

Orbital and Magnetic Physics in Vanadium Spinel

Yukitoshi Motome
(Univ. of Tokyo)

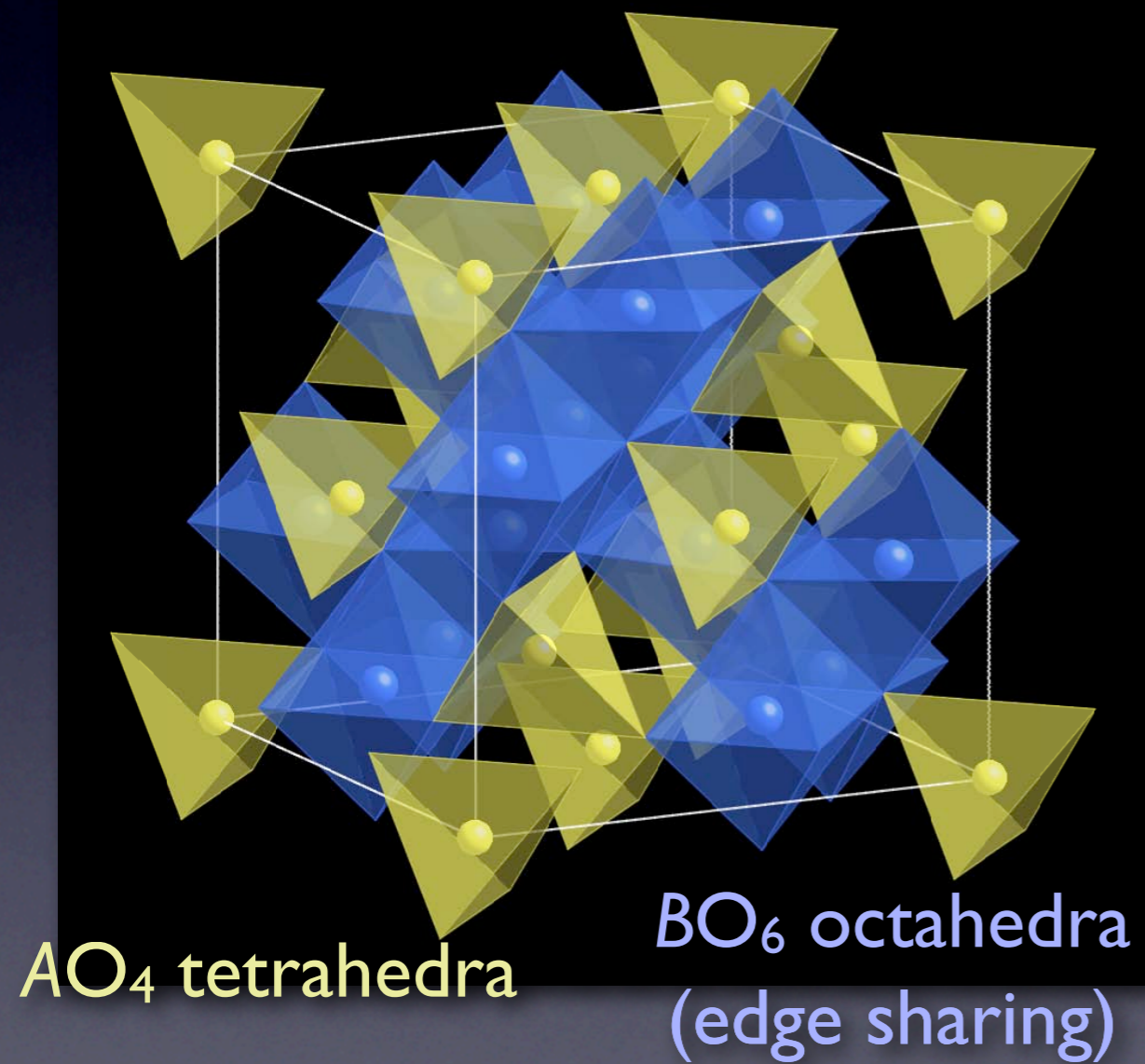
KITP Program
Moments and Multiplets in Mott Materials
Sep. 25, 2007

Outline

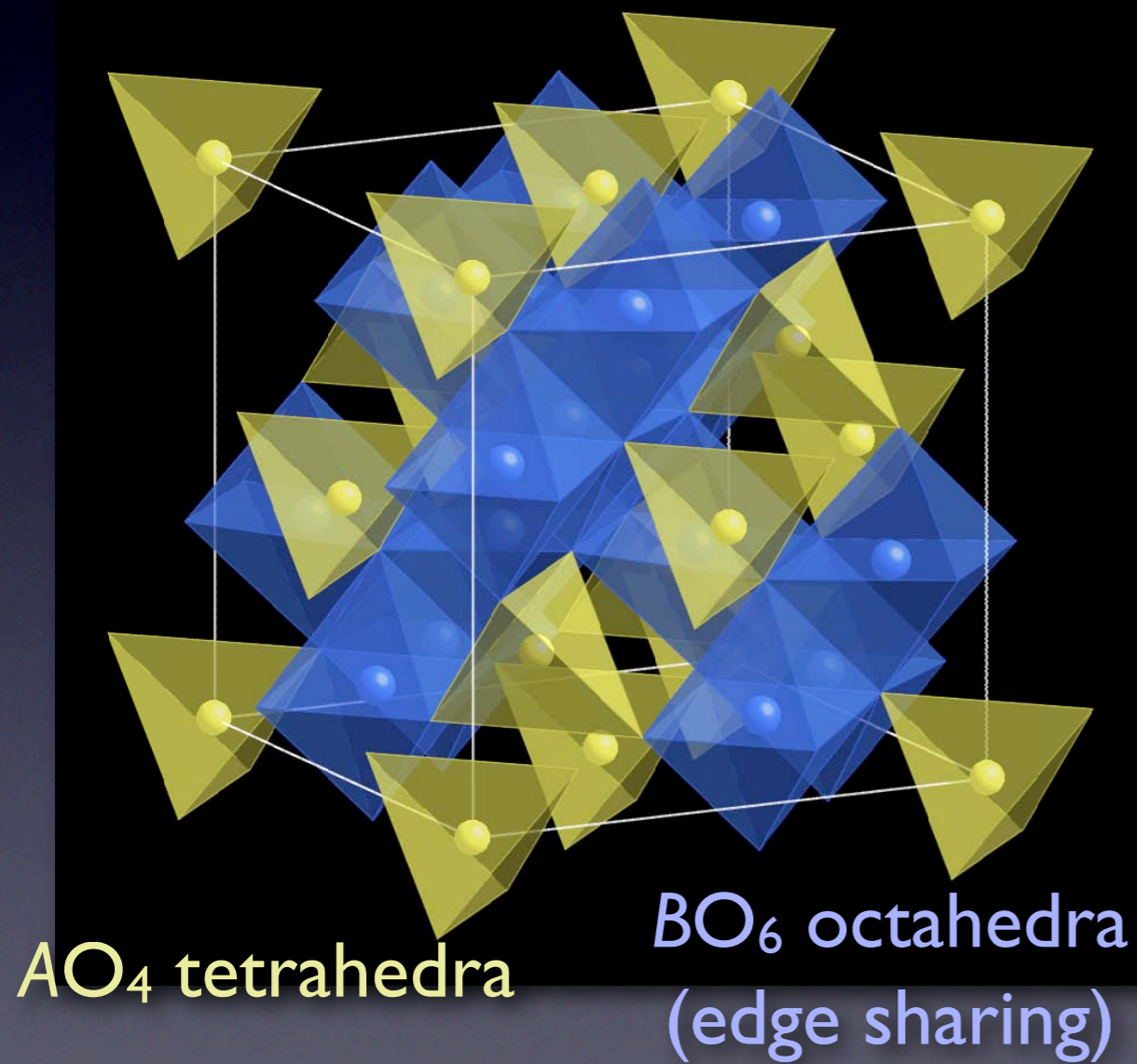
Outline

- introduction to spinels and t_{2g} orbital physics
- controversy on orbital ordering in ZnV_2O_4
 - different models for spin/orbital order in ZnV_2O_4 : relative importance of Kugel-Khomskii superexchange, Jahn-Teller and relativistic spin-orbit couplings
 - symmetry analysis: lesson from experiments in MnV_2O_4
- self-organized 7-site cluster (heptamer) in AlV_2O_4
 - heptamer scenario: 'molecule' of bonding states with anisotropic t_{2g} orbitals
 - implication to heavy-fermion compound LiV_2O_4

Lattice Structure of Spinel AB_2O_4

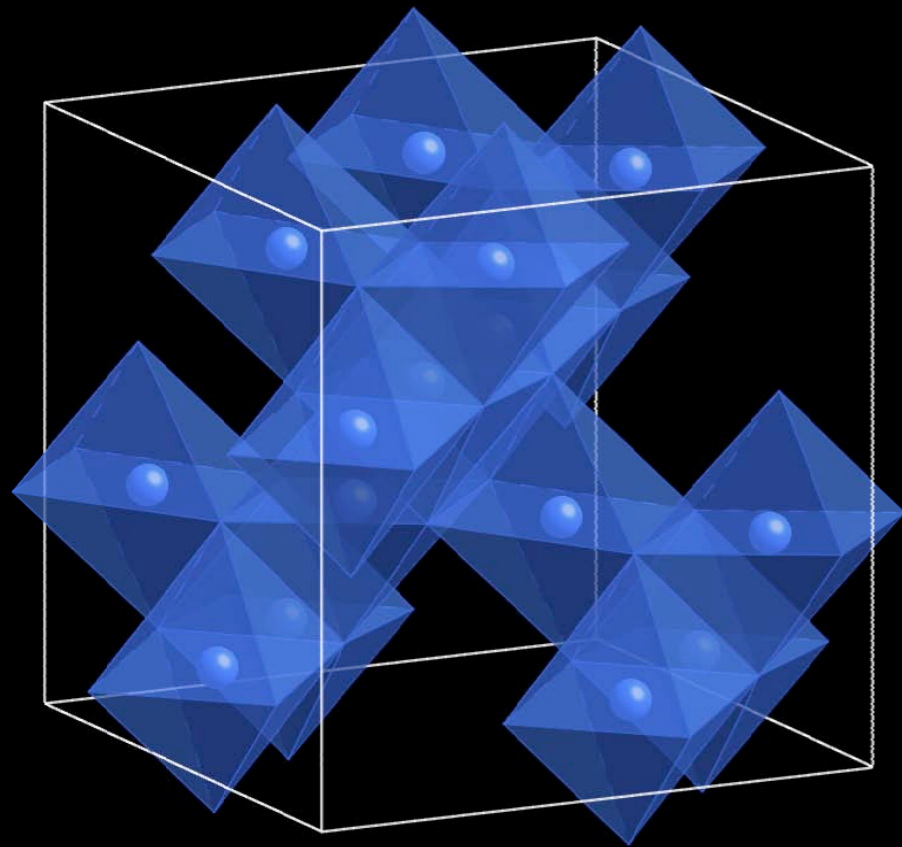


Lattice Structure of Spinel AB_2O_4



- *B* spinels: A-site cations are nonmagnetic

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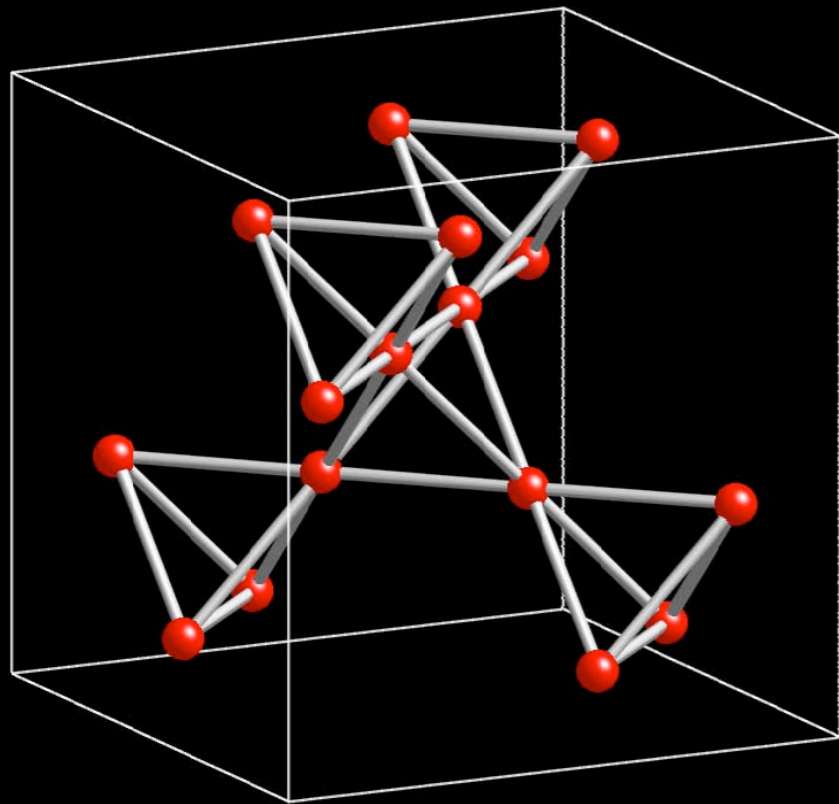


BO_6 octahedra
(edge sharing)

- B spinels: A-site cations are nonmagnetic
- 3D network of **edge-sharing** BO_6 octahedra

Lattice Structure of Spinel AB_2O_4

pyrochlore lattice



- B spinels: A-site cations are nonmagnetic
- 3D network of **edge-sharing** BO_6 octahedra
- 3D network of corner-sharing B_4 tetrahedra → pyrochlore lattice: **strong geometrical frustration**

B Spinelns with t_{2g} Electrons

| d^1 MgTi_2O_4 | d^2 AV_2O_4 (A=Zn,Mg) | d^3 ACr_2O_4 (A=Cd,Hg,Zn) |
|---|--|--|
| <ul style="list-style-type: none"> • metal-insulator transition • spin-singlet ground state • helical dimerization • orbital-Peierls scenario | <ul style="list-style-type: none"> • two successive transitions • complicated AF ordering • dimensionality reduction • competition between spin and orbital degrees of freedom | <ul style="list-style-type: none"> • single transition • half-magnetization plateau • spin-lattice coupling (spin Jahn-Teller mechanism) • self-organized 'hexamer' in high-T para phase |
| $d^{0.5}$ LiTi_2O_4 | $d^{1.5}$ LiV_2O_4 | $d^{2.5}$ AlV_2O_4 |
| <ul style="list-style-type: none"> • superconductivity below 12.4 K (BCS mechanism) | <ul style="list-style-type: none"> • metallic down to 300 mK • absence of any transition • heavy-fermion behavior • metal-insulator transition by applying pressure | <ul style="list-style-type: none"> • structural transition with spin-singlet formation • self-organized 7-site cluster 'heptamer' ? |

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Three main questions related to the physics of orbital degrees of freedom came to the fore in the discussion of Wednesday (biased view --JvdB).

We know that orbitals can order and that they couple to the lattice, but the questions are:

1. is there any material in which the *quantum* character of orbital degrees of freedom become relevant?
2. are there any cases where orbital *fluctuations*, either quantum or classical are relevant?
3. does orbital ordering have interesting *textures*, *symmetries* and/or *excitations*?

Also 15 more or less detailed discussion topics came up:

1. What is the role of vibronic coupling in cooperative Jahn-Teller systems
2. The importance of relativistic spin orbit coupling in eg and t2g systems
3. Orbital and frustration: frustration due to orbital degrees of freedom --- orbitals in frustrated lattices
4. Relative importance of electron-lattice effects (Jahn Teller) versus electronic effects (superexchange).
5. Role of geometry: differences for the situation of 180 degree O-TM-O bonds, 90 degree O-TM-O bonds and edge sharing octahedra
6. Reduced dimensionality due to orbitals
7. Importance of direct d-d electronic hopping versus d-oxygen-d hopping, especially in t2g spinels
8. Orbitals in charge transfer insulators
9. Role of orbital degrees of freedom at metal-insulator transitions
10. Orbital liquids -- quantum effects
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12. Importance of long-range interactions in short-range orbital (cooperative Jahn Teller) models
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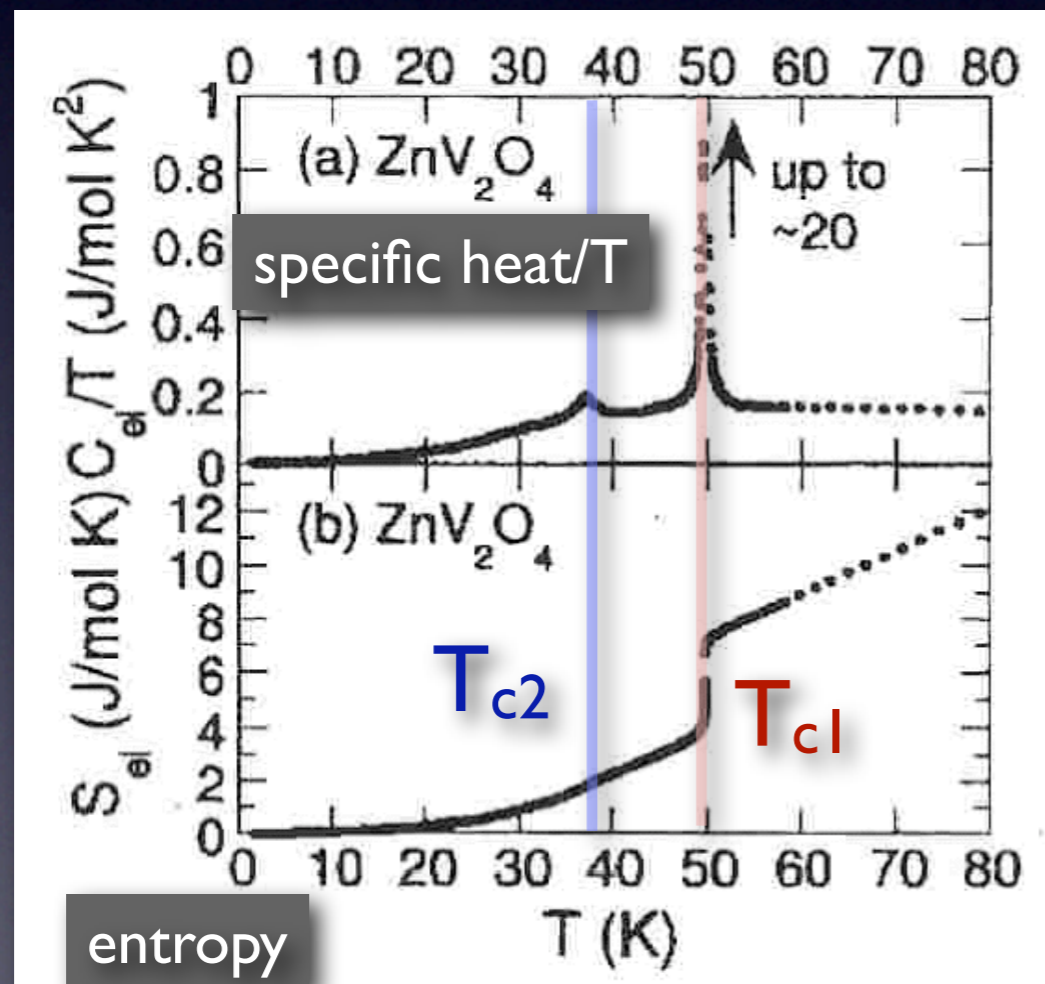
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Two Transitions and Controversy on Orbital Ordering in ZnV_2O_4

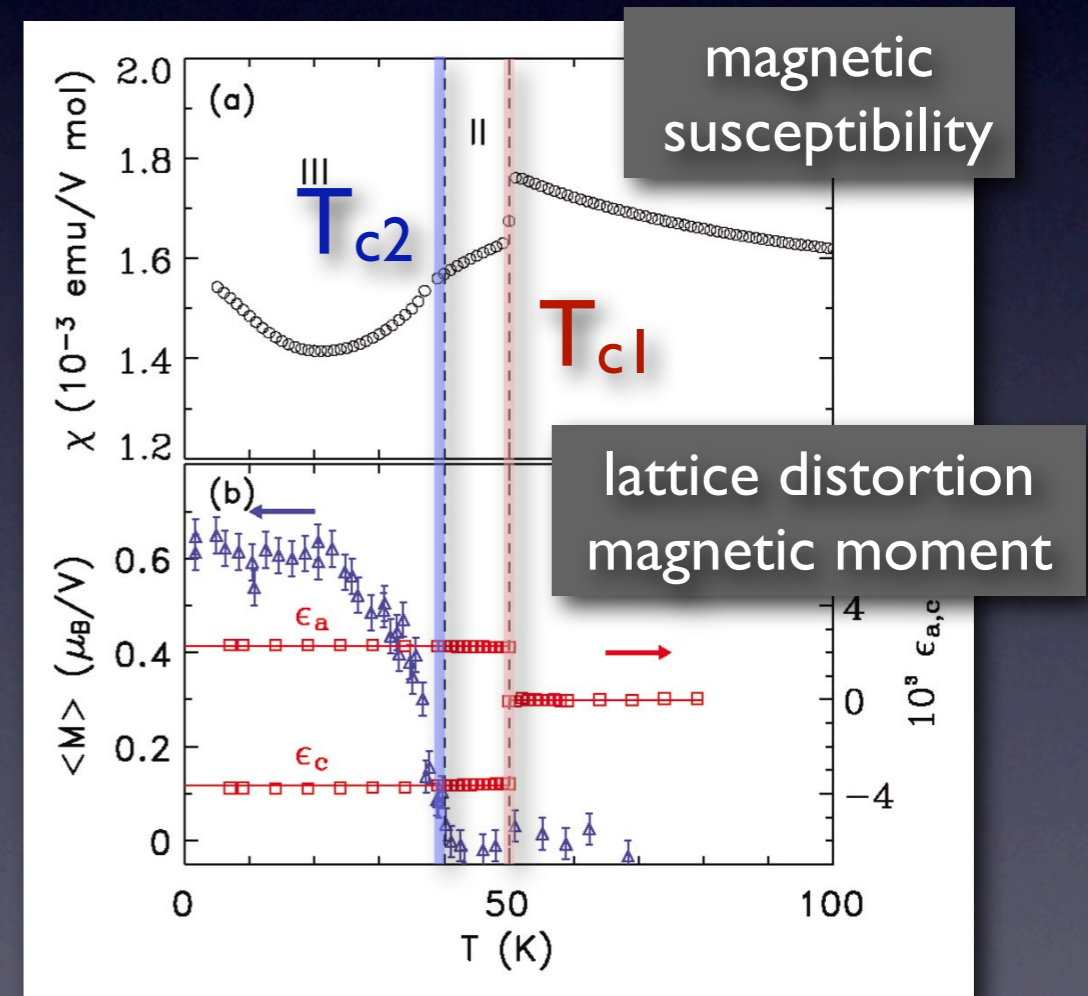
in collaboration with Hirokazu Tsunetsugu

Two Transitions in ZnV_2O_4

- cubic to tetragonal transition at $T_{c1} \sim 50\text{K}$ (1st order)
- antiferromagnetic transition at $T_{c2} \sim 40\text{K}$ (2nd order)



Kondo et al., 2000

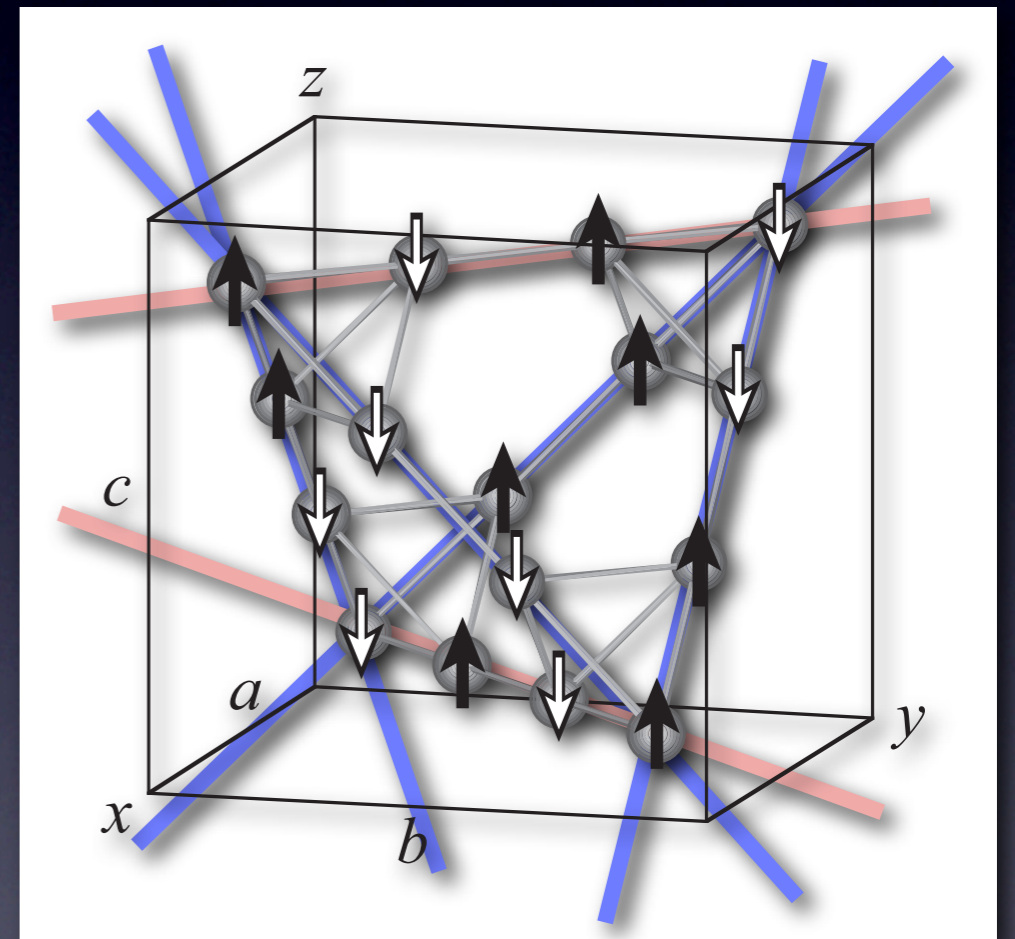


Ueda et al., 1997

Lee et al., 2004

Lattice symmetry and Magnetic Order

- lattice symmetry: $I4_1/amd$
(powder sample)
- orbital order: undetermined
- spin order: antiferromagnetic
 $\uparrow-\downarrow-\uparrow-\downarrow-\dots$ in the xy chains
 $\uparrow-\uparrow-\downarrow-\downarrow-\dots$ in the yz/zx chains
- moment at $T=0 \sim 0.6\mu_B$



Niziol, 1973

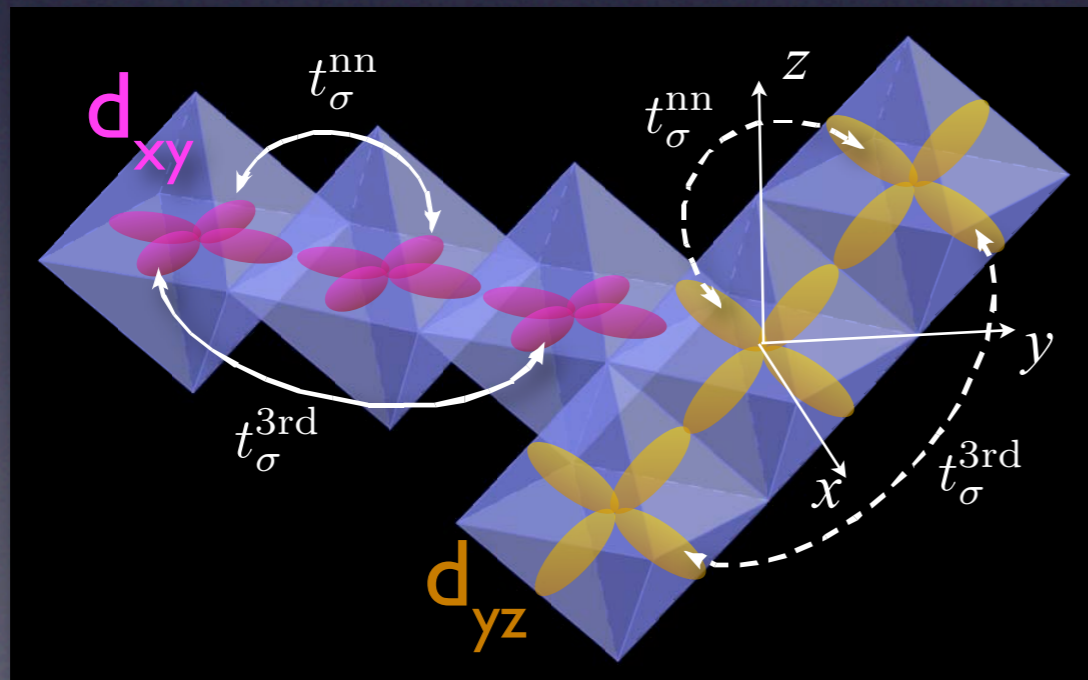
Questions

- 📌 What is the microscopic mechanism of two transitions? Who is the main player? Kugel-Khomskii superexchanges, Jahn-Teller or relativistic spin-orbit coupling?
- 📌 How is the complex AF ordering stabilized? Why is the moment at $T=0$ reduced so largely?
- 📌 What is the role of orbital degree of freedom? Is there orbital ordering? If yes, what type of ordering sets in?

Model

Tsunetsugu and Motome (2003, 2004, 2005)

- Kugel-Khomskii type model derived from 3-fold multi-orbital Hubbard model + tetragonal Jahn-Teller coupling
- assumptions: σ -type transfer integrals only, classical phonon, neglecting spin-orbit coupling and trigonal distortion



$$t_{\sigma}^{nn} = \sim -0.32\text{eV}$$

$$t_{\sigma}^{3rd} = \sim -0.045\text{eV}$$

(Matsuno *et al.*, 1999: for LiV₂O₄)

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$$H_{\text{SO}}^{\text{nn}} = -J \sum_{\langle ij \rangle} [h_{\text{o-AF}}^{(ij)} + h_{\text{o-F}}^{(ij)}] \quad \text{: nearest neighbor term}$$
$$H_{\text{SO}}^{\text{3rd}} = -J_3 \sum_{\langle\langle ij \rangle\rangle} [h_{\text{o-AF}}^{(ij)} + h_{\text{o-F}}^{(ij)}] \quad \text{: 3rd neighbor term}$$
$$J = (t_{\sigma}^{\text{nn}})^2 / U \quad A = (1 - \eta) / (1 - 3\eta)$$
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$$\bar{n}_{i\alpha} = 1 - n_{i\alpha} \quad C = (1 + \eta) / (1 + 2\eta)$$
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$$h_{\text{o-AF}}^{(ij)} = (A + B \vec{S}_i \cdot \vec{S}_j) (n_{i\alpha(ij)} \bar{n}_{j\alpha(ij)} + \bar{n}_{i\alpha(ij)} n_{j\alpha(ij)})$$

$$h_{\text{o-F}}^{(ij)} = C (1 - \vec{S}_i \cdot \vec{S}_j) n_{i\alpha(ij)} n_{j\alpha(ij)}$$

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spin part: Heisenberg / orbital part: 3-state Potts

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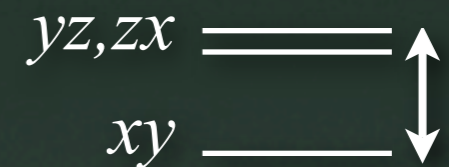
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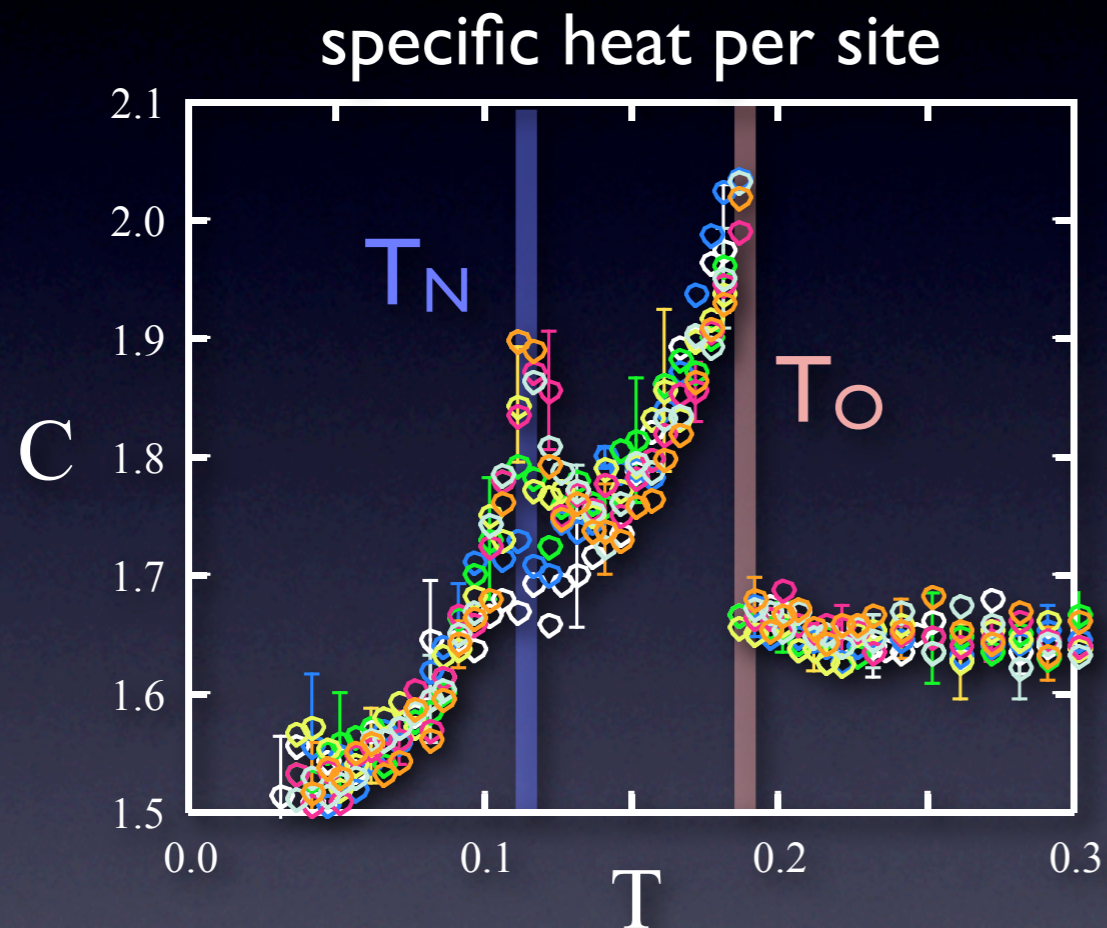
spin part: Heisenberg / orbital part: 3-state Potts

+ tetragonal Jahn-Teller coupling

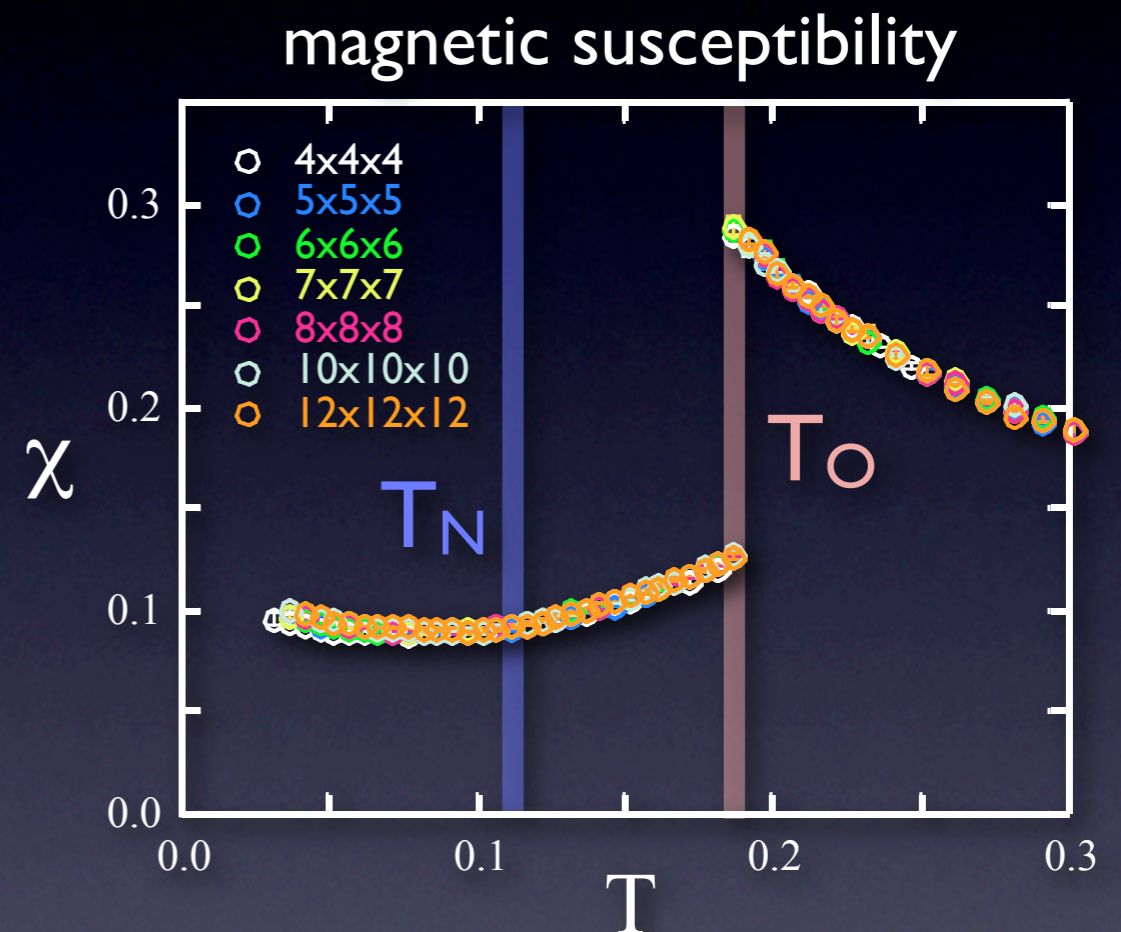
$$H_{\text{JT}} = g \sum_i Q_i (n_{i,yz} + n_{i,zx} - 2n_{i,xy}) + \sum_i Q_i^2 / 2 - \lambda \sum_{\langle ij \rangle} Q_i Q_j$$



Monte Carlo Results

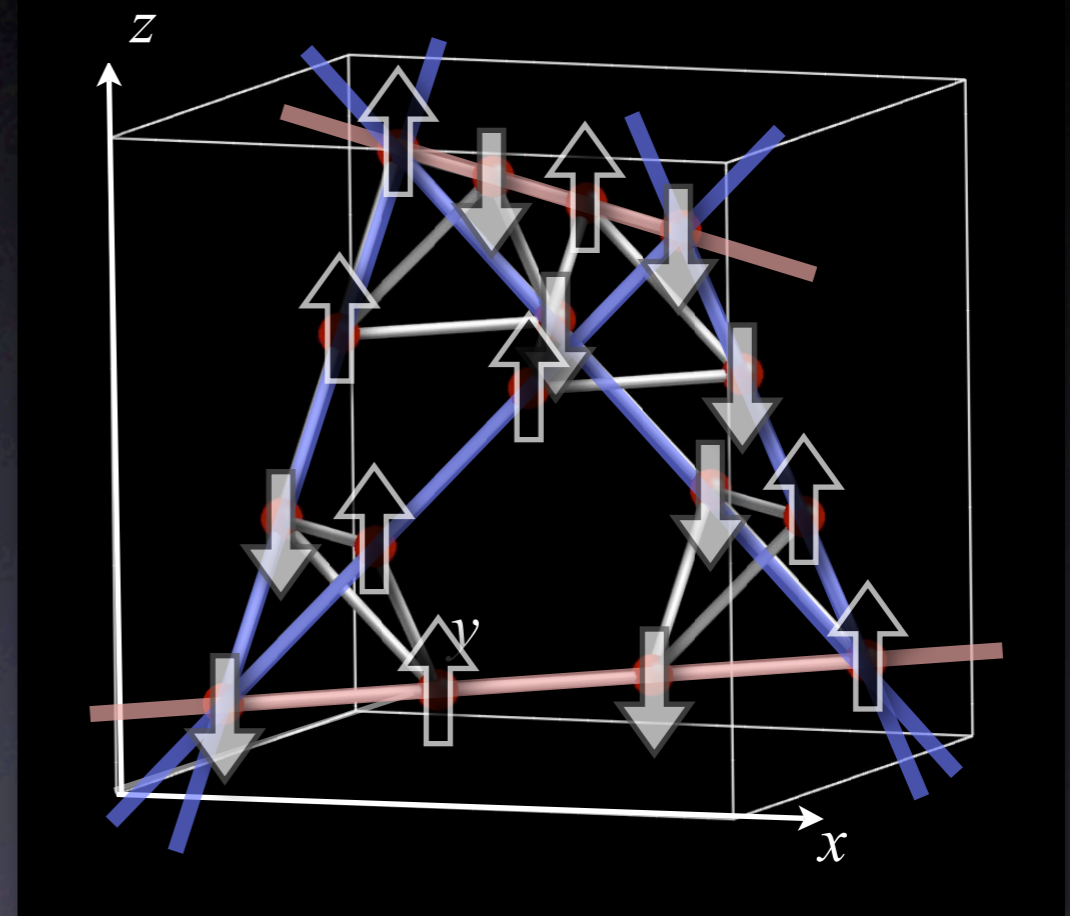
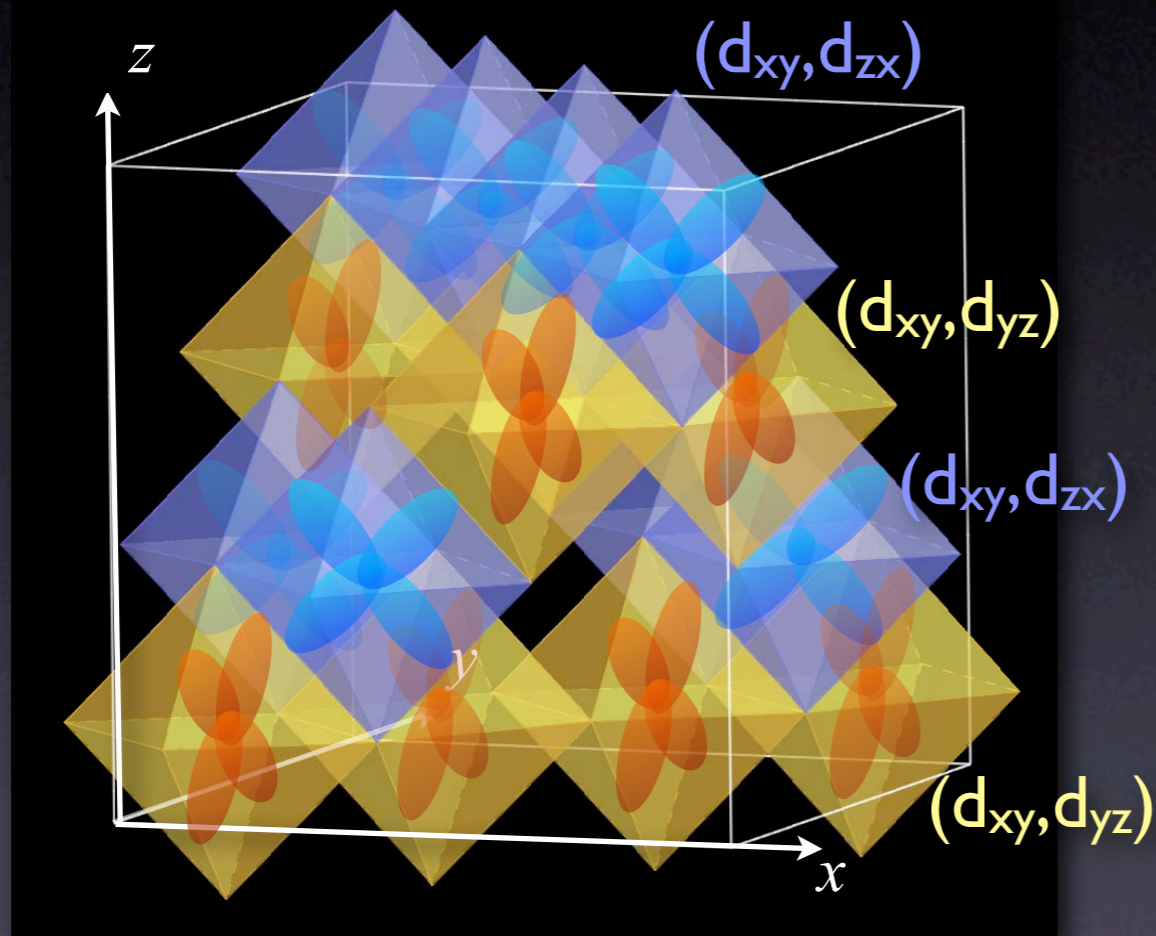


- 1st order at $T=T_O$,
2nd order at $T=T_N$
- consistent estimates
of entropy changes



- sudden drop at $T=T_O$
- tiny change at $T=T_N$

Orbital and Spin Structure



- **orbital:** alternative stacking of (d_{xy}, d_{zx}) and (d_{xy}, d_{yz}) states
- **spin:** $\uparrow-\downarrow-\uparrow-\downarrow-$ in the xy chains and $\uparrow-\uparrow-\downarrow-\downarrow-$ in the yz/zx chains

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spin correlations hardly develop
by themselves alone

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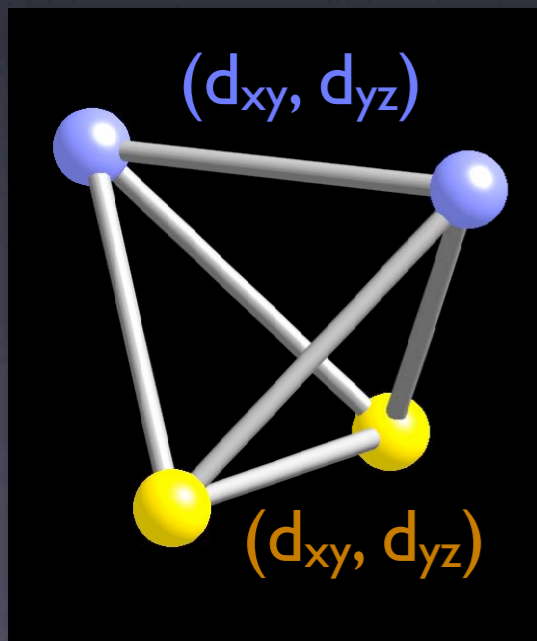
3-state Potts model with AF interactions which depend on both the bond direction and the orbital states

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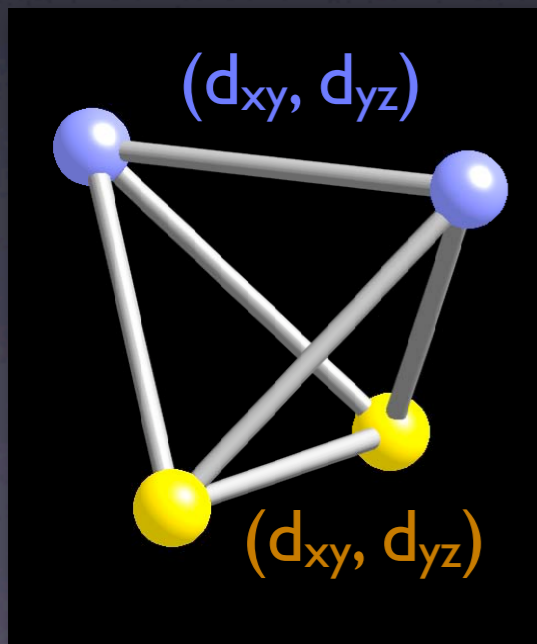
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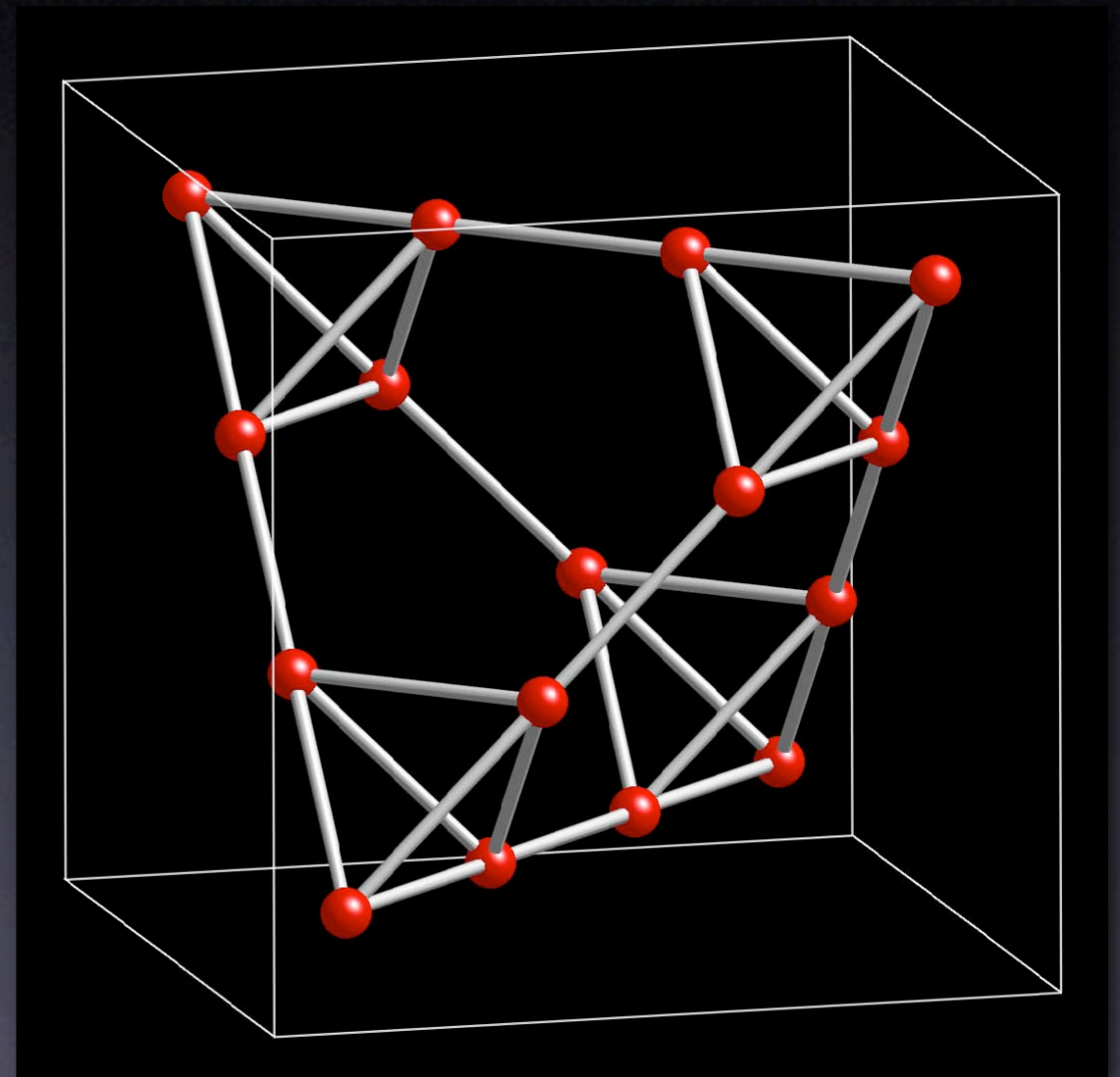
3-state Potts model with AF interactions which depend on both the bond direction and the orbital states

→ (partial) lifting of degeneracy

tetragonal Jahn-Teller distortion assists to stabilize this orbital configuration

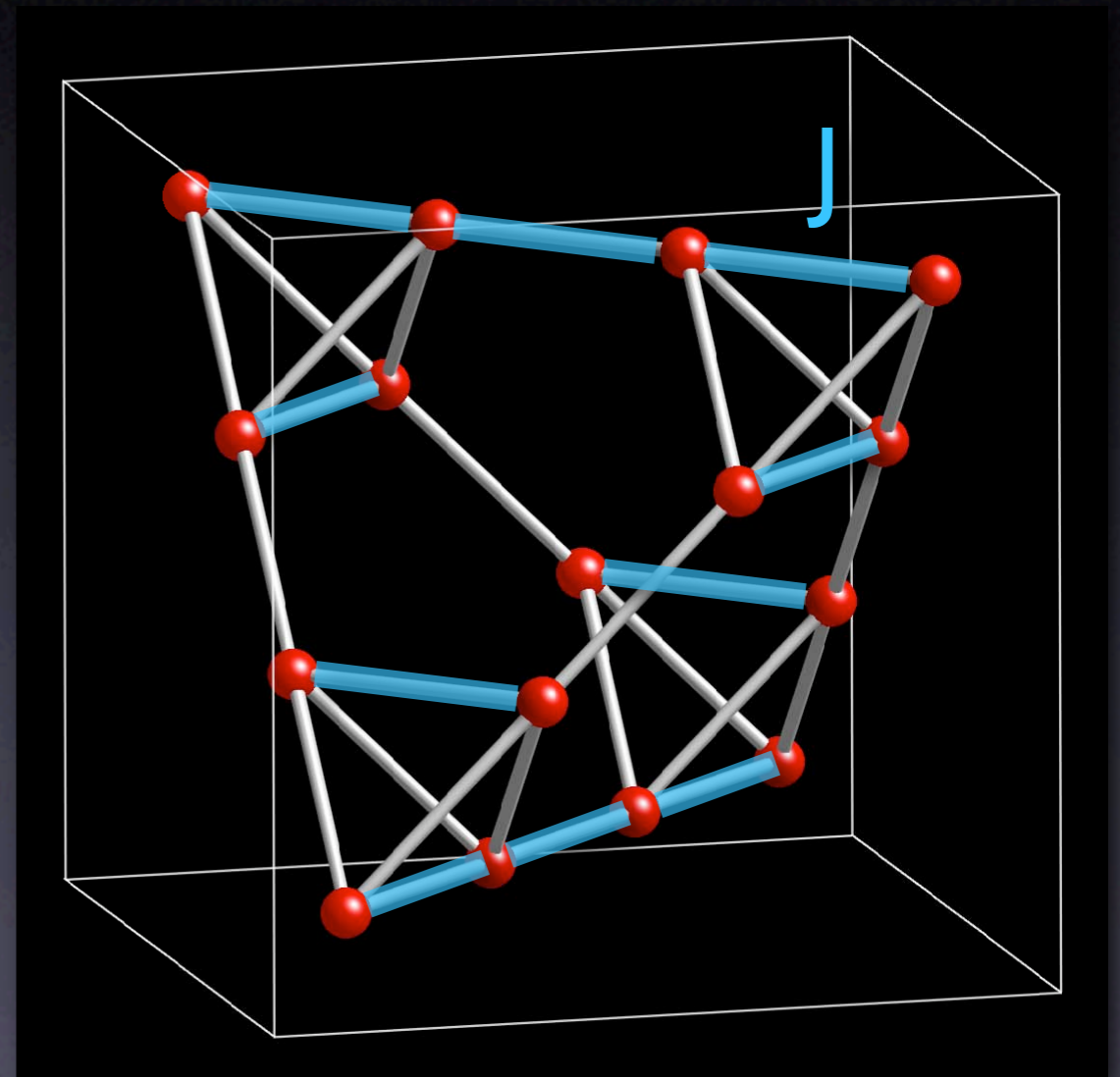


Effective Spin Exchanges under the Orbital Order



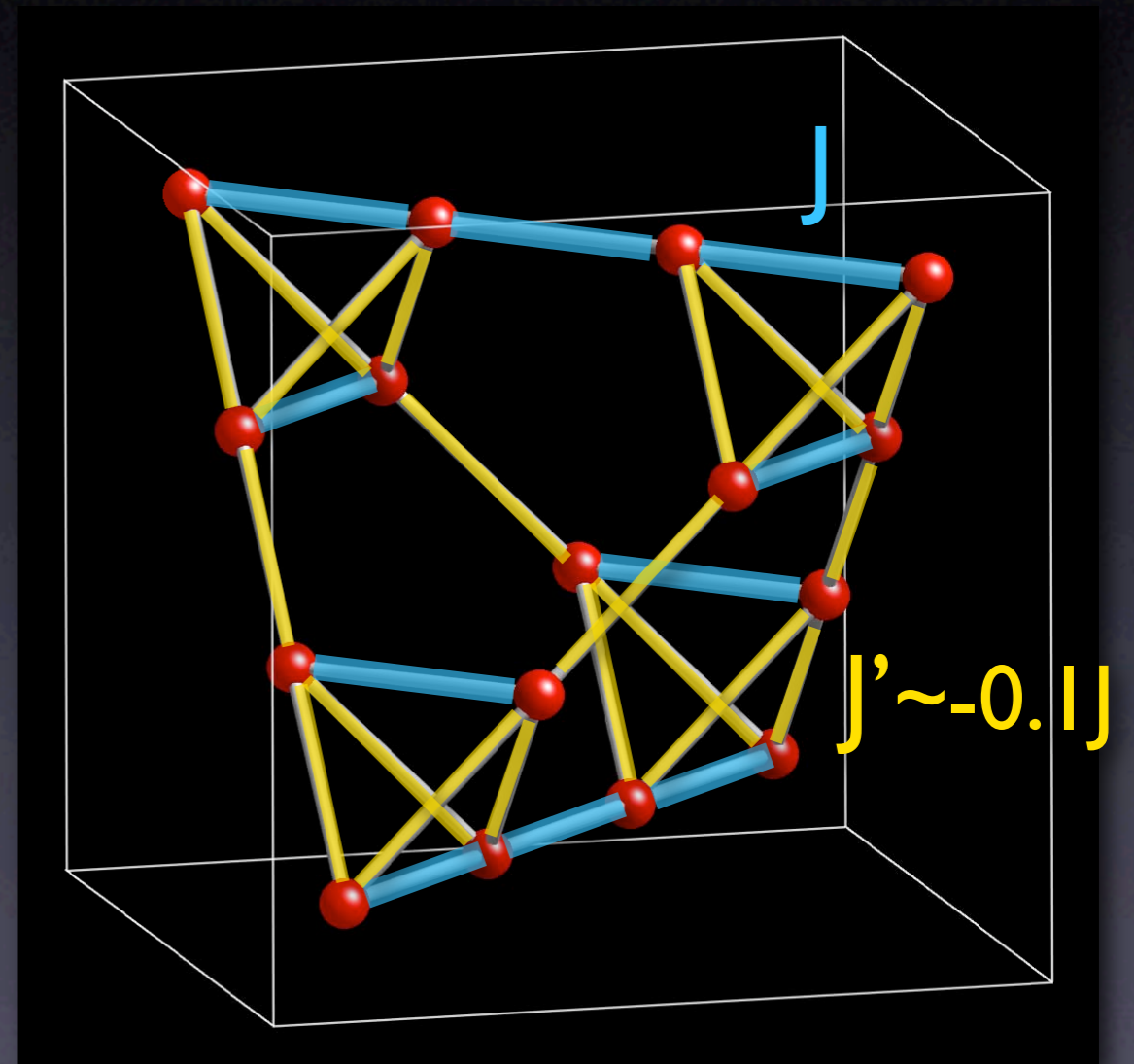
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- d_{xy} is singly occupied at all the sites \rightarrow strong AF exchange in the xy chains J



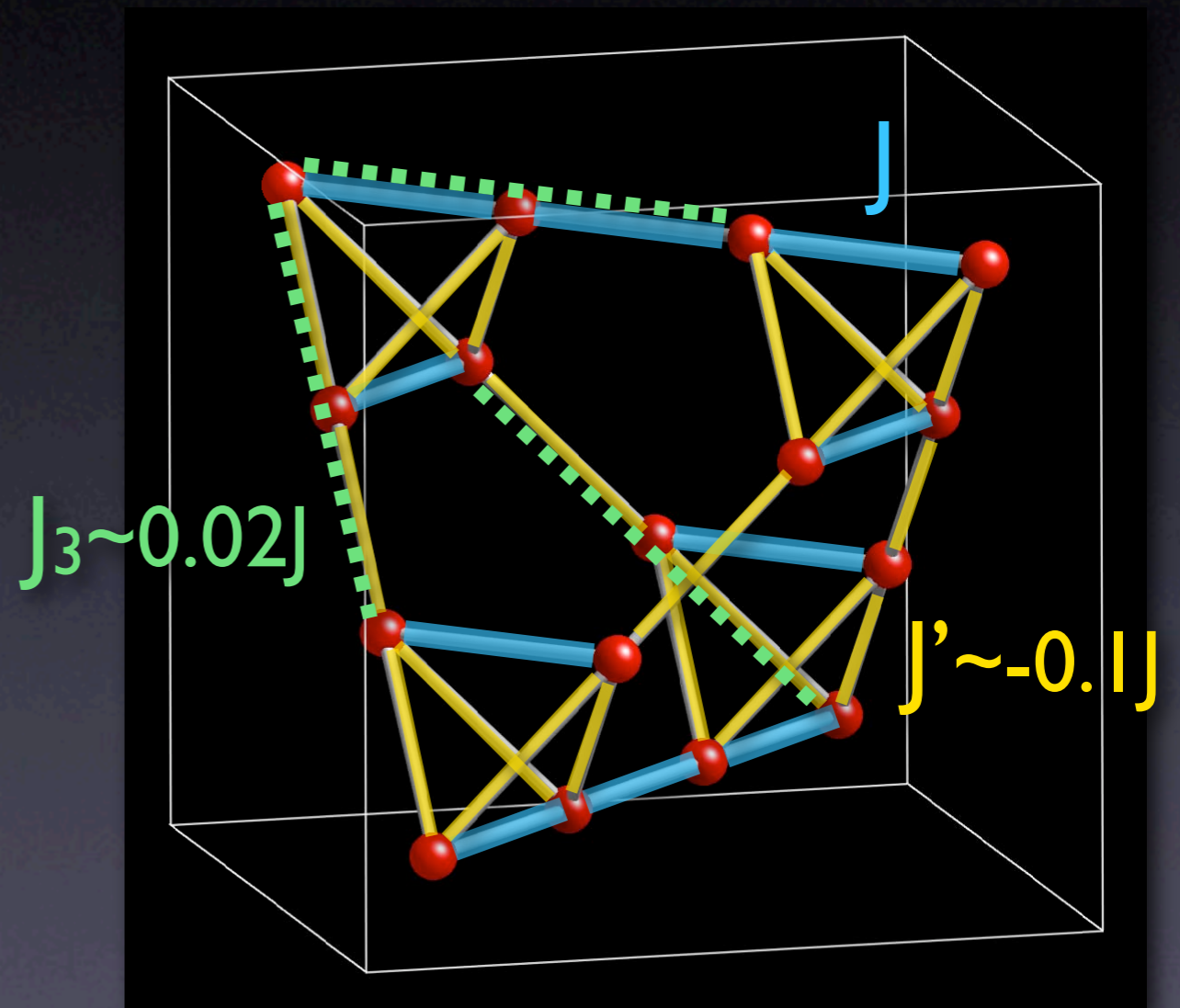
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- n.n. exchange couplings in the yz/zx chains J' are ferromagnetic and about **10 times weaker** than the AF exchange in the xy chains J



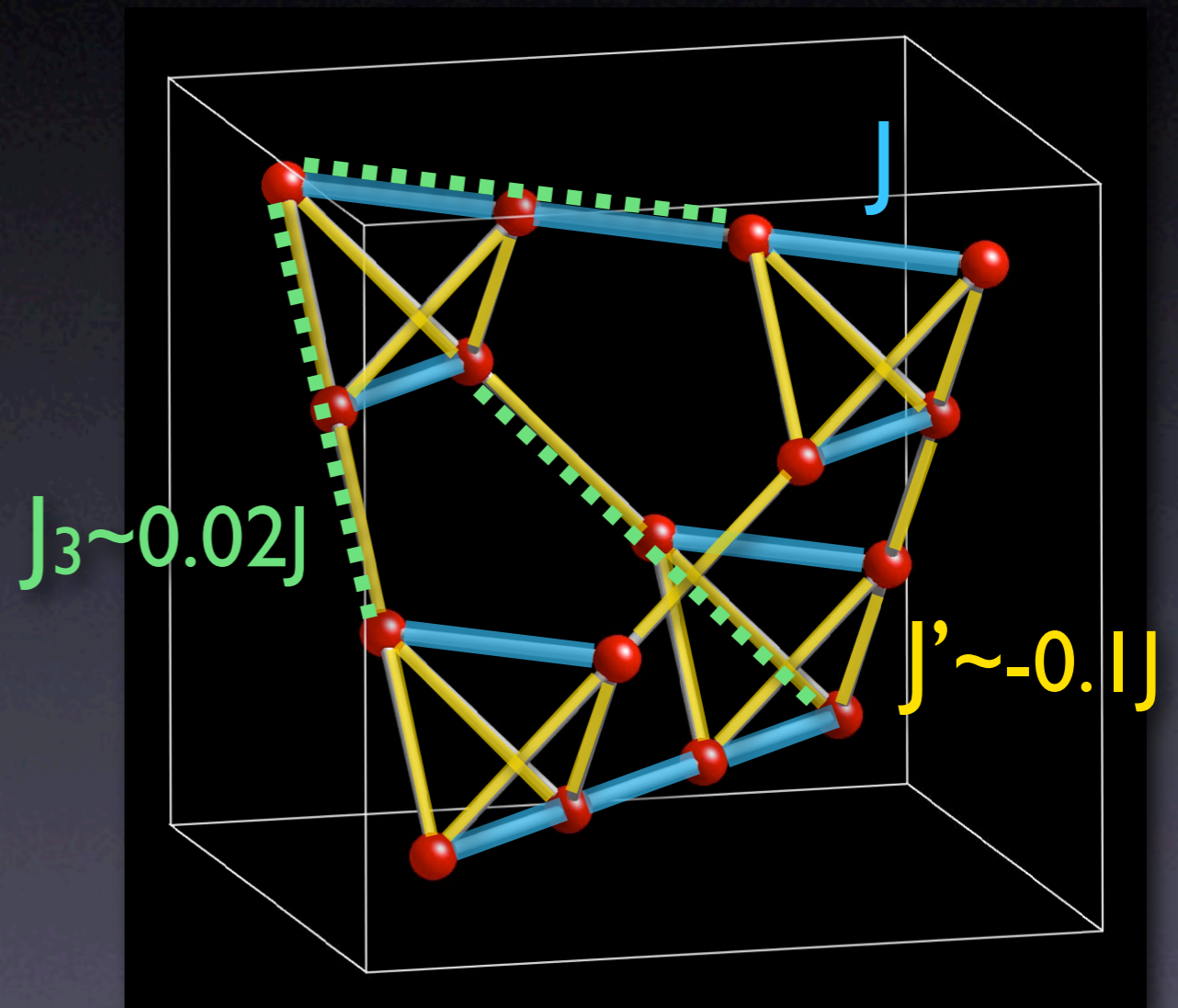
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- n.n. exchange couplings in the yz/zx chains J' are ferromagnetic and about **10 times weaker** than the AF exchange in the xy chains J
- 3rd-neighbor exchange J_3 is $\sim 0.02J$ \rightarrow AF order at T_N



Effective Spin Exchanges under the Orbital Order

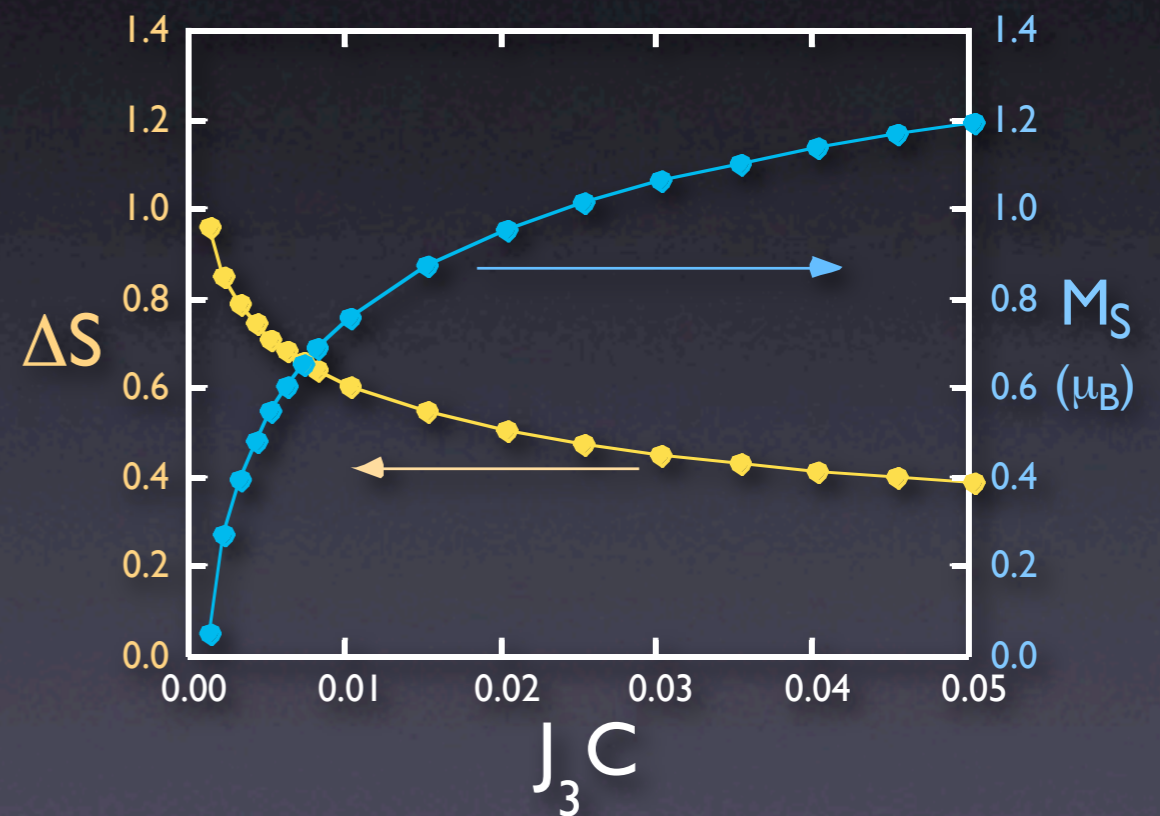
- d_{xy} is singly occupied at all the sites \rightarrow **strong AF exchange in the xy chains J**
- n.n. exchange couplings in the yz/zx chains J' are ferromagnetic and about **10 times weaker** than the AF exchange in the xy chains J
- 3rd-neighbor exchange J_3 is $\sim 0.02J$ \rightarrow AF order at T_N



weakly-coupled 1D spin chains (dimensionality reduction)

Quasi-1D Quantum Fluctuation: Large Reduction of AF Moment

- linear spin-wave analysis for the spin and orbital ordered ground state
- moment reduction ΔS diverges logarithmically at $J_3=0$ due to the zero modes
- ΔS is large in the small J_3 region:
 $M_S \sim 1 \mu_B$ at $J_3 \sim 0.02$
consistent with the experimental result $\sim 0.6 \mu_B$ (Lee *et al.*, 2004)



Short Summary...

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- Kugel-Khomskii spin-orbital exchange + tetragonal Jahn-Teller
- classical Monte Carlo simulation and mean-field type analysis
- linear spin-wave analysis of effective spin model

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- Kugel-Khomskii spin-orbital exchange + tetragonal Jahn-Teller
- classical Monte Carlo simulation and mean-field type analysis
- linear spin-wave analysis of effective spin model
- ☑ two transitions with reasonable estimates of transition temperatures as well as entropy changes
- ☑ T-dep of magnetic susceptibility consistent with experiment
- ☑ magnetic order consistent with the neutron scattering result
- ☑ reduced magnetic moment at $T=0$
- ☑ A-type antiferro orbital order with tetragonal distortion

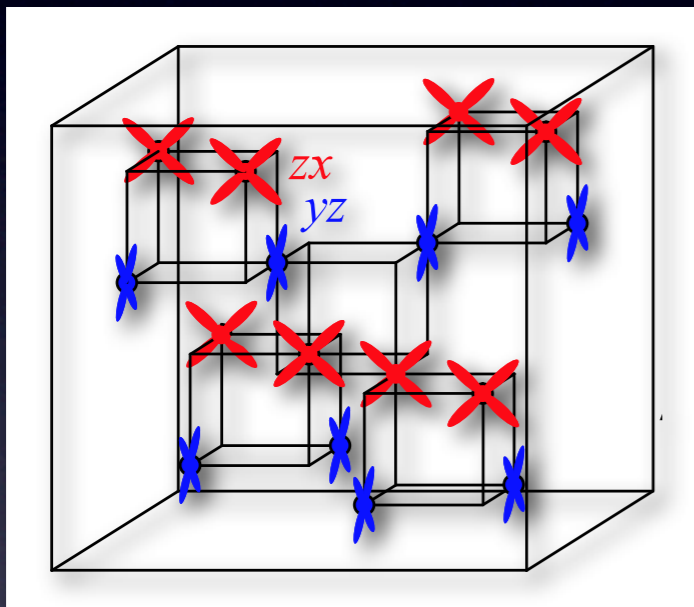
Three Different Models

In all models, xy orbital is singly occupied at all the sites (not shown in the figures)

Three Different Models

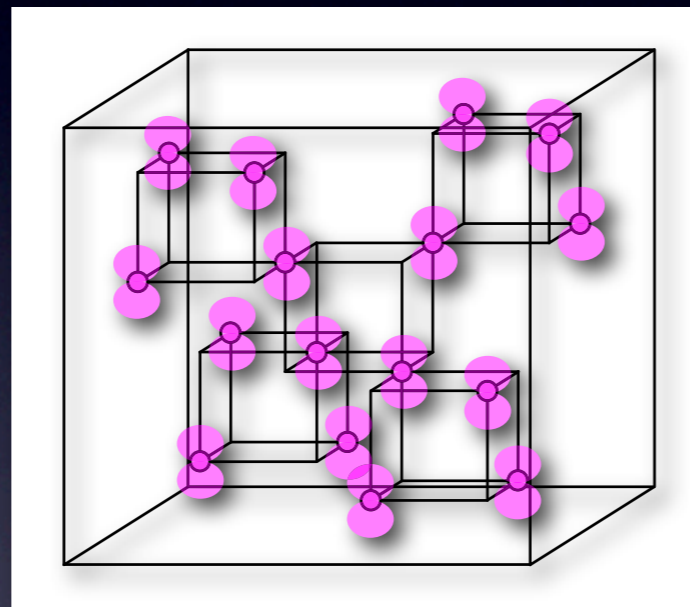
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Tsunetsugu-Motome, 2003



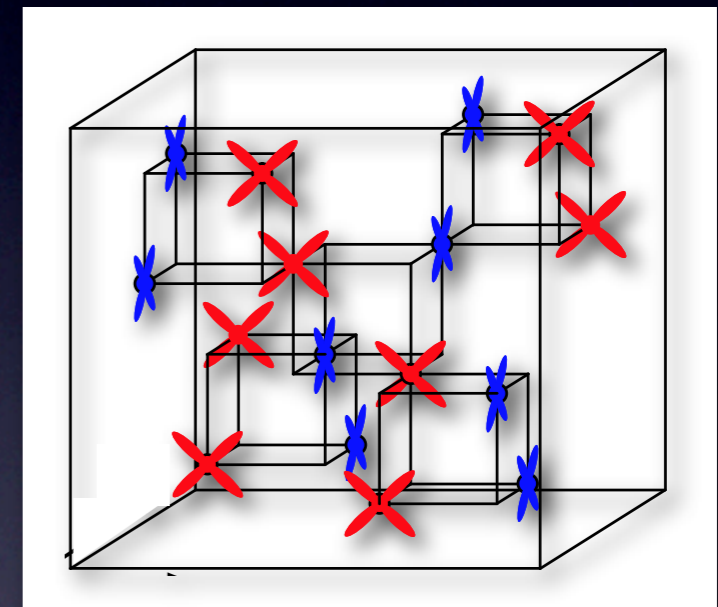
- A-type orbital order
- $I4_1/a$
- spin-orbital superexchanges

Tchernyshyov, 2004



- uniform orbital order
- $I4_1/amd$
- relativistic spin-orbit coupling

Khomskii-Mizokawa, 2005

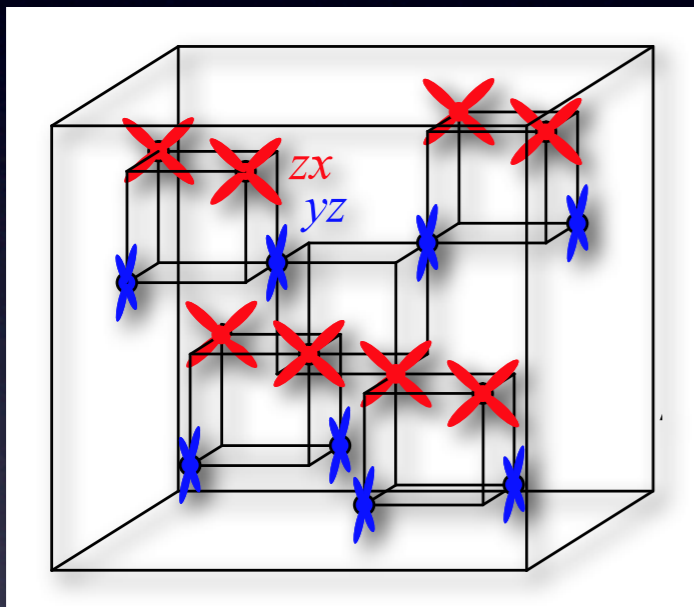


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- $P4_12_12$
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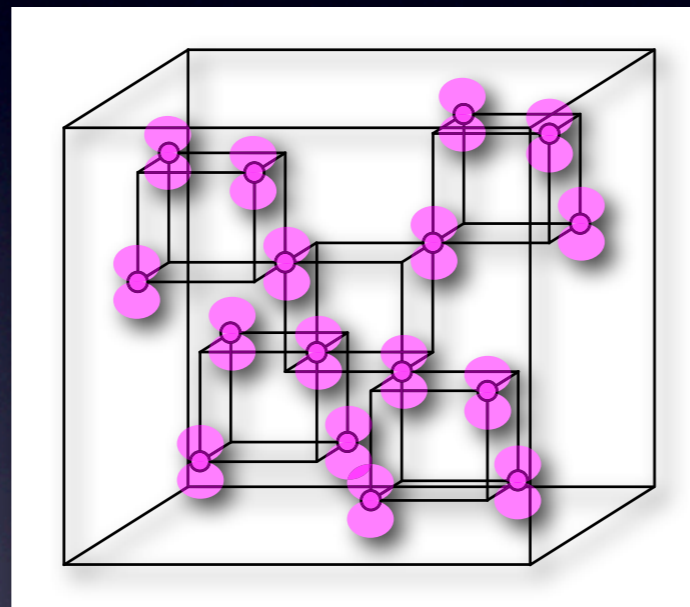
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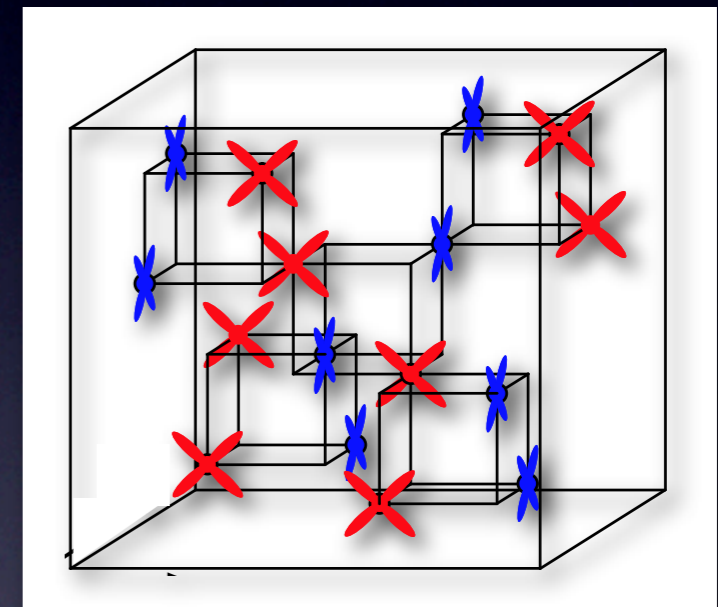
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- orbitally-driven Peierls order
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mean-field (Di Matteo *et al.*)
LSDA+U+SO (T. Maitra and R. Valenti)

Issue...

Issue...

-  role of relativistic spin-orbit interaction
 - orbital ordering at $T=0$: mean-field analysis and first-principle calculation suggest the relevant role
 - thermodynamics: single or two transitions? In general, systems with dominant spin-orbit coupling shows a single transition with concomitant ordering of spin and orbital.
 - reduced AF moment: due to dimensionality reduction and/or L-S coupling?

Issue...

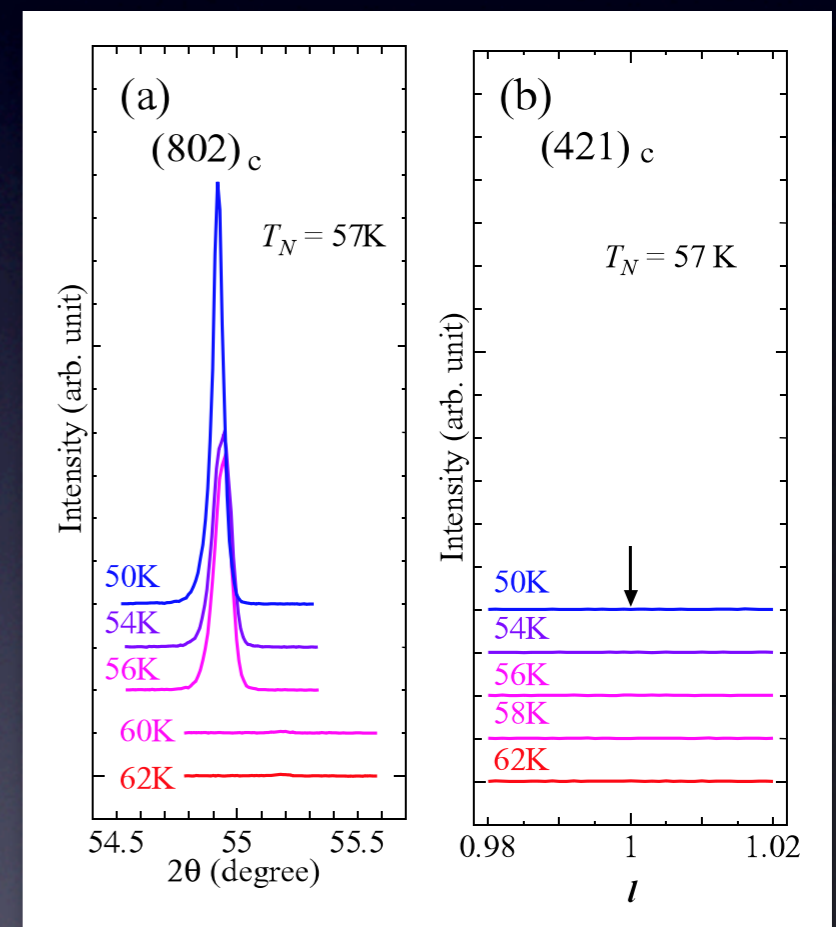
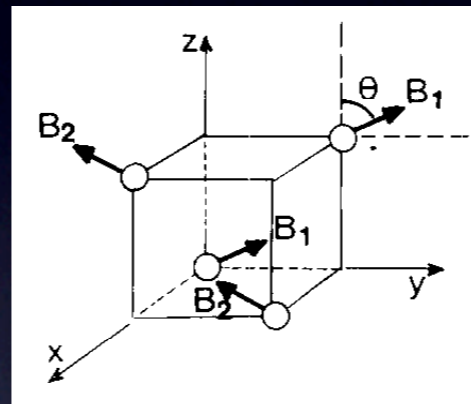
- 📌 role of relativistic spin-orbit interaction
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- Remark: X-ray diffraction has been done only for powder samples...

Lesson from Related Spinel

MnV₂O₄

- Mn²⁺ = (3d)⁵, V³⁺ = (3d)²
- single transition at 57K
 - cubic → tetragonal
 - non-collinear ferri
- low-T phase: *I*4₁/*a* (*large single crystal*)
 - diamond-glide symmetry is broken, but face-center symmetry is hold
 - ▶ peak intensity is ~10⁻⁴ times smaller compared to the fundamental peaks, difficult to observe in powder samples

Plumier and Sougi, 1987



Suzuki et al., 2007

Other Issues...

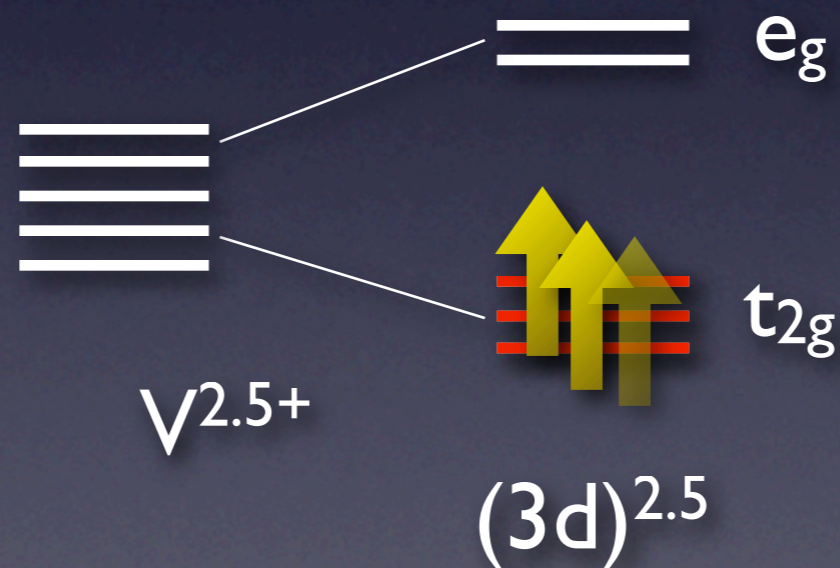
- 📌 role of trigonal distortion
 - quantitative difference in Cd compound
- 📌 d-d direct vs d-p-d (d-p-p-d) indirect transfers
- 📌 orbital and spin ordering in MnV_2O_4
- 📌 single crystal of ZnV_2O_4 !

Self-organized 7-site Cluster (heptamer) in AlV_2O_4

in collaboration with Keisuke Matsuda and Nobuo Furukawa

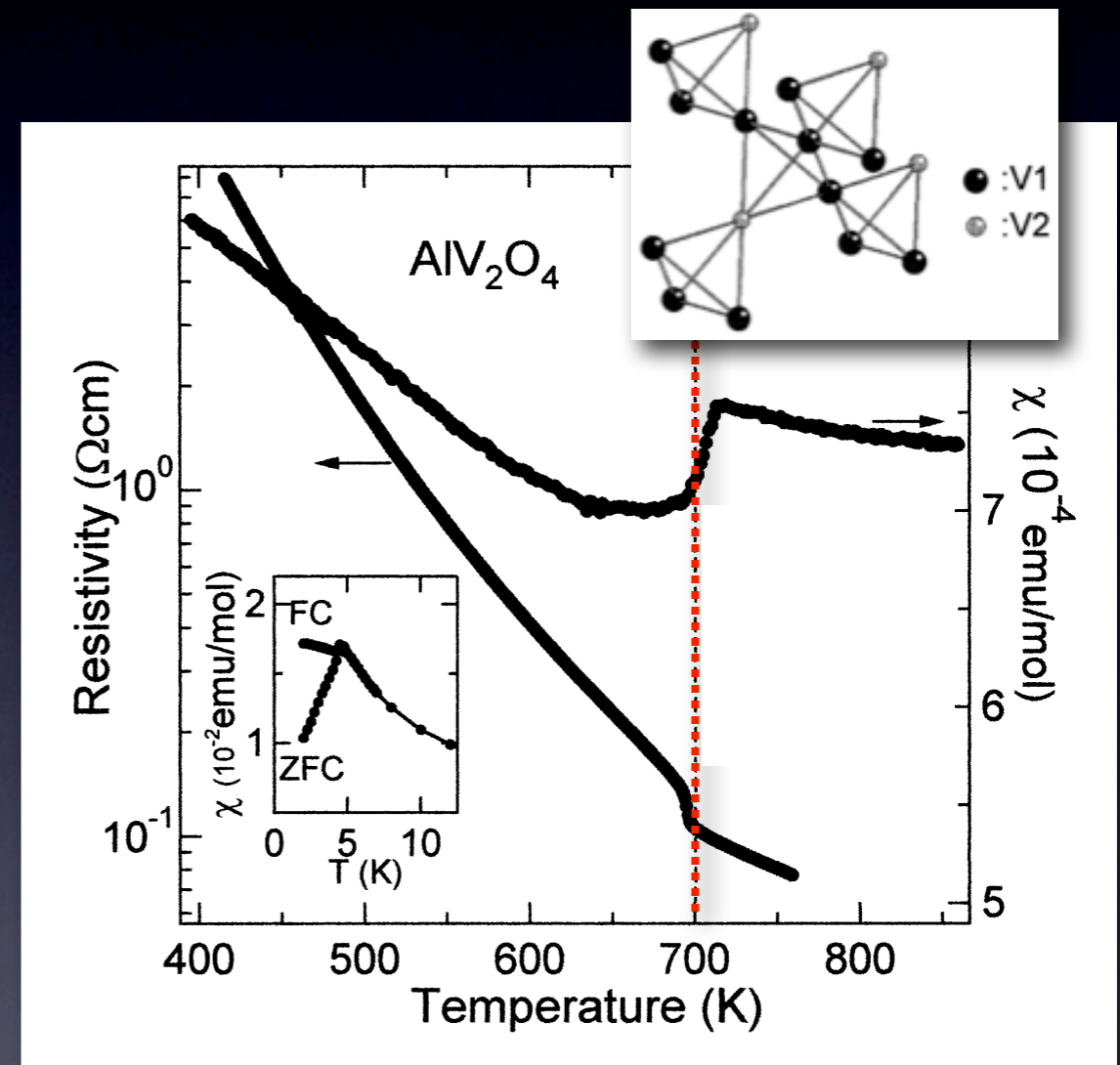
(Atomic) Electronic Structure in AlV_2O_4

- mixed valence: $\text{V}^{2.5+} = (3d)^{2.5}$
- charge, spin and orbital degrees of freedom are all active



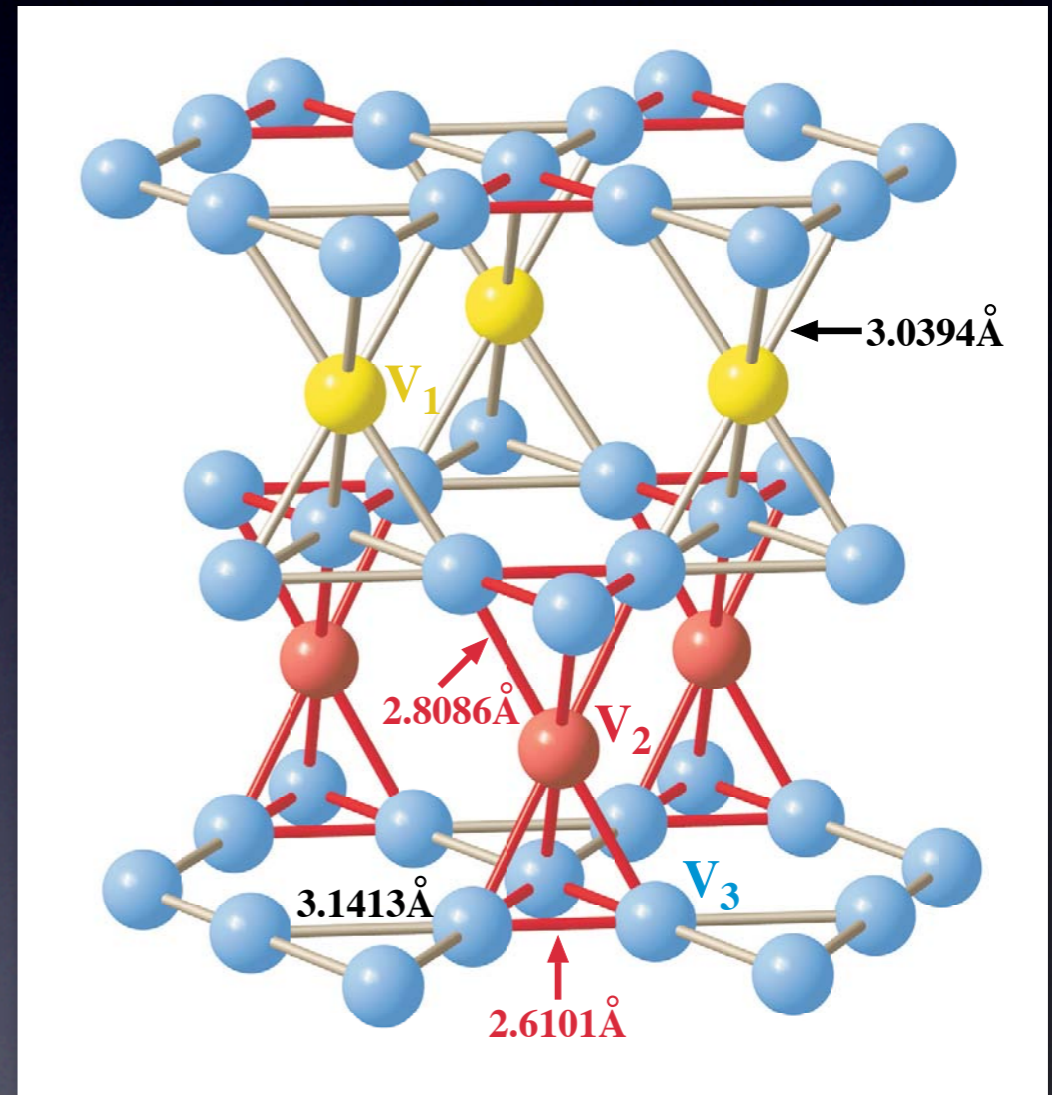
Phase Transition at $T \sim 700\text{K}$

- structural change: doubling of the unit cell along the $[111]$ direction
- shoulder in the resistivity
- sudden drop in the magnetic susceptibility followed by Curie behavior at lower temperatures
- valence-skipping-type charge ordering ?



K. Matsuno et al., 2001

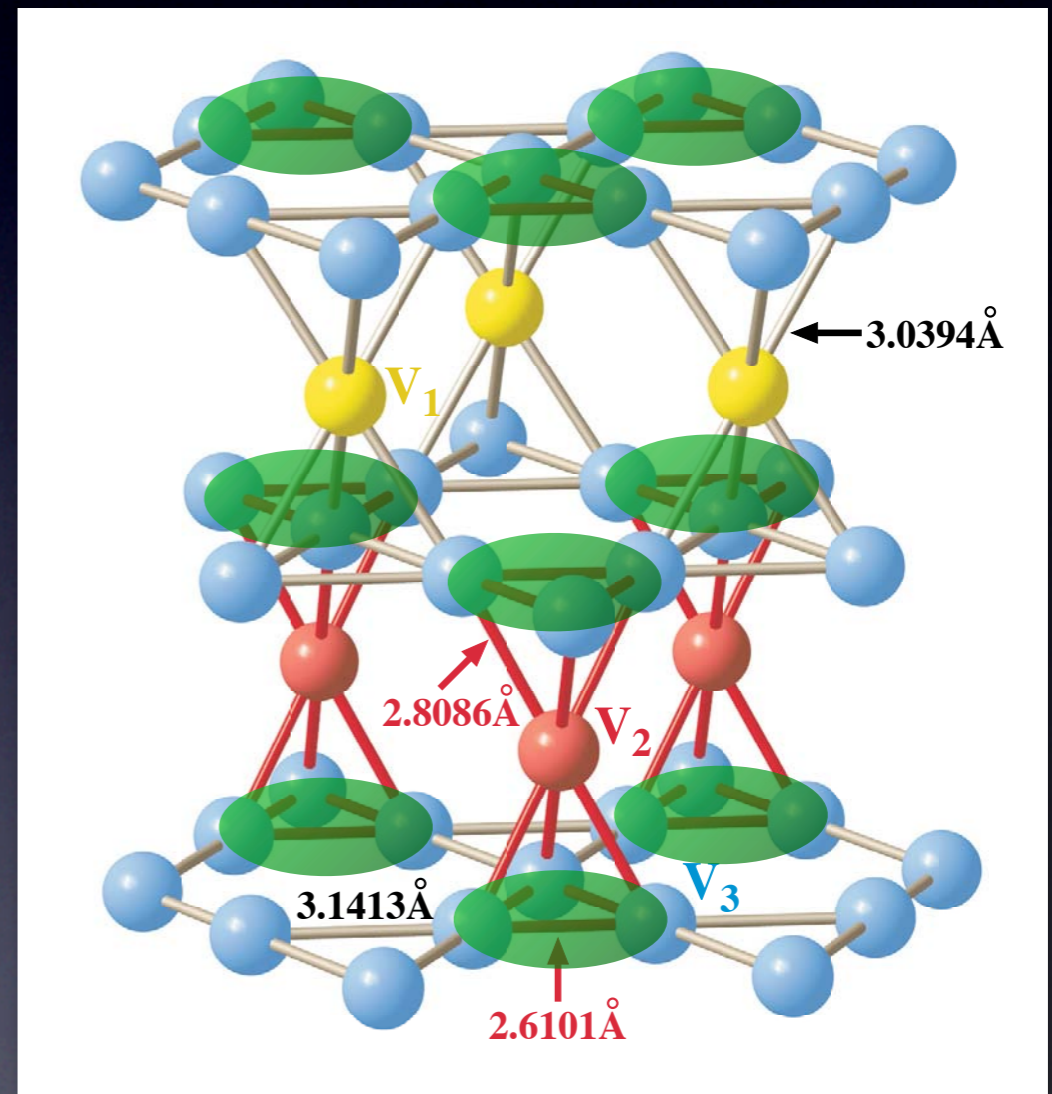
Heptamer Scenario



Y. Horibe *et al.*, 2006

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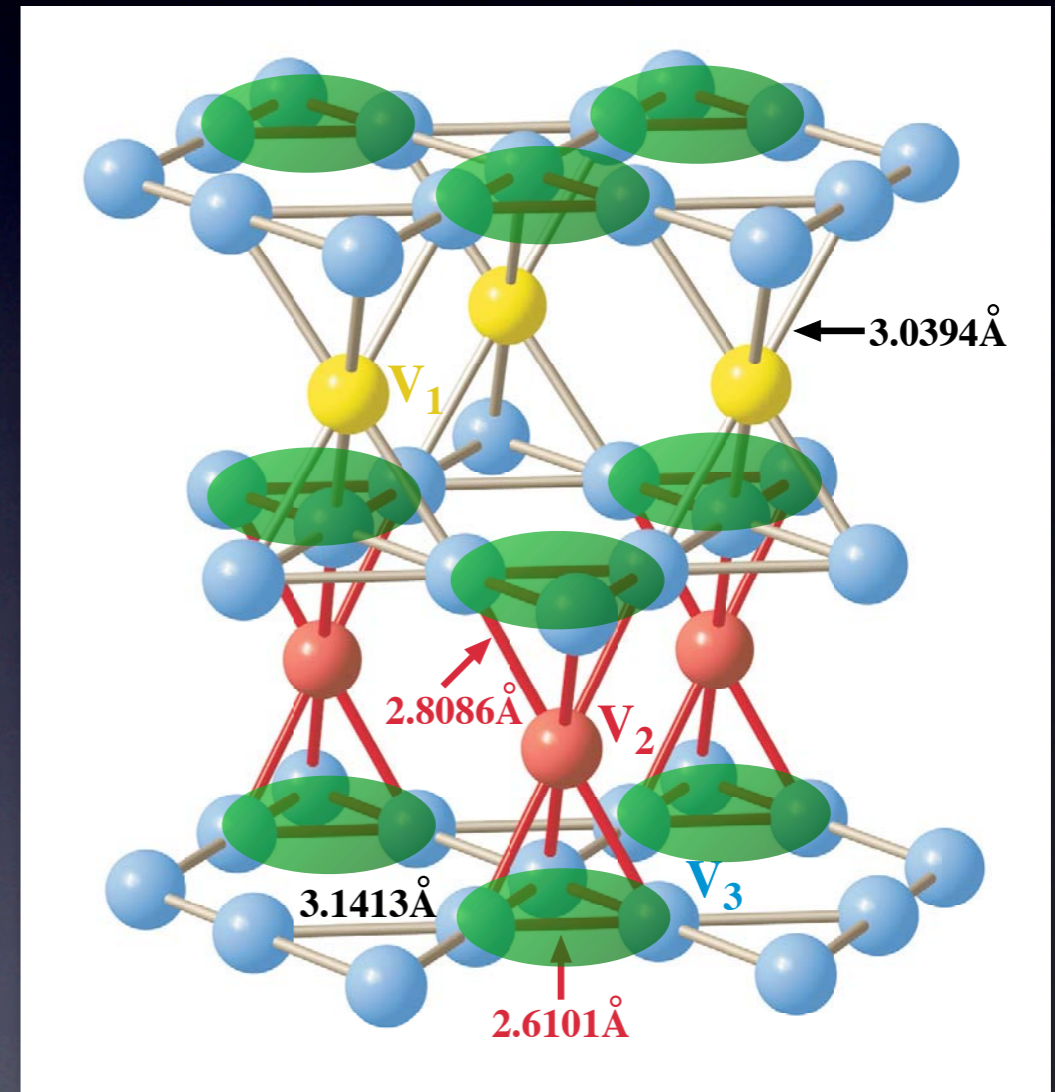
- new experimental finding: trimer formation in Kagome layers below T_c



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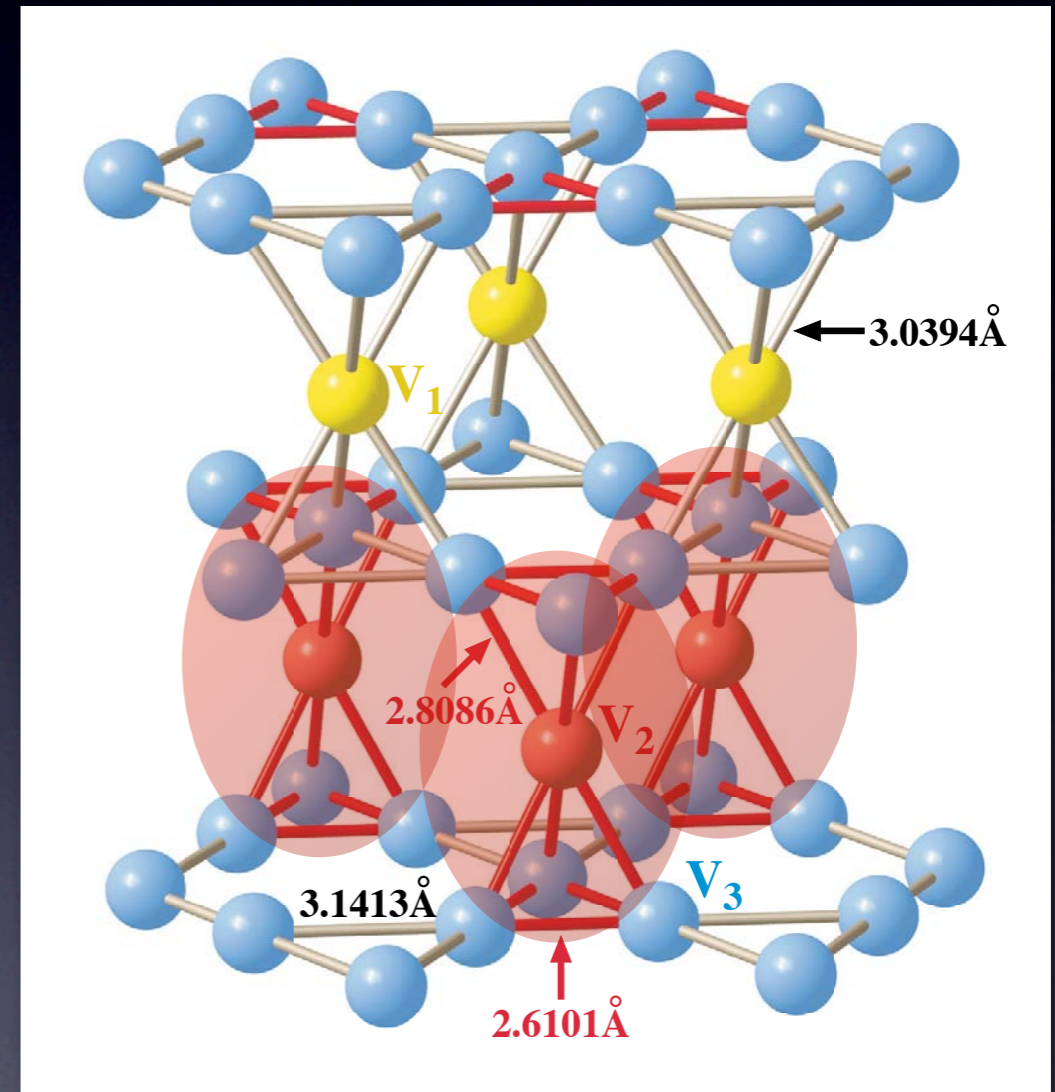
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- spin-singlet formation in trimers? → sharp drop of the magnetic susceptibility?



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Heptamer Scenario

- new experimental finding: trimer formation in Kagome layers below T_c
- spin-singlet formation in trimers? → sharp drop of the magnetic susceptibility?
- We propose a singlet state emerging from the 7-site clusters (heptamers)

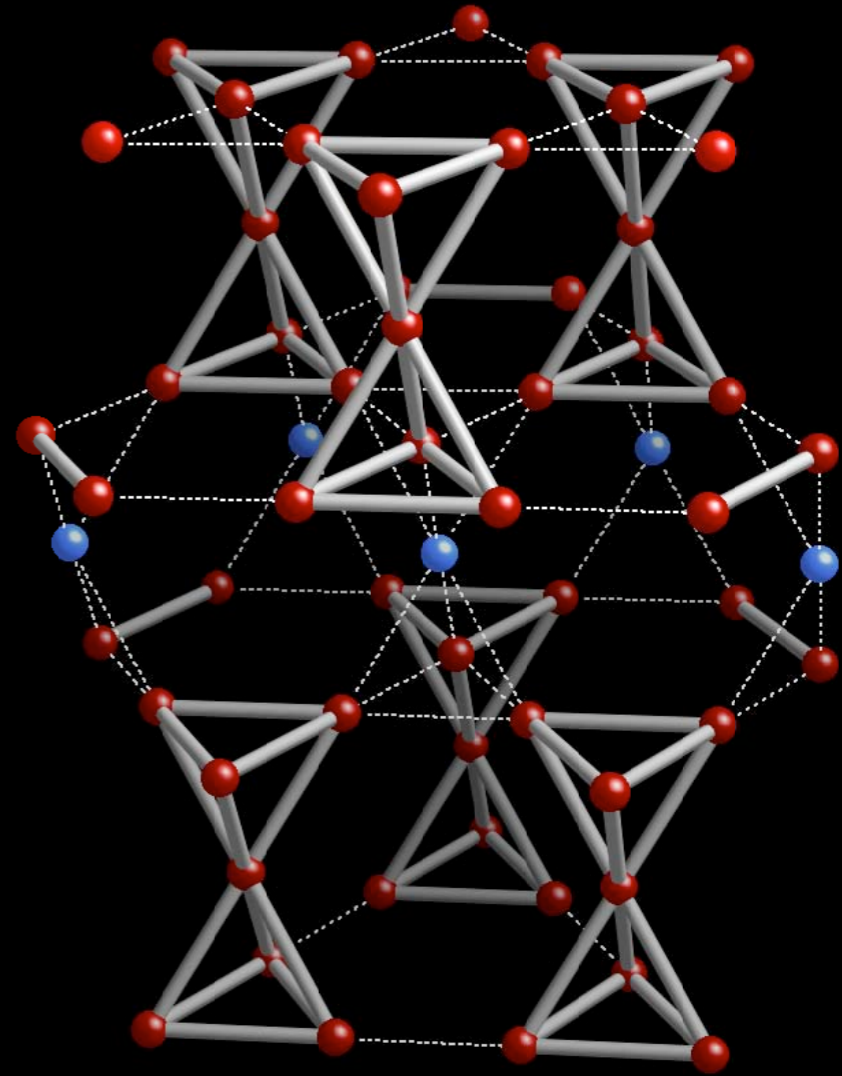


Y. Horibe *et al.*, 2006

Questions

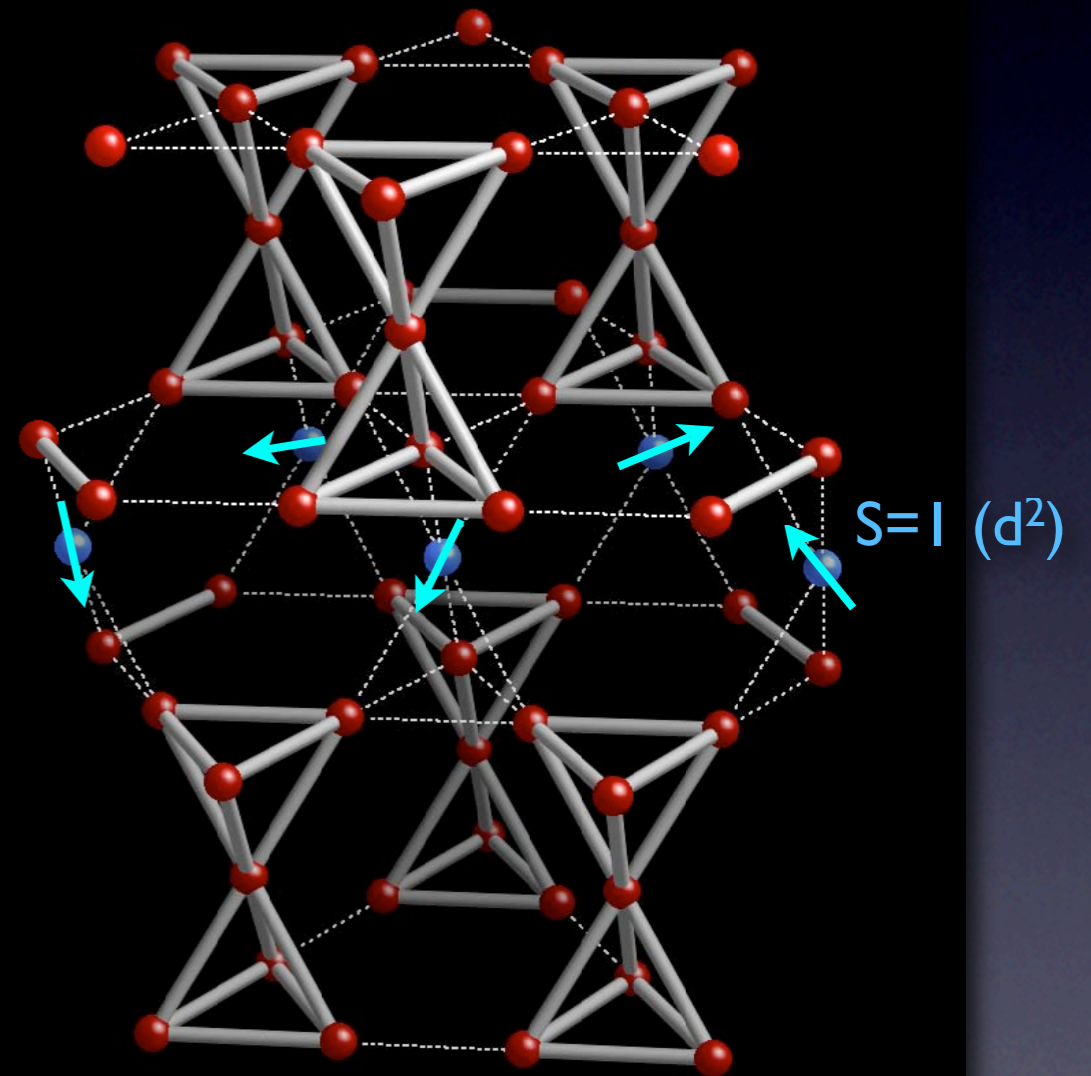
- 📌 What is the mechanism of the heptamer formation? How is the degeneracy in the frustrated pyrochlore system lifted?
- 📌 Is the heptamer in a spin-singlet state? How does the singlet state emerge in each heptamer?

Multi-orbital Heptamer Model



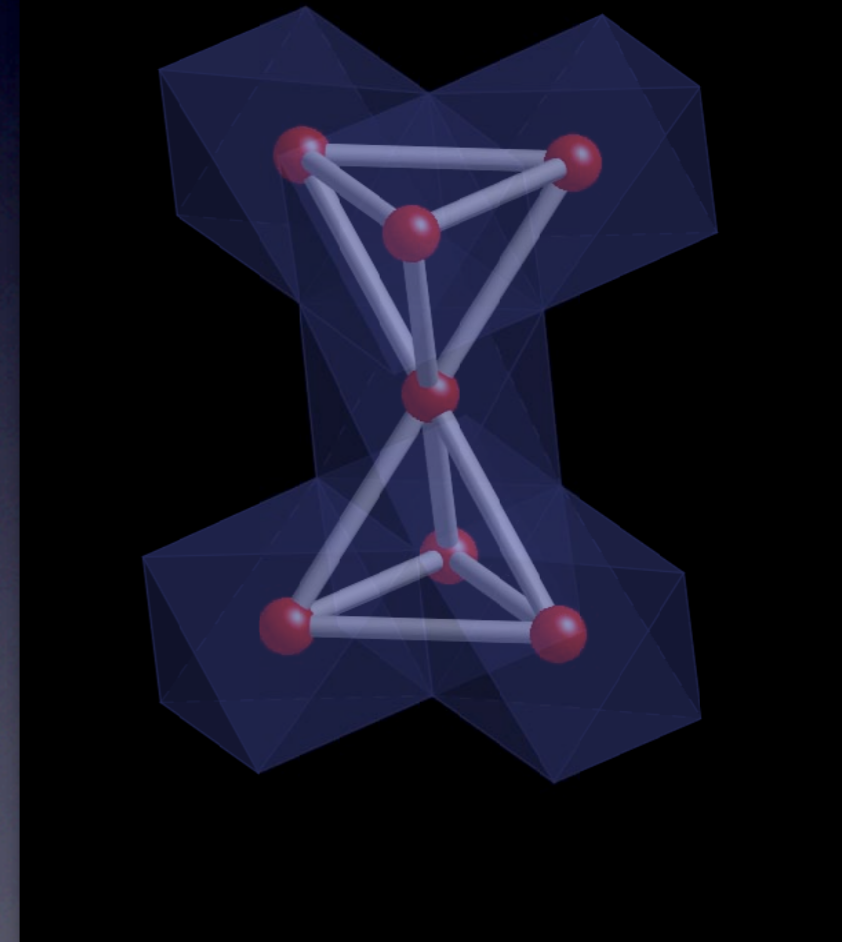
Multi-orbital Heptamer Model

- *assumption*: $S=1$ localized moments at *isolated* V sites (leading Curie behavior at low T)
→ 18 electrons per heptamer



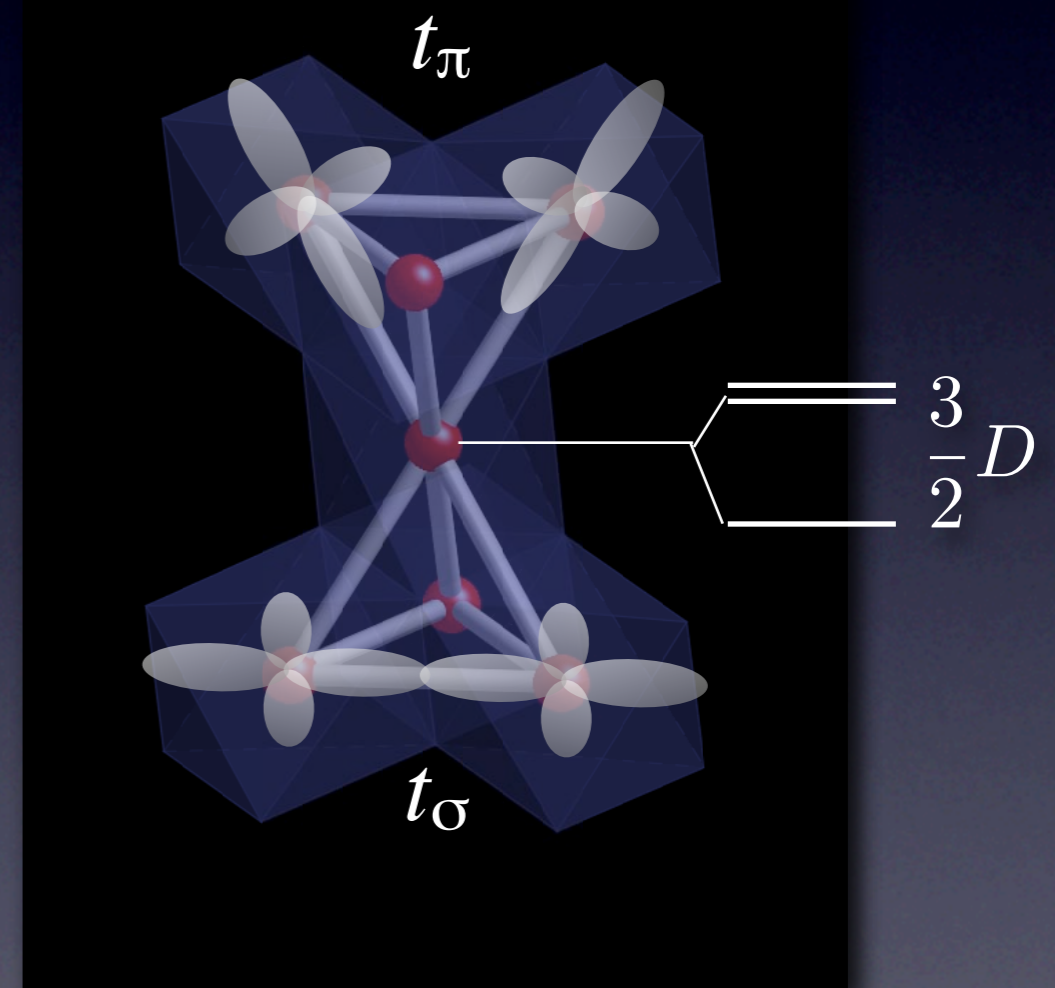
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 - σ and π transfer integrals
 - trigonal lattice distortion at the central site
 - Coulomb interactions



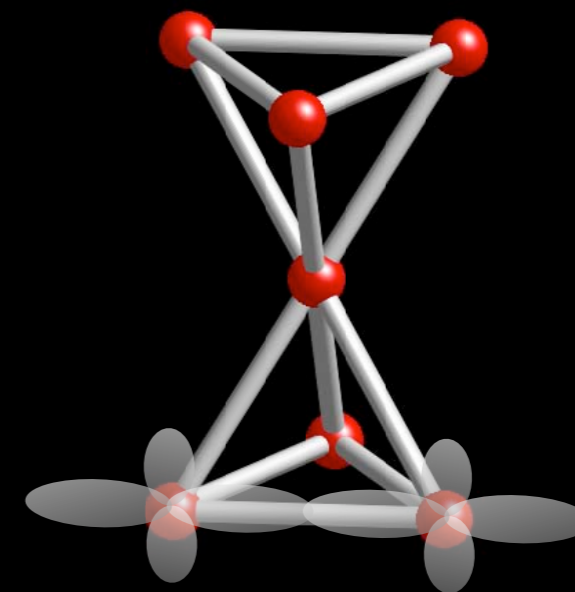
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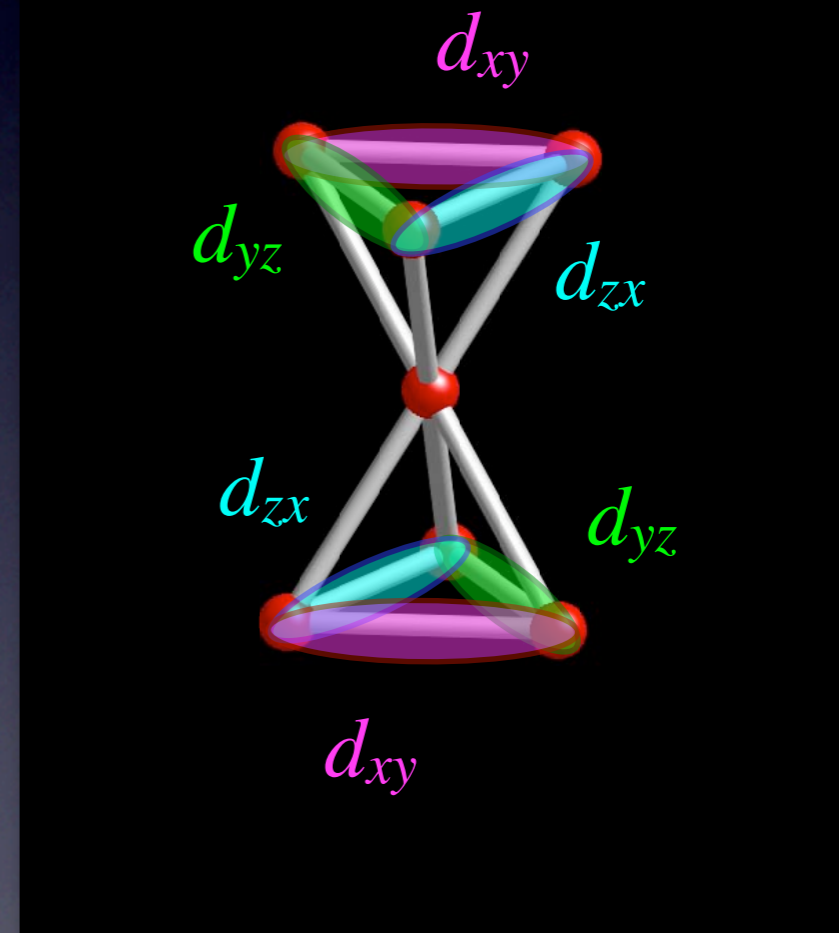
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bonding state:
2 electrons per bond

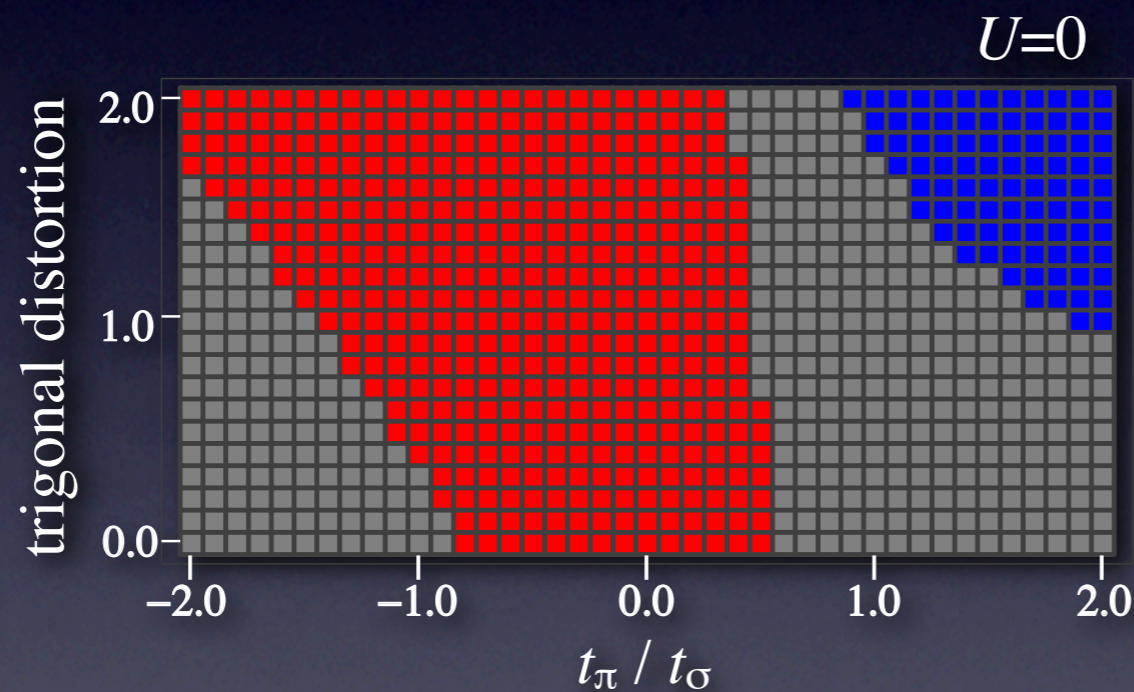
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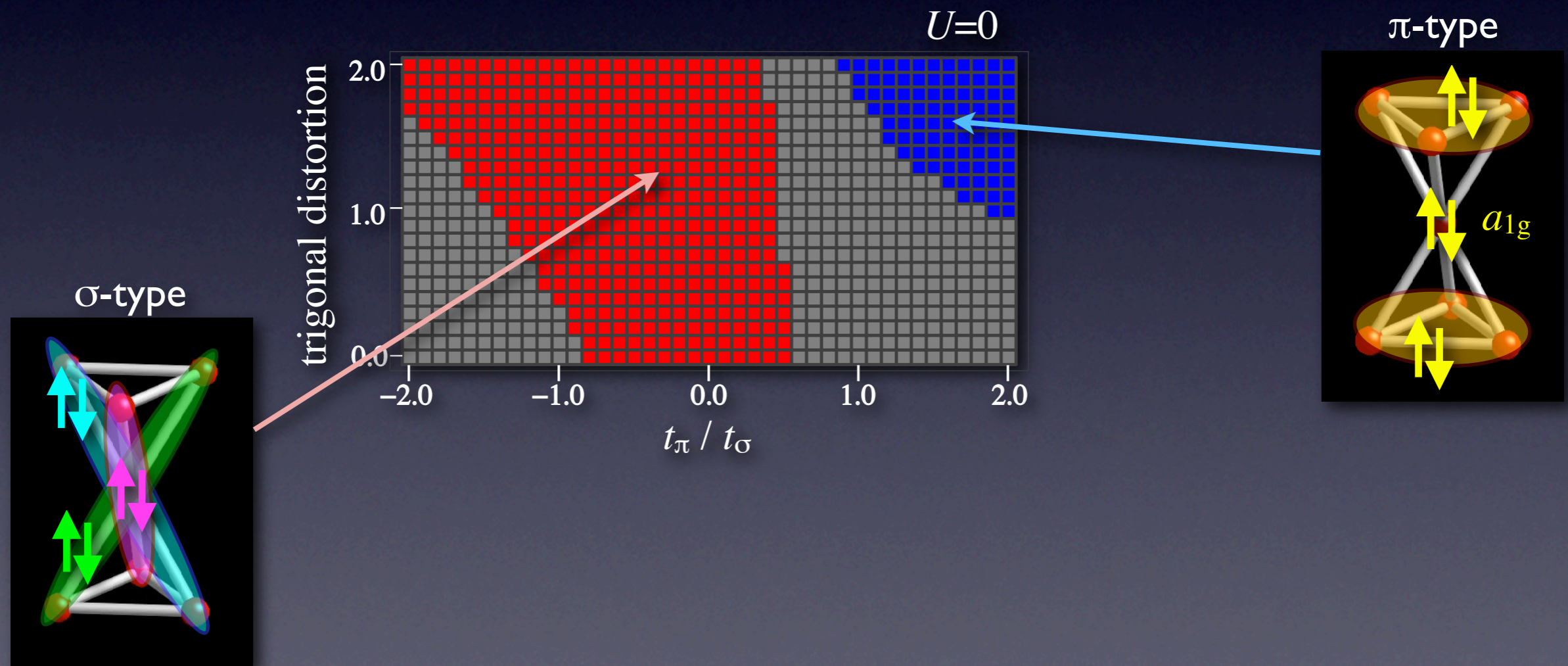
Ground-state Degeneracy

- exact diagonalization of the effective heptamer model
- two different singlet regimes: σ -type and π -type



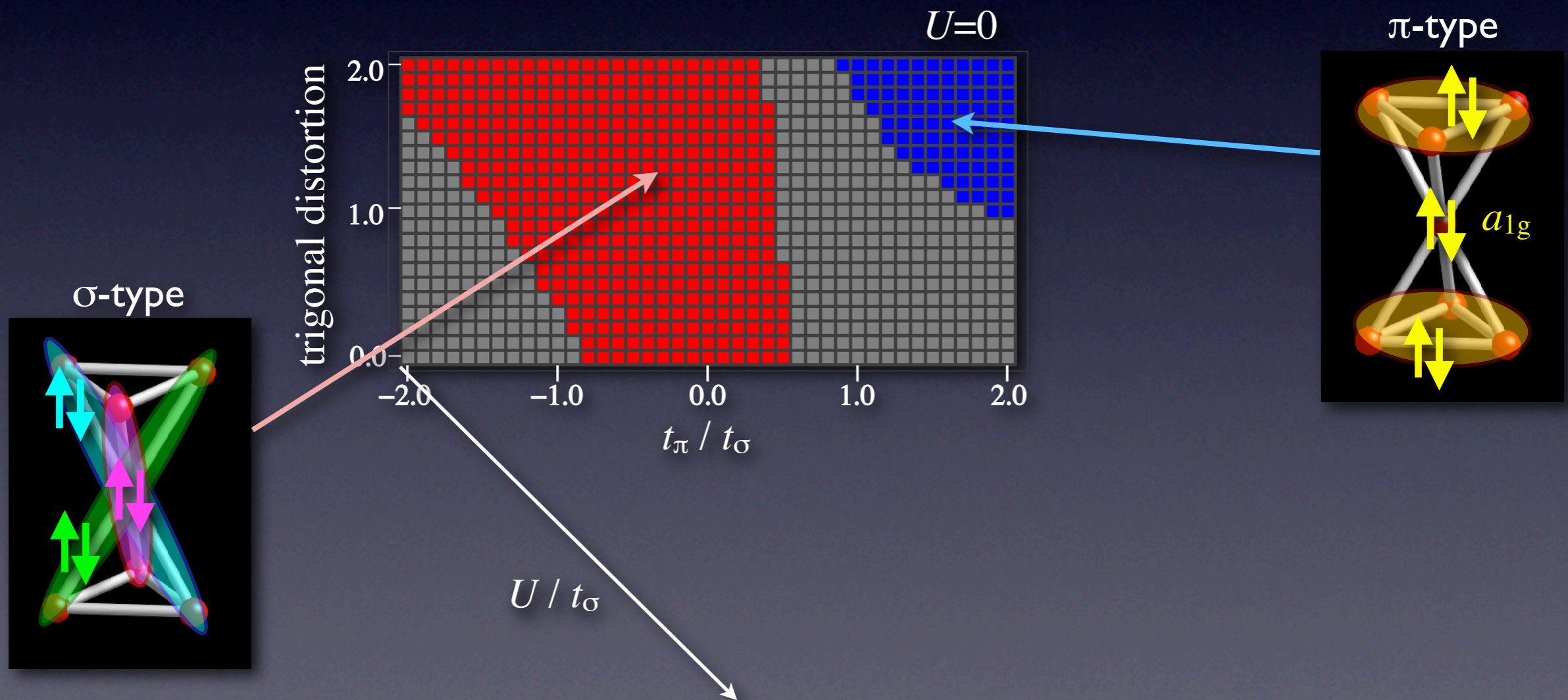
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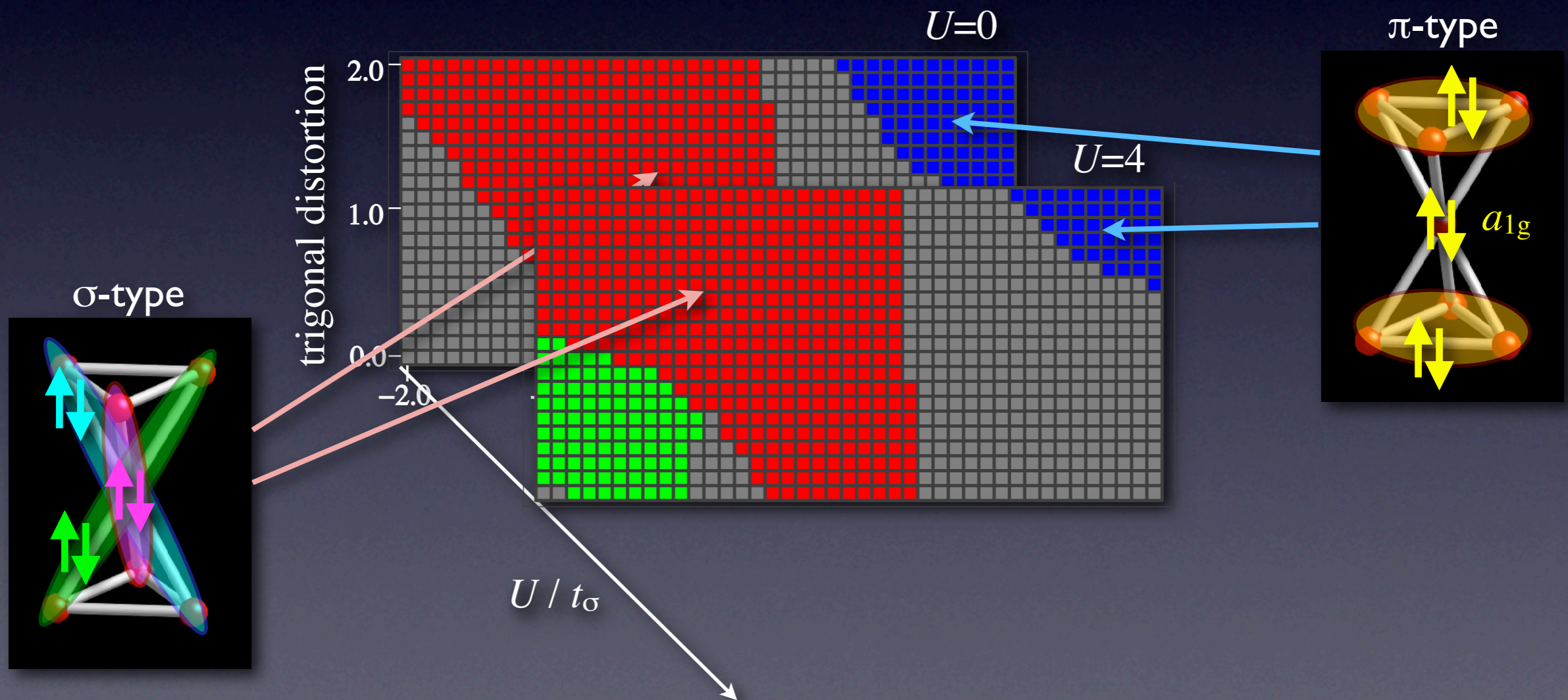
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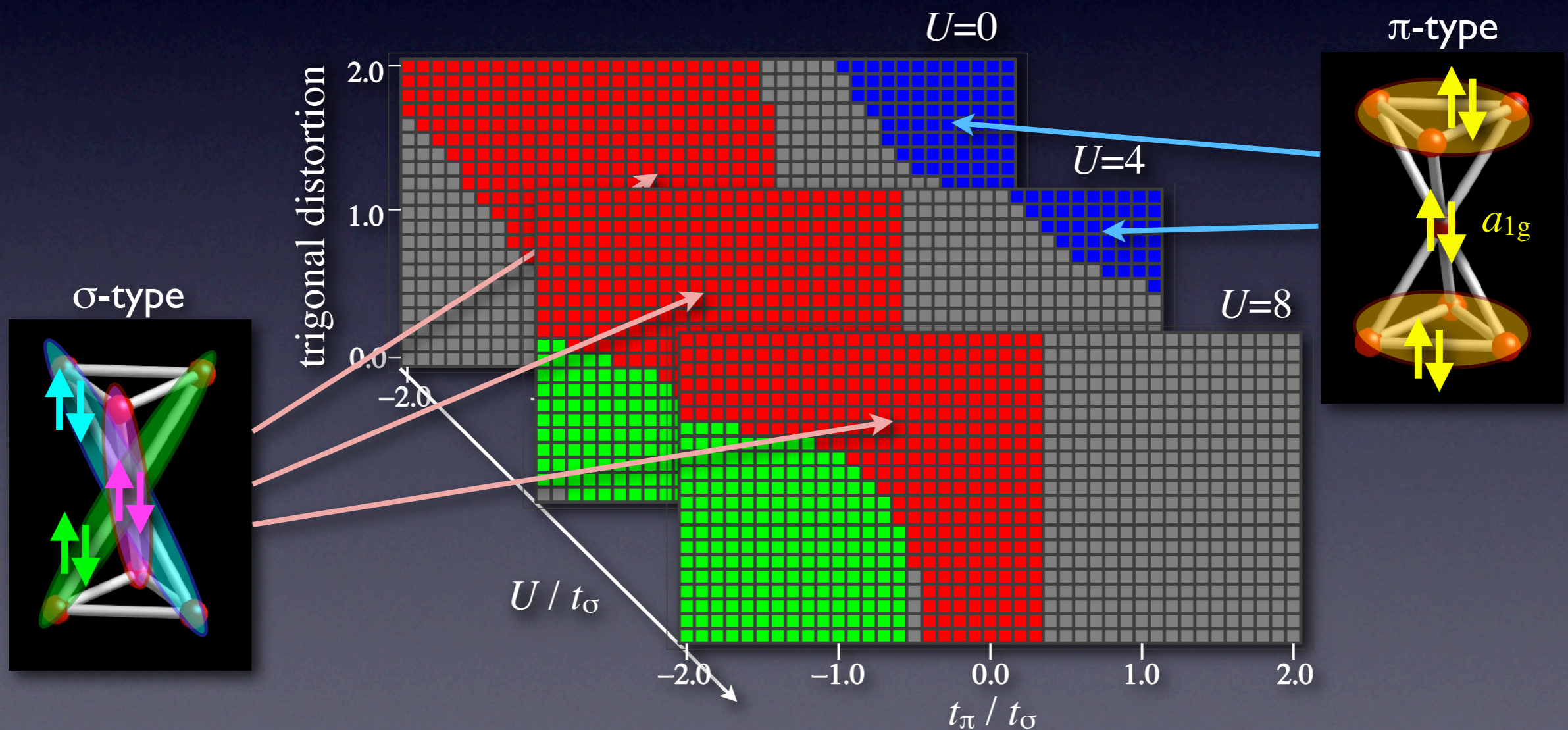
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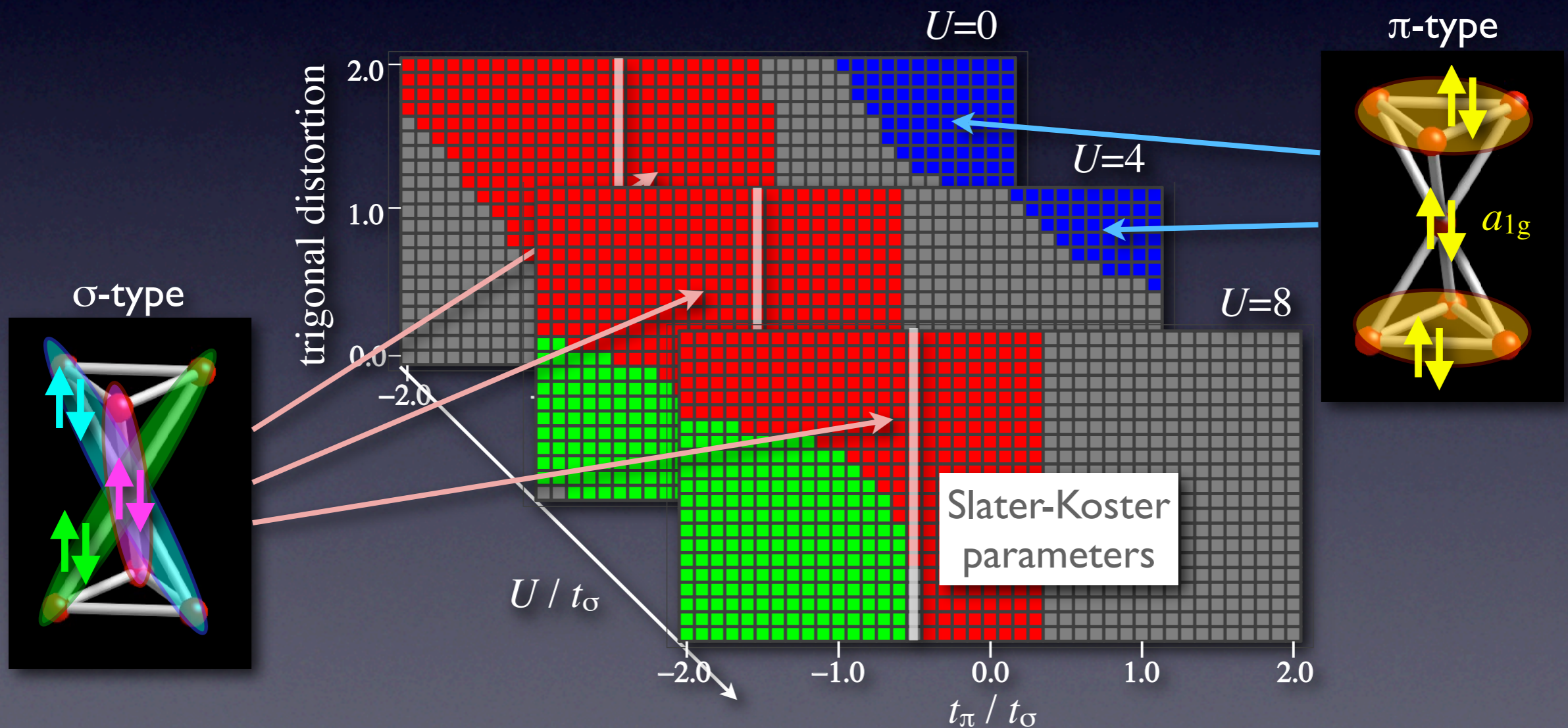
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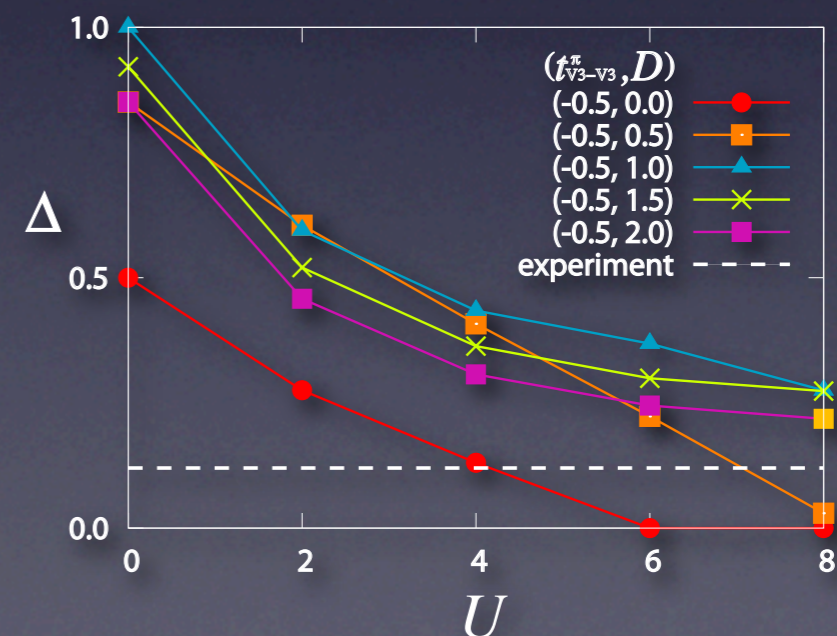
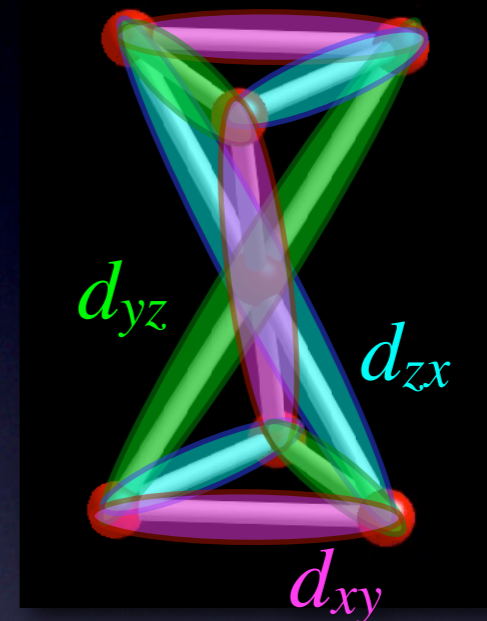
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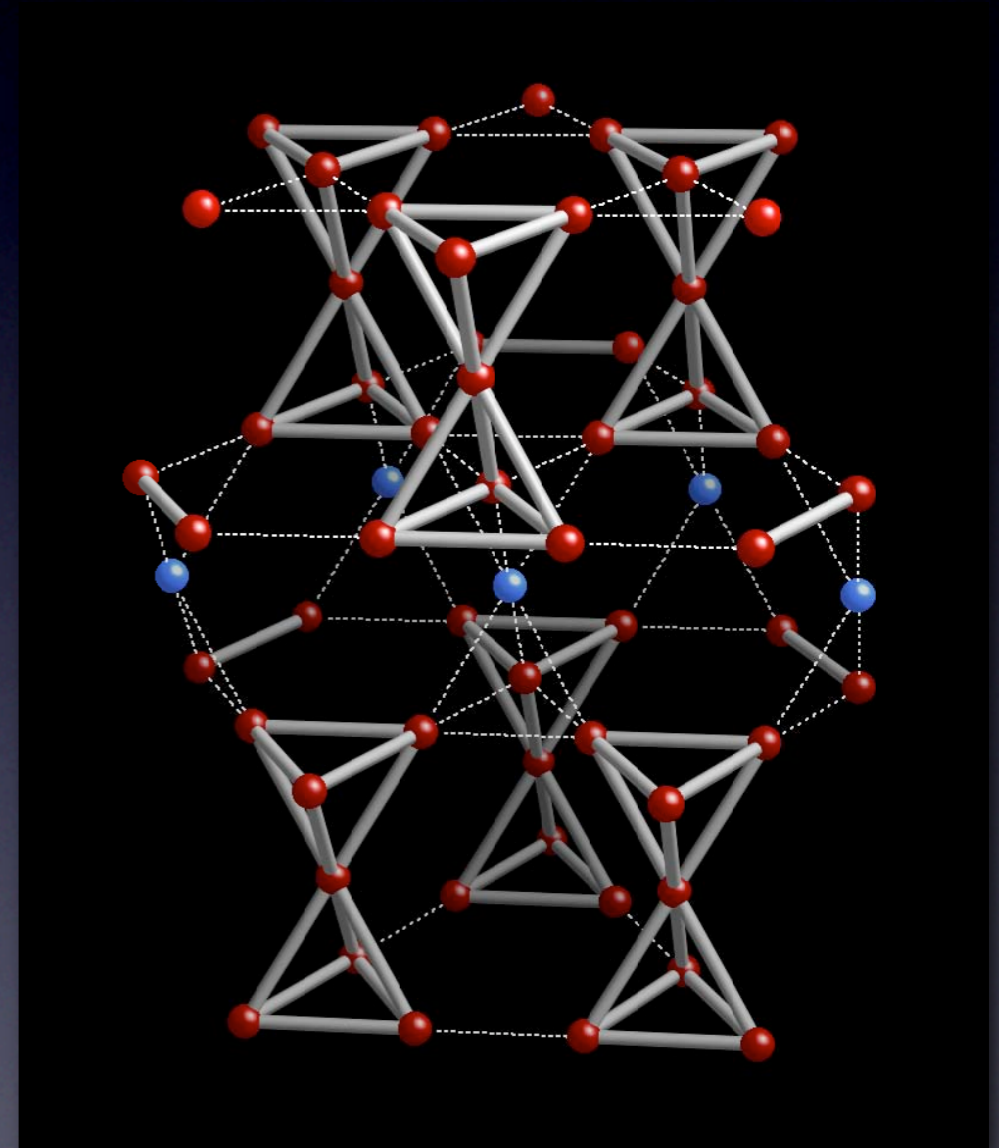
Singlet State in Heptamer

- singlet ground state for realistic parameters = σ -type
‘molecule’ of the bonding states with three t_{2g} orbitals
- estimate of the spin gap is larger than the experimental one: heptamer-heptamer coupling?
- comprehensive understanding of the T-dependence of magnetic susceptibility



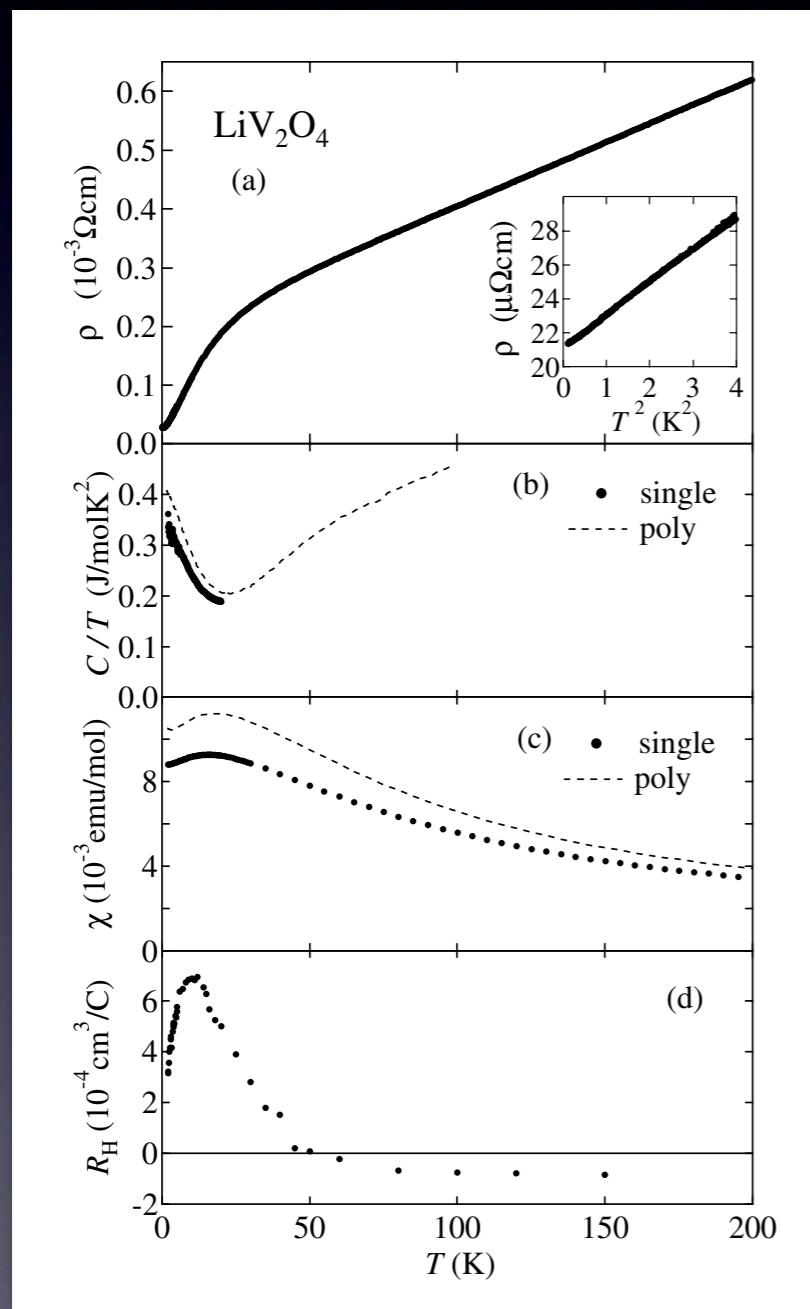
Open Issues...

- Once the heptamers are assumed to be stable, experimental results at low-T phase are explained comprehensively.
- 📌 What is the mechanism of the heptamer formation? How is the degeneracy in the frustrated pyrochlore system lifted?
- 📌 Is similar phenomenon seen in other mixed-valence compounds?



Another Mixed-Valence Compound

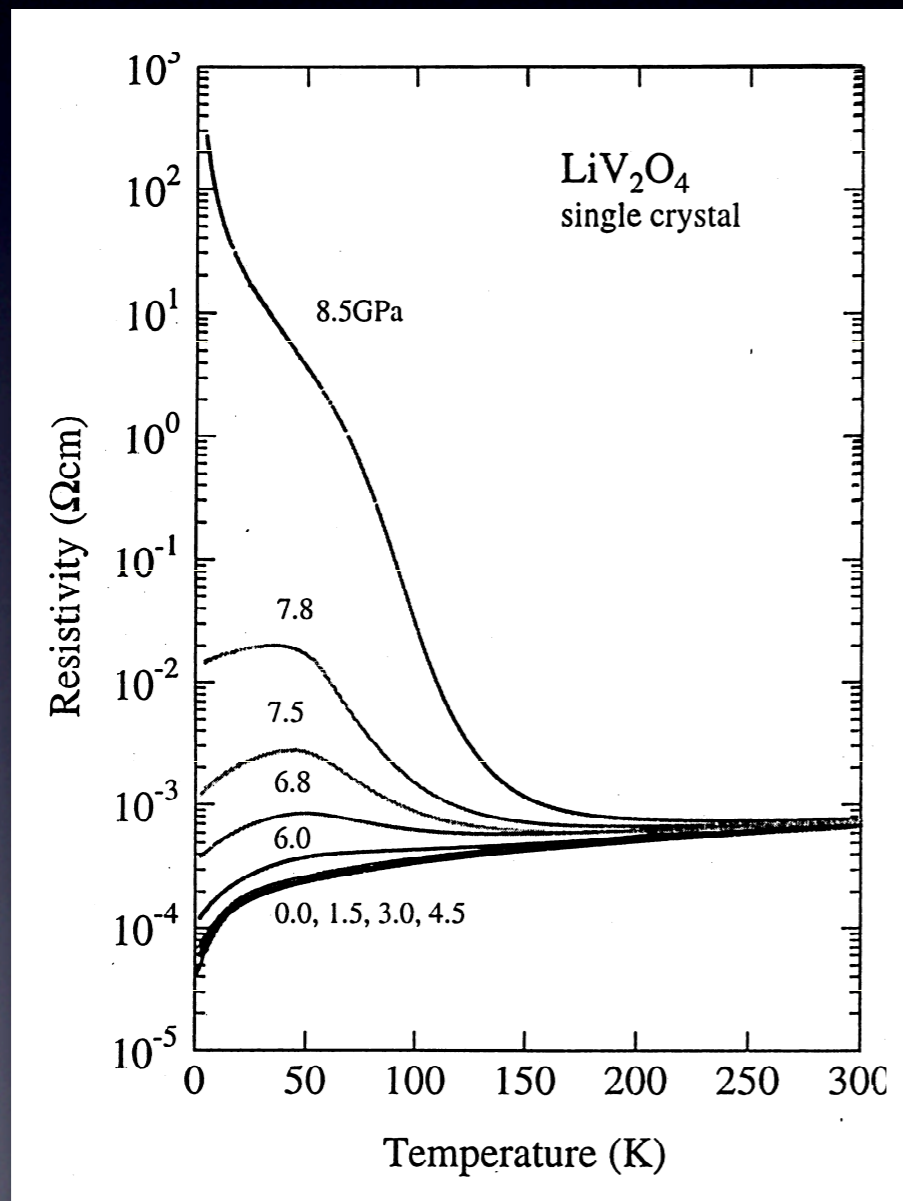
LiV_2O_4 : Heavy-Fermion Behavior



Urano *et al.*, 2000

- mixed valence: $\text{V}^{3.5+} = (3d)^{1.5}$
- heavy mass (Kondo *et al.*, 1997)
- cubic, metallic, no magnetic ordering (Rogers *et al.*, 1967; Chmaissem *et al.*, 1997; Mahajan *et al.*, 1997; Merrin *et al.*, 1998)
- only t_{2g} electrons: new mechanism for heavy fermion behavior?
 - Kondo scenario
 - geometrical frustration + correlation

Implication of AlV_2O_4 ?



Urano, PhD Thesis

- metal-to-insulator transition by applying pressure: *opposite to usual pressure effect*
- short and long V-V bonds in the insulating state (EXAFS by Niitaka *et al.*, unpublished)
- possibility: some cluster formation similar to AlV_2O_4

Summary

- introduction to spinels and t_{2g} orbital physics
- controversy on orbital ordering in ZnV_2O_4
 - different models for spin/orbital order in ZnV_2O_4 : relative importance of Kugel-Khomskii superexchange, Jahn-Teller and relativistic spin-orbit couplings
 - symmetry analysis: lesson from experiments in MnV_2O_4
- self-organized 7-site cluster (heptamer) in AlV_2O_4
 - heptamer scenario: 'molecule' of bonding states with anisotropic t_{2g} orbitals
 - implication to heavy-fermion compound LiV_2O_4