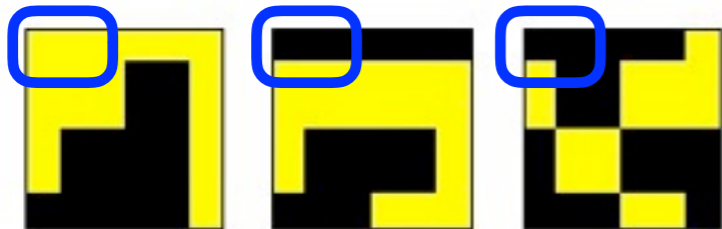
$\xi_1$  $\xi_2$  $\xi_3$  $\dots$ Average over noise history  $\xi$ : $\sigma_1$ 

$$\langle s_0 s_1 \rangle(\sigma_1)$$

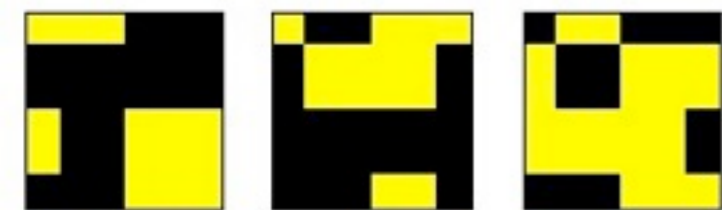
 $\sigma_2$  $\sigma_3$  $\vdots$

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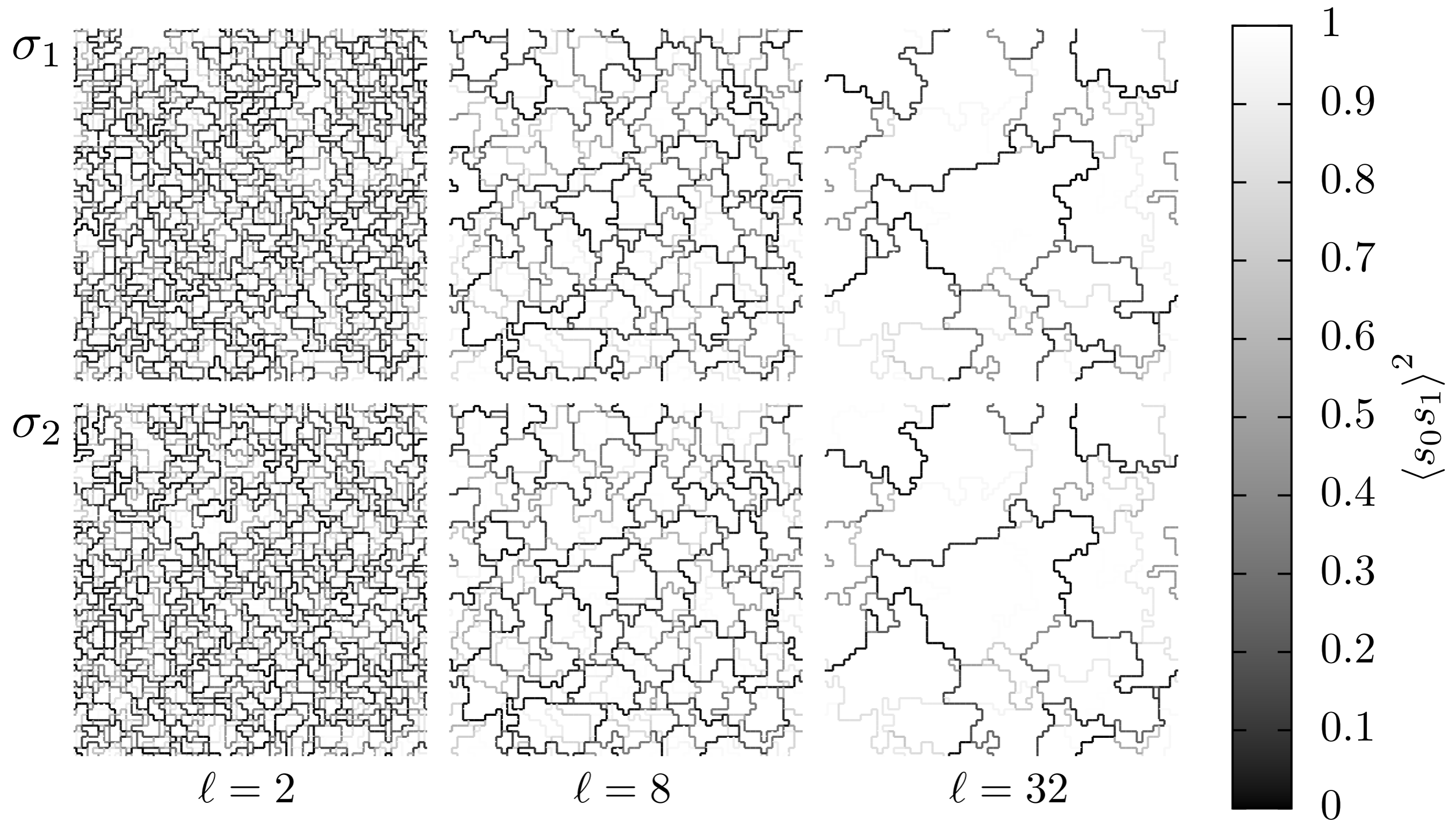
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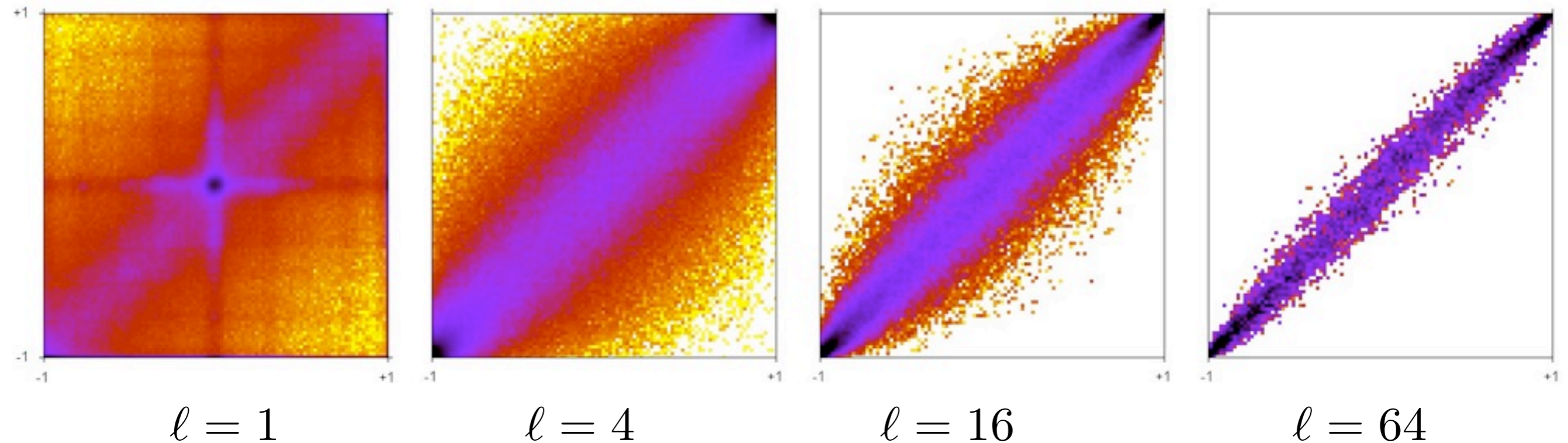
 $\sigma_3$  $\vdots$

# Two initial configs / Multiple noise histories

dark (light) bonds = “floppy” (rigid) with respect to noise



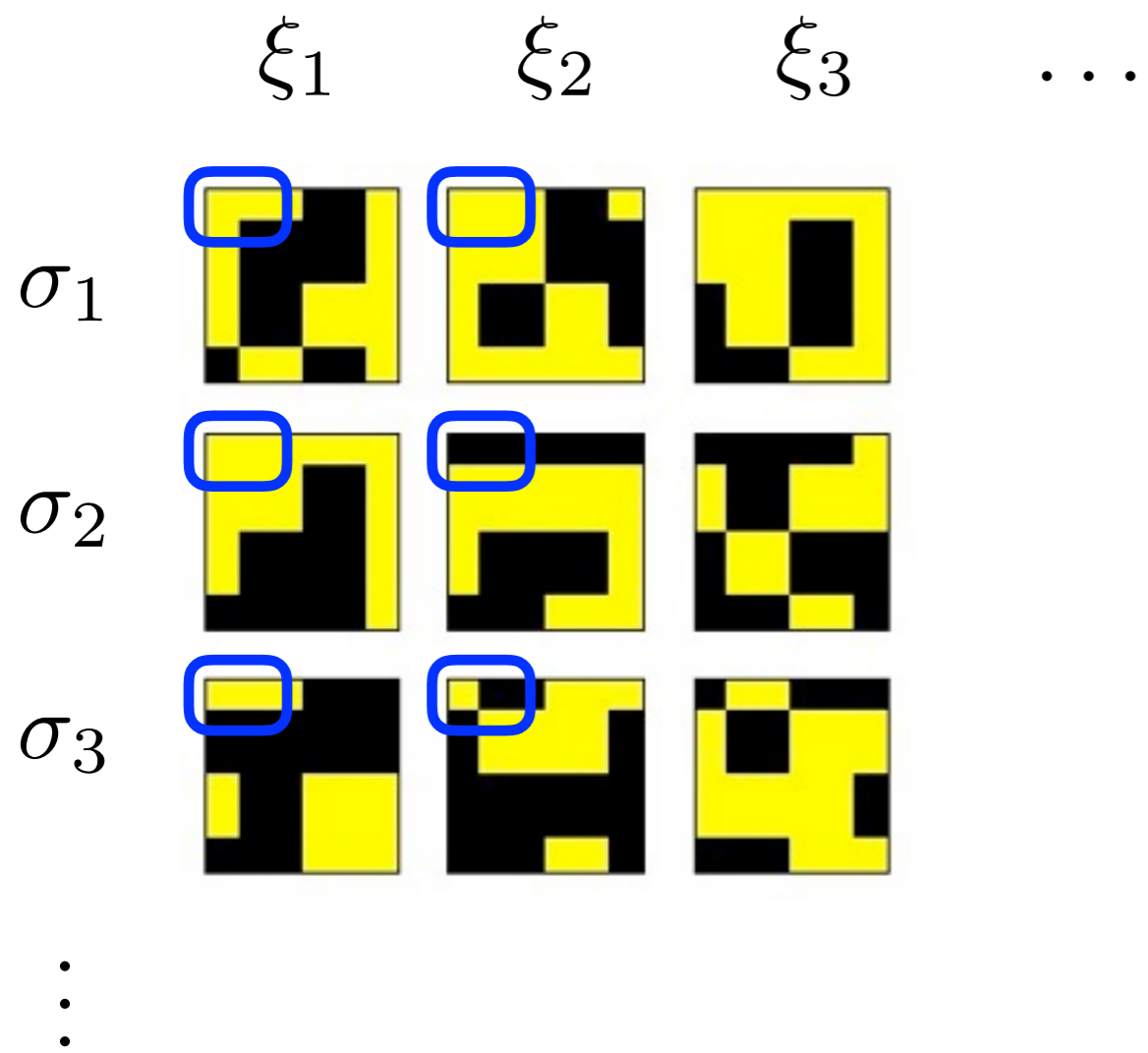
Noise-history-averaged bond correlations for two initial  $\sigma$ ,  
 $\langle s_i s_j \rangle(J_{ij}, \sigma_2)$  vs.  $\langle s_i s_j \rangle(J_{ij}, \sigma_1)$



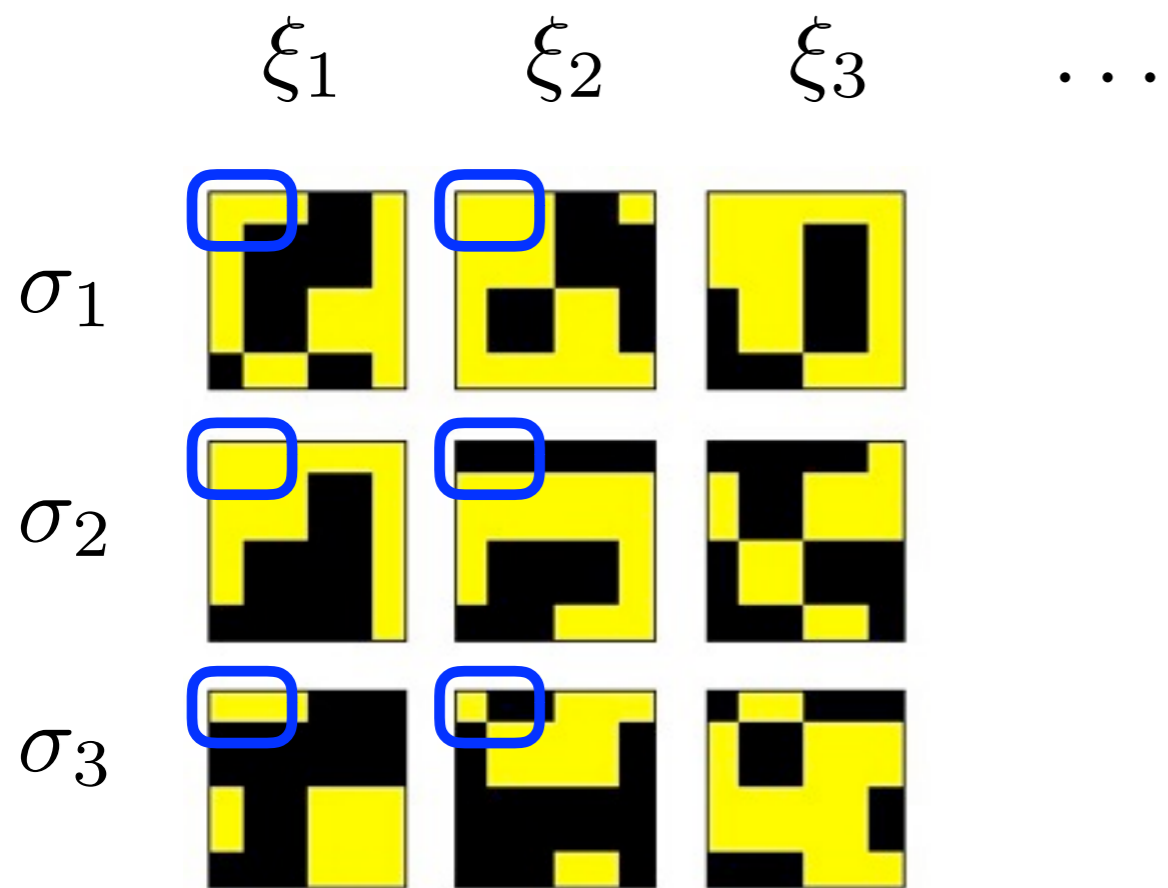
(all bonds  $s_i s_j$ , 50 samples  $J_{ij}$  with  $256^2$  spins,  
2 initial conditions  $\sigma_{1,2}$ , 1000 noise histories  $\xi$ )



Average over initial conditions  $\sigma$ :



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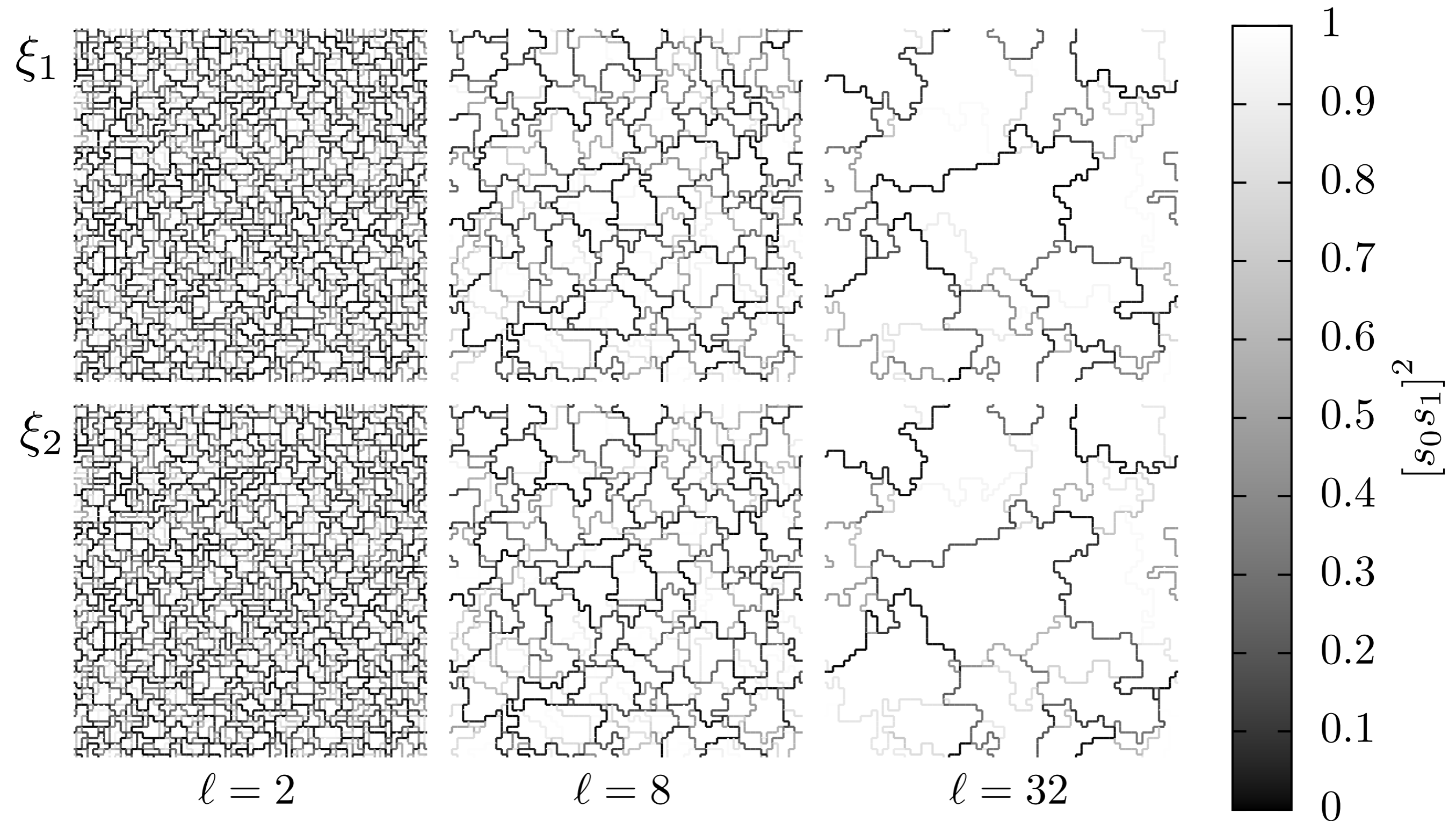
$\vdots$

Average over initial conditions  $\sigma$ :

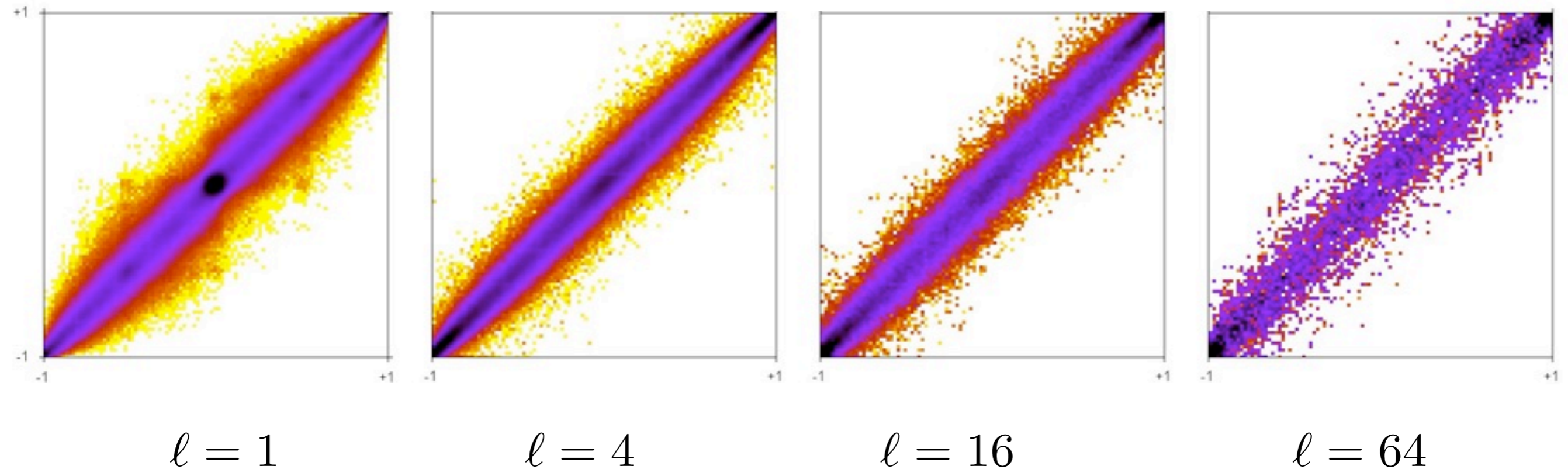
$$[s_0 s_1] (J_{ij}, \xi_1) \quad [s_0 s_1] (J_{ij}, \xi_2)$$

# Two noise histories / Multiple initial configs

dark (light) bonds = “floppy” (rigid) with respect to initial spins



Initial-configuration-averaged bond correlations for two noise histories  $\xi$ ,  
 $\langle s_i s_j \rangle(J_{ij}, \xi_2)$  vs.  $\langle s_i s_j \rangle(J_{ij}, \xi_1)$



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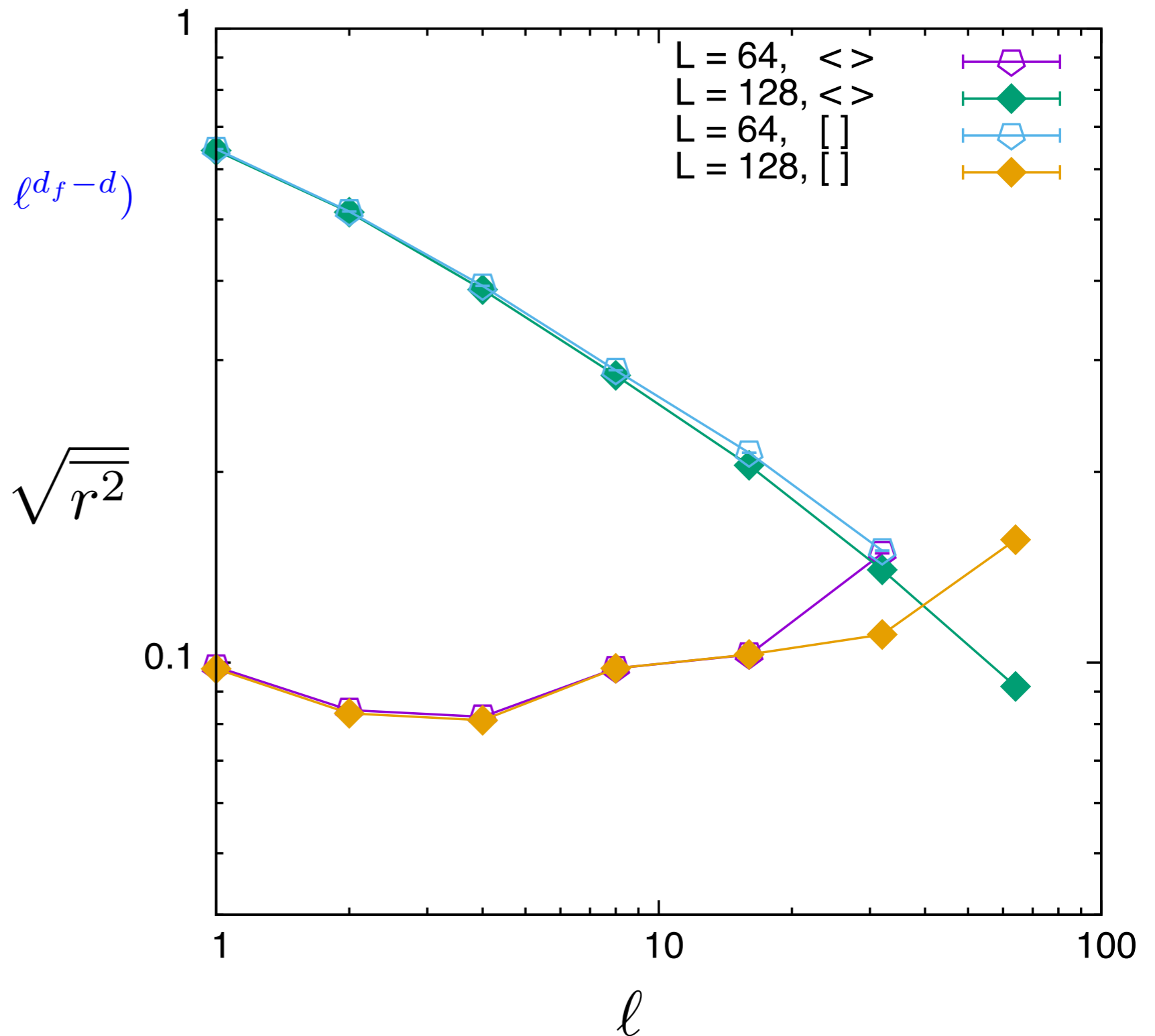
For “non-rigid” bonds, define

$$r = \langle s_0 s_1 \rangle(\sigma) - \langle s_0 s_1 \rangle(\sigma')$$

or

$$r = [s_0 s_1](\xi) - [s_0 s_1](\xi')$$

(Fraction of non-rigid bonds  $\sim \ell^{d_f-d}$ )



# Noise, genesis, environment w/frustration

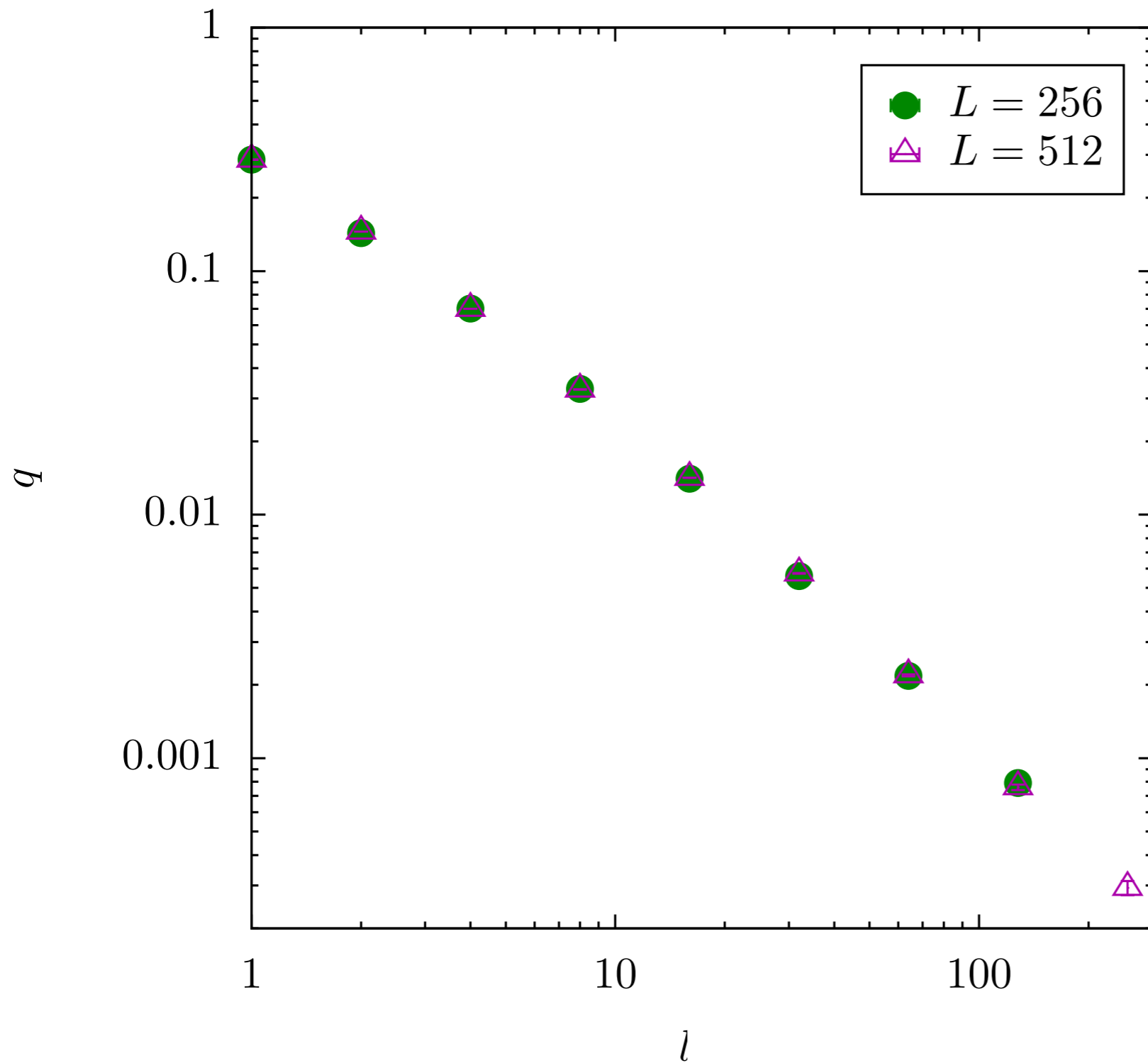
- Both  $\langle \dots \rangle$  and  $[\dots]$ : same rigid domains, floppy walls, given  $J_{ij}$ .
- Average over histories  $\xi$ : floppiness independent of initial configuration  $\sigma$ .
- Average over starts  $\sigma$ : floppiness depends on history  $\xi$ .
- Even for  $[\dots]$ , domain walls not fixed by history.
- ... scale-invariant uncertainty arising from initial conditions.
- Cf. partial-ordering/coupling-from-the-past:  $\xi \rightarrow$  outcome.

# Memory test

1. Given  $J_{ij}$ , set spins to global ground state  $s_i^A(\{J_{ij}\})$ .
2. Scramble disorder - independent  $J'_{ij}$ .
3. Grow patches to scale  $\ell$  under new landscape.
4. Reset to  $J_{ij}$ .
5. Recover by growing patches to scale  $r$ .
6. Track overlap  $q = N^{-1} \sum_i s_i^A s_i$  all along.

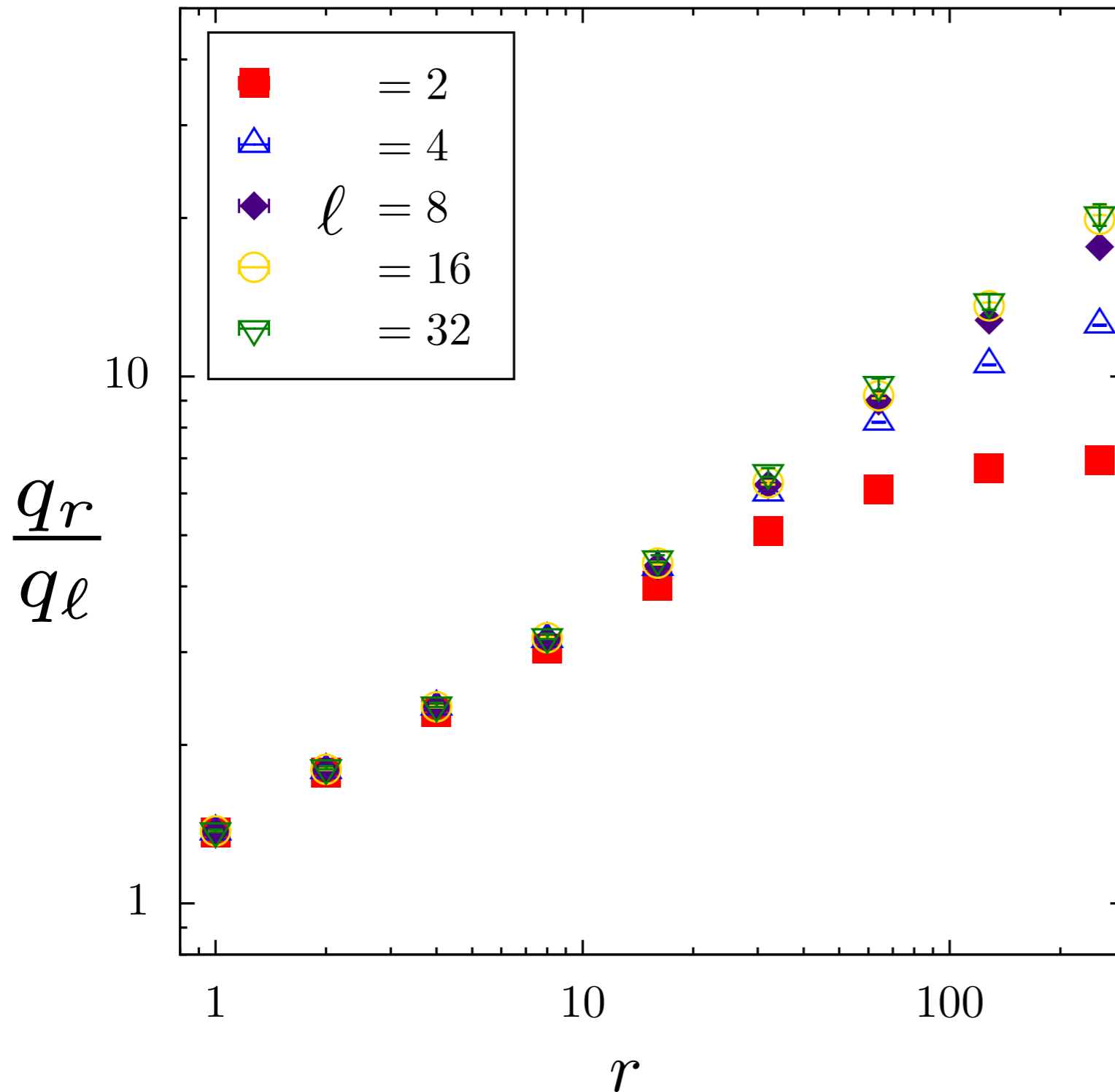


# 2D aging, couplings $J'$



$$q \sim \ell^{-1.4}$$

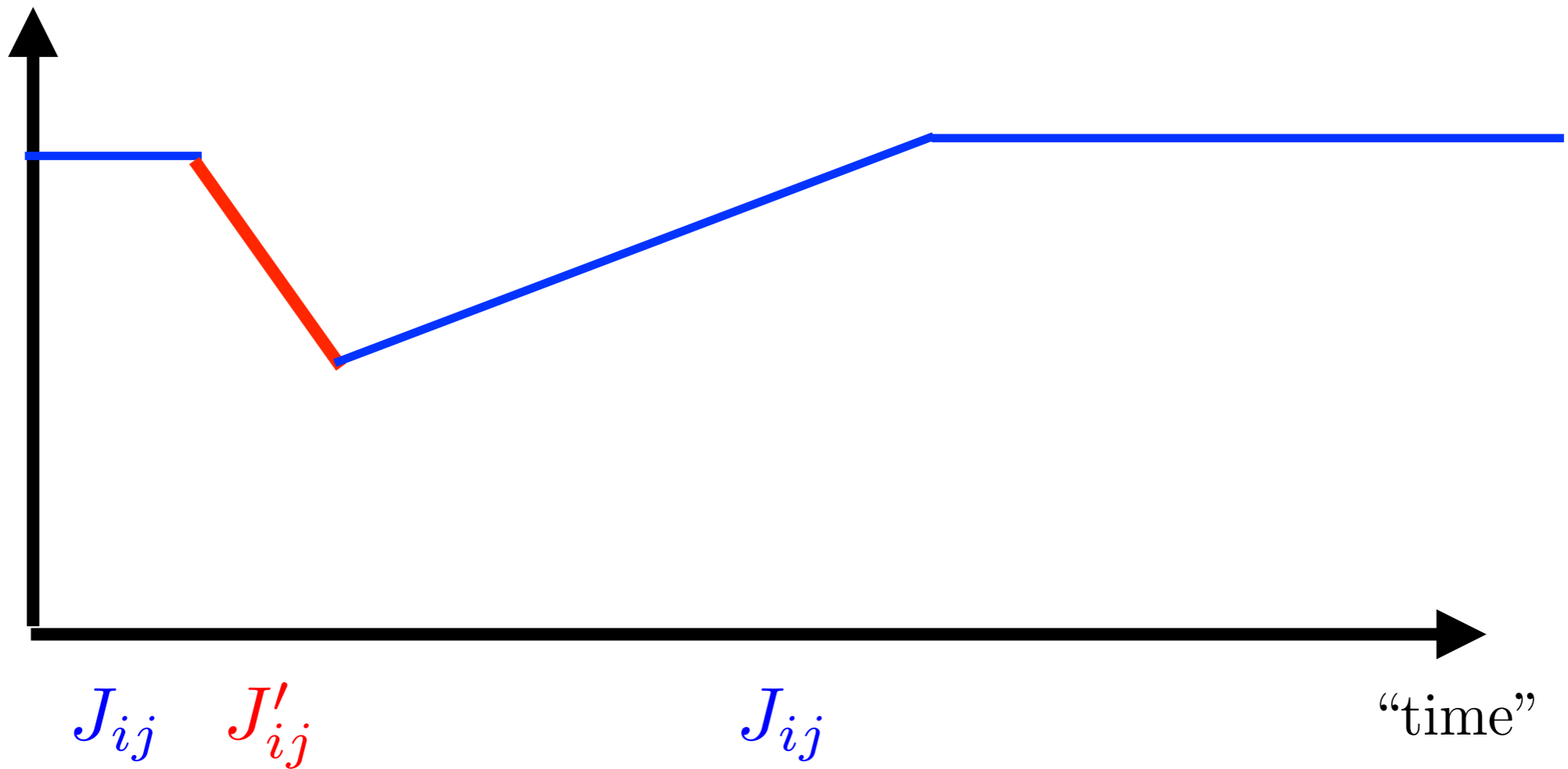
# 2D recovery, couplings J



$$\frac{q_r}{q_l} \sim r^{0.5 \pm 0.05}$$

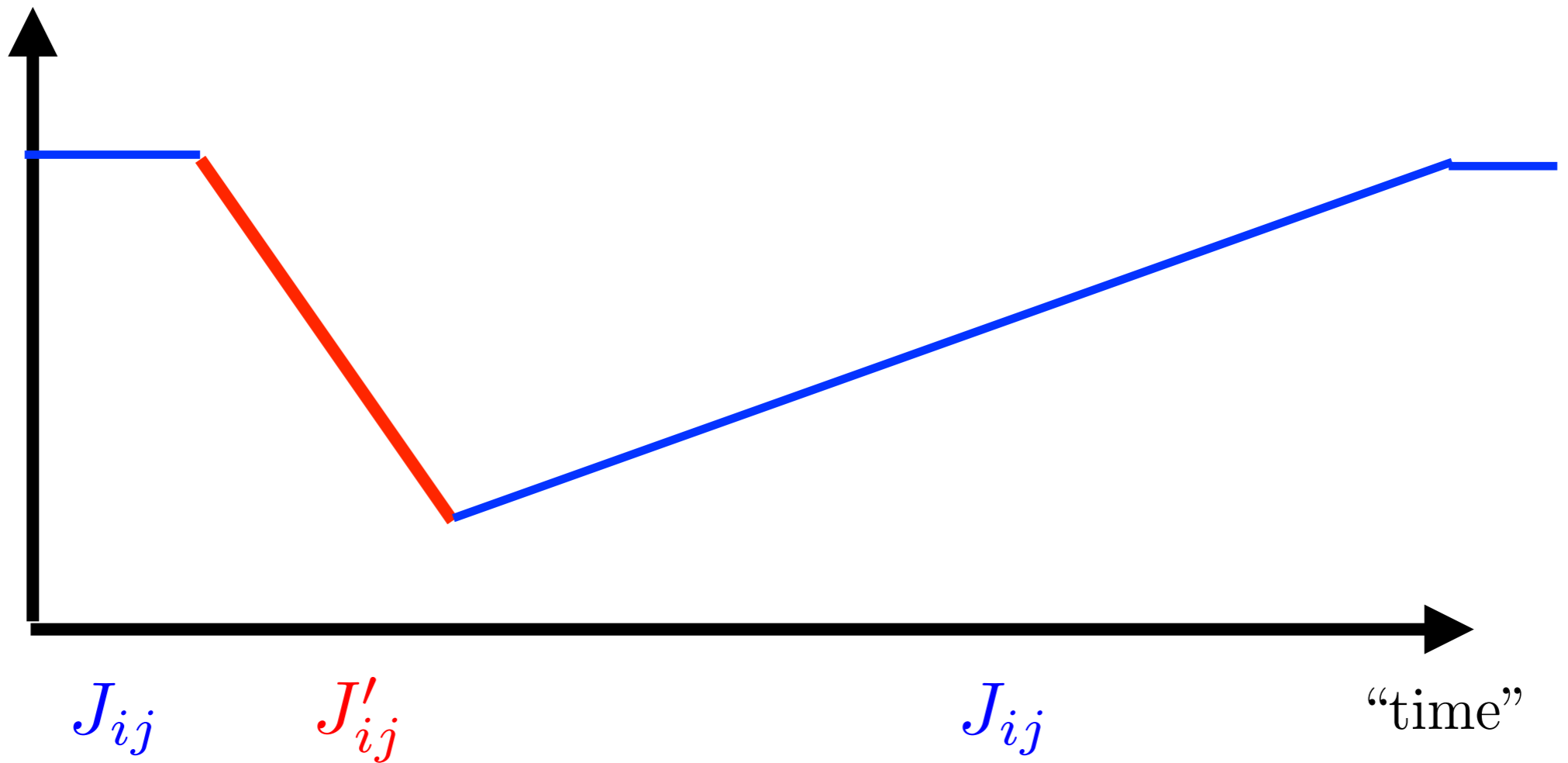
$$d = 2$$

$q = \text{overlap with A for } J_{ij}$



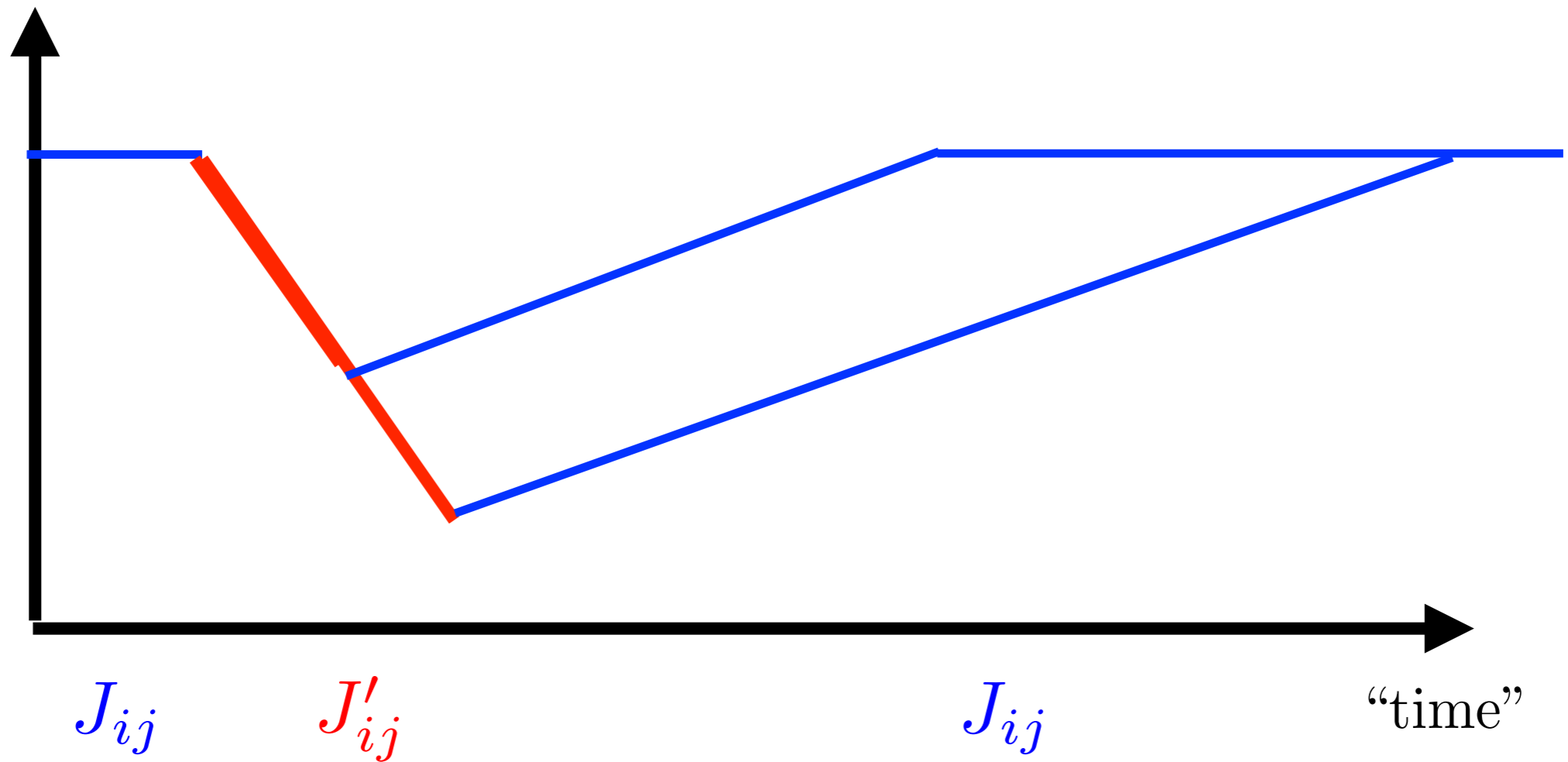
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# Recovery length for 2D patches

Scaling argument:

Full recovery at

$$r_c \sim \ell^{1.4/0.5} = \ell^{2.8}$$

⇒ Recovery at scales  $r_c \gg$  than aging scale  $\ell$

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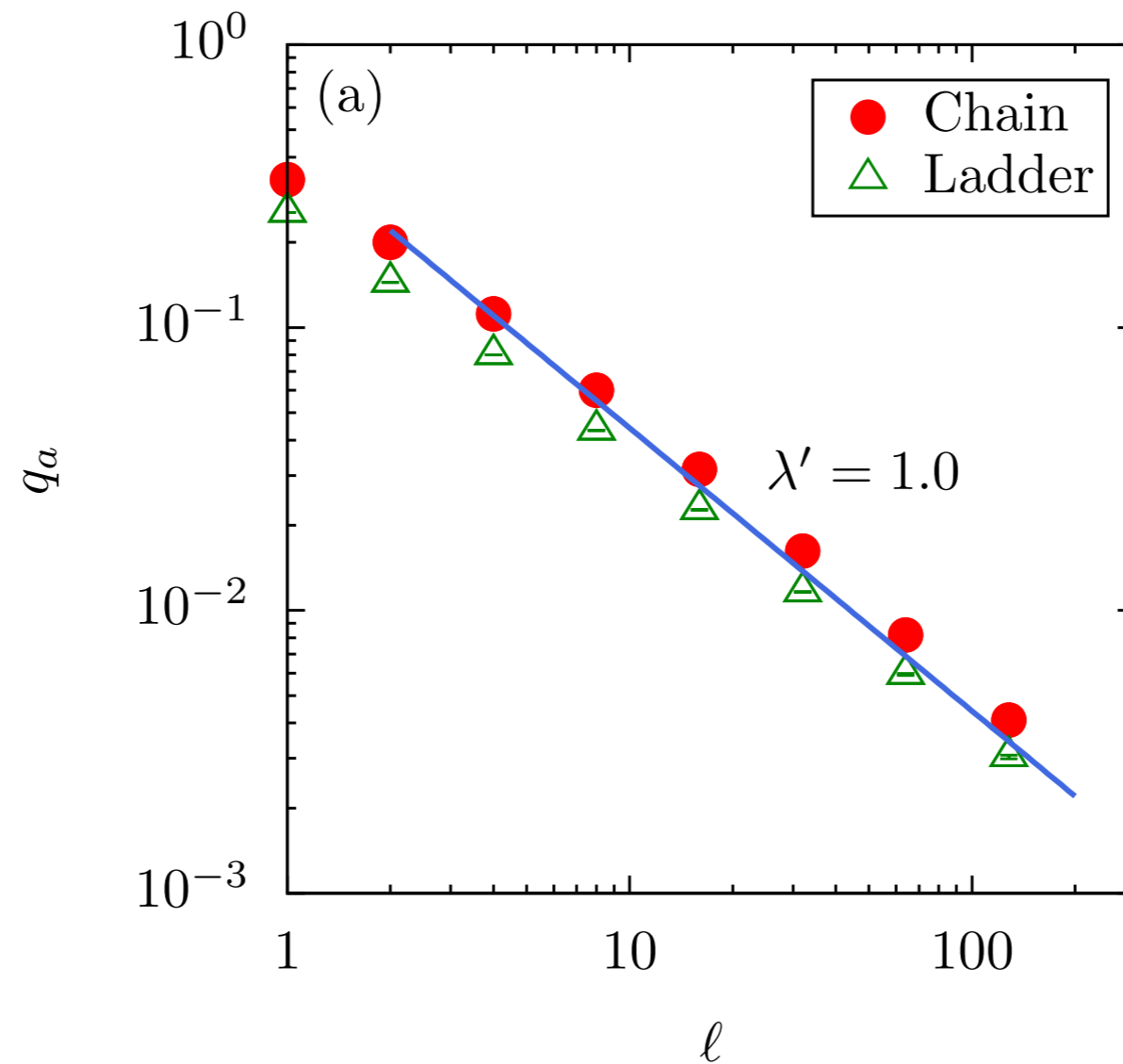
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*Why?*

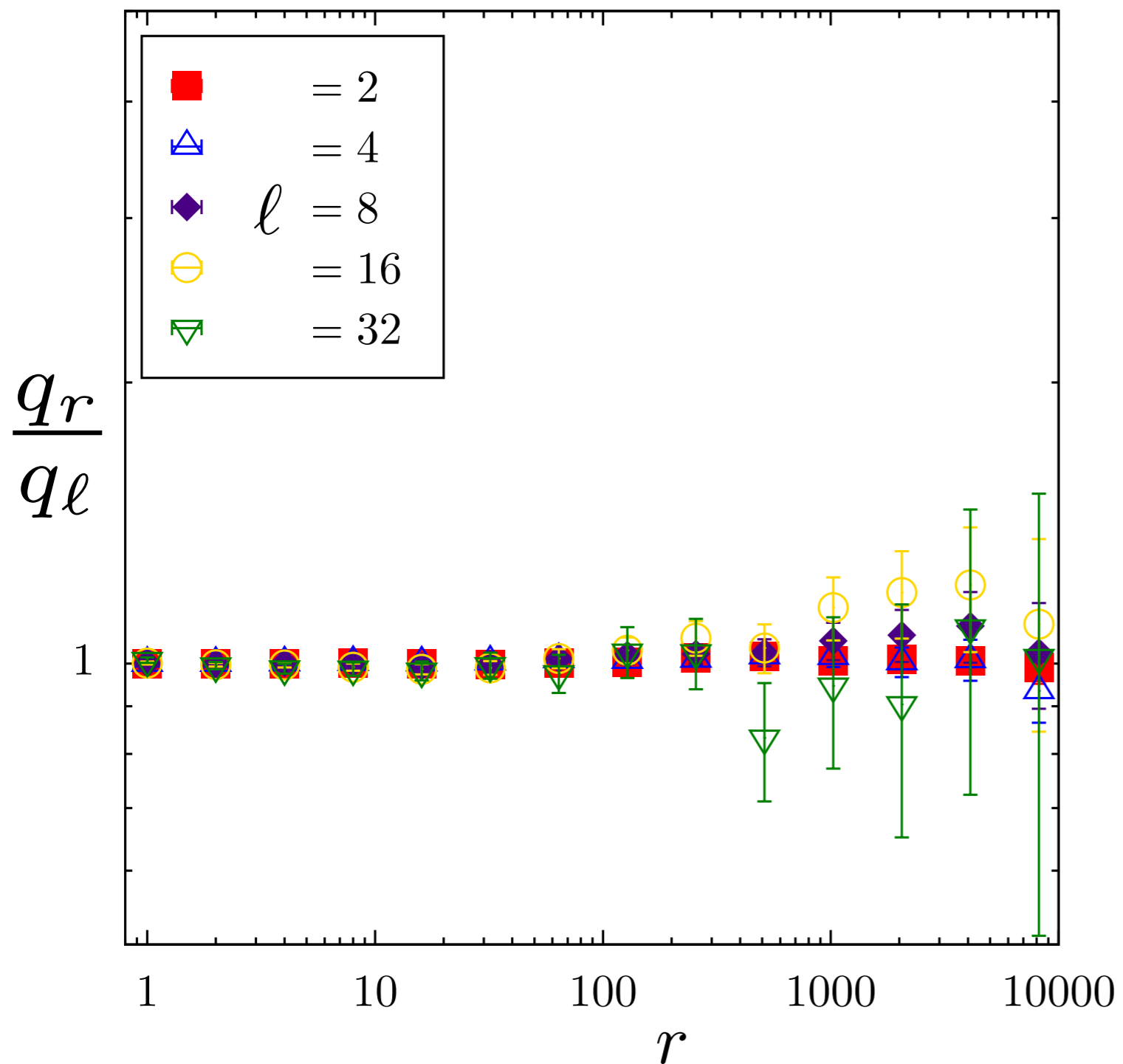
# ID aging



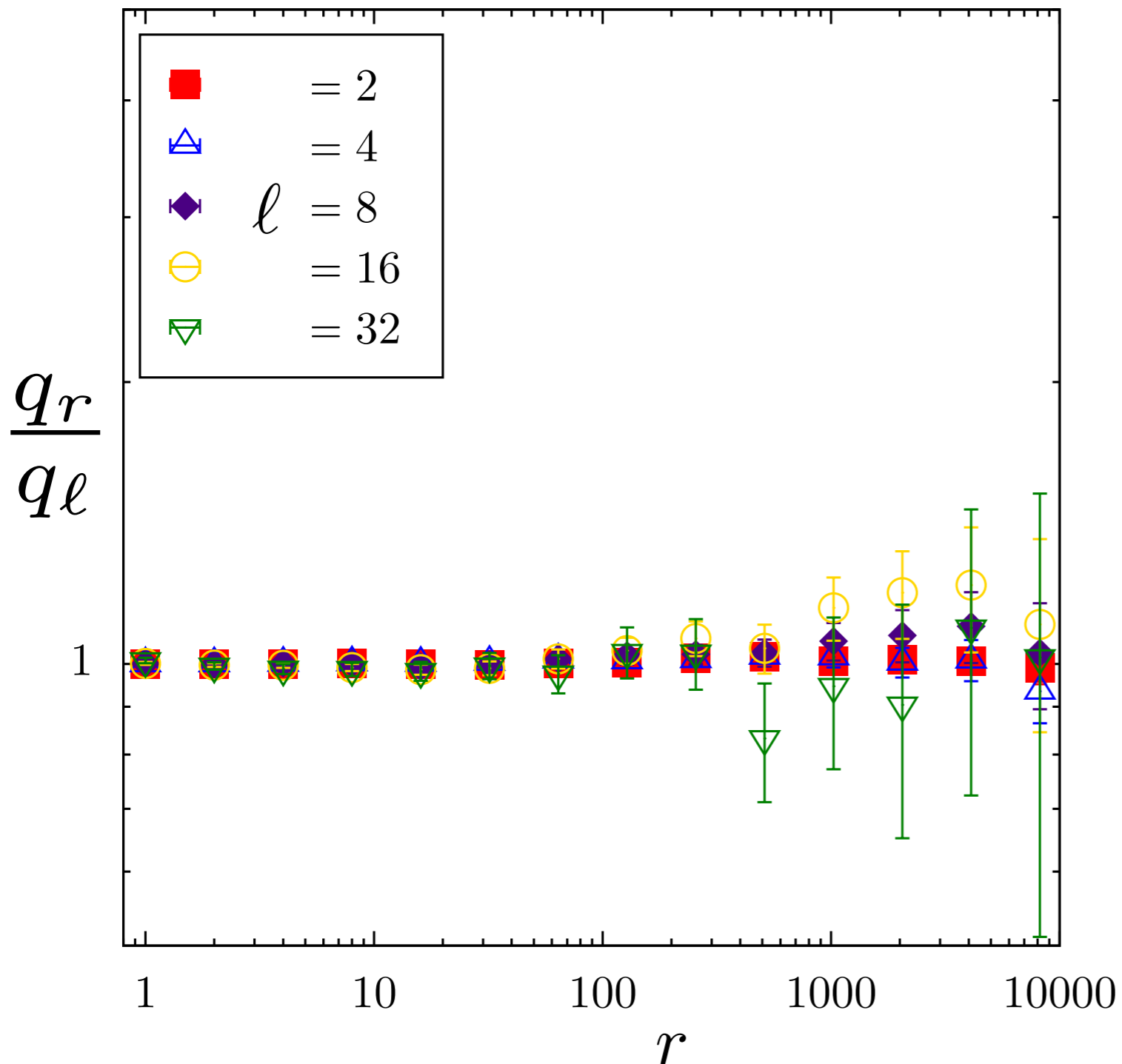
$$q_\alpha \sim l^{-1.0}$$



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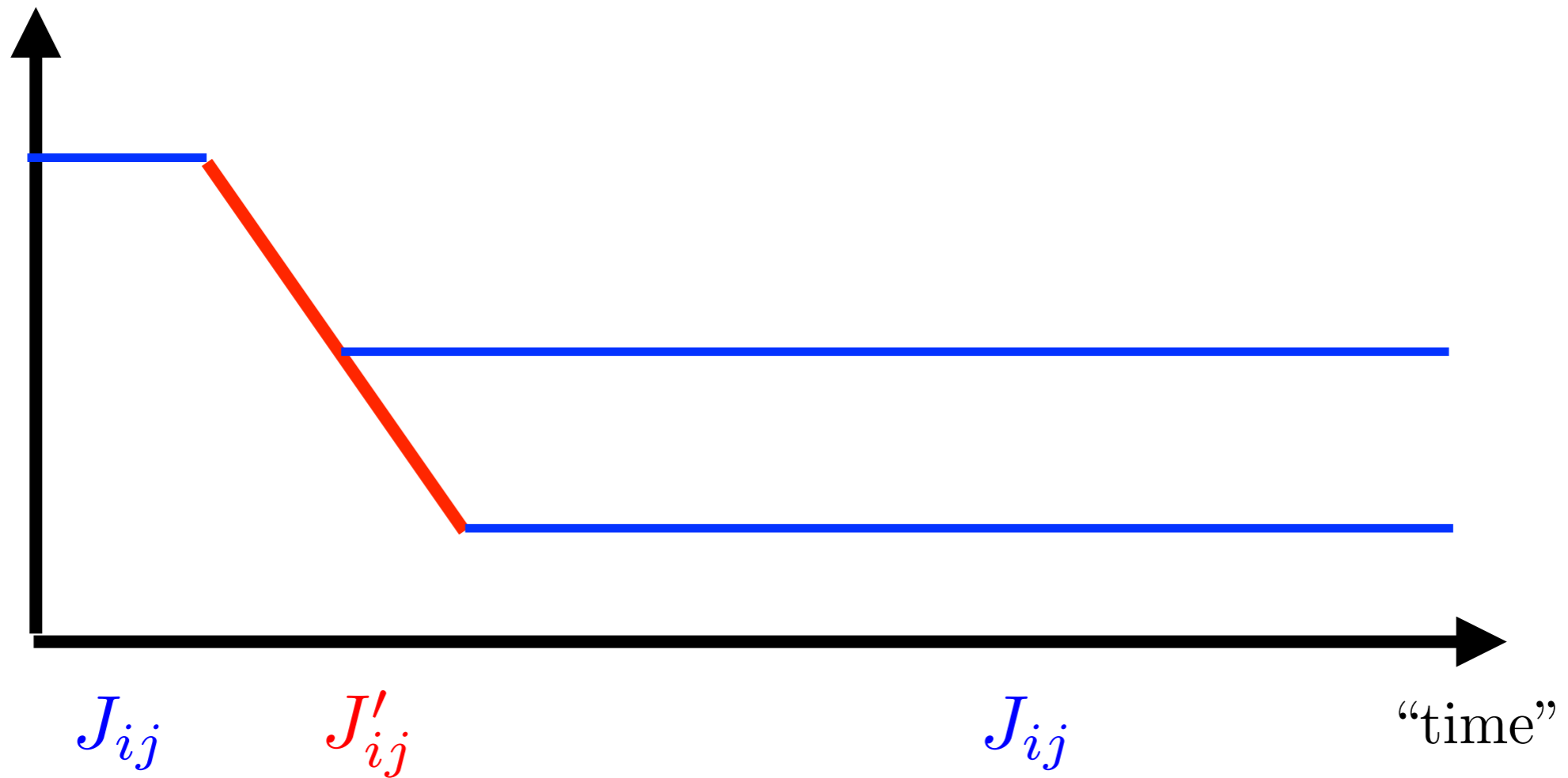
Can prove.

Coarsening while maintaining  $A$  to  $\bar{A}$  ratio.

Flips to  $A$  more frequently, but being “wrong” causes more damage.

$$d = 1$$

$q = \text{overlap with A for } J_{ij}$



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  - recovery scale  $\gg$  erasure scale (unexpected? explain?)

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- No design - random, frustrated interactions
- Finite  $d$ : coarsening domains, pinned walls for memory
- Heuristic, but fast & exact, simulations
- Noise history dominates, but genesis ensemble gives (subtle? unexpected?) uncertainty in outcome (frustration)
- Memory:
  - switched between random (hierarchical) “landscapes”
  - recovery scale  $\gg$  erasure scale (unexpected? explain?)

# Scaling test

